Monopolistic Provision of Excludable Public Goods under Private Information

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This paper characterizes the optimal contract designed by a profit-maximizing monopolist, who can provide an indivisible and excludable public good to a group of n potential consumers, whose valuations are private information. The analysis takes distribution costs and congestion effects into account. The second-best allocation rule, which is welfare-maximizing under the constraint of non-negative profits, is characterized. Properties of the optimal mechanism in the case of many potential consumers are analyzed and it is shown that in this case the monopolist can use simple posted-price contracts. Finally, implications for public intervention are discussed.

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I. Introduction

Several years ago Burns and Walsh (1981) observed a growing interest in the private provision of commodities known as price-excludable public goods or joint goods. These commodities possess the characteristic of non-rivalness in consumption, "so that all consumers simultaneously may consume the total production", but "price exclusion is possible, and in the limit costless, just as for conventional private goods" (Brennan and Walsh (1985)).

Examples of such goods are abundant. Some of the earliest papers dealt with the private provision of subscription TV[1]. Other examples of varying purity include computer software; recreational facilities such as parks, theatres and museums; trips by planes, trains and buses; or at a more general level R&D and the creation of information.

It is interesting to note that Samuelson (1958) repeatedly emphasized that such goods are quite different from "true-blue private goods". According to him the possibility of scrambling a TV signal does not enable us to convert a public good into a private good, because the marginal costs of having one extra person tune in on the program are still zero. Samuelson (1964) pointed out that even if exclusion is possible "we should still be faced with an instance of intrinsic increasing returns and that in all such cases there is an element of the public good dilemma".[2]

Modern formalizations of the public good dilemma for the case of pure public goods include Güth and Hellwig (1986), Rob (1989) and Mailath and Postlewaite (1990). We investigate whether their strong inefficiency results also prevail in an excludable public goods framework and clarify the relations to models of monopolistic supply of private goods (cf. Bulow and Roberts (1989), Cairns (1993) and Cornelli (1996)). By doing so, this note ties up a few loose ends and thus attempts to fill some small gaps in that literature.

Our analysis is related to earlier contributions to the literature on excludable public goods in several ways. Contradicting Buchanan (1967), Auster (1977) claimed that a perfectly discriminating monopolist would not produce an optimal output of an excludable public good. Brennan and Walsh (1985) showed that this claim is not correct, given that the monopolist knows the preferences of the consumer(s). It is however exactly this assumption that has been sharply criticized by Samuelson (1967). He said that a perfectly discriminating monopolist "is just another name for God", because the consumers have incentives not to reveal their true preferences.[3] This important observation is a starting point of our analysis: We abandon the assumption that the monopolist has perfect information[4] about the consumers' preferences.

Brito and Oakland (1980) convincingly argue why we should be interested in the monopolistic provision of excludable public goods and also take the privacy of information into account. However, their analysis is restrictive, because they only consider simple posted-price mechanisms, i.e. there are exogenous constraints on the class of contracts that the monopolist can offer to the consumers. In contrast, we do not impose any such constraints on the class of admissible contracts. Moreover, we explicitly allow for 'impure' cases of excludable public goods in two ways: We consider the existence of distribution costs[5] in addition to production costs and we consider 'congestion effects' by limiting the number of agents to whom the public good can be provided.

Our analysis is also related to the recent literature on contract theory.[6] Hart and Holmström (1987) pointed out that optimal contracts are often very complicated, if there are no ad-hoc constraints on the class of admissible contracts. On the other hand, contracts in the real world seem to be relatively simple. The optimal contracts derived in our model are indeed more complicated than the simple posted-price contracts analyzed by Brito and Oakland (1980). From a contract-theoretic point of view, it is thus desirable to explain why actual contracts often have a simple structure, without imposing this structure as an ad-hoc assumption. We therefore show that the monopolist can indeed choose a very simple contract if the number of potential consumers converges to infinity.

The paper is organized as follows. In section 2 we present the basic setup. In section 3 we characterize the monopolist's profit-maximizing production and selling decision. The second-best allocation rule, which is obtained by maximizing social welfare under the constraint of non-negative profits, is characterized in section 4. In section 5 we analyze properties of the optimal contract, given that the number of potential consumers converges to infinity. Implications for public intervention are discussed in section 6. Finally, we conclude with some remarks on possible modifications and extensions.

II. The model

There are n potential consumers who may enjoy the benefits of an indivisible, excludable public good. Let $q_i \in \{0,1\}$ denote the quantity consumed by agent $i \in \{1,...,n\}$. The allocation of the public good is thus given by a vector $q = (q_1,...,q_n)$. Let v_i denote agent i's privately known willingness-to-pay (or valuation) for the public good. The valuations are independent random variables distributed according to commonly known cumulative distribution functions F_i. The support of a corresponding density function f_i is given by the interval $V_i = [\underline{v}_i, \overline{v}_i] \subset R_+$. The valuations of the n potential consumers are thus denoted by a vector $v = (v_1,...,v_n) \in \prod V_i$. The following standard assumption restricts our attention to what Myerson (1981) has called the "regular case":

Assumption 1. *The term* $v_i - \dfrac{1-F_i(v_i)}{f_i(v_i)}$ *is increasing in* v_i.

For reasons which will become clear later, the term in assumption 1 can be called agent i's "virtual willingness-to-pay". The assumption simplifies the analysis considerably from a technical point of view; it is satisfied by many distribution functions, including the uniform, normal and exponential distributions.

A monopolist (referred to as agent 0) can produce $Q \in \{0,1\}$ units of the public good by incurring costs $KQ \geq 0$. She derives no intrinsic utility from the public good. Note that we must have $q_i \leq Q \ \ \forall i$, with the strict inequality holding whenever exclusion is exercised. When the monopolist provides the good to agent i, distribution costs $c_i \geq 0$ arise. For technological reasons (congestion effects) the public good can be provided to no more than $\overline{n} \in \{1,2,...,n\}$ agents, so that $\sum q_i \leq \overline{n}$ must be satisfied. The following assumption says that in order to qualify as a potential consumer, an agent's maximal willingness-to-pay must be larger than the distribution cost and furthermore restricts our attention to the non-trivial case in which production might actually be efficient:

Assumption 2. $c_i \le \bar{v}_i$, $K + \sum c_i \le \sum \bar{v}_i$.

The monopolist can propose a complicated contract in order to specify the levels of production, distribution and payments. Since the monopolist derives no intrinsic utility from the public good, we can focus on the choice of q, which automatically determines Q by $Q(q) = \max\{q_1,...,q_n\}$. Moreover, we know from the revelation principle that we can restrict our attention to direct revelation mechanisms: The potential consumers are asked to report their valuations. The announcements of the agents determine who may consume the public good by the allocation rule $q: \prod V_i \to \{0,1\}^n$ and who has to pay how much to the monopolist by the transfer rule $x: \prod V_i \to R_+^n$.

The expected utility of potential consumer i given his valuation v_i and the mechanism $(q(v), x(v))$ s defined by $U_i(v_i) = \mathrm{E}_{-i}\{v_i q_i(v) - x_i(v)\}$.

The incentive-compatibility and individual rationality constraints are:[7]

(IC) $$U_i(v_i) \ge \mathrm{E}_{-i}\{v_i q_i(v_{-i}, \tilde{v}_i) - x_i(v_{-i}, \tilde{v}_i)\} \qquad \forall v_i, \tilde{v}_i, i$$

(IR) $$U_i(v_i) \ge 0 \qquad \forall v_i, i$$

Given the constraints (IC) and (IR) the monopolist chooses functions $q(v)$ and $x(v)$ in order to maximize his expected profits, which are given by the difference between the expected total surplus and the rents of the potential consumers, $U_0(q(v)) = \mathrm{E}\{\sum (v_i - c_i) q_i(v) - KQ - \sum U_i(v_i)\}$. Finding a solution to this problem is considerably simplified by the following three lemmas. The proofs are by now standard and can easily be adapted from Myerson (1981).

Lemma 1: The constraints (IC) are equivalent to the following two conditions:

(1) $\mathrm{E}_{-i}\{q_i(v_{-i}, \tilde{v}_i)\}$ *is increasing in* $\tilde{v}_i$ $\qquad \forall \tilde{v}_i, i$

(2) $$U_i(v_i) = U_i(\underline{v}_i) + \int_{\underline{v}_i}^{v_i} \mathrm{E}_{-i}\{q_i(v_{-i}, \tilde{v}_i)\}\, \mathrm{d}\tilde{v}_i \qquad \forall v_i, i$$

Note that as long as $x(v)$ leads to a given value of $U_i(\underline{v}_i)$, expression (2) does not depend on the transfer rule $x(v)$. As a consequence, the expected profits $U_0(q(v))$ are also independent of $x(v)$. Indeed, using lemma 1 and partial integration it is easy to prove the following result.

Lemma 2: Given an incentive-compatible mechanism $(q(v), x(v))$*, the expected profits of the monopolist are*

(3) $$U_0(q(v)) = \mathrm{E}\left\{\sum\left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)} - c_i\right)q_i(v) - KQ\right\} - \sum U_i(\underline{v}_i).$$

Given that due to (IR) the expected utility of a consumer must not be negative, and given that $U_i(v_i)$ is increasing in v_i due to lemma 1, the monopolist will choose $x(v)$ so that $U_i(\underline{v}_i) = 0$. Given this restriction on the class of transfer rules, $x(v)$ does no longer influence the monopolist's profits. We can thus restrict our search for an optimal contract to the choice of an allocation rule $q(v)$. The following lemma specifies a suitable transfer rule.

Lemma 3: An allocation rule which satisfies condition (1) can be paired with the following transfer rule, so that the resulting mechanism meets (IC) and (IR) and leads to $U_i(\underline{v}_i) = 0$*:*

(4) $$x_i(v) = v_i q_i(v) - \int_{\underline{v}_i}^{v_i} q_i(v_{-i}, \tilde{v}_i)\, \mathrm{d}\tilde{v}_i$$

Note that the fact that in the above mentioned sense the expected profits are independet of the concrete specification of $x(v)$ is reminiscent of Myerson's (1981) "revenue equivalence" result in auction theory.

III. Profit- maximization

Consider first the contract a monopolist would propose if she knew the valuations of the consumers. In this case the profit of the monopolist would obviously be given by the social surplus $\sum(v_i - c_i)q_i - KQ$ minus the sum of the consumer rents $\sum(v_i q_i - x_i)$. The monopolist could extract the consumer rents by setting $x_i = v_i q_i$. If $\overline{n} = n$, she would thus maximize profits by setting $q_i = 1$ if (a) $v_i \geq c_i$ and (b) $\sum \max\{(v_i - c_i), 0\} \geq K$, and $q_i = 0$ otherwise. If $\overline{n} < n$, only the $\bar{n}$ consumers with the highest values of $v_i - c_i$ would be considered and q would be chosen in an analogous way. The monopolist would thus implement the first-best allocation rule, absorbing the total surplus through perfect price-discrimination.

However, if the consumers have private information, these simple findings are no longer true. In order to make the consumers reveal their true valuations, the monopolist must pay them information rents. This fact is reflected by the additional term $(1 - F_i(v_i)) / f_i(v_i)$ in expression (3). We now understand why v_i minus this term can be called "virtual willingness-to-pay". If only the true willingness-to-pay v_i appeared in (3), the monopolist could implement the first-best and absorb the total surplus. But due to the privacy of information, this is no longer possible. The willingness-to-pay is distorted by a term which arises because of the need to provide the consumers with sufficient incentives to reveal their types.

The following proposition characterizes the allocation rule which a rational, profit maximizing monopolist will choose. Although this contract appears to be very complex, it is a straightforward generalization of the optimal contract in a perfect information environment, which we have just described.

Proposition 1. Define $\overline{M}(v)$ *as the set of indices* $j_1,..., j_{\bar{n}} \in \{1,..,n\}$ *corresponding to the* $\bar{n}$ *largest*[8] *elements of* $\left\{ v_j - \frac{1 - F_j(v_j)}{f_j(v_j)} - c_j \middle| j \in \{1,...,n\} \right\}$.

Define $$M(v) := \left\{ j \middle| \ v_j - \frac{1 - F_j(v_j)}{f_j(v_j)} \geq c_j \right\} \cap \overline{M}(v).$$

The monopolist maximizes profits by choosing $q^M(v) = (q_1^M(v),..., q_n^M(v))$, *with*[9]

$$(5) \qquad q_i^M(v) = 1 \quad \textit{iff} \quad i \in M(v) \quad \textit{and} \quad \sum_{j \in M(v)} \left(v_j - \frac{1 - F_j(v_j)}{f_j(v_j)} - c_j \right) \geq K.$$

Proof. Recall that Q is given by $Q(q) = \max\{q_1,..., q_n\}$. All we have to do is thus to choose $q_i(v)$ in order to maximize (3) subject to (1) and $\sum q_i \leq \bar{n}$. As a first step, ignore the side constraints. The monopolist maximizes (3) by setting $q_i = 1$ for those consumers, whose virtual willingness-to-pay exceeds the distribution costs, given that the production costs can be covered. If

$\bar{n} < n$, only the $\bar{n}$ consumers with the highest virtual valuations minus distribution costs are considered. Finally, we must check that condition (1) is satisfied. This is obviously the case, because due to assumption 1 the virtual willingness-to-pay and thus $q_i^M(v)$ is increasing in v_i.

Q.E.D.

If $\bar{n} = n$, i.e. if there are no congestion effects, $\overline{M}(v)$ equals the set of all agents $\{1,...,n\}$, and $M(v)$ is simply the set of those agents whose virtual willingness-to-pay is at least as large as the relevant distribution cost. Agents who do not fall in this category never get a chance to enjoy the benefits of the public good. Agents who do fall in this category may consume the public good, but only if the total virtual willingness-to-pay of these agents net of distribution costs is large enough to cover the production costs K. If $\bar{n} < n$, the set $M(v)$ is further restricted so that only the $\bar{n}$ consumers with the highest virtual willingness-to-pay net of distribution costs are considered.[10]

Note that for the special case $K = 0$, $c_i = c \;\; \forall i$ and $\bar{n} < n$, proposition 1 reproduces the optimal multiple goods auction as described by Bulow and Roberts (1989) and Cairns (1993), while the special case $K > 0$, $F_i = F \;\; \forall i$, $c_i = 0 \;\; \forall i$ and $\bar{n} = n$ corresponds to Cornelli (1996). Obviously there are strong analogies between selling strategies for private goods on the one hand and excludable public goods on the other hand. In contrast, the following remark refers to the case of pure public goods, i.e. the case in which all consumers must consume the same amount, because exclusion is impossible.

Remark. Consider the case $c_1 = ... = c_n = 0$, $\bar{n} = n$, *and impose the additional constraint* $q_i = q_j \;\; \forall i, j$. *In this case, a profit-maximizing monopolist chooses*

(6) $$q_i^M(v) = 1 \quad \textit{iff} \quad \sum \left(v_j - \frac{1 - F_j(v_j)}{f_j(v_j)} \right) \geq K .$$

The proof of this result is straightforward given lemma 1 (see Güth and Hellwig (1986) and Rob (1989)). We will come back to this result in section 5.

IV. Welfare

In the trivial case $K = 0$ and $\bar{n} = n$, the first-best allocation rule is given by $q_i(v) = 1$ iff $v_i \geq c_i$. This rule can be implemented by posting personal prices c_i and obviously leads to zero profits if $\underline{v}_i = 0 \;\; \forall i$. With $K > 0$ it can thus happen that a monopolist would make negative profits if she were forced to choose the first-best allocation rule.[11] If public funds to subsidize the monopolist can only be raised by introducing distortions, even a welfare-maximizing government could not achieve the first best in this case. We therefore characterize the second-best allocation rule, which is welfare-maximizing under the condition of non-negative expected profits.

Proposition 2. For a given $\lambda \geq 0$, define $\overline{L}(v)$ as the set of indices $j_1,..., j_{\bar{n}} \in \{1,..,n\}$ corresponding to the $\bar{n}$ largest elements of $\left\{ v_j - \frac{\lambda}{1+\lambda}\frac{1-F_j(v_j)}{f_j(v_j)} - c_j \middle| j \in \{1,...,n\} \right\}$.

Define $L(v) := \left\{ j \middle| \; v_j - \frac{\lambda}{1+\lambda}\frac{1-F_j(v_j)}{f_j(v_j)} \geq c_j \right\} \cap \overline{L}(v)$. *Define $q^\lambda(v)$ in the following way:*

$$(7) \qquad q_i^\lambda(v) = 1 \quad \textit{iff} \quad i \in L(v) \quad \textit{and} \quad \sum_{j \in L(v)} \left(v_j - \frac{\lambda}{1+\lambda}\frac{1-F_j(v_j)}{f_j(v_j)} - c_j \right) \geq K$$

There exists $\tilde{\lambda} \geq 0$, so that $\tilde{\lambda} = \inf\{\lambda | U_0(q^\lambda(v)) \geq 0\}$. The second-best allocation rule is given by $q^S(v) = (q_1^S(v),...,q_n^S(v))$ with $q_i^S(v) = q_i^{\tilde{\lambda}}(v)$.

Proof. Maximization of social surplus under the constraint of non-negative expected profits is equivalent to the maximization of the following Lagrangian:

$$(8) \quad \sum (v_i - c_i) q_i(v) - KQ + \lambda \left(\sum \left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)} - c_i \right) q_i(v) - KQ \right)$$

Dividing by $1+\lambda$ and re-arranging terms, we see that the maximizer of this Lagrangian is equal to the maximizer of the following expression:

$$(9) \qquad \sum \left(v_i - \frac{\lambda}{1+\lambda}\frac{1-F_i(v_i)}{f_i(v_i)} - c_i \right) q_i(v_i) - KQ$$

By arguments similar to those in the proof of proposition 1, it is now easy to see that this expression is maximized by $q_i^{\lambda}(v)$. Note that for $\overline{n} = n$, $c_i = 0 \ \forall i$, the second-best allocation rule for excludable public goods is similar to the one for non-excludable public goods, which has already been analysed by Güth and Hellwig (1986). The existence of $\tilde{\lambda}$ can therefore be established by a straightforward modification of their lemma 5.5.

Q.E.D.

It is not surprising that proposition 2 is affiliated to results that were earlier obtained for non-excludable public goods, while proposition 1 is more similar to results of the private goods literature. A welfare-maximizer has less intention to exclude any agents, as long as congestion effects and distribution costs do not force her to do so. On the other hand, the possibility of exclusion certainly is important. There is indeed a fundamental difference between the allocation rules (5) and (6), which will become clear when we consider the case of many consumers.

V. Many consumers

The monopolist's profit-maximizing behavior as derived in proposition 1 is characterized by two properties, which deserve further considerations. First, the optimal contract must be written before production takes place. Second, the optimal contract is more complex than a simple fixed price mechanism. In this section we demonstrate that when the number of consumers converges to infinity, the monopolist can write a simple contract after production, and we thus provide a foundation for models which assume a continuum of consumers, such as Chae (1990). For simplicity, from now on we assume that $F_i = F \ \forall i$, $c_i = c \ \forall i$, and $\overline{n} = n$. The optimal allocation rule according to proposition 1 is then given by $q_i^M(v) = 1$ iff

(10) (a) $i \in M(v) = \left\{ j \middle| \ v_j \geq c + \dfrac{1 - F(v_j)}{f(v_j)} \right\}$ and (b) $\displaystyle\sum_{j \in M(v)} \left(v_j - \frac{1 - F(v_j)}{f(v_j)} - c \right) \geq K$.

First note that if condition (b) in (10) were satisfied with probability one, the profit-maximizing allocation rule would be given by $q_i = 1$ iff $v_i - \frac{1-F(v_i)}{f(v_i)} \geq c$. It is straightforward to see that this rule can easily be implemented by simply setting a fixed price. Moreover, if n converges to infinity and K remains finite, condition (b) indeed will be trivially satisfied with probability one. It is therefore more interesting to compare economies of different sizes, when K also increases with n. We choose the same normalization as Rob (1989) and assume that $K = \kappa n$, with $\kappa \in (0,1)$.

Proposition 3. If the number of potential consumers converges to infinity, the monopolist in the limit cannot improve upon the following simple mechanism:
The monopolist chooses $Q = 1$ if and only if κ is not larger than the expected profit which she could make using the optimal mechanism of proposition 1 in the case of one consumer and $K = 0$. Then the monopolist sets the same price as she would set in the case $n = 1,\ K = 0$.

Proof. Note that

(11a) $$\lim_{n\to\infty} \text{Prob}\left\{\sum_{j\in M(v)} \left(v_j - \frac{1-F(v_j)}{f(v_j)} - c \right) \geq \kappa n \right\} = 1$$

(11b) $$\Leftrightarrow \quad \lim_{n\to\infty} \mathrm{E}\left\{ \frac{1}{n} \sum_{j\in M(v)} \left(v_j - \frac{1-F(v_j)}{f(v_j)} - c \right) \right\} \geq \kappa$$

(11c) $$\Leftrightarrow \quad \mathrm{E}\left\{ \max\left(0, v_i - \frac{1-F(v_i)}{f(v_i)} - c \right) \right\} \geq \kappa .$$

The left-hand side of the last inequality is the monopolist's expected profit for $K = 0$ and $n = 1$, as can easily be seen from (3) and (5). So whenever this expression is larger than κ, condition (b) in (10) will in the limit be satisfied with probability one. In this case, the monopolist will produce the public good. It is straightforward to see that the optimal mechanism for the case $K = 0$, $n = 1$ can easily be implemented by simply posting a price.

Q.E.D.

Note that our result sharply contrasts with the findings of Rob (1989) and Mailath and Postlewaite (1990) who consider non-excludable public goods. They show that if $\kappa > \underline{v}_i$, the probability of provision converges to zero for $n \to \infty$. Indeed, from the remark in section 3 we know that the allocation rule in the case of pure public goods is given by

$$(12) \qquad q_i^M(v) = 1 \ \text{ iff } \quad \sum \left(v_i - \frac{1 - F(v_i)}{f(v_i)} \right) \geq n\kappa .$$

The probability that this condition is satisfied converges to zero for $n \to \infty$, since

$$(13) \qquad \mathrm{E}\left\{ v_i - \frac{1 - F(v_i)}{f(v_i)} \right\} = \underline{v}_i < \kappa .$$

This shows that non-excludability, i.e. the additional restriction $q_i = q_j \ \ \forall i, j$, is responsible for the negative results of Rob (1989) and Mailath and Postlewaite (1990), but not the fact that public goods in the sense of Samuelson (1964) are considered.

VI. Implications for public intervention

So far it has been assumed that the excludable public good is provided by a profit-maximizing monopolist. It is useful to discuss in more detail the role of the public sector. As has been argued in section 3, if there were no private information, then the monopolist would choose the first-best allocation rule since he could absorb the total surplus through perfect price-discrimination. This means that if there were no private information, intervention of the government could only be justified by distributional concerns.

However, we have seen that if there is private information, a monopolist will deviate from the first-best allocation rule in order to maximize his profits. There may hence be efficiency reasons for government intervention. It is then natural to ask whether the excludable public good should be provided by the public sector or whether the government should intervene through regulation (which means subsidization in the case where the non-negative profit constraint binds). Since the analysis presented in this paper derives from the more general literature on the theory of complete contracts, one cannot expect to get a clear answer to this question. Indeed, it is a well-known fact that if comprehensive contracts can be written, it is always possible to implement the same outcome in both organizational modes (see Schmidt (1996), p. 2, and Hart, Shleifer and Vishny (1997), p. 1128). Hence, from a contract-theoretic point of view, the costs and benefits of public provision versus regulation can only be understood if only incomplete contracts in the sense of Grossman and Hart (1986) are feasible.

For example, assume that the cost structure is initially only known to the agent responsible for the production of the excludable public good. This can now either be the manager of a private enterprise or a public employee. Following Shapiro and Willig (1990), it may make sense to assume that the government has better access to information of public employees. Indeed, access to the monopolist's inside information may be a residual right of control in the sense of Grossman and Hart (1986). In this case, if the government is a welfare-maximizer, public provision is certainly optimal. However, regulation may be optimal if the government is not always benevolent. In this case, giving the government less information may be better since this reduces the government's possibilities to pursue its private agenda (cf. Shapiro and Willig (1990)).

It can certainly be misleading to model the government as a benevolent, unitary decision-maker. For example, in the real world there are conflicts of interest between politicians, ministries, and regulatory agencies (see Tirole (1994)). Hence, further research along the lines of incomplete contracting is clearly desirable in order to better understand the role of the public sector in the provision of excludable public goods.

VII. Conclusion

We have characterized the optimal behavior of a profit-maximizing monopolist who can provide an excludable public good to a group of n potential consumers, who have private information with regard to their valuations.

Several aspects deserve further investigation. We have simplified the analysis by assuming that the public good is indivisible. A more general model would take continuous quantities into consideration, at the expense of tractability. We have also made the simplifying assumptions of the independent private valuations framework. The analysis certainly becomes far more complicated when correlated valuations are considered. Moreover, the mechanism design approach can be criticized. We assumed that the monopolist can commit himself not to renegotiate after the consumers have revealed their information, and we have ruled out the possibility of coalitional manipulations. Finally, our approach belongs to the Bayesian literature. It is of course questionable whether it is appropriate to make excessive use of the assumption that the distribution functions are common knowledge.[12]

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Footnotes

1 See Minasian (1964), Samuelson (1964, 1967) and Buchanan (1967).

2 Demsetz (1970) uses the term 'collective goods' for non-excludable public goods. The property of non-rivalness in consumption is also known as 'supply jointness' or 'undepletability', cf. Baumol and Ordover (1977).

3 Samuelson (1967) continues: "For Him to look into the hearts of 200 million, or 20 thousand, or 2 consumers, and guess right requires miracles."

4 For a recent contribution to the literature on excludable public goods under the assumption of perfect information see Bigman (1992).

5 Distribution costs arise with each additional consumer who is permitted to enjoy the benefits of the public good. Such costs have also been considered by Chae (1992) in a more recent application to subscription TV.

6 See Hart and Holmström (1987) or Schweizer (1996).

7 As long as the agents cannot leave the economy, it is not necessary to consider mechanisms in that certain types choose not to participate, because one could instead use equivalent mechanisms in which everyone participates and the types that did not participate in the original mechanisms have to pay nothing.

8 A tie occurs with probability zero. In this case, choose randomly among the largest elements.

9 We adopt the convention that " $q(v)=1$ iff condition C is met" is a shorthand for " $q(v)=1$ if C is met, whereas $q(v)=0$ if C is not met".

10 As an illustration consider a simple example. Assume that there are only $n=2$ potential consumers, and that the types of these two agents are uniformly distributed on the unit

interval. We further assume that there are no congestion effects ($\bar{n} = 2$) and that the distribution costs are equal to c for both agents.

In this case the profit-maximizing allocation rule $q^M(v_1, v_2)$ is given by

$$q_i^M(v) = 1 \text{ iff } \begin{cases} v_i \geq \dfrac{K+c+1}{2} & \text{and} \quad v_j \leq \dfrac{c+1}{2} \\ v_i \geq \dfrac{K}{2} + c + 1 - v_j & \text{and} \quad \dfrac{c+1}{2} \leq v_j \leq \dfrac{K+c+1}{2} \\ v_i \geq \dfrac{c+1}{2} & \text{and} \quad v_j \geq \dfrac{K+c+1}{2} \end{cases}$$

Messy calculations show that the expected profits of the monopolist are:

$$U_0(q^M(v)) = \begin{cases} \dfrac{1}{24}K^3 + \dfrac{1}{4}K^2 - \dfrac{3}{4}K + \dfrac{1}{2}c^2 + \dfrac{1}{4}Kc^2 + \dfrac{1}{4}K^2c + \dfrac{1}{2}Kc - c + \dfrac{1}{2} & \text{if } K + c \leq 1 \\ -\dfrac{1}{24}K^3 + \dfrac{1}{4}K^2 - \dfrac{1}{2}K + c^2 - \dfrac{1}{2}Kc^2 - \dfrac{1}{4}K^2c + Kc - c + \dfrac{1}{3} & \text{if } K + c \geq 1 \end{cases}$$

It is easy to see that if $K + c \geq 1$, the good is produced only if it can be sold to both consumers, since even the best type of one consumer alone would not be willing to pay enough to cover the costs. However, if $K + c \leq 1$, there are constellations of v_1 and v_2, so that exclusion will be exercised.

11 Recall that the case $K = 0$, $c_i = c \quad \forall i$, $\bar{n} < n$ corresponds to the usual private goods auction framework. In this case the first-best can be achieved while giving the seller positive expected profits, so that a slight increase in K will generally not lead to negative expected profits. Note, however, that (in contrast to what is conjectured in Cornelli, 1996, p.25) in the case $F_i = F \quad \forall i$, $n\bar{v}_i > K > 0$, $c_i = \underline{v}_i = 0$, $\bar{n} = n$ the first-best allocation rule cannot be implemented without expected losses for the monopolist. This follows immediately from Güth and Hellwig (1986, proposition 5.4).

12 The reader interested in the non-Bayesian literature on (excludable) public goods is referred to Moulin (1994) and Deb and Razzolini (1996).

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