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Abstract : *In this paper, we extend the Romer (1990) model by introducing an embodied technological change and by removing the scale effects. We show that this model can still generate steady state growth in which the embodied technical change has an positive and permanent effect on growth in the long-run.*

Keywords : Endogenous growth, Information technology, Embodied technical change, Scale effects

JEL Classification : 031, 041

1 Introduction

The role of information and communication technologies (hereafter ICT) has recently attracted a great attention in the literature. This paper concentrates exclusively on the role of ICT on growth. This particular focus is motivated by a recent empirical studies which show that the embodied technical change

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accounted for about 60% of the growth in consumption per hour over the period 1954-1990 (e.g., Greenwood, Hercowitz, and Krusell (1997), Hercowitz (1998) and Licandro, Ruiz-Castillo, and Duran (2002)).

There are few attempts in the literature to include embodied technical change in a R&D driven growth model. Boucekkine, Del Rio, and Licandro (2005) introduce embodied technical change in a schumpeterian endogenous growth model *à la* Aghion and Howitt (1992). They conclude that the positive effect of modernization generally compensates more than the negative effect of obsolescence. However, embodied technical change means only in this case improvement in quality of goods.

The papers that are closest to our are the ones of Krusell (1998), Hsieh (2001) and Boucekkine, de la Croix, and Vailakis (2002)[3] which are built on Romer (1990). Krusell (1998) develops a model in which the productivity of a given capital good increases over time and the variety of capital goods is constant. Hsieh (2001) presents a model that features the endogenous obsolescence of existing capital goods as a result of the introduction of new capital goods of higher quality. The improvement of the quality is exogenous and grows at a constant growth rate. Boucekkine, de la Croix, and Vailakis (2002) develops a model in which the productivity of capital goods is exogenous and constant. They show that the embodied technical change has no effect on growth. In our model, we introduce an embodied technical change using a formulation *à la* van Zon and Yetkiner (2003). This specification allows us to determine endogenously the productivity of capital goods which is linked to the quality and the variety of a capital good. Moreover, contrary to all the previous models, by using the specification of Dinopoulos and Thompson (1999) in order to formalize the R&D process, we eliminate the scale effects. We show that an increase of the embodied technical change affects positively the economic growth rate in this framework.

The paper is organized as follows. Section 2 presents the model. Section 3 describes the effects of embodied technical change on growth. Section 4 concludes.

2 The model

We consider a closed economy with a growing population $g_H = n > 0$. The economy is structured by three sectors : final good sector, intermediate goods sector and R&D sector. The final output sector produces output that

[3]Boucekkine and de la Croix (2003) is a more general case than Boucekkine, de la Croix, and Vailakis (2002) but they are very much concerned with the productivity slowdown and with the income inequality consequences of ICT revolution.

can be used for consumption and investment using efficient capital services and skilled labor. The intermediate goods sector produces efficient capital using raw capital. The R&D sector creates the blueprints for new varieties of intermediate goods. These blueprints are sold to the intermediate goods sector.

2.1 Preference and technology

The representative individual maximizes the utility function :

$$U = \int_0^\infty e^{(n-\rho)t} \log(c) dt, \tag{1}$$

subject to the common intertemporal budget constraint, where $c = \frac{C}{H}$ is per-capita private consumption and $\rho > 0$ the discount rate. The solution implies :

$$g_C = r - \rho + n, \rho > n^4, \tag{2}$$

where r denotes the interest rate.

Final output Y is produced in a competitive manner according to :

$$Y = BH_Y^{1-\alpha} \int_0^A x_{ei}^\alpha di, \tag{3}$$

where $\alpha \in [0, 1]$ is the partial output elasticity of effective capital services, $1 - \alpha$ is the partial output elasticity of skilled labor, H_Y is the skilled labor, x_{ei} is the *ith* intermediate good and A is the number of blueprint invented. Normalizing the price in the final good sector to one, profit maximizing firm implies an inverse demand function and a demand function of skilled labor that are respectively :

$$\frac{\partial \pi_y}{\partial x_{ei}} = BH_Y^{1-\alpha} x_{ei}^{\alpha-1} \alpha - p_{ei} = 0, \tag{4}$$

$$\frac{\partial \pi_y}{\partial H_Y} = (1-\alpha)\frac{Y}{H_Y} - w_Y = 0, \tag{5}$$

where w_Y is the wage rate of skilled labor and p_{ei} is the rental price of the effective capital services of the *ith* intermediate good.

In each intermediate good industry, raw capital x_i is the only input :

$$x_{ei} = \lambda_i x_i, \tag{6}$$

[4] This condition guarantees that the intertemporal utility function of the household does not explode.

where λ_i is the productivity of the raw capital. We assume also that the improvements of intermediate goods after their invention is impossible. This implies that λ_i can only change over time for $i = A$, i.e. for the latest intermediate. By doing so, we break the symmetry between intermediates which is contained in Romer (1990)'s original model. By consequence, technical change provides opportunities for the division of production tasks between intermediate goods. As in van Zon and Yetkiner (2003), we assume that the productivity of the ith intermediate good is determined by :

$$\lambda_i = \lambda_0 i^{\zeta}, \tag{7}$$

where $\zeta > 0$ and $\lambda_0 > 0$ is the productivity of the first intermediate good. In the equation (7), the parameter ζ measures implicitly the embodied technical change. Indeed, we link the productivity of intermediate goods to an increase in the quality as in Aghion and Howitt (1992). However, contrary to Aghion and Howitt (1992), the creative destruction is not complete and total since all varieties will live forever although they fade away in time. Finally, the quality or the productivity of the latest intermediates good is bigger than the previous ones. In this sense, we can say that there is an embodied technical change. The profit function of firms is given by :

$$\pi_{ei} = p_{ei} x_{ei} - r x_i. \tag{8}$$

Profit maximizing firm implies the value of effective capital services price :

$$p_{ei} = \frac{i^{-\zeta} r}{\lambda_0 \alpha}. \tag{9}$$

We find one result which is really close to Greenwood, Hercowitz, and Krusell (1997). Indeed, the equation (9) shows that the relative price of effective capital services is negatively related to the embodied technical change if the parameters that determine the competition in the intermediate goods sector through the mark-up $\frac{1}{\alpha} - 1$ and the interest rate are constant over time. However, one important difference with Greenwood, Hercowitz, and Krusell (1997) is that in our model the relative price is determined endogenously.

There are competitive research firms undertaking R&D. Following Dinopoulos and Thompson (1999), we assume that new blueprints are produced using old blueprints A, an amount of R&D skilled labor H_A and the skilled labor force H :

$$\frac{\partial A}{\partial t} = \frac{\delta A H_A}{H}. \tag{10}$$

This formalization of the firm research process allows us to eliminate the scale effect which is inconsistent with time series evidence (Jones (1995))[5]. Because of the perfect competition in the R&D sector, we can obtain the real wage in this sector in function of the profit flows associated to the latest intermediate in using the zero profit condition :

$$w_A H_A = \frac{\partial A}{\partial t} P_A, \tag{11}$$

where w_A represents the real wage earned by R&D skilled labor and P_A is the real value of such a blueprint which is equal to :

$$P_A = \int_t^{\infty} \pi_{ei} e^{-r(\tau - t)} d\tau, \tau > t. \tag{12}$$

2.2 Balanced growth path

In equilibrium there is full employment. The fraction of the skilled labor force devoted to R&D and to the production are fixed. As a result, the following two conditions must simultaneously be satisfied :

$$1 = s_Y + s_A, \tag{13}$$

$$w_Y = w_A, \tag{14}$$

where s_Y and s_A represent the shares of the skilled total labor supply devoted respectively to final good production and research activity. On the final good market, a firm produces a good which can be consumed (C) or invested (I) :

$$\dot{K} = I = Y - C, \tag{15}$$

where the stock of raw capital is define by :

$$K = \int_0^{\infty} x_i di. \tag{16}$$

Equation (15) is a resource constraint on the final good sector. Finally, we can determine the equilibrium on the effective capital services market which is described by the supply (equation (6)) and the demand (equation (4)). It follows from the previous equations, that the steady state growth rate of output is given by :

$$g_Y = g_C = g_K = \frac{g_{H_Y}(\alpha - 1) + g_A(-\zeta\alpha + \alpha - 1)}{\alpha - 1}. \tag{17}$$

[5]For a survey about this question see Dinopoulos and Thompson (1999) and Jones (2005).

Equation (17) shows that the steady state growth rate will exceed the growth rate of the number of blueprint if $\zeta > 0$. This is due to the fact that growth does not only come from an increase in the number of intermediates but also from the intrinsic productivity improvements embodied in the latest intermediate. Equation (10) can be substituted into equation (17), the result of which can be associated with the assumption of a constant growth rate of skilled labor $g_{H_Y} = n$, we can obtain the supply side of the economy :

$$g_Y = \frac{n\left((\zeta-1)\alpha^2 - \delta\alpha + \alpha + \delta\right) - \delta(\alpha(\zeta-1)+1)\left((\zeta-1)\alpha^2 + \alpha + r(\alpha-1)\right)}{\alpha(\alpha(\zeta-1) - \delta\zeta + 1)}. \tag{18}$$

We need to calculate the equilibrium value of the real interest rate in order to can compute the economic growth rate. The interest rate is obtained in equalizing the supply (equation (18)) and the demand (equation (2)) :

$$r = \frac{\alpha(\alpha(\zeta-1) - \delta\zeta + 1)\rho - \delta(\alpha(\zeta-1)+1)\left(\zeta\alpha^2 + (1-\alpha)(n+\alpha)\right)}{\left((\zeta-1)\alpha^2 - \delta\alpha + \alpha + \delta\right)}. \tag{19}$$

By substituting equation (19) into equation (18), we find the corresponding equilibrium value of the growth rate[6] :

$$g_Y = n - \rho + \frac{(\alpha-1)\alpha(\alpha(\zeta-1) - \delta\zeta + 1)\rho - \delta(\alpha(\zeta-1)+1)\left(\zeta\alpha^2 + (1-\alpha)(n+\alpha)\right)}{(\alpha-1)\left((\zeta-1)\alpha^2 - \delta\alpha + \alpha + \delta\right)}. \tag{20}$$

The economic growth rate is a function of technical and preference parameters (α, ρ, ζ, n and δ). By using the previous equations, we obtain the equilibrium allocation of skilled labor in the final good sector :

$$s_Y = \frac{(1-\alpha)(-n+\delta+\rho)}{(\zeta-1)\alpha^2 - \delta\alpha + \alpha + \delta}. \tag{21}$$

The equilibrium allocation of skilled labor in the final good sector is affected negatively by the embodied technical change. By substituting equation (21) into equation (13), we obtain the equilibrium allocation of skilled labor in the R&D sector :

$$s_A = \frac{(\alpha-1)(-n+\delta+\rho)}{(\zeta-1)\alpha^2 - \delta\alpha + \alpha + \delta} + 1. \tag{22}$$

Contrary to the previous one, the equilibrium allocation of skilled labor in the R&D sector depends positively on the embodied technical change.

[6] In order to have all the variables positive, we assume that $n < \rho < \frac{-\zeta\alpha^2+\alpha^2+n\alpha-\alpha-n}{\alpha-1}$.

3 Effects of technical change on growth

Proposition 1 *The embodied technical change affects positively and permanently the growth.*

Proof.

$$\frac{\partial g_Y}{\partial \zeta} = \frac{\alpha\delta\left(-\zeta^2\alpha^4 + 2(\alpha-1)(\alpha+\delta)\zeta\alpha^2 - (\alpha-1)^2\left(\alpha^2+2\delta\alpha+n\delta\right) + (\alpha-1)^2\delta\rho\right)}{(\alpha-1)\left((\zeta-1)\alpha^2 - \delta\alpha + \alpha + \delta\right)^2} > 0.$$

As $\delta > 0$, $\left((\zeta-1)\alpha^2 - \delta\alpha + \alpha + \delta\right)^2 > 0$ and $0 < \alpha < 1$ then the sign of the derivative is given by the opposite sign of $(-\zeta^2\alpha^4 + 2(\alpha-1)(\alpha+\delta)\zeta\alpha^2$ $-(\alpha-1)^2\left(\alpha^2+2\delta\alpha+n\delta\right) + (\alpha-1)^2\delta\rho$. $(-\zeta^2\alpha^4 + 2(\alpha-1)(\alpha+\delta)\zeta\alpha^2 - (\alpha-1)^2$ $\left(\alpha^2+2\delta\alpha+n\delta\right) + (\alpha-1)^2\delta\rho < 0$ if and only if $\rho < \frac{\zeta^2\alpha^4 - 2(\alpha-1)(\alpha+\delta)\zeta\alpha^2 + (\alpha-1)^2\left(\alpha^2+2\delta\alpha+n\delta\right)}{(\alpha-1)^2\delta}$. Or, we assume that $\rho < \frac{-\zeta\alpha^2+\alpha^2+n\alpha-\alpha-n}{\alpha-1}$. As $\frac{\zeta^2\alpha^4 - 2(\alpha-1)(\alpha+\delta)\zeta\alpha^2 + (\alpha-1)^2\left(\alpha^2+2\delta\alpha+n\delta\right)}{(\alpha-1)^2\delta} > \frac{-\zeta\alpha^2+\alpha^2+n\alpha-\alpha-n}{\alpha-1}$, we always have $\frac{\partial g_Y}{\partial \zeta} > 0$. ■

This implies that an positive shock on the embodied technical change accelerates the growth contrary to Boucekkine, de la Croix, and Vailakis (2002) and Boucekkine and de la Croix (2003). The embodied technical change affects positively and permanently the productivity of the latest intermediate good which implies a substitution between capital and skilled labor allocated in the final good sector. Finally, due to the labor constraint, we obtain an increase of skilled labor allocated to the R&D sector which explains the growth rate of output.

Proposition 2 *The total factor productivity has no impact on growth.*

Proof.

$$\frac{\partial g_Y}{\partial B} = 0.$$

■

As in Boucekkine, de la Croix, and Vailakis (2002) and Boucekkine and de la Croix (2003), it is obviously that a positive shock on the total factor productivity has no effect on growth. Indeed, this productivity shock does no affect directly the sector responsible for long term growth which is here the R&D sector. Consequently, there may be a no road to long term growth.

Proposition 3 *The R&D productivity affects positively the growth.*

Proof.

$$\frac{\partial g_Y}{\partial \delta} = \frac{\alpha(\alpha(\zeta-1)+1)^2(-\alpha n+n-\rho+\alpha(\alpha(\zeta-1)+\rho+1))}{(1-\alpha)\left((\zeta-1)\alpha^2-\delta\alpha+\alpha+\delta\right)^2} > 0.$$

As $0<\alpha<1$, we have $1-\alpha>0$. We also know that $(\alpha(\zeta-1)+1)^2>0$ and $\left((\zeta-1)\alpha^2-\delta\alpha+\alpha+\delta\right)^2>0$. So, the sign of the derivative is given by the sign of $(-\alpha n+n-\rho+\alpha(\alpha(\zeta-1)+\rho+1))$. Or, the assumption $\rho < \frac{-\zeta\alpha^2+\alpha^2+n\alpha-\alpha-n}{\alpha-1}$ implies obviously that $(-\alpha n+n-\rho+\alpha(\alpha(\zeta-1)+\rho+1))>0$. Consequently, we obtain $\frac{\partial g_Y}{\partial \delta}>0$.

■

This implies that an positive shock on the R&D productivity accelerates the growth. As in Romer (1990), Boucekkine, de la Croix, and Vailakis (2002) and Boucekkine and de la Croix (2003), a rise in R&D productivity implies an increase of skilled labor allocated to this activity which pushes up the growth rate of output.

4 Conclusion

This paper has been devoted to the presentation of an endogenous growth model that is an extension of the Romer (1990) model. Indeed, first, we introduce an embodied technical change following van Zon and Yetkiner (2003) by assuming that the technological change does not only add new intermediates but simultaneously leads to intrinsic productivity improvements. Secondly, we remove scale effects. We show that the growth rate now depends positively on the rate of embodied technical change.

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