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Bouoiyour, Jamal and Selmi, Refk

CATT, University of Pau, France, ESC, Tunis Business School of Tunis, Tunisia

13 July 2015

Online at https://mpra.ub.uni-muenchen.de/65580/ MPRA Paper No. 65580, posted 14 Jul 2015 13:19 UTC

Bitcoin Price: Is it really that New Round of Volatility can be on way?

Jamal BOUOIYOUR CATT, University of Pau, France. E-mail: jamal.bouoiyour@univ-pau.fr

Refk SELMI ESC, Tunis Business School, Tunisia. E-mail: <u>s.refk@yahoo.fr</u>

Abstract: To the mass public, Bitcoin is well known since its creation by its extreme volatility. However, Bitcoin's declining fluctuations since the start 2015 has revived our attention to assess whether there is a coming Bitcoin market phase. Using an optimal GARCH model on daily data, we show that the volatility of Bitcoin price decreases notably when comparing the periods [December 2010-June 2015] and [January 2015-June 2015]. During the first interval, the Threshold- GARCH estimates reveal that there is a great duration of persistence and thus tends to follow a long memory process. For the second period, the chosen specification (Exponential-GARCH) displays less volatility persistence. Despite this remarkable volatility's decrease, we cannot argue that Bitcoin market is mature, since the degree of asymmetry remains strong; Specifically, Bitcoin is likely to be driven by negative rather than positive shocks.

Keywords: Bitcoin; volatility; optimal GARCH model.

1. Introduction

Bitcoin and other so called digital currencies seem phenomena of the present. There are now 450 crypto-currencies circulating on the Internet. Bitcoin is the most popular of what are known as virtual cryptocurrencies. Its popularity is receiving in the media appears much more enormous that other virtual moneys and its use spread more largely and more rapidly around the world. Every passing day, the number of companies who accept Bitcoin increases, thereby making the perceived value of this crypto-currency real. This decentralized online payment system that enables anonymous transactions using cryptography¹, celebrates 6 years old this year. One of the main criticisms thrown at bitcoin is that it's highly volatile to be used as money. It is obvious that when each price has the potential to skyrocket or crash by more than 20 percent in a matter of hours, not many people will be interested to serve Bitcoin as their day-to-day purchases. The story (short) shows that it experienced periods of enthusiasm and euphoria, from some pennies in 2009 to 1100 dollars in 2013, without overlooking some periods of relative stability. This great volatility has increased the speculative nature of this virtual currency. In 2015, something different and interesting has happened to the price of bitcoins: it is gradually gotten less volatile. A stabilization of its value is well depicted, which can encourage more people to use it as a business income or effective transaction tool. The crisis in the eurozone has been a tremendous boost for this nascent currency. Indeed, the interest to Bitcoin was increased substantially when the Cypriot investors were informed of the bailout of Cyprus. As leaders prepared to tax bank deposits in order to prevent a default, Cypriots panicked heavily. Therefore, the Bitcoin has become a safe haven, given its anonymity. The Greek crisis has produced the same phenomenon, but in smaller proportions (Bouoiyour and Selmi, 2015 a).

If the fragile countries (Cyprus, Island, Greece,...) seem more interested to this famous currency, other powerful countries (Germany, Switzerland,...) have shown that they are not insensitive to the evolution of Bitcoin. They did not hesitate to adapt their legislation so as not to miss empowering technology which Bitcoin is giving birth to. Canada, the open minded and progressive country at least as far as digital currencies are concerned went in the same direction. Canadian senators recommend that the Canadian government should adopt a hands-off approach, to limit monitoring of the development of digital currencies and impose regulation only if it is required.

This positive development has undergone serious repercussions where the most famous are the closing website of the Road Silk in October 2013 and the MtGox affair. In the first case it was a site where illegal operations are uncovered thanks to anonymity. In the second case, it was a platform that ran up to 80 % of transactions in Bitcoin. In February 2014, this platform has disappeared causing a loss of 750,000 Bitcoins(\$ 440 million). This clearly highlights the fragility of Bitcoin. Given these considerations, a number of countries have banned this crypto-

¹ Cryptography is the cornerstone of every payment system used for the transfers of electronic money. It allows the creation of the currency itself.

currency (Russia, India, China and Thailand). For example, the Central Bank of Russia stated that "the issue of any other monetary units (than roubles) or quasi-money shall be prohibited in the Russian Federation" (State Duma, 2002). In the same context, the Bank of Thailand argued that Bitcoin is not a legal tender and any Bitcoin payment can be rejected by a merchant. Also it warns the Bitcoiners of its sizable volatility and possible theft.

Despite these problems, the Bitcoin's value becomes more stable since January 2015. Is this the beginning of a maturity of this young currency, or it is a tranquil period that hides great upheavals? Only time will tell. But we hope through this contribution to accurately assess the volatility of Bitcoin from its creation to today, but also since the beginning of 2015. The hypothesis that we support here is that Bitcoin has mellowed and a new phase began.

The remainder of the article proceeds as follows: Section 2 presents the followed methodology while trying to briefly outline several conditional volatility extensions. Section 3 summarizes and discusses the results and section 4 concludes.

2. Methodology

The volatility is an important property of any financial asset. It is the basic measure of risk to which is the investor exposed when buying an asset. For this reason it is crucial for any Bitcoin investor to assess the potential threats arising from Bitcoin volatility especially when we we've seen bitcoin go from being worth more than \$1,200 to less than \$300. The easiest way to observe the volatility of an asset is to determine an appropriate econometric technique to capture properly the unobservable process. At this stage, we should be cautious in choosing the best volatility measurement.

The conventional models (standard deviation, moving average deviation, among others) consider the distribution of asset returns as stable, implying that economic agents formulate their expectations at the same way over time. It is of course well known that this assertion is far from reality, since during periods of great agitation (adverse changes, crisis, political tensions and sudden shocks, among others), the variance-covariance of returns may be very volatile. Statistically, financial markets data seem distinguished during "volatility clustering" in which time series show periods of high volatility and periods of low one. In fact, time-varying is more common than constant volatility. In that context, GARCH (General Autoregressive Conditional Heteroskedasticity) extensions are considered more appropriate to define volatility (Bollerslev et al. 1993). These models are efficient for describing the volatility of the conditional variance by taking into account the characteristics of series using the past errors in estimates. They allow us to take into account the transitory and permanent components, the volatility clustering (the sum of ARCH and GARCH effects), the reaction to shocks and the structural breaks and the leverage effect. The first important set of models used for modelling of volatility clusters was introduced by Engle (1982) and then extended by Bollerslev (1986), assuming that the conditional variance follows an ARMA process. It allows a representation of the autoregressive conditional variance process. It may be expressed as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_j \sigma_{t-j}^2$$
(1)

Where α_i , β_i and ω are the parameters to estimate.

Other extensions followed, Nelson (1991), Engle and Bollerslev (1986), Bollerslev et al. (1993), Zakoin (1994), Bollerslev (2008), Bauwens and Storti (2008), Bouoiyour and Selmi (2014), among others. Since no single measure of volatility has dominated the existing empirical literature, the more parsimonious techniques able to clearly and appropriately depict the volatile behaviors of the focal variables may be selected using standard criteria such as the Akaike information criterion (AIC), the Bayesian (BIC) and Hannan-Quinn information criteria (HQ). These criteria are sufficient to judge the quality of estimation for Bitcoin price conditional variance, because they allow comparing different models in terms of trade-off between goodness of fit and model parsimony.Table-A.1 (Appendices) reports the GARCH specifications used in our study, while Table A.2 (Appendices) represents the tests used in order to reach the best measure.

For empirical context, we use daily time-series data related to Bitcoin and over the period from December 2010 to June 2015. The long time range of our time series data may help policymakers to reach accurate information and fully picture about to what extent Bitcoin is volatile (persistence and volatility clustering). The Bitcoin is collected from Blockchain (https://blockchain.info/). We transform the focal variable by taking natural logarithms to correct for potential heteroskedasticity. Then, we first-difference the time series studied to generate daily-on-daily Bitcoin price (BTP). Ultimately, we standardize the central series to exhibit a zero mean and variance of one. Figure 1 depicts that Bitcoin experienced several jumps and excessive swings over the period spanning between 2010 to 2015. Bitcoin's price has been volatile since its creation in 2009, mainly due to greater appreciations and precipitous depreciations in its value. By mid-2013, Bitcoin's dollar exchange rate increase considerably from \$50 to \$350 before falling back then to \$70. During 2014, Bitcoin's price showed large day-to-day variations, which appear generally trended down. Since the start of the year, something interesting has happened to the price of Bitcoins and it has gradually gotten much less volatile than the previous years. After a period of great volatility especially during 2013 and 2014, having been less than \$20 in January 2013, and reaching \$1,100 in December 2013, and falling then to \$320 in mid-December 2014, Bitcoin seems to some extent stable. From mid-January 2015, a single Bitcoin was valued at around \$220 and does not exceed \$320 over the period spanning between 15/01/2015-15/06/2015, highlighting the occurrence of new phase (calm period).



Figure 1. The evolution of Bitcoin price and the number of transactions

3. Volatility of Bitcoin: Main results via optimal GARCH model **3.1.Preliminary analysis**

Before starting to choose the optimal model via discrimination tests and to estimate Bitcoin volatility, we tested the autocorrelations and the occurrence of asymmetry on conditional volatility in Bitcoin. Simple diagnostic can be used to see whether there we should account for asymmetry when analyzing the focal series is the correlation between squared returns and lagged returns (i.e. $corr(r^2, r_{t-1})$). A negative value of the correlation coefficient implies the existence of potential leverage effect. The results of autocorrelation, ARCH and leverage effects tests are summarized in Table 1. The findings from DW test imply that there is no evidence of autocorrelation between the squared returns and lagged returns has negative value, implying the presence of asymmetric effect.

| | DW test (r_{t-1}) | ARCH LM test (r_{t-1}) | $\rho(r^2,r_{t-1})$ |
|---------------|---------------------|--------------------------|---------------------|
| | | | |
| Bitcoin price | 1.98 | 39.56***[0.0000] | -0.4789 |

| Table 1. | Test of | leverage | effect on | conditional | volatility |
|-----------|----------|-------------|-----------|--------------|-------------|
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Note: *r*: the returns of Bitcoin price.

3.2.Bitcoin volatility for the period [December 2010-June2015]

As mentioned above, to choose the best model, we will use standard criteria such as the Akaike, the Bayesian and the Hannan-Quinn criterion. These criteria evaluate the models based on the history of volatility². Whatever the criterion of historical evaluation (AIC, BIC, HQ), the optimal model is the T-GARCH or the Threshold-GARCH (Table A.2, Appendices). This model was developed by Zakoin (1994) accommodates structural breaks in volatility, while volatility follows a GARCH process within each regime. Basically, the return process in the threshold model is strictly stationary, as well as conditions for the existence of different moments (Wu, 2010). It is expressed as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \left(\alpha_i \left| \varepsilon_{t-i} \right| + \gamma_i \left| \varepsilon_{t_{-i}}^+ \right| \right) + \sum_{i=1}^p \beta_j \sigma_{t-j}$$
(2)

where α_i , β_i and ω and γ are the parameters to estimate.

The main results of estimates are reported in Table 2. The volatility appears persistent and tends towards long memory process since it seems not far from one $(\alpha + \beta + 0.5\gamma = 0.98)$. The asymmetrical effect is positive and statistically significant implying that the effect of bad news on the conditional variance is stronger than that of good news. Indeed, the degree of asymmetry, which measures the relative influence of bad news on volatility seems strong (1.95). Notably, the intensity of positive shocks appears more important than that of positive shocks. The ARCH coefficient is lower than 1, so stationarity of the model is confirmed. The alpha coefficient measuring the dependence of current period volatility on the past period disturbance seems significantly positive and it amounts 0.23. This means that large part of today's volatility can be explained by past volatility and disturbances.

² The formula of these different criteria are as follows:

Akaike information criterion :-2log (vraisemblance) + 2kBayesian information criterion :-2log (vraisemblance) + log(N).kHannan-Quinn information criterion :-2log (vraisemblance) + 2k.log (log(N))

where k the degree of freedom and N is the number of observations.

| Dependent | variable: (r_i) | | |
|---|-------------------|--|--|
| Mean I | Equation | | |
| <u>C</u> | -0.3604 | | |
| | (-0.8590) | | |
| r_{t-1} | -0.6325*** | | |
| | (-3.7151) | | |
| Variance | Equation | | |
| ω | -0.4313* | | |
| | (-2.1727) | | |
| α | 0.2309*** | | |
| | (14.341) | | |
| β | 0.7308*** | | |
| | (23.820) | | |
| γ | 0.2412* | | |
| | (2.1104) | | |
| Duration of persistence: $\alpha + \beta + 0.5\gamma$ | 0.9823 | | |
| ARCH and GARCH effects: $\alpha + \beta$ | 0.9617 | | |
| Leverage effect: γ | 0.2412 | | |
| Degree of asymmetry: $\frac{\alpha + \gamma}{\alpha}$ | 1.9572 | | |
| Intensity of negative shock: $-\alpha + \gamma$ | 0.0103 | | |
| Intensity of positive shock: $\alpha + \gamma$ | 0.4721 | | |

Table 2. Bitcoin volatility' parameters and persistence for the period[December 2010-June 2015]

Notes: *r*: the returns of Bitcoin price; ω : The reaction of conditional variance; α : ARCH effect; β : GARCH effect; γ : Leverage effect.

Figure 1 confirms the great volatile behavior of Bitcoin depicted through the excessive ups and downs, which tends to be less important in the mid-2015. Since this stabilization period does not appear clearly in this figure, we try in the following to see whether the behavior of this nascent

currency really changed from 2015 and if this seems temporary or tend to follow a long memory process.



Figure 1. The conditional variance of Bitcoin for the period [December 2010-June 2015]

3.3.Bitcoin volatility for the period [January 2015-June2015]

Using information criteria, we show that for Bitcoin price between January and mid-2015, the optimal model is the E-GARCH (Exponential GARCH) developed by Nelson (1991). This technique accounts essentially for asymmetry or leverage effect and thus on the sign of shock in the process of conditional volatility, not only the magnitude. It is expressed as following:

$$\log(\sigma_{t}^{2}) = \omega + \sum_{i=1}^{q} (\alpha_{i} z_{t-i} + \gamma_{i} (|z_{t-i}| - \sqrt{2/\pi})) + \sum_{i=1}^{p} \beta_{j} \log(\sigma_{t-j}^{2})$$
(3)

where α_i , β_i and ω and γ are the parameters to estimate; z_t is the standardized value of error.

Table 3 reports the estimated parameters. We show that the leverage effect is positive and significant for Bitcoin price returns, indicating that bad news have more impact than good news. But the effect of bad news appears minor ($\gamma = 0.096$). Furthermore, the intensity of negative and positive shocks appear weaker and duration of persistence seems much less strong than that of the whole period from December 2010 to June 2015, which amount respectively 0.98 and 0.54 (almost the half) implying to some extent, that a new phase of Bitcoin's dropping variability emerges. This appears clearer in Figure 2. Nevertheless, this period appears transitory since the volatility clustering or the sum of ARCH and GARCH effects seems far from one. One element of explanation of this outcome may be that as more time passes people have fresh insights and more information about the use of Bitcoin, and are therefore more confident in their valuation.

This obviously will decreases uncertainty gradually. But this stills conditioning upon sudden speculative attacks that drive intensely this famous crypto market (Bouoiyour and Selmi 2015 b).

| Dependent v | rariable: (r_t) | | |
|---|-------------------|--|--|
| Mean E | quation | | |
| С | 0.0006*** | | |
| | (7.2177) | | |
| r_{t-1} | 0.1777*** | | |
| | (7.9562) | | |
| Variance | Equation | | |
| ω | -0.3334*** | | |
| | (-14.665) | | |
| α | 0.0022* | | |
| | (2.0708) | | |
| β | 0.4953*** | | |
| | (4.8130) | | |
| γ | 0.0965*** | | |
| | (15.979) | | |
| Duration of persistence: $\alpha + \beta + 0.5\gamma$ | 0.5457 | | |
| ARCH and GARCH effects: $\alpha + \beta$ | 0.4975 | | |
| Leverage effect: γ | 0.0965 | | |
| Degree of asymmetry: $\frac{\alpha + \gamma}{\alpha}$ | 49.250 | | |
| Intensity of negative shock: $-\alpha + \gamma$ | -0.0945 | | |
| Intensity of positive shock: $\alpha + \gamma$ | 0.0985 | | |
| | | | |

Table 3. Bitcoin volatility' parameters and persistence for the period[January2015-June2015]

Notes: *r*: the returns of Bitcoin price; ω : The reaction of conditional variance; α : ARCH effect; β : GARCH effect; γ : Leverage effect.

Figure 2. The conditional variance of Bitcoin for the period [January2015-June2015]



4. Conclusion

Price volatility has been extensively investigated on financial markets, but due to the recent emergence of Bitcoin market, researchers have started to scratch the surface in this area. Hence, the excessive volatility of bitcoin and how determine it properly has not yet been sufficiently studied providing for an extensive research gap. This article has aimed to offer a discussion into Bitcoin price volatility by using an optimal GARCH model chosen among several extensions. By doing so, the findings suggest an extreme volatility of Bitcoin price. The conditional variance tends to follow a long memory process over the period spanning between December 2010 and June 2015. We note a period of less volatility in terms of persistence and clustering between January and June 2015, but this seems temporary (the sum of ARCH and GARCH effects is far from one). Remarkably, for the two sub-periods considered, Bitcoin volatility process seems more influenced by negative (bad news) than positive shocks (good news). Not surprisingly, the Bitcoin market is highly driven by self-fulfilling expectations. It consists deeply on nonprofessional noise traders whose actions can lead to heavy bubble behavior of the Bitcoin price increasing volatility (Bouoiyour et al. 2015). It is well known that initial bitcoin users consisted essentially of technology enthusiasts, liberalists and criminals (Yermack 2014), while today it consists substantially of individual noise traders and speculators (Bouoiyour and Selmi 2015b). This highlights consistently that the Bitcoin market is far from mature. Its lack of regulation and transparency reinforces the uncertainty surrounding this crypto market. If it is difficult therefore to predict the future of the currency. We are aware that we are at a point of no return in terms of the technology behind this digital currency. Its philosophy also is not to lose sight of the crypto- currencies generally and the associated technologies for electronic transactions. As technology becomes increasingly integrated into our everyday lives, cryptocurrencies will obviously continue to grow and Bitcoin may probably be displaced by better digital currencies.

Hope that this study succeeds to offer better information about the extent of Bitcoin volatility and good luck to traders and all Bitcoin believers with the coming market phase.

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Appendices

Table A.1. GARCH extensions used in this study

| Extensions | linear | nonlinear | symmetrical | Asymmetrical |
|---|--------|-----------|-------------|--------------|
| GARCH-M (GARCH in mean, | X | nommeai | x | Asymmetrical |
| Bollerslev et al. 1993) | Λ | | л | |
| $r_t = \mu_t + \varepsilon_t + \lambda \sigma_t^2$ | | | | |
| C-GARCH (Component GARCH, Ding | x | | х | |
| et al. 1993) | А | | A | |
| $(\sigma_t^2 - \sigma^2) = \alpha(\varepsilon_{t-1}^2 - \sigma^2) + \beta(\sigma_{t-1}^2 - \sigma^2)$ | | | | |
| I-GARCH (Integrated GARCH, | X | | Х | |
| Bollerslev et al. 1993) | | | | |
| $\sigma_{t}^{2} = \omega + \varepsilon_{t-1}^{2} + \sum_{i=1}^{q} \alpha_{i} (\varepsilon_{t-i}^{2} - \varepsilon_{t-1}^{2}) + \sum_{i=1}^{p} \beta_{j} (\sigma_{t-j}^{2} - \varepsilon_{t-1}^{2})$ | | | | |
| T-GARCH (Threshold GARCH, | | Х | | Х |
| Zakoian, 1994) | | | | |
| $\sigma_t^2 = \omega + \sum_{i=1}^q (\alpha_i \varepsilon_{t-i} + \gamma_i \varepsilon_{t-i}^+) + \sum_{i=1}^p \beta_j \sigma_{t-j}$ | | | | |
| E-GARCH | | | | Х |
| (Exponential GARCH, Nelson, 1991) | | | | |
| $\log(\sigma_{t}^{2}) = \omega + \sum_{i=1}^{q} (\alpha_{i} z_{t-i} + \gamma_{i} (z_{t-i} - \sqrt{2/\pi})) + \sum_{i=1}^{p} \beta_{j} \log(\sigma_{t-j}^{2})$ | | | | |
| P-GARCH (Power GARCH, Higgins | х | | Х | |
| and Bera, 1992) | | | | |
| $\sigma_t^{\varphi} = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^{\varphi} + \sum_{i=1}^p \beta_j \sigma_{t-j}^{\varphi}$ | | | | |
| A-PGARCH (Asymmetric power | | | | Х |
| GARCH, Ding et al., 1993) | | | | |
| $\sigma_{\iota}^{\varphi} = \omega + \sum_{i=1}^{q} \alpha_{i} (\varepsilon_{\iota-i} + \gamma_{i}\varepsilon_{\iota-i})^{\varphi} + \sum_{i=1}^{p} \beta_{j}\sigma_{\iota-j}^{\varphi}$ | | | | |
| CMT-GARCH (Component with | | Х | | х |
| Multiple Thresholds GARCH, | | | | |
| Bouoiyour and Selmi, 2014) | | | | |
| $\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta(\omega + (\alpha + \gamma_{(\varepsilon_{t-2-0})})\varepsilon_{t-2}^2 + \beta \sigma_{t-2}^2)$ | | | | |

Notes: σ_t^2 : conditional variance, α_0 : reaction of shock, α_1 : ARCH term, β_1 : GARCH term, \mathcal{E} : error term; I_t : denotes the information set available at time t; z_t : the standardized value of error term where $z_t = \varepsilon_{t-1}/\sigma_{t-1}$; μ : innovation, γ : leverage effect; φ : power parameter.

| The period [December 2010-June 2015] | | | | |
|--------------------------------------|---------|---------|---------|--|
| Models | AIC | BIC | HQ | |
| GARCH | 0,8729 | 0,9745 | 0,9088 | |
| GARCH-M | 0,7756 | 0,8434 | 0,7872 | |
| I-GARCH | 0,7799 | 0,8672 | 0,8106 | |
| C-GARCH | 0,8223 | 0,9107 | 0,8321 | |
| CMT-GARCH | 0,8311 | 0,9037 | 0,8567 | |
| T-GARCH | 0,9348 | 1,0509 | 0,9757 | |
| E-GARCH | 0,8729 | 0,9745 | 0,9088 | |
| P-GARCH | 0,7756 | 0,8434 | 0,7875 | |
| AP-GARCH | 0,7799 | 0,8672 | 0,8106 | |
| The period [January 2015-June 2015] | | | | |
| GARCH | -1,3894 | -1,3099 | -1,3617 | |
| GARCH-M | -1,3675 | -1,2720 | -1,3342 | |
| I-GARCH | -1,5213 | -1,4736 | -1,5047 | |
| C-GARCH | -1,3560 | -1,2447 | -1,3172 | |
| CMT-GARCH | -1,3923 | -1,2968 | -1,3590 | |
| T-GARCH | -1,4180 | -1,3066 | -1,3791 | |
| E-GARCH | -1,3626 | -1,2353 | -1,3181 | |
| P-GARCH | -1,3954 | -1,3159 | -1,3677 | |
| AP-GARCH | -1,3809 | -1,2854 | -1,3476 | |

Table A.2. Bitcoin volatility: Optimal GARCH model chosen by information criteria