Governmentally amplified output volatility

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Abstract

Predominant government behavior is decomposed by frequency into several periodic components: updating cycles of infrastructure, Kuznets cycles, fiscal policy over business cycles, and election cycles. Little is known, however, about the theoretical impact of such cyclical behavior in public finance on output fluctuations. Based on a standard neoclassical growth model, this study intends to examine the frequency at which public investment cycles are relevant to output fluctuations. We find an inverted U-shaped relationship between output volatility and length of cycle in public investment. Moreover, with a numerical setting, we show that periodic behavior in public investment at low frequencies—such as updating cycles of infrastructure and Kuznets cycles—can cause aggravated output resonance.

Keywords: Output volatility; Public investment; Resonance; Frequency

JEL classification codes: E32, E62

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1 Introduction

Previous works have pointed out that public investment consists of multiple components, but they provide no details about the effects of the individual factors on output movement. In the context of spectral analysis, also known as frequency-domain analysis, behavior in public investment can be decomposed by frequency. That is, the spectrum of public investment resembles that depicted in Figure 1, and the underlying components are considered in order of frequency as follows.

First, many researchers have been interested in the business-cycle components of fiscal policies so far. As in Baxter and King (1999) and Christiano and Fitzgerald (2003), the range of business cycles is supposed to be 1.5–8 years in the United States. As for the short run, previous works empirically found that public sectors are prone to react to business cycles. For example, Sorensen et al. (2001), Lane (2003), Hines (2010), and Afonso and Jalles (2013) estimated some policy reaction functions for broad categories of government spending and demonstrated how governments have reacted to economic fluctuations (i.e., countercyclical, procyclical, or acyclical).

Second, numerous authors have examined the existence of opportunistic political business cycles (e.g., Nordhaus, 1975; Alesina and Roubini, 1992; Alesina et al., 1992, 1997).[1] They shed light on the particular political aspect of policymakers manipulating macroeconomic policies in order to be re-elected, and consequently, there is a tendency to be expansionary as an election approaches and to be austere after the election.[2] This means that election cycles in public investment are based in reality. In the US case, the cycle of presidential elections is determined de jure, and 4 years strictly are included in business-cycle frequencies.

Third, as relatively low-frequency components on approximately 20-year cycles, the Kuznets cycle regarding the public sector is well known (e.g., Hansen, 1941). Lastly, updating cycle of infrastructure can be considered as more low-frequency components (i.e., trend components), such as 50-year cycles.

In this study, we focus on such cyclical behavior of public investment and attempt to clarify the effects on macroeconomic fluctuations. To be more precise, we examine transitional dynamics by incorporating our observational hypotheses that public

[1] As in Alesina and Roubini (1992), the literature can be classified into two types of theories, that is, opportunistic and partisan. The present study pertains only to opportunistic types of political cycles that are inspired by Nordhaus' (1975) pioneering work, rather than partisan types that can be traced to Hibbs (1977). Milani (2010) studies these political business-cycle models in a dynamic stochastic general equilibrium framework.

[2] While the present study does not consider monetary policies, their independence from politics is an ongoing subject of controversy. For recent empirical evidence of the relationship between presidential elections and monetary policy in the United States, see Abrams and Iossifov (2006) and Funashima (2013).

finance patterns are cyclical into Ramsey's (1928) classic growth model. In doing so, as a first attempt, we consider the case in which the government invests cyclically by having access to lump-sum taxes.

There are several predecessors closely related to our model.[3] Turnovsky and Fisher (1995), Fisher and Turnovsky (1998), and Turnovsky (2004) provided models that include public-capital dynamics, and they are closely related to the framework we propose in this study. However, their models are more general than ours in certain aspects while these works naturally differ in accordance with their purposes. For instance, Turnovsky and Fisher (1995) considered two types of government expenditure to investigate these effects: consumption expenditure, which improves utility, and which we do not consider; and investment expenditure, which raises private productivity, and which we do consider. On the other hand, although Fisher and Turnovsky (1998) did not consider government consumption expenditure, instead, they studied investment expenditure under congestion, that is, when infrastructure is not a pure public good and is attended by a certain degree of rivalry. Furthermore, Turnovsky (2004) examined both expenditure types under various tax finances.[4] In order to focus on the public-investment cycle (hereafter PIC), we reasonably consider only lump-sum tax and omit government consumption expenditure from our model for the time being.

The remainder of the paper is organized as follows. Section 2 sets up the model economy. Section 3 approximately log-linearizes our model, provides the analytical solution, which is consistent with the transversality condition, and presents the quantitative analysis with a numerical example. Section 4 concludes.

2 The model economy

Following Barro and Sala-i-Martin (2004), we briefly present the extended Ramsey model. Time (denoted as t) is continuous.

[3] Since a seminal work by Barro (1990), inquiries into the relationship between public investment and economic growth have been conducted widely along with various developments (e.g., Futagami et al., 1993; Cassou and Lansing, 1998; Turnovsky, 2004). From some empirical perspectives, Aschauer (1989) and many others investigated the contribution of public capital to economic growth for the United States. More recently, Marrero (2008) offered an elaborate, calibrated model in which public investment-to-output ratios as observed in developed countries are accounted for with adequate precision.

[4] See also Barro and Sala-i-Martin (2004) for some extensions of the Ramsey model to include government spending with various taxes.

2.1 Firm

Let L be the total population of the economy and A be effective labor. They are assumed to increase at constant rates n and e, respectively: $L = \exp(nt), A = \exp(et)$. A representative firm's technology in intensive form is assumed to be

$$y = k^{\alpha} (K_g)^{\beta},$$

where y is output per effective labor, k is capital per effective labor, K_g is public capital, and α and β are the elasticities of output with respect to capital and public capital, respectively.

The first-order conditions with profit maximization are

$$r + \delta = \alpha k^{\alpha-1} (K_g)^{\beta}, \quad w = (1-\alpha)Ak^{\alpha} (K_g)^{\beta}, \tag{1}$$

where r is the real interest rate, w is the real wage rate, and δ is the depreciation rate of private capital.

2.2 Household

An infinitely lived representative household has preference to maximize

$$U = \int_0^{\infty} e^{-\rho t} \frac{c^{1-\theta}}{1-\theta} L \, dt,$$

where c is per capita consumption, ρ is the rate of time preference, and θ is a relative risk-aversion measure and satisfies $\theta > 0$. The household's budget constraint in per capita representation is

$$\dot{a} = w + ra - c - \tau - (n + e + \delta)a,$$

where τ denotes lump-sum tax.

The familiar first-order condition, the Euler equation, is given by:

$$\frac{\dot{c}}{c} = \frac{r - \rho - \theta e}{\theta}. \tag{2}$$

Note that in (2), c is consumption per effective labor.

2.3 Government

Next, we turn to the original assumption, that is, the PIC:

$$G = B^c \sin \frac{2\pi}{T} t + B^s \equiv B^c \sin \omega t + B^s, \tag{3}$$

where B^c is the amplitude, π is the circular constant, T is the cycle, and $\omega \equiv (2\pi)/T$ is angular frequency. This formulation is a generalization to include the constant public investment. In other words, the constant public investment can be obtained as the limit, that is, $B^c \to 0$.

It should be noted that the PICs are given exogenously in our model. First, regarding the business-cycle frequencies, the acyclical behavior to business cycles is relevant to the present formulation, but on the other hand, the countercyclical or procyclical behavior cannot be considered. Second, if the PIC stems from election-motivated behavior of policymakers, this can result in an opportunistic political business cycle á la Nordhaus (1975, figure 8). In this case, the exogenous PIC is rationalized in the presidential system because the duration regarding particular elections is fixed strictly by the law. For example, the cycle of presidential elections in the United States is determined de jure, and it is strictly 4 years.[5]

Then public capital evolves as follows to

$$\dot{K}_g = (B^c \sin \omega t + B^s) - \delta_g K_g, \tag{4}$$

where δ_g is the depreciation rate of public capital.

2.4 Equilibrium

The government's budget constraint in per effective labor representation satisfies $\tau = g$, where $g \equiv G/(AL)$ is public investment per effective labor. Using the market-clearing condition of capital market $a = k$ and (1), in the equilibrium, we obtain the following dynamic system:

$$\dot{c} = \frac{\alpha k^{\alpha-1} (K_g)^{\beta} - \delta - \rho - \theta e}{\theta} c, \tag{5}$$

$$\dot{k} = k^{\alpha} (K_g)^{\beta} - c - (n + e + \delta)k - g. \tag{6}$$

[5] In fact, Nordhaus (1975) and many others essentially supposed a presidential system, and the length of the electoral period was assumed to be exogenous. It should be noted, however, that our analysis is not really relevant to some countries, such as Japan, where a parliamentary system is adopted and the incumbent government can determine the timing of the elections. In this case, the timing of the elections is endogenous and the exogenous PICs mentioned above are unreasonable. For example, Ito and Park (1988) and Ito (1990) discussed and investigated the hypotheses of political cycles in the Japanese parliamentary system.

3 Analytical solutions in log-linearized system

3.1 Log-linearization

To solve the model analytically, we now log-linearize the system of differential equations, (4)-(6). It should be noted that the strict steady state does not exist because public investment follows an exogenous sinusoidal process. As detailed below, we consider some approximate steady state by ignoring the fluctuation of public investment around the steady state.

We insert a hat above a variable to remind ourselves that this quantity is a log deviation from the steady state: $\hat{x} \equiv \ln x - \ln x^*$ for $\forall x$, where the asterisk denotes the steady state.

Since the steady state is expressed by $\dot{c}/c = d(\ln c)/dt = 0$ and $\dot{k}/k = d(\ln k)/dt = 0$, it holds in the steady state that

$$\alpha (k^*)^{\alpha-1} \left(K_g^*\right)^{\beta} - (\delta + \rho + \theta e) = 0, \tag{7}$$

$$(k^*)^{\alpha-1} \left(K_g^*\right)^{\beta} - \frac{c^*}{k^*} - (n + e + \delta) - \frac{g^*}{k^*} = 0. \tag{8}$$

To ensure the existence of K_g^*, we postulate that the ratio of public investment to public capital is so small that the fluctuation of public investment can be negligible around the steady state: $\dot{K}_g = B^s - \delta_g K_g$. In other words, $g^* = B^s$ is assumed. Consequently, in what follows, the influence of PIC upon public capital is excluded approximately. It then follows from $\dot{K}_g/K_g = d(\ln K_g)/dt = 0$ in the steady state that

$$\frac{B^s}{K_g^*} - \delta_g = 0. \tag{9}$$

Moreover, we assume (a) that the ratio of public investment to private capital is sufficiently small (i.e., $g^*/k^* \ll 1$) and (b) $g^* = B^s$ because evaluating g^* only is troublesome. According to Marrero's (2008) calibration, in fact, G^*/K_g^* and g^*/k^* are calculated to be 0.069 and 0.038, respectively. Hence, $G^*/K_g^* \ll 1$ and $g^*/k^* \ll 1$ seem satisfactory assumptions.

Using an approximate formula $x \simeq x^*(1 + \hat{x})$ for c, k and K_g, that is accurate if x is near x^*, we approximately obtain the following log-linearization system around the steady state:

$$\begin{aligned}
\dot{\hat{c}} &= (\alpha-1)\gamma\hat{k} + \beta\gamma\hat{K}_g, &(10)\\
\dot{\hat{k}} &= \left(n+e+\delta+\frac{g^*}{k^*}-\frac{\theta}{\alpha}\gamma\right)\hat{c} + \left(\zeta-\frac{g^*}{k^*}\right)\hat{k} + \frac{\beta\theta}{\alpha}\gamma\hat{K}_g - \frac{g^*}{k^*}\frac{g}{g^*}, &(11)\\
\dot{\hat{K}}_g &= -\delta_g\hat{K}_g, &(12)
\end{aligned}$$

where $\gamma \equiv (\rho+\theta e+\delta)/\theta, \zeta \equiv \rho - n - e(1-\theta)$.

3.2 Characteristics of analytical solution

As shown in the Appendix, solving the system (10)–(12) yields

$$\begin{aligned}
\hat{y} &= \alpha\hat{k} + \beta\hat{K}_g \\
&= \frac{\alpha\lambda_2 C_2}{(\alpha-1)\gamma}\exp(\lambda_2 t) \\
&\quad + \left\{\frac{\alpha\Gamma_1\delta_g}{(1-\alpha)\gamma\Phi(-\delta_g)} + \frac{\beta}{1-\alpha}\hat{K}_g(0)\right\}\exp(-\delta_g t) \\
&\quad + \frac{\alpha g^*(e+n)}{k^*\Phi(-(e+n))}\exp\{-(e+n)t\} \\
&\quad + \frac{\alpha g^* B^c\tilde{\Theta}}{k^* B^s}\cos(\omega t+\phi_0+\phi_1)\exp\{-(e+n)t\},
\end{aligned}$$

where

$$\begin{aligned}
\tilde{\Theta} &\equiv \sqrt{\omega^2+(e+n)^2}\left[\left\{\omega^2+\frac{(e+n+\lambda_1)^2+(e+n+\lambda_2)^2}{2}\right\}^2\right. \\
&\quad \left. -\left\{\frac{(e+n+\lambda_1)^2-(e+n+\lambda_2)^2}{2}\right\}^2\right]^{-1/2}, \\
\Phi(x) &\equiv (x-\lambda_1)(x-\lambda_2), \\
\phi_0 &\equiv \tan^{-1}\left[-\frac{(e+n+\lambda_1)(e+n+\lambda_2)-\omega^2}{\omega\{2(e+n)+\lambda_1+\lambda_2\}}\right], \\
\phi_1 &\equiv \tan^{-1}\left(-\frac{\omega}{e+n}\right),
\end{aligned}$$

$$\begin{aligned}
\lambda_1 &\equiv \frac{\left[\zeta-\frac{g^*}{k^*}+\sqrt{\left(\zeta-\frac{g^*}{k^*}\right)^2+4(1-\alpha)\gamma\left\{\frac{\rho+\theta e+\delta-\alpha(g^*/k^*)}{\alpha}-(n+e+\delta)\right\}}\right]}{2}, \\
\lambda_2 &\equiv \frac{\left[\zeta-\frac{g^*}{k^*}-\sqrt{\left(\zeta-\frac{g^*}{k^*}\right)^2+4(1-\alpha)\gamma\left\{\frac{\rho+\theta e+\delta-\alpha(g^*/k^*)}{\alpha}-(n+e+\delta)\right\}}\right]}{2}.
\end{aligned}$$

If $e+n>0$ and $\lambda_2<0$, this particular solution is stable, converging to zero as $t\to\infty$.

Our main concern is the relationship between PIC and the transitional dynamics. When focusing on the cycle T (or ω), which is included in $\tilde{\Theta}$, we can obtain a few implications. First, it turns out at once that $\tilde{\Theta}$ converges to $(e+n)/\{(e+n+\lambda_1)(e+n+\lambda_2)\}$ as $T\to\infty$. Moreover, $\tilde{\Theta}$ can be a monotone increasing function in certain order of ω where ω^2 is dominant to $\tilde{\Theta}$. This tells us that $\tilde{\Theta}$ is maximized at certain particular values of cycle T, implying that a kind of "resonance" can occur.[6] Henceforth, we call the volatile transitional dynamics "output resonance." We restate the outcome as Proposition 1:

Proposition 1. *There exists an inverted U-shaped relationship between output volatility and length of cycle in public investment.*

Up to this point, we consider the single cycle of public investment. We now turn to the case in which public investment consists of multiple cycles. Because the dynamic system is linear, the above outcome can be generalized readily. Insofar as the economy is around the steady state, similar consequences would emerge in more general cases owing to the principle of superposition:

Corollary 1. *In the log-linearlized system, even when we consider a generalized public investment, $G(t)=\sum_{p=1}^{q}\left(B_p^c\sin\omega_p t+B_p^s\right)$, certain particular types of cycles that public investment is composed of can be crucial, and thereby, the transitional dynamics radically fluctuate.*

Corollary 1 provides an important implication from spectral analysis, also known as frequency-domain analysis (e.g., Hamilton, 1994). In particular, the band-pass filter perspective, such as that presented in Baxter and King (1999) and Christiano and Fitzgerald (2003), is relevant to our results. That is, the corollary implies that even when public investment G has multiple trend components (i.e., T is large) and noise components (i.e., T is small), the output resonance occurs at certain frequencies. In other words, the resonance phenomenon is independent of the other cyclical components of public investment.

[6] The present term "resonance" is common in the field of physics or engineering. As an example of the resonance phenomenon, a well-known occurrence is the wind-induced collapse of the "Tacoma Narrows Bridge" (November 1940). It is widely recognized that this incident occurred because the cycle of the wind takes particular values, although the amplitude of wind is within expectations. The experience teaches us a lesson that it is important to take account not only of the amplitude of wind (PIC) but also of the cycle.

3.3 Numerical illustration

In order to identify the frequency at which the effect of PIC on the transitional dynamics is significant, we next present a numerical example. The values mostly follow Marrero's (2008) benchmark calibration supposing the annual model of the US economy (see Table 1). Hence, the model's time period is defined as 1 year.

Our main findings are exhibited in Figure 2, which illustrates $\tilde{\Theta}$ as a function of T. In this figure, $\tilde{\Theta}$ is on the vertical axis and T is on the horizontal axis. The numerical result is consistent with Proposition 1 and we can observe the inverted U-shaped relationship between output volatility and length of cycle in public investment.

The amplitude of output dynamics is maximized in the vicinity of $T = 50$. On the other hand, compared with this case, the Kuznets cycle with $T = 20$ has less impact on output volatility. Moreover, in the election-motivated PIC (i.e., around $T = 4$) and at business-cycle frequencies ($T = 1.5 - 8$), the degree is negligible. From Corollary 1, we conclude that relatively low-frequency (trend) components, such as updating the cycle of infrastructure, are dominant in output volatility.

4 Conclusion

This study employed a standard neoclassical growth framework to clarify the effects of public-finance cycles on economic fluctuations. We showed that relatively low frequency components of PICs can cause aggravated output resonance. This acts as a warning against a government's periodic fiscal management under which an economy could radically shake even to collapse, as if the Tacoma Narrows Bridge had collapsed (see footnote 6).

While our numerical result suggests that business-cycle behavior in public investment is relatively irrelevant in output volatility, this suggestion depends on the exogeneous PIC. In this sense, our analysis disregards the response of government behavior to output, and it focuses on acyclical behavior. Countercyclical or procyclical behavior should be investigated formally in some endogenous settings.

Appendix

In this appendix, we show the derivation of analytical solutions in the log-linearized system around the steady state. First, from the differential equation of $\hat{K}_g$, we can directly obtain the particular solution:

$$\hat{K}_g = \hat{K}_g(0) \exp(-\delta_g t),$$

where $\hat{K}_g(0)$ refers to an initial value of $\hat{K}_g$. By substituting this and $g(t) = (B^c \sin \omega t + B^s) \exp\{-(e+n)t\}$ into differential equation of $\hat{c}$ and $\hat{k}$, the model is represented by the following system of inhomogeneous linear differential equations of first order with constant coefficient with $\hat{c}$ and $\hat{k}$, also called the forced system:

$$\frac{d}{dt}\begin{bmatrix} \hat{c} \\ \hat{k} \end{bmatrix} = \boldsymbol{A}\begin{bmatrix} \hat{c} \\ \hat{k} \end{bmatrix} + \boldsymbol{b}(t),$$

where

$$\boldsymbol{A} \equiv \begin{bmatrix} 0 & (\alpha-1)\gamma \\ n+e+\delta+\frac{g^*}{k^*}-\frac{\theta}{\alpha}\gamma & \zeta-\frac{g^*}{k^*} \end{bmatrix},$$

$$\boldsymbol{b}(t) \equiv \begin{bmatrix} \beta\gamma\hat{K}_g(0)\exp(-\delta_g t) \\ \frac{\beta\theta}{\alpha}\gamma\hat{K}_g(0)\exp(-\delta_g t) - \frac{g^*}{k^*}(B^c \sin \omega t + B^s)\exp\{-(e+n)t\}/g^* \end{bmatrix}.$$

Without loss of generality, we now eliminate $\hat{k}$ and obtain an inhomogeneous linear differential equation of second order with constant coefficient with $\hat{c}$:

$$\begin{aligned} &\ddot{\hat{c}} - \left(\zeta - \frac{g^*}{k^*}\right)\dot{\hat{c}} + (\alpha-1)\gamma\left\{\frac{\rho+\theta e+\delta}{\alpha} - (n+e+\delta) - \frac{g^*}{k^*}\right\}\hat{c} \\ &= \Gamma_1 \exp(-\delta_g t) + \Gamma_2 \exp\{-(e+n)t\} + \Gamma_3 \exp\{-(e+n)t\}\sin\omega t, \end{aligned}$$

where $\Gamma_1 \equiv -\beta\gamma\left\{\frac{(1-\alpha)\theta}{\alpha}\gamma + \delta_g + \zeta - \frac{g^*}{k^*}\right\}\hat{K}_g(0)$, $\Gamma_2 \equiv (1-\alpha)\gamma\frac{g^*}{k^*}$, $\Gamma_3 \equiv (1-\alpha)\gamma\frac{g^*}{k^*}\frac{B^c}{B^s}$. In this representation of Γ_3, assumption $g^* = B^s$ is used.

Now let us consider the general solutions of the case of homogeneous linear differential equation, $\ddot{\hat{c}} - \left(\zeta - \frac{g^*}{k^*}\right)\dot{\hat{c}} + (\alpha-1)\gamma\left\{\frac{\rho+\theta e+\delta}{\alpha} - (n+e+\delta) - \frac{g^*}{k^*}\right\}\hat{c} = 0$. Thus, the characteristic equation that is equal to the eigen-equation of $\boldsymbol{A}$ is given by

$$\begin{aligned} \Phi(\lambda) \equiv \left|\boldsymbol{A} \quad -\lambda\boldsymbol{E}\right| &= \lambda^2 - \left(\zeta - \frac{g^*}{k^*}\right)\lambda + (\alpha-1)\gamma\left\{\frac{\rho+\theta e+\delta}{\alpha} - (n+e+\delta) - \frac{g^*}{k^*}\right\} \\ &= 0, \end{aligned}$$

where $\boldsymbol{E}$ denotes the 2×2 identity matrix. Factorizing this equation yields $\Phi(x) = (x-\lambda_1)(x-\lambda_2)$. Because of $\zeta > 0$ and $0 < \alpha < 1$, we only have to consider the case of the real number root, provided that $\zeta - \frac{g^*}{k^*}$ is positive. Moreover, we immediately confirm that λ_1 is positive and λ_2 is negative because the part of square root is larger than $\zeta - \frac{g^*}{k^*}$.[7]

[7] In fact, these conditions hold under our specified parameter on the basis of Marrero's (2008)

Using the so-called differential operator method, we next turn to a derivation of the particular solutions of the case of inhomogeneous linear differential equation. We now write the differential operator as $D \equiv d/dt$. From the useful formula $\phi(D)\exp(at) = \phi(a)\exp(at)$ for $\forall a \in \boldsymbol{C}$, $\phi(D)\exp(at)f(t) = \exp(at)\phi(D+a)\, f(t)$ for $\forall a \in \boldsymbol{C}$, and the principle of superposition, the general solution of $\hat{c}$ is given by:

$$\begin{aligned}
\hat{c}(t) &= C_1 \exp(\lambda_1 t) + C_2 \exp(\lambda_2 t) + \frac{\Gamma_1}{\Phi(-\delta_g)} \exp(-\delta_g t) \\
&+ \frac{\Gamma_2}{\Phi(-(e+n))} \exp\{-(e+n)t\} \\
&+ \Gamma_3 \Theta(\omega) \cos(\omega t + \phi_0) \exp\{-(e+n)t\}.
\end{aligned}$$

Thus, from $\hat{k} = \frac{1}{\gamma(\alpha-1)}\dot{\hat{c}} - \frac{\beta}{\alpha-1}\hat{K}_g(0)\exp(-\delta_g t)$, the general solution of $\hat{k}$ is

$$\begin{aligned}
\hat{k}(t) &= \frac{C_1\lambda_1}{(\alpha-1)\gamma} \exp(\lambda_1 t) + \frac{C_2\lambda_2}{(\alpha-1)\gamma} \exp(\lambda_2 t) \\
&- \left\{ \frac{\Gamma_1 \delta_g}{(\alpha-1)\gamma\Phi(-\delta_g)} + \frac{\beta}{\alpha-1}\hat{K}_g(0) \right\} \exp(-\delta_g t) \\
&- \frac{\Gamma_2(e+n)}{(\alpha-1)\gamma\Phi(-(e+n))} \exp\{-(e+n)t\} \\
&- \frac{\Gamma_3}{(\alpha-1)\gamma} \tilde{\Theta}(\omega) \cos(\omega t + \phi_0 + \phi_1) \exp\{-(e+n)t\},
\end{aligned}$$

where C_1 and C_2 denote arbitrary constants, and

$$\begin{aligned}
\Theta(\omega) &\equiv \left[\left\{ \omega^2 + \frac{(e+n+\lambda_1)^2 + (e+n+\lambda_2)^2}{2} \right\}^2 \right. \\
&\quad \left. - \left\{ \frac{(e+n+\lambda_1)^2 - (e+n+\lambda_2)^2}{2} \right\}^2 \right]^{-1/2}, \\
\tilde{\Theta}(\omega) &\equiv \sqrt{\omega^2 + (e+n)^2}\,\Theta(\omega), \\
\phi_0 &\equiv \tan^{-1}\left[-\frac{(e+n+\lambda_1)(e+n+\lambda_2) - \omega^2}{\omega\{2(e+n) + \lambda_1 + \lambda_2\}} \right], \\
\phi_1 &\equiv \tan^{-1}\left(-\frac{\omega}{e+n} \right).
\end{aligned}$$

Lastly, we impose initial and boundary conditions. To exclude the divergency solution, we set $C_1 = 0$ as the boundary condition when t approaches infinity since $\lambda_1 > 0$ and $\lambda_2 < 0$. Furthermore, in order to impose the initial condition, we now

benchmark calibration (see Table 1). Hence, we consider only the case in which $\lambda_1 > 0$ and $\lambda_2 < 0$ are satisfied.

substitute $t = 0$ to the solution of $\hat{k}$, and solving about C_2 establishes

$$\begin{aligned} C_2 &= \frac{1}{\lambda_2}\Bigg\{\gamma(\alpha-1)\hat{k}(0) + \frac{\Gamma_1\delta_g}{\Phi(-\delta_g)} + \beta\gamma\hat{K}_g(0) \\ &\quad + \frac{\Gamma_2(e+n)}{\Phi(-(e+n))} + \Gamma_3\tilde{\Theta}(\omega)\cos(\phi_0+\phi_1)\Bigg\}. \end{aligned}$$

Substituting these conditions for the general solution, we obtain the particular solution presented in Section 3.

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Table 1: Calibration

Parameter	Value
B^c	1.000
B^s	1.000
e	0.020
n	0.010
α	0.327
β	0.093
θ	2.000
δ	0.094
δ_g	0.038
ρ	0.036
g^*/k^*	0.038

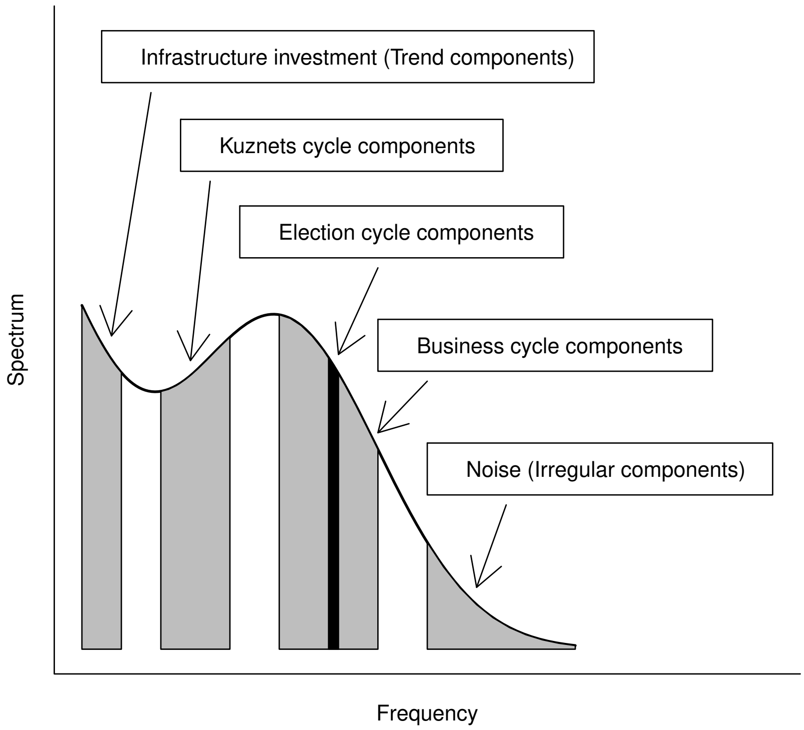

Figure 1: Spectrum of government spending

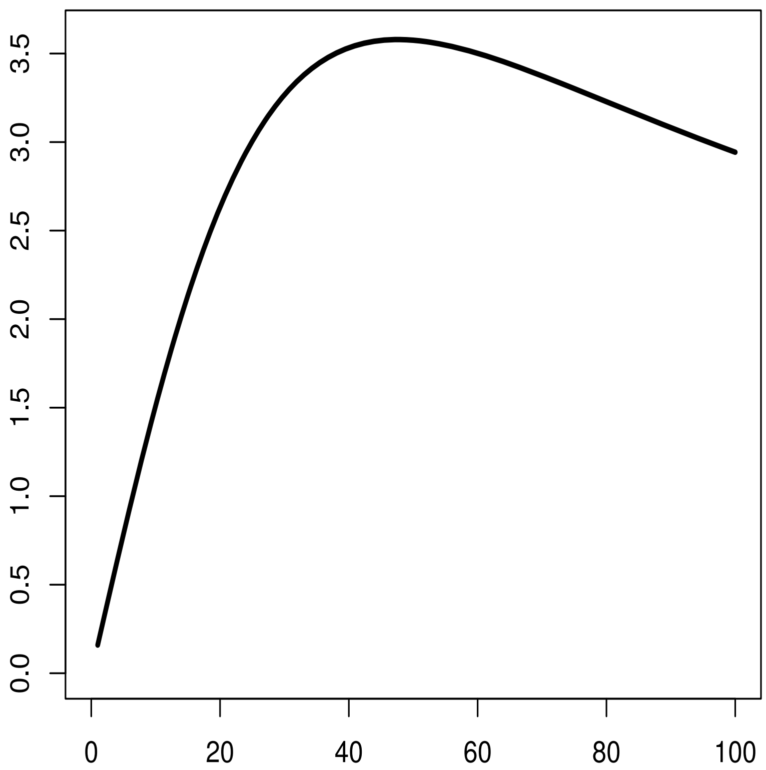

Figure 2: Plot of $\tilde{\Theta}(T)$ in a numerical setting