



Munich Personal RePEc Archive

Disaster risk and preference shifts in a New Keynesian model

Marlène Isoré and Urszula Szczerbowicz

University of Helsinki, HECER, CEPII

15. July 2015

Online at <http://mpra.ub.uni-muenchen.de/65643/>

MPRA Paper No. 65643, posted 17. July 2015 15:17 UTC

Disaster Risk and Preference Shifts in a New Keynesian Model ^{*}

Marlène Isoré^{*} Urszula Szczerbowicz[‡]

July 15, 2015

Abstract

This paper analyzes the effects of a change in a small but time-varying “disaster risk” à la Gourio (2012) in a New Keynesian model. In a real business cycle framework, the disaster risk has been successful in replicating observed moments of equity premia. However, responses of macroeconomic variables critically depend on the value of the elasticity of intertemporal substitution (EIS). In particular, we show here that an increase in the probability of disaster causes a recession only in case of an EIS larger than unity, which may be arbitrarily large. Nevertheless, we also find that incorporating sticky prices allows to conciliate recessionary effects of the disaster risk with a plausible value of the EIS. A higher disaster risk is then also associated with an increase in the discount factor and with deflation, making it consistent with the preference shock literature (Christiano et al., 2011).

keywords: disaster risk, rare events, uncertainty, asset pricing, DSGE models, New Keynesian models, business cycles

JEL classification: D81, D90, E20, E31, E32, E44, G12, Q54

^{*}We thank George-Marios Angeletos, Guido Ascari, Pierpaolo Benigno, Nick Bloom, Luca Dedola, Martin Ellison, Xavier Gabaix, François Gourio, Sébastien Jean, Antonio Mele, Salvatore Nisticò, Juan Carlos Parra-Alvarez, Antti Ripatti, Kjetil Storesletten, Fabien Tripier, Natacha Valla, Philippe Weil, and Francesco Zanetti, as well as numerous participants at Bank of Finland, Bank of Norway, CEPPII, the IIBEO workshop, and Oxford’s Macroeconomics group, for fruitful discussions at different stages of this work. We are grateful to the Yrjö Jahnsson Foundation for financial support on this project.

^{*}E-mail: marlene.isore@helsinki.fi. Address: HECER, Arkadiankatu 7 (P.O Box 17), FI-00014 University of Helsinki, Finland.

[‡]Email: urszula.szczerbowicz@ceprii.fr. Address: CEPPII, 113 rue de Grenelle, 75007 Paris, France.

1 Introduction

In the recent years, interest in the economic impact of ‘rare events’ has been renewed. In particular, Gabaix (2011, 2012) and Gourio (2012) have introduced in real business cycle models a small but time-varying probability of ‘disaster’, defined as an event that destroys a large share of the existing capital stock. The key feature is that an increase in the probability of disaster, without occurrence of the disaster itself, suffices to trigger a recession and replicate some asset pricing regularities.

However, these effects crucially rely on the assumption of an intertemporal elasticity of intertemporal substitution (EIS) being greater than unity. In Gourio (2012), an unexpected increase in the probability of disaster is equivalent, under some assumptions, to a decrease in agents’ discount factor, i.e a lower degree of patience. Agents then save and invest less in the risky capital and instead choose to increase their current consumption. In the RBC economy, this in turn causes a recession and an increase in risk premia. However, this initial response of the discount factor holds only under the assumption of an EIS larger than unity. Indeed, as shown by Leland (1968) and Sandmo (1970), an increase in interest rate risk leads agents to reduce their savings if and only if the EIS is larger than 1. This is because a large EIS increases the propensity to consume. On the contrary, when the EIS is low, income effects overcome substitution effects and savings go up.¹

In other words, Gourio (2012)’s disaster shock predictions would not hold should the parameter value chosen for the EIS be smaller than 1. Indeed, savings would then increase, surprisingly driving the economy into a boom. The empirical evidence on the EIS is quite mixed, yet mostly supports a value below one.² Macroeconomic models have thus largely adopted this range in their calibration, whether they feature Epstein-Zin-Weil preferences or not.³

This paper introduces a small time-varying probability of disaster à la

¹Weil (1990) shows that a large EIS implies that the elasticity of savings to a ‘certainty-equivalent’ interest rate is positive, i.e savings decrease in the aggregate interest rate risk. On the contrary, a small EIS implies that savings go up with interest rate risk.

²The seminal paper by Hall (1988) finds it close to zero and the subsequent literature has often supported values smaller than one. Heterogeneity across agents matters as rich households and stockholders tend to have an EIS larger than other agents (Mankiw and Zeldes (1991), Blundell et al. (1994), Attanasio and Browning, (1995), Vissing-Jorgensen (2002)). A recent meta-analysis by Havránek et al. (2015) gives a mean estimate of 0.5.

³As for a few examples, Piazzesi et al. (2007) use a value of 0.2, Rudebusch and Swanson (2012) 0.5, Smets and Wouters (2007) 0.66.

Gourio (2012) into a New Keynesian model. To the best of our knowledge, we are the first to do so. The contribution is threefold. First, we aim at shedding some light on the critical role of the EIS in driving the disaster risk results. In particular, Gourio (2012)'s mechanisms generate a boom with an EIS taking a plausible value. Here, we are able to nest these results in our decentralized economy with flexible prices. Second, we show that introducing sticky prices provide a solution to restore and generalize Gourio (2012)'s predictions for the disaster risk when the EIS is below unity. This way, we obtain recessionary effects of the disaster risk shock when the low EIS is at play. Third, we show that the variations in the discount factor caused by the disaster risk shock can thus be conciliated with the preference shock literature (Smets and Wouters, 2003, Christiano et al., 2011). In the latter, exogenous increases in the discount factor cause recession and deflation.

Developing a full-fledged New Keynesian model is critical here, not just in order to create a richer macroeconomic setting and broaden the spectrum of potential policy analysis, but because it literally inverts most of the macroeconomic effects associated with a change in the disaster risk, for a given EIS. The reason for that is particularly intuitive. Consider for instance a low EIS, in which case the discount factor goes up with the probability of disaster, such that agents do choose to save more. In a flexible price setup, the economy is mostly driven by this supply-side effect: savings and therefore investment increase due to precautionary motives, as well as does the labor supply, such that the economy enters a boom. In contrast, sticky prices make the output fluctuations more sensitive to the demand-side effect of the shock: more savings imply lower current consumption, and thus a recession. Hence, the New Keynesian structure totally changes the macroeconomic dynamics caused by the disaster risk shock for a given value of the EIS.⁴ Another interesting finding is that the depressed consumption, stemming from the rise in disaster risk in the sticky prices version only, also causes deflation and lowers firms' demand for production factors (and thus a downside effect on wages), along with the recession and a rise in the risk premium. This seems to be

⁴In the same spirit, Basu and Bundick (2014) consider a volatility shock to the discount factor and also show that the New Keynesian structure changes qualitatively the responses of the macroeconomic variables compared to RBC setups. Yet, they focus on a second-moment preference shock from an exogenous discount factor, while we study shifts in the level of the discount factor capturing changes in the disaster risk, à la Gourio (2012). Moreover, in their case, the value of the EIS is not critical, while it is in ours.

particularly consistent with the literature on preference shocks, recently revived as a potential source of zero lower bound on nominal interest rates (e.g. Christiano et al., 2011).

The remainder of this paper is as follows. Section 2 presents the model. Section 3 gives the calibration values and discusses the steady-state, in particular whether Tallarini (2000)’s “observational equivalence” holds or not when the disaster risk is present in the economy. Section 4 shows the responses to an unexpected increase in the disaster risk. Gourio (2012)’s results are nested when prices are flexible and the EIS is larger than 1, while reversed when the EIS is smaller than 1. Then, with sticky prices and an EIS smaller than 1, we find that the disaster risk causes a recession, deflation, and increase in the risk premium in particular. Section 5 reviews the literature and further discusses the soundness of our results. Section 6 concludes.

2 Model

2.1 Households

2.1.1 The household problem with disasters

Let us consider households with Epstein-Zin-Weil preferences given by

$$\tilde{V}_t = \left[[C_t (1 - L_t)^\varpi]^{1-\psi} + \beta_0 \left(E_t \tilde{V}_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \right]^{\frac{1}{1-\psi}}$$

where C is consumption, L labor supply, γ the coefficient of risk aversion, and $1/[1 - (1 + \varpi)(1 - \psi)]$ the elasticity of intertemporal substitution (EIS). Households invest in capital, with a law of accumulation given by

$$K_{t+1} = \left[(1 - \delta_0 u_t^\eta) K_t + S \left(\frac{I_t}{K_t} \right) K_t \right] e^{x_{t+1} \ln(1-\Delta)}$$

where x is an indicator variable capturing the occurrence of a “disaster” destroying a large share of the existing capital stock. Specifically, there is a time-varying probability θ_t that a disaster occurs in the next period, $x_{t+1} = 1$, in which case a share Δ of capital is destroyed. Otherwise, $x_{t+1} = 0$ and the capital accumulation is in line with standard New Keynesian models, with a variable utilization rate of capital, u , and a convex capital adjustment cost function, $S(\cdot)$, with specific forms given further below. Moreover, the

probability of disaster, θ , follows a first-order autoregressive process as

$$\log \theta_t = (1 - \rho_\theta) \log \bar{\theta} + \rho_\theta \log \theta_{t-1} + \sigma_{\theta,t} \varepsilon_{\theta_t}$$

where $\bar{\theta}$ is the mean disaster risk, ρ_θ the persistence, and ε_θ i.i.d innovations.⁵

In addition, households can buy one-period bonds issued by a public authority. As Gabaix (2012) and Gourio (2013), we assume that bonds are also subject to the disaster risk. Sovereign debt can indeed be risky during tail events in the sense that it becomes subject to partial default, as we have observed for Greece in the last financial crisis, Argentina in the early 2000s, or UK and US in the Great Depression, as for a few examples. Conditional on no disaster, bonds are however riskfree, unlike capital. It is worth mentioning that, following Gourio (2012)'s approach, we assume that the destruction share in case of a disaster, Δ , is the same for the assets (both capital and bonds here) and productivity. Although this may appear as a strong assumption, this is essentially a trick used to solve the model with perturbation methods: when detrended by productivity, the system will then not be directly impacted by the *large* disaster event (x) itself but only by the *small* probability of disaster (θ), which is our variable of interest.⁶

Finally, households rent their capital and labor force to monopolistic competition firms. They own these firms, hence earn profits. They pay lump-sum taxes to the public authority. Thus, their budget constraint is

$$C_t + I_t + \frac{B_{t+1}}{p_t} + T_t \leq \frac{W_t}{p_t} L_t + \frac{B_t(1 + r_{t-1})}{p_t} e^{x_{t+1} \ln(1-\Delta)} + \frac{P_t^k}{p_t} u_t K_t + D_t$$

where W denotes the (nominal) wage rate, p the good price, B the bonds and r the corresponding interest rate, P_t^k is the (nominal) rental rate of capital, u the utilization rate of capital, I the investment, T the taxes, and D the (real) dividends from monopolistic firms' (real) profits.

The households want to maximize utility subject to their capital accumulation and the budget constraints. However, because the indicator variable is present in this optimization problem and thus in the equilibrium condition,

⁵These parameters will be calibrated following empirical estimations of disaster risk (see Section 3). Our qualitative results yet hold also without any degree of persistence.

⁶It is possible to release this constraint by using projection methods to solve the model, as Gourio (2012) also does with a smaller set of variables. Another alternative would be to make an assumption on whether the economy is currently in a disaster state, $x = 1$, or not $x = 0$, when generating the impulse response functions for the disaster risk, θ .

we cannot directly use the perturbation methods to solve the model as such. Therefore, we follow Gourio (2012) in detrending the system such that the disaster risk only, and not the disaster event, impacts the detrended system.

2.1.2 Detrending the household's problem

Let us thus assume, as in Gabaix (2012) and Gourio (2012), that productivity, denoted z , is also partly destroyed in case of a disaster, and follows

$$\frac{z_{t+1}}{z_t} = e^{\mu + \varepsilon_{z,t+1} + x_{t+1} \ln(1-\Delta)}$$

where μ is a trend and $\varepsilon_{z,t+1}$ i.i.d normally distributed innovations with zero mean.⁷ As mentioned above, assuming that the share Δ is the same as for the physical assets, the *detrended* variables will not depend on the disaster *event* anymore, but will still be affected by the disaster *risk*. In particular, the household's budget constraint in detrended terms reads as

$$c_t + i_t + \frac{b_{t+1}}{p_t} e^{\mu + \varepsilon_{z,t+1}} + \frac{T_t}{z_t} \leq \frac{w_t}{p_t} L_t + \frac{b_t(1 + r_{t-1})}{p_t} + \frac{P_t^k}{p_t} u_t k_t + \frac{D_t}{z_t} \quad (1)$$

while the capital accumulation becomes

$$k_{t+1} = \frac{(1 - \delta_t)k_t + S\left(\frac{i_t}{k_t}\right)k_t}{e^{\mu + \varepsilon_{z,t+1}}} \quad (2)$$

where lower case letters denote the detrended variables ($k_t = K_t/z_t$, etc), and where, in line with the New Keynesian literature, the capital depreciation rate is considered as a function of the capital utilization rate

$$\delta_t = \delta_0 u_t^\eta \quad (3)$$

while the capital adjustment cost function, with usual properties, reads as

$$S\left(\frac{i_t}{k_t}\right) = \frac{i_t}{k_t} - \frac{\tau}{2} \left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}}\right)^2 \quad (4)$$

⁷Labor productivity may indeed decrease during financial crises (e.g Hughes and Saleheen, 2012), as well as during wars or natural disasters as people may find themselves not necessarily matched with jobs requiring their specific skills. Total factor productivity may also decrease as firms facing severe financing constraints may reduce their R&D expenditures (Millard and Nicolae, 2014).

Note that indeed, the disaster indicator x_{t+1} finds itself canceled out from this part of the model. Then, as far as objective function is concerned, let us define $v_t = \frac{V_t}{z_t^{1-\psi}}$, with $\tilde{V}_t = V_t^{\frac{1}{1-\psi}}$, such that we get

$$v_t = [c_t(1 - L_t)^\varpi]^{1-\psi} + \beta(\theta_t)e^{(1-\psi)\mu} \left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi} \right]^{\frac{1}{1-\chi}} \quad (5)$$

where $\chi = 1 - \frac{1-\gamma}{1-\psi}$ is a combination of parameters, and where the discount factor becomes a function of the time-varying disaster risk as given by

$$\beta(\theta) = \beta_0 \left[1 - \theta_t + \theta_t e^{(1-\gamma)\ln(1-\Delta)} \right]^{\frac{1-\psi}{1-\gamma}} \quad (6)$$

which is similar to Gourio (2012)'s expression.⁸ The households will thus maximize (5) subject to (1)-(4) and (6).⁹

The beauty of Gourio (2012)'s detrending approach is that the disaster risk, θ , affects *in fine* the macroeconomic quantities and asset prices through changes in the discount factor only. In that respect, an unexpected change in the disaster risk is expected to generate the same effects as exogenous preference shocks à la Smets and Wouters (2003) or Christiano et al. (2011). A closer look at the expression (6) above makes it clear that the value of the EIS determines the sign of the effect of the probability of disaster (θ) on the discount factor, and thus agents' propensity to save or consume in response to such a shock. In particular, agents become more patient (higher $\beta(\theta)$) whenever the EIS is below unity, and on the contrary, more impatient (lower $\beta(\theta)$) for all values of the EIS larger than unity. Note that this holds for all degrees of risk aversion, including risk neutrality.¹⁰ In the specific case where the EIS is exactly equal to unity, $\beta(\theta)$ boils down to β_0 , i.e the probability of disaster does not have any impact on the macroeconomic quantities, but only on asset pricing. Thus, here as in Gourio (2012), the so-called 'Tallarini (2000)'s equivalence', i.e the quantities being determined irrespectively of the level of aggregate risk or risk aversion, holds if and only if EIS = 1.

⁸Gourio (2012) also makes the size of the disaster, Δ , a random variable. We consider it constant here for the sake of simplicity, but could easily introduce this feature as well.

⁹The first-order conditions and calculation details are provided in Appendix.

¹⁰Rewriting (6) as $\beta(\theta) = \beta_0 \left[1 - \theta_t \left(1 - e^{(1-\gamma)\ln(1-\Delta)} \right) \right]^{\frac{1-\psi/EIS}{(1-\gamma)(1+\varpi)}}$ and taking the derivative with respect to θ , it is the case that $\partial\beta(\theta)/\partial\theta$ is positive (respectively, negative) for any EIS smaller (respectively larger) than one. This holds for all degrees of risk aversion (including risk neutrality), i.e $\forall\gamma \geq 0$. See more about this in Appendix.

In the partial equilibrium built so far, the sign of the preference shift in response to the disaster risk shock suffices to determine the sign of the output variation: an increase in the probability of disaster causes a recession when the EIS is larger than one, a boom otherwise. Intuitively, higher impatience makes agents save less, thus invest less, such that the output falls. Yet, an EIS smaller than unity is empirically sound, and it seems surprising that disaster risk is positively correlated with output in that case. This is the puzzle we solve here in general equilibrium by introducing sticky prices. The nominal rigidity does not alter the relationship between the value of the EIS and the sign of the preference shift discussed above. However, it makes output respond primarily to consumption rather than savings, and thus allows to conciliate recessionary effects of the disaster risk with an EIS lower than unity.¹¹

2.1.3 The stochastic discount factor

The (real) stochastic discount factor is defined under Epstein-Zin preferences as $Q_{t,t+1} \equiv \frac{\partial \tilde{V}_t / \partial C_{t+1}}{\partial \tilde{V}_t / \partial C_t}$. For the non detrended model, this gives us

$$Q_{t,t+1} = \beta_0 \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varpi(1-\psi)} \frac{V_{t+1}^{-\chi}}{(E_t V_{t+1}^{1-\chi})^{\frac{-\chi}{1-\chi}}}$$

which is identical to Gourio (2012)'s, and from the detrended terms

$$Q_{t,t+1} = \frac{z_t}{z_{t+1}} \left(\frac{c_{t+1}}{c_t} \right)^{-\psi} \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varpi(1-\psi)} \beta(\theta_t) e^{(1-\psi)\mu} \frac{e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{-\chi}}{\left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi} \right]^{\frac{-\chi}{1-\chi}}}$$

Since the disaster event (x) is present within the productivity growth term here, we also need to define a “detrended” stochastic discount factor as $\tilde{Q}_{t,t+1} \equiv \frac{z_{t+1}}{z_t} Q_{t,t+1}$ that we can use, together with the first-order condition on bonds so as to solve for the macroeconomic quantities

$$E_t \tilde{Q}_{t,t+1} = E_t \left(\frac{1 + \pi_{t+1}}{1 + r_t} e^{\mu + \varepsilon_{z,t+1}} \right) \quad (7)$$

¹¹It can be the case that some other model specifications allow for the same results. In particular, a real business cycle model with price determinacy for some periods or a simpler New Keynesian model without capital could potentially reach the same conclusions.

For determining the asset prices in the following subsection, we however still use the proper stochastic discount factor as given by

$$E_t Q_{t,t+1} = E_t \left(\frac{1 + \pi_{t+1}}{1 + r_t} \frac{1}{e^{x_{t+1} \ln(1-\Delta)}} \right) \quad (8)$$

2.1.4 The risk premium

From asset pricing orthogonality condition we can define the following rates

(i) The *riskfree rate*, R^f , is obtained from

$$E_t \left[Q_{t,t+1} R_{t+1}^f \right] = 1$$

Note that this is not the yield on bonds, which are only riskfree conditional on no disaster here, but rather a “natural” (gross) interest rate.

(ii) the (real) *rate of return on capital* $R_{t+1}^{k,real}$, from $E_t \left[Q_{t,t+1} R_{t+1}^{k,real} \right] = 1$, which can be rewritten, replacing $Q_{t,t+1}$ from equation (8), as

$$R_{t+1}^{k,real} = \frac{z_{t+1}}{z_t} \frac{1}{e^{\mu + \varepsilon_{z,t+1}}} \frac{1 + r_t}{1 + \pi_{t+1}} = e^{x_{t+1} \ln(1-\Delta)} \frac{1 + r_t}{1 + \pi_{t+1}}$$

Using the first-order condition on capital and non-detrended terms, we get

$$R_{t+1}^{k,real} = e^{x_{t+1} \ln(1-\Delta)} \left\{ \frac{P_{t+1}^k}{p_{t+1}} \frac{u_{t+1}}{q_t} + \frac{q_{t+1}}{q_t} \left[1 - \delta_0 u_{t+1}^\eta + \tau \frac{i_{t+1}}{k_{t+1}} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right) - \frac{\tau}{2} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right)^2 \right] \right\}$$

which reminds both Gourio (2012)’s centralized economy version, as the disaster event directly affects the return on capital, and the expression in DSGE models in the absence of disaster risk (e.g Benigno and Paciello, 2014).

(iii) The *risk premium* of holding capital is defined, in gross terms, as the ratio of the real return on capital (ii) to the riskfree rate (i), i.e

$$E_t(\text{Risk premium}_{t+1}) \equiv E_t(R_{t+1}^{k,real} / R_{t+1}^f)$$

Note that the risk premium is nil in a first-order approximation, constant in the second order, and time-varying in the third and higher orders.

The value of the EIS has a partial equilibrium effect on the responses of these asset returns to the disaster risk shock. In order to better understand it, let us have a look at the expression of these asset returns in the balanced growth path of our economy. First, the stochastic discount factor is

$$Q(x') = \beta_0 \frac{e^{-\psi\mu - \gamma x' \ln(1-\Delta)}}{\left(E e^{(1-\gamma)x' \ln(1-\Delta)} \right)^{\frac{\psi-\gamma}{1-\gamma}}}$$

which is a function of the disaster state, x . From the orthogonality condition $E(M(x')R^k(x')) = 1$, the return on capital is

$$E(R^k(x')) = \frac{E(e^{x' \ln(1-\Delta)})}{\beta_0 e^{-\psi\mu} \left(E e^{(1-\gamma)x' \ln(1-\Delta)} \right)^{\frac{1-\psi}{1-\gamma}}}$$

while the riskfree rate is

$$R^f = \frac{\left(E e^{(1-\gamma)x' \ln(1-\Delta)} \right)^{\frac{\psi-\gamma}{1-\gamma}}}{\beta_0 e^{-\psi\mu} E \left(e^{-\gamma x' \ln(1-\Delta)} \right)}$$

Note that the riskfree rate decreases in the disaster risk (along the balanced growth path), and the smaller the EIS, the larger the drop.¹² This results is well known in the literature and often justifies the need for a use of an EIS larger than unity in order to limit the fall in the riskfree rate (Tsai and Watcher (2015)).¹³ However, in our general equilibrium setup, the nominal rigidity modifies the effect of the EIS: the drop in the riskfree rate is then larger when the EIS is above unity. Asset pricing effects of the disaster risk can thus be restored with an EIS smaller than unity.

Finally, the risk premium along the balanced growth path is given by

¹²See the Appendix for calculation details.

¹³An increase in disaster risk directly reduces the price of equities as it lowers expected cash flows. But meanwhile, it causes an increase in precautionary savings which diminishes the risk-free rate, and in turn tends to increase the price of equities by increasing demand relatively to the supply of equities. Whether this latter effect offsets the former depends on the value of EIS. Indeed, the smaller the EIS, the larger the precautionary savings and the drop in the risk free rate. For example, Berkman et al. (2011) show that the probability of disasters, defined as political crises, is negatively correlated with stock prices. Evidence of this kind encouraged the asset pricing literature to adopt an EIS larger than one.

$$\frac{E(R^k(x'))}{R^f} = \frac{E(e^{x' \ln(1-\Delta)})E(e^{-\gamma x' \ln(1-\Delta)})}{E(e^{(1-\gamma)x' \ln(1-\Delta)})}$$

As expected, the risk premium depends positively on the disaster risk, and the larger the risk aversion, the larger the effect. If agents were risk neutral, i.e when $\gamma = 0$, the risk premium is unaffected by changes in the probability of disaster. Note that the EIS does not directly impact the value of the risk premium along the balanced growth path, in line with Gourio (2012).¹⁴ However, in the general equilibrium, the EIS will affect the responses of the risk premium to the disaster risk shock (See Section 4).

2.2 Firms

The structure of production considered here is quite standard as for a New Keynesian model. However, it plays crucial role for our results: unlike Gourio (2012)'s centralized economy flexible-price model, the decentralized economy featuring monopolistic competition and sticky prices allows to obtain recessionary effects from a disaster risk shock when the EIS is smaller than unity. The nominal price friction makes the response of output affected mostly by the demand side (consumption) rather than the supply side (savings) of the economy. Thus, the drop in consumption associated with a rise in disaster risk when the EIS is below unity will generate here recession and deflation.

Firms are operating in two sectors, final good production and intermediate good production. The former market is competitive, while the latter is monopolistic. They are briefly described below, see Appendix for details.

2.2.1 Final good production

The final good is an aggregate of intermediate goods j as given by

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

where ν is the elasticity of substitution among intermediate goods. Profit maximization gives a demand curve which is decreasing in the price of intermediate good j relative to the aggregate price index ($p_{j,t}/p_t$) as

¹⁴Gourio (2012) finds the same expression for the risk premium along the balanced growth path (Proposition 5). However, general equilibrium effects will differ here.

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

2.2.2 Intermediate sector

Intermediate sector firms use households' capital and labor to produce goods j , according to a Cobb-Douglas function with labor-augmenting productivity. In each period, they optimize the quantities of factors they want to use, taking their prices as given, subject to the production function and the aggregate demand function at a given output price. With frequency determined by a constant Calvo probability, they also set their price optimally.

The intra-temporal problem (cost minimization problem) is thus

$$\begin{aligned} \min_{L_{j,t}, \tilde{K}_{j,t}} \quad & W_t L_{j,t} + P_t^k \tilde{K}_{j,t} \\ \text{s.t.} \quad & \tilde{K}_{j,t}^\alpha (z_t L_{j,t})^{1-\alpha} \geq \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t \end{aligned}$$

where W_t^{nom} is the (non-detrended) nominal wage rate and P_t^k the capital rental rate. The first-order conditions, expressed in detrended terms, are

$$\begin{aligned} (L_{j,t} :) \quad & w_t = mc_{j,t}^{nom} (1 - \alpha) \left(\frac{\tilde{k}_{j,t}}{L_{j,t}} \right)^\alpha \\ (\tilde{K}_{j,t} :) \quad & P_t^k = mc_{j,t}^{nom} \alpha \left(\frac{\tilde{k}_{j,t}}{L_{j,t}} \right)^{\alpha-1} \end{aligned}$$

in which the Lagrange multiplier denoted $mc_{j,t}^{nom}$ can be interpreted as the (nominal) marginal cost associated with an additional unit of capital or labor. Rearranging further gives an optimal capital to labor ratio which is the same for all intermediate firms in equilibrium.

Let's now consider the inter-temporal problem of a firm that gets to update its price in period t and wants to maximize the present-discounted value of future profits. Given the (real) profit flows that read as $\frac{p_{j,t}}{p_t} Y_{j,t} - mc_t^* Y_{j,t}$ and the demand function $Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$, the maximization problem is

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\left(\frac{p_{j,t}}{p_{t+s}} \right)^{1-\nu} Y_{t+s} - mc_{t+s}^* \left(\frac{p_{j,t}}{p_{t+s}} \right)^{-\nu} Y_{t+s} \right)$$

where discounting includes both households' stochastic discount factor, $Q_{t,t+s}$, and the probability ζ^s that a price chosen at time t is still in effect at time s . After some simplification, the first-order condition is

$$p_{j,t}^* = \frac{\nu}{\nu - 1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu} Y_{t+s} mc_{t+s}^*}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s}}$$

which depends on aggregate variables only, so that $p_t^* = p_{j,t}^*$. Increases in this optimal price from one period to another will give us the reset inflation rate while increases in the current price level p_t defines the current inflation rate. (see Appendix for more details.)

2.3 Public authority

Bonds clears with public debt issued by a public authority which raises taxes from the households. The public authority also sets up the nominal interest rate on bonds following a Taylor type rule as

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) [\Phi_{\pi}(\pi_t - \bar{\pi}) + \Phi_Y(y_t - y^*) + r^*] \quad (9)$$

2.4 Equilibrium

The optimality conditions for the three representative agents' problem described above are derived in Appendix. They are solved together with the aggregate constraints also described in Appendix.

3 Calibration and steady-state analysis

Table 1 summarizes our main calibration values. We follow the literature estimating disasters on historical data. In particular Barro and Ursúa (2008) estimated that the size of disasters is 22% while the probability of such events is 0,9% quarterly.¹⁵ Following Gourio (2012), we consider a persistence of 0.9.

¹⁵Evidence on disasters' size and frequency is quite mixed and highly dependent on the methods used for the estimation. We take Barro and Ursúa (2008)'s values which seem to be in the middle of the range of other studies. Barro (2006) have larger disasters (43%)

We also consider alternative values in Section 4.2 and find similar patterns, suggesting that the sign of the responses of interest is quite robust to lower degree of persistence in disaster risk, only the size of the responses differs.

Since our utility function is identical to Gourio (2012), we use the same valuation of leisure ($\varpi = 2.33$) and risk aversion coefficient ($\gamma = 3.8$). In Section 4.2, we test alternative values of the risk aversion parameter, which is however constrained numerically by the fact that the endogenous discount factor $\beta(\theta)$ should be kept below one for solving the model.

All the other values are in line with the New Keynesian literature, in particular parameters entering the production function, the capital accumulation, and the Taylor-type rule are conventional. We compare the standard value for the Calvo probability of firms not changing their price ($\zeta = 0.8$) to a purely flexible price case with $\zeta = 0$, for two different values of the EIS, either 0.5 (as a plausible value) or 2 (as in Gourio, 2012). Here as well, we try intermediate degrees of price stickiness in Section 4.2.

Table 2 shows the steady-state values obtained under our calibration for some selected variables. In particular, we compare the economy without disaster, i.e having either a probability of disaster (θ) or a size of disaster (Δ) equal to zero, to the economy with disaster (for two example sizes, $\Delta = 0.22$ and $\Delta = 0.40$). This is reported here for three different cases: flexible prices ($\zeta = 0$) and EIS = 2 (economy à la Gourio), flexible prices and EIS = 0.5, sticky prices ($\zeta = 0.8$) and EIS = 0.5 (baseline scenario).

The role of the EIS is particularly worth discussing here. In the economy with an EIS below 1, agents have a high propensity to consume the certainty-equivalent income (see Weil (1990)). Thus, steady-state consumption has to be lower in the economy with disaster risk than the economy without. Intuitively, one can think that agents make precautionary savings if they expect a potential disaster to arrive. The same reasoning holds for providing more labor and capital initially in an economy that will be potentially affected by a disaster. Thus current output is higher. One can also see this higher ‘degree of patience’ in the (time-varying) discount factor and the stochastic discount factor. This holds whether prices are flexible or sticky.

On the contrary, with an EIS larger than 1, agents do not make so much precautionary savings and precautionary labor supply. Thus investment and

with a probability of 0,72% quarterly, whereas Gourio (2013) have smaller disasters (15%) with a probability of 0,5% quarterly.

Table 1: **Baseline calibration values (quarterly)**

Disaster risk		
$\bar{\theta}$	disaster risk	0.009
Δ	disaster size	0.22
ρ_θ	disaster risk persistence	0.9
Utility function		
β_0	discount factor	0.99
$1/\tilde{\psi}$	elasticity of intertemporal substitution	0.5
γ	risk aversion coefficient	3.8
ϖ	leisure preference	2.33
Investment		
δ_0	capital depreciation rate	0.02
τ	investment adjustment costs	1.7
\bar{u}	utilization rate of capital	1
Production		
α	capital share of production	0.33
ζ_0	Calvo probability	0.8
ν	elasticity of substitution among goods	6
μ	trend growth of productivity	0.005
Public authority		
ψ_π	Taylor rule inflation weight	1.5
ψ_Y	Taylor rule output weight	0.5
$\bar{\pi}$	target inflation rate	0.005
ρ_r	interest rate smoothing parameter	0.85

output are lower, and by wealth effect so is consumption, when disaster risk is present in the economy versus not. Note that in both cases, the return on capital is of course decreasing in disaster risk. As one can also expect, the risk premium is nil in all cases as the agents make financial arbitrage with perfect foresight at the steady-state. The Tobin's q remains unchanged since the disaster risk does not affect directly the macroeconomic quantities of the detrended system, unlike a capital depreciation shock for instance.

Only in case when the EIS tends to unity, steady-state values tend to be equal in economies with and without disaster risk. Indeed, when the EIS is equal to 1, the time-varying discount factor ($\beta(\theta)$) boils down to the usual discount factor β_0 (in equation (6)), and the disaster risk does not affect the economic outcomes anymore. This results is referred to as Tallarini (2000)'s "observational equivalence", stating that the macroeconomic quantities are unaffected by the amount of risk in the economy. Again, here as in Gourio

	no disaster risk $\Delta = 0.22$ and $\theta \rightarrow 0$ or $\Delta = 0$ and $\theta = 0.9\%$	baseline disaster risk $\Delta = 0.22$ $\theta = 0.9\%$	large disaster risk $\Delta = 0.4$ $\theta = 0.9\%$
EIS = 0.5, sticky prices ($\zeta = 0.8$)			
output (detrended)	0.614	0.625	0.651
consumption (detrended)	0.499	0.505	0.518
investment (detrended)	0.115	0.121	0.133
labor	0.228	0.229	0.232
capital (detrended)	4.608	4.820	5.332
$\beta(\theta)$	0.990	0.991	0.993
Tobin's q	1	1	1
wage	1.505	1.525	1.570
capital rental rate	0.037	0.036	0.034
stochastic discount factor	0.984	0.986	0.990
return on capital	1.017	1.014	1.010
risk premium	1	1	1
EIS = 0.5, flexible prices ($\zeta = 0$)			
output (detrended)	0.614	0.626	0.652
consumption (detrended)	0.499	0.505	0.518
investment (detrended)	0.115	0.120	0.133
labor	0.228	0.229	0.232
capital (detrended)	4.604	4.818	5.333
$\beta(\theta)$	0.990	0.991	0.993
Tobin's q	1	1	1
wage	1.506	1.526	1.572
capital rental rate	0.037	0.036	0.034
stochastic discount factor	0.984	0.986	0.990
return on capital	1.017	1.014	1.010
risk premium	1	1	1
EIS = 2, flexible prices ($\zeta = 0$)			
output (detrended)	0.642	0.635	0.623
consumption (detrended)	0.513	0.510	0.504
investment (detrended)	0.128	0.125	0.119
labor	0.230	0.230	0.229
capital (detrended)	5.129	5.008	4.766
$\beta(\theta)$	0.990	0.990	0.989
Tobin's q	1	1	1
wage	1.554	1.543	1.521
capital rental rate	0.034	0.035	0.036
stochastic discount factor	0.986	0.987	0.988
return on capital	1.014	1.013	1.012
risk premium	1	1	1

Table 2: Steady-state values

(2012), this holds if and only if the EIS is equal to one. In all other cases, quantities differ from the economy without disaster risk.

4 Impulse Response Functions

In this Section, we simulate the effect of a rise in the small probability of disaster (θ) from the stochastic steady-state. Unless specified, the approximation is made at the third-order such that asset pricing and macroeconomic variables interact with each other.¹⁶ Our aim here is essentially qualitative, and consists in comparing the effect of the shock in the following four cases:

- (i) With flexible prices and $EIS = 2$, a decentralized version of Gourio (2012)'s economy, in order to nest his results as a particular case;
- (ii) Still under flexible prices but with $EIS = 0.5$, i.e a value more in line with the standard RBC and New Keynesian literatures and with micro estimates. Gourio (2012)'s results are then found to be reversed;
- (iii) With sticky prices and the same $EIS = 0.5$, as for our baseline scenario. The recessionary effect is then restored and generalized;
- (iv) With sticky prices and $EIS = 2$, the mirroring case of (iii).

Since we use perturbation methods here, we consider a very small deviation from the itself very small probability of disaster at the steady-state, more specifically a change from $\bar{\theta} = 0.009$ to 0.01. As a consequence, the size of the responses that we get will naturally be small as well. A larger shock could of course give a better feel for the magnitude of the effects we are describing here, yet this would be at the price of potentially large errors.¹⁷ In addition, we try some alternative calibration values, and finally simulate the responses of the model to a standard monetary policy shock. This allows to check the accuracy of our model in replicating well-known perturbation responses in spite of the presence of disaster risk.

¹⁶Indeed, the risk premium is nil in the first-order, constant at the second-order, but fluctuates in response to the disaster risk shock with a third-order approximation.

¹⁷Using projection methods, Gourio (2012) considers a deviation from 0.72% to 4% for the probability of disaster. His results indicate the magnitude of the effects of a disaster risk shock, while we mostly focus on the sign of the responses here.

4.1 Effects of a disaster risk shock: qualitative effects

4.1.1 Case 1: flexible prices and EIS = 2 (à la Gourio, 2012)

Figure 1 shows the effect of the disaster risk shock in an economy à la Gourio (2012), here in a decentralized market version instead of the RBC model, with flexible prices ($\zeta = 0$) and an EIS of 2, according to his calibration. The shock makes the agents more impatient ($\beta(\theta)$ decreases). Hence, they save and thus invest less, such that the economy enters a recession, while the risk premium goes up. However, agents then consume more and work less, so the wage increases. These results are identical to Gourio (2012)'s and partly consistent with the evidence on disaster risk (see Barro (2006) or Gourio (2008) for instance). However, these predictions rely on accepting an EIS greater than 2, which contrasts with commonly accepted values. We propose to look for the responses with a different EIS in Case 2 below.

4.1.2 Case 2: flexible prices and EIS = 0.5

In this case, we just changed the value of the EIS to 0.5 compared to the previous case. As one can see on Figure 2, this is enough to make the sign of most variables completely opposite. Contrary to Case 1, a low EIS implies that agents' propensity to save increases with the disaster risk. This is captured here by an increase in the discount factor, that can be interpreted as a higher degree of patience. This makes the agents save more and invest more. The lower consumption on impact does not have much effect on the total output response. The price of goods drops on impact but rises immediately after (since there is no price rigidity in this case). Hence firms expect the deflation to be short and want to increase their demand for production factors, as well as the utilization rate of capital. Therefore, the rental rate of capital goes up, making the households willing to invest more. Overall, the rise in investment is higher than the drop in consumption, such that the economy enters a boom. As consumption decreases, the marginal utility increases, so the labor supply increases and the wage goes down despite the boom.

As far as asset pricing is concerned, we can see here that the risk premium still goes up because the disaster risk shock makes it more risky to invest in the capital stock. But, in this case, it implies that the risk premium becomes procyclical, which is highly counterfactual. Also note that, by making the EIS smaller, the magnitude of the increase is now larger than in Case 1.

Figure 1: Responses (in percentage change) to a rise in θ from 0.9% to 1% ($\zeta = 0$ and EIS = 2).

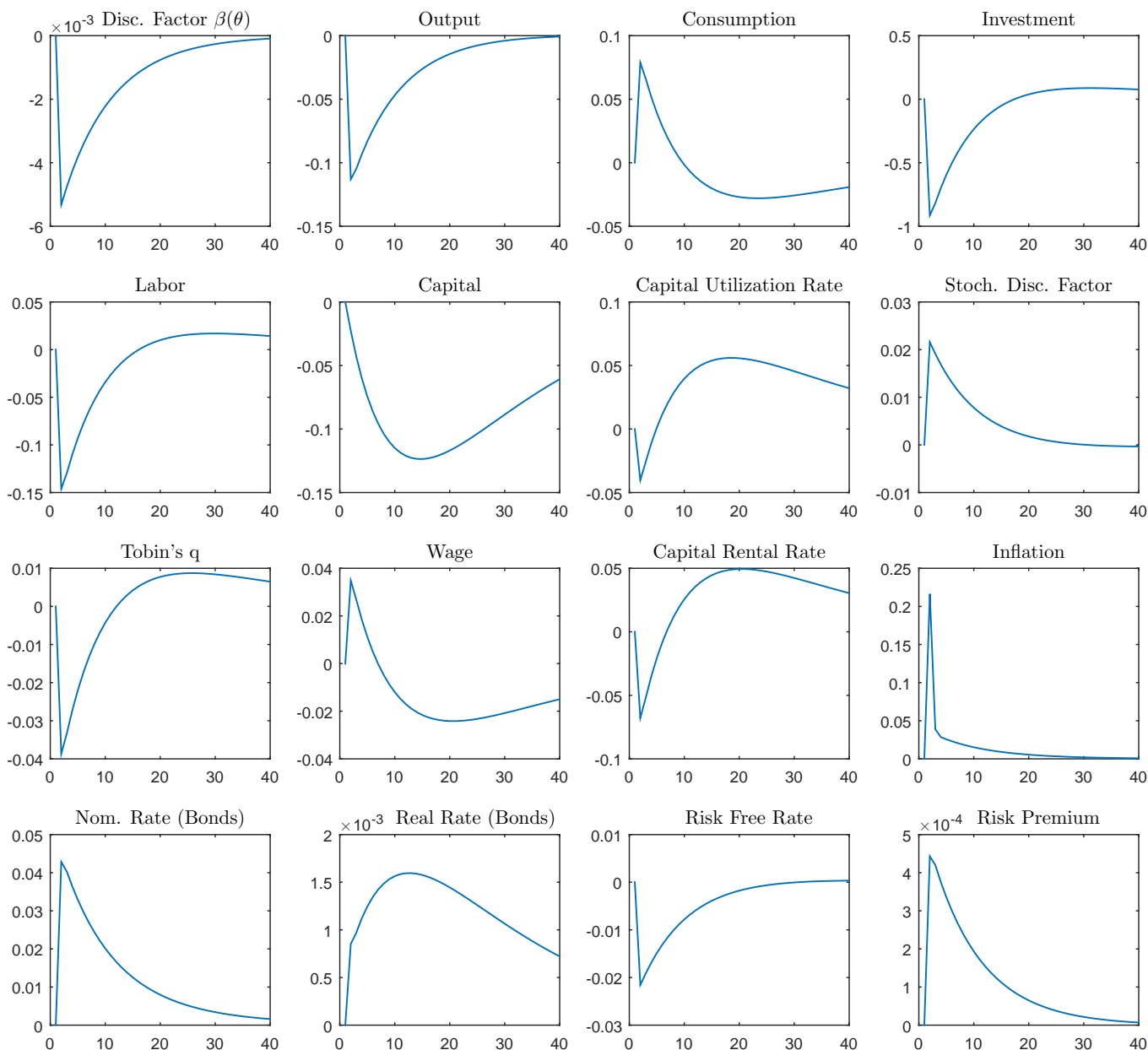
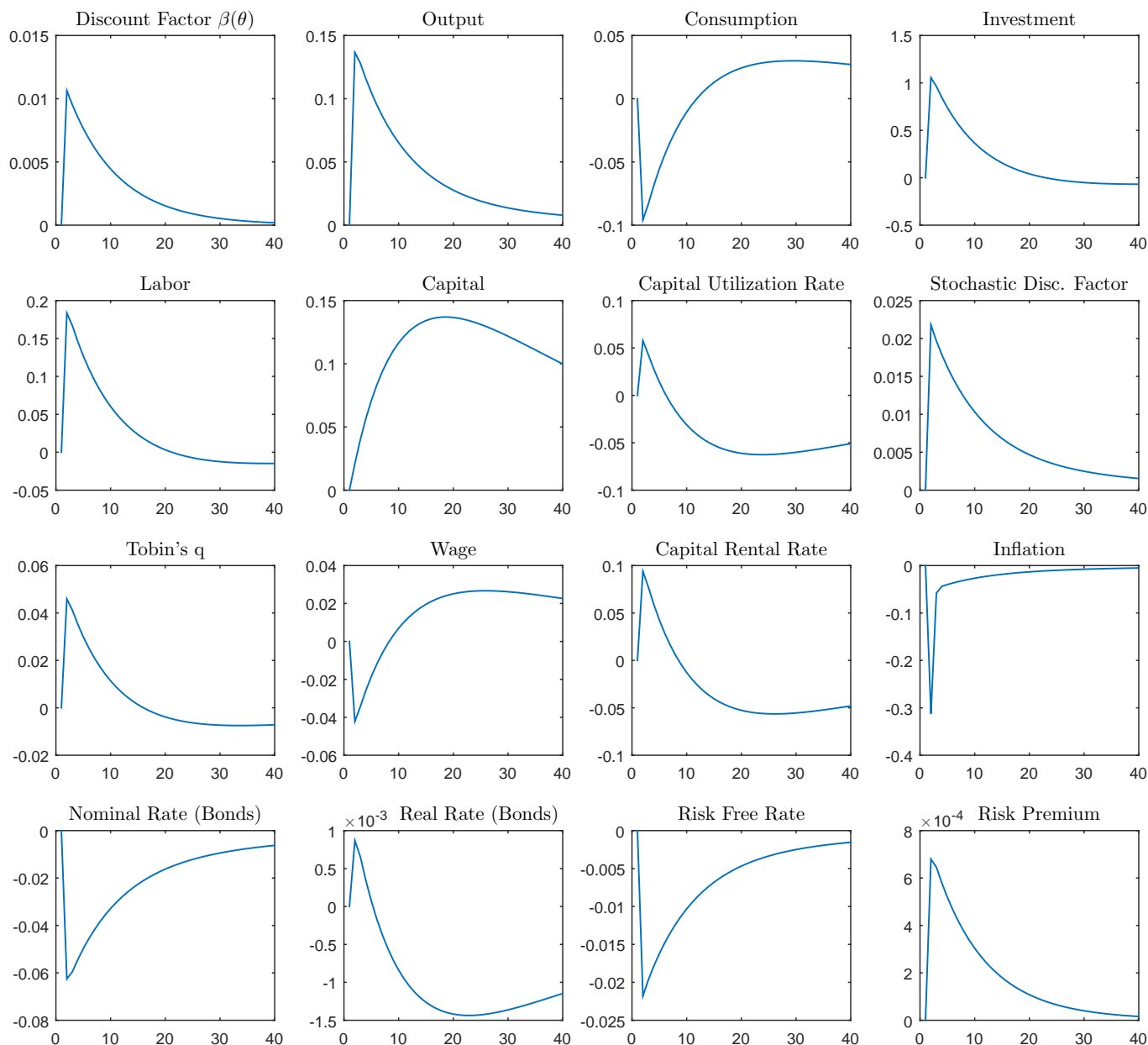


Figure 2: Responses (in percentage change) to a rise in θ from 0.9% to 1% ($\zeta = 0$ and EIS = 0.5).



4.1.3 Case 3: sticky prices and EIS = 0.5 (baseline scenario)

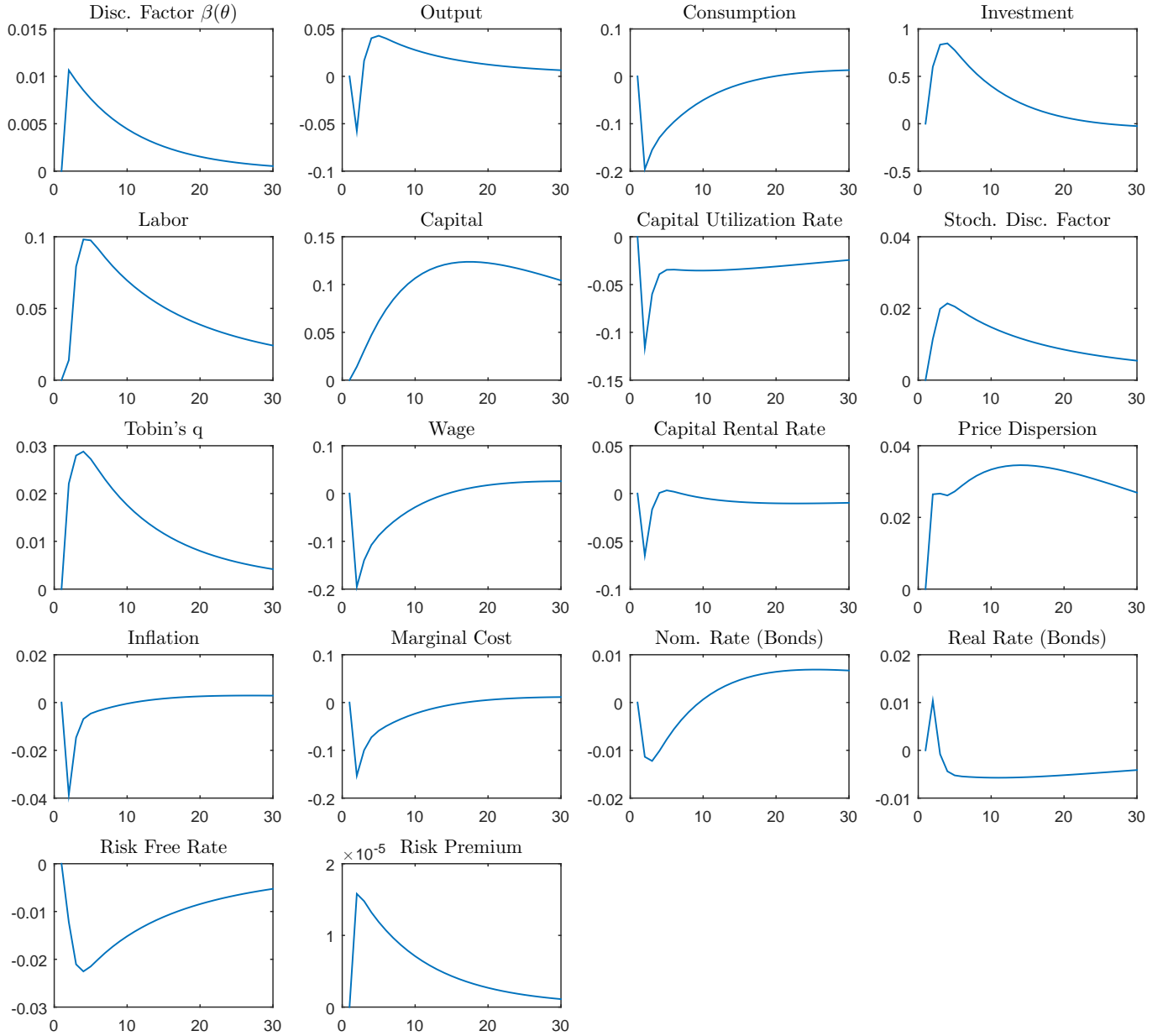
As in Case 2, a low EIS makes the agents more patient (they have a higher $\beta(\theta)$) following the disaster risk shock, which gives them an incentive to save, and thus to invest more (Figure 3). As before too, consumption drops. However, the price stickiness makes the relative size of these responses different from Case 2. Indeed, consumption decreases more than investment increases here, such that recession and deflation ensue.

Two specific responses are worth being discussed more carefully here, namely the rise in investment and labor quantity. Both variables are here subject to a precautionary motive from the households when the disaster risk goes up. The agents want to limit the decrease in their consumption by acquiring more capital and increasing their labor supply when a disaster becomes more likely (under the assumption of an EIS below one). Simultaneously, there is also a downward pressure on investment and labor that stems from firms' lower demand for production factors. However, in our case, the former effect overweighs the latter, such that the net effect on investment and labor is positive, for several reasons. First, firms do not control directly for the level of investment in our model but for the capital utilization rate, which indeed drops in Figure 3. Hence, investment is mostly driven by households' precautionary savings rather than firms' lower demand for capital.¹⁸ Second, recall that the households cannot buy the riskfree asset in our economy, such that investment and risky bonds have to go up since they are the only available vehicles for savings. Thus, the larger the precautionary motive, the larger the increase in investment.¹⁹ Third, it may be the case that considering nonconvex capital adjustment costs is sufficient to make the investment decrease while still preserving the sign of our other responses unchanged, as discussed in Bloom (2009) and here in Section 5.4. Finally,

¹⁸Basu and Bundick (2014)'s responses to a volatility shock on the discount factor have some similarities with our shift in the level of the discount factor caused by the disaster risk shock. In particular, they also obtain precautionary labor and investment. Yet, because investment is realized by firms instead of the capital utilization rate as we have here, they overall find a negative net effect of investment to the shock.

¹⁹A way to overcome this effect would be to allow the agents to acquire the riskfree asset in our economy. However then, the detrending method à la Gourio (2012) could not be perfectly applied, and we would have to make an assumption about the state of the economy today being in a 'disaster' regime or not when generating the impulse response functions. It would be straightforward to do so, and the approach that we choose here is just a question of preference for unconditional impulse response functions, closer to the spirit of Gourio (2012).

Figure 3: Responses (in percentage change) to a rise in θ from 0.9% to 1% ($\zeta = 0.8$ and EIS = 0.5).



whether the increase in supply or the decrease in demand of investment and labor is the dominant effect is also highly sensitive to the calibration choices (see Section 4.2). Note finally that both effects contribute here to the drop in wages, which is not observed in the economy à la Gourio (Case 1).

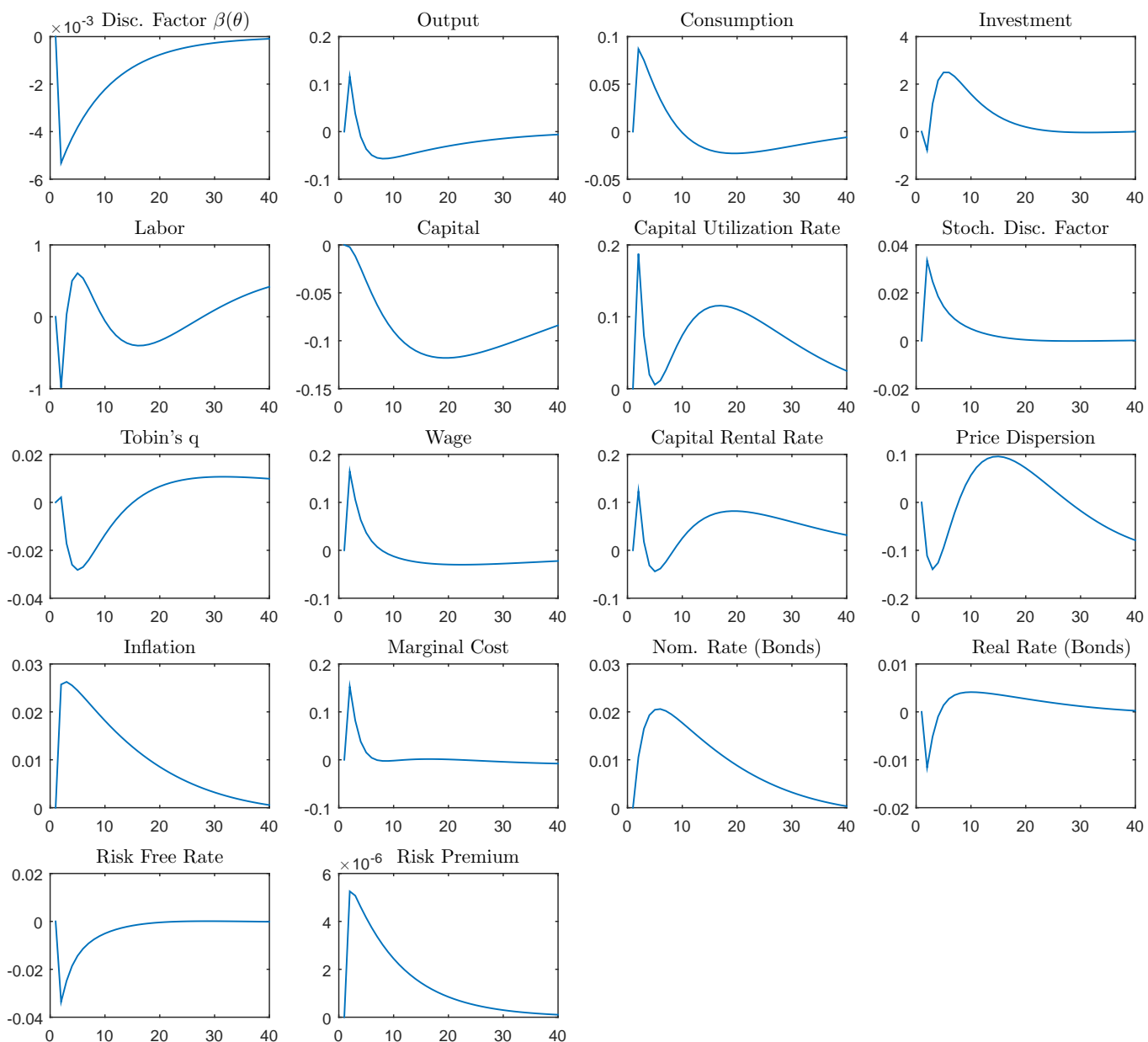
As for asset prices, we observe a ‘flight-to-quality’ effect that is visible through the drop in the riskfree rate when the disaster risk shock hits. This drop in the riskfree rate is of similar magnitude as in Case 1, suggesting that sticky prices may provide a solution to avoid an excessive change in the riskfree rate despite having an EIS smaller than one, as discussed before. Finally, the risk premium also increases here due the disaster risk shock. When compared to the economy à la Gourio (Case 1), the magnitude of this increase is lower here. However, moving from an EIS larger than one to an EIS smaller than one always increases the response of the risk premium to the disaster risk shock, for a given degree of price stickiness/flexibility (comparing Cases 1 and 2 on one hand, and Cases 3 and 4 on the other).

Overall, the responses of both macroeconomic quantities and asset prices to the disaster risk shock in this Case happen to be very similar to those obtained from exogenous preference shocks in the literature (see e.g Smets and Wouters (2003) and Christiano et al. (2011)). They are also reminiscent of the literature on uncertainty shocks. In particular, in giving a short sharp recession followed by an “overshooting” in the recovery, as in Bloom (2009). In that respect, the disaster risk can provide a potential explanation for exogenous shifts in preferences or changes in aggregate uncertainty.

4.1.4 Case 4: sticky prices and $EIS = 2$

In this last scenario, we change again the EIS for a value larger than unity, but still under sticky prices. As we can observe in Figure 4, the same increase in θ now makes the time-varying discount factor drop again. All the other responses are thus the exact opposite to those in Figure 3. In particular, there is a boom (as in Case 2) in output, driven by consumption increase (unlike Case 2). This Case is not a realistic scenario for a disaster risk shock, yet it provides an interesting counterfactual exercise to confirm (i) the effect of the EIS on the discount factor and the propensity to consume/save, and (ii) the fact that the model with nominal rigidity is driven primarily by the response in consumption to the disaster risk shock while the flexible-price version is primarily driven by the supply-side of the economy for a given value

Figure 4: Responses (in percentage change) to a rise in θ from 0.9% to 1% ($\zeta = 0.8$ and $EIS = 2$).



of the EIS. We can also observe that the riskfree rate happens to decrease more than in Case 3 where the EIS was low. This implies that the disaster risk argument about a high value of the EIS being necessary to limit the fall of the riskfree rate does not hold anymore when prices are sticky.

Figures 5 and 6 summarize together the responses in the economy à la Gourio (2012) and in our baseline economy in the same graphs, for easier comparison. This is realized successfully at first-order (Figure 5) and third-order (Figure 6) approximations.

4.2 Sensitivity analysis

We run some sensitivity analysis to alternative calibration values when simulating the effect of our disaster risk shock on the benchmark economy (small EIS, sticky prices). The responses are given in Figure 7.

The first setup of responses is generated against different values of the risk aversion coefficient. Here, we can see that for normal values of risk aversion with Epstein-Zin specification, typically between 3 and 4 (Barro and Jin, 2011), the effects that we describe in Case 3 holds, i.e a recession driven by the drop in output. For an economy closer to risk neutrality (on Figure 8, $\gamma = 1.5$), the effects essentially remain, except for labor, which decreases. This is because the upward effect of the ‘precautionary labor supply’ is now lower than the downward effect of the lower demand of labor from firms when the disaster becomes more likely. Only for very high values of risk aversion (here, $\gamma = 6$), the response of precautionary savings on investment by households is so high that it overcomes the drop in consumption and prevents the recession. Here again, should the households be allowed to buy the riskfree asset in our model setup, the effect would vanish.

Second, we test for different values of the discount factor that would hold without disaster risk in the economy, β_0 . This can be interpreted as a kind of fundamental degree of patience. This is useful to observe that, with agents initially less patient than in our standard calibration ($\beta_0 = 0.99$), the drop in consumption is mitigated while investment also now drops on impact (the precautionary motive for investment turns to be small). Actually, in that case, the downward effect of investment, stemming from a higher chance of destruction of capital by a disaster, overweights the precautionary motive, such that investment decreases on impact, and subsequently limits the overshooting in output following the recession.

Third, we run experiments for the parameters relative to the disaster risk. When the steady-state probability of disaster is tried against alternative values, the qualitative effects are unchanged, except for the labor. Indeed, for a lower probability of disaster, the lower demand effect overweights the higher supply effect, such that the net quantity of labor decreases. The same idea holds for changes of the size of the disaster. Only for large disasters (30%), the precautionary motive is so important that, given agent's incapability to buy the riskfree asset here, investment rises so much that it drives the economy into a boom. To finish with, the persistence of the disaster risk just mitigates the effects but leaves the qualitative responses unchanged.

Finally, Figure 8 shows the responses to a standard monetary policy shock for the baseline scenario. These responses are quite standard and inform us about the validity of the model to conventional shocks.

5 Discussion

5.1 The literature on disaster risk

The literature on rare events has emerged in the 1980s when macroeconomic models were struggling to explain the dynamics of asset prices and their related risk premia. Rietz (1988) has shown that introducing a low probability of an economic disaster into an endowment economy was able to address the the equity premium puzzle (Mehra and Prescott, 1985).²⁰

Estimating economic disasters on US data over the twentieth century, Barro (2006) found a frequency of 1.7% per year and an average size of 29%. Using these results to calibrate an extended version of Rietz (1988)'s model, Barro confirmed that rare disasters could capture high equity premium and low risk-free rate puzzles. Barro and Ursúa (2008) have further extended Barro (2006) by including data on consumption, more relevant for asset pricing models. They assembled international time series since 1870 and found disaster probabilities of around 3.6% per year with a mean disaster

²⁰Other solutions that have been proved able to improve asset pricing of macroeconomic models include notably consumption habits and heteroscedastic shocks (Campbell and Cochrane, 1999) and Epstein-Zin-Weil preferences, separating the risk aversion coefficient from the EIS, combined with stochastic volatility in consumption growth (Bansal and Yaron, 2004). Epstein-Zin-Weil preferences alone contribute only partially to the solution of the equity premium puzzle: in Weil (1989) for instance the risk aversion coefficient must be set at 45 and EIS at 0.1 for a reasonable match with the data to be obtained.

size of 22%. They simulate a Lucas-tree model with i.i.d. growth shocks and Epstein-Zin-Weil preferences and obtain plausible equity premium on levered equity, with a coefficient of relative risk aversion of 3.5.²¹

Despite encouraging results, Rietz (1988), Barro (2006) and Barro and Ursúa (2008)'s models were not able to explain some asset pricing moments such as the volatility of price-dividend ratios for stocks, the volatility of bond risk premia, and return predictability.²² This is essentially due to the fact that they assume the probability of disaster to be constant. Thus, Gabaix (2008) made it time-varying and introduced it in an endowment economy, before Gabaix (2011, 2012) and Gourio (2012, 2013) further considered real business cycle frameworks. A time-varying probability of disaster is indeed able to explain volatility patterns of asset prices and return predictability (Wachter, 2013, Seo and Wachter, 2013). It also solves a number of macro-finance puzzles such as the risk-free rate puzzle or the upward-sloping nominal yield curve (Gabaix, 2012).

Gabaix's (2012) framework is constructed such that variations in the probability of disaster have no impact on macroeconomic quantities. This is reminiscent of Tallarini (2000)'s theorem that macroeconomic dynamics, unlike asset prices, are basically unaffected by changes in aggregate risk. In contrast, Gourio (2012) found that Tallarini (2000)'s "observational equivalence" holds for the presence of disaster risk if and only if the EIS is exactly equal to unity. However, he shows that, as soon as the EIS differs from unity, a shock to the probability of disaster is equivalent, under some assumptions, to a preference shock. His model thus predicts a correlation between asset prices and macroeconomic quantities which is supported empirically.

We here build on Gourio (2012)'s modeling of the disaster risk and incorporate it into a full-fledged New Keynesian model, featuring monopolistic competition, capital adjustment costs, sticky prices, and a Taylor-type rule. To the best of our knowledge, we are the first paper to do so. Thereby, we show that Gourio (2012)'s results can be conciliated with a value of the EIS smaller than unity, and thus generalize them in a framework more suitable for future policy analysis.

²¹Barro and Jin (2011) estimate the risk aversion coefficient from dataset on disasters. They found a mean close to 3, with a 95% confidence interval for values from 2 to 4.

²²In Barro and Ursúa (2008), the price-dividend ratio and the risk-free rate are constant.

5.2 Evidence and calibration of the EIS

There is no clear consensus about the value of EIS in the literature. It is standard in macroeconomic calibrations to take a value smaller than one, whether the utility function is Epstein-Zin and thus the EIS chosen independently from the risk aversion coefficient (Rudebusch and Swanson (2012), Caldara et al. (2012), use both a value of 0.5) or with standard time-additive preferences (Piazzesi et al. (2007) choose 0.2, Smets and Wouters (2007) 0.66). In contrast, macrofinance real business cycles often choose a value higher than unity to match asset pricing moments (Barro and Ursúa (2008), Gourio (2012) and Nakamura et al. (2012) use a value of 2).

This dispersion is due to the fact that empirical evidence on the value of the EIS is yet not conclusive. An influential paper by Hall (1988) finds that this parameter is close to zero, and the subsequent literature has also provided further support for values smaller than one²³, although Hall's results have sometimes been challenged on methodological grounds.²⁴ Moreover, some studies suggest that heterogeneity across agents is an important factor for the EIS estimation.²⁵ As an attempt to explore estimation differences across countries and methodologies, Havránek et al. (2015) have collected 2,735 estimates of EIS reported in 169 studies. From meta-analysis methods, they find a mean estimate around 0.5, and typically lying between 0 and 1. Among six countries for which more than 50 estimates exist, the mean EIS is 0.9 for Japan, 0.6 for the US, 0.5 for the UK, 0.4 for Canada, 0.2 for Israel, and 0.1 for Sweden. While their results suggest that the type of utility function does not affect the reported estimates of the EIS in a systematic way, the cross-country differences are important, yet essentially below unity. Households in countries with higher income per capita and higher stock market participation show larger values of the elasticity.

The fact that the EIS is higher for countries with high stock market participation is important for asset pricing models. Indeed, the disaster risk literature relies on the assumption that EIS is larger than one in order to

²³See for instance Campbell and Mankiw (1989), Ludvigson (1999), Yogo (2004).

²⁴Bansal and Yaron (2004) argue that ignoring time-varying consumption volatility leads to a downward bias in the macro estimates of the EIS, but Beeler and Campbell (2012) question the extent of the bias.

²⁵Blundell et al. (1994) and Attanasio and Browning (1995) find that rich households tend to show a larger EIS. Mankiw and Zeldes (1991) and Vissing-Jorgensen (2002) find a larger EIS for stockholders than for non-stockholders. Bayoumi (1993) finds that liquidity-constrained households show a smaller EIS.

reproduce the key asset pricing behavior (Nakamura et al., 2013). In particular, a high value of EIS is important to prevent the risk-free rate from declining too much following the disaster risk. However, in a recent empirical estimation of Wachter (2013)’s time-varying disaster risk framework, Irarrazabal and Parra-Alvarez (2015) find that a low EIS is consistent with appropriate levels of the equity premium and volatility of government bonds, while an EIS greater than one can generate higher price-dividend volatility and stock market volatility, and hence a lower Sharpe Ratio.

In asset pricing models, an EIS larger than 1 reduces the precautionary saving effect and hence limits the decline in risk-free rate and the counter-intuitive increase in asset prices. Nevertheless, we show here that, when the model features sticky prices, the decrease in the risk free rate following a disaster risk shock is smaller with an EIS smaller than one. The New Keynesian framework thus plays a crucial role in determining the effect of the EIS on macroeconomic dynamics and asset pricing variables.

5.3 Preference shocks

As disaster risk is captured by a shift in preferences, our effects resemble those from exogenous shocks to the discount rate.

The asset pricing literature argues that supply-driven shocks alone cannot account for the observed movements in asset prices and thus points out the needs for considering shocks to ‘preference shocks’ (Campbell and Ammer (1993), Cochrane (2011)). More generally, by changing the demand for assets, preference shocks have been successful in matching the equity premium, the bond term premium, and the weak correlation between stock returns and fundamentals by generating a good fit for risk-free rate variations independently of cash flows (see for instance the early work of Campbell (1986) and recent papers by Schorfheide et al. (2014) and Albuquerque et al. (2015)).

However, these models, just as Gourio (2012), generally consider a *negative* shock to the discount factor, i.e an increase in agents’ impatience: they suddenly want to consume more and hold fewer assets. On the contrary, the New Keynesian literature generally studies the effects of *positive* preference shocks, i.e a decrease in agents’ patience. This has also been successful in some respects, in particular lately in making the zero lower bound (ZLB) for policy interest rate binding (see Eggertsson and Woodford (2003), Eggerton et al. (2014), and Erceg and Linde (2012)). An increase in the

discount factor decreases agents' propensity to consume and puts downward pressure on real factor prices, real marginal cost and inflation. The real interest rate must diminish to reduce deflationary pressures. Empirically, variance decomposition shows that positive preference shocks is one of the main determinants of the nominal interest rate (Smets and Wouters (2003) and Ireland (2004)).

One of our contributions here is to conciliate the asset pricing partial equilibrium effects of a *negative* preference shock, by considering the exact same mechanism as in Gourio (2012), with the general equilibrium effects obtained with exogenous preference shocks in the New Keynesian literature. This is far from trivial as both literatures have found a recession and an increase in risk premia (or flight-to-quality) to be associated with opposite movements in the discount factor. Yet, we show here that Gourio's definition of the disaster risk can give responses compatible with the predictions for exogenous positive preference shocks whenever an EIS below unity and sticky prices are adopted. In line with the ZLB literature, we find that preference shifts lower interest rates on bonds, cause deflation and recession.²⁶

5.4 Uncertainty shocks

Another class of shocks related to ours is changes in aggregate uncertainty, or second-moment shocks. The shock we consider here affects the *level* of the probability of disaster. However, its effects are very similar to a volatility shock, and can be understood as a potential source of it.²⁷ Bloom (2009) finds that an increase in uncertainty generates an immediate drop in output, as well as in labor and productivity as firms wait before hiring and reallocation from low to high productivity firms is impeded. In the medium term

²⁶The only apparent difference with Christiano et al. (2011) concerns the response of investment. Given that capital accumulation is included in our model, a decrease in the real interest rate drives investment up following the shock. In a model extension, Christiano et al. (2011) also considers capital accumulation but because the ZLB prevents the nominal interest rate from declining following a preference shock, and deflation arrives, the real interest rate increases, such that investment naturally cannot rise. Our model does not impose such a constraint but it would be straightforward to do so. In that case, a fall in investment would deepen the recessionary effects of disaster risk.

²⁷Baker and Bloom (2013) use rare events, such as natural disasters, terrorist attacks, political coups d'état and revolutions to instrument for changes in the level and volatility of stock-market returns. They argue that some shocks, like natural disasters, lead primarily to a change in stock-market levels (first-moment shocks), while other shocks like coups d'état lead mainly to changes in stock-market volatility (second-moment shocks).

however, the economy bounces back as firms address their pent-up demand for labor and capital. With this respect, a disaster risk shock has a similar effect to uncertainty shock as it produces a short sharp recession followed by an “overshooting” in the recovery.

As for the response of investment, Saltari and Ticchi (2007) show that, when the EIS is low, higher uncertainty is associated with an increase in investment even if the risk aversion is high. This effect holds for a disaster risk shock in our model. In the same vein, Bansal and Yaron (2004) show that an increase in uncertainty increases precautionary savings and diminishes output in the short run. In the long run however, an increase in savings can have positive impact on output via an increase in investment. Unlike an open-economy setting, where excess savings translates into holdings of foreign assets rather than domestic investment, Fernández-Villaverde et al. (2011a) show that an increase in uncertainty makes investment go up in a closed-economy version. Yet output goes down because sticky prices preventing an immediate adjustment. The same intuition holds in our model.

Finally, a related but slightly separated literature has considered ‘disasters’ as rare events stemming from non-Gaussian shock distributions. Cúrdia et al. (2014) and Chib and Ramamurthy (2014) show evidence that models with a multivariate t -distributed shock structure are favored by the data over standard Gaussian models. Auray et al (2012) show that limiting distributions of several aggregate macroeconomic time series, such as GDP, real wages and capital stock, exhibit fat tails if the returns to scale episodically increase. Andreasen (2012) studies rare disasters and uncertainty shocks, through skewed shock distributions, affect risk premia in a DSGE model. Weitzman (2012) also examines the effects of nonnormalities and rare disasters on risk premia. He finds that, with a higher probability weight on very bad outcomes, tail fattening reduces the magnitude of equity premium and riskfree rate puzzles. Finally, Wachter (2013) analyzes the effects of a time-varying probability of a consumption disaster and assumes that the conditional distribution of consumption growth becomes highly non-normal when a disaster is relatively likely.

6 Conclusion

This paper has developed a New Keynesian model featuring a small but time-varying probability of rare events à la Gourio (2012). The purpose was twofold. First, we aimed at conciliating the recessionary effects of an unexpected rise in disaster risk with a standard value of the elasticity of intertemporal substitution. Indeed, we have first shown that Gourio (2012)'s flexible-price results hold if and only if the EIS is above unity. However, we then argued that the presence of sticky prices provides a solution by making the response of output primarily impacted by the response of consumption, instead of savings in real business cycle models.

Second, we aimed at conciliating the macroeconomic effects of the disaster risk with the preference shock literature. Indeed, under Gourio (2012)'s assumptions, the disaster risk is equivalent to a shift in the discount factor. However, recession is associated with agents becoming more impatient in Gourio (2012) but positive shocks to an exogenous discount factor, and thus agents becoming more patient in the New Keynesian literature (Smets and Wouters (2003), Christiano et al. (2011)). Here, we showed that Gourio (2012)'s model actually drives the agents to be more patient whenever the EIS is below unity. In that case and when combined with sticky prices, we can yet obtain a decrease in consumption, wage, and output, as well as deflation, and an increase in risk premia all together.

This model could easily be used for further research. In particular, it would be interesting to look at the optimal policies to be implement in the face of increased disaster risk. Also, it would be informative to study variations in the term premium due to the disaster risk, and how the short-term risk premium interacts with the long-term yield curve. The impact of unconventional monetary policies would then be particularly worth investigating.

References

- [1] Martin Andreasen. On the Effects of Rare Disasters and Uncertainty Shocks for Risk Premia in Non-Linear DSGE Models. *Review of Economic Dynamics*, 15(3):295–316, July 2012.
- [2] Orazio P. Attanasio and Martin Browning. Consumption over the Life Cycle and over the Business Cycle. *American Economic Review*,

85(5):1118–37, December 1995.

- [3] Stéphane Auray, Aurélien Eyquem, and Frédéric Jouneau-Sion. Modeling tails of aggregate economic processes in a stochastic growth model. *Computational Statistics & Data Analysis*, 76(C):76–94, 2014.
- [4] Scott R. Baker and Nicholas Bloom. Does Uncertainty Reduce Growth? Using Disasters as Natural Experiments. NBER Working Papers 19475, National Bureau of Economic Research, Inc, September 2013.
- [5] Ravi Bansal and Amir Yaron. Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance*, 59(4):1481–1509, 2004.
- [6] Robert J. Barro. Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics*, 2006.
- [7] Robert J. Barro and Tao Jin. On the size distribution of macroeconomic disasters. *Econometrica*, 79(5):1567–1589, 2011.
- [8] Robert J. Barro and Jose F. Ursúa. Consumption disasters in the twentieth century. *The American Economic Review*, pages 58–63, 2008.
- [9] Susanto Basu and Brent Bundick. Uncertainty shocks in a model of effective demand. Research Working Paper RWP 14-15, Federal Reserve Bank of Kansas City, November 2014.
- [10] Tamim A Bayoumi. Financial Deregulation and Consumption in the United Kingdom. *The Review of Economics and Statistics*, 75(3):536–39, August 1993.
- [11] Jason Beeler and John Y. Campbell. The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment. *Critical Finance Review*, 1(1):141–182, January 2012.
- [12] Pierpaolo Benigno and Luigi Paciello. Monetary policy, doubts and asset prices. *Journal of Monetary Economics*, 64(C):85–98, 2014.
- [13] Henk Berkman, Ben Jacobsen, and John B. Lee. Time-varying rare disaster risk and stock returns. *Journal of Financial Economics*, 101(2):313–332, August 2011.

- [14] Ben S Bernanke. Irreversibility, Uncertainty, and Cyclical Investment. *The Quarterly Journal of Economics*, 98(1):85–106, February 1983.
- [15] Nicholas Bloom. The Impact of Uncertainty Shocks. *Econometrica*, 77(3):623–685, 05 2009.
- [16] Nicholas Bloom. Fluctuations in Uncertainty. *Journal of Economic Perspectives*, 28(2):153–76, 2014.
- [17] Nicholas Bloom, Stephen Bond, and John Van Reenen. Uncertainty and Investment Dynamics. *Review of Economic Studies*, 74(2):391–415, 2007.
- [18] Richard Blundell, Martin Browning, and Costas Meghir. Consumer Demand and the Life-Cycle Allocation of Household Expenditures. *Review of Economic Studies*, 61(1):57–80, January 1994.
- [19] Dario Caldara, Jesus Fernández-Villaverde, Juan Rubio-Ramírez, and Wen Yao. Computing DSGE Models with Recursive Preferences and Stochastic Volatility. *Review of Economic Dynamics*, 15(2):188–206, April 2012.
- [20] John Y Campbell. Bond and Stock Returns in a Simple Exchange Model. *The Quarterly Journal of Economics*, 101(4):785–803, November 1986.
- [21] John Y. Campbell and John Ammer. What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns. *The Journal of Finance*, 48(1):3–37, 1993.
- [22] John Y. Campbell and John Cochrane. Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy*, 107(2):205–251, April 1999.
- [23] John Y. Campbell and N. Gregory Mankiw. Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence. In *NBER Macroeconomics Annual 1989, Volume 4*, NBER Chapters, pages 185–246. National Bureau of Economic Research, Inc, May 1989.
- [24] Siddhartha Chib and Srikanth Ramamurthy. DSGE Models with Student- t Errors. *Econometric Reviews*, 33(1-4):152–171, June 2014.

- [25] Lawrence Christiano, Martin Eichenbaum, and Sergio Rebelo. When Is the Government Spending Multiplier Large? *Journal of Political Economy*, 119(1):78 – 121, 2011.
- [26] John H Cochrane. Presidential address: Discount rates. *The Journal of Finance*, 66(4):1047–1108, 2011.
- [27] Vasco Cúrdia, Marco Del Negro, and Daniel L. Greenwald. Rare Shocks, Great Recessions. *Journal of Applied Econometrics*, 29(7):1031–1052, November 2014.
- [28] Gauti B. Eggertsson, Andrea Ferrero, and Andrea Raffo. Can structural reforms help europe? *Journal of Monetary Economics*, 61:2–22, 2014.
- [29] Gauti B. Eggertsson and Michael Woodford. The Zero Bound on Interest Rates and Optimal Monetary Policy. *Brookings Papers on Economic Activity*, 34(1):139–235, 2003.
- [30] Larry G. Epstein and Stanley E. Zin. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica*, 57(4):937–69, July 1989.
- [31] Christopher J. Erceg and Jesper Linde. Fiscal Consolidation in an Open Economy. *American Economic Review*, 102(3):186–91, May 2012.
- [32] Jesús Fernández-Villaverde, Pablo Guerron-Quintana, Keith Kuester, and Juan Francisco Rubio-Ramírez. Fiscal Volatility Shocks and Economic Activity. Technical Report 8528, C.E.P.R. Discussion Papers, August 2011b.
- [33] Jesús Fernández-Villaverde, Pablo Guerron-Quintana, Juan F. Rubio-Ramírez, and Martin Uribe. Risk Matters: The Real Effects of Volatility Shocks. *American Economic Review*, 101(6):2530–61, October 2011a.
- [34] Xavier Gabaix. Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macro-finance. *American Economic Review*, 98(2):64–67, 2008.
- [35] Xavier Gabaix. Disasterization: A Simple Way to Fix the Asset Pricing Properties of Macroeconomic Models. *American Economic Review*, 101(3):406–09, May 2011.

- [36] Xavier Gabaix. Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance. *The Quarterly Journal of Economics*, 127(2):645–700, 2012.
- [37] Xavier Gabaix and Matteo Maggiori. International Liquidity and Exchange Rate Dynamics. NBER Working Papers 19854, National Bureau of Economic Research, Inc, January 2014.
- [38] François Gourio. Disasters and recoveries. *American Economic Review*, 98(2):68–73, May 2008.
- [39] François Gourio. Disaster Risk and Business Cycles. *American Economic Review*, 102(6):2734–66, October 2012.
- [40] François Gourio. Credit Risk and Disaster Risk. *American Economic Journal: Macroeconomics*, 5(3):1–34, July 2013.
- [41] Robert E Hall. Intertemporal Substitution in Consumption. *Journal of Political Economy*, 96(2):339–57, April 1988.
- [42] Richard Hartman. The effects of price and cost uncertainty on investment. *Journal of Economic Theory*, 5(2):258–266, October 1972.
- [43] Tomas Havránek, Roman Horvath, Zuzana Irsova, and Marek Rusnak. Cross-country heterogeneity in intertemporal substitution. *Journal of International Economics*, 96(1):100–118, 2015.
- [44] Abigail Hughes and Jumana Saleheen. UK labour productivity since the onset of the crisis ? an international and historical perspective. *Bank of England Quarterly Bulletin*, 52(2):138–146, 2012.
- [45] Alfonso Irarrazabal and Juan Carlos Parra-Alvarez. Time-varying disaster risk models: An empirical assessment of the Rietz-Barro hypothesis. CREATES Research Papers 2015-08, School of Economics and Management, University of Aarhus, February 2015.
- [46] Peter N. Ireland. Technology shocks in the new keynesian model. *Review of Economics and Statistics*, 86(4):923–936, 2004.
- [47] Sylvain Leduc and Zheng Liu. Uncertainty Shocks are Aggregate Demand Shocks. Working Paper Series 2012-10, Federal Reserve Bank of San Francisco, 2012.

- [48] Hayne E. Leland. Saving and Uncertainty: The Precautionary Demand for Saving. *The Quarterly Journal of Economics*, 82(3):465–473, August 1968.
- [49] Sydney Ludvigson. Consumption And Credit: A Model Of Time-Varying Liquidity Constraints. *The Review of Economics and Statistics*, 81(3):434–447, August 1999.
- [50] Gregory N. Mankiw and Stephen P. Zeldes. The Consumption of Stockholders and Nonstockholders. *Journal of Financial Economics*, 29(1):97–112, 1991.
- [51] Robert McDonald and Daniel Siegel. The Value of Waiting to Invest. *The Quarterly Journal of Economics*, 101(4):707–27, November 1986.
- [52] Rajnish Mehra and Edward C. Prescott. The equity premium: A puzzle. *Journal of monetary Economics*, 15(2):145–161, 1985.
- [53] Stephen Millard and Anamaria Nicolae. The effect of the financial crisis on TFP growth: a general equilibrium approach. Bank of England working papers 502, Bank of England, June 2014.
- [54] Emi Nakamura, Jón Steinsson, Robert J. Barro, and José Ursúa. Crises and Recoveries in an Empirical Model of Consumption Disasters. *American Economic Journal: Macroeconomics*, 5(3):35–74, July 2013.
- [55] Anna Pavlova and Roberto Rigobon. Asset Prices and Exchange Rates. *Review of Financial Studies*, 20(4):1139–1180, 2007.
- [56] Monika Piazzesi, Martin Schneider, and Selale Tuzel. Housing, consumption and asset pricing. *Journal of Financial Economics*, 83(3):531–569, March 2007.
- [57] Thomas A. Rietz. The equity risk premium: a solution. *Journal of Monetary Economics*, 22(1):117–131, July 1988.
- [58] Glenn D. Rudebusch and Eric T. Swanson. The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks. *American Economic Journal: Macroeconomics*, 4(1):105–43, January 2012.

- [59] Enrico Saltari and Davide Ticchi. Risk aversion, intertemporal substitution, and the aggregate investment-uncertainty relationship. *Journal of Monetary Economics*, 54(3):622–648, April 2007.
- [60] Agnar Sandmo. The Effect of Uncertainty on Saving Decisions. *Review of Economic Studies*, 37(3):353–60, July 1970.
- [61] Frank Schorfheide, Dongho Song, and Amir Yaron. Identifying Long-Run Risks: A Bayesian Mixed-Frequency Approach. NBER Working Papers 20303, National Bureau of Economic Research, Inc, July 2014.
- [62] Sang Byung Seo and Jessica A. Wachter. Option Prices in a Model with Stochastic Disaster Risk. NBER Working Papers 19611, National Bureau of Economic Research, Inc, November 2013.
- [63] Frank Smets and Rafael Wouters. An Estimated Dynamic Stochastic General Equilibrium Model of the EURO Area. *Journal of the European Economic Association*, 1(5):1123–1175, 09 2003.
- [64] Frank Smets and Rafael Wouters. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3):586–606, 2007.
- [65] Thomas D. Tallarini Jr. Risk-sensitive real business cycles. *Journal of Monetary Economics*, 45(3):507–532, June 2000.
- [66] Linda L. Tesar and Alan C. Stockman. Tastes and technology in a two-country model of the business cycle: Explaining international. *American Economic Review*, 85(1):168–185, 1995.
- [67] Jerry Tsai and Jessica A. Wachter. Disaster Risk and its Implications for Asset Pricing. NBER Working Papers 20926, National Bureau of Economic Research, Inc, February 2015.
- [68] Annette Vissing-Jorgensen. Limited Asset Market Participation and the Elasticity of Intertemporal Substitution. *Journal of Political Economy*, 110(4):825–853, August 2002.
- [69] Jessica A. Wachter. Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance*, 68(3):987–1035, 2013.

- [70] Philippe Weil. The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics*, 24(3):401–421, November 1989.
- [71] Philippe Weil. Nonexpected utility in macroeconomics. *Quarterly Journal of Economics*, 105(1):29–42, February 1990.
- [72] Martin L. Weitzman. Rare Disasters, Tail-Hedged Investments, and Risk-Adjusted Discount Rates. NBER Working Papers 18496, National Bureau of Economic Research, Inc, October 2012.
- [73] Motohiro Yogo. Estimating the elasticity of intertemporal substitution when instruments are weak. *Review of Economics and Statistics*, 86(3):797–810, 2004.
- [74] Joseph Zeira. Cost uncertainty and the rate of investment. *Journal of Economic Dynamics and Control*, 14(1):53–63, February 1990.

7 Appendices

A Households’ problem with disaster risk

A.1 Capital accumulation with disaster risk

Let us consider that the law of motion for capital is

$$K_{t+1} = \left[(1 - \delta_0 u_t^\eta) K_t + S\left(\frac{I_t}{K_t}\right) K_t \right] e^{x_{t+1} \ln(1-\Delta)}$$

where the depreciation rate of capital given by

$$\delta_t = \delta_0 u_t^\eta \tag{10}$$

with u the utilization rate of capital and η a parameter, where $S(\cdot)$ is a capital adjustment cost function featuring the usual properties as given by

$$S\left(\frac{I_t}{K_t}\right) = \frac{I_t}{K_t} - \frac{\tau}{2} \left(\frac{I_t}{K_t} - \frac{\bar{I}}{\bar{K}}\right)^2 \tag{11}$$

and where the last term expresses that capital accumulation is affected by the occurrence of a “disaster” captured by the indicator variable x_{t+1} . Specifically, if a disaster occurs, we have $x_{t+1} = 1$, with a time-varying probability

denoted θ_t , a fraction $1 - \Delta$ of capital is destroyed. Otherwise, $x_{t+1} = 0$, and the law of motion is standard.

Following Gourio (2012)'s spirit, we assume that productivity is subject to the same disaster risk and follows

$$\frac{z_{t+1}}{z_t} = e^{\mu + \varepsilon_{z,t+1} + x_{t+1} \ln(1-\Delta)}$$

This allows to write a law of motion for the *detrended* capital stock as

$$k_{t+1} = \frac{(1 - \delta_t)k_t + S\left(\frac{i_t}{k_t}\right)k_t}{e^{\mu + \varepsilon_{z,t+1}}} \quad (12)$$

where lower case letters denote the detrended variables ($k_t = K_t/z_t$, etc). This way, the disaster event itself x_{t+1} does not affect the detrended capital, while the disaster risk θ_t will however do (indirectly).

A.2 Bonds with disaster risk and the budget constraint

In addition, households can also buy one-period bonds issued by a public authority. These assets are also subject to the same disaster risk, i.e

$$B_{t+1} = [B_t(1 + r_{t-1})]e^{x_{t+1} \ln(1-\Delta)}$$

or, reexpressed in detrended terms as $b_{t+1} = \frac{b_t(1+r_{t-1})}{e^{\mu + \varepsilon_{z,t+1}}}$. The households' (detrended) budget constraint is thus given by

$$\frac{w_t}{p_t}L_t + \frac{b_t(1 + r_{t-1})}{p_t} + \frac{P_t^k}{p_t}u_t k_t + \frac{D_t}{z_t} = i_t + c_t + \frac{b_{t+1}}{p_t}e^{\mu + \varepsilon_{z,t+1}} + \frac{T_t}{z_t} \quad (13)$$

where w stands for the (detrended) nominal wage rate, p the good price, P^k the (nominal) rental rate of capital received from the firms, D the monopolistic firms' real profits, T lump-sum taxes.

A.3 Epstein-Zin preferences under disaster risk

Epstein-Zin (1989) preferences are given by:

$$\tilde{V}_t = \left[[C_t(1 - L_t)^{\varpi}]^{1-\psi} + \beta_0 \left(E_t \tilde{V}_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \right]^{\frac{1}{1-\psi}}$$

By setting $\tilde{V}_t = V_t^{\frac{1}{1-\psi}}$ and $\chi = 1 - \frac{1-\gamma}{1-\psi}$, we obtain

$$V_t = [C_t(1 - L_t)^\varpi]^{1-\psi} + \beta_0 \left(E_t V_{t+1}^{1-\chi} \right)^{\frac{1}{1-\chi}}$$

and finally defining $v_t \equiv \frac{V_t}{z_t^{\frac{1}{1-\psi}}}$, we get

$$v_t = [c_t(1 - L_t)^\varpi]^{1-\psi} + \frac{\beta_0}{z_t^{\frac{1}{1-\psi}}} \left(E_t \left(z_{t+1}^{1-\psi} v_{t+1} \right)^{1-\chi} \right)^{\frac{1}{1-\chi}}$$

with $c_t = C_t/z_t$ stands for the detrended consumption. Since we have assumed that productivity evolves as $z_{t+1}/z_t = e^{\mu + \varepsilon_{z,t+1} + x_{t+1} \ln(1-\Delta)}$, we can rewrite the previous equation as

$$\begin{aligned} v_t &= [c_t(1 - L_t)^\varpi]^{1-\psi} + \frac{\beta_0}{z_t^{\frac{1}{1-\psi}}} \left(E_t \left(z_t^{(1-\psi)} e^{[\mu + \varepsilon_{z,t+1} + x_{t+1} \ln(1-\Delta)](1-\psi)} v_{t+1} \right)^{1-\chi} \right)^{\frac{1}{1-\chi}} \\ &= [c_t(1 - L_t)^\varpi]^{1-\psi} + \beta_0 \left(E_t e^{[\mu + \varepsilon_{z,t+1} + x_{t+1} \ln(1-\Delta)](1-\psi)(1-\chi)} v_{t+1}^{1-\chi} \right)^{\frac{1}{1-\chi}} \end{aligned}$$

that can further be decomposed, in Gourio (2012)'s spirit, as

$$v_t = [c_t(1 - L_t)^\varpi]^{1-\psi} + \beta_0 E_t \left[e^{(1-\gamma)x_{t+1} \ln(1-\Delta)} \right]^{\frac{1}{1-\chi}} e^{(1-\psi)\mu} E_t \left[e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi} \right]^{\frac{1}{1-\chi}}$$

with $(1-\gamma) = (1-\psi)(1-\chi)$ from earlier definition. Then, since there is a disaster ($x = 1$) with probability θ and no disaster ($x = 0$) with probability $(1-\theta)$, we can decompose in the expression above the term

$$\beta_0 E_t \left[e^{(1-\gamma)x_{t+1} \ln(1-\Delta)} \right]^{\frac{1}{1-\chi}} = \beta_0 \left[(1-\theta_t) + \theta_t e^{(1-\gamma) \ln(1-\Delta)} \right]^{\frac{1}{1-\chi}}$$

where the first expectation operator is conditional on the disaster risk and information at time t whereas the second expectation operator is only conditional on information at time t .

Thus, redefining the discount factor as a function of the (time-varying) disaster risk (à la Gourio, 2012) as

$$\beta(\theta) \equiv \beta_0 \left[1 - \theta_t + \theta_t e^{(1-\gamma) \ln(1-\Delta)} \right]^{\frac{1}{1-\chi}} \quad (14)$$

our objective function can finally be rewritten as

$$v_t = [c_t(1 - L_t)^\varpi]^{1-\psi} + \beta(\theta_t)e^{(1-\psi)\mu} \left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi} \right]^{\frac{1}{1-\chi}} \quad (15)$$

A.4 Solving for the household's problem

Households want to maximize (5) subject to (1)-(4) and (6). The Lagrangian for this problem can be written as

$$\begin{aligned} \mathcal{L} = & [c_t(1 - L_t)^\varpi]^{1-\psi} + \beta(\theta_t)e^{\mu(1-\psi)} \left(E_t e^{\varepsilon_{z,t+1}(1-\gamma)} v_{t+1}^{(1-\chi)} \right)^{\frac{1}{1-\chi}} \\ & + E_t \Lambda_t^B \left(\frac{w_t^{nom}}{p_t} L_t - \frac{b_{t+1}}{p_t} e^{\mu+\varepsilon_{z,t+1}} + \frac{b_t(1+r_{t-1})}{p_t} + \frac{P_t^k}{p_t} u_t k_t + \frac{D_t}{z_t} - i_t - c_t - \frac{T_t}{z_t} \right) \\ & + E_t \Lambda_t^C \left[(1 - \delta_0 u_t^\eta) k_t + S \left(\frac{i_t}{k_t} \right) k_t - k_{t+1} e^{\mu+\varepsilon_{z,t+1}} \right] \end{aligned}$$

with $\delta_t = \delta_0 u_t^\eta$ and $S \left(\frac{i_t}{k_t} \right) = \frac{i_t}{k_t} - \frac{\tau}{2} \left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}} \right)^2$, and where Λ_t^B and Λ_t^C are the Lagrangian multipliers associated with the budget constraint and capital accumulation constraint respectively. The first-order conditions are thus

$$\begin{aligned} (c_t :) \quad & (1 - \psi) c_t^{-\psi} (1 - L_t)^\varpi (1 - \psi) = \Lambda_t^B \\ (c_{t+1} :) \quad & E_t \left[\beta(\theta_t) e^{(1-\psi)\mu} \frac{e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{-\chi}}{\left(E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi} \right)^{\frac{1-\chi}{1-\chi}}} (1 - \psi) c_{t+1}^{-\psi} (1 - L_{t+1})^\varpi (1 - \psi) \right] = E_t \Lambda_{t+1}^B \\ (L_t :) \quad & \frac{1 - L_t}{c_t} = \frac{\varpi}{w_t} \\ (b_{t+1} :) \quad & \Lambda_t^B e^{\mu+\varepsilon_{z,t+1}} = E_t \left(\Lambda_{t+1}^B \frac{1 + r_t}{1 + \pi_{t+1}} \right) \\ (k_{t+1} :) \quad & E_t \left\{ \Lambda_{t+1}^B \frac{P_{t+1}^k}{p_{t+1}} u_{t+1} + \Lambda_{t+1}^C \left[1 - \delta_0 u_{t+1}^\eta + \tau \frac{i_{t+1}}{k_{t+1}} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right) \right. \right. \\ & \left. \left. - \frac{\tau}{2} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right)^2 \right] \right\} = \Lambda_t^C e^{\mu+\varepsilon_{z,t+1}} \\ (u_t :) \quad & \Lambda_t^B \frac{P_t^k}{p_t} = \Lambda_t^C \delta_0 \eta u_t^{\eta-1} \end{aligned}$$

$$(i_t :) \quad \Lambda_t^B = \Lambda_t^C \left[1 - \tau \left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}} \right) \right]$$

Finally, substituting out the Lagrange multipliers, we get the optimality conditions expressed in detrended terms.

A.5 The stochastic discount factor

The stochastic discount factor is defined as

$$Q_{t,t+1} = \frac{\partial \tilde{V}_t / \partial C_{t+1}}{\partial \tilde{V}_t / \partial C_t}$$

and, recalling that $\tilde{V}_t = V_t^{\frac{1}{1-\psi}}$ and $\chi = 1 - \frac{1-\gamma}{1-\psi}$, we get

$$Q_{t,t+1} = \beta_0 \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \left(\frac{1-L_{t+1}}{1-L_t} \right)^{\varpi(1-\psi)} \frac{V_{t+1}^{-\chi}}{(E_t V_{t+1}^{1-\chi})^{\frac{-\chi}{1-\chi}}}$$

Then, to further express it as a function of the detrended variables, let us use $v_t \equiv \frac{V_t}{z_t^{\frac{1}{1-\psi}}}$ and the expression above to get

$$Q_{t,t+1} = \frac{z_t}{z_{t+1}} \left(\frac{c_{t+1}}{c_t} \right)^{-\psi} \left(\frac{1-L_{t+1}}{1-L_t} \right)^{\varpi(1-\psi)} \beta(\theta_t) e^{(1-\psi)\mu} \frac{e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{-\chi}}{\left[E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi} \right]^{\frac{-\chi}{1-\chi}}}$$

Note that we cannot use this expression as such for using the perturbation methods since the term $\frac{z_t}{z_{t+1}}$ still contain the disaster variable x . However, recall the first-order condition on bonds as

$$\begin{aligned} \frac{E_t \Lambda_{t+1}^B}{\Lambda_t^B} &= e^{\mu + \varepsilon_{z,t+1}} \frac{E_t (1 + \pi_{t+1})}{1 + r_t} \\ &= E_t \left[\beta(\theta_t) e^{(1-\psi)\mu} \frac{e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{-\chi}}{\left(E_t e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi} \right)^{\frac{-\chi}{1-\chi}}} \left(\frac{c_{t+1}}{c_t} \right)^{-\psi} \left(\frac{1-L_{t+1}}{1-L_t} \right)^{\varpi(1-\psi)} \right] \end{aligned}$$

Finally, ‘detrending’ the Lagrange multipliers, $\lambda_t^B \equiv \frac{\Lambda_t^B}{z_t}$, we get an equilibrium condition as

$$Q_{t,t+1} = \frac{z_t}{z_{t+1}} \frac{\Lambda_{t+1}^B}{\Lambda_t^B} = \frac{\lambda_{t+1}^B}{\lambda_t^B}, \quad \text{or}$$

$$\tilde{Q}_{t,t+1} \equiv Q_{t,t+1} \frac{z_{t+1}}{z_t} = \frac{\Lambda_{t+1}^B}{\Lambda_t^B} = e^{\mu + \varepsilon_{z,t+1}} \frac{1 + \pi_{t+1}}{1 + r_t} \quad (16)$$

A.6 The risk premium

The standard asset pricing orthogonality condition reads as

$$E_t [Q_{t,t+1} R_{t+1}^i] = 1$$

where R^i is the real return on asset i . Thus, the *riskfree rate*, R^f , is

$$E_t [Q_{t,t+1} R_{t+1}^f] = 1$$

Moreover, from the first-order condition on bonds, we know that

$$E_t [\tilde{Q}_{t,t+1}] = E_t \left[Q_{t,t+1} \frac{z_{t+1}}{z_t} \right] = E_t \left[e^{\mu + \varepsilon_{z,t+1}} \frac{1 + \pi_{t+1}}{1 + r_t} \right]$$

such that the (real) *rate of return on capital* can be written as

$$R_{t+1}^{k,real} = \frac{z_{t+1}}{z_t} \frac{1}{e^{\mu + \varepsilon_{z,t+1}}} \frac{1 + r_t}{1 + \pi_{t+1}} = e^{x_{t+1} \ln(1-\Delta)} \frac{1 + r_t}{1 + \pi_{t+1}}$$

Further replaced into the (non detrended) condition on capital, we get

$$R_{t+1}^{k,real} = e^{x_{t+1} \ln(1-\Delta)} \left\{ \frac{P_{t+1}^k}{p_{t+1}} \frac{u_{t+1}}{q_t} + \frac{q_{t+1}}{q_t} \left[1 - \delta_0 u_{t+1}^\eta + \tau \frac{i_{t+1}}{k_{t+1}} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right) - \frac{\tau}{2} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right)^2 \right] \right\}$$

Finally, the *risk premium* is defined in gross terms as the ratio of the real return on capital to the riskfree rate, i.e $E_t(\text{Premium}_{t+1}) \equiv E_t(R_{t+1}^{k,real} / R_{t+1}^f)$.

A.7 The role of the EIS on households' decisions

A.7.1 The response of the discount factor to the disaster risk

The EIS is given by the following combination of parameters in our model

$$EIS = \frac{1}{1 - (1 + \varpi)(1 - \psi)}$$

so that the time-varying discount factor (6) can be rewritten as

$$\beta(\theta) = \beta_0 \left[1 - \theta_t \left(1 - e^{(1-\gamma) \ln(1-\Delta)} \right) \right]^{\frac{1-1/EIS}{(1-\gamma)(1+\varpi)}}$$

Taking the derivate with respect to the probability of disaster gives

$$\frac{\partial \beta(\theta)}{\partial \theta} = \beta_0 \underbrace{\frac{1-1/EIS}{(1-\gamma)(1+\varpi)}}_A \underbrace{\left[e^{(1-\gamma) \ln(1-\Delta)} - 1 \right]}_B \underbrace{\left[1 - \theta_t \left(1 - e^{(1-\gamma) \ln(1-\Delta)} \right) \right]^{\frac{1-1/EIS}{(1-\gamma)(1+\varpi)} - 1}}_C$$

The sign of this expression crucially depends on the value of the EIS. Given $\varpi > 0$, $\Delta > 0$, $\theta > 0$, $\beta_0 > 0$, we have:

- With EIS < 1 and $\gamma > 1$, $A > 0$, $B > 0$, $C > 0$, so $\frac{\partial \beta(\theta)}{\partial \theta} > 0$;
- With EIS < 1 and $0 \leq \gamma < 1$, $A < 0$, $B < 0$, $C > 0$, so $\frac{\partial \beta(\theta)}{\partial \theta} > 0$;
- With EIS > 1 and $\gamma > 1$, $A < 0$, $B > 0$, $C > 0$, so $\frac{\partial \beta(\theta)}{\partial \theta} < 0$;
- With EIS > 1 and $0 \leq \gamma < 1$, $A > 0$, $B < 0$, $C > 0$, so $\frac{\partial \beta(\theta)}{\partial \theta} < 0$;
- With $\lim_{EIS \rightarrow 1} \frac{\partial \beta(\theta)}{\partial \theta} \rightarrow 0$.

Overall, an increase in the probability of disaster thus makes agents more patient (higher $\beta(\theta)$) when the EIS is below unity, and inversely, more impatient (lower $\beta(\theta)$) when the EIS is above unity. This holds for all degrees of risk aversion (all values of γ), including risk neutrality.

A.7.2 The response of the riskfree rate to the disaster risk (along the balanced growth path)

Along the balanced growth path, the riskfree rate is given by

$$R^f = \frac{\left[1 - \theta \left(1 - e^{(1-\gamma) \ln(1-\Delta)} \right) \right]^{\frac{\varpi(1-\gamma)+1/EIS-\gamma}{(1+\varpi)(1-\gamma)}}}{\beta_0 e^{-\mu \frac{\varpi+1/EIS}{1+\varpi}} \left[1 - \theta \left(1 - e^{-\gamma \ln(1-\Delta)} \right) \right]}$$

The derivative $\partial R^f / \partial \theta$ is always negative, i.e the riskfree rate decreases in the disaster risk for all values of the EIS and risk aversion. However, the magnitude of the slump is sensitive to the value of the EIS: the riskfree rate decreases more with the disaster risk for an EIS below unity than for an EIS above unity, given the degree of risk aversion (including risk neutrality). For instance, with the baseline calibration we find

- With EIS = 0.5 and $\gamma = 3.8$, $\frac{\partial R^f}{\partial \theta} \approx -0.666$;
- With EIS = 2 and $\gamma = 3.8$, $\frac{\partial R^f}{\partial \theta} \approx -0.504$;
- With EIS = 0.5 and $\gamma = 0.5$, $\frac{\partial R^f}{\partial \theta} \approx -0.324$;
- With EIS = 2 and $\gamma = 0.5$, $\frac{\partial R^f}{\partial \theta} \approx -0.217$;
- With EIS = 0.5 and $\gamma = 0$, $\frac{\partial R^f}{\partial \theta} \approx -0.291$;
- With EIS = 2 and $\gamma = 0$, $\frac{\partial R^f}{\partial \theta} \approx -0.190$

Note again that this is not a general equilibrium effect.

A.7.3 The response of the return on capital to the disaster risk (along the balanced growth path)

Along the balanced growth path, the return on capital is given by

$$R^k = \frac{1 - \theta (1 - e^{\ln(1-\Delta)})}{\beta_0 e^{-\mu \frac{\varpi+1/EIS}{1+\varpi}} [1 - \theta (1 - e^{(1-\gamma) \ln(1-\Delta)})]^{\frac{1-1/EIS}{(1-\gamma)(1+\varpi)}}$$

The derivative $\partial R^k / \partial \theta$ is also always negative, i.e the rate of return on capital decreases in the disaster risk. However, just as for the riskfree rate, the decrease is larger when the EIS is below unity (rather than above), for all values of risk aversion (including risk neutrality). For instance, we have

- With EIS = 0.5 and $\gamma = 3.8$, $\frac{\partial R^k}{\partial \theta} \approx -0.332$;
- With EIS = 2 and $\gamma = 3.8$, $\frac{\partial R^k}{\partial \theta} \approx -0.169$;
- With EIS = 0.5 and $\gamma = 0.5$, $\frac{\partial R^k}{\partial \theta} \approx -0.295$;
- With EIS = 2 and $\gamma = 0.5$, $\frac{\partial R^k}{\partial \theta} \approx -0.188$;
- With EIS = 0.5 and $\gamma = 0$, $\frac{\partial R^k}{\partial \theta} \approx -0.291$;
- With EIS = 2 and $\gamma = 0$, $\frac{\partial R^k}{\partial \theta} \approx -0.190$

A.7.4 The response of the risk premium to the disaster risk (along the balanced growth path)

Finally, along the balanced growth path, the risk premium is given by

$$Premium = \frac{[1 - \theta (1 - e^{\ln(1-\Delta)})] [1 - \theta (1 - e^{-\gamma \ln(1-\Delta)})]}{1 - \theta (1 - e^{(1-\gamma) \ln(1-\Delta)})}$$

The derivative, calculated under our calibration values, gives

- With $\gamma = 3.8$, $\frac{\partial E(R^k)/R^f}{\partial \theta} \approx 0.333$;
- With $\gamma = 0.5$, $\frac{\partial E(R^k)/R^f}{\partial \theta} \approx 0.029$;
- With $\gamma = 0$, $\frac{\partial E(R^k)/R^f}{\partial \theta} = 0$.

The risk premium reacts positively to the disaster risk, and the larger the risk aversion the larger its magnitude. It does not directly depend on the value of the EIS along the balanced growth path, in line with Gourio (2012). However, in general equilibrium, the EIS plays a qualitative role: the larger the EIS, the smaller the risk premium in response to the disaster risk shock (see the impulse response functions, comparing Figures 1 and 2 (flexible prices) on one hand, and Figures 3 and 4 (sticky prices) on the other hand).

B Firms' problem

B.1 Production aggregation

The aggregate of intermediate goods is given by

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

so that the representative firm in the final sector maximizes profits as

$$\max_{Y_{t,j}} p_t \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} - \int_0^1 p_{j,t} Y_{j,t} dj$$

The first-order condition with respect to $Y_{t,j}$ yields a downward sloping demand curve for each intermediate good j as

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

The nominal value of the final good is the sum of prices times quantities of intermediates

$$p_t Y_t = \int_0^1 p_{j,t} Y_{j,t} dj$$

in which Y_t is substituted to give the aggregate price index as

$$p_t = \left(\int_0^1 p_{j,t}^{1-\nu} dj \right)^{\frac{1}{1-\nu}}$$

B.2 Cost minimization

Firms are price-takers in the input markets, facing (non-detrended) nominal wage W_t^{nom} and capital rental rate P_t^k . They choose the optimal quantities of labor and capital given the input prices and subject to the restriction of producing at least as much as the intermediate good is demanded at the given price. The intra-temporal problem is thus

$$\begin{aligned} \min_{L_{j,t}, \tilde{K}_{j,t}} \quad & W_t L_{j,t} + P_t^k \tilde{K}_{j,t} \\ \text{s.t.} \quad & \tilde{K}_{j,t}^\alpha (z_t L_{j,t})^{1-\alpha} \geq \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t \end{aligned}$$

The (detrended) first-order conditions are

$$\begin{aligned} (L_{j,t} :) \quad & w_t = m c_{j,t}^{nom} (1 - \alpha) \left(\frac{\tilde{k}_{j,t}}{L_{j,t}} \right)^\alpha \\ (\tilde{K}_{j,t} :) \quad & P_t^k = m c_{j,t}^{nom} \alpha \left(\frac{\tilde{k}_{j,t}}{L_{j,t}} \right)^{\alpha-1} \end{aligned}$$

in which the Lagrange multiplier denoted $m c_{j,t}^{nom}$ can be interpreted as the (nominal) marginal cost associated with an additional unit of capital or labor. Rearranging gives the optimal capital over labor ratio as

$$\left(\frac{\tilde{k}_{j,t}}{L_{j,t}} \right)^* = \frac{w_t}{P_t^k} \frac{\alpha}{(1 - \alpha)}$$

in which none of the terms on the right hand side depends on j , and thus holds for all firms in equilibrium, *i.e.*, $\frac{\tilde{k}_t}{L_t} = \frac{\tilde{k}_{j,t}}{L_{j,t}}$. Replacing in one of the

first-order conditions above gives

$$mc_t^{nom*} = \left(\frac{P_t^k}{\alpha} \right)^\alpha \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha}$$

Reexpressing in real terms $mc_t^* = mc_t^{nom*}/p_t$, we finally have

$$mc_t^* = \left(\frac{P_t^{k,real}}{\alpha} \right)^\alpha \left(\frac{w_t^{real}}{1-\alpha} \right)^{1-\alpha}$$

where $P_t^{k,real}$ and w_t^{real} are the real capital rental rate and (detrended) wage.

B.3 Profit maximization

Let us now consider the pricing problem of a firm that gets to update its price in period t and wants to maximize the present discounted value of future profits. The (nominal) profit flows read as

$$p_{j,t}Y_{j,t} - W_tL_{j,t} - P_{j,t}^k\tilde{K}_{j,t} = (p_{j,t} - mc_t^{nom})Y_{j,t}$$

which can be reexpressed in real terms as $\frac{p_{j,t}}{p_t}Y_{j,t} - mc_t^*Y_{j,t}$. These profit flows are discounted by both the stochastic discount factor, $Q_{t,t+s}$, and by the probability ζ^s that a price chosen at time t is still in effect at time s . Finally, given $Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$, the maximization problem is thus

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\left(\frac{p_{j,t}}{p_{t+s}} \right)^{1-\nu} Y_{t+s} - mc_{t+s}^* \left(\frac{p_{j,t}}{p_{t+s}} \right)^{-\nu} Y_{t+s} \right)$$

which can be further simplified, using $mc_t^* = \frac{mc_t^{nom}}{p_t}$ and factorizing, as

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s} \left(p_{j,t}^{1-\nu} - mc_t^{nom} p_{j,t}^{-\nu} \right)$$

The first-order condition is then

$$E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s} \left((1-\nu)p_{j,t}^{-\nu} + \nu mc_t^{nom} p_{j,t}^{-\nu-1} \right) = 0$$

which simplifies as

$$p_{j,t}^* = \frac{\nu}{\nu-1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu} Y_{t+s} m c_{t+s}^*}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s}}$$

Note that this optimal price depends on aggregate variables only, so that $p_t^* = p_{j,t}^*$. Expressed as a ratio over the current price, we thus have

$$\frac{p_t^*}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t}\right)^{\nu} Y_{t+s} m c_{t+s}^*}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t}\right)^{\nu-1} Y_{t+s}}$$

which can be written recursively as

$$\frac{p_t^*}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\Xi_{1t}}{\Xi_{2t}} \quad \text{with}$$

$$\Xi_{1t} = Y_t m c_t^* + \zeta E_t Q_{t,t+1} \left(\frac{p_{t+1}}{p_t}\right)^{\nu} \Xi_{1,t+1}$$

$$\Xi_{2t} = Y_t + \zeta E_t Q_{t,t+1} \left(\frac{p_{t+1}}{p_t}\right)^{\nu-1} \Xi_{2,t+1}$$

Replacing $Q_{t,t+1} \equiv \tilde{Q}_{t,t+1} \frac{z_t}{z_{t+1}}$, and detrending, these are simplified as

$$\tilde{\Xi}_{1t} = y_t m c_t^* + \zeta E_t \tilde{Q}_{t,t+1} \left(\frac{p_{t+1}}{p_t}\right)^{\nu} \tilde{\Xi}_{1,t+1} \quad (17)$$

$$\tilde{\Xi}_{2t} = y_t + \zeta E_t \tilde{Q}_{t,t+1} \left(\frac{p_{t+1}}{p_t}\right)^{\nu-1} \tilde{\Xi}_{2,t+1} \quad (18)$$

where $\tilde{\Xi}_t \equiv \Xi_t / z_t$, the detrended variable.

C Aggregation

C.1 Bonds market

Market-clearing requires that the public debt equals the quantity of bonds at each time, $Debt_t = B_t$, and is thus symmetrically affected by disasters. Moreover, we assume that the public budget has to be balanced every period, i.e the sum of tax revenues and new debt insurance to equal the current debt

insurance to be repaid with interest rates (as for the non-disaster part), i.e

$$T_t p_t + B_{t+1} = [B_t(1 + r_{t-1})]e^{x_{t+1} \ln(1-\Delta)}$$

or, in detrended terms,

$$\frac{T_t}{z_t} + \frac{b_{t+1}}{p_t} = \frac{b_t(1 + r_{t-1})}{p_t e^{\mu + \varepsilon_{z,t+1}}}$$

C.2 Aggregate demand

Replacing the tax level above into the household's budget constraint gives

$$c_t + i_t + \left(\frac{b_t(1 + r_{t-1})}{p_t e^{\mu + \varepsilon_{z,t+1}}} - \frac{b_{t+1}}{p_t} \right) = w_t^{real} L_t + \left(\frac{b_t(1 + r_{t-1})}{p_t e^{\mu + \varepsilon_{z,t+1}}} - \frac{b_{t+1}}{p_t} \right) + P_t^{k,real} u_t k_t + \frac{D_t}{z_t}$$

which just simplifies as

$$c_t + i_t = w_t^{real} L_t + P_t^{k,real} \tilde{k}_t + \frac{D_t}{z_t}$$

i.e, in non-detrended terms, as

$$C_t + I_t = W_t^{real} L_t + P_t^{k,real} \tilde{K}_t + D_t$$

where we now have to verify that the RHS is equal to Y_t . Total dividends (or profits) D_t must be equal to the sum of dividends (or profits) from intermediate good firms, i.e

$$D_t = \int_0^1 D_{j,t} dj$$

The (real) dividends (or profits) from intermediate good firms j are given by

$$D_{j,t} = \frac{p_{j,t}}{p_t} Y_{j,t} - W_t^{real} L_{j,t} - P_t^{k,real} \tilde{K}_{j,t}$$

Substituting $Y_{j,t}$, we have

$$D_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t - W_t^{real} L_{j,t} - P_t^{k,real} \tilde{K}_{j,t}$$

Therefore, knowing that $D_{t(real)} = \int_0^1 D_{j,t(real)} dj$, we get

$$\begin{aligned}
D_t &= \int_0^1 \left(\left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t - W_t^{real} L_{j,t} - P_t^{k,real} \tilde{K}_{j,t} \right) dj = \int_0^1 \left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t dj \\
&\quad - \int_0^1 W_t^{real} L_{j,t} dj - \int_0^1 P_t^{k,real} \tilde{K}_{j,t} dj \\
D_t &= \int_0^1 \left(\left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t - W_t^{real} L_{j,t} - P_t^{k,real} \tilde{K}_{j,t} \right) dj = Y_t \frac{1}{p_t^{1-\nu}} \int_0^1 (p_{j,t})^{1-\nu} dj \\
&\quad - W_t^{real} \int_0^1 L_{j,t} dj - P_t^{k,real} \int_0^1 \tilde{K}_{j,t} dj
\end{aligned}$$

Given that (i) the aggregate price level is $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$, (ii) aggregate labor demand must equal supply, i.e $\int_0^1 L_{j,t} dj = L_t$, and (iii) aggregate supply of capital services must equal demand $\int_0^1 \tilde{K}_{j,t} dj = \tilde{K}_t$, the aggregate (real) dividend (or profit) is

$$D_t = Y_t - W_t^{real} L_t - P_t^{k,real} \tilde{K}_t$$

Replaced into the household's budget constraint, this finally gives the aggregate accounting identity as

$$Y_t = C_t + I_t$$

or in detrended terms

$$y_t = c_t + i_t$$

C.3 Inflation

Firms have a probability $1 - \zeta$ of getting to update their price each period. Since there are an infinite number of firms, there is also the exact fraction $1 - \zeta$ of total firms who adjust their prices and the fraction ζ who stay with the previous period price. Moreover, since there is a random sampling from the entire distribution of firm prices, the distribution of any subset of firm prices is similar to the entire distribution. Therefore, the aggregate price index, $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$, is rewritten as

$$p_t^{1-\nu} = \int_0^{1-\zeta} p_t^{*1-\nu} dj + \int_{1-\zeta}^1 p_{j,t-1}^{1-\nu} dj$$

which simplifies as

$$p_t^{1-\nu} = (1 - \zeta)p_t^{*1-\nu} + \zeta p_{t-1}^{1-\nu}$$

Let us divide both sides of the equation by $p_{t-1}^{1-\nu}$

$$\left(\frac{p_t}{p_{t-1}}\right)^{1-\nu} = (1 - \zeta) \left(\frac{p_t^*}{p_{t-1}}\right)^{1-\nu} + \zeta \left(\frac{p_{t-1}}{p_{t-1}}\right)^{1-\nu}$$

and define the *gross inflation rate* as

$$1 + \pi_t \equiv \frac{p_t}{p_{t-1}}$$

and the *gross reset inflation rate* as

$$1 + \pi_t^* \equiv \frac{p_t^*}{p_{t-1}}$$

we get

$$(1 + \pi_t)^{1-\nu} = (1 - \zeta)(1 + \pi_t^*)^{1-\nu} + \zeta$$

Finally, since we know that

$$\frac{p_t^*}{p_t} = \frac{\nu}{\nu - 1} E_t \frac{\Xi_{1t}}{\Xi_{2t}}$$

we have the *reset inflation rate* as

$$(1 + \pi_t^*) = (1 + \pi_t) \frac{\nu}{\nu - 1} E_t \frac{\Xi_{1t}}{\Xi_{2t}}$$

with the expressions given previously for Ξ_1 and Ξ_2 (see Appendix B.3), or equivalently with the detrended terms as

$$(1 + \pi_t^*) = (1 + \pi_t) \frac{\nu}{\nu - 1} E_t \frac{\tilde{\Xi}_{1t}}{\tilde{\Xi}_{2t}}$$

C.4 Aggregate supply

We know that the demand to individual firm j is given by

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

and that firm j hires labor and capital in the same proportion than the aggregate capital to labor ratio (common factor markets). Hence, substituting in the production function for the intermediate good j we get

$$\left(\frac{\tilde{K}_t}{z_t L_t}\right)^\alpha z_t L_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

which can be rewritten with detrended variables as

$$\left(\frac{\tilde{k}_t}{L_t}\right)^\alpha L_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} y_t$$

Then, summing up across the intermediate firms gives

$$\left(\frac{\tilde{k}_t}{L_t}\right)^\alpha \int_0^1 L_{j,t} dj = y_t \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj$$

Given that aggregate labor demand and supply must equal, i.e. $\int_0^1 L_{j,t} dj = L_t$, we have

$$\int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj y_t = \tilde{k}_t^\alpha L_t^{1-\alpha}$$

Thus, the aggregate production function can be written as

$$y_t = \frac{\tilde{k}_t^\alpha L_t^{1-\alpha}}{\Omega_t}$$

where $\Omega_t = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj$ measures a distortion introduced by the dispersion in relative prices.²⁸ In order to express Ω_t in aggregate terms, let decompose it according to the Calvo pricing assumption again, so that

$$\begin{aligned} \Omega_t &= \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj = p_t^\nu \int_0^1 p_{j,t}^{-\nu} \\ p_t^\nu \int_0^1 p_{j,t}^{-\nu} &= p_t^\nu \left(\int_0^{1-\zeta} p_t^{*- \nu} dj + \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj \right) \\ p_t^\nu \int_0^1 p_{j,t}^{-\nu} &= p_t^\nu (1-\zeta) p_t^{*- \nu} + p_t^\nu \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj \end{aligned}$$

²⁸This distortion is not the one associated with the monopoly power of firms but an additional one that arises from the relative price fluctuations due to price stickiness.

$$\begin{aligned}
p_t^\nu \int_0^1 p_{j,t}^{-\nu} &= (1 - \zeta) \left(\frac{p_t^*}{p_t} \right)^{-\nu} + p_t^\nu \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj \\
p_t^\nu \int_0^1 p_{j,t}^{-\nu} &= (1 - \zeta) \left(\frac{p_t^*}{p_{t-1}} \right)^{-\nu} \left(\frac{p_{t-1}}{p_t} \right)^{-\nu} + p_t^\nu \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj \\
p_t^\nu \int_0^1 p_{j,t}^{-\nu} &= (1 - \zeta)(1 + \pi_t^*)^{-\nu}(1 + \pi_t)^\nu + p_{t-1}^{-\nu} p_t^\nu \int_{1-\zeta}^1 \left(\frac{p_{j,t-1}}{p_{t-1}} \right)^{-\nu} dj
\end{aligned}$$

Given random sampling and the fact that there is a continuum of firms

$$\Omega_t = (1 - \zeta)(1 + \pi_t^*)^{-\nu}(1 + \pi_t)^\nu + \zeta(1 + \pi_t)^\nu \Omega_{t-1}$$

D Full set of equilibrium conditions

Households' optimality conditions:

$$\tilde{Q}_{t,t+1} = \left(\frac{c_{t+1}}{c_t} \right)^{-\psi} \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varpi(1-\psi)} \beta(\theta_t) e^{(1-\psi)\mu} \frac{e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{-\chi}}{E_t \left[e^{(1-\gamma)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi} \right]^{\frac{-\chi}{1-\chi}}} \quad (19)$$

$$\frac{1 - L_t}{c_t} = \frac{\varpi}{w_t^{real}} \quad (20)$$

$$\frac{E_t[1 + \pi_{t+1}]}{1 + r_t} e^{\mu + \varepsilon_{z,t+1}} = E_t \tilde{Q}_{t,t+1} \quad (21)$$

$$\begin{aligned}
E_t \tilde{Q}_{t,t+1} P_{t+1}^{k,real} \left\{ u_{t+1} + \frac{1}{\delta_0 \eta u_{t+1}^{\eta-1}} \left[1 - \delta_0 u_{t+1}^\eta + \tau \frac{i_{t+1}}{k_{t+1}} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right) \right. \right. \\
\left. \left. - \frac{\tau}{2} \left(\frac{i_{t+1}}{k_{t+1}} - \frac{\bar{i}}{\bar{k}} \right)^2 \right] \right\} = \frac{P_t^{k,real}}{\delta_0 \eta u_t^{\eta-1}} e^{\mu + \varepsilon_{z,t+1}} \quad (22)
\end{aligned}$$

$$\frac{\delta_0 \eta u_t^{\eta-1}}{P_t^{k,real}} = 1 - \tau \left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}} \right) \quad (23)$$

Households' constraints

$$v_t = [c_t(1 - L_t)^\varpi]^{1-\psi} + \beta(\theta_t) e^{(1-\psi)\mu} E_t \left[e^{(1-\psi)(1-\chi)\varepsilon_{z,t+1}} v_{t+1}^{1-\chi} \right]^{\frac{1}{1-\chi}} \quad (24)$$

$$k_{t+1} = \frac{(1 - \delta_t)k_t + S\left(\frac{i_t}{k_t}\right)k_t}{e^{\mu + \varepsilon_{z,t+1}}} \quad (25)$$

Processus for the disaster risk

$$\log \theta_t = (1 - \rho_\theta) \log \bar{\theta} + \rho_\theta \log \theta_{t-1} + \sigma_{\theta,t} \varepsilon_{\theta_t} \quad (26)$$

Time-varying discount factor

$$\beta(\theta) = \beta_0 \left[1 - \theta_t + \theta_t e^{(1-\gamma) \ln(1-\Delta)} \right]^{\frac{1}{1-\alpha}} \quad (27)$$

Tobin's q

$$q_t = \frac{1}{1 - \tau \left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}} \right)} \quad (28)$$

Asset pricing

$$E_t Q_{t,t+1} = \frac{E_t(1 + \pi_{t+1})}{1 + r_t} \frac{1}{1 - \theta_t \Delta} \quad (29)$$

$$E_t \left[Q_{t,t+1} R_{t+1}^f \right] = 1 \quad (30)$$

$$E_t R_{t+1}^{k,real} = (1 - \theta_t \Delta) \frac{1 + r_t}{E_t(1 + \pi_{t+1})} \quad (31)$$

$$E_t(\text{Premium}_{t+1}) = \frac{E_t R_{t+1}^{k,real}}{R_{t+1}^f} \quad (32)$$

Firms' constraints

$$y_t = \frac{\tilde{k}_t^\alpha L_t^{1-\alpha}}{\Omega_t} \quad (33)$$

$$\tilde{k}_t = u_t k_t \quad (34)$$

$$w_t^{real} = mc^* (1 - \alpha) \left(\frac{\tilde{k}_t}{L_t} \right)^\alpha \quad (35)$$

$$P_t^{k,real} = mc^* \alpha \left(\frac{\tilde{k}_t}{L_t} \right)^{\alpha-1} \quad (36)$$

$$(1 + \pi_t^*) = (1 + \pi_t) \frac{\nu}{\nu - 1} E_t \frac{\tilde{\Xi}_{1t}}{\tilde{\Xi}_{2t}} \quad (37)$$

$$\tilde{\Xi}_{1t} = y_t m c_t^* + \zeta E_t \tilde{Q}_{t,t+1} \tilde{\Xi}_{1,t+1} (1 + \pi_{t+1})^\nu \quad (38)$$

$$\tilde{\Xi}_{2t} = y_t + \zeta E_t \tilde{Q}_{t,t+1} \tilde{\Xi}_{2,t+1} (1 + \pi_{t+1})^{\nu-1} \quad (39)$$

Price distortion:

$$\Omega_t = (1 - \zeta)(1 + \pi_t^*)^{-\nu} (1 + \pi_t)^\nu + \zeta (1 + \pi_t)^\nu \Omega_{t-1} \quad (40)$$

Price index:

$$(1 + \pi_t)^{1-\nu} = (1 - \zeta)(1 + \pi_t^*)^{1-\nu} + \zeta \quad (41)$$

Taylor rule:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) [\Phi_\pi (\pi_t - \bar{\pi}) + \Phi_Y (y_t - y^*) + r^*] \quad (42)$$

Aggregate resource constraint:

$$y_t = c_t + i_t \quad (43)$$

This is a system of 25 equations in 25 unknowns: $\{y, c, i, L, k, \tilde{k}, u, \beta, q, \theta, w^{real}, P^{k,real}, \tilde{Q}, Q, R^{k,real}, R^f, Premium, \Omega, \pi, \pi^*, \tilde{\Xi}_1, \tilde{\Xi}_2, m c^*, v, r\}$. We solve for the steady-state and then use Dynare to simulate the responses to the disaster risk shock with a third-order approximation (unless otherwise specified).

Figure 5: Responses (in percentage change) to a rise in θ from 0.9% to 1%, with a first-order approximation, comparing Gourio's calibration ($\zeta = 0$ and $EIS = 2$) and our baseline calibration ($\zeta = 0.8$ and $EIS = 0.5$).

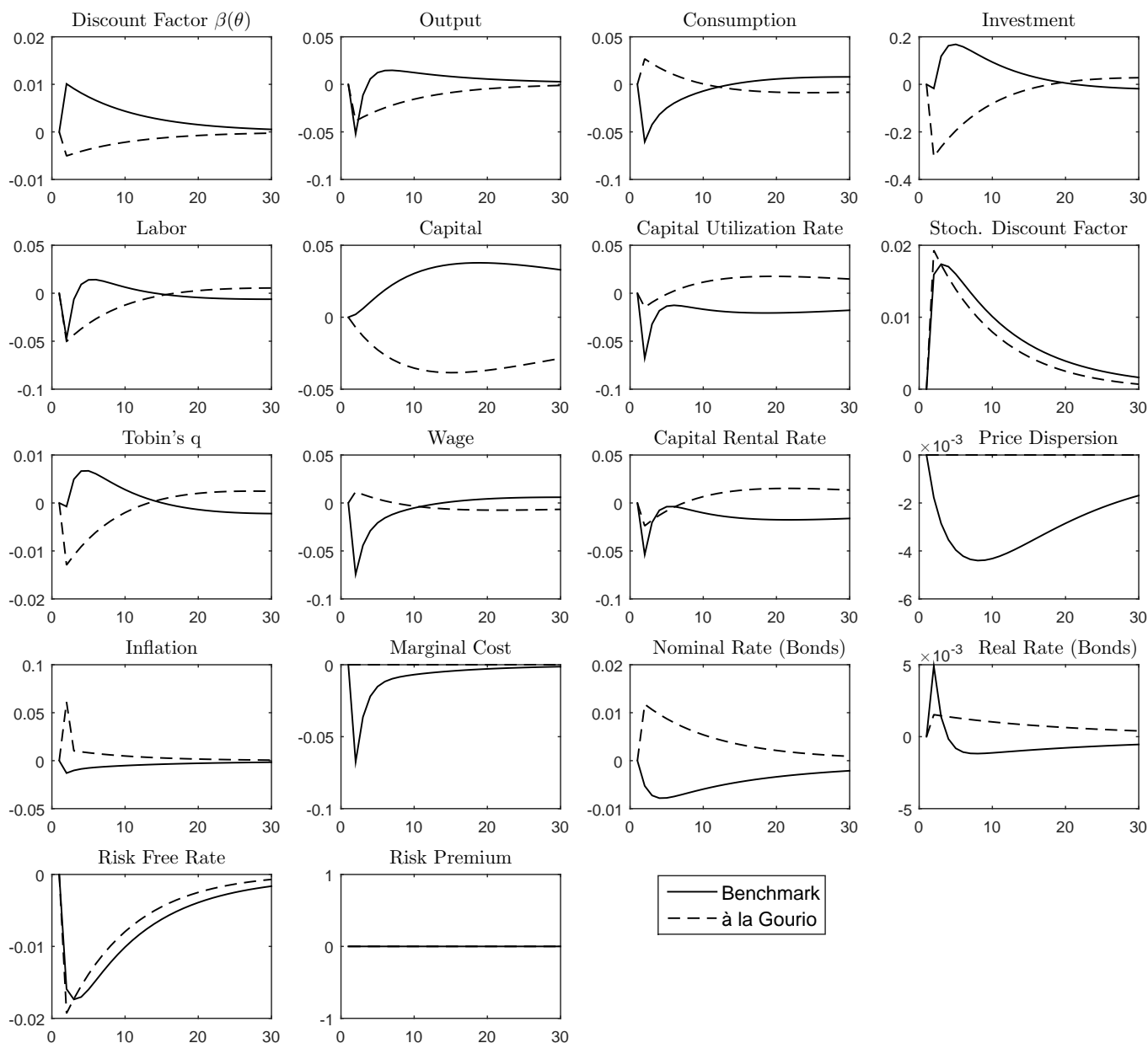


Figure 6: Responses (in percentage change) to a rise in θ from 0.9% to 1%, with a third-order approximation, comparing Gourio's calibration ($\zeta = 0$ and $EIS = 2$) and our baseline calibration ($\zeta = 0.8$ and $EIS = 0.5$).

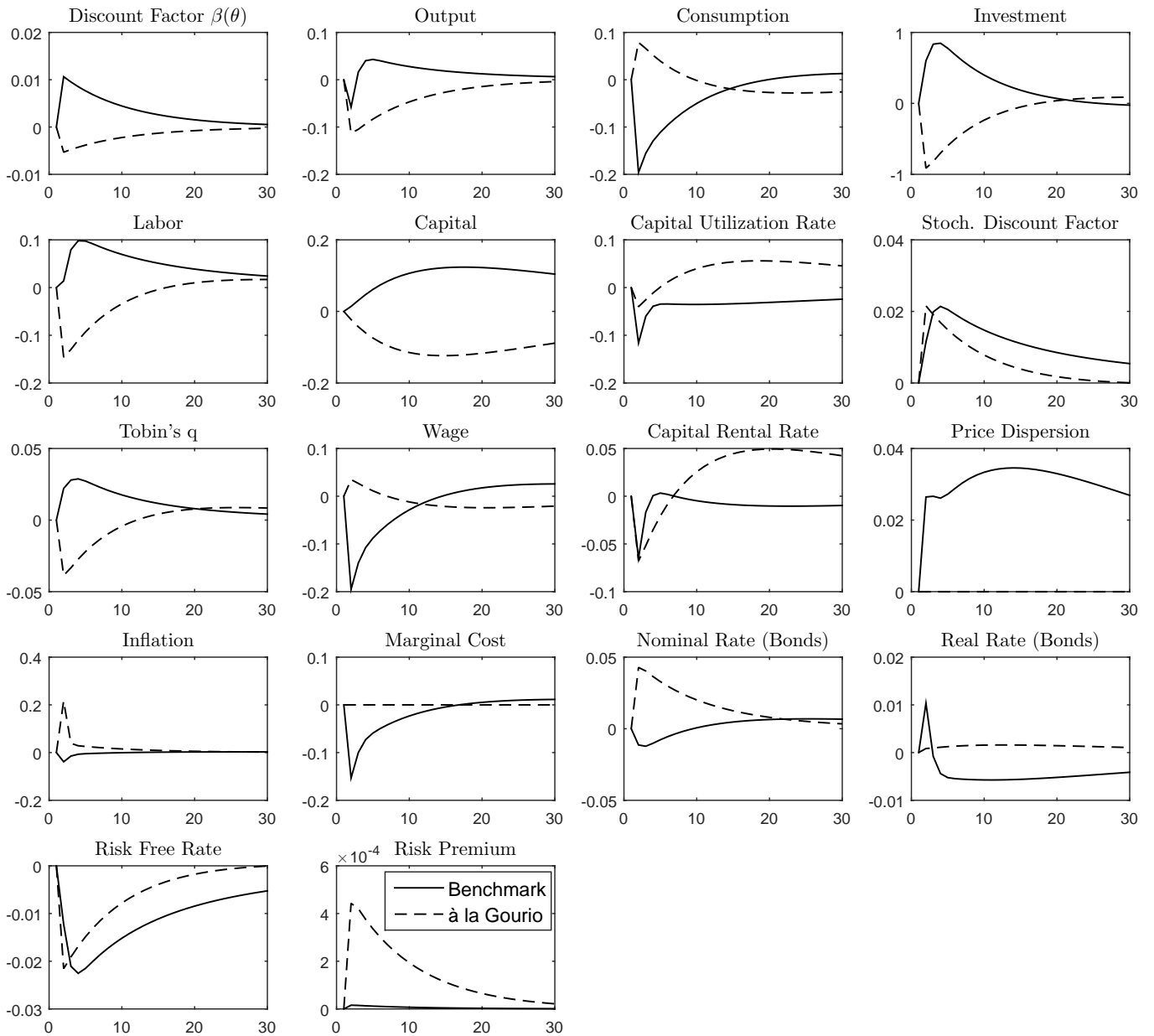
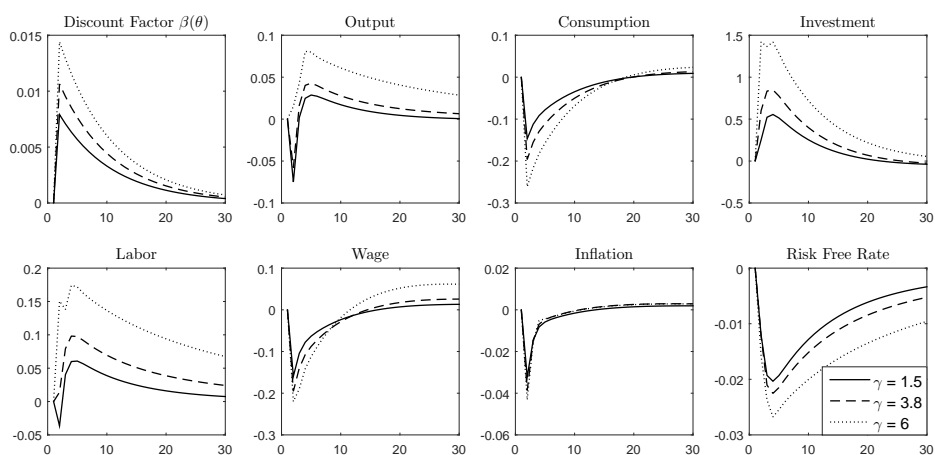
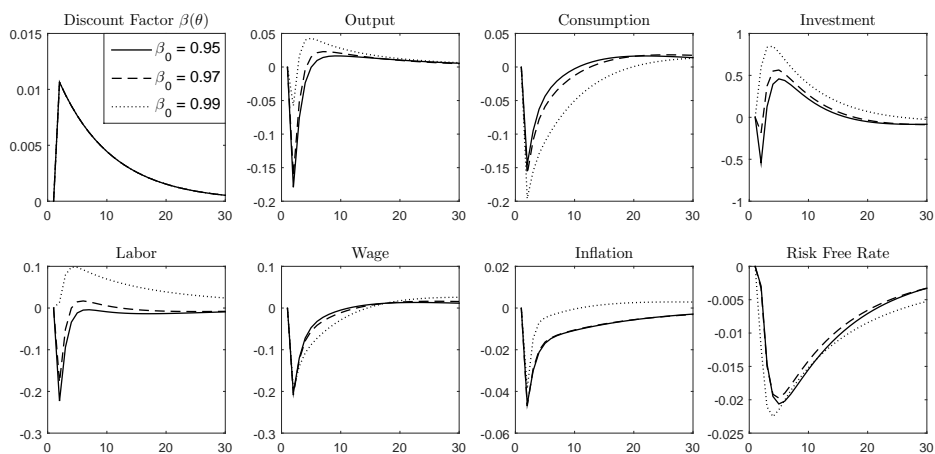


Figure 7: Responses (in percentage change) to a rise in θ from 0.9% to 1% (baseline calibration, third-order), for alternative levels of parameters.

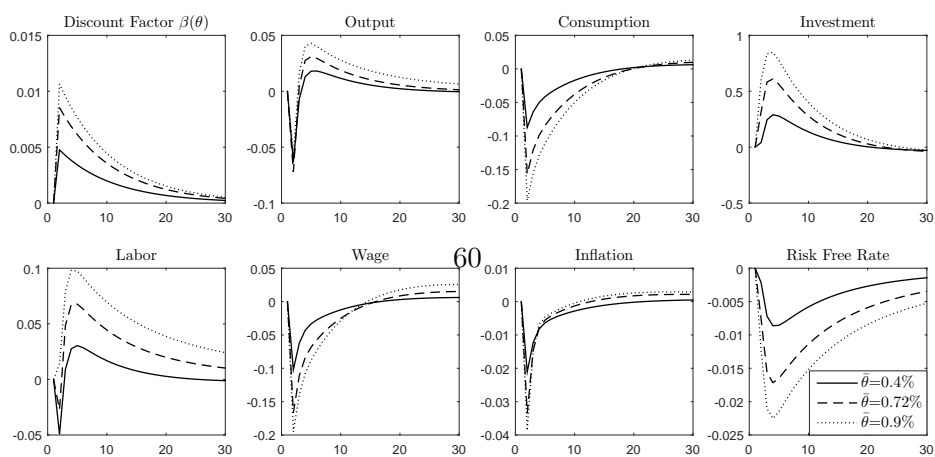
- Risk aversion coefficient, γ



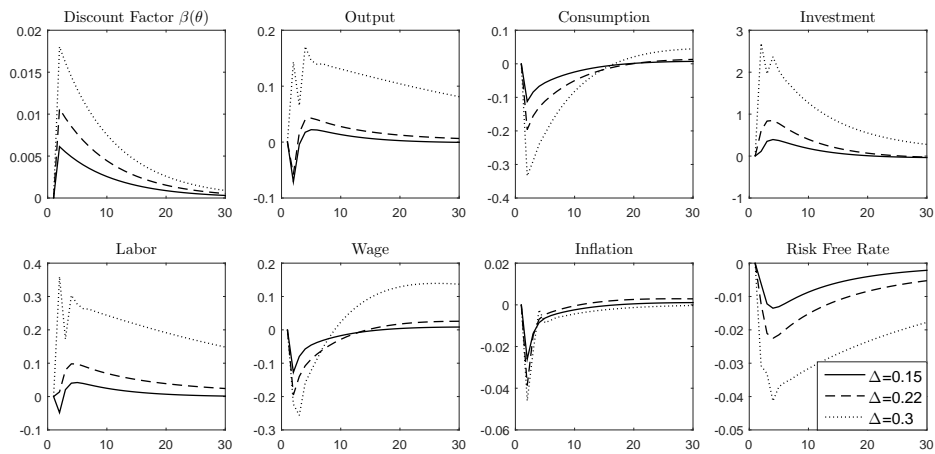
- Discount factor in the absence of disaster risk, β_0



- Steady-state probability of disaster, $\bar{\theta}$



- Size of disaster, Δ



- Persistence of disaster probability, ρ_θ

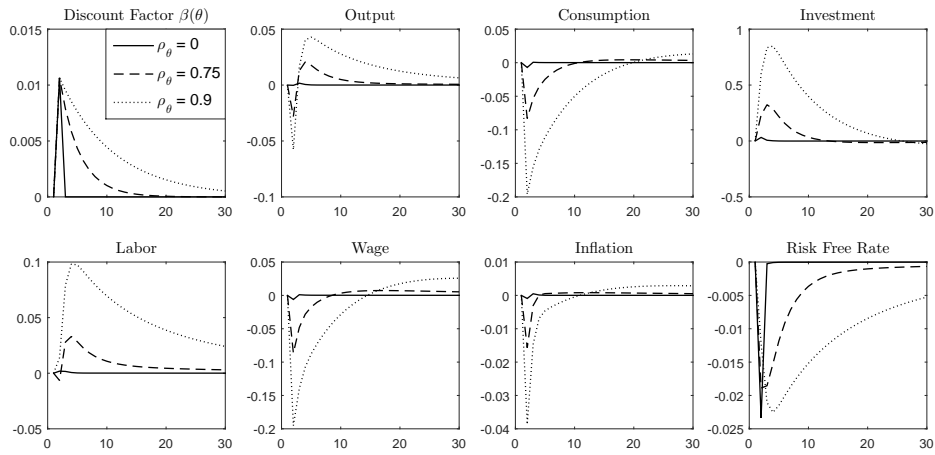


Figure 8: Responses (in percentage change) to a 1% change in the nominal interest rate on bonds (baseline calibration)

