The Rate of Time Preference of Government

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2 July 2015

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Abstract

Here, I examine the mechanism by which the rate of time preference (RTP) of government is formed and present a model of government RTP. The RTP of government has been largely neglected in the study of economics, but it is an important factor in inflation acceleration. The model presented indicates that the RTP of government is determined by the expected RTPs of the median voter and the representative household, as well as the strength of the government's fluid intelligence. The model also indicates, however, that households actually use "beliefs" or heuristics to generate their expected RTPs of government.

JEL Classification code: B20, E31, H10
Keywords: Time preference; Government; Inflation; Rationality; Fluid intelligence

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1. INTRODUCTION

The rate of time preference (RTP) of government has been largely neglected in the study of economics. This neglect has probably continued because it has been widely believed that the RTP of government is naturally identical to the RTP of the representative household; therefore, emphasizing the RTP of government is a meaningless exercise. However, to the best of my knowledge, there is no evidence that the RTP of government is identical to the RTP of the representative household. Rather, it is more natural to think that they are intrinsically different because, according to the median voter theorem (e.g., Downs, 1957), a government basically represents the median voter, which is different from the mean voter (i.e., the representative household).

In addition, Harashima (2004, 2015) showed that the RTP of government plays an essential role in inflation. Harashima indicated that differences between the RTPs of government and the representative household are the main cause of inflation acceleration. Therefore, households must know the RTP of government to behave rationally. However, because the RTP of government is not openly visible or otherwise discernable, households must expect its future value, and to expect it, they need a structural model of the government's RTP that describes how it is formed. Again, to the best of my knowledge, no such model has been presented. The purposes of this paper are to examine the mechanisms by which the RTP of government is formed and to present a model of its formation.

An important feature of government RTP is that, if a government behaves on the basis of its intrinsic RTP, inflation generally accelerates, as Harashima's (2004, 2015) model of inflation shows. However, if households do not want inflation to accelerate and ask their government not to accelerate inflation, how should the government behave? Unless the government changes its RTP to be equal to the RTP of the representative household, no future path that satisfies all optimality conditions of government exists under the condition of no inflation acceleration. Hence, if a government adheres to its intrinsic preference, rational behavior in the sense that, given the available information, optimal decisions to achieve the objective are taken, is impossible. In this case, preference and rationality conflict. If rationality prevails over preference, a government will change its RTP, but if not, the RTP of government will remain at its intrinsic value. Households, which need to generate expected values for the RTP of government, must first decide whether preference or rationality prevails in government. Models of the government's RTP should include the mechanism by which households evaluate how the government deals with this inherent contradiction between preference and rationality.

Another important feature of government RTP is that, even if the functional form of a model of a government's RTP is known, it is difficult to generate the expectation of a government's RTP because it is difficult to know the parameter values in the model. To determine the model parameter values of government RTP, it is necessary to obtain a sufficiently large amount of data on the value of the median voter's RTP as well as the RTPs of the representative household, but in practice this is a very difficult task. Therefore, households have to use "beliefs" or heuristics to determine the expected RTP of the government.

2 NECESSITY OF EXPECTING GOVERNMENT RTP

2.1 Law of motion for inflation

The model constructed by Harashima (2015) is used as the model of inflation in this paper. The details of the model are shown in the Appendix. The model indicates that the law of motion for inflation is described by

$$\int_{t-1}^{t}\int_{s}^{s+1}\pi_{\upsilon}\,d\upsilon\,ds=\pi_t+\theta_G-\theta_P \quad , \tag{1}$$

where π_t is the inflation rate at time t, θ_G is the RTP of government, and θ_P is the RTP of the representative household. θ_G and θ_P are not necessarily identical. Equation (1) is the same as Equation (A19) in the Appendix. Equation (1) indicates that inflation accelerates or decelerates as a result of the government and the representative household reconciles the contradiction in heterogeneous RTPs. A solution of the integral Equation (1) for given θ_G and θ_P is

$$\pi_t=\pi_0+6(\theta_G-\theta_P)t^2 \quad . \tag{2}$$

Generally, the path of inflation that satisfies Equation (1) for $0\le t$ is expressed as

$$\pi_t=\pi_0+6(\theta_G-\theta_P)\exp[z_t\ln(t)] \quad ,$$

where z_t is a time-dependent variable. The stream of z_t varies, depending on the boundary condition. However, if π_t satisfies Equation (1) for $0\le t$, and $-\infty<\pi_t<\infty$ for $-1<t\le 1$, then

$$\lim_{t\to\infty} z_t = 2 \quad .$$

A proof is shown by Harashima (2008). Any inflation path that satisfies Equation (1) for $0\le t$ therefore asymptotically approaches the path of Equation (2).

2.2 Need to construct a structural model of θ_G to generate expectations

All households behave (i.e., choose their optimal paths) on the basis of the expectation of future inflation. The model shown in Section 2.1 indicates that, to expect future inflation, households must know the future value of the government's RTP (θ_G). There is no guarantee that θ_G and the RTPs of households will be constant across time; rather, the RTPs of government and households will be intrinsically temporally variable. However, households cannot even know the present value of θ_G directly, because households and the government are different entities and do not inherently know each other's preferences. Therefore, households must somehow generate expectations of the future values of θ_G by calculating them on the basis of a structural model of the government's RTP. Thereby, they must first construct such a model.

3 PREFERENCE VS. RATIONALITY

The law of motion for inflation discussed in Section 2.1 indicates that, if the government behaves on the basis of its intrinsic RTP, inflation will accelerate. On the other hand, if people strongly dislike inflation acceleration, a government has to behave so as to not accelerate inflation; however, this conflicts with its own intrinsic preference.

3.1 The conflict between preference and rationality

Behaving on the basis of its own intrinsic preferences does not mean that a government acts in a stupid, foolish, or irrational manner; rather, it behaves quite normally by naturally adhering to its intrinsic preferences. A fundamental question arises, however: Even if the government is acting quite normally, is this behavior rational? In economics, rationality usually means that, given the available information, optimal decisions are made to achieve an objective, and rational behavior is generally assumed. However, can rational behavior still prevail when a government cannot optimize its behavior to achieve its objective? This special situation emerges if the central bank is perfectly independent and is firmly determined to stabilize inflation and if, at the same time, the intrinsic time preference rate of government is unchangeable. In this situation, the economy will become severely destabilized because it is impossible to satisfy Equation (1). Therefore, the government cannot achieve its objective (i.e., cannot maximize its expected utility) and can behave only irrationally. Conversely, if the government wants to optimize its objective and behave rationally, it must change its time preference. Clearly, trade-offs between rationality and time preference exist in some situations, and either rationality or time preference must be endogenized.

Nevertheless, it is highly unlikely that people will not optimize their behavior to meet their objectives (i.e., maximize utility) if they have complete knowledge of the optimal path. Hence, rationality should prevail over preferences, and time preference will be endogenized when a clash between rationality and time preference occurs. If time preference is endogenized, rational decisions become possible. Even though rationality should eventually prevail over preferences, governments will not easily change their own preferences. They will resist endogenizing them and search for options to escape from doing so—it is this stubborn nature that drives a government to deviate from the path specified by its central bank. Even though unfavorable consequences are expected if no change is made, it can be very difficult to change one's own preferences alone. Controlling preferences therefore usually requires the help of other people or institutions; this is one of the reasons why independent central banks were established to stabilize inflation.

If a central bank is not sufficiently independent, the government must change its RTP on its own so as to not accelerate inflation. A government must then rein in its preferences on its own. The RTP of government, therefore, is determined through the struggle between preference and rationality inside the government. If rationality prevails, inflation does not accelerate, but if preference prevails, inflation will accelerate.

3.2 Two environments

Models are simplified representations of reality. Therefore, models can be classified by how far the chosen model simplifies reality. In particular, models are classified by whether they are based on the assumption that all agents are homogeneous (i.e., a homogeneous environment) or on the assumption that agents are heterogeneous (i.e., a heterogeneous environment).

In models based on a homogeneous environment, it is usually assumed that rationality always prevails over preference, because it has generally been regarded that there is no conflict between preference and rationality in a homogeneous environment. In general, the dominance of rationality in a homogeneous environment has been undoubted (i.e., the rational expectation hypothesis has been accepted).

On the other hand, dominance of rationality in a heterogeneous environment is not necessarily guaranteed because, unlike in a homogeneous environment, serious contradictions between preference and rationality arise in a heterogeneous environment. For example, Becker (1980) showed that, if the RTPs of households are heterogeneous, the most patient household will eventually own all the capital in an economy and the other households cannot achieve optimality. That is, all households except the most patient household cannot behave rationally in the sense that rational households behave in such a way to achieve optimality, if they adhere to their own intrinsic RTPs. Harashima (2004, 2015) showed another case. If a government adheres to its own intrinsic RTP that is higher than the RTP of the representative household, inflation accelerates. If people dislike inflation acceleration and thereby the government has to behave under the condition that it does not accelerate inflation, there is no path that satisfies all optimality conditions for the government as long as it adheres to its own intrinsic RTP. In a heterogeneous environment, therefore, conflicts between preference and rationality can occur.

3.3 Necessary intelligence

The struggle between preference and rationality is dealt with in the human brain. To resolve conflicts, humans need particular powers or functions—that is, different types of intelligence.

3.3.1 Sustainability in a union or society

Properly dealing with the struggle between preference and rationality is essential for humans because humans do not live alone—they are social and live in groups. However, the struggle has the potential to destroy a society. In a heterogeneous environment, if preference prevails over rationality, there is no guarantee that a political union or society is sustainable because some members of society cannot achieve optimality. In theory, this problem does not exist in a homogeneous environment, because the conflict basically does not exist and competitive equilibria are optimal for all people. On the other hand, in a heterogeneous environment, competitive equilibria are not necessarily optimal for all people because people have heterogeneous preferences, as discussed in Section 3.2. Many of the people who cannot achieve optimality will strongly oppose the government or other people, and it is likely that the political union or society will collapse, possibly violently.

A political union or society is formed and maintained because it provides benefits to its members. Behaviors that support a union or society are important for humans to survive. The type of potential vulnerability in heterogeneous environments that is discussed above indicates that various types of intelligence are essential to properly manage the struggle between preference and rationality.

3.3.1.1 Calculations

In a heterogeneous environment, relationships among people are far more complicated than in a homogeneous environment because people do not all behave in the same way in a heterogeneous environment. Humans must possess the intelligence to cope with these complicated relationships. They need to be able to calculate the outcomes of various activities in a heterogeneous group of people, evaluate the outcomes, and select the best action to take among many options in their brains.

The number of calculations required to reach an optimal solution is far larger in a heterogeneous environment than in a homogeneous one because the number of types of people that must be considered and the number of interconnections among heterogeneous people are far greater in a heterogeneous environment. If each person's brain can cope with this extremely large number of calculations, people can behave rationally (i.e., always take the best actions that are calculated to be optimal, that is, the ones that are consistent with the model) even in a heterogeneous environment. If this does not occur, rationality may not prevail over preference.

3.3.1.2 Evaluation

After a variety of potential outcomes are calculated, many options are evaluated on the basis of the results of calculations to select the optimal option. Therefore, people must have the intelligence to evaluate options. The optimal future path is more complicated in a heterogeneous environment than in a homogeneous environment, because households act differently. The intelligence needed for evaluation allows people to accurately identify the optimal future path by comparing and evaluating various aspects of many different complicated paths.

3.3.1.3 Self-control

In addition, another type of intelligence is required—that which allows people to align their preferences so as to follow the optimal option. Even if an optimal option is appropriately calculated and evaluated, the optimal option cannot be implemented if people's preferences are not properly controlled. That is, people must exercise self-control. This type of intelligence applies to other activities as well—for example, when a person is on a diet. Children often have difficulty exercising self-control because this type of intelligence is not yet fully developed in childhood. In addition, it seems highly likely that it is also not necessarily sufficiently developed in many adults, and even adults will often lose the battle when forced to choose an option that is against their own preferences.

3.3.2 Intelligence needed when the three types of subordinate intelligence are deficient

It remains unclear whether humans are sufficiently equipped with the necessary types of intelligence to deal with the calculation, evaluation, and self-control aspects of decision-making in a heterogeneous environment. For example, the capacity of a human's brain may be insufficient to process the extremely large number of calculations necessary in a heterogeneous environment. If this first type of intelligence is insufficient, it will be even more difficult to evaluate which option is appropriate to prevent disrupting the political union or society. Furthermore, even if the intelligence needed for calculations is sufficient, actions taken will not be optimal if the evaluation process is biased or poor.

If any part of the three subordinate intelligences is deficient, however, humans still have alternative methods to employ. For example, they can use approximations. The number of calculations needed will be significantly reduced if an appropriate approximation method is used. The intelligence needed for approximation is likely basically different from the three types of subordinate intelligence, although there may be partial overlap between them. For appropriate approximations, the concept of "fluid intelligence" will be particularly important.

3.3.3 Fluid intelligence

In psychology and psychometrics, many types of intelligence have been considered, including fluid intelligence, crystallized intelligence, short-term memory, long-term storage and retrieval, reading and writing ability, and visual processing. Among these, the importance of the difference between fluid intelligence and crystallized intelligence has been particularly emphasized. According to Cattell (1963, 1971), fluid intelligence is the capacity to solve new kinds of problems logically, independent of acquired knowledge. This type of intelligence signifies the ability to deal with new situations without relying on knowledge gained at school or through experience. With the help of fluid intelligence, people can flexibly adapt their thinking to new kinds of problems or situations. By contrast, crystallized intelligence is the capacity to acquire and use previously obtained knowledge.

Fluid intelligence is essential when people make approximate calculations and need to judge which approximation is the best among many choices. These judgments are very difficult because we do not know the true values. Therefore, judgments must be made after comprehensive consideration of various choices. Such judgments represent "something new" in the sense that they will not necessarily be judged as best in future periods and under different circumstances. People need to make new judgments in any future period. That is, we must solve an "unknown problem" on each occasion to make the best approximation. Thus, these judgments are innovations that are made by using a person's fluid intelligence. Fluid intelligence is therefore essential in a heterogeneous environment.

These types of judgments are similar to decisions made in politics. Political conditions change from moment to moment. Yesterday's optimal political decision may be a non-optimal political decision today. Furthermore, nobody knows for certain whether today's political decision is truly optimal. Historians examine whether past political decisions were optimal, but there are many political decisions over which even historians cannot reach consensus about their optimality.

3.4 *The degree of rationality in a heterogeneous environment*

3.4.1 The item response theory

Fluid intelligence can be modeled on the basis of the item response theory, which is used widely in psychometric studies (e.g., Lord and Novick, 1968; van der Linden and Hambleton, 1997). In particular, the item response function is used to describe the relationship between abilities and item responses.

A typical item response function is

$$\tilde{p}(\tilde{\mu}) = \tilde{c} + \frac{1-\tilde{c}}{1+e^{-\tilde{a}(\tilde{\mu}-\tilde{b})}} \quad ,$$

where $\tilde{p}$ is the probability of a correct response (e.g., answer) to an item (e.g., test or question), $\tilde{\mu}$ ($\infty > \tilde{\mu} > -\infty$) is a parameter that indicates an individual's ability, $\tilde{a}$ (> 0) is a parameter that characterizes the slope of the function, $\tilde{b}$ ($\infty \geq \tilde{b} \geq -\infty$) is a parameter that represents the difficulty of an item, and $\tilde{c}$ ($1 \geq \tilde{c} \geq 0$) is a parameter that indicates the probability that an item can be answered correctly by chance.

3.4.2 The probability of dominance of rationality

How frequently rationality prevails over preference can be modeled with an item response function. Let *FI* be the degree of fluid intelligence in a person. Larger values of *FI* indicate stronger fluid intelligence in the sense that a person more correctly grasps (approximates) a situation by using fluid intelligence. Let also p_{HE} be the probability that rationality prevails over preference in a heterogeneous environment. On the basis of the item response theory, p_{HE} can be modeled as a function of *FI* such that

$$p_{HE}(FI) = \hat{c} + \frac{1-\hat{c}}{1+e^{-\hat{a}(FI-\hat{b})}} \quad , \qquad (3)$$

where $\hat{a}$ (> 0) is a parameter that characterizes the slope of the function, $\hat{b}$ ($\infty \geq \hat{b} \geq -\infty$) is a parameter that represents the difficulty and complexity of a situation, and $\hat{c}$ ($1 \geq \hat{c} \geq 0$) is a parameter that indicates the probability that rationality prevails over preference by exogenous factors. If *FI* is sufficiently large, rationality almost always prevails over preference in a heterogeneous environment, but if it is very small, preference almost always prevails over rationality.

An important implication of Equation (3) is that the rational expectation hypothesis is not necessarily acceptable in a heterogeneous environment. If *FI* is small (i.e., fluid intelligence is weak), preference will often prevail over rationality and thus the rational expectation hypothesis cannot be unconditionally accepted.

3.5 *Fluid intelligence of government*

According to the median voter theorem (e.g., Downs, 1957), a government behaves just as the median voter prefers in a one-person one-vote democratic political system. This theorem suggests that the fluid intelligence of government is equal to that of the median voter. On the other hand, the top-level positions in government are usually occupied by the best and brightest in a country, and they will almost certainly have stronger fluid intelligence than the median voter. However, does that mean these officials will make decisions that are different from those of the median voter? If they do so, they will be forced to step down in the next election according to the median voter theorem. Only politicians who make the same decisions as the median voter will be able to occupy top-level positions. Hence, it is likely that the fluid intelligence of government is practically equal to that of the median voter when dealing with issues in which preference and rationality conflict.

3.6 The nature of $\hat{c}$

The value of $\hat{c}$ is affected by exogenous factors. For example, if the central bank is sufficiently independent and capable, $\hat{c}$ becomes unity—that is, the central bank makes rationality always prevail over preference with regard to the RTP of government. The government is always forced to change its RTP as the central bank orders. It is likely that many institutions or mechanisms work to raise the value of $\hat{c}$. For example, constitutions, laws, treaties, and many government and international organizations will raise the value of $\hat{c}$ by urging governments to maintain rationality. Such institutions and mechanisms have probably been adopted in many societies, because experience has taught us that they help ensure that rationality prevails over preference in a heterogeneous environment. As new institutions or mechanisms were invented and adopted, the probability that rationality prevails over preference may have gradually increased (by increasing the value of $\hat{c}$) through time. Therefore, it is likely that, as civilization has progressed, $\hat{c}$ has increased, and rationality more frequently prevails over preference in a heterogeneous environment.

4 A MODEL OF GOVERNMENT RTP

4.1 Determinants of θ_G

The value of θ_G will usually be equal to the RTP of the median voter, as discussed in Section 3.5. However, in some cases, other elements will also affect the value of θ_G. The determinants of θ_G will be basically classified into the following two elements.

4.1.1 Preference element

In this paper, I call the determinant that is equal to that of the median voter's RTP the "preference element." This element usually determines the main body of θ_G. Let $\theta_{G,pre}$ be the preference element component of θ_G, and $\theta_{P,med}$ be the intrinsic RTP of the median voter. As discussed in Section 3.5, the intrinsic $\theta_{G,pre}$ is basically equal to the intrinsic $\theta_{P,med}$ in a one-person one-vote democratic political system. Therefore, in the following sections, I assume that $\theta_{G,pre} = \theta_{P,med}$.

4.1.2 Political element

The determinant that is peculiar to the government's RTP is the "political element." Let $\theta_{G,pol}$ be the political element component of θ_G. If a political system is maintained and stable forever, the political element will be nil, and θ_G will be determined only by the preference element. However, if a political system is unstable, the political element component is not zero, and it increases as the political system becomes more unstable. Although rare, it is possible for a political system to collapse. There are many historical examples of the collapse of a political system. These have been often observed, for example, after a defeat in a large-scale war or after a revolution. The political element is of great significance when a political system is on the brink of collapse. Faced with an impending collapse of the system, the incumbent government will do anything possible to survive the crisis. From the government's perspective, the far future is meaningless—survival is the primary objective. It imposes taxes and increases expenditures so as to avoid immediate collapse. As a result, its actions become increasingly myopic and impatient in the sense that it does not concern itself with future economic conditions. This behavior indicates an increase in $\theta_{G,pol}$.

For most democratic countries, the probability of an imminent collapse of the political system will be negligible, and we may assume that $\theta_{G,pol}$ is zero in those countries, but the political element is very important in politically unstable countries.

4.2 The model

Section 3 indicates that p_{HE} needs to be expected to generate an expected $\theta_{G,pre}$. Let $p_{HE,G}$ be the p_{HE} of the government and $p_{HE,P}$ be the p_{HE} of the median voter. Because basically $\theta_{G,pre} = \theta_{P,med}$ as discussed in Section 4.1, $p_{HE,G} = p_{HE,P}$ generally, and thereby it is reasonable to assume that $p_{HE,G} = p_{HE,P}$. Therefore, in a one-person one-vote political system,

$$p_{HE,G} = p_{HE,P} = \hat{c} + \frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}} \quad , \tag{4}$$

where $FI_{P,med}$ is the FI of the median voter. Equation (4) indicates that the smaller $FI_{P,med}$ is, the smaller $p_{HE,G}$ is and the higher the probability of inflation acceleration.

Suppose that the central bank is not independent of the government. Thereby, the government has to control its RTP by itself, that is, without being forced to so by the central bank. (The case for an independent central bank is discussed in Section 4.4.) Suppose also for simplicity that the probability that a political system is on the brink of collapse is p_{inst} and $\theta_{G,pol}$ takes a unique positive value, and the probability of a stable political system is then $1- p_{inst}$ and $\theta_{G,pol} = 0$. The model of the government's RTP that is used to generate the expected RTP of government is therefore

$$\begin{aligned}\theta_G &= p_{HE,G}\theta_P + \left(1-p_{HE,G}\right)\left(p_{inst}\theta_{G,pol}+\theta_{G,pre}\right)\\ &= \left(\hat{c}+\frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\theta_P + \left[1-\left(\hat{c}+\frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\right]\left(p_{inst}\theta_{G,pol}+\theta_{G,pre}\right) \quad .\end{aligned} \tag{5}$$

Equation (5) indicates that the RTP of government is equal to θ_P when rationality prevails over preference with the probability $p_{HE,G}$. When preference prevails over rationality with the probability $1 - p_{HE,G}$, the RTP of government is equal to the intrinsic RTP of government. The intrinsic RTP of government consists of $\theta_{G,pol}$ with the probability p_{inst} and $\theta_{G,pre}$.

Because $\theta_{G,pre} = \theta_{P,med}$ (as assumed in Section 4.1.1), then by Equation (5),

$$\begin{aligned}\theta_G &= p_{HE,G}\theta_P + \left(1-p_{HE,G}\right)\left(p_{inst}\theta_{G,pol}+\theta_{P,med}\right)\\ &= \left(\hat{c}+\frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\theta_P + \left[1-\left(\hat{c}+\frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\right]\left(p_{inst}\theta_{G,pol}+\theta_{P,med}\right) \quad .\end{aligned} \tag{6}$$

In most democratic countries, the probability of the occurrence of extreme political instability is very low. For those countries, therefore, the model is reduced to a more simple form by assuming $p_{inst} = 0$ such that

$$\begin{aligned}\theta_G &= p_{HE,G}\theta_P + \left(1-p_{HE,G}\right)\theta_{P,med}\\ &= \left(\hat{c}+\frac{1-\hat{c}}{1+e^{-\hat{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\theta_P + \left[1-\left(\hat{c}+\frac{1-\hat{c}}{1+e^{-\tilde{a}\left(FI_{P,med}-\hat{b}\right)}}\right)\right]\theta_{P,med} \quad .\end{aligned} \tag{7}$$

Note that if the people and the government have sufficiently strong fluid intelligences, an independent central bank may not be necessary. However, p_{HE} will not be unity even in a country whose people have the highest p_{HE} in the world. Therefore, it is possible for $\theta_G > \theta_P$ in some period in any country; thus, an independent central bank is still important for all countries.

4.3 Generating an expected θ_G by using heuristics

4.3.1 Difficulty in expecting θ_G

Specifying the functional form of the structural model of θ_G is only half of the problem of generating an expected θ_G. Although we have the functional form of the model, as shown in Equation (7), we still cannot generate an expected θ_G unless we specify appropriate values of the parameters $\hat{a}$, $\hat{b}$ and $\hat{c}$. Furthermore, to generate the expected θ_G, we must also know the expected values of θ_P, $\theta_{P,med}$, and $FI_{P,med}$.

We may roughly specify the parameter values of $\hat{a}$, $\hat{b}$ and $\hat{c}$ through the results of some type of social experiment, or we may use the estimates derived from other kinds of model concerning fluid intelligence. By substituting these values for the parameter values in the structural model of θ_G, the model could be calibrated. However, expectations based on these estimates will most likely be rather inaccurate and therefore problematic in terms of decision-making on future actions.

A far more serious problem is obtaining the expected future values of θ_P, $\theta_{P,med}$, and $FI_{P,med}$. It is not certain whether the values of θ_P and $\theta_{P,med}$ are constant across time; in fact, many researchers have posited that it is much more likely that they are temporally variable (e.g., Uzawa, 1968; Epstein and Hynes, 1983; Lucas and Stokey, 1984; Parkin, 1988; Obstfeld, 1990; Becker and Mulligan, 1997). Therefore, there is no guarantee that the future values of θ_P and $\theta_{P,med}$ will equal past ones, so the past values cannot be used as substitutes for the expected future values of θ_P and $\theta_{P,med}$. Hence, to generate the expected future values of θ_P and $\theta_{P,med}$, we have to calculate them on the basis of structural models of θ_P and $\theta_{P,med}$. Even if we knew the functional forms of these structural models, we would still need to determine the parameter values for the models. To determine them, however, we would need to obtain a sufficiently large amount of data on the past values of θ_P and $\theta_{P,med}$—that is, the intrinsic RTPs of the representative household and the median voter. Although a household knows its own RTP, it cannot directly observe θ_P and $\theta_{P,med}$ in the same way that it can obtain data on aggregate consumption, investment, production, inflation, trade, and other indicators at relatively low cost. Without data on the past values of θ_P and $\theta_{P,med}$, we cannot estimate the parameter values. Therefore, we cannot generate the expected future values of θ_P and $\theta_{P,med}$ on the basis of their structural models.

Past data on the real interest rate may be used as a substitute for past θ_P because θ_P is basically equal to the real interest rate at steady state (Fisher, 1930). However, during a transition period after θ_P changes, θ_P is not equal to the rate of real interest. Therefore, unless θ_P is constant across time, this substitution does not seem to be sufficiently useful. In addition, if $\theta_{P,med}$ is constant across time, we may approximate the value of $\theta_{P,med}$ on the basis of historical economic and political (election) data. However, as stated in the previous paragraph, it is not known whether θ_P and $\theta_{P,med}$ are constant across time.

Note that, if we assume that RTP is identical for all households, an expected θ_P and $\theta_{P,med}$ are no longer necessary because the RTP of any household is equal to both θ_P and $\theta_{P,med}$. This assumption is very problematic, however, because it is not merely expedient for the sake of simplicity. It is also a critical requirement to eliminate the need for generating an expected θ_P and $\theta_{P,med}$. Therefore, any rationale for assuming identical RTPs should be validated; that is, it should be demonstrated that identical RTPs do exist and are universally observed. In any case, RTP is unquestionably not identical among households. Therefore, households must generate the expected values of θ_P and $\theta_{P,med}$.

4.3.2 Expectations based on beliefs

Faced with the difficulty of generating expected values of θ_P and $\theta_{P,med}$ and knowing the parameter values in the model of θ_G, households may have to use the concept of bounded rationality to make decisions. One of a few alternatives available for a household to use is its "beliefs" in θ_P and $\theta_{P,med}$ as well as in $\hat{a}$, $\hat{b}$, $\hat{c}$, and $FI_{P,med}$. The use of beliefs does not mean that households deviate from rationality; rather, it is the most rational option in an environment where insufficient information is available.

Belief is merely that, however—belief. There is no guarantee that the value a household believes to be true is actually the correct value. Therefore, it may often change, but it will be changed only if forward-looking information becomes available. In some cases, a household will change its belief when new data are obtained, but in other cases the household will not, depending on how it interprets the new information. This is particularly true when the household believes that it has extracted forward-looking information about θ_P and $\theta_{P,med}$ from the newly obtained data.

4.3.3 Heuristics

When households interpret the information extracted from new data, they may use heuristic methods such as a simplified linear reduced form model of θ_G. Studies of the use of heuristics and bounded rationality in this context would be useful for better understanding the interpretation mechanism. Heuristic methods will be implemented through the use of fluid intelligence. Hence, the value of $FI_{P,med}$ will also be important in improving the accuracy of expectations generated on the basis of heuristics.

There may be many possible simplified linear reduced form models of θ_G that could be used as heuristic methods, although most of them may be *ad hoc*. Even though such reduced form models are far less credible than a structural model, they may be utilized as a heuristic method of interpretation. Although simplified linear reduced form models may often result in misleading conclusions, they may sometimes provide useful information.

4.4 Independent central bank

4.4.1 Generating expected $\theta_G - \theta_P$ through the actions of a central bank

A heuristic way of generating an expected θ_G is to use information about $\theta_G - \theta_P$. The model of inflation acceleration presented in Section 2 indicates that inflation acceleration and deceleration are governed by the value of $\theta_G - \theta_P$. Therefore, what people really need to know is not the expected θ_G but the expected $\theta_G - \theta_P$. If the central bank is sufficiently independent, θ_G is determined by the central bank. In this case, people do not need to know the RTP of the government, but rather the responses of the central bank to $\theta_G - \theta_P$. If an easy method exists to know the response of the central bank to $\theta_G - \theta_P$, households will not have to generate expected θ_G; they need only observe the decisions of the central bank.

Of course, people cannot directly observe the value of $\theta_G - \theta_P$, but they can observe the response of the central bank to $\theta_G - \theta_P$. An independent central bank will raise interest rates if it judges that $\theta_G - \theta_P > 0$. Households can then adjust their expectations accordingly.

4.4.2 Guaranteed $\theta_G = \theta_P$

If the central bank is sufficiently independent and capable, and successfully controls θ_G, then it is not even necessary for households to generate an expected value for $\theta_G - \theta_P$ because, in this case, θ_G will also equal θ_P. As discussed in Section 3.6, if the central bank is sufficiently independent and capable, then $\hat{c} = 1$ in Equation (4) and thereby, by Equations (6) and (7), $\theta_G = \theta_P$. The central bank ensures that rationality always prevails over preference with regard to the RTP of government. If the independence of the central bank is very credible, households will always expect that $\theta_G = \theta_P$ at all times in the future.

5 EXPLANATIONS FOR VARIOUS TYPES OF INFLATION BASED ON THE MODEL

5.1 Hyperinflation

5.1.1 Hyperinflation in chaotic times

The political element discussed in Section 4.1.2 will play an important role in chaotic political and economic times. In these situations, people may judge that the probability of collapse of the incumbent political system (p_{inst}) is currently very high. If they do so, the political element term $p_{inst}\theta_{G,pol}$ in Equation (6) will substantially increase. In addition, the central bank will become largely powerless in these unstable periods, and $p_{HE,G}$ will be left at less than unity. Equation (6) indicates that θ_G is far larger than θ_P in these cases. As a result, according to the law of motion for inflation shown in Section 2, hyperinflation will be generated. The hyperinflation scenarios in Germany after its defeat in WWI, in Japan after its defeat in WWII, and in Russia after the collapse of Soviet Union are typical examples. Nevertheless, if some external or internal forces can successfully establish political stability immediately after these types of major political upheaval, hyperinflation may be deterred.

5.1.2 Hyperinflation in periods of peace

The preference element can also generate hyperinflation. In other words, hyperinflation can be generated even in periods of peace. Equations (6) and (7) indicate that a small value of $p_{HE,G}$ indicates a high probability of inflation acceleration. A small value of $p_{HE,G}$ indicates weak fluid intelligence, and in this case preference prevails over rationality more often and for longer periods of time. Hence, inflation accelerates gradually and persistently because relatively small but positive values of $\theta_G - \theta_P$ are successively generated over a long period. Initially, the acceleration of inflation is slower than in the case of hyperinflation caused by the political element, but according to the law of motion for inflation shown in Section 2, this type of inflation also eventually grows to hyperinflation.

The hyperinflation caused by a relatively small value of $p_{HE,G}$ requires the additional condition that the central bank is not sufficiently independent. If the central bank is sufficiently independent and capable, $\hat{c}$ in Equation (3) is unity, and thereby rationality always prevails over preference and hyperinflation is not generated.

5.2 Modest or creeping inflation

If the value of $p_{HE,G}$ is not low but also not sufficiently high, a relatively high inflation rate but not hyperinflation may be generated. In this case, a small positive $\theta_G - \theta_P$ is periodically generated for a short period, and moderate inflation will be observed. Note that this kind of inflation also requires the condition that the central bank is not sufficiently independent, because if it is sufficiently independent, inflation does not accelerate.

The Great Inflation observed in developed countries in 1960s and 1970s is a typical example of modest or creeping inflation. $p_{HE,P}$ (= $p_{HE,G}$) in developed countries during this period may have been sufficiently high, but a Great Inflation was nonetheless generated. This appears to be a puzzle, and there are two possible solutions. The first possibility is that $p_{HE,P}$ (= $p_{HE,G}$) in those countries was not sufficiently high enough, and in addition, central banks were not sufficiently independent during those periods. If $p_{HE,P}$ (= $p_{HE,G}$) is not sufficiently high, a small positive $\theta_G - \theta_P$ can be periodically generated. If this small $\theta_G - \theta_P$ is not prevented by the central bank, inflation will gradually accelerate.

The second possibility is that $p_{HE,P}$ (= $p_{HE,G}$) in the developed countries was sufficiently high, but people in those countries were using the wrong models to generate expectations of the future economy. That is, expectations may have been systematically biased for some reason. An important difference between this period and later periods is that Keynesian thought was dominant in this period, but not in others. In the 1960s and 1970s, Keynesian thought was accepted by most economists, government officials, and the general public. Part of the Keynesian point of view was that inflation is a necessary evil to maintain a low unemployment rate. Many economists at the time took this as an authoritative theorem. Because rationality means that optimal decisions are made from the choices calculated consistently with the model used, and because most people believed in Keynesian thought, it would have been rational for most people to use models that were consistent with Keynesian thought when generating expectations of the future economy.

According to Keynesian theory, the government's behavior in maintaining low unemployment rates indicates that higher inflation will be allowed by the government; thus, θ_G that is higher than θ_P must be optimal. People who believe in the Keynesian way of thinking therefore should expect high values of θ_G and positive values of $\theta_G - \theta_P$. As a result, even if $p_{HE,P}$ (= $p_{HE,G}$) is sufficiently high, a positive $\theta_G - \theta_P$ can periodically be expected. In addition, if the central bank is not independent, the expectation of a positive $\theta_G - \theta_P$ cannot be prevented, and thereby a modest or creeping inflation can be generated. Even if the central bank is sufficiently independent, this kind of inflation may be generated if the central bank itself also follows the Keynesian line of thought, because it will not prevent the generation of expected positive values of $\theta_G - \theta_P$.

At the present time, the conjecture that inflation is a necessary evil to maintain a low unemployment rate is much less common than in the 1960s and 1970s. Therefore, most people will not use models that are constructed on the basis of this conjecture. It seems unlikely that a phenomenon such as the Great Inflation will occur again in developed countries.

5.3 Disinflation

After a period of high inflation, if a negative $\theta_G - \theta_P$ is maintained, inflation will decline. In the 1980s, after a prolonged period of inflation, inflation gradually declined in many developed countries. This is often called the "Volcker" disinflation (named after the twelfth Chairman of the U.S. Federal Reserve, Paul Volcker). It is highly likely that the central banks during this period forced governments to lower their θ_G to ensure that $\theta_G - \theta_P$ was negative until inflation stabilized. There are two possible reasons for this successful disinflation, depending on the two possible sources of the Great Inflation discussed above: Central banks may have become sufficiently independent of their governments in the 1980s, or Keynesian ideology was largely cast away. Most likely, both contributed to the disinflation.

6 CONCLUDING REMARKS

The RTP of government has been largely neglected in the study of economics. However, it is an important factor in inflation acceleration. In this paper, I examined the mechanisms by which the RTP of government is formed and presented a model of the government's RTP. The model explicitly incorporates the mechanism of how a government deals with the contradiction between preference and rationality.

The model indicates that the RTP of government is determined by the expected RTPs of the median voter and the representative household, as well as by the strength of the government's fluid intelligence. It is, however, difficult in practice to use the model to generate the expected government RTP because it is difficult to generate the expected RTPs of the median voter and the representative household. Therefore, households most likely use "beliefs" or heuristics to generate the expected RTP of government. If the central bank is sufficiently independent and capable, households will always expect that $\theta_G = \theta_P$.

APPENDIX

A1 The law of motion for inflation

A1.1 The government

A.1.1.1 The government budget constraint

The government budget constraint is

$$\dot{B}_t = B_t i_t + G_t - X_t - \vartheta_t \quad ,$$

where B_t is the nominal obligation of the government to pay for its accumulated bonds, i_t is the nominal interest rate for government bonds, G_t is the nominal government expenditure, X_t is the nominal tax revenue, and ϑ_t is the nominal amount of seigniorage at time t. The tax is assumed to be lump sum, the government bonds are long term, and the returns on the bonds are realized only after the bonds are held during a unit period (e.g., a year). The government bonds are redeemed in a unit period, and the government successively refinances the bonds by issuing new ones at each time t. Let $b_t = \frac{B_t}{P_t}$, $g_t = \frac{G_t}{P_t}$, $x_t = \frac{X_t}{P_t}$, and $\varphi_t = \frac{\vartheta_t}{P_t}$, where P_t is the price level at time t. Let also $\pi_t = \frac{\dot{P}_t}{P_t}$ be the inflation rate at time t. By dividing by P_t, the budget constraint is transformed to $\frac{\dot{B}_t}{P_t} = b_t i_t + g_t - x_t - \varphi_t$, which is equivalent to

$$\dot{b}_t = b_t i_t + g_t - x_t - \varphi_t - b_t \pi_t = b_t (i_t - \pi_t) + g_t - x_t - \varphi_t \quad . \tag{A1}$$

Because the returns on government bonds are realized only after holding the bonds during a unit period, investors buy the bonds if $\bar{i}_t \geq E_t \int_t^{t+1} (\pi_s + r_s) ds$ at time t, where $\bar{i}_t$ is the nominal interest rate for bonds bought at t and r_t is the real interest rate in markets at t. Hence, by arbitrage, $\bar{i}_t = E_t \int_t^{t+1} (\pi_s + r_s) ds$ and if r_t is constant such that $r_t = r$ (i.e., if it is at steady state), then

$$\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r \quad .$$

The nominal interest rate $\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r$ means that, during a sufficiently small period between t and $t + dt$, the government's obligation to pay for the bonds' return in the future increases not by $dt(\pi_t + r)$ but by $dt\left(E_t \int_t^{t+1} \pi_s \, ds + r\right)$. If π_t is constant, then $E_t \int_t^{t+1} \pi_s \, ds = \pi_t$ and $\bar{i}_t = \pi_t + r$, but if π_t is not constant, these equations do not necessarily hold.

Since bonds are redeemed in a unit period and successively refinanced, the bonds the government is holding at t have been issued between t - 1 and t. Hence, under perfect foresight, the average nominal interest rate for all government bonds at time t is the weighted sum of $\bar{i}_t$ such that

$$i_t = \int_{t-1}^{t} \bar{i}_s \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r \quad ,$$

where $\overline{B}_{s,t}$ is the nominal value of bonds at time t that were issued at time s. If the weights $\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ between t - 1 and t are not so different from each other, then approximately $i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r$. To be precise, if the absolute values of π_s for $t-1 < s \leq t+1$ are sufficiently smaller than unity, the differences among the weights are negligible and then approximately

$$i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r \tag{A2}$$

(see Harashima, 2008). The average nominal interest rate for the total government bonds, therefore, develops by $i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r$. If π_t is constant, then $\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ $= \pi_t$; thus, $i_t = \pi_t + r$. If π_t is not constant, however, the equations $\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t$ and $i_t = \pi_t + r$ do not necessarily hold.

A1.1.2 An economically Leviathan government

Under a proportional representation system, the government represents the median household whereas the representative household from an economic perspective represents the mean household.[1] Because of this difference, they usually have different preferences. To account for this essential difference, a Leviathan government is assumed in the model.[2] There are two extremely different views regarding government's behavior in the literature on political economy: the Leviathan view and the benevolent view (e.g., Downs 1957; Brennan and Buchanan 1980; Alesina and Cukierman 1990). From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household, but a Leviathan government does not. Whereas the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government's own policy objectives.[3] For example, if a Leviathan government considers national security to be the most important political issue, defense spending will increase greatly, but if improving social welfare is the top political priority, spending on social welfare will increase dramatically, even though the increased expenditures may not necessarily increase the economic utility of the representative household.

Is it possible, however, for such a Leviathan government to hold office for a long period? Yes, because a government is generally chosen by the median of households under a

[1] See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Alesina and Drazen 1991; Cukierman et al. 1992).
[2] The most prominent reference to Leviathan governments is Brennan and Buchanan (1980).
[3] The government behavior assumed in the fiscal theory of the price level reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.

proportional representation system (e.g., Downs 1957), whereas the representative household usually presumed in the economics literature is the mean household. The economically representative household is not usually identical to the politically representative household, and a majority of people could support a Leviathan government even if they know that the government does not necessarily pursue only the economic objectives of the economically representative household. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people, whereas the conventional economically benevolent government maximizes the economic utility of people. In addition, because the politically and economically representative households are different (the median and mean households, respectively), the preferences of future governments will also be similarly different from those of the mean representative household. In this sense, the current and future governments presented in the model can be seen as a combined government that goes on indefinitely; that is, the economically Leviathan government always represents the median representative household.

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related activities in the government's political utility function (e.g., Edwards and Keen 1996). Because an economically Leviathan government derives political utility from expenditure for its political purposes, the larger the expenditure is, the happier the Leviathan government will be. But raising tax rates will provoke people's antipathy, which increases the probability of being replaced by the opposing party that also nearly represents the median household. Thus, the economically Leviathan government regards taxes as necessary costs to obtain freedom of expenditure for its own purposes. The government therefore will derive utility from expenditure and disutility from taxes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the representative household. Consumption and labor hours are both control variables, and as such, the government's expenditure and tax revenue are also control variables. As a whole, the political utility function of economically Leviathan government can be expressed as $u_G(g_t, x_t)$.[4] In addition, it can be assumed on the basis of previously mentioned arguments that $\frac{\partial u_G}{\partial g_t} > 0$ and $\frac{\partial^2 u_G}{\partial g_t^{\ 2}} < 0$, and therefore that $\frac{\partial u_G}{\partial x_t} < 0$ and $\frac{\partial^2 u_G}{\partial x_t^{\ 2}} > 0$.[5] An economically Leviathan government therefore maximizes the expected sum of these utilities discounted by its time preference rate under the constraint of deficit financing.

A1.1.3 The optimization problem

The optimization problem of an economically Leviathan government is

$$Max\ E\int_0^\infty u_G(g_t, x_t)\exp(-\theta_G t)dt$$

[4] It is possible to assume that governments are partially benevolent. In this case, the utility function of a government can be assumed to be $u_G(g_t, x_t, c_t, l_t)$, where c_t is real consumption and l_t is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government's policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be $u_G(g_t, x_t)$.

[5] Some may argue that it is more likely that $\frac{\partial u_G}{\partial x_t} > 0$ and $\frac{\partial^2 u_G}{\partial x_t^{\ 2}} < 0$. However, the assumption used is not an important issue here because $-x_t\left[\frac{\partial u_G(g_t, x_t)}{\partial x_t}\right]^{-1}\frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^{\ 2}}\frac{\dot{x}_t}{x_t} = 0$ at steady state, as will be shown in the solution to the optimization problem later in the paper. Thus, the results are not affected by which assumption is used.

subject to the budget constraint

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \varphi_t \quad , \tag{A3}$$

where u_G is the constant relative risk aversion utility function of the government, θ_G is the government's rate of time preference, and E is the expectation operator. All variables are expressed in per capita terms, and population is assumed to be constant. The government maximizes its expected political utility considering the behavior of the economically representative household that is reflected in i_t in its budget constraint.

A1.2 Households

The economically representative household maximizes its expected economic utility. Sidrauski (1967)'s well-known money in the utility function model is used for the optimization problem. The representative household maximizes its expected utility

$$E\int_0^\infty u_P(c_t, m_t)\exp(-\theta_P\, t)dt$$

subject to the budget constraint

$$\dot{a}_t = (r_t\, a_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t)m_t] - g_t \quad ,$$

where u_P and θ_P are the utility function and the time preference rate of the representative household, c_t is real consumption, w_t is real wage, σ_t is lump-sum real government transfers, m_t is real money, $a_t = k_t + m_t$, and k_t is real capital. It is assumed that $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$, $u_P' > 0$, $u_P'' < 0$, $\frac{\partial u_P(c_t, m_t)}{\partial m_t} > 0$, and $\frac{\partial^2 u_P(c_t, m_t)}{\partial m_t^2} < 0$, where $f(\cdot)$ is the production function. Government expenditure (g_t) is an exogenous variable for the representative household because it is an economically Leviathan government. It is also assumed that, although all households receive transfers from a government in equilibrium, when making decisions, each household takes the amount it receives as given, independent of its money holdings. Thus, the budget constraint means that the real output $f(k_t)$ at any time is demanded for the real consumption c_t, the real investment $\dot{k}_t$, and the real government expenditure g_t such that $f(k_t) = c_t + \dot{k}_t + g_t$. The representative household maximizes its expected economic utility considering the behavior of government reflected in g_t in the budget constraint. In this discussion, a central bank is not assumed to be independent of the government; thus, the functions of the government and the central bank are not separated. This assumption can be relaxed, and the roles of the government and the central bank are explicitly separated in Section A2.

Note that the time preference rate of government (θ_G) is not necessarily identical to that of the representative household (θ_P) because the government and the representative household represent different households (i.e., the median and mean households, respectively). In addition, the preferences will differ because (1) even though people want to choose a government that has the same time preference rate as the representative household, the rates may differ owing to errors in expectations (e.g., Alesina and Cukierman 1990); and (2) current voters cannot bind the choices of future voters and, if current voters are aware of this possibility, they may vote more myopically as compared with their own rates of impatience in private economic activities (e.g., Tabellini and Alesina 1990). Hence, it is highly likely that the time preference rates of a government and the representative household are heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, an economically Leviathan government behaves based only on its own time preference rate, without hesitation.

A1.3 The simultaneous optimization

First, I examine the optimization problem of the representative household. Let Hamiltonian H_P be $H_P = u_P(c_t, m_t)\exp(-\theta_P t) + \lambda_{P,t}[r_t a_t + w_t + \sigma_t - c_t - (\pi_t + r_t)m_t - g_t]$, where $\lambda_{P,t}$ is a costate variable, c_t and m_t are control variables, and a_t is a state variable. The optimality conditions for the representative household are;

$$\frac{\partial u_P(c_t, m_t)}{\partial c_t}\exp(-\theta_P t) = \lambda_{P,t} \quad , \tag{A4}$$

$$\frac{\partial u_P(c_t, m_t)}{\partial m_t}\exp(-\theta_P t) = \lambda_{P,t}(\pi_t + r_t) \quad , \tag{A5}$$

$$\dot{\lambda}_{P,t} = -\lambda_{P,t}\, r_t \quad , \tag{A6}$$

$$\dot{a}_t = (ra_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t)m_t - g_t] \quad , \tag{A7}$$

$$\lim_{t \to \infty} \lambda_{P,t}\, a_t = 0 \quad . \tag{A8}$$

By conditions (A4) and (A5), $\left[\frac{\partial u_P(c_t, m_t)}{\partial c_t}\right]^{-1} \frac{\partial u_P(c_t, m_t)}{\partial m_t} = \pi_t + r_t$, and by conditions (A4) and (A6),

$$-c_t\left[\frac{\partial u_P(c_t, m_t)}{\partial c_t}\right]^{-1} \frac{\partial^2 u_P(c_t, m_t)}{\partial c_t^2}\frac{\dot{c}_t}{c_t} + \theta_P = r_t \quad . \tag{A9}$$

Hence,

$$\theta_P = r_t = r \tag{A10}$$

at steady state such that $\dot{c}_t = 0$ and $\dot{k}_t = 0$.

Next, I examine the optimization problem of the economically Leviathan government. Let Hamiltonian H_G be $H_G = u_G(g_t, x_t)\exp(-\theta_G t) + \lambda_{G,t}[b_t(i_t - \pi_t) + g_t - x_t - \varphi_t]$, where $\lambda_{G,t}$ is a costate variable. The optimality conditions for the government are;

$$\frac{\partial u_G(g_t, x_t)}{\partial g_t}\exp(-\theta_G t) = -\lambda_{G,t} \quad , \tag{A11}$$

$$\frac{\partial u_G(g_t, x_t)}{\partial x_t}\exp(-\theta_G t) = \lambda_{G,t} \quad , \tag{A12}$$

$$\dot{\lambda}_{G,t} = -\lambda_{G,t}(i_t - \pi_t) \ , \tag{A13}$$

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \varphi_t \ , \tag{A14}$$

$$\lim_{t\to\infty} \lambda_{G,t}\, b_t = 0 \ . \tag{A15}$$

Combining conditions (A11), (A12), and (A13) and Equation (A2) yields the following equations:

$$-g_t\left[\frac{\partial u_G(g_t,x_t)}{\partial g_t}\right]^{-1}\frac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}\frac{\dot{g}_t}{g_t} + \theta_G = i_t - \pi_t = r_t + \int_{t-1}^{t}\int_{s}^{s+1}\pi_\upsilon\, d\upsilon\ ds - \pi_t \tag{A16}$$

and

$$-\ x_t\left[\frac{\partial u_G(g_t,x_t)}{\partial x_t}\right]^{-1}\frac{\partial^2 u_G(g_t,x_t)}{\partial x_t^{\ 2}}\frac{\dot{x}_t}{x_t} + \theta_G = i_t - \pi_t = r_t + \int_{t-1}^{t}\int_{s}^{s+1}\pi_\upsilon\, d\upsilon\ ds - \pi_t \ . \tag{A17}$$

Here, $g_t\left[\frac{\partial u_G(g_t,x_t)}{\partial g_t}\right]^{-1}\frac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}\frac{\dot{g}_t}{g_t} = 0$ and $x_t\left[\frac{\partial u_G(g_t,x_t)}{\partial x_t}\right]^{-1}\frac{\partial^2 u_G(g_t,x_t)}{\partial x_t^{\ 2}}\frac{\dot{x}_t}{x_t} = 0$ at steady state such that $\dot{g}_t = 0$ and $\dot{x}_t = 0$; thus,

$$\theta_G = r_t + \int_{t-1}^{t}\int_{s}^{s+1}\pi_\upsilon\, d\upsilon\, ds - \pi_t \ . \tag{A18}$$

Hence, by Equation (A10),

$$\int_{t-1}^{t}\int_{s}^{s+1}\pi_\upsilon\, d\upsilon\, ds = \pi_t + \theta_G - \theta_P \tag{A19}$$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$.[6]

Equation (A19) is a natural consequence of simultaneous optimization by the economically Leviathan government and the representative household. If the rates of time preference are heterogeneous between them, then

$$i_t - r = \int_{t-1}^{t}\int_{s}^{s+1}\pi_\upsilon\, d\upsilon\, ds \neq \pi_t \ .$$

This result might seem surprising because it has been naturally conjectured that $i_t = \pi_t + r$. However, this is a simple misunderstanding because π_t indicates the instantaneous rate of

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6 If and only if $\theta_G = -\frac{g_t - x_t - \varphi_t}{b_t}$ at steady state, then the transversality condition (A15) $\lim_{t\to\infty}\lambda_{G,t}\, b_t = 0$ holds. The proof is shown in Harashima (2008).

inflation at a point such that $\pi_t = \frac{\dot{P}_t}{P_t}$, whereas $\int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ roughly indicates the average inflation rate in a period. Equation (A19) indicates that π_t develops according to the integral equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds - \theta_G + \theta_P$. If π_t is constant, the equations $i_t = \pi_t + r$ and $\int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t$ are true. However, if π_t is not constant, the equations do not necessarily hold. Equation (A19) indicates that the equations $i_t = \pi_t + r$ and $\int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t$ hold only in the case where $\theta_G = \theta_P$ (i.e., a homogeneous rate of time preference). It has been previously thought that a homogeneous rate of time preference naturally prevails; thus, the equation $i_t = \pi_t + r$ has not been questioned. As argued previously, however, a homogeneous rate of time preference is not usually guaranteed.

A1.4 The law of motion for trend inflation

Equation (A19) indicates that inflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in heterogeneous rates of time preference. If π_t is constant, the equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ holds; conversely, if $\pi_t \neq \int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$, then π_t is not constant. Without the acceleration or deceleration of inflation, therefore, Equation (A19) cannot hold in an economy in which $\theta_G \neq \theta_P$. In other words, it is not until $\theta_G \neq \theta_P$ that inflation can accelerate or decelerate. Heterogeneous time preferences ($\theta_G \neq \theta_P$) bend the path of inflation and enables inflation to accelerate or decelerate. The difference of time preference rates ($\theta_G - \theta_P$) at each time needs to be transformed to the accelerated or decelerated inflation rate π_t at each time.

Equation (A19) implies that inflation accelerates or decelerates nonlinearly in the case in which $\theta_G \neq \theta_P$. For a sufficiently small period dt, π_{t+1+dt} is determined with π_s $(t-1 < s \leq t+1)$ that satisfies $\int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds - \pi_t = \theta_G - \theta_P$, so as to hold the equation $\int_{t}^{t+dt}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \int_{t-1}^{t-1+dt}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + \pi_{t+dt} - \pi_t$. A solution of the integral equation (A19) for given θ_G and θ_P is

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)t^2 \quad . \tag{A20}$$

Generally, the path of inflation that satisfies Equation (A19) for $0 \leq t$ is expressed as

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)\exp[z_t \ln(t)] \quad ,$$

where z_t is a time dependent variable. The stream of z_t is various depending on the boundary condition, i.e., the past and present inflation during $-1 < t \leq 0$ and the path of inflation during $0 < t \leq 1$ that is set to make π_0 satisfy Equation (A19). However, z_t has the following important property. If π_t satisfies Equation (A19) for $0 \leq t$, and $-\infty < \pi_t < \infty$ for $-1 < t \leq 1$, then

$$\lim_{t \to \infty} z_t = 2 \quad .$$

Proof is shown in Harashima (2008). Any inflation path that satisfies Equation (A19) for $0 \leq t$ therefore asymptotically approaches the path of Equation (A20). The mechanism behind the law of motion for inflation (Equation [A20]) is examined more in detail in Harashima (2008).

A2 The central bank

A central bank manipulates the nominal interest rate according to the following Taylor-type instrument rule in the model;

$$i_t = \bar{\gamma} + \gamma_\pi \left(\pi_t - \pi^*\right) + \gamma_x x_t \quad , \tag{A21}$$

where π^* is the target rate of inflation and $\bar{\gamma}$, γ_π, and γ_x are constant coefficients. $\bar{\gamma} = \pi^* + r$ as is usually assumed.

In Section A1, central banks are not explicitly considered because they are not assumed to be independent of governments. However, in actuality, central banks are independent organizations in most countries even though some of them are not sufficiently independent. Furthermore, in the conventional inflation model, it is the central banks that control inflation and governments have no role in controlling inflation. Conventional inflation models show that the rate of inflation basically converges at the target rate of inflation set by a central bank. The target rate of inflation therefore is the key exogenous variable that determines the path of inflation in these models.

Both the government and the central bank can probably affect the development of inflation, but they would do so in different manners, as Equation (A20) and conventional inflation models indicate. However, the objectives of the government and the central bank may not be the same. For example, if trend inflation is added to conventional models by replacing their aggregate supply equations with Equation (A20), inflation cannot necessarily converge at the target rate of inflation because another key exogenous variable (θ_G) is included in the models. A government makes inflation develop consistently with the Equation (A20), which implies that inflation will not necessarily converge at the target rate of inflation. Conversely, a central bank makes inflation converge at the target rate of inflation, which implies that inflation will not necessarily develop consistently with Equation (A20). That is, unless either θ_G is adjusted to be consistent with the target rate of inflation or the target rate of inflation is adjusted to be consistent with θ_G, the path of inflation cannot necessarily be determined. Either θ_G or the target rate of inflation need be an endogenous variable. If a central bank dominates, the target rate of inflation remains as the key exogenous variable and θ_G should then be an endogenous variable. The reverse is also true.

A central bank will be regarded as truly independent if θ_G is forced to be adjusted to the one that is consistent with the target rate of inflation set by the central bank. For example, suppose that $\theta_G > \theta_P$ and a truly independent central bank manipulates the nominal interest rate according to the Taylor-type instrument rule (Equation [21]). Here,

$$i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r = \theta_G + \pi_t \tag{A22}$$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$ by Equations (A2), (A10), and (A19). If the accelerating inflation rate is higher than the target rate of inflation, the central bank can raise the nominal interest rate from $i_t = \theta_G + \pi_t$ (Equation [A22]) to

$$i_t = \theta_G + \pi_t + \psi$$

by positive ψ by intervening in financial markets to lower the accelerating rate of inflation. In this case, the central bank keeps the initial target rate of inflation because it is truly independent. The government thus faces a rate of increase of real obligation that is higher than θ_G by the extra rate ψ.[7] If the government lowers θ_G so that $\theta_G < \theta_P$ and inflation stops accelerating, the central bank will accordingly reduce the extra rate ψ. If, however, the government does not accommodate θ_G to the target rate of inflation, the extra rate ψ will increase as time passes because of the gap between the accelerating inflation rate and the target rate of inflation widens by Equation (A20) and γ_x in Taylor-type instrument rules is usually larger than unity, say 1.5. Because of the extra rate ψ, the government has no other way to achieve optimization unless it lowers θ_G to one that is consistent with the target rate of inflation. Once the government recognizes that the central bank is firmly determined to be independent and it is in vain to try to intervene in the central bank's decision makings, the government would not dare to attempt to raise θ_G again anymore.

Equation (A20) implies that a government allows inflation to accelerate because it acts to maximize its expected utility based only on its own preferences. A government is hardly the only entity that cannot easily control its own preferences even when these preferences may result in unfavorable consequences. It may not even be possible to manipulate one's own preferences at will. Thus, even though a government is fully rational and is not weak, foolish, or untruthful, it is difficult for it to self-regulate its preferences. Hence, an independent neutral organization is needed to help control θ_G. Delegating the authority to set and keep the target rate of inflation to an independent central bank is a way to control θ_G. The delegated independent central bank will control θ_G because it is not the central bank's preference to stabilize the price level—it is simply a duty delegated to it. An independent central bank is not the only possible choice. For example, pegging the local currency with a foreign currency can be seen as a kind of delegation to an independent neutral organization. In addition, the gold standard that prevailed before World War II can be also seen as a type of such delegation.

Note also that the delegation may not be viewed as bad from the Leviathan government's point of view because only its rate of time preference is changed, and the government can still pursue its political objectives. One criticism of the argument that central banks should be independent (e.g., Blinder 1998) is that, since the time-inconsistency problem argued in Kydland and Prescott (1977) or Barro and Gordon (1983) is more acute with fiscal policy, why is it not also necessary to delegate fiscal policies? An economically Leviathan government, however, will never allow fiscal policies to be delegated to an independent neutral organization because the Leviathan government would then not be able to pursue its political objectives, which in a sense would mean the death of the Leviathan government. The median household that backs the Leviathan government, but at the same time dislikes high inflation, will therefore support the delegation of authority but only if it concerns monetary policy. The independent central bank will then be given the authority to control θ_G and oblige the government to change θ_G in order to meet the target rate of inflation.

Without such a delegation of authority, it is likely that generally $\theta_G > \theta_P$ because θ_G represents the median household whereas θ_P represents the mean household. Empirical studies indicate that the rate of time preference negatively correlates with permanent income (e.g., Lawrance 1991), and the permanent income of the median household is usually lower than that of the mean household. If generally $\theta_G > \theta_P$, that suggests that inflation will tend to accelerate unless a central bank is independent. The independence of the central bank is therefore very important in keeping the path of inflation stable.

Note also that the forced adjustments of θ_G by an independent central bank are exogenous

[7] The extra rate ψ affects not only the behavior of government but also that of the representative household, in which the conventional inflation theory is particularly interested. In this sense, the central bank's instrument rule that concerns and simultaneously affects both behaviors of the government and the representative household is particularly important for price stability.

shocks to both the government and the representative household because they are planned solely by the central bank. When a shock on θ_G is given, the government and the representative household must recalculate their optimal paths including the path of inflation by resetting θ_G, π_t, and φ.

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