Dynamic Model of Markets of Homogenous Non-Durables

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Abstract

A new microeconomic model is presented that aims at a description of the long-term unit sales and price evolution of homogeneous non-durable goods in polypoly markets. It merges the product lifecycle approach with the price dispersion dynamics of homogeneous goods. The model predicts a minimum critical lifetime of non-durables in order to survive. Under the condition that the supply side of the market evolves much faster than the demand side the theory suggests that unsatisfied demands are present in the first stages of the lifecycle. With the growth of production capacities these demands disappear accompanied with a logistic decrease of the mean price of the good. The model is applied to electricity as a non-durable satisfying the model condition. The presented theory allows a deeper understanding of the sales and price dynamics of non-durables.

Keywords: non-durables, homogeneous goods, product lifecycle, economic growth, price dispersion, electricity market

1. Introduction

A microeconomic model is presented that aims at a description of the unit sales and price evolution of homogeneous non-durable goods in polypoly markets. The unit sales evolution is also called the product lifecycle of a good (Saaksvuori and Immonen 2008). The idea of a product lifecycle implies that non-durables are analogous to the life of organisms subject to characteristic stages in their sales evolution. The lifecycle of durable goods was studied intensively and a number of models turned out to be suitable to predict the evolution of durable markets (Mahajan et al. 2000; Rogers 2003). In similarity to durables the life cycle of non-durables can be characterized by four stages, the introduction, growth, maturity and decline phase (Chitale and Gupta 2010). While in the introduction period a non-durable enters the market, the market penetration is related to the growth phase. This process saturates in the maturity phase. In the decline phase a current version of the non-durable is replaced by an innovation. The product lifecycle concept is applied both, as a forecasting tool and a guideline for a corporate marketing strategy (Hollensen 2010, Chitale and Gupta 2010). In difference to durables the product lifecycle of non-durables is essentially determined by the evolution of the number of adopters (market diffusion) and the repurchase demand per adopter (repurchase rate). While the price is a key factor in classic microeconomic models (Varian 2006, Cantner 2010), including the price to the lifecycle dynamics turned out to be difficult. The main approach is to treat the price as a perturbation variable in the diffusion process (Chandrasekaran and Tellis 2007).

The presented model, however, applies the lifecycle concept for the sales evolution and takes advantage from a previously published work on the price dispersion of homogeneous goods (Kaldasch 2014, Kaldasch 2015). It suggests that the price dispersion of a homogenous good can be approximated for a short time period by a Laplace distribution, while the mean price is governed by a Walrus equation. That means, the mean price increases if there is an excess growth of demanded (required) units and decreases for the case of an excess growth of supplied (available) units (Zhang 1991). Based on these ideas a dynamic approach to the product lifecycle of non-durables is established here, where the mean price is a direct consequence of the supply and demand dynamics of the market. The model predicts that in the first stages of the lifecycle the market suffers from not-satisfied demands. The growth of the production capacities in time increases the number of available units and leads as a consequence to a decrease of the mean price. That means not all demanded units can meet supplied units during the lifetime of a good in the initial stages of the product lifecycle. The model predicts a critical lifetime of a non-durable in order to survive the introduction phase. Analytic relations are derived describing the evolution of the unit sales and mean price which can be compared with empirical data.

The comparison is performed here with electricity as a homogeneous good. In difference to other non-durable commodities electricity can hardly be stored. Therefore, the electricity market is based on day-ahead auctions while market clearing of the total amount of demanded and supplied units defines the spot price (Bunn 2004; Sioshansi 2013; Biggar and Hesamzadeh 2014). The presented theory yields a dynamic interpretation of the long term evolution of an electricity market.

The paper starts with the presentation of the dynamics of a non-durable market. The unit sales and mean price evolution is established for the case that the number of supplied units evolves much faster than the number of demanded units. The model is compared with empirical results of an electricity market. In the following chapters the general evolution of a non-durable market is discussed and completed by some conclusions.

2. The Model

2.1. The Market Dynamics of a Homogeneous Good

The dynamic microeconomic model presented here is established for the case of homogeneous non-durable goods in polypoly markets. The model is based on two major presumptions:

i.) While potential buyers generate demanded units the supply side of the non-durable market offer supplied product units. The purchase process of the good can be considered as the meeting of demanded and supplied product units. Defining the total number of demanded (desired) units at time step *t* by $\tilde{x}(t)$ and the total number of supplied (available) units by $\tilde{z}(t)$, purchase events must disappear if one of these numbers vanishes. Hence, the total unit sales $\tilde{y}(t)$ can be written up to the first order as a product of both variables (Kaldasch 2011):

$$\tilde{y}(t) \cong \eta \tilde{z}(t)\tilde{x}(t) \tag{1}$$

with the unknown rate *η*. This rate characterizes the mean frequency by which the meeting of demanded and supplied product units generates successful purchase events. The rate *η* is termed meeting rate. Since $\tilde{x}(t), \tilde{z}(t), \tilde{y}(t) \geq 0$, we demand that also $\eta \geq 0$.

ii.) Both $\tilde{x}(t)$ and $\tilde{z}(t)$ are governed by conservation relations. They have the form[1]:

[1] In order to establish a continuous model the integer variables are scaled by a large constant number such that they can be treated as small real numbers. We demand that this scaling leads to $\tilde{x}(t)$, $\tilde{z}(t) < 1$.

$$\frac{d\tilde{x}(t)}{dt} = \tilde{d}(t) - \tilde{y}(t)$$
(2)

and

$$\frac{d\tilde{z}(t)}{dt} = \tilde{s}(t) - \tilde{y}(t)$$
(3)

Eq.(2) suggests that the total number of demanded units increases with the total demand rate $\tilde{d}(t)$ which characterizes the generation rate of demanded units. It decreases by the purchase of product units with the total unit sales rate $\tilde{y}(t)$. The relation Eq.(3) states that the total number of supplied units increases with the supply of units described by the total supply rate $\tilde{s}(t)$ and decreases with the total unit sales rate $\tilde{y}(t)$. The supply numbers can be obtained from a sum over the total number of suppliers *N(t)*:

$$\tilde{z}(t) = \sum_{k=1}^{N(t)} z_k(t); \qquad \tilde{s}(t) = \sum_{k=1}^{N(t)} s_k(t); \qquad \tilde{y}(t) = \sum_{k=1}^{N(t)} y_k(t)$$
(4)

In order to specify the time evolution of demanded and supplied product units the following processes are taken into account.

I) The evolution of the total supply rate $\tilde{s}(t)$ is governed by the growth of the production capacities of suppliers. We want to characterize this growth process with variable $\gamma(t)$ defined by the relation between supply flow and unit sales: [2]

$$\gamma(t) = \frac{\tilde{s}(t)}{\tilde{y}(t)} - 1$$
(5)

The growth of the number of supplied units is essentially determined by the mean magnitude of the variable $\gamma(t)$. It expresses implicitly the amount of investments in new production capacities.

II) The main feature of non-durables (e.g. food) is that they have a finite mean lifetime τ. It is the maximum time product units of the good can be offered

[2] This variable is also called reproduction parameter, since it characterizes the growth process of the output in the reproduction process.

by the suppliers. Not purchased units disappear after this duration. Taking this effect into account Eq.(3) can be rewritten with Eq.(5) as:

$$\frac{d\tilde{z}(t)}{dt} = \gamma(t)\tilde{y}(t) - \frac{\tilde{z}(t)}{\tau}$$
(6)

where the first term characterizes the output evolution and the last term the disappearance of the current number of units with mean lifetime τ.

III) Also taken into account is that the number of demanded product units $\tilde{x}(t)$ may decrease not merely by purchase events, but also by a time dependent change of the demand of potential buyers. We assume that demanded units of a non-durable do not exist forever after they are generated, but have also a finite mean lifetime Θ.[3] In other words, demanded units disappear on average after a time period Θ. Interpreting $\tilde{d}(t)$ as an effective total demand rate this effect can be included by writing:

$$\tilde{d}(t) = \tilde{d}_0(t) - \frac{\tilde{x}(t)}{\Theta}$$
(7)

The first term $\tilde{d}_0(t)$ expresses the general evolution of the demand rate. The second term describes the decrease of the number of demanded units $\tilde{x}(t)$ with the rate $1/\Theta$. For later use we introduce the maximum number of demanded units generated by the demand rate $\tilde{d}_0(t)$ by:

$$\tilde{x}_0(t) = \Theta\tilde{d}_0(t)$$
(8)

It characterizes the stationary number of demanded units if no purchase events occur.[4]

IV) The evolution of the demand rate $\tilde{d}_0(t)$ determines the product lifecycle of a good. The lifecycle is governed by first- and repurchase of the non-durable. First purchase is related to the spreading of the good into the market called diffusion. The diffusion process is usually described by the market penetration *n(t)* defined by:

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[3] In other words, potential consumers generate a demand. But if this demand is not satisfied within an average time period Θ it disappears of its own volition. This effect can be expected in particular for non-durables.

[4] The variable $\tilde{x}_0(t)$ can be obtained from the stationary solution of Eq.(2) using Eq.(7) and setting $\tilde{y}(t) = 0$.

$$n(t) = \frac{N_A(t)}{M_A}$$
(9)

where $N_A(t)$ is the cumulative number of adopters and M_A the market potential, i.e. the number of all possible adopters of a good[5]. The evolution of the number of adopters can be written as a conservation relation of the form:

$$\frac{dn(t)}{dt} = \varphi(t)n'(t) - \theta(t)n(t)$$
(10)

The first term indicates the generation of new adopters. It is proportional to the generation rate of adopters $\varphi(t)$ and the number of potential adopter $n'(t)$ not yet adopted the good. This number is determined by the difference between the maximum of adopters n_{max} and the actual number $n(t)$:

$$n'(t) = n_{\max} - n(t)$$
(11)

The second term in Eq.(10) indicates the decline of $n(t)$ with a decline rate $\theta(t)$. In the introduction, growth and maturity phase of the lifecycle the decline rate is $\theta(t)\approx 0$. However, in the decline phase we demand that $\theta(t)\neq 0$.

Expanding the generation rate $\varphi(t)$ as a function of the number of adopters we obtain up to the first order:

$$\varphi(t) \cong \mathrm{A} + \mathrm{B}n(t) + \ldots$$
(12)

with constant coefficients $A,B>0$. Inserting this relation into Eq.(10) yields for $\theta(t)=0$ a standard approach to describe the diffusion processes of goods known as the Bass model (Bass 1969). It has the form:

$$\frac{dn(t)}{dt} = \mathrm{A}(n_{\max} - n(t)) + \mathrm{B}n(t)(n_{\max} - n(t))$$
(13)

The first term is interpreted as a spontaneous purchase by potential adopters, where A is the so-called innovation rate. The second term is due to social learning, where $n(t)$ increases with an imitation rate B. Formally Eq.(13) can be used to distinguish between the characteristic stages of the lifecycle by the

[5] The evolution of M_A is neglected here.

dominant process generating adopters. In the introduction phase the first term dominates over the second. That means in this phase spontaneous purchase governs the diffusion process. Note that the introduction phase is often dominated by a single supplier (monopoly). It is therefore not in the focus of this model. When the second term in Eq.(13) dominates over the first the good is in the growth phase. In this period the main adopter generation process is social learning. In the maturity phase the number of adopters approaches its maximum $n(t) \approx n_{max}$.

The market penetration suggested by the Bass diffusion model has the form:

$$n(t) = \frac{1 - e^{-(A+B)t}}{1 + \frac{B}{A} e^{-(A+B)t}} n_{\max}$$
(14)

While first purchase (compared to repurchase) plays a crucial role for durable goods, for non-durables the impact of first purchase events on the unit sales can be neglected. Repurchase events can be regarded to be proportional to the current number of adopters $n(t)$. The total repurchase sales of a non-durable can be modelled as the product of $n(t)$ and a time dependent repurchase rate $\xi(t)$ characterizing the average number of repurchased units per unit time and adopter. Neglecting first purchase the total unit sales can be approximated by the repurchase of the non-durable described by:

$$\tilde{y}(t) = \xi(t) n(t)$$
(15)

The repurchase rate $\xi(t)$ can be written as the sum of a time independent constant and a time dependent contribution:

$$\xi(t) = \xi_0 + \xi_r(t)$$
(16)

The constant ξ_0 indicates a minimum time average number of units per purchase event. The time dependent repurchase rate $\xi_r(t)$ takes the growth of the repurchase rate with the economic development of a country into account. In order to keep the model simple we assume that the repurchase rate $\xi_r(t)$ cannot grow up to infinity and demand that $\xi_r(t)$ approaches a maximum magnitude b_ξ after sufficient time. Such a constraint growth can be described in a first order approximation by a logistic growth. The repurchase rate is governed in this case by a differential equation of the form:

$$\frac{d\xi_r(t)}{dt} = a_\xi \xi_r(t)\left(1 - \frac{\xi_r(t)}{b_\xi}\right)$$
(17)

where a_ξ and b_ξ are free parameters. The repurchase rate $\xi_r(t)$ can be described by:

$$\xi_r(t) = \frac{b_\xi}{1 + C_\xi e^{-a_\xi t}}$$
(18)

with the unknown constant C_ξ.

2.2. The Sales Evolution of a Homogeneous Good

The total numbers of demand units $\tilde{x}(t)$ and supplied units $\tilde{z}(t)$ fluctuate considerable in time. If time-dependent variations of these variables are of the same order the sales evolution can hardly be predicted. In order to keep the model simple the theory is confined here to markets dominated by the supply side dynamics in a considered time interval Δt. In other words we confine here to free markets where the number of demanded units evolves much slower than the number of supplied units:

$$\frac{d\tilde{x}(t)}{dt} << \frac{d\tilde{z}(t)}{dt}$$
(19)

If the total number of demanded units evolves sufficiently slowly in comparison with the number of supplied units we can approximately write:[6]

$$d\tilde{x}(t)/dt \cong 0$$
(20)

and obtain directly from Eq.(2):

$$\tilde{y}(t) \cong \tilde{d}_0(t) - \frac{1}{\Theta}\tilde{x}(t)$$
(21)

[6] It is the so-called adiabatic approximation.

where we used Eq.(7). That means, the purchase rate is equal to the generation rate of demanded units $\tilde{d}_0(t)$ diminished by the rate $\tilde{x}(t)/\Theta$. Applying Eq.(1) and Eq.(8) we further get for the total number of demanded units:

$$\tilde{x}(t) = \frac{\tilde{x}_0(t)}{1+\Theta\eta\tilde{z}(t)} \cong \tilde{x}_0(t) - \Theta\eta\tilde{x}_0(t)\tilde{z}(t)$$
(22)

where we used in the approximation that $\tilde{z}(t) < 1$. Applying this result in Eq.(1), the relation Eq.(6) turns into:

$$\frac{d\tilde{z}(t)}{dt} = \alpha\tilde{z}(t) - \Theta\eta\alpha'\tilde{z}(t)^2$$
(23)

with the time averaged parameters:

$$\alpha' = \frac{1}{\Delta t}\int_t^{t+\Delta t}\eta\gamma(t')\tilde{x}_0(t')dt'; \qquad \alpha = \alpha' - \frac{1}{\tau}$$
(24)

Note that for a sufficiently long lifetime τ of the good we can approximate $\alpha \approx \alpha'$. Since Eq.(23) is a logistic differential equation the evolution of the number of supplied units can be given by:

$$\tilde{z}(t) = \frac{\tilde{z}_0}{1+C_z e^{-\alpha t}}$$
(25)

where C_z is a constant and the maximum number of supplied units is:

$$\tilde{z}_0 = \frac{\alpha}{\alpha'\eta\Theta}$$
(26)

The sales evolution of a non-durable depends on the magnitude of α. For $\alpha<0$, there is on time average a supply shortage of product units. In this case the number of available units disappears in time $\tilde{z}(t) \rightarrow 0$. Eq.(24) suggests that this may happen if the lifetime τ of the good is very short, since then there is not sufficient time for potential buyers to purchase available units during the time they are offered. The critical lifetime τ_c can be estimated from Eq.(24) by setting

$\alpha=0$. It leads to $\tau_c \geq 1/(\eta\gamma x_0)$. Thus, a non-durable good must have a high mean reproduction parameter γ (output compared to unit sales), a high meeting rate η and a sufficient number of potential consumers x_0 in order to survive the introduction phase of the lifecycle.

We want to confine the discussion here to $\alpha>0$. In this case the supply side extends the production capacities and the number of available units increases in time until $\tilde{z}(t)=\tilde{z}_0$. The total unit sales Eq.(1) evolve with Eq.(22) and Eq.(25) as:

$$\tilde{y}(t)=\frac{\alpha}{\alpha'}\frac{\tilde{d}_0(t)}{1+C_z e^{-\alpha t}}\cong\frac{\tilde{d}_0(t)}{1+C_z e^{-\alpha t}}$$

(27)

This relation suggests that at introduction of the good *(t=0)* the unit sales are smaller than the generation rate of demanded units. That means there are unsatisfied demands in this phase of the lifecycle. The increase of the production capacities for $\alpha>0$, however, decreases the amount of unsatisfied demanded units. As a consequence the total unit sales $\tilde{y}(t)$ increase in time until $\tilde{z}(t)=\tilde{z}_0$, where the sales evolution is completely determined by the demand rate $\tilde{d}_0(t)$. Inserting Eq.(26) in Eq.(22) we can conclude that in this state the number of demanded units is proportional to the maximum number of demanded units $\tilde{x}(t)=\tilde{x}_0(t)(1+\alpha/\alpha')\approx\tilde{x}_0(t)/2$. And since $\tilde{z}_0$ is a constant in this saturated state, we can further conclude from Eq.(6) and the condition $d\tilde{z}_0/dt=0$ that $\tilde{x}_0(t)=\left(\eta\tau\gamma(t)\right)^{-1}$.

The model suggests therefore that the demand for a good is much higher in the initial phases of the lifecycle than the unit sales. The demand rate $\tilde{d}_0(t)$ is, however, not directly available. The only empirically observable variable is $\tilde{y}(t)$. But as will become clear in the next chapter, unsatisfied demands have an impact on the price of a good. In particular the evolution of the number of available units $\tilde{z}(t)$ plays a crucial role in the determination of the mean price of the good.

2.2. The Price Evolution of a Homogeneous Good

We want to study the price evolution of a homogeneous non-durable in a polypoly market as a result of the sales dynamics established in the previous chapter. For this purpose presumption i) is generalized. We assume that the number of purchase events in a given price interval *p* and *p+dp* must disappear if either the number of demanded units at this price *x(t,p)* or the number of supplied units $z(t,p)$ vanishes. Hence the unit sales at a given price $y(t,p)$ can be approximated in similarity to Eq.(1) as:

$$y(t,p) \cong \eta z(t,p)x(t,p)$$
(28)

where the meeting rate η is treated as price independent. The price dispersion is determined by the relative abundance of purchase events in a price interval given by:

$$P(t,p) = \frac{1}{\tilde{y}(t)} y(t,p)$$
(29)

The mean price can be obtained from:

$$\mu(t) = \int_0^\infty P(t,p) p dp$$
(30)

As shown in a previous work and known from empirical investigations the price dispersion of homogeneous goods can be approximately described for short time horizons by a Laplace distribution (Kaldasch 2015):

$$P(p) \cong \frac{1}{2\sigma} e^{-\frac{|p-\mu|}{\sigma}}$$
(31)

where the standard deviation σ is a function of the mean price $\sigma = \sqrt{2}(\mu - \mu_m)$. The minimum price $\mu_m \geq 0$ indicates a price below which the production of the good is not profitable.[7] Further shown is that the mean price is governed by a Walrus equation relation of the form (Kaldasch 2015):

$$\frac{1}{\mu(t) - \mu_m} \frac{d\mu}{dt} = H(t)\left(\frac{d\tilde{x}}{dt} - \frac{d\tilde{z}}{dt} \right)$$
(32)

where:

[7] The minimum mean price μ_m is considered to be the lowest price that can be offered with the applied production technology. The applied technology is assumed to evolve on the very slowly such that μ_m can be treated as constant.

$$H(t) \cong \frac{1}{\sqrt{2}\tilde{x}(t)}$$
(33)

Applying Eq.(2) and Eq.(3) this relation states that an excess total demand rate $\tilde{d}(t)$ increases and an excess total supply rate $\tilde{s}(t)$ decreases the mean price in time (Zhang 1991).

Eq.(32) can be used to determine the evolution of the mean price of a non-durable homogeneous good. Taking advantage from Eq.(20) we obtain:

$$\frac{1}{\mu(t)-\mu_m}\frac{d\mu(t)}{dt} \cong -H\frac{d\tilde{z}(t)}{dt}$$
(34)

while $\tilde{x}(t)$ is approximated by the constant $\tilde{x}_0$. Applying Eq.(23) we further get:

$$\frac{d\mu(t)}{dt} \cong -H\alpha(t)\tilde{z}(t)\left(\mu(t)-\mu_m\right)$$
(35)

while higher order terms in $\tilde{z}(t)$ are neglected.

The evolution of the mean price depends on the sign of *α(t)*. For *α(t)<0*, the supply shortage generates an exponential increase of the price until $\tilde{z}(t)=0$ (inflation). For *α(t)>0*, however, the mean price declines as a result of the excess supply (deflation). It approaches a stationary state given by either $\mu=\mu_m$ or $\tilde{z}=\tilde{z}_0$. In the first case Eq.(35) reduces for $\tilde{z}(t)<\tilde{z}_0$ to:

$$\frac{d\mu(t)}{dt} = -a(t)\left(\mu(t)-\mu_m\right)$$
(36)

with a mean price decline rate $a(t) \cong H\tilde{z}(t)\alpha(t)$. The mean price evolution has then the form:

$$\mu(t) \cong \mu_0 e^{-\int a(t)dt} + \mu_m$$
(37)

where $\mu(0)=\mu_0+\mu_m$. The main trend of the mean price is in this case an exponential decline towards the stationary minimum price μ_m. However, because the standard deviation of the price dispersion disappears at μ_m this case ends up with a monopoly market. Since we focus on polypoly markets here, this case is not further considered.

For $\mu(t)>\mu_m$ Eq.(34) can be rewritten as:

$$\int \frac{d\mu(t)}{\mu(t)-\mu_m} \cong -H\int d\tilde{z}(t)$$
(38)

and we obtain:

$$\mu(t)=\mu_0 e^{-H\tilde{z}(t)}+\mu_m \cong \mu_0\left(1-H\tilde{z}(t)\right)+\mu_m$$
(39)

while we used in the approximation that $\tilde{z}(t)$ has a small magnitude. The mean price declines therefore by a logistic law induced by the function $\tilde{z}(t)$ (Eq.(25)). For $\tilde{z}(t)\rightarrow\tilde{z}_0$ the mean price approaches a floor price $\mu_f>\mu_m$ given by:

$$\mu_f=\mu_0\exp(-H\tilde{z}_0)+\mu_m$$
(40)

The increase of the supply capacities is therefore directly related to the evolution of $\mu(t)$. When $\tilde{z}(t)\rightarrow\tilde{z}_0$, the mean price decreases towards the floor price μ_f expressing a stationary market without unsatisfied demands (the unit sales are equal to the generation rate of demanded units). Note that the previous work by Kaldasch (2015) on the price dispersion of homogeneous goods suggests that the symmetry of the price dispersion implies $\tilde{z}_0\approx\tilde{x}_0$. When the market is at μ_f the mean lifetime of demanded units can therefore approximately obtained from:

$$\Theta\cong\sqrt{\frac{1}{\eta d_0}}$$
(41)

where we used Eq.(8) and Eq.(26).

3. Comparison with the Empirical Results

The presented theory suggests the following characteristics of the evolution of a polypoly market of homogeneous non-durables when output capacities growth sufficiently fast:

1. The price dispersion *P(p)* of homogenous non-durables are given for short time periods by Eq.(31) and the mean price *μ(t)* decreases over long time periods according to the logistic law suggested by Eq.(39).
2. The evolution of the total unit sales $\tilde{y}(t)$ is essentially determined by the repurchase of the non-durable Eq.(15), which is governed by the growth of the repurchase rate Eq.(18).

For a comparison of these statements with empirical data we want to take advantage from investigations on electricity as a non-durable homogeneous good. The electricity market is a polypoly day-ahead market of power generating companies on the supply side, utilities and large industrial consumers on the demand side.[8] Details of this market are discussed for example in Geman 2005 not further outlined here.

The first assertion suggests that the price dispersion of a homogenous good can be approximated by a Laplace distribution for short time periods by Eq.(31). As an example the central part of a filtered price dispersion of the Nordpool electricity market investigated by Sapio 2004 is shown in Fig.1 and fitted with Eq.(31). The good coincidence indicates the applicability of the presented model to electricity markets.

The model further suggests that the long term evolution of the mean price for a fast growing output can be described by Eq.(39). In order to compare this statement with empirical investigations, we take advantage from historical data of the US non-industrial electricity price of the last century presented by Ayres and Warr 2005. They are displayed in Fig.2 and taken as a measure of the mean price *μ(t)* indicated by dots in this graph. Note that in the introduction phase the electricity market was separated into different grids of single suppliers (monopolies). The electricity act of 1926 led to the setting up of the national electricity grid. In order to satisfy the model conditions the fit of Eq.(39) with empirical price data is therefore confined here to the time period after 1926.

The theory suggests that if suppliers expand their capacities even faster than the rise of demanded units the freely available amount of units increases with time. This expansion process is related to the decrease of the mean price as can be found in the empirical price evolution. The fit of the logistic mean price decline with Eq.(39) suggests a time evolution of $\tilde{z}(t)$ (Eq.(25)) as displayed in the insert of Fig.2 with saturation at $\tilde{z}_0 = 1$. The mean price converges to a constant floor price, while Eq.(39) yields $\mu_f \approx 6$ *Cent/kWH*. Unfortunately the available data do not allow the specification of the lifetime *Θ* by the application of Eq.(41).

Also displayed in Fig.2 is the evolution of the total unit sales $\tilde{y}(t)$. The empirical data of the electricity lifecycle are given in terms of an index which characterizes the electricity unit sales in relation to the *1902* magnitude (Ayres and Warr 2005). The second assertion suggests that the unit sales are determined

[8] The lifetime τ is therefore 1 day for this good.

by the repurchase of the non-durable. Repurchase is in this model proportional to the market penetration $n(t)$ displayed in Fig.3. The empirical diffusion process of electricity follows the expected S-curve (solid line). The fit of the data with Eq.(14) of the Bass model (dashed line) ignores perturbations of the diffusion process from the expected mean growth in the roaring twenties and the following depression (since we set $\theta=0$ in this model). A fit of the logistic growth of the repurchase rate $\xi(t)$ applying the market penetration in Fig.3 leads to the total unit sales $\tilde{y}(t)$ (Eq.(15)) indicated by the dashed line in Fig.1. The data suggest that the growth phase of the life cycle ended around 1970, since the market penetration was completed there. In the maturity phase the main growth of electricity demand is due to the increase of the repurchase rate $\xi(t)$.

From the application of the model to the electricity market we can conclude that the highest growth of capacities took place after World War II around 1950, as can be seen by the rapid increase of available units $\tilde{z}(t)$ in the insert of Fig.2. It generates not only a considerable decrease of the mean price for electricity, but leads also to the convergence of the market penetration $n(t)$ to its maximum magnitude.

4. Discussion

For the case that the market is dominated by a fast growing supply side the presented model predicts the unit sales and mean price evolution of a non-durable. It suggests that the unit sales are governed on the one hand by the spreading (diffusion) of the good into the market, described by here by the Bass model, and on the other hand by the amount of repurchased units due to the economic growth of the country modelled by a logistic law. For electricity as a non-durable the unit sales and mean price evolution is illustrated in Fig.4. Shown is the price dispersion $P(p)$ at a time step in the growth phase of the lifecycle indicated by the dotted line. Generally the price dispersion has the form of a Laplace distribution as found empirically (see Fig.1). It is a consequence of the meeting of the cumulated number of demanded units $x(p,t)$ and the cumulated number of supplied units $z(p,t)$ also shown in Fig.4. The chance that demanded and supplied units meet at a given price p has its maximum at mean price $\mu(t)$ where the functions $x(p,t)$ and $z(p,t)$ have maximum overlap (Kaldasch 2015). The function $x(p,t)$ can be interpreted as a demand function and $z(p,t)$ as a supply function in the neoclassic view of a market. The supply function is governed by the costs per unit for the generation of electricity, which depends on the applied production technology. The smallest prices are offered by nuclear power stations while the highest prices are generally demanded by gas power stations indicated in the figure.

The main idea to understand the price evolution of the electricity market is that there are unsatisfied demands in the growth phase of the product lifecycle. For the case $\alpha>0$, the production capacities increase faster than the unit sales. As a result the number of available units $\tilde{z}(t)$ growth in time. However, this growth leads to the decline of the mean price governed by a logistic law as displayed in the insert of Fig.4. When $\tilde{z}(t)$ approaches $\tilde{z}_0$ unsatisfied demands disappear and the mean price $\mu(t)$ approaches the constant floor price μ_f. Since the variance of the price dispersion depends explicitly on the mean price, $P(p)$ becomes a sharp distribution around μ_f in this state.

4. Conclusion

The presented microeconomic theory is aimed at a deeper understanding of the market dynamics of homogeneous non-durable goods. Although the model is applied here merely to the electricity market it can be expected that it is also applicable to other fast growing non-durable markets, if it is not dominated by speculation. The dynamic theory suggests that non-durable goods must have a minimum lifetime in order to survive. Its magnitude depends on the availability and demand for the good. Due to limited production capacities in the first stages of the lifecycle the presented approach predicts the existence of unsatisfied demands. They are hidden since empirically observable are the unit sales, governed essentially by the repurchase of the non-durable. However, with increasing production capacities unsatisfied demands decrease associated with a logistic decline of the mean price.

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Figures

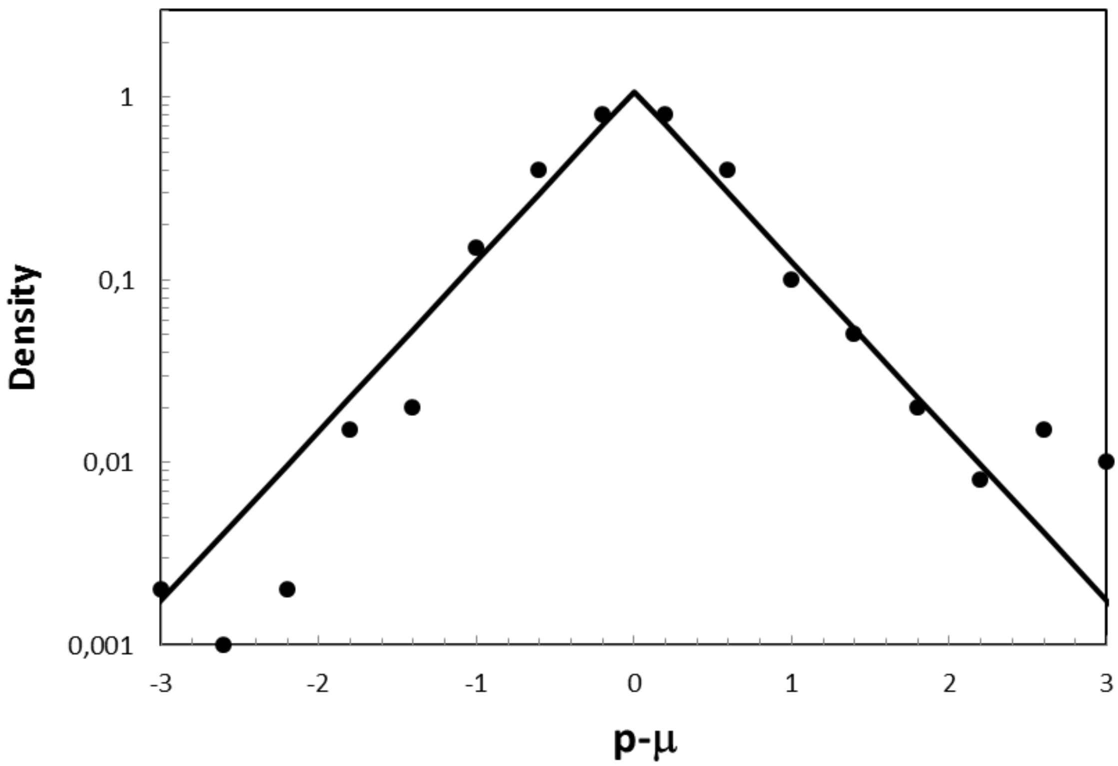

Figure 1: The price distribution *P(p)* of the Noordpool electricity market (at 1 am) around a scaled mean price $\mu=0$. The dots indicate empirical data (Sapio 2004). The solid line is a fit of Eq.(31) with $\sigma=0.468$.

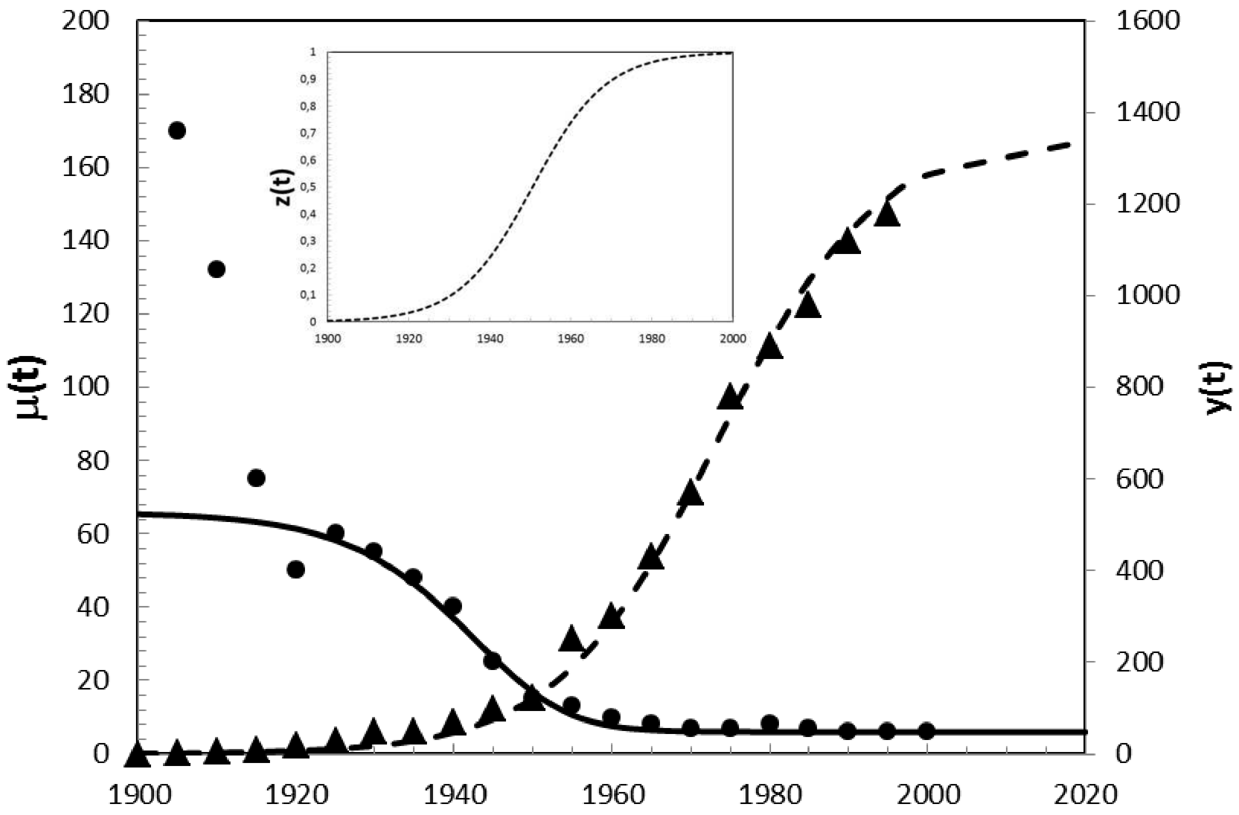

Figure 2: Evolution of the US non-industrial electricity price $\mu(t)$ in cent per kWh (constant 1992 US\$). The dots indicate empirical data (Ayres and Warr 2005). The solid line is a fit of Eq.(39) with μ_m=*1 cent/kWh*, μ_0= *70 cent/ kWh*, $H\tilde{z}_0=2.8$. Displayed in the insert is the fitted function $\tilde{z}(t)$ given by Eq.(25) with $\tilde{z}_0=1$, α=*0.11 per year* and C_z=*260*. Also shown are the historical unit sales $\tilde{y}(t)$ (triangles) in terms of an index indicating the electricity demand in relation to the magnitude in *1902*. The dashed line is a fit of the long term unit sales given by Eq.(15), where we used *n(t)* from Fig.3 and ξ_0=*0.2* a_ξ =*0.1 per year*, b_ξ =*30* and C_ξ=*1500*.

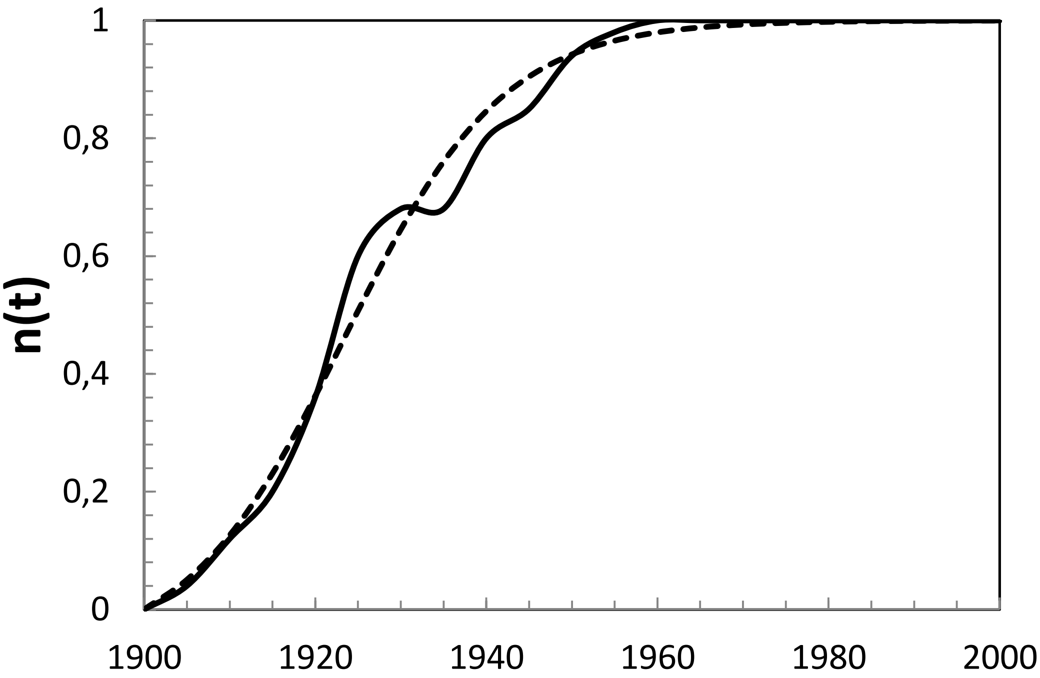

Figure 3: The solid line indicates empirical data of the electricity penetration *n(t)* in the USA (Felton 2008). The dashed line is a fit with Eq.(14) using $n_{max}=1$, $A=0.008$, $B=0.1$.

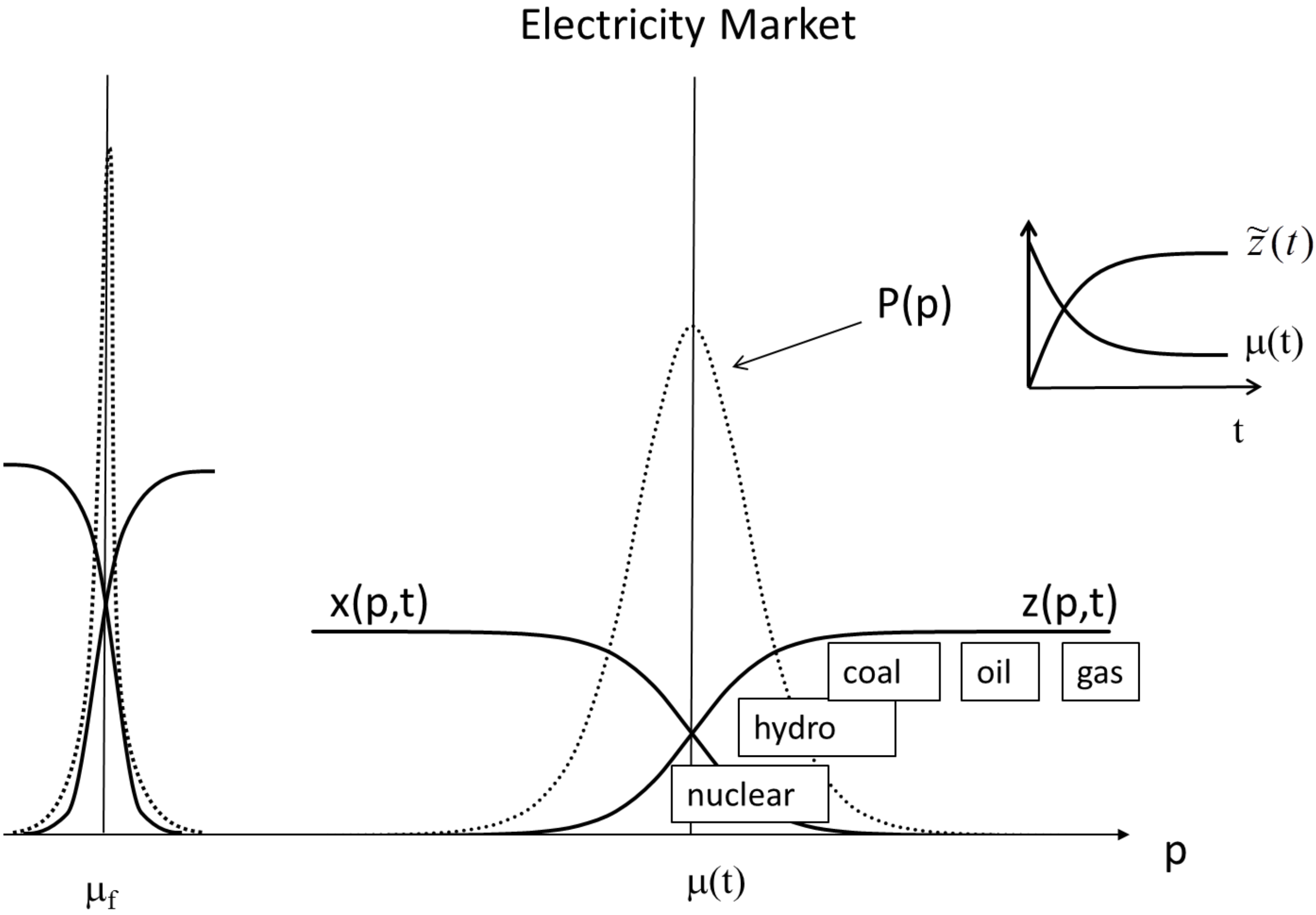

Figure 4: Schematically displayed are the price dependent demand and supply curves *x(p,t)*, *z(p,t)* (solid lines) and the price distributions *P(p)* (dotted lines) of the electricity market at two different time steps. The supply curve *z(p,t)* depends on the costs per unit of the electricity generation which depends on the applied generation technology. The model suggests that with the increase of the number of available units $\tilde{z}(t) \rightarrow \tilde{z}_0$ the mean price $\mu(t)$ declines until $\mu(t) \rightarrow \mu_f$ (insert). For $\tilde{z}(t) = \tilde{z}_0$ is $\mu = \mu_f$ while the price dispersion becomes a narrow peak around μ_f.