Belief merging and revision under social influence: An explanation for the volatility clustering puzzle

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Abstract

A share price in a stock market can be thought of as arising out of an aggregation procedure. The price of a stock aggregates many individual beliefs into a collective one, the collective will of the market, so to speak. How does this aggregation come about? And is this aggregation fair in the sense that it correctly reflects the value? Furthermore, in the context of a stock market, it becomes immediately clear that belief merging cannot be separated from belief revision since investors in the market have a direct stake in what others think and clearly find it optimal to revise their beliefs in the light of the information about what others believe. We show that if investors are revising their beliefs not only after receiving new exogenous information but also after their social interactions with other investors and these revised beliefs are getting merged to generate the stock price under the accepted principles of finance (no arbitrage) then the resulting price dynamics explain a long standing puzzle in finance, the volatility clustering puzzle.

Keyword: *Social Influence, Knightian Uncertainty, Ambiguity, Volatility Clustering*

JEL G12, D70, D71

[1] Previously titled "Can social influence explain volatility clustering?"

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Introduction

This paper studies the financial market implications of social influences on probability judgments. Our results indicate that the social influence may be playing a role in generating a key stylized fact (volatility clustering) observed in financial markets.

Most macroeconomic time series relations are non-stationary with structural instability.[2] It implies the relevance of Knightian uncertainty (ambiguity) where a single objective probability distribution cannot be assigned to future outcomes; rather, a set of distributions is available any of which can be true.[3]

How do people act under Knightian uncertainty? With multiple distributions there is no given anchor for expectations and without an anchor macroeconomic equations cannot give meaningful results. Gilboa and Schmeidler (1989) propose that the conservative nature of agents provides an anchor for expectations and this is the foundation for their maxmin expected utility approach under Knightian uncertainty. In their approach, agents pick the most pessimistic distribution from the set of possible distributions and then act accordingly.

In this paper, we propose that the social context of a person may provide an anchor for expectations and explore the financial market implications of this proposition. Social context is an important factor affecting our judgments and beliefs. Social psychologists have documented both implicit as well as explicit social influences on our judgments and beliefs. The very nature of interpersonal relations involve influencing others and getting influenced by others. Our thoughts, judgments, actions all seem to be greatly influenced by what others think and do. These influences operate both at conscious as well as subconscious level and affect us in a variety of explicit as well as subtle ways.

A decision maker is just a name given to a utility function which is maximized subject to a model, where a model of an agent is his belief regarding the transition law linking the state variable to the control. There is strong empirical evidence that markets have internal dynamics of their own and over the last two decades a series of phenomena

[2] See Clements and Hendry (1999), Stock and Watson (1996), Davidson (1991).
[3] Frank Knight, the father of the Chicago school of economics, is one of the earliest economists to recognize the relevance of Knightian uncertainty. See Knight (1921).

that are anomalies under rational expectations finance have been documented. Furthermore, the conclusions from empirical time series literature is that most macroeconomic time series relations are non-stationary with structural instability, hence making it impossible for any agent to have the kind of structural knowledge that the theory demands. Hence, all decision makers face ambiguity. Social psychologists have extensively studied the link between decisions made by an individual under ambiguity and his social context and have documented that social context has a strong influence on an individual's decisions and especially so under ambiguity. This means that economic phenomena under ambiguity are in essence *socioeconomic phenomena* demanding that we consider both individual economic incentives as well as the social context as the determinant of human behavior. So, a decision maker under ambiguity is a utility function which is maximized subject to a model that is open to social influence.

This paper carries out this modification and puts forward a model of social interactions under ambiguity. The time series generated by the model displays a key stylized fact observed in the financial market time series. The stylized fact, called volatility clustering is considered a puzzling feature for which no convincingly explanation exists.

This paper is organized as follows. First, we present evidence from social psychology that social influences play a crucial role in human behavior. Then, we present our model and simulation results.

1. Examples of Implicit (Subconscious) Social Influence on Judgment from Social and Cognitive Psychology

In the light of what social psychologists tell us, people's judgments and behaviors can be influenced by the most innocuous and subtle manipulations. Thus, people express more favorable opinions about the future of the economy after they have seen a happy rather than a sad movie (see Forgas and Moylan, 1987); they agree more with a proposal to raise tuition after they hear it while nodding their heads in a vertical (yes) rather than a horizontal (no) manner (see Wells and Petty, 1980); and they are more likely to interrupt somebody after they unscramble sentences with rude rather than polite content (see Bargh et al. 1996). In a very interesting study (Steele and Aronson, 1995) black students were

asked to participate in a GRE-style test. In one version of the test, the students were given a pretest in which they were asked to identify their race. Steele and Aronson found that asking black students to identify their race in a pretest question reduced the number of right answers in the actual test by half. This is a startling result indicating that race identification was enough to generate all the negative stereotypes in the unconscious minds of participants. This negative preconditioning or priming (in the jargon of social psychology) greatly reduced the performance of black students. It makes one wonder whether people who go to expensive private schools perform better than others because of the constant positive priming rather than ability.

These and other effects appear to occur without a person's awareness; that is, they are implicit. When people are attempting to be rational, they presumably would not want their judgment of the economy to be influenced by the type of movie they have seen, or their judgment of an important issue to be influenced by their head movements, or their social behavior to be influenced by an irrelevant cognitive task they have just completed, or their test performance to be influenced by a seemingly innocent question.

The fact that people's judgments are influenced by irrelevant events and tasks makes it impossible to dissociate their states of mind from their social contexts. Hence, it is crucial to take into account a decision maker's social context in determining his behavior. It is interesting to speculate that these subtle social influences can potentially provide insights into the different economic performances of different geographic units. Economic development or growth is affected by entrepreneurial activities and any entrepreneurial task requires the entrepreneur to make probability judgments. It can be argued that different social contexts (as an example, cultural optimism or pessimism) would lead to different judgments and different levels of entrepreneurial activity.

2. Examples of Explicit Social Influence on Judgment from Social and Cognitive Psychology

Social psychologists have shown that even in very simple situations, people are willing to follow the majority and abandon their own judgment when provided with information that others think differently from them. In a series of experiments inspired by Asch (1952), Deutsch and Gerard (1955) show that people are greatly influenced by

majority opinion. In their experiments, each subject was asked to answer simple questions based on the length of line segments shown to him or her. Each subject almost always gave correct answers when asked individually in isolation. However, when the false information that a majority of others had answered differently was conveyed to them, about 35% of the time they changed their answers and agreed with the incorrect answers. These experiments are important because they show that even in very simple situations people are willing to abandon their judgment in favor of the majority. The urge to trust majority judgment over one's own judgment is likely to be even stronger when the situation is as highly ambiguous as in financial markets.

Sherif (1936) was one of the first social psychologists to investigate social influence on judgment. He presented participants with a stationary dot of light for 2 seconds in an otherwise dark room. This created an optical illusion known as the *autokinetic effect*: the stationary dot appeared to jump around. When participants were asked to judge how much the light had moved, they typically gave an estimate of around 1 to 10 inches. When group of participants were asked to announce their estimates out loud on consecutive days, a norm emerged. Their estimates gradually converged. In Sherif's study, participants cannot be sure about how much the light moved. Similarly, people cannot be sure about the true probability distribution of an uncertain prospect. In Sherif's experiment the judgments of others affected the judgment of a participant. Similarly, the judgment of others about the probabilities associated with an uncertain prospect may affect a person's belief.

Campbell (1961) examined conformism in a task where the correct answer is highly ambiguous. He formed micro-"societies" of two, three, and four individuals in which only one or two were real subjects, the remainder being confederates. Real and fake subjects were placed together in a darkened room and shown a fixed spot of light, then asked to estimate the distance that the light had traveled. In the experiment the light did not, in reality, travel at all—it was fixed. However, it is well known that due to a consistent optical illusion, people think the light moves about 4 inches: it's called the autokinetic effect, as mentioned before. The confederates gave their estimate first, and they had been instructed to give estimates (16 inches) much higher than the usual estimates. Then the real subjects would give it a try. In the experiment this constituted

the first "generation." For the second "generation," one of the fake subjects was removed and replaced with a real one and all participants then proceeded to make estimates again. This procedure was repeated until the micro-"society" was composed exclusively of real subjects. From then on, in each "generation," a real subject would be removed and replaced with another real subject, for a total of eleven "generations." What did they find? When there is only one confederate (fake subject) and two real subjects, the wildly high estimate of the lonely confederate (16 inches) nevertheless has some influence, as in the first generation the real subjects give estimates higher than 4 inches, though always below 9. When there are two confederates—a 2/3 majority—and only one real subject, the latter is quite strongly influenced and in the first generation gives a very similar estimate, about 14 inches.

3. Towards an Economic Framework Incorporating Social Influence on beliefs under Knightian Uncertainty

Economic literature on social influence can be divided into sections: 1) Preference modification 2) Belief modification, since social influence can affect both preferences as well as beliefs. Belief modification literature uses the idea of information cascades due to Bikhchandani, Hirshleifer, and Welch (1992). In information cascades, decisions are made in a sequence so later decision makers have an opportunity to combine their private information with the information inferred from the actions of others. In equilibrium, they may find it optimal to ignore their private information in favor of the information inferred from others.

In this paper, we assume that investors make decisions simultaneously so information cascades cannot arise. As discussed in the previous section, social psychologists have extensively studied the link between decisions made by an individual under ambiguity and his social context and have documented that social context has a strong influence on an individual's decisions and especially so under ambiguity. This means that economic phenomena under ambiguity are in essence *socioeconomic phenomena* demanding that we consider both individual economic incentives as well as the social context as the determinant of human behavior. This paper carries out this modification and puts forward a model of asset pricing with social influence on probability judgments under Knightian

uncertainty. The time series generated by our model displays a key stylized fact observed in financial market time series. The stylized fact called volatility clustering is a puzzling feature in the real world data. Before presenting our model, we provide a brief description of the volatility clustering phenomenon.

3.1 Volatility Clustering

Volatility clustering is one of the most important "stylized facts" in financial markets. A large number of empirical studies report that while changes in asset prices appear to be random the magnitude (amplitude) of these changes has a structure to it. Changes of large magnitude typically tend to follow changes of large magnitude and changes of small magnitude tend to follow changes of small magnitude. This is called volatility clustering.

Mandelbrot (1963) first discovered this phenomenon in commodity prices. However, it is the pioneering work of Engle (1982) and Bollerslev (1986) on autoregressive conditional heteroskedastic (ARCH) models and their generalization GARCH models that brought this phenomenon to the forefront of economic research. Volatility clustering has been shown to be present in a wide variety of financial assets including stock market indices, as well as exchange rates. In empirical work, volatility clustering is usually modeled by a statistical model such as GARCH or one of its extensions. As noted by Engle (2001), these models are only statistical descriptions of the data and they do not provide any structural explanation as to why the phenomenon arises. Rather, the statistical models postulate that volatility clustering has an exogenous source and is for example caused by the clustered arrival of random news about the economic fundamentals. See Engle (2004).

Theoretical modern finance models based on rational expectations cannot generate volatility clustering. See Bossaerts (2003) for a detailed discussion of the empirical failure of rational expectations hypothesis. The causes of volatility clustering are poorly understood. Engle (2001) writes:

"The goal of volatility analysis must ultimately be to explain the causes of volatility. While, the time series structure is valuable for forecasting, it does not satisfy our need to

explain volatility. Thus far, attempts to find the ultimate cause of volatility are not very satisfactory."[4]

3.2 The Model

We take it to be axiomatic that people disagree about the probabilities of events even when exposed to the same information. In the economics literature, this position is taken, as an example, by Rubinstein (1993), who thinks that it is obvious that agents interpret the same information differently:

"Agents reading the same morning newspapers with the same stock price lists will interpret the information differently."[5]

Empirical evidence also supports this assertion. Kandel and Pearson (1995) test and accept[6] the hypothesis that agents in speculative markets interpret the same information differently. Kandel and Zilberfarb (1999) test and accept the hypothesis that different forecasters interpret the same information differently while forecasting inflation.

Our model has three sets of assumptions:

1) Assumptions about belief formation

2) Assumptions about price formation

3) Assumptions about updating confidence

3.2.1 Assumptions about belief formation

Each period, all agents receive the same information. They use a mental model to convert this information into a belief about the price level next period. There is a commonly known part of the model about which every one agrees and there is an

[4] Engle, R. (2001). GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics. *Journal of Economic Perspectives*, *15(4)*, page 166.

[5] Rubinstein (1993), page 473.

[6] Technically, they reject the null of identical interpretation. However, it is customary to write that the alternative is accepted with the implicit understanding that rejection of the null is implied. As one example, Kurz (1997a), page 3, does the same.

unknown part about which agents disagree.[7] Let θ_{tp} be the interpretation of this information according to the commonly known part of the model. Each agent's belief (expectation about the next period's price level), x_{it} is some perturbation of θ_{tp} : $x_{it} = \theta_{tp} + v_{it} = E_{it}[p_{t+1}]$. This perturbation accounts for the idiosyncratic differences between different agents' mental models.

After forming his belief, each agent interacts with people in his social circle. These interactions influence his belief. This is captured by considering a 2-dimensional lattice and assigning a cell to each agent with neighboring cells as his neighbors. Each agent's belief is affected by his interactions with his neighbors. Let z_{it} represent the belief of agent i after interacting with his neighbors:

$$z_{it} = f\left(neighbors'beliefs, x_{it}\right) = E_{it}[p_{t+1} \mid Social\ Interactions]$$

where f is some function describing how neighbors' beliefs influence an agent's belief.

Obviously, people have a direct interest in discovering and influencing the beliefs of others since stock price is a reflection average market beliefs. Keynes (1936) is one of the earlier economists to explicitly recognize this fact. He compared stock market behavior to a beauty contest:

"Each competitor has to pick not those faces he himself finds prettiest, but those which he thinks are likeliest to catch the fancy of other competitors."[8]

Note that there does not have to be any explicit communication of beliefs since beliefs can be influenced by even the most innocuous and subtle clues as psychological evidence in section 1 shows. As an example, if people in one's social circle are exuberant since their team has won a football match then that exuberance may subconsciously make one an optimist in investment decisions also. Anecdotal evidence of such behavior abound. As one example, the Pakistani Stock Exchanges made significant upward movements after their team won the cricket world cup in 1992.

[7] Manski (2003) argues that econometricians trying to estimate a model are almost always in this position.
[8] Keynes (1936), page 156.

3.2.2 Assumptions about price formation

Each agent optimizes given his belief after social interactions, z_{it}. The standard optimization exercise with one risk-free and one risky asset produces a demand curve for the risky asset of each agent. Assuming that the number of shares outstanding is constant and by equating them with aggregate demand, we can solve for the equilibrium price:[9]

$$p_t = \frac{1}{(1+r)N}\left\{\sum_i^N E_{it}[p_{t+1} \mid Social\ Interactions] + \sum_i^N E_{it}[d_{t+1}]\right\}$$

where r is the one period risk free net return, N is the total number of agents and d_{t+1} is the intervening dividend. Similarly, in the next period, the whole process repeats, new information arrives $(\theta_{(t+1)p})$, new private beliefs $(x_{i(t+1)})$ are formed, new beliefs after social interactions are formed, and the new equilibrium price p_{t+1} is determined.

3.2.3 Assumptions about updating confidence

Once p_{t+1} is known, each agent compares the expectation error of his belief before social interactions, $|p_{t+1} - E_{it}[p_{t+1}]|$ with the expectation errors of his neighbors. If his expectation error is greater than the expectation error of his neighbors, he assigns a greater weight to their opinion in the next period. If his error is smaller, he assigns a lower weight to the opinion of his neighbors in the next period. We describe the exact weighting function, when we set the parameters of the simulation (next section).

In summary, the model works in the following way: All agents receive the same information each period. They interpret this information differently to arrive at different beliefs about the next period's price level. Then, they interact with agents in their social

[9] See the Appendix for a standard derivation of this equation. Brock and Hommes (1998) derive this equation as an extension of asset pricing model to the case of heterogeneous expectations. Some authors such as Arthur et al. (1997) derive this equation from no-arbitrage arguments without any explicit optimization exercise. Also see Chiarella and He (2001, 2002, 2003), Farmer and Joshi (2002), Lebaron (2000),
Lebaron et al. (1999), Lux and Marchesi (1999), and Lesourne (1992). Note that the wealth does not appear in this equation. Individual wealth obviously has an effect on an individual's allocation decision. However, for the determination of stock price aggregate wealth distribution may be relevant and not the individual wealth. In any case, the focus of this paper is on how belief changes caused by social influence cause price changes; therefore we ignore any other variable except beliefs.

circle and these interactions influence their beliefs. Each agent optimizes (by choosing between a risky and a risk free asset) given his belief after social interactions and in this process equilibrium price level is determined. After the determination of equilibrium price next period, each agent calculates his expectation error and compares it with his neighbors' errors and accordingly adjusts the importance he attaches to his neighbors opinions.

3.2.4 Simulation Parameters

In what follows, subscripts ij denote the location of an agent in a 2-dimensional plane divided into cells (lattice). We use the following parameter values for simulation:

$$\theta_{tp} \in [current\ price - 0.05 \times current\ price, current\ price + 0.05 \times current\ price]$$

$$v_{ijt} \in [-0.05 \times current\ price, 0.05 \times current\ price]$$

$$r = 0.05,\ N = 40000,\ E_{ijt}[d_{t+1}] = 1 \forall i, j, t$$

θ_{tp} represents the interpretation of new information according to the commonly known part of the model. Each period, θ_{tp} takes a random value from a uniform distribution with the range defined above. The role of θ_{tp} in the model is to ensure a steady arrival of new information each period. The results presented here are robust to the range of values θ_{tp} can take.[10] v_{ijt} is the idiosyncratic element in each agent's belief. Each period, for each agent, a random value is drawn from a uniform distribution with the range defined above. Together, these two parameters ensure that different agents have different interpretations in accordance with Knightian uncertainty. We use the simplest linear form for the function z_{ijt} (belief after social interactions, that is, $z_{ijt} = E_{ijt}[p_{t+1} \mid Social\ Interactions]$):[11]

[10] Of course, negative values are not allowed since price is a strictly non-negative variable.
[11] Using a linear specification is preferred for the following reasons: Even the simplest of nonlinear specification can generate extremely rich behavior such as bifurcations, strange attractors and chaos. See Gulick (1992), Preston (1983), Collet and Eckmann (1980). So, in a nonlinear specification one cannot be sure whether the results are due to social influence or due to the nature of nonlinearity. For a detailed

$$z_{ijt} = a \times b_{ijt} + c \times x_{ijt} \quad (1)$$

Where a and c are positive parameters, b_{ijt} is the average belief in the neighborhood of agent ij, that is, $b_{ijt} = \frac{x_{(i+1)j} + x_{(i-1)j} + x_{i(j+1)} + x_{i(j-1)}}{4}$ where $x_{(i+1)j}, x_{(i-1)j}, x_{i(j+1)}$, and $x_{i(j-1)}$ denote the beliefs of neighbors immediately to the right, left, above, and below the agent, respectively. We assume that each agent has 4 neighbors.[12] x_{ijt} is the own belief of agent ij. Equation (1) states that the belief after social interactions depends on the average belief in one's social circle as well as on one's own initial predisposition. Parameters a and c control the relative importance that an agent attaches to others' opinion in his social circle. We will refer to a as intensity of social influence and c as own confidence.

For updating confidence, if the expectations error of an agent is greater than the expectation error of average neighborhood belief, that is,

If $\left|p_{t+1} - x_{ijt}]\right| > \left|p_{t+1} - b_{ijt}\right|$

then a goes up by an amount g which is randomly drawn from a uniform distribution: $g \in [0.10 \times a, 0.50 \times a]$.

If the expectation error of an agent is less than the expectation error of average neighborhood belief then a goes down by g. The rationale behind this assumption is as follows: if a person's social circle outperforms him then plausibly he will assign a greater weight to their opinion in the next future. How much greater? That depends on his state

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discussion of rich behavior of nonlinear systems, see Drazin (1992) or May (1976). We suspect that linearity is preferred in social interactions literature (preference modification) for the same reason. See Glaeser and Scheinkman (2000). It may very well be the case that financial markets are nonlinear systems as argued in the econophysics literature. See Wille (2004). However, the implications of nonlinearity (and chaos) for financial markets have been studied elsewhere. See Guanersdorfer et al. (2003), Guanersdorfer (2000), Hommes and Guanersdorfer (2003).

[12] Face-to-face interactions with people with whom one has strong social ties are likely to have the strongest influence on one's judgment. Typically, the number of such people is small (average 3.8 for innermost social circle). Results are similar for either 4 or 8 neighbors. Due to the sampling issue, it is clear that results will get weaker for larger neighborhoods with our model becoming equivalent to modern asset pricing model for very large neighborhoods (social influence will cancel out).

of mind at the moment of decision, which depends on a lot of environmental factors (such as the type of movie he just saw).[13] These environmental factors are essentially random.

3.2.5 Simulation Results

The results depend on the relative importance of neighbors beliefs b_{ijt} versus one's own idiosyncratic predisposition x_{ijt}. That is, on the relative magnitudes of parameters a and c. A number of representative simulations are run:

1. Simulation without social interactions. This simulation is run to establish the benchmark.
2. Simulation with social interactions and the parameter values: a=0.80, c=0.20
3. Simulation with social interactions and the parameter values: a=0.60, c=0.20
4. Simulation with social interactions and the parameter values: a=0.40, c=0.20
5. Simulation with social interactions and the parameter values: a=0.20, c=0.20

3.2.5.1 Simulation without Social Interactions

If there is no social influence, our model reduces to modern asset pricing model (substitute a=0, and c=1 in equation (1)). See Figure 1. Unsurprisingly, Figure 1 is similar to output from a typical modern asset pricing model. See Tsay (2002). Returns are measured as changes in log-price. Of course, there is no volatility clustering.

[13] Gladwell, M. (2005). *Blink: The Power of Thinking without Thinking*. Little, Brown.

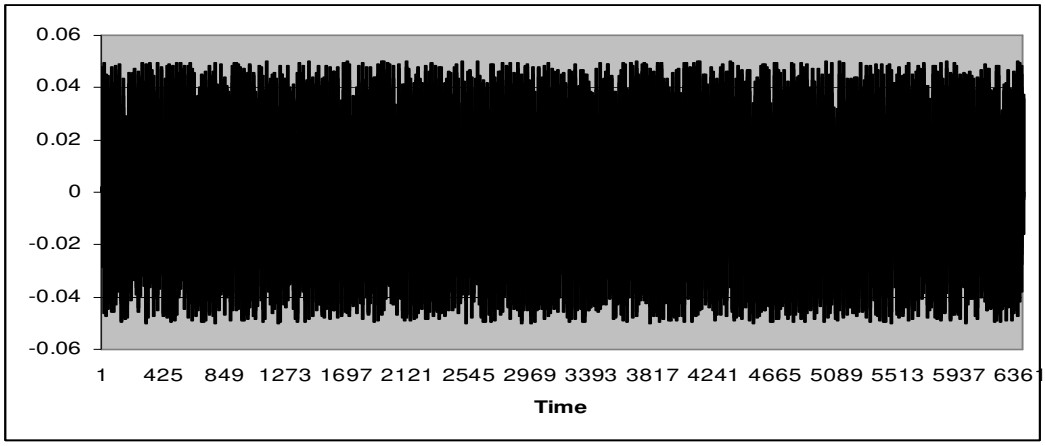

Figure 1. Returns without Social Interactions

3.2.5.2 Simulation with social interactions, a=0.80 and c=0.20

Figure 2 shows the returns generated by our model when a =0.80 and c =0.20

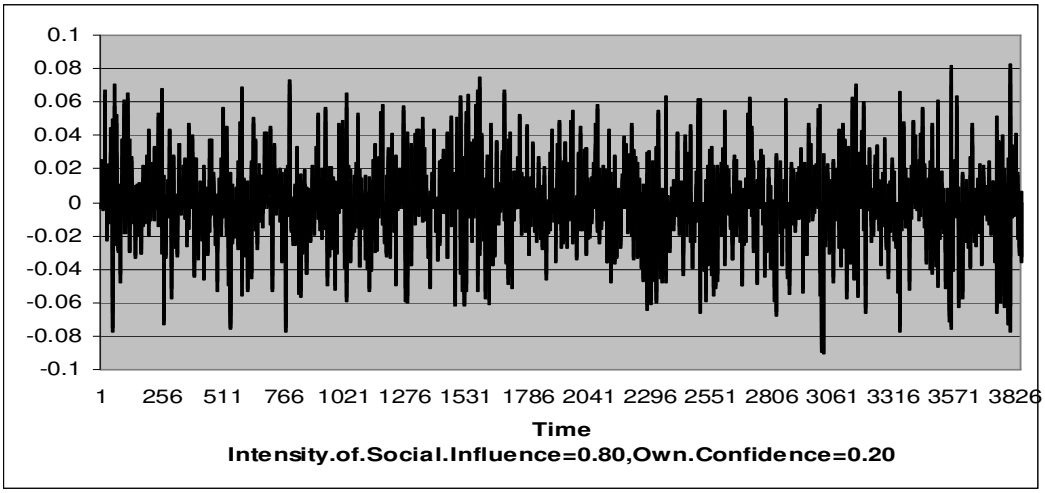

Figure 2. Returns

Volatility clustering can be seen clearly in figure 2. Autoregressive Conditional Heteroscedasticity (ARCH) regression can be employed as a test for volatility clustering. See Tsay (2002), Engle (2001) or any text in financial econometrics (such as Wang (2003). If coefficients are significant than volatility clustering is present. As reported in Table 1, ARCH coefficients are significant, indicating volatility clustering.

ARCH(1) REGRESSION: TEST FOR VOLATILITY CLUSTERING					
	Estimate	**Error**	**t-Value**	**p-Value**	**ARCH EFFECT**
ARCH0	0.000093	1.17E-05	7.94	<.0001	Yes
ARCH1	0.9133	0.0689	13.26	<.0001	Yes

Table 1

3.2.5.3 Simulation with social interactions, a=0.60 and c=0.20

Figure 3 shows the returns generated by our model when a =0.80 and c =0.20.

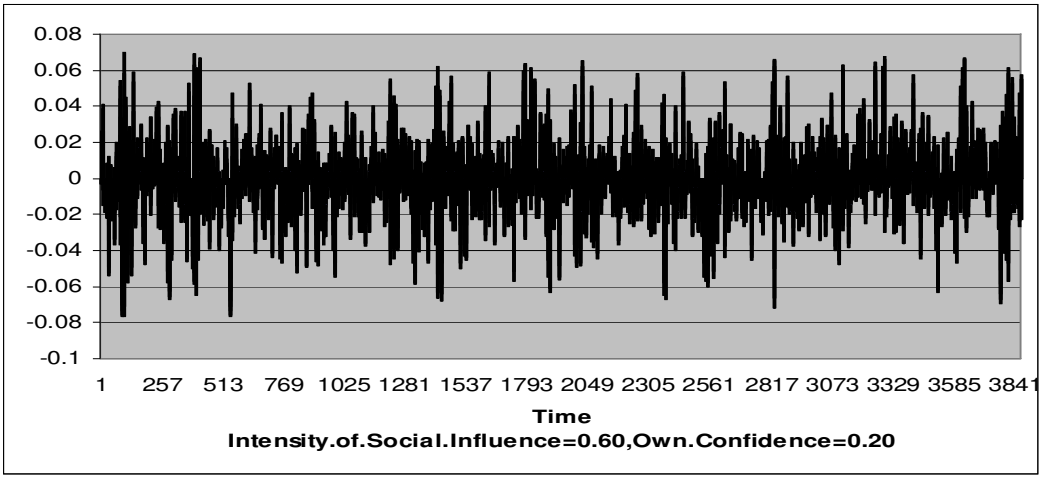

Figure 3. Returns

As before, volatility clustering can be seen.

We test for volatility clustering and find significant volatility clustering (see Table 2).

ARCH(1) REGRESSION: TEST FOR VOLATILITY CLUSTERING					
	Estimate	Error	t-Value	p-Value	ARCH EFFECT
ARCH0	0.000072	8.80E-06	8.18	<.0001	Yes
ARCH1	0.8818	0.0674	13.07	<.0001	Yes

Table 2

3.2.5.4 Simulation with social interactions, a=0.40, c=0.20

As can be seen from figure 4, volatility clustering is present. This is confirmed by the ARCH regression results reported in Table 3.

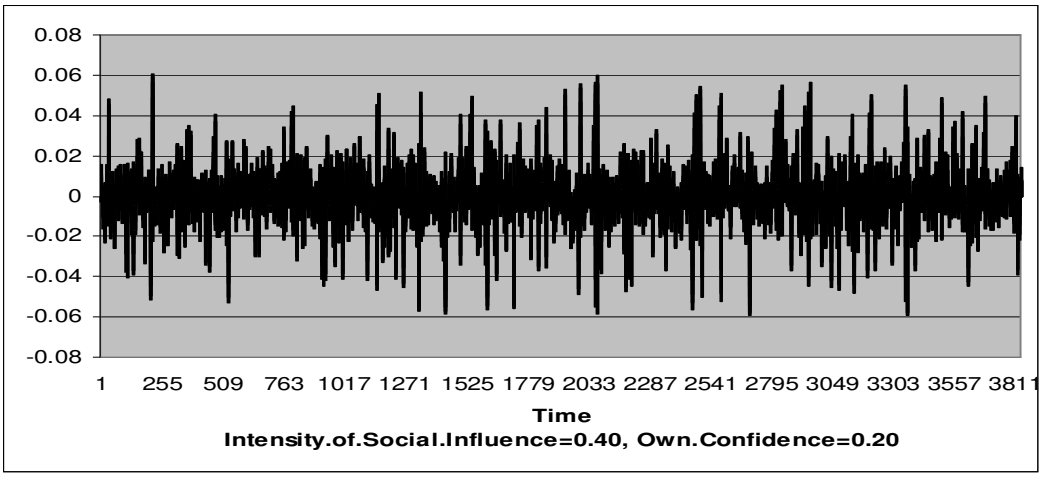

Figure 4. Returns

ARCH(1) REGRESSION: TEST FOR VOLATILITY CLUSTERING

	Estimate	Error	t-Value	p-Value	ARCH EFFECT
ARCH0	4.38E-05	5.27E-06	8.31	<.0001	Yes
ARCH1	0.8147	0.0634	12.85	<.0001	Yes

Table 3

3.2.5.5 Simulation with social interactions, a=0.20, c=0.20

Figure 5 shows the returns generated by our model when a =0.20 and c =0.20.

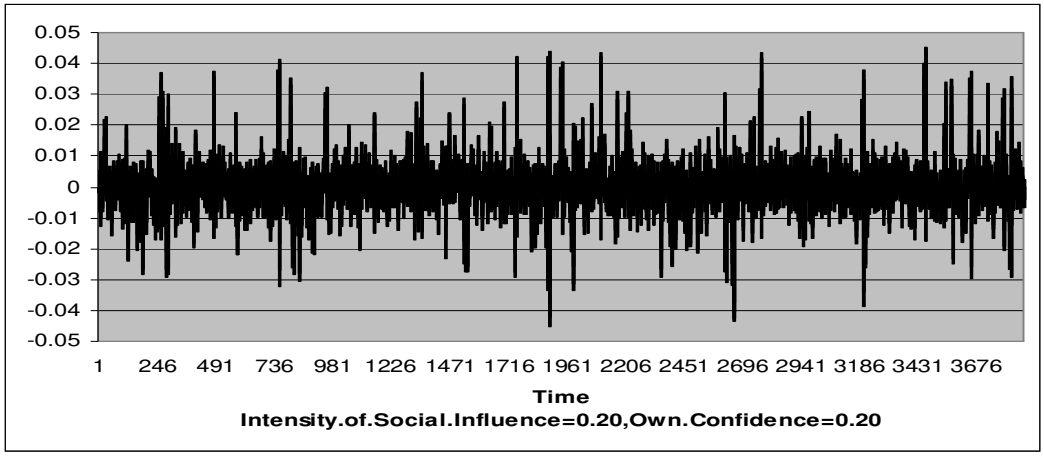

Figure 5

We test for volatility clustering and find significant volatility clustering (see Table 4).

ARCH(1) REGRESSION: TEST FOR VOLATILITY CLUSTERING

	Estimate	Error	t-Value	p-Value	ARCH EFFECT
ARCH0	2.18E-05	2.65E-06	8.23	<.0001	Yes
ARCH1	0.6922	0.057	12.15	<.0001	Yes

Table 4

4. Conclusion

Human are social animals and their social interactions undoubtedly affect their judgments. We present an exploratory model of social influence on judgment. Our results indicate that social influence may be playing a role in generating volatility clustering observed in financial markets.

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APPENDIX

Brock and Hommes (1998) put forward a simple model of asset pricing with heterogeneous beliefs (BH model). Their model is now a workhorse for literature with heterogeneous expectations. Guanersdorfer et al (2003), Guanersdorfer (2000), Chiarella and He (2003), Chiarella and He (2002), Chiarella and He (2001), Farmer and Joshi (2002), Lebaron (2000), Lebaron et al (1999), Lux and Marchesi (1999), and Lesourne (1992) are a few examples of use of this model.

In Brock and Hommes (1998), there are two types of assets, a risk free asset and a risky asset. Risk free asset pays a net return of r, which is between 0 and 1. That is, for a dollar of investment, the gross return is $(1+r)$ after a unit interval. Let p_t denote the price of risky asset that pays dividends, d. The dynamics of wealth of an agent type 'a' is described by

$$W_{a,t+1} = W_{at}(1+r) + R_{t+1}S_{at} \tag{A.1}$$

where R_{t+1} is the excess return (in dollars) per share of risky asset over risk free asset, that is, $R_{t+1} = p_{t+1} + d_{t+1} - (1+r)p_t$ and S_{at} is the number of shares of risky asset bought by an agent of type 'a'.

Let E_t and V_t denote conditional expectation and conditional variance, and let E_{at} and V_{at} denote the beliefs of investor type 'a' about these conditional expectation and variance.

Assume that investors are mean-variance maximizers.[14] The demand for shares of risky asset by an agent of type 'a' can be obtained as follows.

$$Maximize\{E_{at}[W_{a,t+1}] - (e/2)V_{at}[W_{a,t+1}]\}$$

$$\Rightarrow S_{at} = \frac{E_{at}[R_{t+1}]}{eV_{at}[R_{t+1}]} \tag{A.2}$$

where e is interpreted as a risk aversion parameter.[15]

Assume a constant supply of outside shares over time, m. Further, assume that all agents agree about the variance and that the market clears:

[14] Mean-Variance Optimization is a decision making model proposed by Markowitz (1952) as an alternative to Expected Utility decision model. The Expected utility model gives the same results as the Mean-Variance model if the utility function is quadratic or returns are normally distributed. Levy and Markowitz (1979) show that mean-variance analysis can be regarded as a Taylor approximation (second order) of any given utility function (such as power utility) in the Expected Utility model. Rabin (2000) argues that Expected Utility model is absurd as a model of human decision making. The Mean-Variance model is simpler, though less general; however, it does not suffer from serious plausibility issues such as the one raised by Rabin (2000).

[15] Not to be confused with the risk aversion parameter in the Expected Utility Model.

$$\sum_{1}^{N} E_{at}[p_{t+1}] + \sum_{1}^{N} E_{at}[d_{t+1}] - \sum_{1}^{N} p_t(1+r) = m \times e \times V_t[R_{t+1}] \tag{A.3}$$

$$\Rightarrow \sum_{1}^{N} E_{at}[p_{t+1}] + \sum_{1}^{N} E_{at}[d_{t+1}] - \sum_{1}^{N} p_t(1+r) = \sum_{1}^{N} \frac{m}{N} \times e \times V_t[R_{t+1}] \tag{A.4}$$

$$\Rightarrow \sum_{1}^{N} E_{at}[p_{t+1}] + \sum_{1}^{N} E_{at}[d_{t+1}] - \sum_{1}^{N} \frac{m}{N} \times e \times V_t[R_{t+1}] - \sum_{1}^{N} p_t(1+r) = 0 \tag{A.5}$$

$$\Rightarrow \sum_{1}^{N} E_{at}[p_{t+1}] + \sum_{1}^{N} \{E_{at}[d_{t+1}] - \frac{m}{N} \times e \times V_t[R_{t+1}]\} - \sum_{1}^{N} p_t(1+r) = 0 \tag{A.6}$$

Define risk adjusted dividend as, $d`_{t+1} = d_{t+1} - \frac{m}{N} \times e \times V_t(R_{t+1})$:

$$\Rightarrow \sum_{1}^{N} E_{at}[p_{t+1}] + \sum_{1}^{N} \{E_{at}[d`_{t+1}]\} - \sum_{1}^{N} p_t(1+r) = 0 \tag{A.7}$$

$$\Rightarrow \sum_{1}^{N} E_{at}[p_{t+1}] + \sum_{1}^{N} \{E_{at}[d`_{t+1}]\} = N \times p_t(1+r) \tag{A.8}$$

$$\Rightarrow p_t = \frac{1}{(1+r)N} \left\{ \sum_{1}^{N} E_{at}[p_{t+1}] + \sum_{1}^{N} E_{at}[d`_{t+1}] \right\} \tag{A.9}$$