Start Me Up: Modeling of Power Plant Start-Up Conditions and their Impact on Prices

Abrell, Jan and Kunz, Friedrich and Weigt, Hannes

July 2008

Electricity Markets Working Papers

WP-EM-28

Start Me Up
Modeling of Power Plant Start-Up Conditions and their Impact on Prices

Jan Abrell, Friedrich Kunz, Hannes Weigt

July 2008

Dresden University of Technology

Chair for Energy Economics and Public Sector Management

Start Me Up
Modeling of Power Plant Start-Up Conditions and their Impact on Prices[1]

Jan Abrell, Friedrich Kunz, Hannes Weigt

Corresponding author:

Hannes Weigt

Dresden University of Technology

Faculty of Business and Economics

Chair of Energy Economics and Public Sector Management

01069 Dresden, Germany

Phone: +49-(0)351-463-39764

Fax: +49-(0)351-463-39763

hannes.weigt@tu-dresden.de

Abstract:

In this paper we compare different approaches to account for start-up costs when modeling electricity markets. We restrict the model formulation to either linear or mixed integer problems in order to guarantee a robust solution. The results indicate that the choice of the model has a significant impact on the resulting market prices and company profit. The models either calculate higher peak prices or prices below marginal costs in off-peak periods but not both. Furthermore, the models perform differently when we apply a large sample, the number of equations having an important impact. We conclude that different model formulations respond particularly to specific modeling questions.

Key words: electricity, unit commitment, start-up costs, linear programming, mixed integer programming

JEL-code: L94, L51, D61

[1]. We thank William Hogan, Florian Leuthold, Christian von Hirschhausen, and participants of the YEEES meeting April 2008 in Vienna, for their comments and suggestions.

1 Introduction

Electric power industries are experiencing a major restructuring process towards a competitive market environment in which power generators face the fundamental task to determine the optimal dispatch of their power plants. Due to restrictions regarding the start-up and shut-down process, power plants have to be scheduled carefully with respect to the market conditions and the resulting profitability of the plant. In contrast to former monopolistic times generators now have to recover their costs solely through market based prices. Furthermore, politicians, regulators, and competition authorities keep a close eye on the new markets to estimate whether the prices are subject to market power abuse. This opens the questions how to model start-up restrictions within a market framework and to which extend prices are affected by different approaches.

In order to derive the optimal dispatch two main issues have to be regarded: the unit commitment decision, and the calculation of start-up costs. The first problem regards the question when a plant is available for dispatch. As starting up a power plant can take several hours this decision has to be made ahead of the actual dispatch. A list of different approaches regarding the unit commitment is presented in Sheblé and Fahd (1994). Firstly, the unit commitment can be modeled as a mixed integer problem (MIP) using integer variables to determine the status of the power plant. This technique is widely used, but solvability is restricted particularly in large systems. Baldick (1995) formulates a generalized version of the unit commitment problem that can treat minimum up- and down-time constraints, power flow constraints, line flow limits, voltage limits, reserve constraints, ramp limits, and total fuel and energy limits on hydro and thermal units. The authors propose an algorithm for this problem, based on Lagrangian decomposition, and demonstrate the algorithm with reference to a simple model system. Arroyo and Conejo (2000) propose a mixed-integer linear programming approach that allows a rigorous modeling of i) non-convex and non-differentiable operating costs, ii) exponential start-up costs, iii) available spinning reserve taking into account ramp rate restrictions, and iv) minimum up and down time constraints. The problem of solvability can be avoided by using a linear programming approach (LP) to solve the unit commitment problem. The linear programming approach neglects integer status variables and therefore solvability in large systems can be increased. Kuntz and Müsgens (2007) describe the unit commitment problem in terms of a linear program without integer variables simultaneously taking into account start-up costs. However, the linearization leads to a more unrealistic market modeling: technical characteristics of power plants concerning the start-up process are neglected and therefore generation units are assumed as fully dispatchable.

The second problem is the question how start-up costs can be implemented within the unit commitment formulations. Flexibility of power plants is restricted by generation technology, fuel type, and external conditions. According to Dillon et al. (1978), detailed start-up costs of a thermal power plant can be modeled by an exponential function depending on the cooling time of the boiler and the costs of starting the boiler and/or the turbine. To achieve a linear mixed integer programming approach different transformation techniques (e.g. due to the exponential start-up function) are described by Sheblé and Fahd (1994) and applied to the unit commitment problem by e.g. Baldick (1995) and Doorman and Nygreen (2003). However, a linear unit commitment model can also be modeled without an exponential start-up function and further mathematical transformations (e.g. Kuntz and Müsgens, 2007). Start-up costs can be modeled either as fixed cost within a mixed integer unit commitment formulation or related to output within a linear approach. However, one has to consider that the implementation of start-up costs within a unit commitment process results in non-convexities of the marginal costs curve and therefore the resulting market prices can be inconsistent.

An additional problem is the question whether the actual dispatch and the resulting market prices support a decentralized solution in which each generator has at least zero profit. Gribik et al. (2007) illustrate different ways to define and calculate consistent market prices and side payments given the dispatch. As we focus on the actual dispatching process this problem is not addressed in this paper.

In our paper we compare different approaches regarding the implementation of unit commitment and start-up costs into a market framework. We restrict our analysis to linear and mixed integer approaches based on cost minimization. Thus decentralized equilibrium approaches and more complex mathematical algorithms are not taken into account. We apply four different model types to account for start-up costs: first a two stage approach building on a marginal cost based unit commitment (Hirschhausen and Weigt, 2007); second the approach of Kuntz and Müsgens (2007) using a linearized model formulation; the third approach takes up the linearized model and introduces binary choice variables; fourth several pure mixed integer solution are calculated with additional binary variables stating if a plant is started as in Arroyo and Conejo (2000). All models are tested against a simple marginal cost based unit commitment approach. Our results indicate that depending on the chosen model obtained market prices and thus company profits can vary significantly. Furthermore, we test the performance of the models when applied to a large sample set like the German electricity market. The remainder of our paper is structured as follows: Section 2 presents the model formulations and the data used to derive our results. Section 3 presents the results by comparing the different model formulations. Section 4 gives the conclusions.

2 Model formulations and data

The basic objective of our paper is to compare different model approaches with respect to start-up restrictions. The main premise is that the model is either a linear problem (LP) or a mixed integer problem (MIP) in order to obtain a robust solvability when applying the approaches to large markets. All models are coded and solved in GAMS.

2.1 Basic model formulation: Benchmark (MIP approach)

All start-up models are based on basic market characteristics with the same market clearing mechanism. The objective is cost minimization under a given external demand level d subject to the energy balance constraint and capacity restrictions of the power plants:

$$\min \left[\text{costs} = \sum_{s,t} mc_s \, g_{s,t} + startup \right] \quad (1)$$

s.t.

$$\sum_s g_{s,t} = d_t \qquad \text{energy balance constraint} \quad (2)$$

$$g_{s,t} \le on_{s,t} \, g_s^{\max} \qquad \text{upper capacity constraint} \quad (3)$$

$$g_{s,t} \ge on_{s,t} \, g_s^{\min} \qquad \text{lower capacity constraint} \quad (4)$$

$$on_{s,t-1} - on_{s,t} \le 1 - on_{s,\tau} \quad \tau = t,...,t + l_s \qquad \text{start-up constraint} \quad (5)$$

We assume a perfect competitive market; thus generators s bid their true marginal costs mc_s. In order to determine the condition of a power plant the binary variable $on_{s,t}$ is introduced into the model which takes the value of 1 in case the plant s is dispatched in hour t and 0 otherwise. The generation g must remain within the minimum (g^{min}) and maximum capacity (g^{max}) constraints in case the plant is operating. The possibility to start a plant is subject to the status of the plant in the previous periopds. In case a plant goes offline in hour t it is not able to start right again for the next τ hours defined by the start-up time of the plant (l_s) (see Takriti et al., 1998).

The actual dispatch $g_{s,t}$ is determined by a market clearing process equaling demand d_t with the total amount of generation in each hour t minimizing the sum of generation expenses and start-up costs *startup*. How these start-up costs are determined differs for the considered models. The dual variable on the energy balance constraint is considered to represent the hourly market price. The benchmark model assumes that start-up cost are neglected (*startup* is fixed to 0), thus the market is solely driven by short run marginal generation costs. This problem is solved as MIP.

2.2 Adjustments for different start-up approaches

2.2.1 Two stage approach (MIP and LP approach)

The first model to take account of start-up costs is a two stage approach with the unit commitment determined in the first stage and the final dispatch in the second stage.[2] The benchmark model acts as first stage; in the second stage start-up costs are determined according to the plant status:

$$startup = \sum_{s,t} sc_s g_{s,t} (on_{s,t} - on_{s,t-1}) \qquad \text{start-up costs} \quad (6)$$

with sc_s the fuel costs per heated up MW. In case a plant goes online in period t, the condition difference $(on_{s,t} - on_{s,t-1})$ will be 1, output $g_{s,t}$ will be positive and thus start-up costs will be added. In all other cases start-up costs will be 0 as either the condition difference or the actual output will be 0. Thus start-up costs will be added in relation to the generation. This allows the model to have a minor influence on the actual dispatch in the second stage.[3] The first stage is solved as a MIP and the second stage as an LP, as the binary variables are fixed.

2.2.2 Ramping approach (LP approach)

The second approach to take account of start-up costs is based on Kuntz and Müsgens (2007) and corresponds to a ramping cost approach: generation increase is associated with additional costs beyond pure fuel costs. Kuntz and Müsgens extend the choice variables of generation by adding newly started generation $g^{+}_{s,t}$ and the amount shut down $g^{-}_{s,t}$. On the other hand they neglect the conditional variables $on_{s,t}$ and thus also the start-up constraints (equation 5).[4]

The actual generation output in this model is determined by a dynamic equation:

$$g_{s,t} = g_{s,t-1} + g^{+}_{s,t} - g^{-}_{s,t} \qquad \text{generation balance} \qquad (7)$$

Start-up costs are then obtained by taking into account the newly started generation. It is assumed that shutting down a plant does not imply additional costs:

$$startup = \sum_{s,t} sc_s g^{+}_{s,t} \qquad \text{start-up costs} \qquad (8)$$

The model consists of equations (1), (2), (3), (7), and (8) without the conditional variable $on_{s,t}$ and is solved as a LP in GAMS.

2.2.3 Extended ramping approach (MIP approach)

The third start-up approach extends the model of Kuntz and Müsgens (2007) by including the binary condition variables $on_{s,t}$ and making a differentiation between started capacity $g^{start}_{s,t}$ and increased generation output $g^{+}_{s,t}$. The dynamic generation balance is adjusted accordingly:

$$g_{s,t} = g_{s,t-1} + g^{start}_{s,t} + g^{+}_{s,t} - g^{-}_{s,t} \qquad \text{generation balance} \qquad (9)$$

To allow for a differentiation between $g^{start}_{s,t}$ and $g^{+}_{s,t}$. a link to the plant condition is established:

$$g^{start}_{s,t} \leq on_{s,t}\, g^{\max}_{s} \qquad \text{start-up capacity constraint} \qquad (10)$$

$$g^{+}_{s,t} \leq on_{s,t-1}\left(g^{\max}_{s} - g^{\min}_{s}\right) \qquad \text{ramping capacity constraint} \qquad (11)$$

Equation (11) ensures that the increased generation output does not exceed the available plant capacity and is only available in case that plant was online in the previous period.[5] Similarly, equation (10) restricts the quantity started in period t to fulfill the upper capacity restriction. Therefore the only

---

[2] Hirschhausen and Weigt (2007) apply this approach in an analysis of the German wholesale electricity market.

[3] One could also use a fixed quantity (e.g. g_{min}) for the calculation of start-up costs or distribute the occurring cost equally over all periods in which the plant is running. However, in those cases the market prices will remain on fuel costs level, as the start-up block is independent from the dispatch choice in the second stage.

[4] Kuntz and Müsgens (2007) also include minimum generation constraints in their general model formulation. However, they do not allow for a shut-down of a plant in case the minimum constraint can not be fulfilled.

[5] If a plant is shut down in period t, equation 11 does not bind $g^{+}_{s,t}$. However, $g_{s,t}$ is bound by equation (4) to be 0.

possibility to start-up an offline plant in period t is to use $g_{s,t}^{start}$, as the ramping quantity $g^{+}_{s,t}$ is restricted by equation (11). The start-up costs are then obtained by summing the started capacities:[6]

$$startup = \sum_{s,t} sc_s g_{s,t}^{start} \qquad \text{start-up costs} \qquad (12)$$

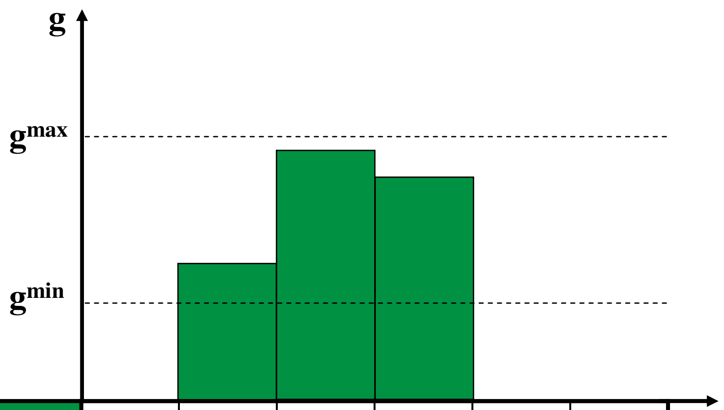
This approach allows to keep the basic model formulation with plant condition variables and start-up restrictions and simultaneously to account for start-up costs. The model consists of equations (1) to (5), and (9) to (12) and is solved as a MIP in GAMS.

2.2.4 Additional choice variables (MIP approach)

The following approaches all introduce additional binary variables compared to the benchmark model thus keeping a MIP approach following Arroyo and Conejo (2000). The approaches have in common that start-up costs are considered as a fixed cost block correlated to the minimum output capacity in the hour of start-up. Thus they differ from the previous approaches where start-up costs depend on the actual plant output. Due to this similarity they are supposed to produce a similar output. However, due to varying equations and binary variables they differ significantly in their computational performance.

The binary plant condition variables cover the start-up ($up_{s,t}$) and shut-down ($down_{s,t}$) of the plant as well as the actual plant condition $on_{s,t}$. Figure 1 provides an overview how the variables are connected: if the plant is started in period t $up_{s,t}$ takes the value of 1; when the plant is shut down again in t+n $down_{s,t}$ becomes 1; the conditional variable $on_{s,t}$ remains 1 from the starting period t until the period before shutdown $t+n-1$. The actual output in each period in which the plant is running has to remain within the minimum and maximum capacity constraints.

Figure 1: Binary plant variables and output



up	0	1	0	0	0	0
down	0	0	0	0	1	0
on	0	1	1	1	0	0

Source: Based on Arroyo and Conejo (2004)

[6] The model can easily be extended by adding a further costs parameter for ramped generation.

2.2.4.1 Start-ups as additional choice (MIP-Up)

Considering start-up costs to be fixed, they can be introduced in the benchmark model by adding a new binary choice variable and an additional constraint. Let $up_{s,t}$ be 1 if plant s is started in period t and 0 else. Fixed start-up cost follow as:

$$startup = \sum_{s,t} sc_s g_s^{\min} up_{s,t} \qquad \text{start-up costs} \qquad (13)$$

In order to ensure consistency with the variable for the plant status, $on_{s,t}$, we introduce the following constraint, which ensures that $up_{s,t}$ becomes 1 if and only if the status of the plant switches from offline to online.

$$up_{s,t} \geq on_{s,t} - on_{s,t-1} \qquad \text{choice restriction} \qquad (14)$$

The model (MIP-Up) consists of equations (1)-(5), (13), and (14), and is solved as MIP in GAMS.

2.2.4.2 Two Binary Choices: start-up and shut-down (MIP-Up-Down)

In this approach we replace the binary choice variable $on_{s,t}$ in the benchmark model by two binary choices, $up_{s,t}$ and $down_{s,t}$: $up_{s,t}$ is 1 if plant s is started in period t and 0 else; $down_{s,t,}$ is 1 if plant s is shut down in period t and 0 else.[7] Again, these choice variables allow to model start-up costs as pure fixed costs. Thus starting costs are defined as in the previous model (equation (13)). The objective function (1) and the energy balance constraint (2) remain unchanged. Previous minimum and maximum capacity constraints (3) and (4) become:

$$g_{s,t} \leq g_s^{\max} \sum_{\tau=0}^{t} \left(up_{s,\tau} - down_{s,\tau} \right) \qquad \text{upper capacity constraint} \qquad (15)$$

$$g_{s,t} \geq g_s^{\min} \sum_{\tau=0}^{t} \left(up_{s,\tau} - down_{s,\tau} \right) \qquad \text{lower capacity constraint} \qquad (16)$$

The sum of the difference of the binary choices from period 0 up to period t expresses the status of plant s in period t.[8] It becomes 1 if the plant is online and 0 if the plant is offline in period t. This sum is restricted by the requirement that it is lower or equal to 1 since plant s can only go online if it is offline. If such a restriction is missing, there is an incentive to choose $up_{s,t}$ equal to 1 in consecutive periods which would lead to a higher upper capacity bound in equation (15).

$$1 \geq \sum_{\tau=0}^{t} \left(up_{s,\tau} - down_{s,\tau} \right) \qquad \text{up-down restriction} \qquad (17)$$

The constraint on the required offline time (5) can be rewritten to:

$$down_{s,t} \leq 1 - \sum_{\tau=t}^{\tau+l_s} up_{s,t} \qquad \text{adjusted start-up constraint} \qquad (18)$$

The model (MIP-Up-Down) consists of equation 1, 2, and 15 to 18 and is solved as MIP in GAMS.

---

[7] This formulation of the choice variables is somehow similar to Arroyo and Conejo (2000).

[8] This sum replicates the previous binary choice $on_{s,t}$.

2.2.4.3 Three Binary Choice Variables: start-up, shut-down and status (MIP-Up-Down-On)

In this model we use all three of the above binary choice variables, $on_{s,t}$, $up_{s,t}$, and $down_{s,t}$. The formulation of the start-up costs and restriction on binary variables to ensure consistency is the same as in the MIP-Up model. However, using the $down_{s,t}$ binary variable allows to incorporate the adjusted start-up constraint (18) instead of the original start-up constraint (5). Put differently, by increasing the number of binary choice variables, we can decrease the number of equations. The (MIP-Up-Down-On) model consists of equations (1)-(4), (13), (14), and (18).

2.3 Data

All models use the same dataset of a hypothetical electricity market without an underlying network (Table 1). We assume 40 plants of four different plant types: base load plants with large capacities, low marginal and high start-up costs (e.g. nuclear plants), medium load plants with medium capacities, relatively low marginal and intermediate start-up restrictions (e.g. coal plants), medium load plants with medium capacities, relatively high marginal costs and low start-up restrictions (e.g. CCGT-plants), and peak plants that have small capacities, high marginal costs and no start-up restrictions (e.g. gas turbines). The cost and restriction data is based on Dena (2005) and Schröter (2004). Table 1 gives a summary of the underlying plant characteristics. For simplicity reasons we assume that there are 10 plants of each type in the market.

The demand curve of the market is constructed to represent a typical day with a noon and an evening peak and the lowest demand in the early morning hours. The values have been chosen to have at least one demand level in each capacity range (Figure 2). The model period is a 24 hour run. As the model formulation needs the previous period for start-up calculations a period 0 has been added to the dataset with the same demand level as in hour 1. Thus the model consists of 25 hours. However, the cost values obtained for period 0 are not accounted in the objective function (equation (1)).

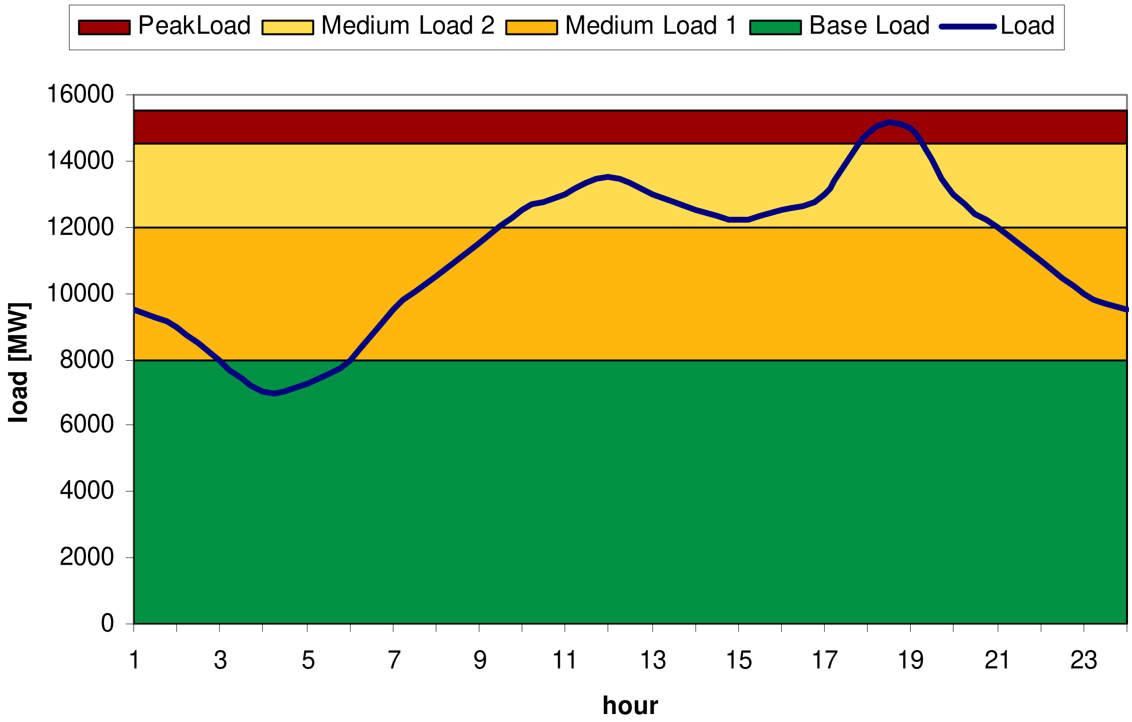
In order to test the performance of the models a second dataset based on the German market in 2006 is used (Hirschhausen and Weigt, 2007). We conduct model tests consisting of one full week (168 hours) for each month of the year.

Table 1: Plant characteristics

Plant type	Maximum Capacity [g^{max}]	Minimum Capacity [g^{min}]	Marginal Costs [mc]	Start-up Costs [sc]	Start-up Time [l]
Base Load	800 MW	320 MW	12.5 €/MWh	66.8 €/MW	8
Medium Load I	400 MW	152 MW	20.0 €/MWh	47.1 €/MW	4
Medium Load II	250 MW	83 MW	30.0 €/MWh	57.8 €/MW	2
Peak Load	100 MW	20 MW	60.0 €/MWh	23.1 €/MW	0

Source: Own estimations based on Dena (2005) and Schröter (2004)

Figure 2: Model load curve and plant capacities



3 Model Results and Discussion

3.1 Model Results

First the models are all tested using the same hypothetical dataset of plants and demand. Thus differences are solely contributed to the model formulation. As we assume a fixed demand level the market price is derived as the dual variable on the energy balance constraint representing the change of the overall costs in case of a marginal demand increase. As can be shown subsequently, the results differ due to specific assumptions of each model approach.

Figure 2 shows the obtained market prices for each hour and each approach. The benchmark case represents the pure marginal generation cost approach. Thus, in each hour the market price equals the fuel costs of the last running unit. The two stage model takes up this solution, fixes the plant conditions and calculates the start-up costs ex-post. Therefore the results indicate when addition units have been started, as the market prices exceed the fuel costs in those hours.

The ramping approach shows price spikes in few hours, negative prices in hour 4 and 15, and price levels equal to fuel costs in the remaining hours. The negative prices signal that plants should be kept online in order to avoid a re-start-up (e.g. base load plants during night times). Surprisingly, price spikes are observed shortly before load and prices go down again. One would assume that start-up costs lead to higher prices in situations where load increases. This is due to the model assumption that each additional MWh of generation is accounted for with fuel and start-up costs whereas stable generation is only accounted with fuel costs. Thus a marginal load increase will only lead to prices above fuel costs if the load will not be started up later anyway which is the case in hour 12 and 19.

The same mechanism holds for negative prices as a marginal load increase will reduce the need for re-startups.[9]

The extended ramping approach shows prices below benchmark fuel prices but no price spikes in hours of load increases.[10] The prices below marginal costs are due to the fact that keeping plants running at minimum output for few hours instead of shutting them down and starting them up again results in lower costs if the start-up costs are significantly high. This can be observed during the night hours where medium load plants are kept at minimum output to avoid the start-up costs in the morning. Therefore the base load plants are not running at full capacity. Thus an additional demand could be satisfied with base load plants keeping the price low. The same holds true in the afternoon where the second medium load plants are kept online at minimum output.

The MIP approaches taking into account start-up with a separate binary variable and with the minimum capacity as cost determiner produces similar market prices, independently from the concrete model formulation. Due to the sunk cost character of start-up cost we do not observe any price spikes. However, we observe prices below marginal fuel costs for some night hours and one afternoon hour due to the same reasons as for the extended ramping approach.

The actual power plant dispatch differs according to the model approach (Figure 4). The benchmark case and the two stage approach have the same generation pattern which represents the least fuel cost solution given the start-up restrictions. The ramping approach shows a slightly altered dispatch in hour 12 and 15. As the peak units have lower total costs (fuel plus start-up) than the medium load II plants the demand peak at 12 is satisfied with peak units whereas the medium load II plants are kept at constant output from hours 11 to 13. In hour 15 the medium load II plants are kept online with constant output whereas the output of the medium load I plants decreases as the total costs of re-starting a medium load I plant are lower.

The three MIP approaches as well as the extended ramping approach all show nearly the same power plant dispatch. In order to avoid the re-startup of the medium load plants I they are kept online at minimum capacity during the night times and the same holds for medium load II plants during hour 15. A small divergence between the ramping and the other MIP approaches can be observed in hours 9 and 17. In hour 9 a small fraction of load is satisfied with medium load II plants and in hour 17 the same holds for peak plants. This is a result of the start-up cost approach that is based on the actual output during the hour of start-up. In order to keep the costs low plants are started an hour before they are actually needed if otherwise they would have to produce significantly above the minimum capacity limit.

[9] Numerical example for hour 4: The demand level is below the maximum capacity of all base load plant. Thus an additional MW of demand will need 12.5 €/MWh fuel costs. However, the additional demand would reduce the need for a re-start-up of the base load unit in the next hour saving 66.8 €/MW. The resulting market price therefore is -54.3 €/MWh.

[10] For this model approach we assume an optimality gap of 1% and the obtained results depend on this gap restriction. In case of a higher gap price spikes will occur and prices below marginal costs can vanish. All other MIP approaches have an optimality gap below 0.1%.

Figure 3: Market Prices

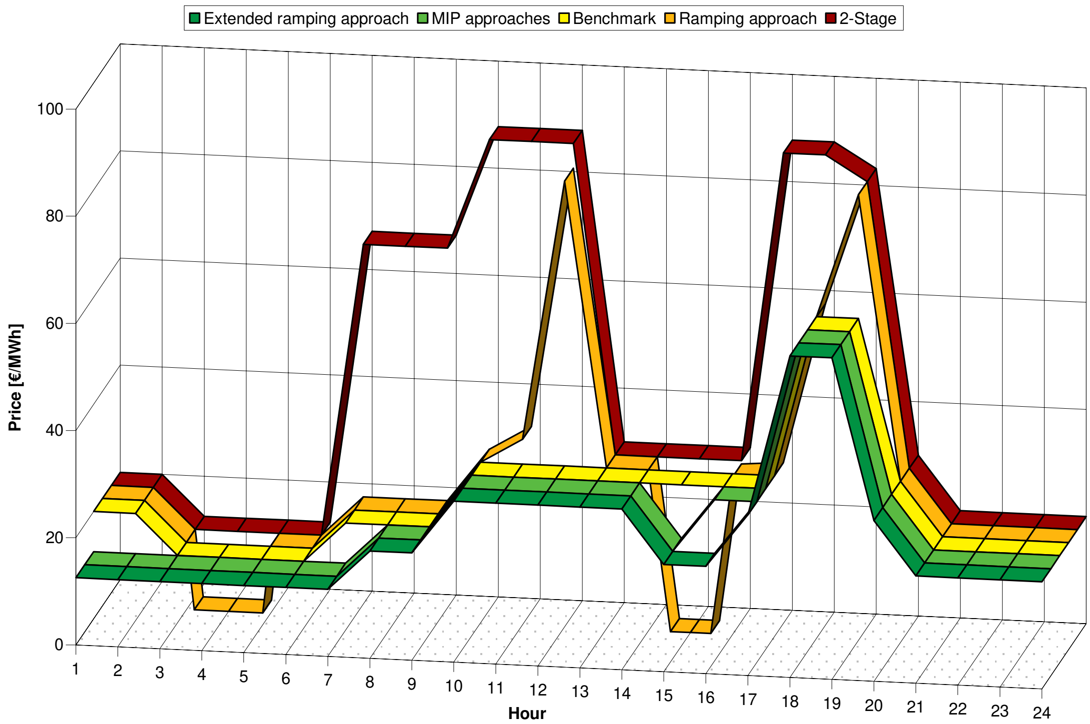
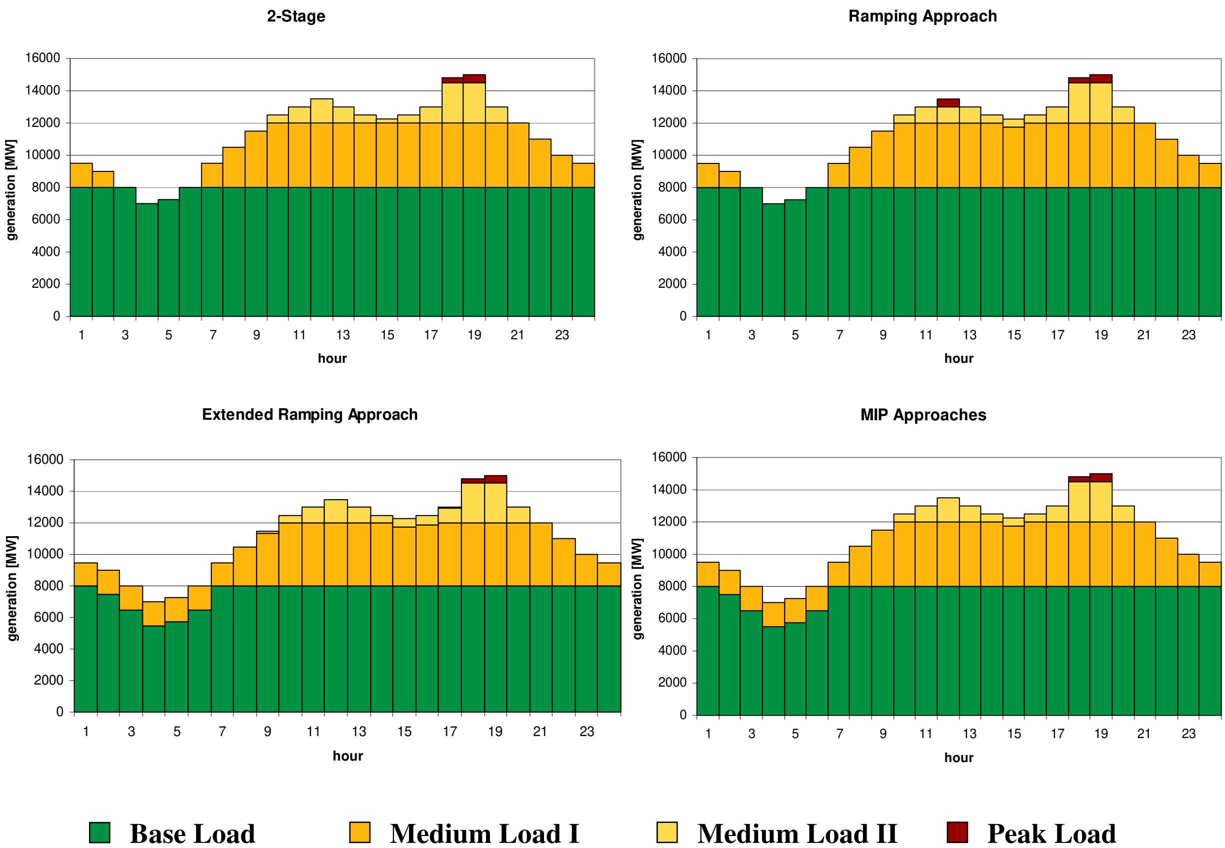


Figure 4: Dispatch



3.2 Discussion

When comparing the overall results we can classify the approaches in two groups (Table 2): The first group results in relatively low total fuel but high start-up expenses and includes the two stage and the ramping approach. The second group leads to low start-up costs but higher total fuel costs and includes the extended ramping and the MIP approaches with start-up choice variables.

The two stage approach takes the unit commitment of the benchmark model and calculates the start-up costs ex-post which does not allow keeping plants running to avoid re-startup. Furthermore the plant output in the hour of start-up is fully accounted with start-up costs. Thus, the resulting market prices are the highest of all approaches and consequently the company profit is the highest as well.

In the ramping approach each additional unit is accounted with fuel and start-up costs leading to the highest overall costs. This is mainly due to the impact of re-starting the base load plants in the morning hours. As all other approaches have an integer consideration of plant condition the base load plants remain online in all hours and no start-up costs occur. However, the linear approach does not take into account plant conditions and obtains costs solely on a dispatch basis. Thus, a part time decrease of output will result in start-up costs if the output increases again even if the plant theoretically has not been shut down. The resulting average market price is the lowest of all approaches. This is due to the high negative prices in cases where a plant should be kept online to avoid a re-startup (e.g. during night times).

Contrary to the first group, the MIP approaches and the extended ramping approach have in common that start-up costs are significantly smaller whereas fuel expenses are higher. This is due to the trade off conducted to keep plants running as long as the increased fuel consumption is offset by the savings of start-up costs. The resulting market prices are either equal to marginal fuel costs of the last running unit or lower in case plants are kept at minimum output to avoid re-startup. Thus the average market price is slightly lower than the benchmark case.

A further important question is whether or not the market prices will allow generators to recover their start-up costs. Given that the 2-stage approach accounts for start-up costs in the hour a plant is going online with the correlating output this approach allows each generator to recover its start-up expenses. Consequently the obtained profit of generators is the highest of all approaches. All other approaches have prices close to marginal fuel cost level which will allow generators to only recover their costs in case more expensive plants are setting the market price. For a more detailed discussion about the relation between unit commitment and company profits see Gribik et al. (2007).

Table 2: Model comparison

Model	Fuel Cost [T €]	Start-up Costs [T €]	Profit [T €]	Average Market Price
Benchmark	4,085.5	-	3,391.3	25.8 €/MWh
2 Stage	4,085.6	377.9	8,259.2	43.5 €/MWh
Ramping	4,103.0	463.6	2,858.8	24.0 €/MWh
Extended Ramping	4,143.0	50.1	2,827.5	24.1 €/MWh
MIP-approaches	4,137.8	49.9	2,956.7	24.5 €/MWh

In order to compare the performance of the models for large markets, we run them for a sample set of the German electricity market in 2006. As the resulting computation time depends on the underlying parameter values we conduct a series of test runs, one for the second week (168 hours) of each month in 2006. All models are run on a standard PC with an Intel Pentium 4, 3 GHz processor, 1 GB RAM, programmed in GAMS, and solved using the CPLEX 10.2 solver. We used CPLEX standard options. Table 3 shows the performance in terms of computation time, the optimality gap for integer problems, and the number of equations and variables for each model. The benchmark MIP shows an average performance of 20 seconds and is close to the optimal solution. As the two stage model is based on the benchmark approach the number of equations and variables is equal and only the computation time increases as the linear problem of the second stage adds to the MIP computation.

The ramping approach has the best performance in terms of computation time and guarantees an optimal solution due to its linear character. This makes the approach a valid option for large scale analyses and extensions of the model regarding network implementation. The extended ramping has the highest computation time of all models and also the highest optimality gap. This is due to the large model size in terms of equations and variables. Nevertheless, the easy implementation of start-up and ramping costs may still make this approach a valid option for smaller problem sets.

The three MIP approaches show a quite different performance. The MIP-Up-Down approach which replaces the condition variables by a start-up and shut-down variable shows computation problems due to workspace limitations. The size of the model becomes to large due to the summation over all periods within equations (15), (16), and (17) leading to memory problems. Thus the relative small number of variables and equations is offset by the equation formulation. The remaining two MIP approaches both produce close to optimal outcomes but differ in their performance. The MIP-Up-Down approach has a higher computation time due to the larger number of equations. Although the MIP-Up-Down-On approach has the largest number of binary variables the general size of the model remains small leading to a fast computation which is comparable to the benchmark case. The results indicate that particularly the number of equations as well as inter-temporal components in equations can lead to problems in performance whereas the number of binary variables does not harm the model performance.

Table 3: Performance comparison

Model	Average Solve Time [sec]	Standard Deviation of Solve Time	Average Relative Gap	Number of Equations	Number of Total Variables	Number of Binary Variables
Benchmark	19.77	11.49	0.01%	278,025	102,146	51,072
2 Stage	22,79	32.07	0.01%	278,025	102,146	51,072
Ramping	3.56	1.00	--	102,314	153,218	--
Extended Ramping	56.70	72.81	0.88%	431,241	255,362	51,072
MIP-Up	30.78	13.40	0.01%	329,097	153,218	102,144
MIP-Up-Down	--	--	--	204,458	153,218	102,144
MIP-Up-Down-On	18.28	8.64	0,01%	204,458	204,290	153,216

4 Conclusion

In this paper we compare different approaches regarding the implementation of start-up costs using either linear or mixed integer formulations. According to the considered start-up costs calculation the results differ significantly: When only start-up restrictions are taken into account, but the dispatch is still solely fuel cost driven the resulting market prices are well above marginal generation costs. When a linearized approach is taken that accounts for start-up costs but neglects start-up restrictions the calculated market prices show price spikes and negative prices. The negative prices correspond to the willingness to keep certain plants running during the night in order to avoid a re-start-up in the morning.

The mixed integer approaches show somehow similar market results. Particularly, we can observe prices below marginal costs of generation as the models make trade-offs between keeping plants online even if the demand level is low and shutting them down and start them up again after few hours. If start-up costs are linked to the output in the hour of start-up we observe price spikes. If these costs are added as a fixed block e.g. related to the minimum output no price spikes will occur.

Comparing the performance of the models under realistic market conditions we observe that the "simple" approaches are faster to solve and can handle large datasets. The more complex mixed integer approaches show limitations regarding dataset size and solving time. However, when keeping the sample small they solve quickly and reliably. Particularly, the number of equations and inter-temporal restrictions drive the solvability.

The results of our paper indicate that none of the applied approaches is capable to obtain price spikes in hours of start-up and simultaneously prices below marginal costs when plants are kept online. Only the linear approach shows spikes and dumps but the low prices are negative. Thus, when the research focus is on obtaining cost minimal dispatch under technical restrictions the MIP approaches are preferable because they allow for the choice between keeping plants running or re-start them. When the research focus is more on competiveness of market prices and thus generally on the peak load situations the two-stage and linear approach tend to produce higher prices and thus can represent an upper boundary. Furthermore, they allow an easy handling of large datasets.

References

Arroyo, José M., and Antonio J. Conejo (2000): Optimal Response of a Thermal Unit to an Electricity Sport Market. In: *IEEE Transactions on Power Systems*, Vol. 15, No. 3, pp. 1098-1104.

Arroyo, José M., and Antonio J. Conejo (2004): Modeling of Start-Up and Shut-Down Power Trajectories of Thermal Units. In: *IEEE Transactions on Power Systems*, Vol. 19, No. 3, pp. 1562-1568.

Baldick, Ross (1995): The Generalized Unit Commitment Problem. In: *IEEE Transactions on Power Systems*, Vol. 10, No. 1, pp. 465-475.

DENA (2005): *dena-Netzstudie, Energiewirtschaftliche Planung für die Netzintegration von Windenergie in Deutschland an Land und Offshore bis zum Jahr 2020*. Deutsche Energie-Agentur.

Dillon, Tharam S., Edwin, Kurt W., Kochs, Hans-Dieter, and Robert J. Taud (1978): Integer Programming Approach to the Problem of Optimal Unit Commitment with Probabilistic Reserve Determination. In: *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-97, No. 6, pp. 2154–2166.

Doormann, Gerard, and Bjorn Nygreen (2003): Market Price Calculations in Restructured Electricity Markets. In: *Annals of Operations Research* Vol. 124, pp. 49-67.

Gribik, Paul R., Hogan, William W., and Susan L. Pope (2007): *Market-Clearing Electricity Prices and Energy Uplift*. Harvard University.

Hirschhausen, Christian v., and Hannes Weigt (2007): *Price Formation and Market Power in the German Wholesale Electricity Market in 2006.* Electricity Markets Working Papers WP-EM-24, Chair of Energy Economics and Public Sector Management, Dresden.

Kuntz, Ludwig, and Felix Müsgens (2007): Modelling Start-up Costs of Multiple Technologies in Electricity Markets. In: *Mathematical Methods of Operations Research.* Vol. 66, pp. 21-32.

Schröter, Jochen (2004): *Auswirkungen des europäischen Emissionshandelssystems auf den Kraftwerkseinsatz in Deutschland*. Diplomarbeit, Berlin University of Technology, Institute of Power Engineering.

Sheblé, Gerald B., and Fahd George N. (1994): Unit Commitment Literature Synopsis. In: *IEEE Transactions on Power Systems*, Vol. 9, No. 1, pp. 128-135.

Takriti, Samer, Krasenbrink, Benedikt, and Lilian S.-Y. Wu (1998): Incorporating Fuel Constraints and Electricity Spot Prices into the Stochastic Unit Commitment Problem, In: *Operations Research*, Vol. 48, pp. 268-280.