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# Capital Tax as a Consequence of Bargaining

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## Abstract

We study an OLG model in which heterogenous agents bargain over capital taxation. In our model, both of the balance of *bargaining power* and *threat point*, that standard median voter models have not considered, are endogenized. We show that the two key features are crucial determinants for political as well as economic outcomes.

*JEL classification:* [E62](#); [P48](#); [H20](#); [H30](#)

*Keywords:* Legislative bargaining; wealth inequality; capital taxation

## 1 Introduction

Inequality is not just consequence of economic forces but policies also play their role in the formation of inequality. Moreover, policies themselves reflect the will of the people to some degree. The standard approach to illustrate this interaction is to suppose that the median voter can choose his/her most preferred policy without any negotiation ([Meltzer and Richard, 1981](#)). In the real democratic countries, however, policies are chosen by bargaining among strategic legislators. This paper, apart from the median voter approach, studies an overlapping generation(OLG) model where consumers explicitly bargain over capital taxation. We show several crucial determinants for political and economic outcomes in which the median voter models were not taken into account.

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One of the key determinants is the balance of *bargaining power* among legislators<sup>1</sup>. For example, the more share of conservative people in legislature might induces lower tax as a result of voting. We endogenize the bargaining power assuming that consumers can directly bargain over taxation, so the ideological distribution of legislators is wealth distribution of consumers. This *direct democracy* setting makes the model to capture the *pure* result without considering political frictions such as disproportional election, political campaign, or rent-seeking legislators.

The other important element is the *threat point* (the utility from the default option). For instance, if the default option is always zero tax, it might strengthen the power of rich who prefer lower tax. Although the median voter models have omitted it, we endogenize the threat point by allowing the current policy to be a default option tomorrow (*status quo*), so forward looking young consumers chose current policy taking into account that it will be the status quo tomorrow while old ones do not.

One of the most related paper is [Bassetto \(2008\)](#) that studies an OLG model where consumers cooperatively Nash bargaining over public policies. Due to the direct democracy feature, his model shows a political economic interaction: fiscal policy affects capital accumulation of consumers and it changes the bargaining power balance in the next period. However, as he exogenously fixes the threat point, the model cannot study the effect of status quo.

The other important paper is [Piguillem and Riboni \(2013\)](#) which studies a Neoclassical growth model where exogenous “legislators” bargain over capital taxation with endogenous threat point (status quo). In their model, the expected tax is increasing in threat point due to its endogeneity. We show that this result is not robust if the balance of bargaining power is endogenously formed.

According to our knowledge, this is the first paper to study the dynamic general equilibrium model with both endogenous bargaining power and threat point.

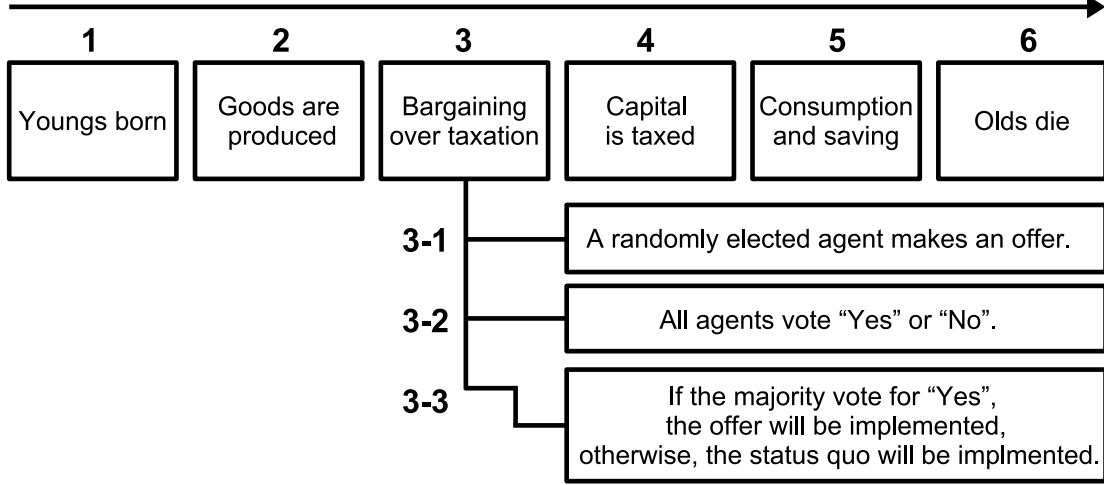
## 2 The Model

The model is based on life-cycle overlapping generation model without private information. A new generation is born in each period and all of them live for two periods without bequest motives. Each consumer is indexed by generation  $g \in \{Y, O\}$

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<sup>1</sup>There are numerous literature of the bargaining game. See, for example [Rubinstein \(1982\)](#), [Baron and Ferejohn \(1989\)](#), and [Baron \(1996\)](#).

Figure 1: Timing of Events within per Period



where  $Y$  denotes young;  $O$  denotes old, and intra-generational share of capital holdings  $i$ . Young consumers are endowed identical amount of wage  $w$  and unequal capital  $k^{y,i}$ . We suppose that the distribution of  $k^{y,i}$ :  $f(k^{y,i})$  is exogenously fixed across periods.

**Technology and Redistribution Rule.** The private good is produced by a technology using aggregate capital stock  $K_t$  and aggregate labor supply  $L_t$ . We assume that the total population  $N_t$  and aggregate labor supply  $L$  are constant:  $N_t = N$  for all  $t$ ;  $L = \frac{N}{2}$ . We also assume that capital fully depreciates after each use because the length of per period equals per generation. The aggregate technology is represented by a linear technology in which factor prices  $(R, w)$  are exogenously given

$$F(K_t, L_t) = RK_t + wL. \quad (1)$$

The revenue from capital tax is used to finance a commonly distributed lump-sum rebate  $T_t$

$$T_t = \frac{\tau_t RK_t}{N}. \quad (2)$$

**Preferences.** In period  $t$ , a young consumer endowed  $k_t^{y,i}$  has a preference described by

$$u(c_t^{y,i}) + \beta E_t [u(c_{t+1}^{o,i})] \quad (3)$$

where  $c_t^{y,i}$  and  $c_{t+1}^{o,i}$  are consumption when he is young and old and  $\beta$  is the discount factor. He faces budget constraints at date  $t$  and  $t + 1$

$$c_t^{y,i} + k_{t+1}^{o,i} \leq w + (1 - \tau_t) Rk_t^{y,i} + T_t \quad (4)$$

$$c_{t+1}^{o,i} \leq (1 - \tau_{t+1}) Rk_{t+1}^{o,i} + T_{t+1} \quad (5)$$

where  $R_t$  is the gross rate of return to capital,  $\tau_t$  is the capital tax rate, and  $T_t$  is equally distributed lump-sum rebate. Consumers are uncertain for  $\tau_{t+1}$ . The expectation for  $\tau_{t+1}$  evolves following Markov process

$$\Gamma(\tau|\tau^Q) = \Gamma \begin{pmatrix} p(\tau^0|\tau^0) & \dots & p(\tau^0|\tau^1) \\ \vdots & \ddots & \vdots \\ p(\tau^1|\tau^0) & \dots & p(\tau^1|\tau^1) \end{pmatrix}, \quad (6)$$

where  $\tau^Q$  is the tax in the prior period: *status-quo*;  $p(\tau^m|\tau^n) = \text{Prob}(\tau = \tau^m | \tau^Q = \tau^n)$ . The next period aggregate capital  $K_{t+1}$ , which affects the expectation for the next period lump-sum rebate  $T_{t+1}$ , is given by the law of motion of aggregate capital

$$K_{t+1} = g(K_t(\tau_{t-1})). \quad (7)$$

Old agents just consume all of their disposable income. In period  $t$ , an old consumer with capital share  $i$  has preferences described by

$$u(c_t^{o,i}). \quad (8)$$

He / she maximizes his / her utility (8) subject to the budget constraints

$$c_t^{o,i} \leq (1 - \tau_t) Rk_t^{o,i} + T_t. \quad (9)$$

**Legislature.** In each period, one of the consumers is randomly elected to be *agenda setter* who proposes take-it-or-leave-it offer for current tax  $\tau$ . Then, all consumers

cast a vote “Yes” or “No”, if the majority accepts it, it becomes the current tax, otherwise the status quo will be implemented again. Consumers vote for the proposal  $\tau^A$  when it provide higher (expected) utility than the status quo. The voting rule  $\phi$  for consumer  $g, i$  is described by

$$\phi^{g,i}(\tau^A|\tau^Q) = \begin{cases} \text{“Yes”} & \text{if } V^{g,i}(\tau^A) \geq V^{g,i}(\tau^Q) \\ \text{“No”} & \text{otherwise} \end{cases} \quad (10)$$

where  $V^{g,i}$  is her value function given tax  $\tau^A$  of  $\tau^Q$ . We let  $\Phi^{Yes}(\tau^A|\tau^Q)$  and  $\Phi^{No}(\tau^A|\tau^Q)$  denote the share of consumers to total population who support and who don’t. Then, the majority rule implies that the proposal will be accepted if and only if

$$\Phi^{Yes}(\tau^A|\tau^Q) \geq \Phi^{No}(\tau^A|\tau^Q). \quad (11)$$

An agenda setter  $g, i$  proposes a current tax  $\tau^{A,g,i}(\tau^Q)$  to maximize his inter-temporal utility

$$\arg \max_{\tau} V^{g,i}(\tau|\tau^Q), \quad (12)$$

subject to the Markov process (6) and the majority rule constraints (11). Also, an optimal tax for each agent  $\tau^{*g,i}(\tau^Q)$  is easily delivered by the expression (12) subject to the Markov process (6). Note that the Markov process (6) is not binding for old agenda setters because the next period tax does not affect their utilities.

All agents know which tax will be accepted. Therefore, if there exist a policy which increases his expected utility and majority of agents will accept it, he proposes the policy, otherwise he proposes the status quo. We let  $\Phi^A(\tau^A|\tau^Q)$  denote the share of consumers who propose tax  $\tau^A$  if she become agenda setter and with status-quo  $\tau^Q$ . Then, the  $(\tau^A, \tau^Q)$  element of  $\Gamma(\tau|q) : p(\tau^A|\tau^Q)$  is

$$p(\tau^A|\tau^Q) = \Phi^A(\tau^A|\tau^Q). \quad (13)$$

**Definition: Politico-economic Equilibrium.** For given factor prices  $\{w, R\}$  and the wealth distribution of young  $f(k^y, i)$ , a Politico-economic Equilibrium is a stochastic sequence of redistribution policies  $\{T_t, \tau_t\}_{t=0}^{\infty}$ , allocations  $\{c_t^{g,i}, k_t^{g,i}\}_{t=0}^{\infty}$  for all  $g$  and  $i$ , the law of motion of aggregate capital  $g(K_t(\tau_{t-1}))$ , and the wealth distribution of old agents’  $f(k_t^{o,i}(\tau_{t-1}))$  for all  $t$  such that:

1. Value functions are generated by consumers’ optimal decisions.

2. The law of motion of aggregate capital  $g(K_t(\tau_{t-1}))$  wealth distribution of old consumers  $f(k_t^{o,i}(\tau_{t-1}))$  are stationary.
3. The sequence of policies is generated by the Markov process for taxes  $\Gamma(\tau|\tau^Q)$  and the government's budget constraint.
4. Markets clear:  $\sum_g \sum_i c_t^{g,i} + \sum_i k_{t+1}^{o,i} = F(K_t, L_t)$ , for all  $t$ .
5. Given the optimal decisions,
  - (a) Decision shall be taken by a majority given by the majority rule constraints (11).
  - (b) The tax proposal is delivered by the expression (12) subject to the Markov process (6) and the majority rule constraints (11).
  - (c) The Markov process for tax is generated by equation (13).

**Table 1: Parameter Values**

Parameter	Description	Value				
$u(c)$	Utility Function	$\log c$				
$\beta$	Discount Rate	$0.6355 \approx 0.985^{30}$				
$w$	Wage	1				
$R$	Rental Cost of Capital	$5.743 \approx 1.06^{30}$				
$N^g$	Population of each Generation	100				
$f(k^{y,i})$	Wealth Distribution of Young	<table style="width: 100%; border: none;"> <tr> <td style="text-align: center;"><i>Inter</i></td> <td style="text-align: center;"><i>Uniform</i></td> </tr> <tr> <td style="text-align: center;"><i>Inter and Intra</i></td> <td>Wealth distribution in the US in 2010.</td> </tr> </table>	<i>Inter</i>	<i>Uniform</i>	<i>Inter and Intra</i>	Wealth distribution in the US in 2010.
<i>Inter</i>	<i>Uniform</i>					
<i>Inter and Intra</i>	Wealth distribution in the US in 2010.					

**Calibration.** The calibrated parameters are described in Table 1. The length of period is assumed to be 30 years. In the first experimental case *Inter*, all young consumers are endowed same level of wealth. In the case of *Inter and Intra*, following Piketty (2014), we divided a generation to 4 groups: (1) *Super Rich* (top1% whose  $k^{y,i}$  is 35% of  $K^y$ ), (2) *Rich* (top2-10%:  $k^{y,i}$  is 35% of  $K^y$ ), (3) *Middle* (top 11-50%,  $k^{y,i}$  is 25% of  $K^y$ ), and (4) *Poor* (bottom 50%,  $k^{y,i}$  is 5% of  $K^y$ ). Essentially, we divide the one generation into the four types of consumers with different share of population.<sup>2</sup>

<sup>2</sup>The detailed calculation and computational algorithm are discussed in the online appendix <<https://sites.google.com/site/yutasaitoecon/>>.

**Table 2:** Equilibrium Political Outcomes: “*Inter*” Economy

Markov $\Gamma(\tau \tau^Q)$	$\begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}$		
Expected Tax $E[\tau \tau^Q]$	$(0.5 \quad 0.5 \quad 0.5)$		
	Young	Old	
Votes $\Phi^{Yes}(\tau \tau^Q)$	$\begin{pmatrix} 100 & 0 & 0 \\ 100 & 100 & 0 \\ 100 & 100 & 100 \end{pmatrix}$	$\begin{pmatrix} 100 & 100 & 100 \\ 0 & 100 & 100 \\ 0 & 0 & 100 \end{pmatrix}$	
Preferred Tax $\tau^*(\tau^Q)$	$(H \quad H \quad H)$	$(L \quad L \quad L)$	
Proposed Tax $\tau^A(\tau^Q)$	$(H \quad H \quad H)$	$(L \quad L \quad L)$	

**Note:** The order of the matrixes and vectors are:  $L, M, H$ .

### 3 Results

Table 2 shows the equilibrium result of *Inter* case where there exists only intergenerational heterogeneity. From now on, we show that the result of the case where capital tax takes only three values:  $L = 0.0$ ,  $M = 0.5$ , and  $H = 1.0$ <sup>3</sup>. The equilibrium Markov process shows “bang-bang” result: the tax will randomly be  $H$  or  $L$  with any status quo. This is because both generations have majority so every agenda setter proposes their most preferred tax.

The result is much different in the case with both intra and inter-generational inequality called *Inter and Intra*(Table 3). As no types of consumers have majority, all of them need support from the other consumers. Several agenda setters, whose  $\tau^*(\tau^Q)$  and  $\tau^A(\tau^Q)$  are different, propose their second-best policy because they know their first-best will be rejected. The equilibrium Markov shows that the expected tax is increasing in status quo when  $\tau^Q$  is smaller than  $H$  as studied in Piguillem and Riboni (2013). However, when  $\tau^Q$  is  $H$ , the expected tax dramatically declines, because the middle and poor old consumers prefer  $L$  who were perfectly redistributed when they were young. In our calibrated economy,  $k^{o,p}(H)$  is larger than average capital holdings, so they prefer  $L$ . This effect is caused by the endogeneity

<sup>3</sup>The fundamental insights are robust in the case with more smaller grids. See the online Appendix in more detail.



**Table 3:** Equilibrium Political Outcomes: “*Inter and Intra*” Economy

Markov $\Gamma(\tau \tau^Q)$	$\begin{pmatrix} 0.1 & 0 & 0.55 \\ 0 & 0.055 & 0 \\ 0.9 & 0.945 & 0.45 \end{pmatrix}$							
Expected TaxE [ $\tau \tau^Q$ ]	$(0.9 \quad 0.9725 \quad 0.45)$							
	Young				Old			
Votes $\Phi^{Yes}(\tau \tau^Q)$	$\begin{pmatrix} 100 & 50 & 10 \\ 50 & 100 & 10 \\ 90 & 90 & 100 \end{pmatrix}$			$\begin{pmatrix} 100 & 1 & 100 \\ 90 & 100 & 100 \\ 90 & 99 & 100 \end{pmatrix}$				
	<i>Super Rich</i>	<i>Rich</i>	<i>Middle</i>	<i>Poor</i>	<i>Super Rich</i>	<i>Rich</i>	<i>Middle</i>	<i>Poor</i>
Preferred Tax $\tau^*(\tau^Q)$	$(L L L)$	$(L L L)$	$(H L H)$	$(H H H)$	$(L L L)$	$(L H L)$	$(H H L)$	$(H H L)$
Proposed Tax $\tau^A(\tau^Q)$	$(L M L)$	$(L M L)$	$(H H H)$	$(H H H)$	$(L M L)$	$(L H L)$	$(H H L)$	$(H H L)$

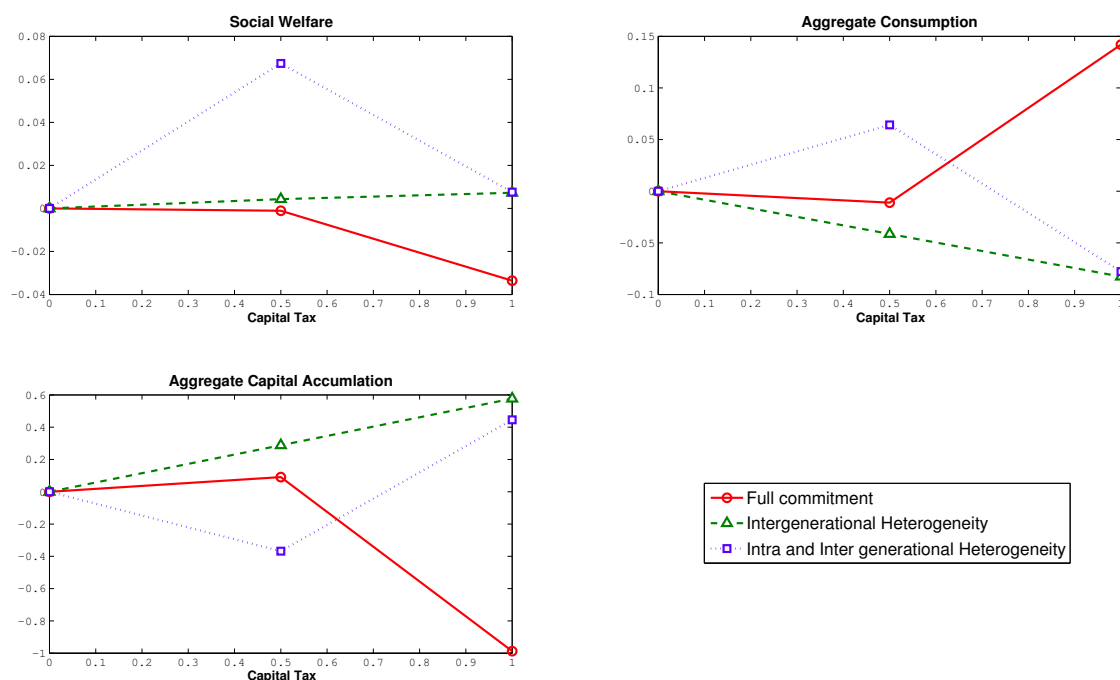
**Note:** The order of the matrixes and vectors are:  $L, M, H$ .

of the bargaining power and it has never been seen in the model with exogenous legislators.

Figure 2 shows that the effect of taxation to the economy is heavily depends on the expectation of the next period tax. In addition to the two equilibria, we add *Full-commitment* case where *Inter* economy with fixed Markov process:  $\Gamma(\tau|q) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  which can be interrupted as the standard life-cycle OLG model. In this case, high current capital tax induces high amount of current consumption for young because accumulated wealth is perfectly taxed in the next period. However, in the case of *Intra and Inter*, high current tax makes the people to expect lower future tax, so the consumption will decrease.

“Social Welfare” in Figure 2 shows the effect on the utilitarian social welfare:  $\sum_i V^{y,i}(\tau|\tau^Q) + \sum_i V^{o,i}(\tau|\tau^Q)$ . In the case of *Full-commitment*, the tax  $H$  decreases the social welfare because the young consume all of their disposable income today which significantly decreases the utility in the next period. In *Inter* case, tax  $H$  increases the young’s utility through the increase of their disposable income and it is larger than the utility decrease of olds. In the case of *Intra and Inter*, this effect is larger because the marginal utility benefit of poor agents is much higher.

**Figure 2:** Effect of Taxation to Economy (Y label: % increase of the variable from the  $\tau = L$  case)



**Note:** This figure shows the case where  $\tau^Q = L$ .

## 4 Conclusion

We studied an OLG model with legislative bargain process which illustrates how inequality, bargaining power and threat point affect the policy and economy in some broader perspective.

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