Price Indexes are a Problem for Testing PPP

Wallace, Frederick

Gulf University for Science and Technology

27 July 2015

Price Indexes are a Problem for Testing PPP

by

Frederick H. Wallace
Department of Economics and Finance
Gulf University for Science and Technology
West Mishref, Kuwait
wallace.f@gust.edu.kw

Abstract:
This note illustrates a problem in purchasing power parity studies that test for stationarity of the real exchange rate. If the real rate series is constructed using price indexes then the real exchange rate may not be stationary even if the law of one price always holds for every good in the indexes.

Forming a real exchange rate series using price indexes and the nominal exchange rate then testing for stationarity of the real rate using unit root test is a common approach in empirical studies of purchasing power parity. If the real exchange rate is stationary, that is reverts to a constant mean, then researchers assume this constant mean is the real exchange rate when PPP holds. Taylor (2002) is probably the most cited study of this type. The purpose of this note is to show that the using price indexes to construct the real exchange rate means that a change in a single price in the index may alter the real exchange rate even if the law of one price always holds. Stated differently, purchasing power parity may hold even if the real exchange rate is nonstationary.

Consider purchasing power parity when price indexes are used. For definiteness, Mexico, denoted by the M superscript, and the United States are the two countries. Suppose the price indexes for the two countries contain the prices of n goods and services all of which are traded. The time t nominal exchange rate is E_t is the U.S. dollar price of a Mexican peso. Further suppose the law of one price (LOOP) always holds at time t for each good i in the indexes so that $E_t P_{it}^M = P_{it}$, that is the price of good i at time t denominated in dollars is the same in both countries. Since LOOP holds for each good in the index, PPP holds as well. Following standard practice in empirical studies, when PPP holds the two countries price indexes must be related as in equation (1) where the π_{it}^M, π_{it} are possibly different, fixed weights of each good in each country's index.[1]

$$\sum_{i=1}^{n} \pi_i^M E_t P_{it}^M = A \sum_{i=1}^{n} \pi_i P_{it} \tag{1}$$

[1] Empirical studies usually use historical price index data. Although details of construction of these indexes are often unavailable, one suspects that most are either Paasche or Laspeyres. The argument in this note applies to indexes constructed by either weighting system.

The weights sum to one in the indexes for both countries. A is the real exchange rate. Forming the real exchange rate from equation (1) gives equation (2). If the weights in the index are the same in both countries and LOOP holds then the sums in the denominator and the numerator will be identical and A = 1. If the weights are different, the value of A will depend on the weights in the indexes.

$$\frac{\sum_{i=1}^{n} \pi_i^M E_t P_{it}^M}{\sum_{i=1}^{n} \pi_i P_{it}} = A \qquad (2)$$

Now suppose there is an initial small change in the price of good 1 in the indexes but the law of one price continues to hold so that $E_t P_{1t}^M = P_{1t}$. The effect of this change on A can be found by totally differentiating the real exchange rate expression holding constant the weights in the indexes. Relative prices may adjust causing some or all of the other price components of the index to change. Assume that LOOP is maintained for all prices.

$$\frac{[\sum_{i=1}^{n} \pi_i^M d(E_t P_{it}^M)](\sum_{i=1}^{n} \pi_i P_{it}) - (\sum_{i=1}^{n} \pi_i^M E_t P_{it}^M)(\sum_{i=1}^{n} \pi_i dP_{it})}{(\sum_{i=1}^{n} \pi_i P_{it})^2} = dA$$

Using the definition of A and canceling some terms gives equation (3).

$$\frac{[\sum_{i=1}^{n} \pi_i^M d(E_t P_{it}^M)] - A(\sum_{i=1}^{n} \pi_i dP_{it})}{(\sum_{i=1}^{n} \pi_i P_{it})} = dA \qquad (3)$$

The sign of dA will depend on the numerator, the denominator is obviously positive. Assuming LOOP is constantly maintained then $d(E_t P_{it}^M) = dP_{it}$ for every good i giving

$$\frac{[\sum_{i=1}^{n} (\pi_i^M - A\pi_i) dP_{it}]}{(\sum_{i=1}^{n} \pi_i P_{it})} = dA \qquad (4)$$

As previously noted, if LOOP holds for all goods and each good i has the same weight in both price indexes, that is $\pi_i^M = \pi_i \ \forall \ i$, then A = 1 and all terms in parentheses in the numerator are zero so dA = 0, the real rate is stationary. Otherwise the real exchange rate is likely to change suggesting that testing for PPP by looking for mean reversion of the real rate may fail even if LOOP always holds. Alternatively, studies like Gómez et al. that find structural breaks in the real rate and reversion to a changing mean may be capturing the effects of price changes causing the real rate to shift.

So, what is happening? For simplicity imagine that only the price of good 1 changes in both countries maintaining the LOOP but the weights are different. In this case the previous expression reduces to $\frac{(\pi_1^M - A\pi_1)dP_{1t}}{(\sum_{i=1}^{n} \pi_i P_{it})} = dA$. In this case $dA \neq 0$ unless, coincidentally, $\pi_1^M = A\pi_1$. Note that $\pi_i^M = A\pi_i$ cannot hold for all i unless the weights in both countries are the same for each good. There must be some goods for which $\pi_i^M \neq A\pi_i$. To understand suppose that there is a good i such that $\pi_i^M > \pi_i$ and $\pi_i^M = A\pi_i$, thus A > 1. But if $\pi_i^M > \pi_i$ there must be at least one other good j in the indexes for which $\pi_j^M < \pi_j$, because the weights in each index sum to one, so that $\pi_j^M < A\pi_j$. Thus, in at least some cases a change in the price of a good will alter the real exchange rate constructed using price indexes even if LOOP always holds.

Summarizing, the behavior of the real exchange rate will depend on how the prices of goods and services comprising the index change and on the weights assigned to those prices. The real exchange rate will revert to a constant mean only in the exceptional situations.

References

Gómez-Zaldivar, M.; Ventosa-Santàularia, D. and Wallace, F.H. (2013) The PPP hypothesis and structural breaks: The case of Mexico, *Empirical Economics* 45, 1351-1359.

Taylor, A. (2002) A century of purchasing power parity, *Review of Economics and Statistics* 84, 139-150.