Process innovations, patent litigation and time effects

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PROCESS INNOVATIONS, PATENT LITIGATION AND TIME EFFECTS

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Abstract. In this work we extend the model developed in (Aoki and Hu, 2003) in order to cover cost reduction innovations, instead of product innovations originally developed on that article. The results show that smaller innovations are more licensable. Regarding the time factors, infringers like faster innovation and patentees prefer bigger innovations and longer imitation periods. Under some suitable situations, litigation time could support innovation and discourage infringement. However the patent life has ambiguous effects and may promote infringement.

1. Introduction

Patents are, by their own right, an interesting topic of study, in part because there is no clear conclusion about the balance between the positive effects (promote innovations) and the negative ones (market power for example) of having a patent system. However, there are other points to take into account since nowadays, some of them relate to the complexity of patent rights. Such complexities are derived from the actual development of science and the efficiency of the legal system to determine whether an invention is in fact a nontrivial improvement of knowledge.

The other dimension is related with the complexity of the actual technology, i.e. a cell phone needs more than one hundred different patented technologies. So in many cases a developer of a product faces several patent holders in order to develop a final product, and several of those technologies could complement one of the others (fragmented patents).

The term probabilistic patents has its origins on the possibility that a patent can be declared invalid in a court. This happens because the control of the patent office is not absolute and sometimes that institution endorses patents to innovations that do not fulfill the requirements to be patented (most commonly inventive step). Even firms dealing with market competition could hold these weak patents (patent with

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high probability to be declared invalid under litigation)and license its competitors taking together with the other firms the market prize of the patent (Farrell and Shapiro, 2008).

There are several studies about patent litigation; the first ones study their relationship with settlements by using bargaining models. Inside these models they try several variations of consequences on the information of probabilistic patents. (Bebchuk, 1984; Meurer, 1989), in these earlier models the failure to reach agreements are mere consequence of information failures.

Another group of important studies come from Aoki and Hu (1999) that analyze the effect of the legal system on licensing and litigation, they characterize the legal system by the strength of patents and legal costs. They conclude that a legal system that induces a monopoly power incentive research, also they found that longer litigation is better for innovator and imitators (Aoki and Hu, 2003); those results were found for the case of product innovations.

Nevertheless, some analysts think that in the last step, the strength of a patent comes from a position of the responsible court that in some cases could be pro patent protection (Bessen and Meurer, 2005). This proposition has important consequences on markets and social welfare in the actual system of incentives to innovation and in all sectors and systems related with innovations. Many of these arguments and results of several years of empirical and theoretical research summarized in Bessen and Meurer (2008), show the importance of the study of the actual patent litigation policies and its deficiencies in some cases.

Consequently, there is a gap related to patent litigation and its relationship with innovation, licensing and settlement when the innovation is a process innovation. When an innovation is a process innovation there are several more difficulties, because in the case of product innovations markets with two competitors are duopolies with the same technology or just a monopoly. However in the case of process innovations, the competitors stay in the market even if they do not exploit the innovation and have an inferior technology.

In a market with two firms, firm 1 (patentee) and firm 2 (competitor, potential infringer or licensee) this work attempts to explore the effects of time factors in specific litigation and imitation times on the licensing, litigation and settlement of process innovations. In this way we use the base model by Aoki and Hu (2003). However it is necessary to work on some specific duopoly and simple games, in order to explain what is going to happen in those scenarios that differ from those that product innovations produce. In the section 2 we solve some static duopoly games that help out to describe a model that models licensing, imitation, litigation and settlement of a cost reduction innovation in a one shot Cournot game. Afterwards in the section 3 we describe the main model, that is basically a game in extensive form. This model is solved by backward induction throughout the subsections 4.1 to 4.4. After then we use the results found in the previous sections and compare these results in the sense of social welfare (consumer surplus added to the profits of the firms). Finally we finish with concise conclusions.

2. Particular equilibria

It is important before considering our main model, to study some simple models which are later going to be used in order to extend the model by Aoki and Hu (2003) and to include process innovations instead of product innovations. So in this section we solve three simple duopoly models with linear demands, the setup being as follows:

- There are two firms: firm 1 and firm 2
- firm 1 and firm 2, both produce a homogeneous good and face an inverse linear demand given by:

$$p = 1 - q_1 - q_2 \tag{2.1}$$

  - both firms compete in this markets choosing quantities (Cournout).
- Without loss of generality, we assume that the firms initially produce under a constant marginal cost of $c_i = 0$;
- The firm 1 obtains a cost reduction innovation of size $\epsilon \in (0, 1)$, in this way its marginal cost is now $c_1 = -\epsilon$
- Such cost reduction innovation is patented.
- In this way the profit function for the firm i is:

$$\pi_i(q_i, q_j) = (1 - q_i - q_j - c_i)q_i$$

  - where q_j represents the offered quantity by the other firm.

2.1. **Duopoly under same technology.**

At this case both firms produce goods with the same technology, and it is important to characterize a situation where an infringer uses the patented technology without any consequence. By solving the game, the equilibrium quantities are:

$$q_1^a = q_2^a = \frac{1+\epsilon}{3} \tag{2.2}$$

and the equilibrium profits are

$$\pi_1^a = \left(\frac{1+\epsilon}{3}\right)^2 \qquad \pi_2^a = \left(\frac{1+\epsilon}{3}\right)^2 \tag{2.3}$$

2.2. **Duopoly under different technologies.** In this case there is no licensing and firms just compete as Cournot with different costs, so firm 1 enjoys the innovation and firm 2 produces with the old technology. We are going to use this case when the potential infringer decides not to infringe and produce under the inferior technology, so by solving the game we have:

$$q_1^b = \frac{1+2\epsilon}{3} \qquad q_2^b = \frac{1-\epsilon}{3} \tag{2.4}$$

and the the equilibrium profits under this setup are easily calculated as

$$\pi_1^b = \left(\frac{1+2\epsilon}{3}\right)^2 \qquad \pi_2^b = \left(\frac{1-\epsilon}{3}\right)^2 \tag{2.5}$$

2.3. **Duopoly under the same technology and an expected reasonable royalty rate.** We should also consider the cases where there is infringement and a suit from the patentee, so the quantities chosen by the firms in the Cournot competition are made under the shadow of damage payments and injunctions. In this case both firms produce under the same technology, but in the case of firm 2, it infringes the patented technology and as consequence, firm 1 sues the other firm. If the patent is declared valid and infringed, then the infringer (firm 2) has to pay a reasonable royalty rate τ by each unit sold; the probability that the innovation will be declared valid is common knowledge $\theta \in (0,1)$

So in this case the payoff functions are:

$$\pi_1(q_1,q_2) = (1-q_1-q_2+\epsilon)q_1 + \theta\tau q_2 \tag{2.6}$$

$$\pi_2(q_1,q_2) = (1-q_1-q_2+\epsilon-\theta\tau)q_2 \tag{2.7}$$

where $\theta\tau q_2$ is the expected rent for the firm 1,so in this case the equilibrium quantities are:

$$q_1^c = \frac{1+\epsilon+\tau\theta}{3} \qquad q_2^c = \frac{1+\epsilon-2\tau\theta}{3} \tag{2.8}$$

and the equilibrium profits are

$$\pi_1^c = \left(\frac{1+\epsilon+\tau\theta}{3}\right)^2 + \theta\tau\frac{1+\epsilon-2\tau\theta}{3} \tag{2.9}$$

$$\pi_2^c = \left(\frac{1+\epsilon-2\tau\theta}{3}\right)^2 \tag{2.10}$$

We assume that $\tau \in (0,1)$ and in some cases we also are going to assume that $\tau \leq \epsilon$

3. Model

The main model is based in the model of (Aoki and Hu, 2003); that model studies the effect of time factors on the licensing of a product innovation. Here we will use the same structure in order to evaluate the impact of time factors on the imitation and litigation when the innovation is a process innovation[1].

The temporal setting and the description of the game is as follows:

(1) At the very beginning firm 1 gets a cost reduction innovation with a patent life of γ periods, A license is offered to firm 2 in the form of a "take or leave it" offer. This offer is a fixed fee F, firm 2 then has two options: accept the license an produce with the same technology or reject it. If firm 2 accepts the offer the game ends and both firms produce under the same technology, for γ periods .

(2) If there is no licensing firm 2 has to decide whether to imitate the technology or just stay with the old technology. If firm 2 decides not to imitate, then the game ends with a duopoly where the firms produce under different technologies, for γ periods.

(3) If firm 2 decided to imitate, imitation needs an investment of h and takes α periods. when the imitation is complete, firm 1 can litigate in order to stop infringement or just leave the things as they are. if the firm 1 decides not to litigate the game ends, with both firms producing under the same technology. So then during the period of imitation firms produce under different technologies and after the $\gamma - \alpha$ remaining periods, they produce with the same technology.

(4) If litigation is chosen by firm 1, the trial is going to end β periods after (where $\beta \leq \gamma - \alpha$). At the very beginning of the legal process, firm 1 can offer a settlement by a fixed fee

[1]In a difference of the model by Aoki and Hu (2003), our model is discrete, also we do not consider the effect of the temporal discount factor without loss of generality.

K, this offer again is a "take or leave it" offer. If the firm 2 accepts, both firms end producing under the same technology for the remain $\gamma - \alpha$ periods.

(5) If there is no settlement, both firms continue producing but under the shadow of a royalty rate τ that should be paid if the patent is declared valid and infringed. The probability that this happens is a common knowledge value $\theta \in (0, 1)$. Also, if the patent is declared valid and infringed the infringer firm 2 stops using the technology till the end of the life of patent, and produces under the old technology. Consequence of this legal process each firm pays a litigation cost by ℓ_1 and ℓ_2.

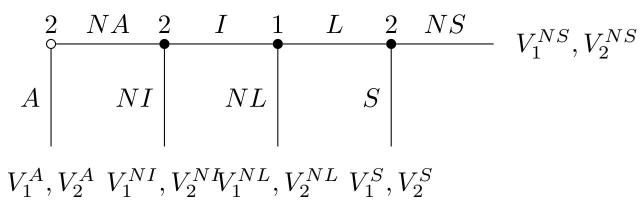

FIGURE 1. Game tree of the process

The figure 1 shows the structure of the game, unfortunately the real game tree should be a lot bigger the one that is shown, even so we will call the figure 1 as tree of the game, because this graph gives a good reference of the over all game.

In order to define the values for all cases, we are going to use our basic results obtained in the last section,and we characterize the values of payoffs for firm i under scenario k as V_i^k.

Now we proceed to characterize the payoffs, using the equilibrium profits of section 2, utilizing a finite repeated game[2]

(1) In the case that firm 2 accepts the license, it pays a fixed fee of F, and both firms produce under the same technology as a Duopoly during the patent life γ. We have the following payoffs:

$$V_1^A = \gamma \pi_1^a + F \tag{3.1}$$

$$V_2^A = \gamma \pi_2^a - F \tag{3.2}$$

(2) In the case where there is no licensing and where firm 2 decides not to imitate the innovation, we see that firm 1 produces with

[2]In a finite repeated game the equilibrium in each period is the Nash solution of the one shot game, so we just multiply the equilibrium profits on section 2 in order to calculate the payoffs

the new technology and firm 2 produces with the old technology for the γ periods. In consequence payoffs are:

$$V_1^{NI} = \gamma \pi_1^b \tag{3.3}$$

$$V_2^{NI} = \gamma \pi_2^b \tag{3.4}$$

(3) If firm 1 decides to imitate and makes it at a cost of h, after which if firm 1 (patent holder) decides not to litigate, both firms act as a duopoly with the same costs for the last $\gamma - \alpha$ periods, and in the first α periods (time required to imitate) firm 1 has lower costs than firm 2, so we have the following payoffs:

$$V_1^{NL} = \alpha \pi_1^b + (\gamma - \alpha)\pi_1^a \tag{3.5}$$

$$V_2^{NL} = \alpha \pi_2^b + (\gamma - \alpha)\pi_2^a - h \tag{3.6}$$

(4) In the case where firm 1 decides to litigate, firm 1 makes a take it or leave it offer by K. If a settlement is achieved firm 2 produces under the same costs as firm 1 for the remaining periods, and pays K to the firm 1, then:

$$V_1^S = \alpha \pi_1^b + (\gamma - \alpha)\pi_1^a + K \tag{3.7}$$

$$V_2^S = \alpha \pi_2^b + (\gamma - \alpha)\pi_2^a - h - K \tag{3.8}$$

(5) The most complex case emerges when there is no settlement after infringement and after both firms have been unable to reach a licensing accord. So after α periods, both firms produce under the shadow of expected cost and expected benefits of τq_2 for β periods. With the probability of θ that firm 1 will win, it is going to produce with lower costs than firm 2 for the remaining $\gamma - \alpha - \beta$ periods; with a probability of $(1-\theta)$ both will produce with the same costs for the remaining $\gamma - \alpha - \beta$ periods and in this case, both firms pay litigation costs by ℓ_i each period. So:

$$V_1^{NS} = \alpha \pi_1^b + \beta \pi_1^c + \theta(\gamma - \alpha - \beta)\pi_1^b + (1-\theta)(\gamma - \alpha - \beta)\pi_1^a - \beta \ell_1 \tag{3.9}$$

$$\begin{aligned} V_2^{NS} = &\alpha \pi_2^b + \beta \pi_2^c + \theta(\gamma - \alpha - \beta)\pi_2^b + (1-\theta)(\gamma - \alpha - \beta)\pi_2^a \\ &- \beta \ell_2 - h \end{aligned} \tag{3.10}$$

4. Equilibrium of the model

In this section we solve the model by backward induction. First we determinate the equilibrium solution for the fixed fee optimal for settlement, and then with these results, we can explore the optimal choices of litigation, imitation and licensing recursively.

4.1. **Settlement.** As previously described firm 1 makes a "take-it-or-leave-it" option of a fixed fee settlement. If we assume that this fixed fee exists that value should be enough to compensate the payoff that firms should receive in the case of no settlement[3].

The settlement conditions depends of the payoff of firm 2, by assuming a take it or leave it offer, $K^\star$ should be such that makes $V_2^{S\star} = V_2^{NS\star}$, so:

$$\begin{aligned}
K^\star &= -\beta\pi_2^c - \theta(\gamma-\alpha-\beta)\pi_2^b - (1-\theta)(\gamma-\alpha-\beta)\pi_2^a + \beta\ell_2 + (\gamma-\alpha)\pi_2^a \\
&= (\theta(\gamma-\alpha-\beta)+\beta)\pi_2^a - \theta(\gamma-\alpha-\beta)\pi_2^b - \beta\pi_2^c + \beta\ell_2 \\
&= \theta(\gamma-\alpha-\beta)(\pi_2^a-\pi_2^b) + \beta(\pi_2^a-\pi_2^c) + \beta\ell_2 \geq 0
\end{aligned} \tag{4.1}$$

Now because $\pi_2^a \geq \pi_2^b$ and $\pi_2^a \geq \pi_2^c$ (the better situation happens when firm 2 uses the innovation without paying any fee or royalty), so $K^\star \geq 0$, it implies that the settlement condition depends only on the payoff of firm 1. In this case we should note that the equilibrium payoffs should hold :

$$V_1^{S\star} \geq V_1^{NS\star}$$

- by developing this condition

$$\begin{aligned}
\alpha\pi_1^b + (\gamma-\alpha)\pi_1^a + K^\star &\geq \alpha\pi_1^b + \beta\pi_1^c + \theta(\gamma-\alpha-\beta)\pi_1^b + \\
&\quad (1-\theta)(\gamma-\alpha-\beta)\pi_1^a - \beta\ell_1 \\
K^\star &\geq \beta(\pi_1^c-\pi_1^a) + \theta(\gamma-\alpha-\beta)(\pi_1^b-\pi_1^a) - \beta\ell_1
\end{aligned}$$

by using the value of $K^\star$

$$\begin{aligned}
&\theta(\gamma-\alpha-\beta)(\pi_2^a-\pi_2^b) + \\
&\qquad \beta(\pi_2^a-\pi_2^c) + \beta\ell_2 \geq \beta(\pi_1^c-\pi_1^a) + \theta(\gamma-\alpha-\beta)(\pi_1^b-\pi_1^a) - \beta\ell_1 \\
&\qquad\qquad \beta(\ell_1+\ell_2) \geq \beta(\pi_1^c+\pi_2^c-\pi_1^a-\pi_2^a) + \\
&\qquad\qquad\qquad \theta(\gamma-\alpha-\beta)(\pi_1^b+\pi_2^b-\pi_1^a-\pi_2^a)
\end{aligned}$$

- in consequence, the settlement condition is:

$$\sum_i \ell_i \geq \left(\sum_i \pi_i^c - \sum_i \pi_i^a\right) + \theta\left(\frac{\gamma-\alpha}{\beta} - 1\right)\left(\sum_i \pi_i^b - \sum_i \pi_i^a\right) \tag{4.2}$$

Now we have:

[3] $\star$ symbol is used to denote the equilibrium solution.

$$\sum_i \pi_i^c - \sum_i \pi_i^a = \frac{\theta\tau}{9}(1+\epsilon-\theta\tau) \geq 0 \tag{4.3}$$

which is positive.

About the other term we have that

$$\sum_i \pi_i^b - \sum_i \pi_i^a = -\frac{\epsilon}{9}(2-3\epsilon) \tag{4.4}$$

when the effect of the innovation is lower than 2/3, the term is negative and is going to be positive where $\epsilon > 2/3$.

With such facts it is easy to get to know the effects of some variables as the patent life and others on the suitability of litigation using the eq. 4.2 , just some few comments are necessary for the case of θ and τ and in the other ones the result is direct.

Because

$$\frac{\partial \sum \pi_i^c}{\partial \theta} = \frac{\tau}{9}(1+\epsilon-2\theta\tau) \geq 0 \tag{4.5}$$

bigger patent strength makes less suitable settlement just for bigger innovations because the term $\sum_i \pi_i^b - \sum_i \pi_i^a$ in the equation 4.2 is just positive for $\epsilon > 2/3$.

About the royalty rate we have that

$$\frac{\partial \sum \pi_i^c}{\partial \tau} = \frac{\theta}{9}(1+\epsilon-2\theta\tau) \geq 0 \tag{4.6}$$

so more royalty rate is less suitable to have a settlement. These facts are summarized in the following proposition.

Proposition 4.1. *It is more suitable to have settlement when:*

1. *litigation costs are higher,*
2. *patent life γ is longer when $\epsilon < 2/3$ and shorter when $\epsilon > 2/3$*
3. *imitation time α is shorter when $\epsilon < 2/3$ and longer when $\epsilon > 2/3$*
4. *litigation time β is shorter when $\epsilon < 2/3$ and longer when $\epsilon > 2/3$*
5. *the patent strength θ is lower if $\epsilon > 2/3$*
6. *the reasonable rate τ is lower*

It is important to see the results of proposition 4.1, and the effects that change in relation to the size of the innovation. This is because when an innovation is substantial, the cumulated profit of both firms producing under the same technology is lower than the rent when they produce under different technologies. There is a scenario to have a settlement accord in order that the patentee has more bargaining power (because it makes the offer in the take it or leave scheme), the patentee will ask for a bigger share in order to settle, and these effects are amplified by the patent life and decreased by the imitation time and the litigation time. For the case of smaller innovations ($\epsilon < 2/3$), the effects are reversed.

Now when there is a settlement, firm 2 pays $K^\star$ to firm 1, this value is positive as we saw before, the value of this is:

$$K^\star = \theta(\gamma - \alpha - \beta)(\pi_2^a - \pi_2^b) + \beta(\pi_2^a - \pi_2^c) + \beta\ell_2 \tag{4.7}$$

By doing easy calculations we get:

$$\frac{\partial K^\star}{\partial \alpha} = -\theta(\pi_2^a - \pi_2^b) \leq 0$$

Because $(\pi_2^a - \pi_2^b) > 0$,this means that if the time for imitation is longer the settlement should be lower. This happens as a consequence that the firm has less time to explore the benefits of innovation along bigger profits, also

$$\frac{\partial K^\star}{\partial \gamma} = \theta(\pi_2^a - \pi_2^b) \geq 0$$

by the oppossing reason, so bigger patent lifetime makes the settlement fee bigger. Now, about the effect of litigation time it comes

$$\begin{aligned}\frac{\partial K^\star}{\partial \beta} &= -\theta(\pi_2^a - \pi_2^b) + (\pi_2^a - \pi_2^c) + \ell_2 \\ &= -\frac{4}{9}\theta\left(\epsilon - \tau - \epsilon\tau + \theta\tau^2\right) + \ell_2\end{aligned}$$

So by trying to characterize an expected royalty rate (or desired), it should be fair to assume that $\tau = \epsilon$, it ends as:

$$\frac{\partial K^\star}{\partial \beta} = \frac{4}{9}\epsilon^2(1-\theta)\theta + \ell_2 \geq 0 \quad \text{if } \tau = \epsilon \tag{4.8}$$

So when the reasonable royalty rate is fair [4] litigation time has a positive effect on the settlement fee.

As expected

$$\frac{\partial K^\star}{\partial \ell_2} = \beta \geq 0 \tag{4.9}$$

[4]see Farrell and Shapiro (2008) for a discussion of the ratio between ϵ and τ, given that in some cases it could be possible that $\tau > \epsilon$

, so bigger litigation costs increases the settlement fee.The effect of the strength of the patent θ on $K^\star$ is

$$\frac{\partial K^\star}{\partial \theta} = (\gamma - \alpha - \beta)(\pi_2^a - \pi_2^b) - \beta \frac{\partial \pi_2^c}{\partial \theta} \geq 0 \tag{4.10}$$

which is positive because $\frac{\partial \pi_2^c}{\partial \theta} \leq 0$, in that way innovations with more strength will be settled with greater settlement fees. Finally about the effect of the reasonable royalty rate we have that

$$\frac{\partial K^\star}{\partial \tau} = -\beta \frac{\partial \pi_2^c}{\partial \tau} \geq 0 \tag{4.11}$$

this result comes because $\frac{\partial \pi_2^c}{\partial \tau} \leq 0$.

In consequence we get the following proposition

Proposition 4.2. *$\frac{\partial K^\star}{\partial \alpha} \leq 0$; $\frac{\partial K^\star}{\partial \gamma} \geq 0$; $\frac{\partial K^\star}{\partial \beta} \geq 0$ when $\tau = \epsilon$; $\frac{\partial K^\star}{\partial \ell_2} \geq 0$; $\frac{\partial K^\star}{\partial \theta} \geq 0$; $\frac{\partial K^\star}{\partial \tau} \geq 0$*

This proposition reflects the fact that time factors amplify the impacts of rents difference under the different technologies, that are more or less in our setup captured by the patentee, in relation to the legal system (θ, τ, ℓ) these variables improve the bargaining power of the patentee and in consequence affect the settlement fee.

4.2. **Litigation.** By solving the game, we back another stage (see figure 1)to the choice of firm 1 to litigate or not to litigate. At this point we have to consider the two possible scenarios (Settlement and No Settlement). There is no Litigation if $V_1^{NL^*} < V_1^{S^*}$ when settlement takes place in the next stage, or $V_1^{NL^*} < V_1^{NS^*}$ when there is no settlement in the next stage. We consider the payoffs of firm 2 equal in equilibrium under both situations, it is understood the optimal fee is positive (see eq.(4.7)).

If there is settlement we have:

$$V_1^{S^*} - V_1^{NL^*} = K^\star > 0$$

So we have that

$$V_1^{S^*} > V_1^{NL^*} \tag{4.12}$$

Now if there is no settlement in the next stage, it happens because:

$$V_1^{NS^*} > V_1^{S^*}$$

By using the eq. 4.12 we have

$$V_1^{NS^*} > V_1^{NL^*} \tag{4.13}$$

This make us conclude that :

Proposition 4.3. *Litigation is always optimal for firm 1*

This result is quite important because it shows that independently the innovation's size and the patent strength is always optimal for the patentee to litigate when infringement happens.

4.3. **Imitation.** Imitation is going to take place if $V_2^{NI\star} < V_2^{S\star}$. This eventually happens independently of if there is or not settlement. This is because the settlement fixed fee is such that $V_2^{S\star} = V_2^{NS\star}$, so we can obtain the imitation condition as $V_2^{NI\star} < V_2^{S\star}$.

By developing this condition, we obtain

$$\begin{aligned} \alpha\pi_2^b + (\gamma-\alpha)\pi_2^a - h - K^\star > & \quad \gamma\pi_2^b \\ G = (\gamma-\alpha)(\pi_2^a - \pi_2^b) - h - K^\star > & \quad 0 \end{aligned}$$

It means that the imitation condition is $G > 0$. By deriving the time variables we have that:

$$\frac{\partial G}{\partial \alpha} = -(\pi_2^a - \pi_2^b) - \frac{\partial K^\star}{\partial \alpha} = -(1-\theta)(\pi_2^a - \pi_2^b) < 0 \tag{4.14}$$

$$\frac{\partial G}{\partial \gamma} = (\pi_2^a - \pi_2^b) - \frac{\partial K^\star}{\partial \gamma} = (1-\theta)(\pi_2^a - \pi_2^b) > 0 \tag{4.15}$$

$$\frac{\partial G}{\partial \beta} = -\frac{\partial K^\star}{\partial \beta} < 0 \quad \text{if } \tau = \epsilon \tag{4.16}$$

Essentially these effects represent the incentives to imitations because longer patent life, short imitation time and lower litigation time make the infringement premium greater for the potential infringer (firm 2), by summarizing:

Proposition 4.4. *There is more suitability to have imitation, when:*

1. *The imitation time α is lower;*
2. *The patent life γ is bigger;*
3. *The litigation time β is lower when $\tau = \epsilon$*

Also we can take derivatives in respect of the variables that represent the legal system, so:

$$\frac{\partial G}{\partial \theta} = -\frac{\partial K^\star}{\partial \theta} \leq 0 \tag{4.17}$$

$$\frac{\partial G}{\partial \tau} = -\frac{\partial K^\star}{\partial \tau} \leq 0 \tag{4.18}$$

$$\frac{\partial G}{\partial \ell_2} = -\frac{\partial K^\star}{\partial \ell_2} \leq 0 \tag{4.19}$$

If the legal variables have some direction to support the patentee system (higher patent strength, higher royalty rates, and high cost of litigation for the infringer). They reduce the feasibility of infringement, because these directly affect the settlement payment, making it greater and the premium of infringement is made lower, so:

Proposition 4.5. *There is less suitability to have imitation, when:*

(1) *The patent strength θ is higher;*
(2) *The reasonable royalty rate τ is higher;*
(3) *The litigation cost for the firm 2 ℓ_2 is higher.*

One aspect that is quite interesting, is to explore at this stage the impact of time effects on the payoffs of patentee and potential infringer.

By starting with the infringer we proceed to calculate the derivatives V_2^{S*} [5] with respect to the time variables α, β and γ. It is observed in figure 1 given that litigation is active, there are two potential scenarios: Settlement and No Settlement. It was discussed before if the equilibrium payoffs for firm 2 are equal, we can have the impacts of time variables on the incentives to infringe just deriving V_2^{S*} respect to the time variables, so:

$$\frac{\partial V_2^{S*}}{\partial \alpha} = -(\pi_2^a - \pi_2^b) - \frac{\partial K^\star}{\partial \alpha} = -(1-\theta)(\pi_2^a - \pi_2^b) < 0 \tag{4.20}$$

$$\frac{\partial V_2^{S*}}{\partial \beta} = -\frac{\partial K^*}{\partial \beta} \leq 0 \text{ if } \tau = \epsilon \tag{4.21}$$

$$\frac{\partial V_2^{S*}}{\partial \gamma} = \pi_2^a - \frac{\partial K^\star}{\partial \gamma} = \pi_2^a - \theta(\pi_2^a - \pi_2^b) = (1-\theta)\pi_2^a + \theta\pi_2^b > 0 \tag{4.22}$$

Because the impact of the imitation time is negative it is optimal for the infringer to imitate as fast as possible. One counterintuitive result

[5]We obtain this equilibrium payoff by replacing the equilibrium values of $K^\star$ in equation 3.7, we act is similar way for the other equilibrium payoffs.

is the impact of the patent life, the greater the incentive to infringe, because it is possible to get greater rents throughout settlement accord and enjoy the innovation together with the patentee, for a bigger period of time. However the impact of the litigation time is negative when $\tau = \epsilon$ (fair compensation), because longer periods of litigation reduce the premium of infringement along the litigation costs that eventually make the settlement fee greater, as is summarized in the following proposition.

Proposition 4.6. $\dfrac{\partial V_2^{S*}}{\partial \alpha} < 0$, $\dfrac{\partial V_2^{S*}}{\partial \beta} < 0$ *if* $\tau = \epsilon$, $\dfrac{\partial V_2^{S*}}{\partial \gamma} > 0$

By working in the same way as before, only this time for the patentee, we have to compare the two payoffs under both scenarios: 1) Settlement and 2) No Settlement.

Then we start deriving the payoff of the patentee in respect of the imitation time under the first scenario, as

$$\frac{\partial V_1^{S*}}{\partial \alpha} = \pi_1^b - \pi_1^a + \frac{\partial K^\star}{\partial \alpha} = \pi_1^b - \pi_1^a - \theta(\pi_2^a - \pi_2^b) = \frac{1}{9}\epsilon(2 + 3\epsilon - 4\theta)$$

This term in most of the cases is positive, and in particular it is positive if $\epsilon > 2/3$ (bigger innovations). However, it could be negative if the innovation is small enough at least $\epsilon < 2/3$ and with very high patent strength $\theta > \dfrac{2+\epsilon}{4} > 1/2$, so the patentee with bigger innovations benefit from a longer imitation.

$$\frac{\partial V_1^{S*}}{\partial \beta} = \frac{\partial K^\star}{\partial \beta} > 0 \text{ if } \tau = \epsilon \tag{4.23}$$

Litigation time has a direct consequential effect of increasing the settlement fee that is positive under the suitable assumption that $\tau = \epsilon$, meaning that longer periods of litigation benefit the patentee, when a fair royalty rate is applied. Finally making the derivatives of the payoff of patentee respect the patent life we get that:

$$\frac{\partial V_1^{S*}}{\partial \gamma} = \pi_1^a + \frac{\partial K^\star}{\partial \gamma} = \pi_1^a + \theta(\pi_2^a - \pi_2^b) > 0 \tag{4.24}$$

So this equation shows that in the scenario of a settlement that longer patent life benefits the patentee, and summarizing the results, we get the following proposition

Proposition 4.7. $\dfrac{\partial V_1^{S*}}{\partial \alpha} > 0$ *if* $\epsilon > 2/3$, $\dfrac{\partial V_1^{S*}}{\partial \beta} > 0$ *if* $\tau = \epsilon$, $\dfrac{\partial V_1^{S*}}{\partial \gamma} >$ 0

By working under the last scenario "No Settlement" and making the calculations of the derivatives, we get that:

$$\frac{\partial V_1^{NS*}}{\partial \alpha} = (1-\theta)(\pi_1^b - \pi_1^a) > 0 \tag{4.25}$$

$$\frac{\partial V_1^{NS*}}{\partial \gamma} = \theta\pi_1^b + (1-\theta)\pi_1^a > 0 \tag{4.26}$$

This means that imitation and longer patent life improves the payoff of the patentee. By calculating the derivative in respect to the litigation time, we get:

$$\frac{\partial V_1^{NS*}}{\partial \beta} = \pi_1^c - \theta\pi_1^b + (1-\theta)\pi_1^a - \ell_1$$

The first term is

$$\pi_1^c - \theta\pi_1^b + (1-\theta)\pi_1^a = \frac{1}{9}\left(3\epsilon\left(1-\epsilon\theta^2\right) + (2+\epsilon)(1-\theta) + 2\epsilon^2\left(1-\theta^2\right)\right) > 0$$

which is positive, then:

$$\text{if } \tau = \epsilon \begin{cases} \dfrac{\partial V_1^{NS*}}{\partial \beta} > 0, & \text{if } \ell_1 < \pi_1^c - \theta\pi_1^b + (1-\theta)\pi_1^a; \\ \dfrac{\partial V_1^{NS*}}{\partial \beta} < 0, & \text{if } \ell_1 > \pi_1^c - \theta\pi_1^b + (1-\theta)\pi_1^a). \end{cases} \tag{4.27}$$

We conclude that longer litigation benefits the patentee under a fair royalty rate if the litigation costs for the patentee are smaller and reverting the effect if they are bigger or when there is no settlement.

Proposition 4.8. $\dfrac{\partial V_1^{NS*}}{\partial \alpha} > 0$, $\dfrac{\partial V_1^{NS*}}{\partial \beta} > 0$ *if* $\ell_1 < \pi_1^c - \theta\pi_1^b + (1-\theta)\pi_1^a$, $\dfrac{\partial V_1^{NS*}}{\partial \gamma} > 0$

We can collate these results as they impact on the incentives to innovate or to infringement, in the following table:

	Imitation time	Litigation time	Patent Life
Infringement	$-$	$-$ if $\tau = \epsilon$	$+$
Innovation	$+$ if $\epsilon > 2/3$	$+$ if $\tau = \epsilon$, and smaller ℓ_1	$+$

TABLE 1. Impacts of time variables on infringement and innovation.

We can conclude that the effect of patent life have ambiguous effects as it supports innovation (making the patentee payoff better off in all situations) but also supports infringement (because longer patent life make the incentives to infringe greater). The imitation time as we have seen before, infringers like fast innovation; in the case of innovators with greater innovation they will prefer slower imitation. Time of litigations seems to show that it is worse for the infringer in all cases, and in some cases supports innovation on the probability that litigation costs for the patentee are diminished.

4.4. **Licensing.** In the study of licensing there are 3 possible scenarios:
(1) No imitation
(2) Imitation and no settlement
(3) Imitation and settlement

But as we saw in equilibrium, the payoff of firm 2 is the same whether there is settlement or not. For this reason we have two possible fixed fee values, and three different scenarios for the patentee.

4.4.1. *Licensing under not imitation.* If we assume the case that the backward solution is consistent with no imitation, it is certain firm 1 is offered a license fee $F^{NI\star}$ that make $V_2^{NI\star} = V_2^{A\star}$ so we have that:

$$\begin{aligned} V_2^A = \gamma\pi_2^a - F^{NI\star} &= \gamma\pi_2^b = V_2^{NI} \\ F^{NI\star} &= \gamma(\pi_2^a - \pi_2^b) \\ F^{NI\star} &= \frac{4}{9}\epsilon \geq 0 \end{aligned} \tag{4.28}$$

Now we should see if licensing under this condition is feasible for the firm 1,

$$\begin{aligned} &V_1^A - V_1^{NI} = \\ =&\gamma\pi_1^a + F^{NI\star} - \gamma\pi_1^b = \\ =&\gamma(\pi_1^a - \pi_1^b) + \gamma(\pi_2^a - \pi_2^b) = \\ =&\gamma\left(\sum_i \pi_i^a - \sum_i \pi_i^b\right) = \frac{\gamma}{9}(2-3\epsilon)\epsilon \end{aligned} \tag{4.29}$$

When the imitation does not hold , the licensing condition depends solely upon the size of the cost reduction innovation,

Proposition 4.9. *When no imitate is optimal for the firm 2, there is a positive optimal fixed fee $F^{NI\star}$ if there is licensing, and the licensing condition is $\epsilon \leq 2/3$, meaning that big innovations are never licensed and small innovations are always licensed. Also it is more suitable to have licensing when the patent life is longer.*

4.4.2. *Licensing under imitation and no settlement.* Certainly firm 1 is offered a license fee $F^{I\star}$ that makes $V_2^{A\star} = V_2^{S\star} = V_2^{NS\star}$ so we can get this value by developing this equation as:

(4.30)
$$V_2^A = \gamma\pi_2^a - F^{I\star} = \alpha\pi_2^b + (\gamma-\alpha)\pi_2^a - h - K^\star = V_2^S$$
$$F^{I\star} = \alpha(\pi_2^a - \pi_2^b) + h + K^\star$$
$$F^{I\star} = \alpha(\pi_2^a - \pi_2^b) + h + \theta(\gamma-\alpha-\beta)(\pi_2^a - \pi_2^b) + \beta(\pi_2^a - \pi_2^c) + \beta\ell_2 \geq 0$$

Now we explore firm 1's payoffs under licensing V_1^A and no licensing, so the difference is:

(4.31)
$$\begin{aligned}
H =& V_1^A - V_1^{NS} = \\
=& \gamma\pi_1^a + F^{I\star} - (\alpha\pi_1^b + \beta\pi_1^c + \theta(\gamma-\alpha-\beta)\pi_1^b + (1-\theta)(\gamma-\alpha-\beta)\pi_1^a - \beta\ell_1) \\
=& F^{I\star} + \alpha(\pi_1^a - \pi_1^b) + \theta(\gamma-\alpha-\beta)(\pi_1^a - \pi_1^b) + \beta(\pi_1^a - \pi_1^c) + \beta\ell_1 = \\
& \text{by using the value of } F^{I\star} \text{ from eq.4.30} \\
=& \alpha\left(\sum_i \pi_i^a - \sum_i \pi_i^b\right) + \theta(\gamma-\alpha-\beta)\left(\sum_i \pi_i^a - \sum_i \pi_i^b\right) \\
& + \beta\left(\sum_i \pi_i^a - \sum_i \pi_i^c\right) + \sum_i \beta\ell_i
\end{aligned}$$

Unfortunately this expression is not always positive because the term $(\sum_i \pi_i^a - \sum_i \pi_i^c)$ is negative when $\tau = \epsilon$, the other terms being positive, so when H is positive there is licensing and in other cases there is no licensing.

If we derive H in order to the time variables we get that:

$$\begin{aligned}
\frac{\partial H}{\partial \alpha} &= (1-\theta)\left(\sum_i \pi_i^a - \sum_i \pi_i^b\right) = \frac{(1-\theta)}{9}(2-3\epsilon)\epsilon \\
\frac{\partial H}{\partial \gamma} &= \theta\left(\sum_i \pi_i^a - \sum_i \pi_i^b\right) = \frac{\theta}{9}(2-3\epsilon)\epsilon
\end{aligned}$$

So it means in both cases that the impacts are going to depend on the size of the innovation ϵ, so the impact of greater imitation time and longer patent life makes a license more suitable in small innovations and the effect is contrary in a significant innovation.

Litigation time we have that

$$\begin{aligned}\frac{\partial H}{\partial \beta} =& -\theta\left(\sum_i \pi_i^a - \sum_i \pi_i^b\right) + \left(\sum_i \pi_i^a - \sum_i \pi_i^c\right) + \sum_i \ell_i \\ & -\frac{\theta}{9}(2-3\epsilon)\epsilon + \left(\sum_i \pi_i^a - \sum_i \pi_i^c\right) + \sum_i \ell_i\end{aligned} \tag{4.32}$$

Now because we have that

$$\sum_i \pi_i^a - \sum_i \pi_i^c = \frac{1}{9}\theta\tau(-1-\epsilon+\theta\tau) < 0$$

we can say that $\dfrac{\partial H}{\partial \beta} > 0$ if and only if

$$\sum_i \ell_i > \theta\left(\sum_i \pi_i^a - \sum_i \pi_i^b\right) - \left(\sum_i \pi_i^a - \sum_i \pi_i^c\right)$$

Now in the case of θ we have that

$$\begin{aligned}\frac{\partial H}{\partial \theta} =& (\gamma-\alpha-\beta)\left(\sum_i \pi_i^a - \sum_i \pi_i^b\right) - \beta\sum_i \frac{\partial \pi_i^c}{\partial \theta} \\ & (\gamma-\alpha-\beta)\frac{1}{9}(2-3\epsilon)\epsilon - \beta\frac{1}{9}\tau(1+\epsilon-2\theta\tau)\end{aligned} \tag{4.33}$$

which is negative for bigger innovations, and finally calculating the derivative respect τ

$$\begin{aligned}\frac{\partial H}{\partial \tau} =& -\beta\sum_i \frac{\partial \pi_i^c}{\partial \tau} \\ & -\frac{\beta}{9}\theta(1+\epsilon-2\theta\tau) < 0\end{aligned} \tag{4.34}$$

Proposition 4.10. *When there is imitation and no settlement as consistent choices, it is more suitable to have an accord of licensing , when there are:*

1. *Longer (Smaller) imitation periods and patent life for small (big) innovations*
2. *Longer litigation periods if litigation costs are high enough*
3. *Lower patent strength and reasonable royalty rates.*

4.4.3. *Licensing under imitation and settlement.* In this case we consider again the following fixed fee

$$F^{I^\star} = \alpha(\pi_2^a - \pi_2^b) + h + K^\star$$

Using the definitions of the payoffs of the patentee we compare both payoffs in order to get the licensing conditions in this case, so:

$$\begin{aligned}
&V_1^A - V_1^S = \\
&= \gamma\pi_1^a + F^{I^\star} - \alpha\pi_1^b - (\gamma - \alpha)\pi_1^a - K^\star = \\
&= \alpha(\pi_1^a + \pi_2^a - \pi_1^b - \pi_2^b) + h = \\
&= \alpha\left(\sum_i \pi_i^a - \sum_i \pi_i^b\right) + h = \frac{\alpha}{9}(2 - 3\epsilon)\epsilon + h
\end{aligned} \tag{4.35}$$

Proposition 4.11. *In the scenario where imitation and settlement are optimal choices, smaller innovations are licensed and it is more suitable to have licensing when the imitation time is longer and the cost of imitation are higher.*

What we see in the last three propositions is:

(1) Small innovations are more suitable to be licensed
(2) Longer patent life has a limited effect to promote licensing, eventually just in the case of no imitation or in some cases of small innovations.
(3) Improve the patent strength and reasonable royalty rates (penalties for infringement)in some cases are against licensing, because improves the situation of the patentee when licensing contracts are bargained.

5. Welfare Analysis

5.1. **Welfare Analysis of simple games.** The welfare indicator we are going to use is the sum of the consumer surplus plus the sum of profits of the firms, and because the models are linear, the general form of social welfare is

$$SW = \sum_i \pi_i + \frac{\left(\sum_i q_i\right)^2}{2}$$

So in simple cases by using simple substitution of the formulas in the section 2 we have that, the social welfare for the cases: a) Duopoly under same technology; b) Duopoly under different technologies; c)

Duopoly under same technology but under the shadow of reasonable royalty rates:

$$SW^a = \frac{4}{9}(1+\epsilon)^2 \tag{5.1}$$

$$SW^b = \frac{1}{18}\left(8+8\epsilon+11\epsilon^2\right) \tag{5.2}$$

$$SW^c = \frac{1}{18}(2+2\epsilon-\theta\tau)(4+4\epsilon+\theta\tau) \tag{5.3}$$

Now it should be interesting to compare different situations so we have that:

$$SW^a - SW^b = \frac{1}{18}(8-3\epsilon)\epsilon \tag{5.4}$$

$$SW^a - SW^c = \frac{1}{18}\theta\tau(2+2\epsilon+\theta\tau) \tag{5.5}$$

$$SW^c - SW^b = \frac{1}{18}\left(8\epsilon-3\epsilon^2-2\theta\tau-2\epsilon\theta\tau-\theta^2\tau^2\right) \tag{5.6}$$

The first two expressions are positive, but the last one needs an extra condition. Using the condition $\tau = \epsilon$, the three conditions are positive, so we have:

Proposition 5.1. *$SW^a > SW^b$, $SW^a > SW^c$ and $SW^b > SW^c$ if $\tau = \epsilon$*

5.2. **Welfare Analysis.** We are going to have several scenarios, listed below:

(1) Ex ante licensing A, in this case

$$SW^A = \gamma SW^a \tag{5.7}$$

(2) No ex ante licensing and no imitation NI, so

$$SW^{NI} = \gamma SW^b \tag{5.8}$$

(3) No ex ante licensing, imitation, and no settlement NS, so

$$\begin{aligned} SW^{NS} =& \alpha SW^b + \beta SW^c + \theta(\gamma-\alpha-\beta)SW^b + \\ & (1-\theta)(\gamma-\alpha-\beta)SW^a - \beta\sum_i \ell_i - h \end{aligned} \tag{5.9}$$

(4) No ex ante licensing, imitation, and settlement S, so

$$SW^S = \alpha SW^b + (\gamma-\alpha)SW^a - h \tag{5.10}$$

By fast comparisons of $SW^A \geq SW^{NI}$, $SW^A \geq SW^{NS}$, $SW^A \geq SW^S$, it means that the better situations happens when there is ex ante licensing,

Proposition 5.2. *The best situation from the point of view of social welfare is licensing.*

6. Conclusions

Related with the litigation costs of patents, Bessen and Meurer (2005) point out that innovations with small rewards at risk (less than one million of USD) make the median estimate of a half million in total litigation costs and for median rewards at risk (1-25 million) the estimated median of legal costs is 2 million, then these results are compatible with the possibility of firms settling in cases of weak and small patents that are the cause of the greater legal costs. This fact is consistent with the opinion and data of some authors that the greater share of patent disputes settle (see PWC (2014)) , comes as a consequence of the higher legal costs and is consistent with the results of the model.

Time factors are relevant in the sense that they amplify the effects of other variables, such as reasonable royalty rates and patent strength, but in some cases, time factors have direct effects, as can be shown in the table 1. Infringers will minimize the imitation time and innovators with significant innovations prefer longer imitation times. Also under suitable longer litigation times have desirable results in order to promote innovation and discourage infringement. However the patent life has an ambiguous effect because it supports innovation as infringement.

One relevant point is that much of the analysis shows that the legal system variables i.e. legal costs, patent strength and royalty rates have important interference on the bargaining power of patentees. So it is important that the patent strength that represents the probability that the patentee wins in a patent dispute represents the real meaning of innovation as a discovery, and its inventor has to receive an incentive in such a way that continues research.

Some of the results have shown the importance of the size of innovation in order to measure the magnitude of several policies. So we have to consider there should be different economic policies for different combinations of patent size and patent strength.

References

Reiko Aoki and Jin-Li Hu. Licensing vs. litigation: the effect of the legal system on incentives to innovate. *Journal of Economics & Management Strategy*, 8(1):133–160, 1999.

Reiko Aoki and Jin-Li Hu. Time factors of patent litigation and licensing. *Journal of Institutional and Theoretical Economics JITE*, 159 (2):280–301, 2003.

Lucian Arye Bebchuk. Litigation and settlement under imperfect information. *The RAND Journal of Economics*, 15(3):404–415, 1984.

James Bessen and Michael J Meurer. Lessons for patent policy from empirical research on patent litigation. *Lewis & Clark L. Rev.*, 9:1, 2005.

James Bessen and Michael James Meurer. *Patent failure: How judges, bureaucrats, and lawyers put innovators at risk*. Princeton University Press, 2008.

Joseph Farrell and Carl Shapiro. How strong are weak patents? *The American Economic Review*, 98(4):1347–69, 2008.

Michael J Meurer. The settlement of patent litigation. *The RAND Journal of Economics*, 20(1):77–91, 1989.

PWC. 2014 patent litigation study: As case volume leaps, damages continue general decline. Technical report, Price Waterhouse Coopers, 2014.

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