Multiscale systematic risk: Empirical Evidence from Pakistan

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Abstract

This study utilizes the wavelet approach namely Maximal Overlap Discrete Wavelet Transform (MODWT) to examine the multiscale risk-return relationship for Pakistan stock market. The method enables scale-by-scale analysis of CAPM validity and heterogeneous market expectations. Our sample consists of 117 firms listed at Karachi stock exchange for the period January 1, 2006 to December 31, 2013. The empirical findings show that the risk-return relationship is linear at higher (16-128 days) scales and average daily market risk premium is 23.8%. The study, consistent with literature, concludes that systematic risk is a multiscale phenomenon.

Key words: CAPM, Multiscale systematic risk, Pakistan

1. Introduction

Decision making in finance particularly in portfolio management, capital budgeting, and equity valuation requires return estimation on individual firm basis. Capital Asset Pricing Model (CAPM) and Fama and French (1992) are single and three factor models commonly used for the estimation of expected returns. Academic literature favors Fama and French when portfolio perspective is considered. However, CAPM is preferred by practitioners for estimation on individual stock basis (Bruner et al., 1998; Graham and Harvey, 2001). Fama and French model performs no better than CAPM when applied for individual stock's return estimation. It requires more calculation and is not cost effective (Bartholdy and Peare, 2005).

The CAPM developed by Sharpe (1964) and Lintner (1965) based on its simplistic approach is widely used by practitioners and academicians to calculate the required returns for the portfolio management, capital budgeting decisions and assets valuation. CAPM was developed in a hypothetical environment with certain assumptions about the market. The assumption include inter-alia risk-averse attitude of investors, risk free lending and borrowing, assets divisibility and market perfection. The model implies mean-variance efficiency of the market portfolio in a manner that (a) the expected return of a security is a positive linear function of its market beta (the slope in regression of a security's return on the market's return), and (b) the beta describes the cross-section of expected returns. The basic principle of the model is that the investor should be compensated for taking risk in addition to risk free return (time value). CAPM assumes the opportunity cost and justifies any excess return as the risk premium for the risky portfolio. The model divides total risk of an asset in two parts; unsystematic and systematic. There is no risk premium justified for unsystematic risk as it can be eliminated through efficient diversification. CAPM talks about the systematic risk which covers the risk of an economy as a whole.

Application and validity of CAPM posed intellectual challenge ever since foundations for its development were laid by proposing mean-variance framework (Markowitz, 1952). Since inception, CAPM version developed by Sharp (1964) and Lintner (1965) remained under criticism. Model has been modified in many ways to capture the true nature of risk/return relationship and to increase the predictability of returns. However, CAPM is still widely used in financial applications and provides basis for financial decisions. The model has been criticized many times by the researchers however; the research efforts were unable to invalidate the underlying principle of the model. CAPM is a powerful and intuitive tool to measure relationship between expected return and risk. Unfortunately, the empirical record of the model has been unsatisfactory. The CAPM's empirical insufficiency may reflect theoretical limitations and the result of many unrealistic assumptions. However, they may also be caused by difficulties in implementing valid tests of the model.

The empirical evidence on the validity of CAPM in Pakistan is also mixed. Zubairi and Farooq (2011), Masood et al., (2012) and Rizwan et al., (2013), among others concluded that CAPM is not a useful model to predict the stocks return in case of Pakistan. Whereas, Ali et al., (2011) suggest that CAPM predict expected return more accurately on short time horizon. Many variants of CAPM (Javid and Ahmad, 2008; Javid, 2009) including the downside risk based asset pricing framework (Ayub et al., 2015; Akbar et al., 2012) were tested but no significant improvement over original CAPM was identified.

In this paper, we follow the approach proposed by Gencay et al., (2005), namely wavelet based multiscale decomposition, to estimate the scale-by-scale systematic risk using CAPM. The framework allows us to examine the validity of CAPM and to accommodate the heterogeneous market expectations (Muller et al., 1997). The heterogeneous market hypothesis suggests that the market price dynamics differ over time based on participants expectations. The empirical results using Karachi stock exchange data show that the relationship between the risk and return is scale depended and becomes stronger at long-term scale. Therefore, the multiscale framework provides better predictions through CAPM. The rest of the paper is organized as follows. Section 2 summarizes a recent debate on the validity of CAPM. Section 3 discusses econometric methodology. Section 4 explains empirical results. Final section concludes the paper.

2. Recent Debate

Dempsey (2013) claims that CAPM which made finance an appropriate subject for econometric studies may need to be replaced with a *paradigm of market* that is vulnerable to unpredictable behavior. In his words, "The empirical studies of CAPM are nothing more than fitting of data rather than theoretical principles". He also criticized the Black's CAPM, the replacement of risk free rate (R_f) with returns on zero beta portfolio (R_z). The higher value of R_z was fitted as an econometric technique in an effort to improve CAPM results found during the earlier studies. The love affair with data mining emanates from the desire to support one's prior convictions (Moosa, 2013). In conclusion, Dempsey (2013) says "An inexact science becomes even more inexact for academics" claiming finance now to be an econometric exercise in mining data either for confirmation of a particular factor model or for the confirmation of deviations from a model's predictions as anomalies.

Moosa (2013) adding to the Dempsey's discussion concludes that CAPM is an inadequate model. The belief "economics and finance are allegedly similar to physics" has led to excessive and unnecessary mathematization, producing fancy but unrealistic models (including the CAPM). "Finance, after all, is not physics and CAPM is by no means Boyle's law" he concludes.

Claims of CAPM's invalidity routing deep in history of finance (Fama and French, 2004; Lai, 2011) and criticism on Mean-Variance (MV) analysis (Borch, 1969; Borch, 1974; Borch, 1978) were discussed under "Logic and philosophy of finance" by Johnstone (2013). He encourages the CAPM debate started by Dempsey (2013), considering it the philosophical aspect of finance and criticizes the cultural shift within finance from critical philosophical analysis of logic and methods, caused due to more empirical historical focus of researchers in finance. Bornholt (2013) studied an extensive data set (from July 1963 to December 2009) using value-weighted monthly returns and revealed that CAPM will eventually become a reasonable approximation to market reality. He supported the claim by showing weakening of beta anomaly over recent years and argued that it resulted due to continued exploration by practitioners.

Brown and Walter (2013), in response to Dempsey (2013), quoted Roll (1977) that so-called tests of CAPM were invalid due to the use of inefficient benchmark portfolio proxy for market portfolio contrary to what the theory suggests that the benchmark should be efficient. The critics have not sufficiently internalized this requirement (Diacogiannis and Feldman, 2007). They supported their argument based on the results presented by Graham and Harvey (2005) where the average 10-year bond yield (4.6%) is higher than average market risk premium (3.7%), hence market portfolio used was inefficient. In their view, valid test of CAPM requires efficient benchmark, which in the case of CAPM is *world market portfolio* which has been proved elusive so far.

Partington (2013) under the title "*Death where is thy string?*" based his conclusion, "CAPM is the king of asset pricing models", on two arguments. First, CAPM has passed a very important test which is the test of time (Graham and Harvey, 2001; Truong et al., 2008). Second, it is not entirely clear as to what has been tested in empirical tests of the CAPM. He also emphasized the importance of efficient benchmark as proxy for market portfolio. The MV framework was first introduced in 1952 by Professor Henry Markowitz where mean and standard deviation were used as proxies for investment return and risk respectively (Markowitz (1952). In 1960s, Sharpe (1964) and Lintner (1965) extended his work to modern portfolio theory (MPT). MPT resulted in equilibrium expected returns that are linear function of security's and market's risk. The single factor CAPM developed by Sharpe (1964) and Lintner (1965) provided basis for return predictability and equilibrium pricing with several unrealistic theoretical assumptions. He and Ng (1994) and Davis (1994) criticized the model's predictability for providing week empirical evidence on risk-return relationship. Bowers and Heaton (2013) emphasized the importance of CAPM but failed to provide empirical evidence in support. Contrary to the criticism, studies by Lau et al., (1974) and Michailidis et al., (2006) verified linear relationship between risk and return and highlighted the importance CAPM.

3. Methodology

In this section, we first briefly discuss the estimation process of individual stock returns by CAPM and then wavelet based multiscale methodology. The Single factor CAPM calculates the expected return on an asset by:

$$R_i = R_f + \beta_i (R_m - R_f) + \epsilon_i \quad (1)$$

where, R_i, R_f, R_m and β_i represent return on stock *i*, risk free rate, return on market portfolio and beta of stock *i* respectively. Beta is the systematic risk of asset *i* relative to market (world market portfolio) and can be calculated as $\beta_i = Cov_{(r_i, r_m)}/\sigma_m^2$. Firms' and market returns are calculated using natural log of periods' closing prices as $R_i = Ln\ (P_t/P_{t-1})$. The estimate for β_i is typically calculated by running time series regression of market and stock excess returns as follow:

$$R_{it} - R_{ft} = \alpha_i + \beta_i \gamma_{it} + \epsilon_{it} \qquad (2)$$

where, $\gamma_{it} = R_{mt} - R_{ft}$ is known as market risk premium. As the market portfolio[1], which consists of all assets in the market is not observable so it is necessary to use a proxy. We have used Karachi Stock Exchnage (KSE 100), denoted by k, index as the market proxy. The precise equation in case of proxy would be:

$$R_{it} - R_{ft} = \alpha_i + \beta_i\big(R_{kt} - R_{ft}\big) + \epsilon_{it} \qquad t = 1, \dots \dots n. \qquad (3)$$

Eq. (3) can also be calculated by using raw returns instead of excess returns, there is no significant difference in estimates using both returns (Bartholdy and Peare, 2005).

$$\mathrm{R_{it}} = \alpha_\mathrm{i} + \beta_i \mathrm{R_{kt}} + \epsilon_\mathrm{it} \qquad \mathrm{t} = 1, \dots \dots \mathrm{n}. \qquad (4)$$

Fundamental, the wavelet approach[2] is used to simultaneously decompose a time series into time and scale component. The wavelets are obtained by dilation and translation of the so-called father and mother wavelet pair respectively that are specified as follow:

$$\int \varphi(t)\, dt = 1 \qquad (5)$$
$$\int \psi[t]\, dt = 0 \qquad (6)$$

The father function detects the smooth (low-frequency) components of a signal and the mother wavelets approximate the details (high frequency or noisy) constituents. Resulting low and high frequency wavelets can then be defined as under:

$$\varphi_{j,k}(t) = 2^{j/2}\varphi\big(2^j t - k\big) \qquad (7)$$
$$\psi_{j,k}(t) = 2^{j/2}\psi\big(2^j t - k\big) \qquad (8)$$

where, $\mathrm{j} = 1,2,3, \dots \dots \mathrm{J}$ indicate the number of scale crystal (frequencies) and $\mathrm{k} = 1, \dots \dots, 2^\mathrm{j}$ k=1,…,2j show the number of coefficients (translation). The multi-resolution *f*(*t*) representation of the signal can be given by:

$$f(t) = \textstyle\sum_k S_{j,k}\varphi_{j,k}(t) + \sum_k d_{j,k}\psi_{j,k}(t) + \sum_k S_{j-1,k}\psi_{j-1,k}(t) + \ldots + \sum_k S_{1,k}\varphi_{1,k}(t) \qquad (9)$$

where, $\varphi_{j,k}$, $\psi_{j,k}$, $S_{j,k}$ and $d_{j,k}$ are the scaled, translated, smooth and detailed coefficients, respectively. We have used Maximal Overlap Discrete Wavelet Transform (MODWT) with Daubechies Least Asymmetric (LA) filter. This choice is motivated by the fact that MODWT has several advantages over conventional DWT. The MODWT can be applied to any sample size, provides higher resolution at coarser scales, is translation invariance, and provide asymptotically efficient wavelet variance estimator compared to DWT (Percival and Mofjeld, 1997). The scaling $\breve{\varphi}_{\mathrm{j,k}}$ and wavelet $\breve{\psi}_{\mathrm{j,k}}$ filters of MODWT can be obtained directly through the

[1] Estimation through CAPM requires construction of a world market index, representative of all tradable assets in the world, which is unobservable and thus makes estimation impossible. However, the term "estimation based on CAPM" used in this study is consistent with common use.

[2] For a detailed perspective on wavelet, see Gencay et al., (2005).

conventional
DWT filters:

$$\breve{\varphi}_{j,k} = \frac{\varphi_{j,k}}{2^{2/j}} \tag{10}$$

$$\breve{\psi}_{j,k} = \frac{\psi_{j,k}}{2^{2/j}} \tag{11}$$

The jth level MODWT scaling $\tilde{\upsilon}_{j,t}$ and wavelet $\tilde{\omega}_{j,t}$ coefficients for the time series, denoted as X, with an arbitrary sample size N, can be specified as follows:

$$\tilde{\upsilon}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L-1} \tilde{g}_{j,l} X_{t-l} \tag{12}$$

$$\tilde{\omega}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L-1} \tilde{h}_{j,l} X_{t-l} \tag{13}$$

4. Empirical Results

The study utilizes the data of 117 firms listed at Karachi Stock Exchange (KSE) of Pakistan. Sample is based on data availability and survival of firms and time period spans from January 1, 2006 to December 31, 2013. In order to avoid thinly traded stocks (interval effect, see Hawawini, 1983), only the stocks that have traded at least 80% of trading days during the sample period. Data comprises of 1864 daily returns for each stock and KSE-100 index that is approximately 250 days per year.

The wavelet beta $\beta_{ij}^{w} = \left[Cov(\widetilde{W}_{kj}, \widetilde{W}_{ij}\right]/\sigma_{kj}^{2}$ for scales j=1, 2…., 6, of each stock is estimated (Eq. 4) for each subsample year over entire sample period. Since, we used daily data for CAPM estimation, each wavelet scale corresponds to different range of days. The scales and their associated days as follows: scale 1 = 2-4 days, scale 2 = 4-8 days, scale 3 = 8-16 days, scale 4 = 16-32 days, scale 5 = 32-64 days. Finally, the scale 6 is associated with 64-132 days dynamics. Multiscale decomposed return series of KSE 100 index (scale-wise returns and time on vertical and horizontal axis, respectively) is shown in Fig. 1. As explained in the methodology section, high frequency (scale 1-3) and low frequency (scale 4-6) show high to variation in market returns, respectively. Each scale, as it is extracted from the raw series, explains a certain amount of energy. The wavelet variance (y-axis) of market returns at each of the six scales (x-axis) is shown in Fig. 2. It is observed that variance of market returns decrease from scale 1 (high frequency or noisy component) to scale 6 (smooth component). The energy captured by each scale is equal to sum of squares of scale observations divided by sum of squares of raw data. Scale-wise explaining power of decomposed signal is shown on the right side of Fig. 2. From this preliminary analysis, we can conclude that wavelet based scale-by-scale decomposition provides a different set of information at various scale and hence can be utilized to examine the multiscale systematic risk.

<< INSERT FIGURE 1 HERE >>

<< INSERT FIGURE 2 HERE >>

Next, the scale-by-scale betas for each stock and every year were estimated over entire sample period. These betas were ranked from lowest to highest along with next year associated stock returns. These ranked betas were then split into nine equal size portfolios and next year portfolio (equally weighted) returns from the stocks in each portfolio were calculated. The process to update the portfolios in this manner (every year based on beta sizes) and calculating the average portfolio returns for the next year was repeated for the entire sample period[3].

After covering the entire sample period, nine average betas and associated portfolio returns were obtained. These portfolio betas and next year returns were obtained for all six scales and for the raw data, for comparison. Theoretically, the validity of CAPM requires a positive relationship between the average portfolio betas and the corresponding average returns. If the next year portfolio returns increase with the increase in beta, we can conclude that CAPM is valid. We have shown the scatter plots of returns versus corresponding portfolio betas at different wavelet scales in Fig. 3. The average daily portfolio returns were plotted on the y-axis (vertical) and average portfolio betas were plotted on x-axis (horizontal). According to MV framework, a positive linear relationship (an increase in systematic risk (beta) is associated with the increase in expected returns) is expected for the validity of CAPM. The multiscale visualization of average portfolio betas and returns is provided from low (Fig. 3a) to high (Fig. 3f) scales. The low scale (high frequency) scatter plots (Fig. 3a-3c) show that points are well dispersed and hence, no particular pattern can be observed. However, a positive pattern emerges as we proceed to higher scales (low frequency) in Fig. 4-6. Notably, the slop of the points also increases with the increase in scale. Gencay et al., (2005) concluded that this change in relationship from non-linear to linearity suggest that risk-return relationship is a scale dependent phenomenon. In economic interpretation, the market expectations are heterogeneous. We have also plotted the relationship of betas obtained using raw data and returns in Fig. 4. The figure shows a clear positive linear relationship between risk and return. It can be inferred that CAPM provides a reasonable approximation of future returns when applied to KSE.

<< INSERT FIGURE 3 HERE >>

<< INSERT FIGURE 4 HERE >>

To conclude our analysis, we have applied Ordinary Least Square (OLS) regression to estimate the slope co-efficients for all scales and raw data. We regressed average portfolio return (as dependent variable) against average portfolio betas (as independent variable). Results of regression analysis are provided in Table 1. The OLS estimates of the slope co-efficients indicate that the average market risk premium is underestimated at first three scales (-2.40%, -8.70% and -7.70%, respectively). The estimated slopes at first third scales are also statistically insignificant at conventional levels. The slope co-efficients are positive at higher (scale 4-6) three scales and when raw data is used. The average market premium is 1.70%, 2.40% and 15.5% respectively for the fourth, fifth and sixth scale. The raw data shows that average market premium is 23.8% (significant at 10% level). These results confirm our earlier findings and hence, we can conclude a multiscale relationship between risk-return in case of Pakistan.

<< INSERT TABLE 1 HERE >>

[3] The approach of updating betas and testing the CAPM validity is similar to Reinganum (1981) and Gencay et al., (2005).

5. Conclusion

This study provides the empirical evidence on the multiscale systematic risk in Pakistan. We used data of 117 firms listed at Karachi stock exchange for the period January 1, 2006 to December 31, 2013. The multi-scale beta approach (Gencay et al, 2005) using wavelet Maximal Overlap Discrete Wavelet Transform (MODWT) is adopted to test the validity of CAPM and heterogeneous market hypothesis. The approach decomposes a time series into wavelet distributed over time and frequency (scale). The empirical findings indicate that the risk-return relationship becomes linear with the increase of wavelet scales and hence provide evidence on the validity of CAPM. More precisely, the future returns predictions through CAPM are more relevant for the investors with long time horizons compared to investor with short- to medium time horizons. Average daily market risk premium is 23.8% when raw data is used for the OLS estimation. The study, consistent with previous literature, concludes that systematic risk is a multiscale phenomenon.

This research effort, however, suffers for many limitations. First, the availability of relatively larger firms' data spanning over a longer time period, as in developed market, can significantly impact the findings especially when testing the validity of a theoretical model. Second, the selection of wavelet method and filtering techniques can provide different results. However, the method adopted in this study can be extended to the other variants of CAPM like Fama-French three and five factor models to provide further insight on risk-return relationship in Pakistan.

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Figure 1: Scale-by-scale decomposed KSE 100 index returns.

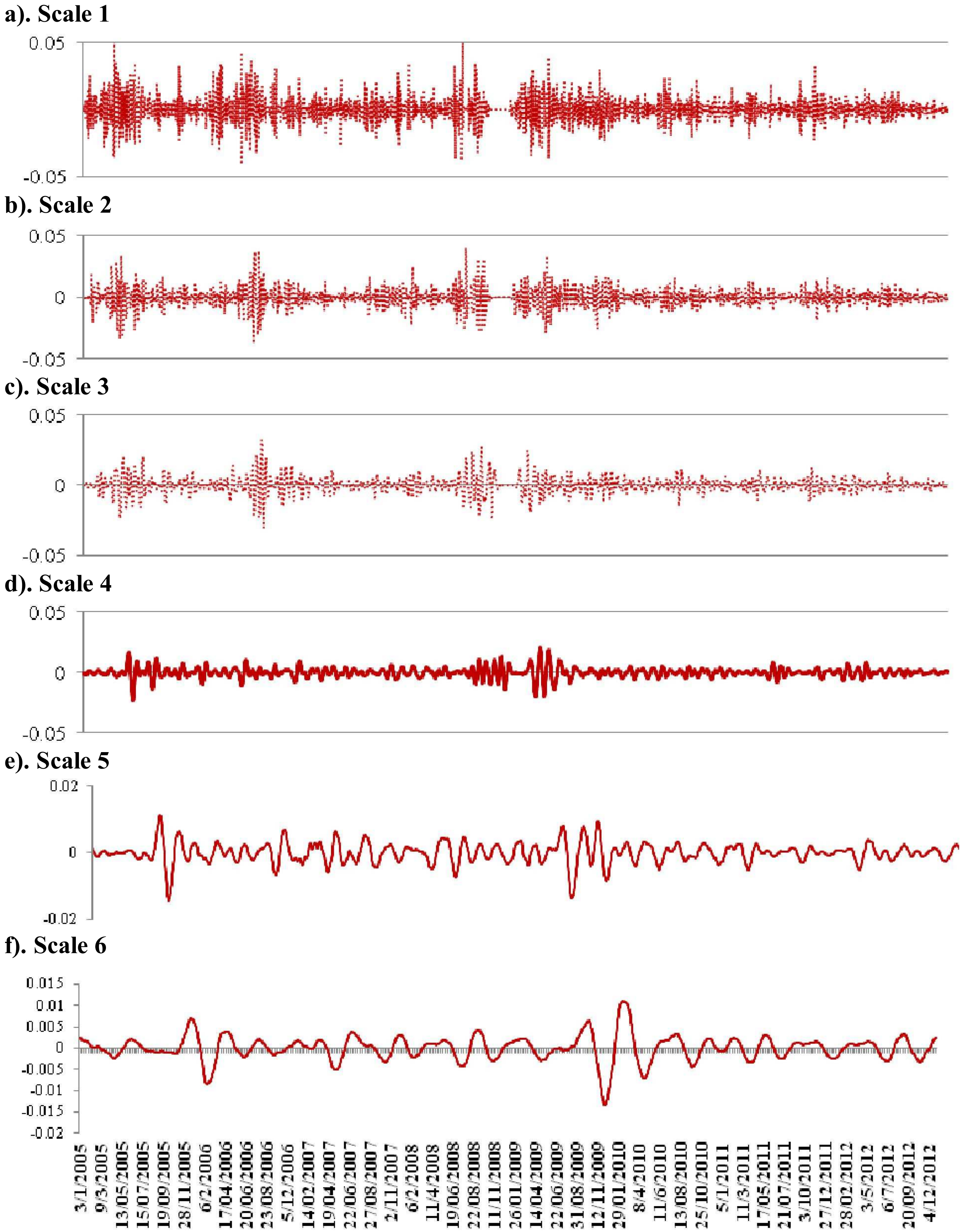
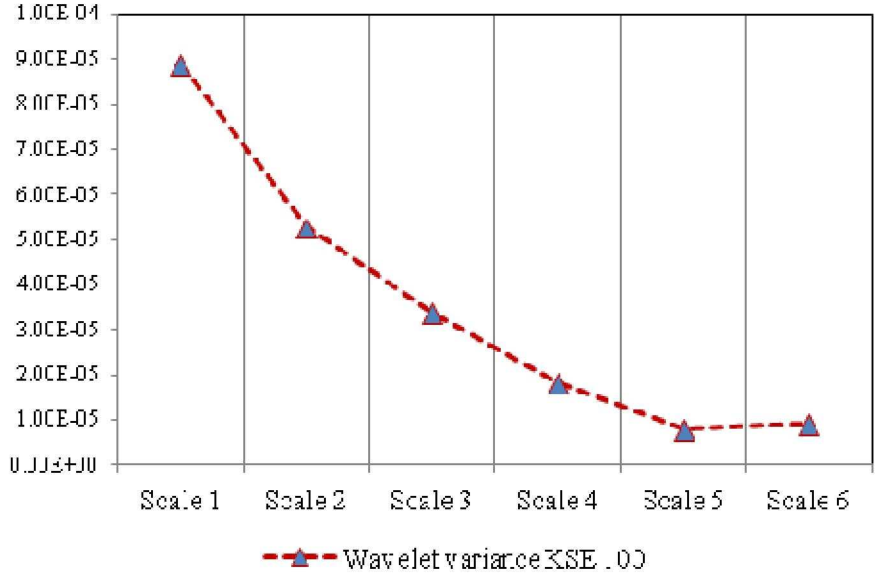


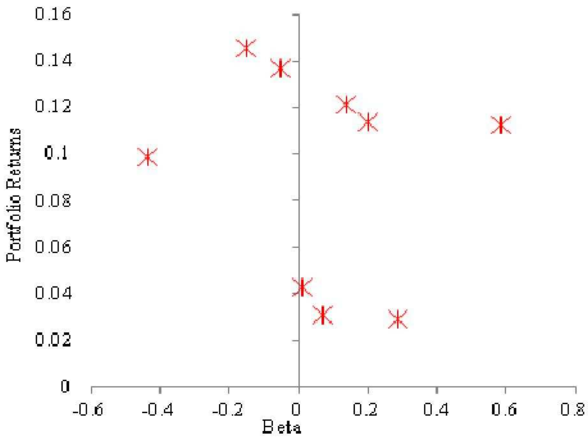
Figure 2: MODWT based wavelet variance of Karachi Stock Exchnage with explained energies at each scale.



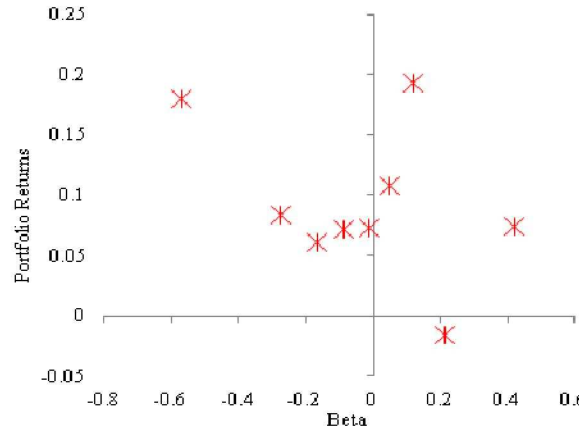
Scales	Energy explained (%)
Scale 1	40.74
Scale 2	24.33
Scale 3	15.32
Scale 4	8.49
Scale 5	4.25
Scale 6	3.97
Total	97.10

<u>Figure 3: Scatter plots of average portfolio betas (x-axis) and average daily returns.</u>

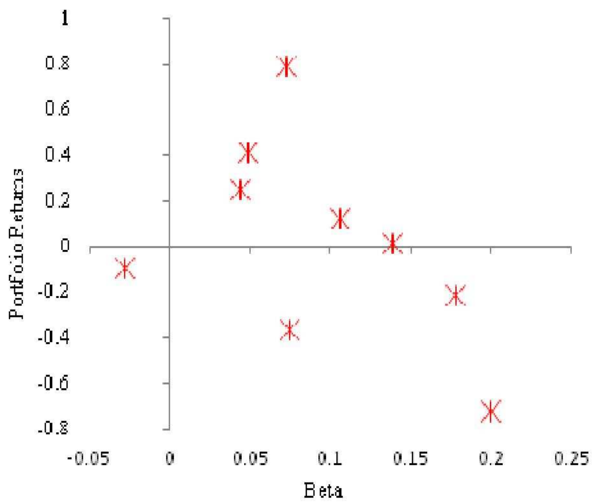
a). Scale 1



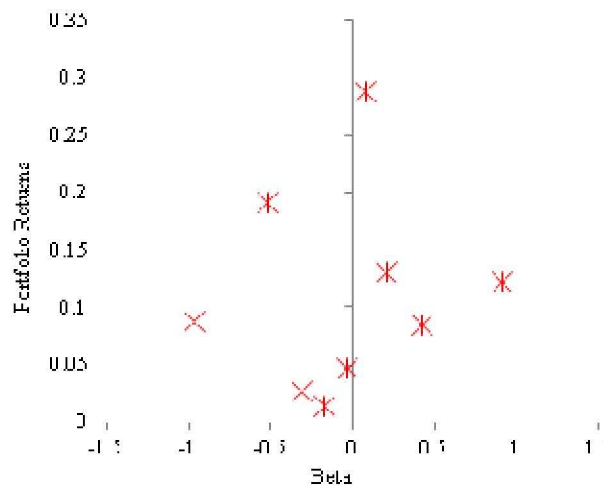
b). Scale 2



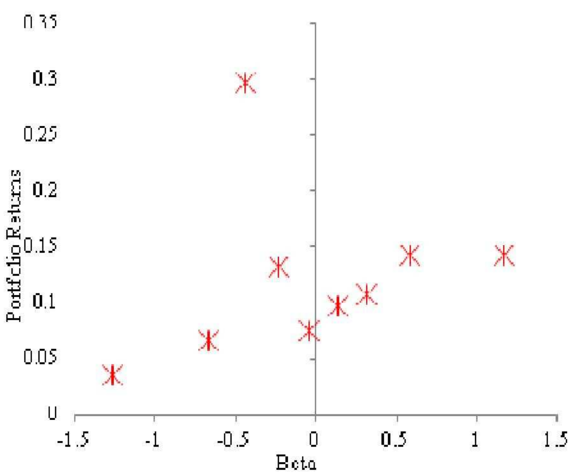
c). Scale 3



d). Scale 4



e). Scale 5



f). Scale 6

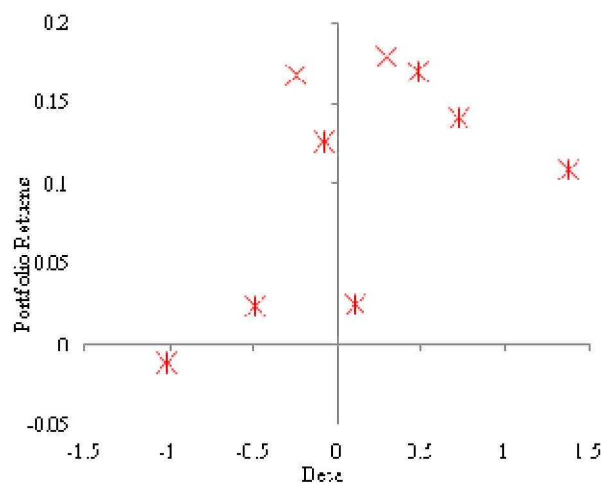


Figure 4: Scatter plots of average portfolio betas (x-axis) and average daily returns (raw).

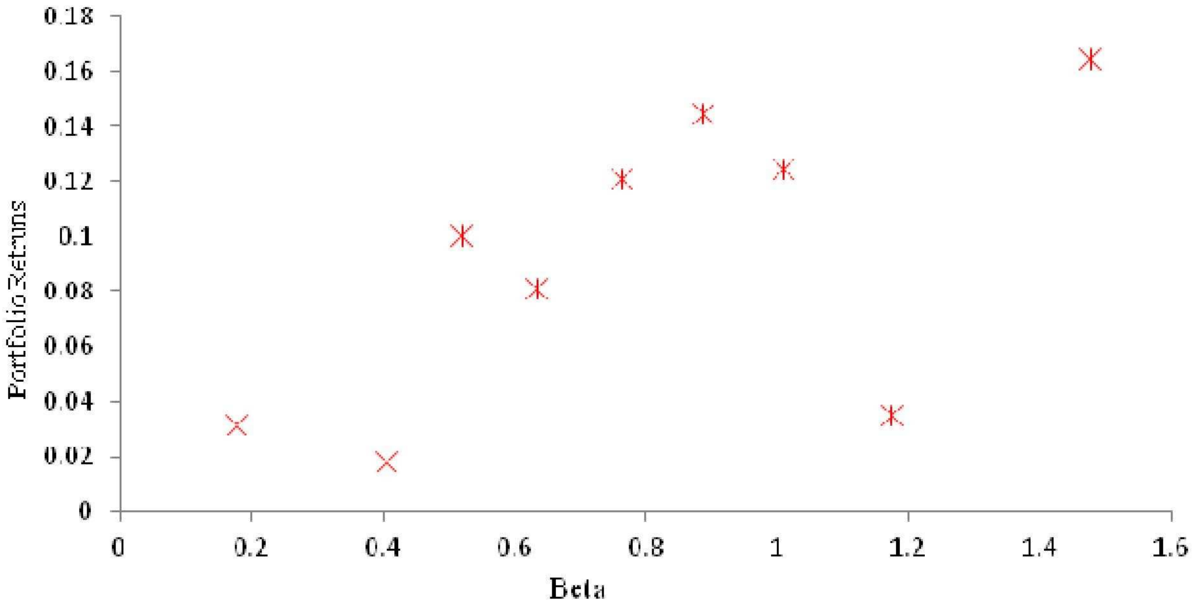


Table 1: The results of OLS estimation at different scales.

	Intercept	Slop	R square
Scale 1	0.094 (5.660)*	-0.024 (-0.410)	0.024
Scale 2	0.089 (4.290)*	-0.087 (-1.155)	0.160
Scale 3	0.094 (4.234)*	-0.077 (-1.450)	0.231
Scale 4	0.112 (3.607)*	0.017 (0.281)	0.011
Scale 5	0.123 (4.718)*	0.024 (0.620)	0.052
Scale 6	0.096 (4.385)*	0.155 (1.675)***	0.286
Raw	0.605 (4.199)*	0.238 (2.044)***	0.360

Note: * and *** indicate significance at 1% and 10% level, respectively. The values in the parenthesis are of t-statistics.