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Innovation and Imitation in a Product-cycle Model with FDI and Cash-in-advance Constraints

Hung-Ju Chen*

ABSTRACT

This paper analyzes the effects of monetary policy on innovation and imitation in a North-South product-cycle model with foreign direct investment (FDI) and separate cash-in-advance (CIA) constraints on innovative R&D, adaptive R&D and imitative R&D. We find that if the CIA constraint is applied to innovative R&D, then an increase in the Northern nominal interest will raise the rate of Northern innovation and the extent of FDI while reducing the rate of Southern imitation and the North-South wage gap. Regarding the effects of the Southern monetary policy, the object that is liquidity-constrained plays a significant role. If adaptive (imitative) R&D is subject to the CIA constraint, then an increase in the Southern nominal interest rate will raise (reduce) the rate of Northern innovation and the extent of FDI while reducing (raising) the rate of Southern imitation. We also examine the responses of social welfare for Northern and Southern consumers to monetary policy.

Keywords: CIA constraint; FDI; Imitation; Monetary policy; R&D.

JEL Classification: F12; F23; O31.

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1. INTRODUCTION

Technological progress resulting from research and development (R&D) has allowed consumers to enjoy goods with better quality. Advances in technology also cause improvements in transportation, making international production through foreign direct investment (FDI) quite common nowadays. When considering the location of production, firms can choose to produce goods domestically or abroad as a means of saving costs. The availability of FDI allows monetary policy in one country to have cross-country influences due to the adjustment of the production pattern for firms in response to these policy changes. In this paper we investigate the long-run macroeconomic effects of monetary policy in a two-country model with FDI and quality improvements of goods.

The macroeconomic effects of monetary policy have long been an important issue in macroeconomics. Based on a descriptive aggregate model, the pioneering paper of Tobin (1965) demonstrates that a higher money growth rate can positively affect the accumulation of physical capital due to the reduction in the real interest rate. Stockman (1981) and Abel (1985) develop a cash-in-advance (CIA) economy where consumption/investment is subject to the CIA constraint to analyze the impact of monetary policy. The CIA model subsequently undergoes various modifications in several studies that examine the effects of monetary policy on economic growth.[1] These studies focus on the effects of monetary policy on economic growth that depends on the accumulation of physical and human capital and ignore the impact on innovation resulting from R&D. However, the empirical evidence suggests that there is a significant relationship between R&D expenditures and cash flows (Hall, 1992; Opler, Pinkowitz, Stulz and Williamson, 1999 and Himmelberg and Petersen, 1994). Recently, several studies introduce the CIA-constrained property into an R&D model to examine the effects of monetary policy. However, most studies in this field tend to restrict their analysis to a closed economy and very few studies examine the cross-country effects of monetary policy. Based on a closed-economy product-cycle model, Chu and Cozzi (2012) and Huang, Chang and Ji (2014) look at how monetary policy affects the market structure and employment. A model with Northern and Southern countries is developed by Chu, Cozzi and Furukawa (2013) to analyze how monetary policy affects R&D and technology transfer via FDI.

This paper introduces the CIA constraint into a North-South product-cycle model with technology transfer via FDI to examine the effects of monetary policy on innovation, imitation, and production pattern.[2] Our product-cycle model presents innovative R&D in the North (a developed

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[1] For example, Suen and Yip (2005) show that indeterminacy may occur in a one-sector CIA model with an *AK* production function. A two-sector model with human capital accumulation and a CIA constraint is found in Marquis and Reffett (1991) and Mino (1997). Wang and Yip (1992) examine the impact of monetary policy under various monetary models.

[2] The North-South product-cycle model is originally introduced by Vernon (1966) and subsequently developed by Segerstrom, Anant and Dinopoulos (1990) and Grossman and Helpman (1991a, 1991b).

country), with adaptive R&D through FDI and imitative R&D in the South (a developing country). Innovation improves the quality of goods and Northern workers can work either in the R&D sector or in the production sector. Northern production firms choose either to carry out the entire production of the goods in the North or allow the goods to be produced through FDI in the South. Multinational firms produce products in the South through the use of state-of-the-art technologies (adaptive R&D) in order to take advantage of the lower Southern wage rate, but they face the risk of imitation by Southern firms. Southern firms can raise their rate of imitation (imitative R&D) by investing in imitation. Once Southern firms succeed at imitation, they are then able to use the state-of-the-art technologies to produce the highest quality products.

There are two features of this paper. First, imitation is costly and the rate of imitation is endogenized. Previous theoretical studies related to R&D and imitation tend to assume that imitation is costless and the rate of imitation is exogenous. Although assuming that imitation is costless can simplify the analysis a lot, empirical studies find that imitation is in fact a costly process. By asking firms to estimate typical costs required to duplicate several categories of innovations if a competitor has developed them, Levin, Klevorick, Nelson and Winter (1987) show that imitation is not free. Their survey data indicate that for a major unpatented new product, the cost of duplication is between fifty-one to seventy-five percent of the innovator's R&D cost for more than half of firms.[3] Using data from firms in the chemical, drug, electronics, and machinery industries, Mansfield, Schwartz and Wagner (1981) report that for 30 out of 48 products, the innovation cost exceeds \$1 million, while for 12 products, it exceeds \$5 million. They also note that on average the ratio of the imitation cost to the innovation cost is about 0.65. Since the cost of imitation is significant, the assumption of costless imitation may be convenient for analysis, but it considerably departs from reality. Besides failing to reflect reality, the lack of an appropriate consideration of the nature of imitation may may not provide a complete picture for policy implications.[4]

Second, R&D activities are subject to CIA constraints. Brown and Petersen (2009, 2011) argue that since R&D has high adjustment costs, it is very expensive for firms to adjust the flow of R&D in response to transitory finance shocks. They provide direct evidence that U.S. firms relied heavily on cash reserves to smooth R&D expenditure during the 1998-2002 boom and bust in stock market returns.[5] Brown, Fazzari and Petersen (2009) estimate a dynamic R&D model for high-tech firms

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[3] Patents tend to raise imitation costs. For a major patented new product, the cost of duplication is between seventy-six to one hundred percent of the innovator's R&D cost for more than half of firms.

[4] For studies considering costly imitation, see Gallini (1992) and Chen (2014). Gallini (1992) develops a closed-economy model with costly imitation and finds that a rival's decision to imitate depends on the length of patent protection awarded to the patentee. Chen (2014) examines the macro effects of the strengthening of intellectual property rights in developing countries in a North-South model with costly imitation.

[5] Brown, Martinsson and Petersen (2012) argue that information friction and the lack of collateral value make R&D more sensitive to financing frictions; thus, R&D-incentive firms tend to hold cash to prevent themselves from financing R&D investment though debt or equity. Using a large sample of European firms,

and find that cash holdings have a significant impact on R&D in young firms. Hall and Lerner (2010) show that in practice fifty percent or more of R&D spending goes to the wages and salaries of highly educated technology scientists and engineers. Because projects often take a long time between conception and commercialization and the departure of these highly educated workers will reduce a firm's profits, firms tend to hold cash in order to smooth their R&D spending over time, in order to avoid having to lay off these workers. These findings suggest that innovative-firms are subject to cash constraints. Furthermore, Mansfield, Schwartz and Wagner (1981) point out that innovators usually have a technological edge over their rivals in the relevant field. Often this edge is due to superior "know-how" - that is, better and more extensive technical information based on highly specialized experience with the development and production of related products. Thus, an imitator has to go through many of the same steps as an innovator. Their results suggest that an imitation-incentive firm, like an innovation-incentive firm, also relies on cash reserves to smooth its imitation spending due to high adjustment costs of imitation or the requirement for hiring highly educated workers.

In order to capture the cash requirements faced by innovative-incentive and imitative-incentive firms, we consider three scenarios based on the setting of CIA constraints: a CIA constraint on innovative R&D in the North, a CIA constraint on adaptive R&D in the South, and a CIA constraint on imitative R&D in the South.[6] While we examine the impact of the Northern monetary policy (an increase in the Northern nominal interest rate) in the first scenario, the impact of the Southern monetary policy (an increase in the Southern nominal interest rate) is considered in the second and third scenarios. In each scenario, besides looking at the effects of monetary policy on key variables such as the rate of Northern innovation, the rate of imitation and the pattern of production, we also examine its effect on social welfare for Northern and Southern consumers.

In the first scenario where the CIA constraint is applied to innovative R&D in the North, an increase in the Northern nominal interest rate raises the rate of Northern innovation while reducing the rate of Southern imitation. Since the employment increase in the R&D sector crowds out Northern labor used in the production sector, Northern firms will shift production from the North to the South. As a result, the extent of Northern production will decrease and the extent of FDI will increase, inducing a reduction in the extent of Southern production. We also find that the North-South wage gap will fall and global expenditure will rise.

Regarding the effects of the Southern monetary policy, our results indicate that the object that is liquidity-constrained plays a significant role. In the second scenario where adaptive R&D is CIA-constrained, an increase in the Southern nominal interest rate raises the rate of innovation while

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they find strong evidence that the availability of finance matters for R&D once they control firm efforts to smooth R&D with cash reserves and a firm's use of external equity finance.

[6] Early theoretical studies, which suggest that R&D investment may be particularly constrained by cash flow, can be found in Leland and Pyle (1977) and Bhattacharya and Ritter (1983).

reducing the rate of imitation. Global expenditure will rise with an increase in the Southern nominal interest rate. The North-South wage gap is immune from the Southern monetary policy. However, in the third scenario where the CIA constraint is applied to imitative R&D, the reverse effects caused by an increase in the Southern nominal interest rate on the rates of Northern innovation and Southern imitation are found. The North-South wage gap is positively correlated with the Southern nominal interest rate while the change in global expenditure is ambiguous.

Concerning the pattern of production, when the CIA constraint is applied to adaptive R&D, an increase in the Southern nominal interest rate will reduce the extents of both Northern and Southern production, causing an increase in the extent of FDI. When the CIA constraint is applied to imitative R&D, the extent of FDI will decrease with an increase in the Southern nominal interest rate. However, the responses of the extents of Northern and Southern production are ambiguous. In both cases, global expenditure will rise with an increase in the Southern nominal interest rate.

In a closely-related paper, Chu, Cozzi and Furukawa (2013) find that changes in the Northern and Southern nominal interest rate may cause different effects on the rate of innovation, the North-South wage gap, and the rate of international technology transfer through FDI based on a North-South model where CIA constraints are applied to innovative and adaptive R&D. This paper differs from their study in two ways. First, the feature of semi-endogenous growth in their model implies that at the steady state, the innovation rate is determined by exogenous parameters and is immune from monetary policy.[7] In this paper we remove the semi-endogenous growth feature and show that changes in the Northern (Southern) monetary policy can have long-run effects on the Northern innovation rate. Second, this paper considers imitation to be costly and the rate of imitation to be endogenized, whereas the rate of imitation is exogenous in their paper. Taking into account costly imitation endogenizes a rival's imitation decisions and allows for the re-allocation of Southern labor between the production sector and the imitation sector in response to changes in monetary policy, thus generating different results from those obtained based on a model with an exogenous rate of imitation. Taking the effect of monetary policy on the rate of innovation when innovative R&D is CIA-constrained as an example, an increase in the Northern nominal interest rate will raise the cost of Northern innovation and cause a decrease in the rate of innovation if the rate of imitation is assumed to be exogenous. However, if the rate of imitation is endogenized, then an increase in the Northern nominal interest rate will also induce the re-allocation of Southern labor between the production sector and the imitation sector, causing a reduction in the North-South wage

[7] They find that an increase in the Northern (Southern) nominal monetary policy induces only a *temporary* decrease in the rate of Northern innovation, leaving the long-run rate of Northern innovation unchanged. They also present that an increase in the Northern nominal interest rate generates a permanent decrease in the North-South wage gap and an ambiguous effect on the rate of international technology transfer, while an increase in the Southern nominal interest rate causes permanent decreases in the North-South wage gap and the rate of international technology transfer.

gap and an increase in the rate of Northern innovation. Therefore, the rate of innovation may increase or decrease, depending on which effect dominates. This indicates that, when considering any policy implication in an R&D model, we cannot ignore the important role of imitation.

We finally examine the effects of monetary policy on social welfare for Northern and Southern consumers.[8] We show that welfare for Northern (Southern) consumers is positively correlated with the rate of innovation and consumer's expenditure in the North (South). In the first two scenarios, although increases in the nominal interest rate raise the innovation rate, which is beneficial to social welfare, the change in monetary policy may also cause a reduction in the Northern (Southern) consumer's expenditure, which reduces welfare for Northern (Southern) consumers. When innovative R&D is subject to the CIA constraint, an increase in the Northern nominal interest rate will lead to a long-run welfare gain for Northern consumers if the Northern nominal interest rate and units of labor required for innovation are sufficiently large, but its effect on welfare for Southern consumers is ambiguous. When the CIA constraint is applied to adaptive R&D, an increase in the Southern nominal interest rate will raise both the rate of innovation and consumer's expenditure in the North, generating a welfare gain for Northern consumers. But its effect on the welfare for Southern consumers is ambiguous. Besides, the effects of Southern monetary policy on the Northern and Southern welfare are ambiguous if imitative R&D is liquidity-constrained.

The remainder of this paper is organized as follows. The next section develops a North-South product-cycle model with three types of CIA constraint: the CIA constraint imposed on innovative R&D, adaptive R&D, or imitative R&D. Section 3 studies the effects of monetary policy under each type of CIA constraint. We also examine the social welfare for Northern and Southern consumers in this section. The final section concludes.

2. THE MODEL

There exist a developed Northern country (*N*) and a developing Southern country (*S*). Each economy ($i = \{N, S\}$) is comprised of L_i households. In every period, each Northerner (Southerner) is endowed with one unit of time and s/he spends all of the time at work to earn the real wage rate w_N (w_S). The wage rate of Southerners (w_S) is normalized to 1, implying that the North-South wage gap (measured by the ratio of the Northern wage rate to the Southern wage rate) is represented by w_N. Time t is continuous, and we suppress the time index throughout the paper.

2.1. Consumers

The lifetime utility of the representative consumer in country i is:

[8] Pepall and Richards (1994) analyze the impact of the cost of copying relative to original development on social welfare based on a closed economy with innovation and imitation.

$$U_i = \int_0^\infty e^{-\rho t} \log u_i \, dt, \quad (1)$$

where ρ denotes the subjective discount factor, and $\log u_i$ is the instantaneous utility faced by a representative household.

Consumers living in either countries care about both the quantity and quality of goods and can choose from a continuum of products $z \in [0,1]$ available at different quality levels (j). Each quality level 'j' is better than quality level '$j-1$' by λ times, where the size of the quality increment λ is constant and greater than 1. This implies that each product of quality j provides quality λ^j. All products begin at time $t = 0$ with a quality level $j = 0$ and a base quality $\lambda^0 = 1$. The instantaneous utility faced by a representative household in country i is:

$$\log u_i = \int_0^1 \log\left[\sum_j \lambda^j q_{ij}(z)\right] dz, \quad (2)$$

where $q_{ij}(z)$ is the household consumption in country i for quality level j of product z at time t.

Each consumer supplies one unit of labor to earn a nominal wage W_i. The inter-temporal budget constraint faced by each consumer is:

$$\dot{A}_i + \dot{M}_i = i_i A_i + i_i B_i + W_i + T_i - P_i E_i,$$

where A_i is the nominal value of financial assets owned by each consumer, M_i is the nominal value of domestic currency held by each consumer, P_i is the price of goods denominated in units of domestic currency in country i, E_i is the real consumption per capita, T_i is the nominal value of lump-sum transfers from the government to each consumer, i_i represents the nominal interest rate, and B_i is the real value of loans of domestic currency borrowed by firms for innovative R&D, adaptive R&D, or imitative R&D activities.[9]

Let r_i represent the real interest rate and π_i represent the inflation rate. We then rewrite the inter-temporal budget constraint in real terms:

$$\dot{a}_i + \dot{m}_i = r_i a_i - \pi_i m_i + i_i b_i + w_i + \tau_i - E_i, \quad (3)$$

where a_i is the real value of financial assets owned by each agent, m_i represents the real value of domestic currency held by each household, τ_i is the real value of lump-sum transfers from the government to each consumer, w_i represents the real wage, and b_i represents the real value of loans of domestic currency borrowed by firms. Following Chu, Cozzi and Furukawa (2013), we assume that the CIA constraint faced by each agent is:

[9] Following Chu, Cozzi and Furukawa (2013), we assume that Northern entrepreneurs need to borrow from Northern consumers to finance investments in innovative R&D and to borrow from Southern consumers to finance adaptive R&D (FDI). Furthermore, we assume that Southern firms may need to borrow from Southern consumers in order to finance investments in imitation.

$$b_i \leq m_i. \tag{4}$$

Equation (4) indicates that the real money balance m_i held by the consumers is required in order to finance the firms' investments in innovative R&D, adaptive R&D, or imitative R&D activities.[10]

The total expenditure for all products with different quality levels under the real price $p_{ij}(z)$ (denominated in units of goods) is:

$$E_i = \int_0^1 \left[\sum_j p_{ij}(z) q_{ij}(z) \right] dz, \tag{5}$$

The consumer's problem is solved in three steps. First, the solution of the within-industry static optimization problem indicates that the expenditure for each product across available quality levels at each instant is allocated in a such way that consumers choose the quality that gives the lowest adjusted price, $\frac{p_{j(z)}}{\lambda^j}$. Thus, consumers are willing to pay λ for a single quality level improvement in a product.

Second, consumers allocate expenditures across products at each instant. Note that the expenditure across all products will be the same since the elasticity of substitution between any two products is constant at unity. This leads to a global demand function for product z of quality j at time t equal to $q_j(z) = E(t)/p_j(z)$, where $E = E_N L_N + E_S L_S$ represents global expenditure.

Third and finally, consumers allocate lifetime wealth across time by maximizing lifetime utility subject to the inter-temporal budget constraint. This gives the optimal expenditure path for the representative agent in each country:

$$\frac{\dot{E}_i}{E_i} = r_i - \rho\,. \tag{6}$$

We assume that there exists a global financial market, indicating that the real interest rates in the two countries must be the same - that is, $r_N = r_S = r$. In this paper we focus on the equilibrium where $r_i = \rho$ holds.

2.2. Producers

Innovation occurs only in the North and all existing products are the targets of innovation. Northern firms engaging in R&D activity hire skilled Northern workers and produce cutting-edge quality products through innovation. A Northern firm in industry z engaged in innovation intensity $\phi_R(z)$ will achieve one level of quality improvement in the final product with a probability $\phi_R(z)dt$ for a time interval dt. In order to achieve this, $a_R \phi_R(z)dt$ units of labor will be required at a total cost of $w_N a_R \phi_R(z)dt$. In order to pay the wage for R&D employment, the Northern firms need to

[10] See Huang, Chang and Ji (2014) for a model where consumer's purchase of goods is cash constrained.

borrow Northern currency from the Northern consumers. The return of borrowing is the nominal interest rate i_N in the North. Let v_N denote the expected discounted value of a Northern firm that has discovered a new product. To generate a finite rate of innovation, expected gains from innovation cannot exceed the costs, with equality being achieved when innovation occurs with positive intensity - that is:

$$v_N \leq (1+\mu_R i_N) a_R w_N, \ \phi_R > 0 \Leftrightarrow v_N = (1+\mu_R i_N) a_R w_N, \tag{7}$$

where $\mu_R = 0, 1$. When $\mu_R = 1$, the Northern firm investing in innovative R&D is subject to the CIA constraint. When $\mu_R = 0$, the CIA constraint is not applied to the Northern firm's investments in innovative R&D.

After succeeding in innovating a higher-level quality product, a Northern firm can undertake its production in the North by hiring Northern workers or carry out its production in the South through FDI, lowering its costs by hiring Southern workers. In order to undertake its production in the South through FDI, a Northern firm needs to hire Southern workers to adopt the cutting-edge technology (adaptive R&D) in the South. Engaging in FDI intensity $\phi_F(z)$ for a time interval, dt, will require $a_F \phi_F(z) dt$ units of labor at a cost of $w_S a_F \phi_F(z) dt$, with the probability of success being $\phi_F(z) dt$. In order to pay the wage to Southern workers to facilitate FDI, the entrepreneurs need to borrow Southern currency from the Southern consumers, with the return of borrowing equal to the nominal interest rate i_S in the South. Let $(v_F - v_N)$ represent capital gains from undertaking production in the South through FDI, and then we have:

$$v_F - v_N \leq (1+\mu_F i_S) a_F w_S, \ \ \phi_F > 0 \Leftrightarrow v_F - v_N = (1+\mu_F i_S) a_F w_S, \tag{8}$$

where $\mu_F = 0, 1$. When $\mu_F = 1$, adaptive R&D is subject to the CIA constraint. When $\mu_F = 0$, the CIA constraint is not applied to adaptive R&D.

Although Northern firms undertaking production in the South through FDI can save production costs, they face the risk of imitation, which is denoted by ϕ_S. A Southern firm engaged in imitation intensity $\phi_S(z)$ for a time interval dt requires $a_S \phi_S(z) dt$ units of labor. With the cost of $w_S a_S \phi_S(z) dt$, the Southern firm can successfully imitate the final product with a probability of $\phi_S(z) dt$. In order to pay the wage to Southern employment in the imitation sector, Southern firms need to borrow Southern currency from the Southern consumers, with the return of borrowing equal the Southern nominal interest rate i_S. Let v_S be the expected gains of imitation, and then we have:

$$v_S \leq (1+\mu_S i_S) a_S w_S, \ \ \phi_S > 0 \Leftrightarrow v_S = (1+\mu_S i_S) a_S w_S \,, \tag{9}$$

where $\mu_S = 0, 1$. When $\mu_S = 1$, imitative R&D is subject to the CIA constraint. When $\mu_S = 0$, the CIA constraint is not applied to imitative R&D.

Old technologies that designs have been improved are available internationally; therefore, Southern firms are able to produce final goods by using old technologies. Then Northern firms which produce through the use of state-of-the-art technologies will charge the price equal to the size of the improvement in quality times the marginal cost of closest rivals since they possess a one quality level lead over the closest rivals; that is, $p = \lambda$ (and make a sale $q = E/\lambda$). Following Howitt (1999), we assume that once a Northern firm has exited the market, it will not reenter the market due to the costly maintaining costs of unused production and R&D facilities.[11] We assume that that one unit of labor will be needed to produce one unit of the final product, regardless of the location of production. Then the cost of Northern firms completing one unit of final production in the North is w_N while the cost of Southern firms completing one unit of final production in the South is 1. When successful at adapting its technology for Southern production, multinationals can earn a higher profit through by charging the price $p = \lambda$ and hiring Southerners for production. To reflect the fact that multinationals face higher production costs relative to Southern firms, we assume that the unit labor requirement for multinational equals $\xi > 1$.[12]

When successful at imitating the technology of multinationals, a Southern firm is able to capture the entire industry market by setting a price that is slightly lower than ξ. As maintaining unused production and R&D facilities are costly, the Northern rival which has exited the market will not reenter, then the Southern firm will raise its price to λ. This price is the Nash equilibrium price since the Southern firm has no incentive to deviate from it and the presence of positive costs for unused production and R&D facilities ensures that the former Northern rival will not reenter the market. In equilibrium, only the highest quality level available will sell.

The instantaneous profits for Northern production are:

$$\Pi_N = \frac{E}{\lambda}(\lambda - w_N). \tag{10}$$

When successful at adapting its technology for Southern production, a Northern firm can earn a higher profit by charging the price $p = \lambda$ and hiring Southerners for production. The instantaneous profits for FDI are therefore:

$$\Pi_F = \frac{E}{\lambda}(\lambda - \xi). \tag{11}$$

When successful at imitating the technology of multinationals, a Southern firm can earn the same profits as multinationals:

$$\Pi_S = \frac{E}{\lambda}(\lambda - 1). \tag{12}$$

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11 Previous studies tend to assume that either it is free to reenter the market for both Northern and Southern firms (Glass and Saggi, 2002) or it is costly to reenter the market for both Northern and Southern firms (Parello, 2008). Since comparing with Southern firms (the imitators), it is more costly for Northern firms (the innovators) to maintain unused production and R&D facilities once they have exited the market, we then follow Howitt (1999) and assume that it is costly for innovators to reenter the market.

12 The same setting of production cost for multinationals is also adopted by Glass and Saggi (2002) and Parello (2008).

The no-arbitrage condition that determines v_N is:

$$r = \frac{\dot{v}_N + \Pi_N - \phi_R v_N}{v_N}. \tag{13}$$

Equation (13) equates the real interest rate to the asset return per unit of asset for Northern production. The asset return includes (i) any potential capital gain $\dot{v}_N(t)$; (ii) profits of successful innovation; and (iii) the expected capital loss $(-\phi_R v_N)$ from creative destruction.

The no-arbitrage condition that determines v_F is:

$$r = \frac{\dot{v}_F + \Pi_F - (\phi_R + \phi_S) v_F}{v_F}. \tag{14}$$

Equation (14) equates the real interest rate to the asset return per unit of asset for FDI. The asset return is the sum of (i) any potential capital gain $\dot{v}_F(t)$; (ii) profits of a successful imitation; (iii) the expected capital loss $(-\phi_R v_F)$ from creative destruction; and (iv) the expected capital loss $(-\phi_S v_F)$ from imitation.

The no-arbitrage condition that determines v_S is:

$$r = \frac{\dot{v}_S + \Pi_S - \phi_R v_S}{v_S}. \tag{15}$$

Equation (15) equates the real interest rate to the asset return per unit of asset for Southern production. The asset return is the sum of (i) any potential capital gain $\dot{v}_S$; (ii) profits of a successful imitation; and (iii) the expected capital loss $(-\phi_R v_S)$ from creative destruction.

2.3. Monetary authority

The Northern (Southern) central bank (exogenously) decides the domestic nominal interest rate i_N (i_S). Given the nominal interest rate, the inflation rate in the North (South) is endogenously determined by the Fisher equation - that is, $\pi_i = i_i - r_i$. Let η_i denote the growth rate of nominal money supply (M_i), then $\frac{\dot{M}_i}{M_i} = \eta_i$. This implies that the growth rate of the real money balance is $\frac{\dot{m}_i}{m_i} = \eta_i - \pi_i$. The money growth rate is adjusted by the monetary authority in order to achieve the targeted nominal interest rate i_i. The monetary authority returns the seigniorage revenues to consumers as a lump-sum transfer τ_i - that is, $\tau_i = \eta_i m_i$.

2.4. Equilibrium

Let n_N, n_F, and n_S respectively denote the proportion of products produced completely in the North (the extent of Northern production), the proportion of the goods for which production is carried out through FDI (the extent of FDI), and the proportion of products produced completely in the South (the extent of Southern production). The sum of these product measures should be one:

$$n_N + n_F + n_S = 1. \tag{16}$$

At the steady-state equilibrium, the flows into FDI activities and Southern production equal the flows out of them:

$$\phi_F n_N = (\phi_R + \phi_S) n_F, \tag{17}$$

$$\phi_S n_F = \phi_R n_S. \tag{18}$$

Since Northern labor is used for the R&D sector and the production sector, the labor-market clearing condition for Northern labor is:

$$a_R \phi_R + n_N \frac{E}{\lambda} = L_N. \tag{19}$$

Since Southern labor is used for the FDI sector of adapting cutting-edge technology, the imitation sector and the production sector, the labor-market clearing condition for Southern labor is:

$$a_F \phi_F n_N + a_S \phi_S n_F + (n_F + n_S) \frac{E}{\lambda} = L_S. \tag{20}$$

Using the condition that $r = \rho$ and the assumption that $\dot{v}_N = \dot{v}_F = \dot{v}_S = 0$, the no-arbitrage conditions of (13)-(15) can be expressed as:

$$v_N = \frac{\Pi_N}{\rho + \phi_R}, \tag{21}$$

$$v_F = \frac{\Pi_F}{\rho + \phi_R + \phi_S}, \tag{22}$$

$$v_S = \frac{\Pi_S}{\rho + \phi_R}. \tag{23}$$

Substituting (7) and (10) into (21) gives us:

$$\frac{E}{\lambda}(\lambda - w_N) = (\rho + \phi_R)(1 + \mu_R i_N) a_R w_N, \tag{24}$$

Substituting (7), (8), and (11) into (22) yields:

$$\frac{E}{\lambda}(\lambda - \xi) = (\rho + \phi_R + \phi_S)[(1 + \mu_R i_N) a_R w_N + (1 + \mu_F i_S) a_F]. \tag{25}$$

Substituting (9) and (12) into (23) yields:

$$\frac{E}{\lambda}(\lambda - 1) = (\rho + \phi_R)(1 + \mu_S i_S) a_S. \tag{26}$$

The steady-state equilibrium is characterized by (16)-(20) and (24)-(26) with eight variables $\{w_N, E, n_N, n_F, n_S, \phi_R, \phi_F, \phi_S\}$. Using (26), the global expenditure (E) can be expressed as a function of R&D intensity (ϕ_R):

$$E(\phi_R; i_S) = \frac{\lambda a_S (\rho + \phi_R)(1 + \mu_S i_S)}{\lambda - 1}, \tag{27}$$

with $\frac{\partial E}{\partial \phi_R} = \frac{E}{\rho + \phi_R} > 0$ and $\frac{\partial E}{\partial i_S} = \frac{\mu_S E}{1 + \mu_S i_S} \geq 0$.

From the no-arbitrage conditions of v_N and v_S (equations (24) and (26)), we have:

$$\frac{\lambda - w_N}{\lambda - 1} = \frac{(1 + \mu_R i_N) a_R w_N}{(1 + \mu_S i_S) a_S}. \tag{28}$$

Using (28), we derive the Northern wage rate as:

$$w_N(i_N, i_S) = \frac{\lambda a_S (1 + \mu_S i_S)}{a_S(1 + \mu_S i_S) + a_R(\lambda - 1)(1 + \mu_R i_N)}. \tag{29}$$

To ensure that $w_N > w_S = 1$, we assume that $a_S(1 + \mu_S i_S) > a_R(1 + \mu_R i_N)$.

Substituting (27) and (29) into the no-arbitrage condition of v_F (equation (25)), we obtain:

$$\frac{(\lambda - \xi)\, a_S}{\lambda - 1}(\rho + \phi_R) = (\rho + \phi_R + \phi_S)\theta, \tag{30}$$

where $\theta = \theta(i_N, i_S) = \frac{\lambda a_S a_R (1 + \mu_R i_N)}{a_S(1 + \mu_S i_S) + a_R(\lambda - 1)(1 + \mu_R i_N)} + a_F \frac{1 + \mu_F i_S}{1 + \mu_S i_S}$ with $\frac{\partial \theta}{\partial i_N} = \frac{\lambda a_S^2 a_R (1 + \mu_S i_S)\mu_R}{[a_S(1 + \mu_S i_S) + a_R(\lambda - 1)(1 + \mu_R i_N)]^2} \geq 0$ and $\frac{\partial \theta}{\partial i_S} = -\frac{\lambda a_S^2 a_R (1 + \mu_R i_N)\mu_S}{[a_S(1 + \mu_S i_S) + a_R(\lambda - 1)(1 + \mu_R i_N)]^2} + a_F \frac{\mu_F - \mu_S}{(1 + \mu_S i_S)^2}$.

From (30), we now express ϕ_S as a function of ϕ_R:

$$\phi_S(\phi_R; i_N, i_S) = \left[\frac{(\lambda - \xi)\, a_S - (\lambda - 1)\theta(i_N, i_S)}{(\lambda - 1)\theta(i_N, i_S)}\right](\rho + \phi_R), \tag{31}$$

with $\frac{\partial \phi_S}{\partial \phi_R} = \frac{\phi_S}{\rho + \phi_R} > 0$, $\frac{\partial \phi_S}{\partial i_N} = -\frac{(\lambda - \xi) a_S (\rho + \phi_R)}{(\lambda - 1)\theta^2}\left(\frac{\partial \theta}{\partial i_N}\right) \leq 0$, and $\frac{\partial \phi_S}{\partial i_S} = -\frac{(\lambda - \xi) a_S (\rho + \phi_R)}{(\lambda - 1)\theta^2}\left(\frac{\partial \theta}{\partial i_S}\right)$. We assume that parameter values satisfy $0 < \phi_S < 1$.

Substituting (27) into (19) yields:

$$n_N(\phi_R; i_S) = \frac{\lambda}{E(\phi_R; i_S)}(L_N - a_R \phi_R), \tag{32}$$

with $\frac{\partial n_N}{\partial \phi_R} = -\frac{n_N}{E}\left(\frac{\partial E}{\partial \phi_R}\right) - \frac{\lambda a_R}{E} < 0$ and $\frac{\partial n_N}{\partial i_S} = -\frac{n_N}{E}\left(\frac{\partial E}{\partial i_S}\right) \leq 0$.

Using (16) and (17), we can respectively replace $(n_F + n_S)$ and $\phi_F n_N$ in (20) by $(1 - n_N)$ and $(\phi_R + \phi_S) n_F$ and rewrite (20) as:

$$a_F(\phi_R + \phi_S) n_F + a_S \phi_S n_F + (1 - n_N)\frac{E}{\lambda} = L_S. \tag{33}$$

Substituting (27), (31), and (32) into (33), we can rewrite (33) as an equation in ϕ_R and n_F:

$$a_F[\phi_R + \phi_S(\phi_R)] n_F + a_S \phi_S(\phi_R) n_F + \frac{E(\phi_R)}{\lambda} - (L_N - a_R \phi_R) = L_S. \tag{34}$$

Inserting (31) and (32) into (17), we can express ϕ_F as a function of ϕ_R and n_F:

$$\phi_F(\phi_R, n_F) = \frac{[\phi_R + \phi_S(\phi_R)]n_F}{n_N(\phi_R)}. \tag{35}$$

Inserting (32) into (16) yields:

$$n_S(\phi_R, n_F) = 1 - n_N(\phi_R) - n_F. \tag{36}$$

Combining (36) and (18) and using (31) and (32), we obtain:

$$[\phi_R + \phi_S(\phi_R)]n_F = \phi_R[1 - n_N(\phi_R)]. \tag{37}$$

The equilibrium is represented by the two equations of (34) and (37), which can be used to implicitly solve for the equilibrium values of $\{\phi_R, n_F\}$. Once one derives the solution of $\{\phi_R, n_F\}$, the remaining endogenous variables can be solved accordingly.

2.5. Social welfare

One may wonder how monetary policy and the CIA constraint affect the long-run welfare for Northern and Southern consumers. To answer this question, we derive the steady-state level of social welfare in the North and in the South, which will be used to evaluate the welfare effects of monetary policy under different settings of the CIA constraint in the next section. Since consumers pay the price of λ for all goods produced by different wages, the average price is constant and equals:

$$\bar{p} = (n_N + n_F + n_S)\lambda = \lambda.$$

Because the expected number of innovations arriving in period t is $\phi_R t$, the instantaneous utility is:

$$\begin{aligned} \log u_i(t) &= logE_i - log\bar{p} + \phi_R tlog\lambda \\ &= logE_i - log\lambda + \phi_R tlog\lambda. \end{aligned}$$

The lifetime utility can then be written as:

$$U_i(0) = \frac{1}{\rho}\left(logE_i - log\lambda + \frac{\phi_R}{\rho}log\lambda\right). \tag{38}$$

To study the effect of monetary policy on the Northern (Southern) welfare, we differentiate (38) with respect to i_i and obtain:

$$\frac{dU_i(0)}{di_i} = \frac{1}{\rho}\left[\frac{1}{E_i}\left(\frac{dE_i}{di_i}\right) + \frac{log\lambda}{\rho}\left(\frac{d\phi_R}{di_i}\right)\right]. \tag{39}$$

Since the average price is constant, monetary policy affects the Northern (Southern) welfare through two channels: E_i and ϕ_R. Equation (39) indicates that Northern (Southern) welfare is positively correlated with the quality of goods and consumer's expenditure in the North (South). An increase in the rate of innovation allows consumers to enjoy a better quality of products, which is beneficial to the welfare of consumers worldwide. An increase in Northern (Southern) consumer's expenditure allows Northern (Southern) consumers to buy more goods and this is beneficial to Northern (Southern) consumers.

We next need to determine the effects of monetary policy on the rate of innovation and Northern (Southern) consumer's expenditure. Note that the rate of innovation is determined by equations (34) and (37). Since $\frac{\dot{M}_i}{M_i} = \pi_i + \frac{\dot{m}_i}{m_i}$, we thus have $\tau_i = \eta_i m_i = \frac{\dot{M}_i}{M_i} m_i = \pi_i m_i + \dot{m}_i$. Substituting this into the inter-temporal budget constraint (equation (3)), the initial Northern (Southern) consumer's expenditure at the steady state can be derived as:

$$E_i = \rho a_i + i_i b_i + w_i. \quad (40)$$

Following Dinopoulos and Segerstrom (2010) and Chu, Cozzi and Furukawa (2013), we assume that the Northern household receives dividends equal to the flow of global profits earned by Northern quality leaders, as well as the flow of global profits from foreign affiliates - that is:

$$a_N = \frac{(n_N + n_F)v_N}{L_N} = \frac{(n_N + n_F)a_R w_N}{L_N}.$$

Note that b_N represents the money borrowed by Northern firms from the household in order to finance the labor cost in innovative R&D. This implies that $b_N = \frac{\mu_R \phi_R a_R w_N}{L_N}$. Therefore, $b_N = \frac{\phi_R a_R w_N}{L_N}$ if the CIA constraint is applied to innovative R&D and $b_N = 0$ otherwise. Substituting a_N and b_N into (40) for $i = N$ and using (16), we can express the Northern consumer's expenditure as follows:

$$\begin{aligned} E_N &= \left\{\frac{a_R[\rho(n_N + n_F) + \mu_R i_N \phi_R]}{L_N} + 1\right\} w_N \\ &= \left\{\frac{a_R[\rho(1 - n_S) + \mu_R i_N \phi_R]}{L_N} + 1\right\} w_N. \end{aligned} \quad (41)$$

Substituting (41) into (38) yields:

$$U_N(0) = \frac{1}{\rho}\left(log\left\{\frac{a_R[\rho(1 - n_S) + \mu_R i_N \phi_R]}{L_N} + 1\right\} + log w_N - log\lambda + \frac{\phi_R}{\rho} log\lambda\right). \quad (42)$$

Equation (42) indicates that monetary policy and the CIA constraint will affect the Northern welfare through three channels: n_S, ϕ_R, and w_N. Once the Northern consumer's expenditure has been derived, the Southern consumer's expenditure can derived from the definition of global expenditure - that is, $E_S = \frac{E - E_N L_N}{L_S}$. The Southern welfare then becomes:

$$U_S(0) = \frac{1}{\rho}\left(\log(E - E_N L_N) - log L_S - log\lambda + \frac{\phi_R}{\rho} log\lambda\right). \quad (43)$$

3. MONETARY POLICY

We are now ready to examine the effects of monetary policies under three scenarios based on the setting of the CIA constraint. In each case, we first analyze the effects of monetary policy on key macroeconomic variables, following the analysis of the effects on social welfare for Northern and Southern consumers.

3.1. A CIA constraint on innovative R&D

We start from considering the scenario where innovative R&D is subject to the CIA constraint. This corresponds to the parameter values of $\mu_R = 1$ and $\mu_F = \mu_S = 0$. Equation (27) becomes:

$$E(\phi_R) = \frac{\lambda a_S(\rho + \phi_R)}{\lambda - 1}, \tag{44}$$

with $\frac{\partial E}{\partial \phi_R} = \frac{E}{\rho + \phi_R} > 0$.

Equation (28) becomes:

$$w_N(i_N) = \frac{\lambda a_S}{a_S + a_R(\lambda - 1)(1 + i_N)}. \tag{45}$$

Equation (45) indicates that the Northern wage depends only on i_N and is independent of ϕ_R.

Equation (31) becomes:

$$n_N(\phi_R) = \frac{\lambda}{E(\phi_R)}(L_N - a_R\phi_R), \tag{46}$$

with $\frac{\partial n_N}{\partial \phi_R} = -\frac{n_N}{E}\left(\frac{\partial E}{\partial \phi_R}\right) - \frac{\lambda a_R}{E} < 0$. Equations (44) and (46) indicate that with the CIA constraint applied to innovative R&D, the Northern nominal interest rate does not directly affect global expenditure and the extent of Northern production. However, it affects global expenditure and the extent of Northern production indirectly by influencing the rate of innovation.

Equation (31) becomes:

$$\phi_S(\phi_R; i_N) = \left[\frac{(\lambda - \xi)\, a_S - (\lambda - 1)\theta(i_N)}{(\lambda - 1)\theta(i_N)}\right](\rho + \phi_R), \tag{47}$$

where $\theta = \theta(i_N) = a_F + \frac{\lambda a_S a_R(1+i_N)}{a_S + a_R(\lambda-1)(1+i_N)}$ with $\frac{\partial \theta}{\partial i_N} = \frac{\lambda a_R a_S^2}{[a_S + a_R(\lambda-1)(1+i_N)]^2} > 0$. Furthermore, we also obtain $\frac{\partial \phi_S}{\partial \phi_R} = \frac{\phi_S}{\rho + \phi_R} > 0$ and $\frac{\partial \phi_S}{\partial i_N} = -\frac{(\lambda-\xi)a_S(\rho+\phi_R)}{(\lambda-1)\theta^2}\left(\frac{\partial \theta}{\partial i_N}\right) < 0$.

We are now ready to examine the effects of an increase in i_N. Since the equilibrium is represented by the two equations of (34) and (37) in $\{\phi_R, n_F\}$, then we totally differentiate (34) and (37) with respect to ϕ_R, n_F, and i_N. As revealed in Appendix A, both equations are affected by an increase in i_N. Appendix A shows that $\frac{d\phi_R}{di_N} > 0$ and $\frac{dn_F}{di_N} > 0$, indicating that an increase in i_N raises both the rate of innovation and the extent of FDI.

An increase in the Northern nominal interest rate causes two opposite effects on the rate of innovation. First, the increase in the Northern nominal interest rate raises the cost of innovation and reduces the rate of innovation. Second, the no-arbitrage conditions of v_N and v_S indicate that $\frac{w_N}{\lambda - w_N} = \frac{a_S}{a_R(\lambda-1)(1+i_N)}$ (equation (28)). This condition implies that there is a negative relationship between w_N and i_N - that is, $\frac{dw_N}{di_N} = -\frac{\lambda a_S a_R(\lambda-1)}{[a_S + a_R(\lambda-1)(1+i_N)]^2} < 0$. Therefore, an increase in i_N reduces the North-South wage gap. Since the Northern labor becomes relatively cheaper, the demand for Northern labor in the innovative R&D sector will increase and this will raise the rate of innovation. Appendix A shows that the second effect dominates the first effect and there is an overall increase in the rate of innovation.[13]

The increase in the rate of innovation will raise global expenditure as indicated by (44). With increases in the rate of innovation and global expenditure, the labor-market clearing condition for Northern labor (equation (19)) implies that there will be a decrease in the extent of Northern production, causing an increase in the extent of FDI. The increase in the Southern labor employed by foreign affiliates crowds out employment in the production sector and the imitation sector for Southern firms, resulting in decreases in the extent of Southern production and the rate of imitation. However, the increase in the rate of innovation means that there will be more newly innovated goods to be imitated and this will raise the rate of imitation. As demonstrated by (47), besides the direct effect of i_N, there will be an indirect effect of i_N on ϕ_S through the channel of ϕ_R. We find that there will be overall decrease in the rate of imitation. Finally, equation (35) indicates that the change of FDI intensity is ambiguous. We summarize our findings in the following proposition.

Proposition 1. With a CIA constraint applied to innovative R&D, an increase in the nominal interest rate in the North causes (a) a decrease in the North-South wage gap; (b) an increase in global expenditure; (c) an increase in the rate of innovation; (d) a decrease in the rate of imitation; and (e) an ambiguous change in FDI intensity. Concerning the production pattern, the extents of Northern and Southern production will decrease while the extent of FDI will increase.

Proof. See Appendix A.

Equation (39) demonstrates that the change in the Northern monetary policy will affect the welfare for Northern (Southern) consumers by impacting E_N (E_S) and ϕ_R. Equation (41) indicates that $E_N = \left\{\frac{a_R[\rho(1-n_S)+i_N\phi_R)]}{L_N} + 1\right\} w_N$ when innovative R&D is CIA-constrained. This implies that an increase in i_N will affect E_N through the three channels of n_S, ϕ_R, and w_N. The decrease in

[13] In a model where imitation is costless and the rate of imitation is assumed to be exogenous, there is no need to consider the no-arbitrage condition that determines v_S and there is no second effect. Therefore, an increase in the Northern nominal interest rate will *reduce* the rate of innovation.

n_S and the increase in ϕ_R will raise E_N while the decrease in w_N will reduce E_N. Therefore, E_N may increase or decrease, depending on which effect dominates. In Appendix B we show that if i_N and a_R are sufficiently large, then the positive effects will outweigh the negative effect and there will be an overall increase in E_N. With increases in E_N and ϕ_R, the welfare for Northern consumers will increase.

Proposition 2. With a CIA constraint applied to innovative R&D, an increase in the Northern nominal interest rate will raise the expenditure and the welfare for Northern consumers if i_N and a_R are sufficiently large.

Proof. See Appendix B.

From the definition of global expenditure, the consumer's expenditure in the South can be derived as $E_S = \frac{E - E_N L_N}{L_S}$. Under the assumption of a sufficiently large i_N, an increase in i_N will raise both E and E_N, leading to an ambiguous change in E_S. Accordingly, the change in the welfare of Southern consumers is ambiguous. If there is an overall increase in E_S, then the welfare of Southern consumers will increase.

3.2. A CIA constraint on adaptive R&D

We now turn to investigate the effects of an increase in the Southern nominal interest rate. We first consider the scenario where the CIA constraint is applied to adaptive R&D. This corresponds to the parameter values of $\mu_F = 1$ and $\mu_R = \mu_S = 0$. Global expenditure is the same as the one in (44) and the extent of Northern production is the same as the one in (46).

Note that w_N is derived from the no-arbitrage conditions of v_N and v_S (equations (24) and (26)). Because only adaptive R&D is CIA-constrained, these two equations are not directly affected by i_S. From (29), we obtain a constant North-South wage gap:

$$w_N = \frac{\lambda a_S}{a_S + a_R(\lambda - 1)}. \tag{48}$$

Equation (31) becomes:

$$\phi_S(\phi_R; i_S) = \left[\frac{(\lambda - \xi)\, a_S - (\lambda - 1)\theta(i_S)}{(\lambda - 1)\theta(i_S)}\right](\rho + \phi_R), \tag{49}$$

where $\theta = \theta(i_S) = (1 + i_S)a_F + \frac{\lambda a_S a_R}{a_S + a_R(\lambda - 1)}$ with $\frac{\partial \theta}{\partial i_S} = a_F > 0$. Furthermore, we also have $\frac{\partial \phi_S}{\partial \phi_R} = \frac{\phi_S}{\rho + \phi_R} > 0$ and $\frac{\partial \phi_S}{\partial i_S} = -\frac{(\lambda - \xi) a_S a_F (\rho + \phi_R)}{(\lambda - 1)\theta^2} < 0$.

To study the effects of i_S, we totally differentiate (34) and (37) with respect to ϕ_R, n_F, and i_S. In Appendix B we show that $\frac{d\phi_R}{di_S} > 0$ and $\frac{dn_F}{di_S} > 0$, indicating that an increase in i_S raises both the rate of innovation and the extent of FDI. An increase in the Southern nominal interest rate raises the cost of FDI and reduces the extent of FDI. Since Southern firms only imitated products produced by foreign affiliates, the reduced extent of FDI implies that fewer products are targeted for imitation, causing an increase in the rate of innovation. With more Northern workers employed in the R&D sector, fewer Northern workers are available for the production sector, leading to a decrease in the extent of Northern production and an increase in the extent of FDI. We find that overall, there will be an increase in the extent of FDI. The increase in the Southern labor employed by foreign affiliates means that fewer Southern workers are released from the FDI sector, resulting in a reduction in the extent of Southern production.

Combining the no-arbitrage conditions of v_N and v_S, equation (29) indicates that the North-South wage gap can be expressed as in (48) and is immune from Southern monetary policy. Equation (49) implies that an increase in the Southern nominal interest rate affects the rate of imitation by influencing $\theta(i_S)$ and ϕ_R. Given the fact that i_S does not directly affect w_N and E, the no-arbitrage condition that determines v_F (equation (25)) implies that a higher i_S *ceteris paribus* reduces ϕ_S - that is, $\frac{\partial\phi_S}{\partial i_S} < 0$ as indicated by (49). However, the increased rate of innovation caused by an increase in the Southern nominal interest rate means that there are more products to be imitated, implying an increase in the rate of imitation - that is, $\frac{\partial\phi_S}{\partial\phi_R} > 0$ as indicated by (49). We find that there will be an overall decrease in the rate of imitation. In fact, the higher rate of innovation generates higher global expenditure as indicated by (44). Similar to the previous case, the change in FDI intensity is ambiguous. The following proposition summarizes these results.

Proposition 3. With a CIA constraint applied to adaptive R&D, an increase in the nominal interest rate in the South causes (a) an increase in global expenditure; (b) an increase in the rate of innovation; (c) a decrease in the rate of imitation; and (d) an ambiguous change in FDI intensity. The North-South wage gap is not affected by Southern monetary policy. Concerning the production pattern, such monetary policy will reduce in the extents of Northern production and FDI while raising the extent of Southern production.

Proof. See Appendix C.

To examine the change in welfare for Northern and Southern consumers in response to an increase in the Southern nominal interest rate, we first use (41) to study the change of Northern consumer's expenditure. When the CIA constraint is applied to adaptive R&D, the Northern consumer's expenditure becomes $E_N = \left[\frac{a_R \rho(1-n_S)}{L_N} + 1\right] w_N$. Since w_N is constant, then a decrease in n_S caused by an increase in i_S will lead to an increase in E_N. Since both E_N and ϕ_R increase, the Northern welfare will increase.

Proposition 4. With a CIA constraint applied to adaptive R&D, an increase in the nominal interest rate in the South will raise the Northern consumer's expenditure and the welfare for Northern consumers.

Proof. See Appendix D.

With both increases in E and E_N, an increase in i_S will cause an ambiguous change in E_S, because $E_S = \frac{E - E_N L_N}{L_S}$. Therefore, the change of the welfare for Southern consumers will be ambiguous. Similar to the previous case, if there is an overall increase in E_S, then the welfare of Southern consumers will increase.

3.3. A CIA constraint on imitative R&D

We finally investigate the effects of an increase in i_S when the CIA constraint is applied to imitative R&D. This corresponds to the parameter values of $\mu_S = 1$ and $\mu_R = \mu_F = 0$. From (27), global expenditure becomes:

$$E(\phi_R; i_S) = \frac{\lambda a_S(\rho + \phi_R)(1 + i_S)}{\lambda - 1}, \tag{50}$$

with $\frac{\partial E}{\partial \phi_R} = \frac{E}{\rho + \phi_R} > 0$ and $\frac{\partial E}{\partial i_S} = \frac{E}{1 + i_S} > 0$.

Equation (29) becomes:

$$w_N(i_S) = \frac{\lambda a_S(1 + i_S)}{a_S(1 + i_S) + a_R(\lambda - 1)}. \tag{51}$$

Equation (32) becomes:

$$n_N(\phi_R; i_S) = \frac{\lambda}{E(\phi_R; i_S)}(L_N - a_R \phi_R), \tag{52}$$

with $\frac{\partial n_N}{\partial \phi_R} = -\frac{n_N}{E}\left(\frac{\partial E}{\partial \phi_R}\right) - \frac{\lambda a_R}{E} < 0$ and $\frac{\partial n_N}{\partial i_S} = -\frac{n_N}{E}\left(\frac{\partial E}{\partial i_S}\right) < 0$. Equations (51) and (52) imply that besides causing direct effects, an increase in i_S also directly influences E and n_N through its effect on ϕ_R.

Equation (31) becomes:

$$\phi_S(\phi_R; i_S) = \left[\frac{(\lambda - \xi)\, a_S - (\lambda - 1)\theta(i_S)}{(\lambda - 1)\theta(i_S)}\right](\rho + \phi_R), \tag{53}$$

where $\theta = \theta(i_S) = \frac{\lambda a_S a_R}{a_S(1+i_S)+a_R(\lambda-1)} + \frac{a_F}{1+i_S}$ with $\frac{\partial\theta}{\partial i_S} = -\frac{\lambda a_S^2 a_R}{[a_S(1+i_S)+a_R(\lambda-1)]^2} - \frac{a_F}{(1+i_S)^2} < 0$.

Furthermore, we also have $\frac{\partial \phi_S}{\partial \phi_R} = \frac{\phi_S}{\rho+\phi_R} > 0$ and $\frac{\partial \phi_S}{\partial i_S} = -\frac{(\lambda-\xi)a_S(\rho+\phi_R)}{(\lambda-1)\theta^2}\left(\frac{\partial\theta}{\partial i_S}\right) > 0$.

Totally differentiating (34) and (37) with respect to ϕ_R, n_F, and i_S, we show that $\frac{d\phi_R}{di_S} < 0$ and $\frac{dn_F}{di_S} < 0$ in Appendix C. These results imply that an increase in i_S reduces both the rate of innovation and the extent of FDI.

Using the no-arbitrage conditions of v_N and v_S (equation (28)), we can derive the North-South wage gap as given in Equation (51). It indicates that the North-South wage gap will increase with a rise in the Southern nominal interest rate since $\frac{dw_N}{di_S} = \frac{\lambda a_S a_R(\lambda-1)}{[a_S(1+i_S)+a_R(\lambda-1)]^2} > 0$. The increase in the North-South wage gap will reduce labor employment in the R&D sector in the North, causing the rate of Northern innovation to decrease. As shown by (50), an increase in the Southern nominal interest rate raises global expenditure directly, but a decrease in the rate of innovation reduces global expenditure indirectly. Therefore, the change in global expenditure is ambiguous.

From the no-arbitrage condition of v_F (equation (25)), an increase in the Southern nominal interest rate will cause a direct increase in the rate of imitation - that is, $\frac{\partial \phi_S}{\partial i_S} > 0$ as indicated by (53). However, the reduced rate of innovation caused by an increase in the Southern nominal interest rate means that there are fewer products to be imitated, implying a reduction in the rate of imitation - that is, $\frac{\partial \phi_S}{\partial \phi_R} > 0$ as indicated by (53). In Appendix E we show that if ξ is sufficiently large, then the positive effect caused by i_S outweighs the negative effects and there will be overall increases in the rate of imitation. Although the increase in the North-South wage gap may reduce the extent of Northern production while encouraging FDI activities, the increase in demand for Southern labor in the imitation sector crowds out Southern labor employed in the FDI sector and in the Southern production sector, causing a reduction in the extent of FDI and an increase in the extent of Northern production. Our calculation in Appendix E demonstrates that if ξ is sufficiently large and $a_R > a_F$, then the extent of FDI will decrease while the change of the extent of Northern production is ambiguous. With the ambiguous change in the extent of Northern production, we are not able to determine the change in the extent of Southern production and FDI intensity. The following proposition summarizes these results.

Proposition 5. Suppose ξ is sufficiently large and $a_R > a_F$. Then with a CIA constraint applied to imitative R&D, an increase in the nominal interest rate in the South will cause (a) an increase in the North-South wage gap; (b) a decrease in the rate of innovation; (c) an increase in the rate of imitation; and (d) ambiguous changes in global expenditure and FDI intensity. Concerning the production pattern, such a monetary policy will reduce the extent of FDI, leaving the changes of the extents of Northern and Southern production ambiguous.

Proof. See Appendix E.

When the CIA constraint is applied to imitative R&D, the Northern consumer's expenditure can expressed as $E_N = \left[\frac{a_R \rho (1-n_S)}{L_N} + 1\right] w_N$. Since an increase in the Southern nominal interest rate will cause ambiguous effects on n_S, the change of E_N is ambiguous; thus, we are not able to determine the change in welfare for Northern consumers. Due to the ambiguous change in E_N, the change of E_S is also ambiguous and we are not able to determine the change in welfare for Southern consumers. Note that in this case, an increase in the Southern nominal interest rate will reduce the rate of Northern innovation. If this change in the monetary policy reduces E_N (E_S), then it will reduce the welfare for Northern (Southern) consumers.

4. CONCLUSION

In this paper we examine the effects of monetary policy on innovation, imitation, the North-South wage gap, and the pattern of production based on a product-cycle model with CIA constraints applied to innovative R&D, adaptive R&D and imitative R&D. Our analysis reveals that the effects of monetary policy on these variables and social welfare for Northern and Southern consumers depend on the object of the CIA constraint.

When the CIA constraint is applied to innovative R&D, an increase in the Northern nominal interest rate will raise the rate of Northern innovation while reducing the rate of Southern imitation. Global expenditure will increase while the North-South wage gap and the extent of Northern production will decrease. We find that an increase in the Southern nominal interest will cause the same effect on the rates of innovation and imitation, global expenditure, and the pattern of production when adaptive R&D is subject to the CIA constraint. When the CIA constraint is applied to imitative R&D, the rate of innovation will decrease and the rate of imitation will increase with an increase in the Southern nominal interest rate. This change in monetary policy will result in a decrease in the extent of FDI, leaving the change in the extents of Northern and Southern production ambiguous. Table 1 summarizes these results.

Concerning social welfare for Northern (Southern) consumers, we find that the Northern (Southern) welfare depends on Northern (Southern) consumer's expenditure and the rate of innovation. Assuming that the Northern household receives dividends equal to the flow of global profits earned by Northern quality leaders, as well as the flow of global profits from foreign affiliates, the Northern consumer's expenditure depends positively on the North-South wage gap and negatively on the extent of Southern production. Given that changes in monetary policy will cause various effects on the North-South wage gap, the extent of Southern production, and the rate of innovation, changes in the Northern and Southern welfare may be ambiguous. We show that under certain conditions, the welfare for Northern consumers will increase with an increase in the Northern nominal interest rate when innovative R&D is subject to the CIA constraint. Besides, a rise in the Southern nominal interest rate will also raise the welfare for Northern consumers when adaptive R&D is CIA-constrained.

We point out two directions for future study. In this paper, labor supply is assumed to be inelastic and the CIA constraint is not applied to household expenditure. It would be interesting to extend our model by endogenizing labor supply or by assuming that household expenditure is subject to the CIA constraint in order to examine the robustness of the results found in this paper.

REFERENCES

Abel, A., 1985. Dynamic behavior of capital accumulation in a cash-in-advance model. *Journal of Monetary Economics* 16, 55-71.

Bhattacharya, S., Ritter, J.R., 1983. Innovation and Communication: Signaling with partial disclosure. *Review of Economic Studies* 50, 331-346.

Brown, J.R., Fazzari, S.M., Petersen, B.C., 2009. Financing innovation and growth: Cash flow, external equity, and the 1990s R&D boom. *Journal of Finance* 64, 151-185.

Brown, J.R., Martinsson, G., Petersen, B.C., 2012. Do financing constraints matter for R&D? *European Economic Review* 56, 1512-1529.

Brown, J.R., Petersen, B.C., 2009. Why has the investment-cash flow sensitivity declined so sharply? Rising R&D and equity market developments. *Journal of Banking and Finance* 33, 971-984.

Brown, J.R., Petersen, B.C., 2011. Cash holdings and R&D smoothing. *Journal of Corporate Finance* 17, 694-709.

Chen, H.-J., 2014. Effects of intellectual property rights in a product-cycle model of skills accumulation. Mimeo. National Taiwan University.

Chu, A.C., Cozzi, G., 2012. R&D and economic growth in a cash-in-advance economy. *International Economic Review* 55, 507-524.

Chu, A.C., Cozzi, G., Furukawa, Y., 2013. Innovation and North-South technology transfer in a cash-in-advance economy. http://sites.google.com/angusccc/North-South.pdf?attredirects=0

Dinopoulos, E., Segerstrom, P., 2010. Intellectual property rights, multinational firms and economic growth. *Journal of Development Economics* 92, 13-27.

Gallini, N., 1992. Patent policy and costly imitation. *Rand Journal of Economics* 23, 52-63.

Glass, A.J., Saggi, K., 2002. Intellectual property rights and foreign direct investment. *Journal of International Economics* 56, 387-410.

Grossman, G.M., Helpman, E., 1991a. Quality ladders and product cycles. *Quarterly Journal of Economics* 106, 557-586.

Grossman, G.M., Helpman, E., 1991b. Endogenous product cycles. *Economic Journal* 101, 1214-1229.

Hall, B.H., 1992. Investment and R&D at the firm level: Does the source of financing matter? NBER Working Paper No. 4096.

Hall, B.H., Lerner, J., 2010. The financing of R&D and innovation, in: *Handbook of the Economics of Innovation*, edited by B.H. Hall and N. Rosenberg, 609-639, Cambridge University Press, UK.

Himmelberg, C.P., Petersen, B.C., 1994. R&D and internal finance: a panel study of small firms in high-tech industries. *Review of Economics and Statistics* 76, 38-51.

Howitt, P., 1999. Steady endogenous growth with population and R&D inputs growing. *Journal of Political Economy* 107, 715-730.

Huang, C.Y., Chang, J.J., Ji, L., 2014. Cash-in-advance constraints in a Shumpeterian growth model with an endogenous market structure. IEAS working paper: academic research 13-A012, Institute of Economics, Academia Sinica, Taiwan.

Leland, H.E., Pyle, D.H., 1977. Informational asymmetries, financial structure, and financial intermediation. *Journal of Finance* 32, 371-387.

Levin, R., Klevorick, A., Nelson, R., Winter, S., 1987. Appropriating the returns from industrial R&D. *Brookings Papers on Economic Activity* 3, 783-831.

Mansfield, E., Schwartz, M., Wagner, S., 1981. Imitation costs and patents: An empirical study. *Economic Journal* 91, 907-918.

Marquis, M.H., Reffett, K.L., 1991. Real interest rates and endogenous growth in a monetary economy. *Economics Letters* 37, 105-109.

Mino, K., 1997. Long-run effects of monetary expansion in a two-sector model of endogenous growth. *Journal of Macroeconomics* 19, 635-655.

Opler, T., Pinkowitz, L., Stulz, R., Williamson, R., 1999. The determinants and implications of corporate cash holding. *Journal of Financial Economics* 52, 3-46.

Parello, C.P., 2008. A North-South model of intellectual property rights protection and skill accumulation. *Journal of Development Economics* 85, 253-281.

Pepall, L.M., Richards, D.J., 1994. Innovation, imitation, and social welfare. *Southern Economic Journal* 60, 673-684.

Segerstrom, P., Anant, T., Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. *American Economic Review* 80, 1077-1091.

Stockman, A.C., 1981. Anticipated inflation and the capital stock in a cash-in-advance economy. *Journal of Monetary Economics* 8, 387-393.

Suen, M.H., Yip, C.K., 2005. Superneutrality, indeterminacy and endogenous growth. *Journal of Macroeconomics* 27, 579-595.

Tobin, J., 1965. Money and economic growth. *Econometrica* 33, 671-684.

Vernon, R., 1966. International investment and international trade in the product cycle. *Quarterly Journal of Economics* 80, 190-20.

Wang, P., Yip, C.K., 1992. Alternative approaches to money and growth. *Journal of Money, Credit and Banking* 24, 553-562.

Table 1 The effects of monetary policy

	CIA on R&D (μ_R=1, μ_F=0, μ_S=0)	CIA on FDI (μ_R=0, μ_F=1, μ_S=0)	CIA on Imitation (μ_R=0, μ_F=0, μ_S=1)
	i_N↑	i_S↑	i_S↑
w_N	↓	→	↑
E	↑	↑	N/A
ϕ_R	↑	↑	↓
ϕ_F	N/A	N/A	N/A
ϕ_S	↓	↓	↑
n_N	↓	↓	N/A
n_F	↑	↑	↓
n_S	↓	↓	N/A

APPENDIX A

Proof of Proposition 1

In this case, we have $\mu_R = 1$ and $\mu_F = \mu_S = 0$. Equation (27) becomes:

$$E(\phi_R) = \frac{\lambda a_S(\rho + \phi_R)}{\lambda - 1}, \tag{A1}$$

with $\frac{\partial E}{\partial \phi_R} = \frac{E}{\rho + \phi_R} > 0$. Equation (29) becomes:

$$w_N(i_N) = \frac{\lambda a_S}{a_S + a_R(\lambda - 1)(1 + i_N)}, \tag{A2}$$

where $\frac{dw_N}{di_N} = -\frac{\lambda a_S a_R(\lambda-1)}{[a_S + a_R(\lambda-1)(1+i_N)]^2} = -\frac{a_R(\lambda-1)w_N}{a_S + a_R(\lambda-1)(1+i_N)} < 0$.

In this case, we have $\theta = \theta(i_N) = a_F + \frac{\lambda a_S a_R(1+i_N)}{a_S + a_R(\lambda-1)(1+i_N)}$ and $\frac{\partial \theta}{\partial i_N} = \frac{\lambda a_R a_S^2}{[a_S + a_R(\lambda-1)(1+i_N)]^2} > 0$. From (31), we then obtain:

$$\phi_S(\phi_R; i_N) = \left[\frac{(\lambda - \xi)\, a_S - (\lambda - 1)\theta(i_N)}{(\lambda - 1)\theta(i_N)}\right](\rho + \phi_R), \tag{A3}$$

where $\frac{\partial \phi_S}{\partial \phi_R} = \frac{\phi_S}{\rho + \phi_R} > 0$ and $\frac{\partial \phi_S}{\partial i_N} = - - \frac{(\lambda-\xi)a_S(\rho+\phi_R)}{(\lambda-1)\theta^2}\left(\frac{\partial \theta}{\partial i_N}\right) < 0$.

Equation (32) becomes:

$$n_N(\phi_R) = \frac{\lambda}{E(\phi_R)}(L_N - a_R\phi_R), \tag{A4}$$

where $\frac{\partial n_N}{\partial \phi_R} = -\frac{n_N}{E}\left(\frac{\partial E}{\partial \phi_R}\right) - \frac{\lambda a_R}{E} < 0$.

To examine the effects of an increase in i_N, we totally differentiate (34) and (37) with respect to ϕ_R, n_F, and i_N to obtain:

$$\underbrace{\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}}_{B}\begin{bmatrix} d\phi_R \\ dn_F \end{bmatrix} = -\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} di_N\,, \tag{A5}$$

where $b_1 = n_F\left[(a_S + a_F)\left(\frac{\partial \phi_S}{\partial \phi_R}\right) + a_F\right] + \frac{1}{\lambda}\left(\frac{\partial E}{\partial \phi_R}\right) + a_R > 0$, $b_2 = (a_S + a_F)\phi_S + a_F\phi_R > 0$, $b_3 = n_F\left(1 + \frac{\partial \phi_S}{\partial \phi_R}\right) - (1 - n_N) + \phi_R\left(\frac{\partial n_N}{\partial \phi_R}\right) = n_F\left(\frac{\partial \phi_S}{\partial \phi_R}\right) - n_S + \phi_R\left(\frac{\partial n_N}{\partial \phi_R}\right) < 0$, $b_4 = \phi_R + \phi_S > 0$, $e_1 = n_F(a_S + a_F)\left(\frac{\partial \phi_S}{\partial i_N}\right) < 0$, and $e_2 = n_F\left(\frac{\partial\, \phi_S}{\partial i_N}\right) < 0$.

Note that the determinant of B is negative since $|B| = b_1 b_4 - b_2 b_3 < 0$. Using (A5), the effects of changes in i_N on ϕ_R and n_F are:

$$\frac{d\phi_R}{di_N} = -\frac{b_4 e_1 - b_2 e_2}{|B|}, \quad \text{(A6)}$$

$$\frac{dn_F}{di_N} = -\frac{b_1 e_2 - b_3 e_1}{|B|}. \quad \text{(A7)}$$

With a few steps of calculation, we obtain that $(b_4 e_1 - b_2 e_2) = -a_S \phi_R n_F \left(\frac{\partial \phi_S}{\partial i_N}\right) > 0$, indicating that $\frac{d\phi_R}{di_S} > 0$. With a few steps of calculation, we can derive that $(b_1 e_2 - b_3 e_1) = n_F \Big\{-(a_S + a_F)\left(\frac{\partial \phi_S}{\partial i_N}\right)\left[n_S - \phi_R\left(\frac{\partial n_N}{\partial \phi_R}\right)\right] - \left[a_F n_F + \frac{1}{\lambda}\left(\frac{\partial E}{\partial \phi_R}\right) + a_R\right]\left(\frac{\partial \phi_S}{\partial i_N}\right)\Big\} > 0$, indicating that $\frac{dn_F}{di_N} > 0$.

We now turn to examine the effects of i_N on other variables. From (A2), we derive that $\frac{dw_N}{di_N} < 0$. From (A1) and (A4), we have:

$$\frac{dE}{di_N} = \underbrace{\frac{\partial E}{\partial \phi_R}}_{+}\underbrace{\frac{d\phi_R}{di_N}}_{+} > 0,$$

$$\frac{dn_N}{di_N} = \underbrace{\frac{\partial n_N}{\partial \phi_R}}_{-}\underbrace{\frac{d\phi_R}{di_N}}_{+} < 0.$$

Since $n_S = 1 - n_N - n_F$, we have:

$$\frac{dn_S}{di_N} = -\frac{dn_N}{di_N} - \frac{dn_F}{di_N} = \frac{\partial n_N}{\partial \phi_R}\frac{d\phi_R}{di_N} - \frac{dn_F}{di_N}. \quad \text{(A8)}$$

Substituting (A6) and (A7) into (A8), we next obtain:

$$\frac{dn_S}{di_N} = -\frac{n_F\left(\frac{\partial \phi_S}{\partial i_N}\right)\left\{a_F\left[n_F + n_S - \phi_R\left(\frac{\partial n_N}{\partial \phi_R}\right)\right] + \frac{1}{\lambda}\left(\frac{\partial E}{\partial \phi_R}\right) + a_R + a_S n_S\right\}}{|B|} < 0.$$

From (47), we have:

$$\frac{d\phi_S}{di_N} = \underbrace{\underbrace{\frac{\partial \phi_S}{\partial \phi_R}}_{+}\underbrace{\frac{d\phi_R}{di_N}}_{+}}_{+} + \underbrace{\frac{\partial \phi_S}{\partial i_N}}_{-}. \quad \text{(A9)}$$

Substituting (A6) into (A8) yields:

$$\frac{d\phi_S}{di_N} = -\frac{1}{|B|}\left(\frac{\partial \phi_S}{\partial i_N}\right)\left\{b_4\left[a_F n_F + \frac{1}{\lambda}\left(\frac{\partial E}{\partial \phi_R}\right) + a_R\right] + b_2\left[n_S - \phi_R\left(\frac{\partial n_N}{\partial \phi_R}\right)\right]\right\} < 0.$$

In order to examine the change in FDI intensity, we use (35) to derive:

$$\frac{d\phi_F}{di_N} = \underbrace{\left(\underbrace{\frac{\partial\phi_F}{\partial\phi_R}}_{+} + \underbrace{\frac{\partial\phi_F}{\partial n_N}}_{-} \underbrace{\frac{dn_N}{d\phi_R}}_{-} \right)}_{+} \underbrace{\left(\frac{d\phi_R}{di_N} \right)}_{+} + \underbrace{\left(\underbrace{\frac{\partial\phi_F}{\partial\phi_S}}_{+} \underbrace{\frac{d\phi_S}{di_N}}_{-} \right)}_{-} + \underbrace{\left(\underbrace{\frac{\partial\phi_F}{\partial n_F}}_{+} \underbrace{\frac{dn_F}{di_N}}_{+} \right)}_{+}. \quad \text{(A10)}$$

Equation (A10) indicates that the sign of $\frac{d\phi_F}{di_N}$ is ambiguous.

□

APPENDIX B

Proof of Proposition 2

From (41), we have $E_N = \left\{\frac{a_R[\rho(1-n_S)+i_N\phi_R]}{L_N} + 1\right\} w_N$. Thus, we can derive:

$$\begin{aligned}
\frac{dE_N}{di_N} &= \frac{a_R w_N}{L_N}\left[-\rho\left(\frac{dn_S}{di_N}\right) + \phi_R + i_N\left(\frac{d\phi_R}{di_N}\right)\right] + \left[\frac{a_R[\rho(1-n_S)+i_N\phi_R)]}{L_N} + 1\right]\frac{dw_N}{di_N} \\
&= \frac{a_R w_N}{L_N}\left\{\left[-\rho\left(\frac{dn_S}{di_N}\right) + \phi_R + i_N\left(\frac{d\phi_R}{di_N}\right)\right] - \frac{\{a_R[\rho(1-n_S)+i_N\phi_R)] + L_N\}(\lambda-1)}{a_S + a_R(\lambda-1)(1+i_N)}\right\} \\
&= \frac{a_R w_N}{L_N}\left\{\left[-\rho\left(\frac{dn_S}{di_N}\right) + i_N\left(\frac{d\phi_R}{di_N}\right)\right] + \frac{\phi_R[a_S + (\lambda-1)a_R] - [a_R\rho(1-n_S) + L_N](\lambda-1)}{a_S + a_R(\lambda-1)(1+i_N)}\right\}.
\end{aligned} \quad \text{(B1)}$$

Since $\frac{dn_S}{di_N} < 0$, $\frac{d\phi_R}{di_N} > 0$, and $0 < 1 - n_S < 1$, then $\frac{dE_N}{di_N} > 0$ if the following condition holds:

$$\phi_R[a_S + (\lambda-1)a_R] > (\lambda-1)(a_R\rho + L_N). \quad \text{(B2)}$$

The condition (B2) implies that:

$$\phi_R > \frac{(\lambda-1)(a_R\rho + L_N)}{a_S + (\lambda-1)a_R}. \quad \text{(B3)}$$

The right-hand side of (B3) is smaller than 1 if a_R is sufficiently large such that $a_R > \frac{(\lambda-1)L_N - a_S}{(\lambda-1)(1-\rho)}$. Because (A6) indicates that $\frac{d\phi_R}{di_N} > 0$, then (B3) can be satisfied if i_N and a_R are sufficiently large. Therefore, $\frac{dE_N}{di_N} > 0$ if i_N and a_R are sufficiently large.

Using (39), we obtain:

$$\frac{dU_N(0)}{di_N} = \frac{1}{\rho}\left[\frac{1}{E_N}\left(\frac{dE_N}{di_N}\right) + \frac{log\lambda}{\rho}\left(\frac{d\phi_R}{di_N}\right)\right].$$

Therefore, $\frac{dU_N(0)}{di_N} > 0$ if $\frac{dE_N}{di_N} > 0$ - that is, $\frac{dU_N(0)}{di_N} > 0$ if i_N and a_R are sufficiently large.

Since $E_S = \frac{E-E_N L_N}{L_S}$, we have:

$$\frac{dE_S}{di_N} = \frac{1}{L_S}\underbrace{\left(\underbrace{\left(\frac{dE}{di_N}\right)}_{+} - L_N \underbrace{\left(\frac{dE_N}{di_N}\right)}_{+}\right)}_{+/-}. \tag{B4}$$

Because $\frac{dE}{di_N} > 0$, equation (B4) indicates that we are not able to determine the sign of $\frac{dE_S}{di_N}$ when $\frac{dE_N}{di_N} > 0$. Finally, we have:

$$\frac{dU_S(0)}{di_N} = \frac{1}{\rho}\left[\frac{1}{E_S}\left(\frac{dE_S}{di_N}\right) + \frac{log\lambda}{\rho}\left(\frac{d\phi_R}{di_N}\right)\right].$$

Because we are not able to determine the sign of $\frac{dE_S}{di_N}$, then the sign of $\frac{dU_S(0)}{di_N}$ is ambiguous.

□

APPENDIX C

Proof of Proposition 3

In this case, we have $\mu_F = 1$ and $\mu_R = \mu_S = 0$. Equation (27) becomes:

$$E(\phi_R) = \frac{\lambda a_S(\rho + \phi_R)}{\lambda - 1}, \tag{C1}$$

with $\frac{\partial E}{\partial \phi_R} = \frac{E}{\rho+\phi_R} > 0$. Equation (29) becomes:

$$w_N = \frac{\lambda a_S}{a_S + a_R(\lambda - 1)}. \tag{C2}$$

In this case, we have $\theta = \theta(i_S) = (1 + i_S)a_F + \frac{\lambda a_S a_R}{a_S+a_R(\lambda-1)}$ with $\frac{\partial \theta}{\partial i_S} = a_F$. From (31), we obtain:

$$\phi_S(\phi_R; i_S) = \left[\frac{(\lambda - \xi)\, a_S - (\lambda - 1)\theta(i_S)}{(\lambda - 1)\theta(i_S)}\right](\rho + \phi_R), \tag{C3}$$

with $\frac{\partial \phi_S}{\partial \phi_R} = \frac{\phi_S}{\rho+\phi_R} > 0$ and $\frac{\partial \phi_S}{\partial i_S} = -\frac{(\lambda-\xi)a_S a_F(\rho+\phi_R)}{(\lambda-1)\theta^2} < 0$.

Equation (32) becomes:

$$n_N(\phi_R) = \frac{\lambda}{E(\phi_R)}(L_N - a_R\phi_R), \tag{C4}$$

where $\frac{\partial n_N}{\partial \phi_R} = -\frac{n_N}{E}\left(\frac{\partial E}{\partial \phi_R}\right) - \frac{\lambda a_R}{E}$ <0.

Totally differentiating (34) and (37) with respect to ϕ_R, n_F, and i_S yields:

$$\underbrace{\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}}_{B}\begin{bmatrix} d\phi_R \\ dn_F \end{bmatrix} = -\begin{bmatrix} e_3 \\ e_4 \end{bmatrix} di_S, \tag{C5}$$

where $e_3 = n_F(a_S + a_F)\left(\frac{\partial\phi_S}{\partial i_S}\right) < 0$ and $e_4 = n_F\left(\frac{\partial\phi_S}{\partial i_S}\right) < 0$. Note that the matrix B is the same as the one in (A5) and the determinant of B is negative.

The effects of changes in i_S on ϕ_R and n_F are:

$$\frac{d\phi_R}{di_S} = -\frac{b_4 e_3 - b_2 e_4}{|B|}, \tag{C6}$$

$$\frac{dn_F}{di_S} = -\frac{b_1 e_4 - b_3 e_3}{|B|}. \tag{C7}$$

With a few steps of calculation, we obtain that $(b_4 e_3 - b_2 e_4) = -a_S \phi_R n_F\left(\frac{\partial\phi_S}{\partial i_S}\right) > 0$, indicating that $\frac{d\phi_R}{di_S} > 0$. With a few steps of calculation, we derive that:

$$(b_1 e_4 - b_3 e_3) = -n_F\left(\frac{\partial\phi_S}{\partial i_S}\right)\left\{(a_S + a_F)\left[n_S - \phi_R\left(\frac{\partial n_N}{\partial\phi_R}\right)\right] + a_F n_F + \frac{1}{\lambda}\left(\frac{\partial E}{\partial\phi_R}\right) + a_R\right\} > 0.$$

From (C7), we obtain $\frac{dn_F}{di_N} > 0$.

Note that as indicated by (45), w_N is constant and is not affected by i_S. From (C1) and (C4), we have:

$$\frac{dE}{di_S} = \underbrace{\frac{dE}{d\phi_R}}_{+}\underbrace{\frac{d\phi_R}{di_S}}_{+} > 0,$$

$$\frac{dn_N}{di_S} = \underbrace{\frac{dn_N}{d\phi_R}}_{-}\underbrace{\frac{d\phi_R}{di_S}}_{+} < 0.$$

Using (C6) and (C7), we can derive:

$$\begin{aligned}\frac{dn_N}{di_S} + \frac{dn_F}{di_S} &= -\left(\frac{\partial n_N}{\partial\phi_R}\frac{d\phi_R}{di_S} + \frac{dn_F}{di_S}\right)\\ &= \frac{n_F}{|B|}\left(\frac{\partial\phi_S}{\partial i_S}\right)\left\{a_S n_S + a_F\left[n_S - \phi_R\left(\frac{\partial n_N}{\partial\phi_R}\right)\right] + a_F n_F + \frac{1}{\lambda}\left(\frac{\partial E}{\partial\phi_R}\right)\right\} > 0.\end{aligned}$$

Then (36) indicates that

$$\frac{dn_S}{di_S} = -\left(\frac{dn_N}{di_S} + \frac{dn_F}{di_S}\right) < 0. \tag{C8}$$

From (40) we obtain:

$$\frac{d\phi_S}{di_S} = \underbrace{\underbrace{\frac{\partial\phi_S}{\partial\phi_R}}_{+}\underbrace{\frac{d\phi_R}{di_S}}_{+}}_{+} + \underbrace{\frac{\partial\phi_S}{\partial i_S}}_{-}. \tag{C9}$$

Substituting (C6) into (C9) yields:

$$\frac{d\phi_S}{di_S} = -\frac{1}{|B|}\left(\frac{\partial\phi_S}{\partial i_S}\right)\left\{b_4\left[a_F n_F + \frac{1}{\lambda}\left(\frac{\partial E}{\partial\phi_R}\right) + a_R\right] + b_2\left[n_S - \phi_R\left(\frac{\partial n_N}{\partial\phi_R}\right)\right]\right\} < 0.$$

In order to examine the change in FDI intensity, we use (34) to derive:

$$\frac{d\phi_F}{di_S} = \underbrace{\left(\underbrace{\frac{\partial\phi_F}{\partial\phi_R}}_{+} + \underbrace{\frac{\partial\phi_F}{\partial n_N}}_{-}\underbrace{\frac{dn_N}{d\phi_R}}_{-}\right)}_{+}\underbrace{\left(\frac{d\phi_R}{di_S}\right)}_{+} + \underbrace{\left(\underbrace{\frac{\partial\phi_F}{\partial\phi_S}}_{+}\underbrace{\frac{d\phi_S}{di_S}}_{-}\right)}_{-} + \underbrace{\left(\underbrace{\frac{\partial\phi_F}{\partial n_F}}_{+}\underbrace{\frac{dn_F}{di_S}}_{-}\right)}_{-}. \tag{C10}$$

Therefore, the sign of $\frac{d\phi_F}{di_N} < 0$ is ambiguous.

□

APPENDIX D

Proof of Proposition 4

From (41), we have $E_N = \left[\frac{a_R\rho(1-n_S)}{L_N} + 1\right]w_N$. Equation (C2) indicates that w_N is constant. We thus have:

$$\frac{dE_N}{di_S} = -\frac{\rho a_R w_N}{L_N}\left(\frac{dn_S}{di_N}\right) > 0. \tag{D1}$$

Using (39), (D1), and (C8) we obtain:

$$\frac{dU_N(0)}{di_S} = \frac{1}{\rho}\left[\frac{1}{E_N}\left(\frac{dE_N}{di_S}\right) + \frac{log\lambda}{\rho}\left(\frac{d\phi_R}{di_S}\right)\right] > 0. \tag{D2}$$

Since $E_S = \frac{E - E_N L_N}{L_S}$, we have:

$$\frac{dE_S}{di_S} = \frac{1}{L_S}\underbrace{\left(\underbrace{\left(\frac{dE}{di_S}\right)}_{+} - L_N\underbrace{\left(\frac{dE_N}{di_S}\right)}_{+}\right)}_{+/-}.$$

Finally, we have:

$$\frac{dU_S(0)}{di_S} = \frac{1}{\rho}\left[\frac{1}{E_S}\left(\frac{dE_S}{di_S}\right) + \frac{log\lambda}{\rho}\left(\frac{d\phi_R}{di_S}\right)\right].$$

Because we are not able to determine the sign of $\frac{dE_S}{di_S}$, then the sign of $\frac{dU_S(0)}{di_S}$ is ambiguous.

□

APPENDIX E

Proof of Proposition 5

This case corresponds to the parameter values with $\mu_S = 1$ and $\mu_R = \mu_F = 0$. From (27), global expenditure becomes:

$$E(\phi_R; i_S) = \frac{\lambda a_S(\rho + \phi_R)(1 + i_S)}{\lambda - 1}, \tag{E1}$$

with $\frac{\partial E}{\partial \phi_R} = \frac{E}{\rho + \phi_R} > 0$ and $\frac{\partial E}{\partial i_S} = \frac{E}{1 + i_S} > 0$.

Equation (29) becomes:

$$w_N(i_S) = \frac{\lambda a_S(1 + i_S)}{a_S(1 + i_S) + a_R(\lambda - 1)}, \tag{E2}$$

with $\frac{dw_N}{di_S} = \frac{\lambda a_S a_R(\lambda - 1)}{[a_S(1+i_S) + a_R(\lambda - 1)]^2} > 0$. Sine $w_N > 1$, then (E2) implies $a_S(1 + i_S) > a_R$.

In this case, we have $\theta = \theta(i_S) = \frac{\lambda a_S a_R}{a_S(1+i_S) + a_R(\lambda - 1)} + \frac{a_F}{1+i_S}$ with $\frac{\partial \theta}{\partial i_S} = -\frac{\lambda a_S^2 a_R}{[a_S(1+i_S) + a_R(\lambda - 1)]^2} - \frac{a_F}{(1+i_S)^2} < 0$. Under the assumption that $a_S(1 + i_S) > a_R$, we have:

$$\theta(i_S) < \frac{\lambda a_S a_R}{a_R + a_R(\lambda - 1)} + \frac{a_F a_S}{a_R} = \frac{a_S(a_R + a_F)}{a_R}.$$

From (31), we obtain:

$$\phi_S(\phi_R; i_S) = \left[\frac{(\lambda - \xi)\, a_S - (\lambda - 1)\theta(i_S)}{(\lambda - 1)\theta(i_S)}\right](\rho + \phi_R), \tag{E3}$$

with $\frac{\partial \phi_S}{\partial \phi_R} = \frac{\phi_S}{\rho + \phi_R} > 0$ and $\frac{\partial \phi_S}{\partial i_S} = -\frac{(\lambda - \xi) a_S(\rho + \phi_R)}{(\lambda - 1)\theta^2}\left(\frac{\partial \theta}{\partial i_S}\right) > 0$.

Equation (32) becomes:

$$n_N(\phi_R; i_S) = \frac{\lambda}{E(\phi_R; i_S)}(L_N - a_R\phi_R), \tag{E4}$$

with $\frac{\partial n_N}{\partial \phi_R} = -\frac{n_N}{E}\left(\frac{\partial E}{\partial \phi_R}\right) - \frac{\lambda a_R}{E} < 0$ and $\frac{\partial n_N}{\partial i_S} = -\frac{n_N}{E}\left(\frac{\partial E}{\partial i_S}\right) < 0$.

To examine the effects of an increase in i_S, we totally differentiate (34) and (37) with respect to ϕ_R, n_F, and i_S to obtain:

$$\underbrace{\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}}_{B} \begin{bmatrix} d\phi_R \\ dn_F \end{bmatrix} = -\begin{bmatrix} e_5 \\ e_6 \end{bmatrix} di_S \,, \tag{E5}$$

where $e_5 = n_F(a_S + a_F)\left(\frac{\partial \phi_S}{\partial i_S}\right) + \frac{1}{\lambda}\left(\frac{\partial E}{\partial i_S}\right) > 0$ and $e_6 = n_F\left(\frac{\partial \phi_S}{\partial i_S}\right) + \phi_R\left(\frac{\partial n_N}{\partial i_S}\right)$. Note that the matrix B is the same as the one in (A5) and the determinant of B is negative. The effects of changes in i_S on ϕ_R and n_F are:

$$\frac{d\phi_R}{di_S} = -\frac{b_4 e_5 - b_2 e_6}{|B|}, \tag{E6}$$

$$\frac{dn_F}{di_S} = -\frac{b_1 e_6 - b_3 e_5}{|B|}. \tag{E7}$$

With a few steps of calculation, we can derive that:

$$\begin{aligned} & b_4 e_5 - b_2 e_6 \\ & = -a_S \phi_R n_F \left(\frac{\partial \phi_S}{\partial i_S}\right) + \phi_R \left(\frac{\partial n_N}{\partial i_S}\right)[(a_S + a_F)\phi_S + a_F \phi_R] - \frac{1}{\lambda}\left(\frac{\partial E}{\partial i_S}\right)(\phi_R + \phi_S) < 0. \end{aligned}$$

Since $|B| < 0$, then (E6) indicates that $\frac{d\phi_R}{di_S} < 0$.

In order to determine the sign of $\frac{dn_F}{di_S}$, we first prove the following Lemma.

Lemma 1. $\frac{\partial \phi_S}{\partial i_S} > \frac{\phi_S}{1+i_S}$ if ξ is sufficiently large.

Since $w_N > 1$, we have $\frac{1}{a_S(1+i_S)+a_R(\lambda-1)} > \frac{1}{\lambda a_S(1+i_S)}$. Thus, we have:

$$\begin{aligned} \frac{\partial \phi_S}{\partial i_S} &= -\frac{(\lambda-\xi)a_S(\rho+\phi_R)}{(\lambda-1)\theta^2}\left(\frac{\partial \theta}{\partial i_S}\right) \\ &= -\frac{(\lambda-\xi)a_S\phi_S}{\theta[(\lambda-\xi)a_S-(\lambda-1)\theta]}\left(\frac{\partial\, \theta}{\partial i_S}\right) \\ &= \frac{(\lambda-\xi)a_S\phi_S}{\theta[(\lambda-\xi)a_S-(\lambda-1)\theta]}\left\{\frac{\lambda a_S^2 a_R}{[a_S(1+i_S)+a_R(\lambda-1)]^2} + \frac{a_F}{(1+i_S)^2}\right\} \\ &> \frac{(\lambda-\xi)a_S\phi_S}{\theta[(\lambda-\xi)a_S-(\lambda-1)\theta]}\frac{\lambda a_S^2 a_R}{[a_S(1+i_S)+a_R(\lambda-1)]^2} \\ &> \frac{(\lambda-\xi)a_S\phi_S}{\theta[(\lambda-\xi)a_S-(\lambda-1)\theta]}\frac{\lambda a_S^2 a_R}{[\lambda a_S(1+i_S)]^2} \\ &= \frac{(\lambda-\xi)a_S\phi_S}{\theta[(\lambda-\xi)a_S-(\lambda-1)\theta]}\frac{a_S a_R}{\lambda(1+i_S)^2}. \end{aligned}$$

Then $\frac{\partial \phi_S}{\partial i_S} > \frac{\phi_S}{1+i_S}$ if:

$$(\lambda-\xi)\, a_S a_R > \theta[(\lambda-\xi)a_S-(\lambda-1)\theta]\lambda(1+i_S). \tag{E8}$$

Since $\theta < \frac{a_S(a_R+a_F)}{a_R}$, then the inequality of (E8) will hold if:

$$(\lambda - \xi)\, a_S a_R > \frac{a_S(a_R + a_F)}{a_R}[(\lambda - \xi)a_S - (\lambda - 1)\theta]\lambda(1 + i_S).$$

That is,

$$\begin{aligned}&(\lambda - \xi)\, a_S a_R \\ &> \frac{a_S(a_R + a_F)}{a_R}\left\{(\lambda - \xi)a_S - (\lambda - 1)\left[\frac{\lambda a_S a_R}{a_S(1 + i_S) + a_R(\lambda - 1)} + \frac{a_F}{1 + i_S}\right]\right\}\lambda(1 + i_S).\end{aligned} \tag{E9}$$

The inequality of (E9) will hold if:

$$(\lambda - \xi)\, a_R > \frac{(a_R + a_F)a_S}{a_R}\left[\lambda - \xi - (\lambda - 1)\frac{\lambda a_R}{a_S(1 + i_S) + a_R(\lambda - 1)}\right]\lambda(1 + i_S).$$

That is,

$$a_R^2 > (a_R + a_F)a_S\left[1 - \frac{\lambda - 1}{\lambda - \xi}\frac{\lambda a_R}{a_S(1 + i_S) + a_R(\lambda - 1)}\right]\lambda(1 + i_S). \tag{E10}$$

The inequality of (E10) will hold if ξ is sufficiently large.

□

We now go back to consider the sign of $\frac{dn_F}{di_S}$. Note that:

$$\begin{aligned}&b_1 e_6 - b_3 e_5 \\ &= n_F(a_S + a_F)\left\{-n_S\left(\frac{\partial\phi_S}{\partial i_S}\right) + \phi_R\left[\left(\frac{\partial\phi_S}{\partial i_S}\right)\left(\frac{\partial n_N}{\partial\phi_R}\right) - \left(\frac{\partial\phi_S}{\partial\phi_R}\right)\left(\frac{\partial n_N}{\partial i_S}\right)\right]\right\} \\ &- \frac{\rho n_S}{\lambda(\rho + \phi_R)}\left(\frac{\partial E}{\partial i_S}\right) - a_F n_F\left[\left(\frac{\partial\phi_S}{\partial i_S}\right)n_F + \left(\frac{\partial n_N}{\partial i_S}\right)\phi_R\right] - \frac{n_F}{\lambda}\left(\frac{\partial E}{\partial\phi_R}\right)\left(\frac{\partial\phi_S}{\partial i_S}\right) \\ &- a_R\left[\left(\frac{\partial\phi_S}{\partial i_S}\right)n_F + \frac{\phi_R(1 - n_N)}{1 + i_S}\right].\end{aligned}$$

Because $\frac{\partial n_N}{\partial\phi_R} < 0$, then under the assumption of $\frac{\partial\phi_S}{\partial i_S} > \frac{\phi_S}{1+i_S}$, we have:

$$\begin{aligned}\left(\frac{\partial\phi_S}{\partial i_S}\right)\left(\frac{\partial n_N}{\partial\phi_R}\right) - \left(\frac{\partial\phi_S}{\partial\phi_R}\right)\left(\frac{\partial n_N}{\partial i_S}\right) &< -\frac{\phi_S}{1 + i_S}\left(\frac{n_N}{\rho + \phi_R} + \frac{\lambda a_R}{E}\right) + \frac{\phi_S n_N}{(\rho + \phi_R)(1 + i_S)} \\ &= -\frac{\phi_S\lambda a_R}{(1 + i_S)E} < 0.\end{aligned}$$

Since $\frac{\partial E}{\partial\phi_R} > 0$, $\frac{\partial E}{\partial i_S} > 0$, $\frac{\partial\phi_S}{\partial i_S} > 0$, and $\frac{\partial\phi_S}{\partial i_S} > \frac{\phi_S}{1+i_S}$, then we have:

$$\begin{aligned}b_1 e_6 - b_3 e_5 &< -a_F n_F\left[\left(\frac{\partial\phi_S}{\partial i_S}\right)n_F + \left(\frac{\partial n_N}{\partial i_S}\right)\phi_R\right] - \frac{a_R\phi_R(1 - n_N)}{1 + i_S} \\ &< -a_F n_F\left(\frac{\phi_S n_F}{1 + i_S} - \frac{\phi_R n_N}{1 + i_S}\right) - \frac{a_R\phi_R(1 - n_N)}{1 + i_S}\end{aligned}$$

$$= -\frac{a_F n_F(\ \phi_S n_F - \phi_R n_N) + a_R \phi_R (n_S + n_F)}{1 + i_S}$$

$$< -\frac{a_F n_F(\ \phi_S n_F - \phi_R n_N) + a_R \phi_R n_F}{1 + i_S}.$$

Using (18), we have:

$$b_1 e_6 - b_3 e_5 < -\frac{a_F n_F(\phi_R n_S - \phi_R n_N) + a_R \phi_R n_F}{1 + i_S}$$

$$= -\frac{n_F \phi_R [a_F(n_S - n_N) + a_R]}{1 + i_S}$$

$$= \frac{n_F \phi_R [a_F(n_N - n_S) - a_R]}{1 + i_S}.$$

Because $n_N - n_S < 1$ and $0 < \phi_S < 1$, we obtain:

$$b_1 e_6 - b_3 e_5 < \frac{n_F \phi_R (a_F - a_R)}{1 + i_S}.$$

Thus, $b_1 e_6 - b_3 e_5 < 0$ if $a_F < a_R$. Since $|B| < 0$, we have $\frac{dn_F}{di_S} < 0$ if $a_R > a_F$ and $\frac{\partial \phi_S}{\partial i_S} > \frac{\phi_S}{1+i_S}$.

Equation (E2) indicates that $\frac{dw_N}{di_S} = \frac{\lambda a_S a_R (\lambda - 1)(1 + i_N)}{[a_S(1+i_S) + a_R(\lambda-1)(1+i_N)]^2} > 0$, implying that an increase in i_S will raise w_N. To examine how changes in i_S affect the rate of imitation, we totally differentiate (E3) with respect to i_S and have:

$$\frac{d\phi_S}{di_S} = \underbrace{\underbrace{\frac{\partial \phi_S}{\partial \phi_R}}_{+} \underbrace{\frac{d\phi_R}{di_S}}_{-}}_{-} + \underbrace{\frac{\partial \phi_S}{\partial i_S}}_{+}. \tag{E11}$$

Substituting (E6) into (E11) and using $\frac{\partial \phi_S}{\partial i_S} > \frac{\phi_S}{1+i_S}$, then we obtain:

$$\frac{d\phi_S}{di_S} > -\frac{1}{|B|} \frac{\phi_S}{1 + i_S} \{(\phi_R + \phi_S)(a_F n_F + a_R) + [(a_S + a_F)\phi_S + a_F \phi_R] n_S\} > 0.$$

This indicates that the rate of imitation will increase with a rise in the Southern nominal interest rate.

From (E1) and (E4), we have:

$$\frac{dE}{di_S} = \underbrace{\underbrace{\frac{\partial E}{\partial \phi_R}}_{+} \underbrace{\frac{d\phi_R}{di_S}}_{-} + \underbrace{\frac{\partial E}{\partial i_S}}_{+}}_{+/-},$$

$$\frac{dn_N}{di_S} = \underbrace{\underbrace{\frac{\partial n_N}{\partial \phi_R}}_{-} \underbrace{\frac{d\phi_R}{di_N}}_{-} + \underbrace{\frac{\partial n_N}{\partial i_S}}_{-}}_{+/-} .$$

Therefore, an increase in i_S causes ambiguous effects on the global expenditure and the extent of Northern production, leading to an ambiguous effect on the extent of Southern production.

From (34), we derive that:

$$\frac{d\phi_F}{di_S} = \underbrace{\left(\underbrace{\frac{\partial \phi_F}{\partial \phi_R}}_{+} + \underbrace{\frac{\partial \phi_F}{\partial \phi_S}}_{+} \underbrace{\frac{\partial \phi_S}{\partial \phi_R}}_{+} + \underbrace{\frac{\partial \phi_F}{\partial n_N}}_{-} \underbrace{\frac{dn_N}{d\phi_R}}_{?} \right)}_{+/-} \underbrace{\left(\frac{d\phi_R}{di_S} \right)}_{-} + \underbrace{\frac{\partial \phi_F}{\partial n_F}}_{+} \underbrace{\frac{dn_F}{di_S}}_{-} .$$

Since the sign of $\frac{dn_N}{di_S}$ is ambiguous, then we are not able to determine the sign of $\frac{d\phi_F}{di_S}$.

□