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Abstract

The author seeks to demonstrate that the price system proposed by Piero Sraffa in his major work Production of Commodities by means of Commodities – Prelude to a Critique of Economic Theory *is compatible with both David Ricardo and Karl Marx ´s labour embodied theory of value and with Adam Smith's labour-commanded theory of value. In reality, Sraffa's measure of prices, the Standard Commodity, satisfies rigorously the mathematical condition of invariability in relation to income distribution between wages and profits. In this sense, it can be a consistent solution to the transformation problem of labour values into production prices. Besides, the Standard ratio or the maximum rate of profits can be used to analyze the evolution of the three major types or forms of technical progress in a capitalist economy, as follows: labour-using, neutral and capital-using techniques.*

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Introduction

The economic literature encompasses an endless number of articles, comments and warm discussions concerning the determination of prices from labour values, especially among the Marxian economists and those later known as Neo-Ricardians, whose *magna opus* is Piero Sraffa's *Production of Commodities by Mean of Commodities - Prelude to a critic of Economic Theory* published in 1960.

In this paper, we will demonstrate that the measure of prices and wages proposed by Sraffa, that is, the *Standard Commodity*, can be a consistent solution to the transformation problem of labour values into prices of production, because it satisfies rigorously the mathematical condition of invariability in relation to income distribution between wages and profits.

We will also show that there is a connection among the measures of value proposed by Smith, Marx and Sraffa. The construction of the *Standard Commodity* is rather closer to the measure proposed by Marx in Book III, chapter IX of *Capital* than to the *average commodity* imagined by Ricardo in the 3rd edition of his *Principles*, published in 1821.

Our major objective in this paper is not a reconciliation of Sraffa with Marx, but rather to show that the Sraffian model is not conflicting with the labour theory of value. Worth mentioning that the labour theory of value has two major approaches, the *labour-embodied* theory of value of Ricardo and Marx and the Smith's *labour-commanded* theory of value. Sraffa was perfectly aware of the difference between these two approaches (see *References to the Literature*, Appendix D of his book) and his work, as we shall see, fits perfectly in these two approaches.

In order to clarify our exposition, it is worth mentioning a very important premise of the classical approach, including the Sraffian model, regarding the determination of values and prices. Such premise is the previous knowledge of the physical output. In both Smith and Ricardo, as well as in the Sraffa's approach, the *surplus* is a magnitude independent on the determination of prices and distribution, contrary to the marginalist and neoclassical theories. In such theories, both prices and the output are simultaneously determined. In the classical theory instead, the production and distribution are two separate phases of the analysis and the adoption of the *output as given* is a preliminary approach to the determination of prices. The methods of production or the set of techniques are previously known by the capitalist and in this respect there is no difference between the classical and neoclassical theories. However, in the real world, the production level of a commodity may be affected by the relative prices of the means of production of such commodity. Although the classical economists knew about this aspect, it was not possible till now to establish a consistent relationship between prices and the quantities produced.

In the neoclassical theories, such connection is represented by both the demand curves and consumers´ preferences. However, according to Garegnani (1984) in the neoclassical theories the demand is not a functional relationship between prices and quantities, but rather, a construction whose aim is to provide the distribution of income among the production factors, capital, labour and land.

The single Labour Theory of Value

Adam Smith had noticed that, when the net income or surplus is fully absorbed by wages, the price of a commodity is exactly equal to its labour-embodied value.

This situation, however, only took place according to Adam Smith, '*in that early and rude state of society...*' (*The Wealth of Nations*, 5th edition, 1789, Book I, chapter VI).

Making use of the matrix notation, we can easily demonstrate the above Smiths *dictum*.

$$\mathbf{vA} + \mathbf{L} = \mathbf{vB} \qquad [1]$$

$$\mathbf{v}(\mathbf{B} - \mathbf{A}) = \mathbf{L} \qquad [1.a]$$

$$\mathbf{v} = \mathbf{L}(\mathbf{B} - \mathbf{A})^{-1} \qquad [1.b]$$

The determination of the absolute value of any commodity, expressed by the row vector of labour-embodied values (**v**), depends uniquely on the order *n* square matrix **A** regarding the quantities of means of production, on the diagonal output matrix **B** and on the row vector of the direct labour quantities **L.** There are n equations and n unknowns (**v**).

However, when we try to determine the prices of production, their solution is not as simple as above. Besides the *n* prices, we have two additional unknowns, the distributive variables *r* and *w*.

Such price system can be expressed as follows:

$$\mathbf{pA}(\delta+\mathrm{r})+\mathrm{w}\mathbf{L} = \mathbf{pB} \qquad [2]$$

p = price row vector.
B = diagonal output matrix.
L = row vector of direct labour quantities.
r = rate of profits (scalar)
δ = depreciation rate (scalar) and its value is between 0 and 1. When δ = 0 it means that the good or the fixed asset has an infinite life expectancy, like land. There is no depreciation.
w = **pb**, the wage rate, where **b** is a column vector comprising a basket of consumption goods (scalar).

The means of production, represented by the square matrix **A,** correspond to the circulating capital, that is, they are fully consumed during the productive period. The fixed capital, comprised of machines and equipments, used up in the course of several production periods, was not included here, due to the need of a complex mathematical treatment. However, if we want to include the fixed capital in the above price system, it should be treated like we did with the circulating capital, that is, it will be entirely consumed in one production period, that is, the depreciation rate δ = 1.

The equation [2] is the Sraffa's price system expressed in matrix notation. As it can be noticed, such system operates in a competitive economy, given a uniform rate of profits.

The difference between the price system proposed by Sraffa and that proposed by Marx and other classical economists lies on the fact that the wages are paid *post-factum,* instead of a wage fund.

Alessandro Roncaglia (1984, chap. II, section 5, page 30), provides two political or ideological reasons. Firstly, the wages will only be paid if the work had been fairly executed. Secondly, as an instrument of domination by capital over labour, causing the impression the capitalist pays exactly the amount for the quantity of labour dispensed in the production of commodities.

Firstly, let see the solution provided by Marx for the determination of prices of production in a competitive economy.

When the rate of profits is null (r = 0) and the wages absorb all the national income (w = 1), the above price system [2] is simply:

$$\mathbf{pA}+\mathbf{L} = \mathbf{pB}$$

$$\mathbf{p}(\mathbf{B} - \mathbf{A}) = \mathbf{L}$$

$$\mathbf{p} = \mathbf{L}(\mathbf{B} - \mathbf{A})^{-1}$$

Therefore, in such case the prices are identical to their correspondent labour-embodied values.

$$p_i = v_i$$

and i = 1, 2, . . ., n;

In a real capitalist economy, however, the rate of profits is positive. The wage rate, by its turn, is at least equal to the subsistence level or reproduction of the labour class. Since *w* and *r* are now unknowns, we need two additional independent equations to the determination of all prices, the wage and the rate of profits.

The Marxian price system can be expressed by the following set of equations:

$$(\mathbf{pA}+w\mathbf{L})(1+r) = \mathbf{pB} \qquad [3]$$

$$w = \mathbf{pb} \qquad [4]$$

$$\mathbf{v}(\mathbf{B} - \mathbf{A}) = \mathbf{p}(\mathbf{B} - \mathbf{A}) \qquad [5]$$

In Book III, chapter IX, of *Capital*, Marx faced this problem by proposing firstly as standard of prices the equality between the surplus value (labour-embodied values) and the net output in terms of prices for the economy as a whole, given by the equation [5].

The above equation [5], necessary to the solution of the price system, shows simply that we can not share or distribute more than was previously created by the society, expressed by the left side of the above identity.

The simultaneous resolution of [3], [4] and [5] will determine the *n* prices and the rate of profits.

The real wage rate (equation 4) is exogenously given and comprises a basket of consumption goods previously defined (column vector **b**). The level of *w* depends both on the degree of political organisation of the labour unions and the historical standard of living of the labour class. Despite the wages in the Marxian system are included as means of production (wage fund), this fact does not change our conclusion on the subject.

Both Smith and Ricardo provided several reasons to support the adoption of a given real wage in the determination of prices. In Smith, the (nominal) wage is the result of a contract made between the master and the worker (bargaining) and he pointed out that it is not difficult to foresee which of the two parts take more advantage in the dispute. As the masters are fewer in number, they can combine more easily (*Wealth of Nations*, chapter VIII, §12). On the other hand, there are also acts and laws which prohibit the combinations among workers. In this sense, we may conclude that, for Smith, the situation was clearly unfavourable for the workers.

Besides this process of bargain, in the long term both the masters and workers depend on each other. The result is that there is a minimum level for the real wage, defined as subsistence minimum. This level of subsistence should be sufficient to maintain the worker and his family. Smith also made a lot of other considerations about wages, comparing, for instance, the real wage in China and in England and concluded that is depends on historical and social conditions.

In Ricardo, the real wage is determined in the long run, that is, by the necessaries and conveniences required to support the reproduction of the labour class (*Principles*, Chapter V, On Wages) In such respect, Ricardo's analysis does not differ from Smith's. For Ricardo, the wages have a tendency to rise in real terms, due to the greater difficulty to produce "*one of the principal commodities by which its natural price is regulated*" (corn). However, as Ricardo recognises that "*the improvements in agriculture, the discovery of new markets, whence provisions may be imported, may for a time counteract the tendency to a rise in the price of necessaries…"*(*Principles*, Chapter V, On Wages, §3).

In Ricardo, as well as in Smith, there is a natural price and a market price for all commodities. "*Labour, like all other things which are purchased and sold, and which may be increased or diminished in quantity, has its natural and its market price*" (*Principles*, Chapter V, On Wages, §1). By natural price Ricardo understands "*the quantity of labour necessary to their production*" (*Principles*, Chapter IV, On Natural and Market Price, §8). The market price of labour may deviate from the natural price and has its origin "*from the natural operation of the proportion of the supply and demand*" (*Principles*, Chapter V, On Wages, §5), but such forces are temporary and not functional. In another passage, Ricardo states: "*wages would fall, if they were regulated only by the supply and demand of labourers; but we must not forget, that wages are also regulated by the prices of the commodities on which they are expended*" (*Principles*, Chapter V, On Wages, §25). These excerpts from Ricardo are a clear position of the classical approach that the conditions of production (labour and technology) play an important role in the determination of the real wage.

On the other hand, the level of the rate of profits expressed in *labour value* terms (r*) is determined as follows:

$$r^*\mathbf{vA} = \mathbf{v}\,(\mathbf{B} - \mathbf{A}) - w^*\mathbf{L} \qquad [6]$$

$$w^* = \mathbf{vb} \qquad [7]$$

In order to obtain r* as a ratio (scalar), all the elements of both sides of the relation [6] were multiplied by a column vector **H** (*n* rows and *one* column, with all elements equal to 1);

$$r^* = \frac{\mathbf{v}(\mathbf{B} - \mathbf{A})\mathbf{H} - w\mathbf{LH}}{\mathbf{vAH}} \qquad [6.1]$$

vA = the value of capital employed
w* = wage (scalar) in value terms
v(**B** - **A**) - **L**w **=** total profits in value terms
H = column vector with all elements equal to 1;

However, contrary to Marx's expectation, the above rate of profits r* is numerically different from that rate (r) obtained from the simultaneous resolution of the price system (equations [3], [4] and [5]).

In other words, if we eliminate the identity given by equation [5] on page 4 and substitute the rate of profits in the system of equations [3] by the value of r*, previously determined by the above relation [6], the relative prices in each industry will be different, which is contrary to the principle of a unique relative price structure.

On the other hand, if we adopt only one relative price structure for all industries, the rate of profits will have to be different in each industry, which is also inconsistent with free competition.

Therefore, we got into a crucial point of the Marxian transformation problem and such inconsistency is the reason by which Marx introduced another *postulate of invariance*, namely, total output equals total value, that is, **pB = vB.** From Marx's own words:"*And in the same way the sum of the prices of production of all commodities produced in society – the totality of all branches of production – is equal to the sum of their values*" (Marx's Capital, Book III, Part II, chapter IX).

However, the simultaneous adoption by Marx of the two standards of value does not provide a unique solution for the determination of commodity prices, as demonstrated by Francis Seton (1957) in his classic article. Therefore, the Marxian transformation of values into production prices remains without a satisfactory solution.

Worth mentioning that, due to a response to Eugen von Böhm-Bawerk, who in 1896 criticised the logical consistence of Marx´solution, several Marxian economists since Ladislaus von Bortkiewicz provided alternative postulates of invariance. I suggest the reader to examine the articles written by Kenneth May (1948), Joseph Winternitz (1948) and Ronald Meek (1956).

Despite their efforts to preserve the basic structure of the Marxian transformation problem, none of such proposals can be accepted, because two limitations arise.

The first one regards the formulation of the problem. In general, the economic system is divided into few sectors, linked to the social classes for which the production is destined. The commodities are classified in terms of aggregates, such as capital goods, labour consumer goods and luxury goods.

The above limitation disappears by splitting the sectors into several industries, each one producing one single commodity. However, the Marxian economists faced a second difficulty, more complex, concerning the adoption of a postulate of invariance. The arguments proposed in favour of the postulates of invariance were in general subjective and quite difficult to be empirically confirmed. Meek, for instance, adopted the premise that the organic composition of capital in the sector which produces the necessary goods for the reproduction of the labour class is identical to the average organic composition of capital of the economic system as whole. Such premise is equivalent to say that the prices in that sector are equal to their correspondent labour values and Ronald Meek was unable to provide a logical explanation for his choice.

Although highly unfeasible, another situation by which the relative prices are equal to their relative values is when the organic composition of capital, expressed in labour-embodied values, is the same in all industries:

$$\begin{gathered}\mathbf{vA}_1+L_1 = v_1\\ \mathbf{vA}_2 + L_2 = v_2\\ \cdots\cdots\\ \mathbf{vA}_n+L_n = v_n\end{gathered}$$

in which:

$$(\mathbf{vA}_1/L_1) = (\mathbf{vA}_2/L_2) = ...= (\mathbf{vA}_n/L_n) \qquad [8]$$

Let us see now the general case of an economy with two industries (n=2) and with the same proportion of *indirect* to *direct* labour:

$$(a_{11}v_1+a_{21}v_2)/L_1 = (a_{12}v_1+a_{22}v_2)/L_2$$

Isolating the technical coefficients from values, we have:

$$v_1/v_2 = (a_{22}L_1 - a_{21}L_2)/(a_{11}L2 - a_{12}L_1)$$

The determination of the relative value of the commodity 1 to commodity 2 depends only on the given technical coefficients of production in each industry.

Since that proportion (expression [8]) is the same in each industry, the above result also applies to the price system:

$$(a_{11}p_1+a_{21}p_2)/L_1 = (a_{12}p_1+a_{22}p_2)/L_2$$

$$p_1/p_2 = (a_{22}L_1 - a_{21}L_2)/(a_{11}L2 - a_{12}L_1)$$

Therefore,

$$v_1/v_2 = p_1/p_2$$

The ratio [8] is also known as the *organic composition of capital* and there are some basic reasons by which it is not equal in all industries. Some commodities, in order to be produced, demand a higher proportion of direct labour in relation to indirect labour (*labour intensive techniques* or *labour-using techniques*), for instance, fine leather consumer products and mechanical watches with several complications. On the opposite side, commodities such as steel, pulp & paper and petrochemicals, for example, demand higher proportion of indirect labour (*capital intensive techniques* or *capital-using techniques*). Although we have a dominant production technique in each industry, depending on specific market conditions, the firms may also use alternative production techniques in order to both increase profits and to offer products according to their target-consumer preferences. Therefore, it is quite reasonable to suppose the existence of different direct labour to indirect labour ratios.

In the next section, we will demonstrate that the Sraffa's Standard Commodity is analogue to the first standard of prices proposed by Marx (equation [5]).

We also believe that both Marx and Sraffa adopted the *surplus value* as a major reference of prices because it is the most important economic magnitude in any economy and, to be consistent with the labour theory of value, the net income should be equal to the total quantity of labour spent during the production period.

Sraffa's Standard Commodity

The choice of the Standard Commodity as the *numéraire* of prices is a fundamental condition for the existence of a linear relationship between the wage and the rate of profits. This linearity, in its turn, is necessary to attend the mathematical condition of an invariable standard of value in relation to the distribution of income.

Sraffa was really concerned about finding an invariable standard for prices and wages, in order to obtain a uniform rate of profits consistent with the concept of value. In other words, the rise of income share by one class should be exactly offset by the reduction of income share by the other class.

The above mathematical condition we have referred to can be described as the first derivative of profits in relation to wages. This expression should be equal to -1 because any rise (or reduction) of total profits should be exactly offset by a reduction (or rise) of total wages. In this sense, we have:

$$\text{d(Profits)} = -\,\text{d(Wages)}$$

or

$$\frac{\text{d(Profits)}}{\text{d(Wages)}} = -1$$

Sraffa obtained the following linear relationship between the rate of profits and the wage rate:

$$r = R(1 - w) \qquad [9]$$

It is worth mentioning the above relation (see the Appendix I for a detailed construction of the Standard Commodity) *apparently* does not depend on the size of the surplus or on the absolute value of income. With the use of multipliers (column vector **Q**), the net income of the real economic system is modified in order to reflect a uniform proportion between surplus and means of production and it becomes the invariable standard of prices and wages (equation [6 A] in the Appendix I on page 15).

Besides, by adopting the whole *surplus value* and its equivalent, the total quantity of labour utilised during the production period, as the standard of prices and wages, Sraffa avoids the subjectivity presented in the postulates of invariance proposed by Marx and other economists.

Since w is the share of net income that goes to wages, (1- w) or r/R represents the fraction of income destined for profits. Thus, it is easy to demonstrate how the Standard Commodity or its equivalent (equation [7 A] in the Appendix I on page 15), fully attends the mathematical condition mentioned above.

Calculating the first derivative of the rate of profits in relation to the wage rate in the equation [9], we have:

$$\frac{d(r/R)}{d(w)} = -1$$

The standard of value proposed by Sraffa also eliminates the tautological reasoning in Adam Smith's *labour commanded* conception (*The Wealth of Nations*, 5th edition, 1789, Book I, chap V), since the relation [9] can also be written as:

$$\frac{1}{w} = \frac{R}{R - r} \qquad [9.a]$$

As r goes to its maximum R, the lower will be the wage and, therefore, the higher will be the quantity of labour which the capitalist is able to buy or *command.* In other words, the notion of *labour commanded* in Smith has now an accurate meaning.

In the work of Smith, the value of any commodity is explained by the quantity of labour it can buy. However, such result depends on the level of the money wage in the labour market. As the wage comprises a basket of consumption goods, firstly we have to know the value of each of such commodities to in order to explain the value of these same commodities. Therefore, we fell into a circular reasoning.

The original price system (equation [2] on page 3) contains n+2 unknowns and n equations. In order to determine the price of commodities, we need two more independent equations.

The $(n+1)^{th}$ equation must be the standard of prices and wages. As there is a straight link between the Standard Commodity (equation [6-A] of the Appendix I on page 15) and the relation [9], the result will be the same if we use one or another as such standard. The linear relation, equation [9] above, has the advantage of being more simple and elegant, allowing a crystal clear perception of the income distribution movements.

The $(n+2)^{th}$ independent equation must reflect the value of one of the two distributive variables, by fixing the value of the wage or the rate of profits. Sraffa himself suggested the rate of profits as the exogenous variable in the price system and it is obtained as a function of the money rate of interest (Sraffa, 1960, chap V, section 44, page 33).

There is no doubt that the Standard Commodity is a commodity *ad hoc*, built with a specific purpose, as Claudio Napoleoni pointed out (1973, page 243).

Although the Standard Commodity is a purely mathematical construction, it rigorously solves *strictu sensu* the problem of income distribution between wages and profits.

In Sraffa's book, contrary to Marx's *Capital*, there are no political or social concerns. His model is restricted to the quantitative aspects of distribution. For instance, when w = ¾, it means simply that the wages account for 75% of the national income.

However, the real wage, constituted by a bundle of commodities, previously determined, has nothing to do with the Sraffa's Standard Commodity. It is highly possible that the physical composition of the real wage will *not* coincide exactly with the composition of the salary in terms of the Standard Commodity. The Standard Commodity has the property to show that the Ricardo's *corn-corn* model "is hidden in any productive system" as pointed out by Marco Lippi (1998). The Standard Commodity is an elegant and pedagogical tool to understand the inverse relationship between the wage and the rate of profits. If any commodity or other composite commodity is used to measure prices and wages, we will not have a straight line between the two distributive variables.

The reduction of a price into dated quantities of labour

The relationship between the price theory formulated by Sraffa and the labour theory of value becomes clearer with the method *Reduction to Dated Quantities of Labour*, presented in the sixth chapter of his book.

The method *Reduction* has its origins in the work of Ricardo - *The Principles of Political Economy and Taxation* - chap.I, section III, 3rd edition, 1821) and also in the book of the Russian economist Vladimir Karpovich Dmitriev (1868-1913) - *Economic Essays on Value, Competition and Utility*, (Cambridge University Press, 1981, Domenico Mario Nuti - editor), originally published in 1902.

The *Reduction* method shows that the price of a commodity is the result of a sum of labour quantities weighted by a profit factor, taking also into account the *dates* or *rounds* of production.

Although the understanding of the aggregation process of direct labour quantities is quite easy, a formal demonstration of the *Reduction* method is relatively complex, so that we will present only a concise exposition of it.

In order to make our explanation easier, let us use the Sraffa own mathematical notation for the price equation of the commodity *a* , as such:

$$(A_a p_a + B_a p_b + \ldots + K_a p_k)(1+r) + wL_a = Ap_a \qquad [10]$$

The above equation can also be written as:

$$A_a p_a(1+r)+B_a p_b(1+r)+...+K_a p_k(1+r)+wL_a = Ap_a \qquad [10a]$$

where A_a, B_a, . . . , K_a, are the physical quantities of commodities *a, b, . . ., k,* annually used in the production of A quantities of the commodity *a.* The corresponding prices are p_a, p_b, . . . , p_k.

Each mean of production in the equation [10] was produced by other means of production and direct labour. This process can be observed in the successive *rounds*, which preceded the current production. As we carry on this process, we accumulate in each *date* a series of direct labour quantities L_{at} ,weighted by the profit factor $(1+r)^t$, where *t* represents the *date* or round of production of the commodity *a*:

The price equation in terms of dated quantities of labour can be reduced into an infinite series of terms expressed as follows:

$$wL_{a0}+wL_{a1}(1+r)+ .. +wL_{an}(1+r)^n + . . . = Ap_a \qquad [10b]$$

Except for the two distributive variables, w and r, the value of all terms in the equation above are known.

One of these two distributive variables will be exogenously given, for instance the rate of profits, as Sraffa himself suggested. Therefore, we need one independent equation to determine the price of commodity *a*.

This additional equation is

$$r = R(1-w)$$

It will be the standard of prices and wages. Each term L_{at} represents the quantity of direct labour used in the date *t,* in order to produce *A* quantities of the commodity *a.*

As we see, the above series may have infinite terms and the accuracy of the calculus will depend on the level of the rate of profits. The closer *r* is of its maximum R, more important will be the *date* t, because the term $wL_{at}(1+r)^t$ is a decreasing function for a rise in the profit rate, given both L_{at} and *t* constant. In this case, we have to bring out direct labour quantities from preceding periods as far as we can.

The equation [10] shows how complex the solution of the transformation problem of values into prices is. Two commodities produced with the same absolute quantities of labour may have two different prices, depending on how the respective quantities of labour are distributed in time or in the successive rounds of production. Except when all the net income goes to wages (w = 1 and r = 0), as Smith pointed out, or when the organic composition of capital expressed in value terms is identical in all industries, the prices coincide with their respective embodied labour values.

It is worth mentioning that the terms L_{a0}, L_{a1}, , L_{an} , of equation [10b] are the quantities of direct and indirect labour necessary to produce A physical quantities of commodity "a". Consequently, there is no logical reason to say that the Sraffa price theory is not consistent with the labour theory of value.

Conclusion

The existence of a connection between the Standard Commodity and the measures proposed by Smith e Marx, is a hint that the Sraffa model could not be dissociated from the classical approach, whose basic characteristics are:

i] the economic system is a circular process, with a defined production cycle.
ii] the existence of a surplus, comprised of heterogeneous goods.
iii] the rate of profits and the wage are uniform in all production branches.
iv] the society is divided in classes.
v] the technology is given previously to price determination.
vi] the human labour is the source of value.

In my interpretation, the lack of a critical approach of the capitalist society in *Production of Commodities* is the main reason why Marxist economists reject the Sraffa's model as a substitute of the Marx's price system. Such rejection is rather ideological than logical.

I believe it is necessary to make a distinction between the figure of Sraffa as an economist, concerned with the logical rigor in the solution of specific problems unsolved by the classical economists, in particular by Ricardo and Sraffa as a citizen. Piero Sraffa supported Antonio Gramsci and Palmiro Togliatti in different ways and occasions, probably the two major figures in the fight against the fascism in Italy during the Mussolini era.

We conclude that from a logical point of view, Sraffa's price system is superior to that of the classical economists and of Marx's. The critics of Sraffa may also state that his model does not back a dynamic analysis of the capitalist economy.

However, in macroeconomic terms, the maximum rate of profits R, as a ratio between surplus and means of production, is equivalent to the *output to capital ratio* (Y/K). For instance, when the output to capital ratio of the economy as a whole is increasing, it means that the value of the net output is rising faster than the value of the means of production. In this case, we have a *capital- saving* technical progress, considering the daily labour journey is fixed or constant. The means of production are, therefore, being used more efficiently, with the introduction of new technologies. There is a reduction in the technical coefficients of production and therefore a reduction in the prices of commodities.

Two other important forms of technical progress are also possible: *capital-using,* with R decreasing and *neutral*, when the ratio R is stable. The *neutral* technical progress has been widely used in the construction of steady growth models, especially by Harrod and von Newmann.

When Marx formulated his law of the falling rate of profits, the *capital-using* technical progress or *mechanisation* was dominant at that time. Direct labour was being substituted by machines and the productivity increased dramatically.

However, the rise of unions, the introduction of mechanisms to preserve both the employment and the competition among industries, lead to a substitution of old machines by new ones, more efficient and cheaper. The result of this change towards a *capital-saving* technical progress provided a relative stability of the general rate of profits during a long period.

Appendix I

Sraffa built a composed commodity by transforming the real economic system into a new price system. With the use of appropriate multipliers, the physical surplus of each commodity produced will be in the same proportion by which such commodity is used as mean of production.

During the elaboration of *Production of Commodities by means of Commodities*, Sraffa had the unavailable mathematical support from prominent mathematicians from the Cambridge University, such as Frank Ramsey (1903-1930) and Abram Samailovitch Besicovitch (1891-1970), as richly exposed by H. Kurz & N. Salvadori (2000).

The surplus or net income of this new price system, called by Sraffa as *Standard Commodity*, will be the measure of prices and wages of the real economic system.

Initially it is necessary to determine the *n* multipliers $Q_1, Q_2, \dots, Q_n$ and the rate of surplus R (a scalar), from the n + 1 independent equations, given below:

$$\mathbf{AQ}(1+R) = \mathbf{BQ} \qquad [1\ A]$$

$$\mathbf{LQ} = 1 \qquad [2\ A]$$

The *n* equations of the above system [1 A] have n+1 unknowns. The equation [2 A] completes the set of independent equations in order to determine the multipliers.

One of the characteristics of the Standard System is the presence of the same rate of surplus R in all industries, calculated in physical terms, that is:

$$\mathbf{A_1Q}(1+R) = B_1\, Q_1$$

$$\mathbf{A_2Q}(1+R) = B_2\, Q_2$$

$$\dots$$

$$\mathbf{A_nQ}(1+R) = B_n\, Qn$$

We shall see that the ratio R is determined by the conditions of production *strictu sensu*, regardless the distribution of income

and

$$\mathbf{Q} = \begin{vmatrix} Q_1 \\ Q_2 \\ \dots \\ \\ Q_n \end{vmatrix} \qquad \text{Column vector of the multipliers } Q_j$$

$$
\mathbf{A} = \begin{vmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ & \ldots & & \\ a_{n1} & a_{n2} & \ldots\ldots & a_{nn} \end{vmatrix} \qquad \text{Means of production square matrix}
$$

$$
\mathbf{B} = \begin{vmatrix} B_1 & 0 & \ldots\ldots & 0 \\ 0 & B_2 & \ldots\ldots & 0 \\ & \ldots & & \\ 0 & 0 & \ldots\ldots & B_n \end{vmatrix} \qquad \text{Product or Outuput diagonal matrix}
$$

The order n square matrix of technical coefficients of production **A**, can also be represented by:

$$
\mathbf{A} = \begin{vmatrix} \mathbf{A_1} \\ \mathbf{A_2} \\ \ldots \\ \mathbf{A_n} \end{vmatrix} \qquad \text{Column vector of the commodities used as means of production}
$$

where:

$$\mathbf{A_1} = [\, a_{11} \;\; a_{12} \;\; \ldots \;\; a_{1n} \,]$$

$$\mathbf{A_2} = [\, a_{21} \;\; a_{22} \;\; \ldots \;\; a_{2n} \,]$$

....

$$\mathbf{A}n = [\, a_{n1} \;\; a_{n2} \;\; \ldots \;\; a_{nn} \,]$$

Each $\mathbf{A}_j$ for j = 1, 2, . . . , n; represents the inputs of commodity j used as mean of production in the several industries.

The rate R is analogue to that imagined by Ricardo in his *Essay on Profits*, written in February 1815, as a ratio of two quantities physically homogeneous (*corn-corn* model), so that the quotient obtained between profits and means of production is a pure number.

As a result, the Standard System is therefore represented by:

$$\mathbf{pAQ}(1+r)+\mathbf{LQ}w = \mathbf{pBQ} \qquad [3\text{ A}]$$

Let’s see in detail the construction of the Standard Commodity.

In both the original price system as in the Standard System, the wages should be distributed in each industry according to the respective quantities of direct labour. Hence the necessity of the identity **LQ** = 1

This condition is fundamental to understand the concept of Sraffa's Standard Commodity as well as the distribution of the net income between wages and profits according to the labour theory of value in Sraffa's model.

From the system [3-A] we obtain the net income or surplus, which is divided between wages and profits:

$$\mathbf{pAQ}r+w\mathbf{LQ} = \mathbf{p}(\mathbf{B} - \mathbf{A})\mathbf{Q} \qquad [3.1\ A]$$

When w = 0, r is maximum and identical to R.

Therefore, the system [3.1 A] can be expressed as follows:

$$\mathbf{pAQ}R = \mathbf{p}(\mathbf{B} - \mathbf{A})\mathbf{Q} \qquad [3.2\ A]$$

On the other hand, the maximum rate of profits R can also be written as:

$$(1+R)\mathbf{AQ} = \mathbf{Q} \qquad [4]$$

In order to determine the maximum rate of profits (a scalar) in the Standard System, we have to multiply all elements of both sides of the above equation [4] by a row vector **I** (one row and *n* columns with all elements equal to 1).

$$R = \frac{\mathbf{IQ} - \mathbf{IAQ}}{\mathbf{IAQ}} \qquad [4\ A]$$

In other words, the maximum rate of profits in the Standard System is identical to the rate of surplus R expressed in physical terms, similar to David Ricardo's corn model in his *Essay on Profits*.

Our next step is to demonstrate how we can obtain a linear relationship between wages and profits which attends the mathematical condition on page 8.

The value of the National Income or *surplus* must be exactly equal to the whole quantity of direct labour dispended in the production period, that is:

$$\mathbf{p}(\mathbf{B} - \mathbf{A})\mathbf{Q} = \mathbf{LQ} \qquad [6\ A]$$

Besides, Sraffa adopted the value of the Standard Commodity as the unit of measure of prices and wage, that is:

$$\mathbf{p}(\mathbf{B} - \mathbf{A})\mathbf{Q} = 1 \qquad [7\ A]$$

Going back to the system [3.1 A], it does not change if we multiply all terms by the scalar R:

$$r\mathbf{pAQ}R + \mathbf{LQ}wR = \mathbf{p}(\mathbf{B} - \mathbf{A})\ \mathbf{Q}R \qquad [3.3\ A]$$

Combining [3.2 A], [6 A] with [3.3A] above, we obtain:

$$r\mathbf{p}(\mathbf{B} - \mathbf{A})\mathbf{Q} + \mathbf{p}(\mathbf{B} - \mathbf{A})\mathbf{Q}wR = \mathbf{p}(\mathbf{B} - \mathbf{A})\mathbf{Q}R \quad [8\ A]$$

Finally we have

$$r + wR = R$$

$$r = R(1-w) \qquad [9\ A]$$

Now we have a crystal clear relationship between the rate of profits and the wage rate. As we can see, both prices and quantities were isolated and the logical inconsistency observed in the Marxian price system was completely eliminated.

The level of efficiency of the economic system is given by the maximum rate of profits R. For example, the higher the level of R, the lower will be the quantity of means of production necessary to produce the same quantity of commodities.

Appendix II

Prices and Labour –Values

A general numerical example of a price system with two industries

In our numerical example there are only two industries and two commodities.

Each industry produces only one commodity.

The first industry produces one consumption good, wheat and the second industry produces iron, for example.

The two commodities are also means of production. Combining these two means of production with labour and technology, each industry produces an output at the end of the period of production

In terms of labour-values, the equations of production are very simple:

$$A_{11}v_1 + A_{21}v_2 + L_1 = A_1v_1$$

$$A_{12}v_1 + A_{22}v_2 + L_2 = A_2v_2$$

where v_1 and v_2 are the respective labour-embodied values of commodity 1 and 2.

As we have only two unknowns v_1 and v_2 and two independent equations, the value of the commodities are easily determined, as shown on page 3.

The equations of production are as follow:

$$(A_{11}p_1 + A_{21}p_2)(\delta + r) + w\Lambda_1 = A_1p_1$$

$$(A_{12}p_1 + A_{22}p_2)(\delta + r) + w\Lambda_2 = A_2p_2$$

where:

A_{11} and A_{21} are the physical quantities of commodities 1 and 2 necessary to produce A units of commodity 1 (wheat), in physical terms;

A_{12} and A_{22} are the physical quantities of commodities 1 and 2 necessary to produce A units of commodity 2 (iron) in physical terms;

In our numerical example, the depreciation rate δ is equal to 1, which means that there is no fixed capital, only circulating capital, which is entirely consumed in the production period. Therefore, the term (δ +r) is reduced to (1+r).

The physical surplus (S) in each industry is

$$A_1-(A_{11}+A_{12}) = S_1$$

$$A_2-(A_{21}+A_{22}) = S_2$$

The size of the physical surplus is a reflection of the technical progress that prevails in the industry. Higher the value of S_1 or of S_2, higher the productivity in the respective industry.

p_1 and p_2 are the respective prices of commodity 1 (wheat) and 2 (iron), for example

Λ_1 and Λ_2 = the quantity of direct labour used to produce A quantities of commodity 1 and 2, respectively.

$\Lambda_1 + \Lambda_2 = \Lambda$, the total quantity of direct labour dispend in the economy to produce the net income.

The price equations above, however, can be simplified into a general case:

$$(a_{11}p_1 + a_{21}p_2)(1+r) + wL_1 = p_1$$

$$(a_{12}p_1 + a_{22}p_2)(1+r) + wL_2 = p_2$$

As the value of the National Income or the total economic surplus must be exactly equal to the total quantity of direct labour dispended in the production period, such quantity must be equal to 1, that is, L = 1.

In other words, each term of the identity below was divided by Λ

$$\Lambda_1 + \Lambda_2 = \Lambda$$

$$L_1 + L_2 = 1$$

where $L_1 = \Lambda_1/\Lambda$ and $L_1 = \Lambda_2/\Lambda$

r = the rate of profits, which we suppose to be equal in both industries (free competition).

w = the wage rate as a proportion of net income, that is, $0 < w < 1$

Therefore, we have:

$W_1 = wL_1$, total wages in industry 1

$W_2 = wL_2$, total wages in industry 2

$W_1 + W_2 = W$, total wages in the whole economy.

$(a_{11}p_1+a_{21}p_2)r = P_1$, total profits in industry 1

$(a_{12}p_1+a_{22}p_2)r = P_2$, total profits in industry 2

$P_1+P_2 = P$, total profits for the whole economy

Another concept very useful to understand Marxian economics is the organic composition of capital (κ), which is derived from the labour-embodied value system:

$$(a_{11}v_1+a_{21}v_2)/L_1 = \kappa_1$$

$$(a_{12}v_1+a_{22}v_2)/L_2 = \kappa_2$$

The determination of prices, however, is more complex and it is interesting to make a comparison between Sraffa and Marx *vis-à-vis* the labour-embodied value theory.

The first variable we have to determine is the maximum rate of profits, R. It is not only a pure number. It reflects the productivity level of the whole economy and it is also the best *proxy* for the output to capital ratio (Y/K). This ratio is important to the analysis of economic growth in the long run.

Remembering that R is identical to the rate of surplus expressed in physical terms, we have:

$$(a_{11}Q_1+a_{12}Q_2)(1+R) = Q_1$$

$$(a_{21}Q_1+a_{22}Q_2)(1+R) = Q_2$$

Where Q_1 and Q_2 are the multipliers to transform the real economic system into the Standard System.

In order to calculate the two multipliers and R, considering we have two independent equations, we need one more independent equation and such equation is:

$$L_1Q_1+L_2Q_2 = 1$$

The total direct labour dispend in the Standard System has to be equal to total direct labour of the real economic system, as mentioned above, that is, $(L_1 + L_2 = 1)$.

$$(1+R) = Q_1/(a_{11}Q_1+a_{12}Q_2) = Q_2/(a_{21}Q_1+a_{22}Q_2)$$

In order to make the determination of the respective values of Q_1 and Q_2 easier, we suggest to make Q_2 as a parameter of Q_1, that is $Z = Q_2/Q_1$.

In this sense, we have

$$1/(a_{11}+a_{12}Z) = Z/(a_{21}+a_{22}Z)$$

The above identity has now only one unknown, Z, which will be determined by solving a quadratic equation (2nd degree equation).

Remembering that $Q_2 = ZQ_1$, the following step is to substitute the value of Q_2 in the equation below

$$L_1Q_1+L_2Q_2 = 1$$

which will have only one unknown (Q_1)

$$L_1Q_1+L_2ZQ_1 = 1$$

With the values of Q_1 and Q_2, we obtain the ratio R from one of the two equations of the preceding page.

As we know, Sraffa provided a very useful equation which connects the two distributive variables w and r, as follows

$$r = R(1-w)$$

Before we go ahead, two major premises of the classical approach arise:

- the previous knowledge of the physical output;
- *surplus* or net income is a magnitude independent on the determination of prices and distribution between capitalists and labourers;

In other words, production precedes distribution, contrary to the neoclassical approach by which production and distribution are determined simultaneously.

Prices are determined given previously the level of one of the two distributive variables, the rate of profits (r) or the wage rate (w).

In our general numerical example, the previously known variables are:

- the coefficients of production a_{ij} (commodity *i* utilized by industry *j*)
- the coefficients of direct labour, L_1 and L_2

And the unknown variables are:

- the maximum rate of profits R
- the multipliers Q_1 and Q_2
- the prices of commodities, p_1 and p_2
- the rate of profits r
- the wage rate w

We have seven unknowns as shown above.

Which are the independent equations that will provide the solution for all unknowns?
- two equations of production on page 17
- three equations regarding the multipliers on page 18 above
- the above equation connecting the rate of profits and the wage rate

Therefore, it is missing more one equation to make the number of unknowns equal to the number of independent equations.

As it is also known, Sraffa himself suggested the rate of profits as the exogenous variable in the price system and it is obtained as a function of the money rate of interest (Sraffa, 1960, chap V, section 44, p. 33).

Sraffa called money rate of interest because there is another type of interest rate, namely, the own rate of interest.

The concept of own rate of interest was idealized by Sraffa in 1932, but was Lord Keynes who provided its accurate meaning in the chapter 17 of *The General Theory of Employment, Interest and Money* (1937). In Keynes's words: "…for every durable commodity there is a rate of interest in terms of itself,..." and "…100 quarters of wheat for *spot* delivery will buy 98 quarters for *forward* delivery, it follows that the wheat-rate of interest is *minus* 2 per cent. per annum."

For a full and clear explanation of the subject, I suggest the reading of the excellent essay *Keynes and Sraffa on the concept of commodity rate of interest,* written by Nerio Naldi (2012).

The money rate of interest is fixed by the monetary authority and its level depends on the objective of the Central Bank. Such objective may be, for instance, the control of inflation provoked by a fast economic growth or to stimulate the consumption of goods and services in the economy.

The level of the rate of profits is the sum of the money interest rate (i) fixed by the monetary authority with a risk premium, both in annual terms, that is,

$$r = i+\sigma$$

where σ is the risk premium in order to offset the uncertainty (business risk), since *a priori* the money invested in government bonds has no risk.

The level of the risk premium is different among industries. Each industry faces different types of uncertainty during the period of production. As a consequence, the rate of profits in one industry may be higher or lower in relation to the general or average rate of profits.

In this sense, the hypothesis of free competition among industries must be abandoned, unless for didactic purposes we adopt the same risk premium rate (σ) in all industries.

Once the rate of profits in fixed *outside* de system of production, the wage will be a residue, because the value of R, the maximum rate of profits, is determined *inside* the production system, that is, by the technical coefficients of production, independent on the determination of prices and distribution of income between wages and profits.

A numerical example

A] Sraffa's price system

Coefficients of production:

Industry 1 | Industry 2
$a_{11} = 0.6114019520$ | $a_{21}= 0.33$

$a_{12} = 0.25$ | $a_{22} = 0.55$

Labour coefficients:

industry 1 | industry 2
$L_1 = 0.60$ | $L_2 =0.40$

$L_1+L_2 = 1$

Therefore, the price system is as follows:

$$(0.6114019520p_1+0.33p_2)(1+r)+0.6w = p_1$$

$$(0.250000000p_1+0.55p_2)(1+r)+0.4w = p_2$$

And the physical surplus (S) in each industry is

$$1-(a_{11}+a_{12}) = S_1 = 0.13859805$$

$$1-(a_{21}+a_{22}) = S_2 = 0.12000000$$

Now it is possible to determine the multipliers Q_1 and Q_2 and also the maximum rate of profits R, according to our explanation on pages 18 and 19.

$Q_1 = 0.98710717$

$Q_2 = 1.01933924$

$R = 0.1500000000$ or 15%

In order to round the value of R to 15%, it was fixed ten decimal places for the physical coefficient a_{11}.

We have four unknowns, p_1, p_2, r and w and three independent equations, two from the price system and a third equation represented by $r = R(1-w)$.

Suppose the annual interest rate or in a modern language, the opportunity cost of capital, is, for instance, 6% and the risk premium 4%. Therefore, the rate of profits is 10% and we suppose that such percentage is in accordance with the capitalist's expectations.

Given r = 10%, it is now possible to determine the share of the labourers in the net income, as follows:

$r = R(1-w)$

$0.10 = 0.15(1 - w)$, therefore, $w = 1/3$

The determination of each commodity price is the most difficult phase of the calculus and it demands great attention.

$$(a_{11}p_1+a_{21}p_2)(1+r)+wL_1= p_1$$

$$(a_{12}p_1+a_{22}p_2)(1+r)+wL_2 = p_2$$

There are basically two methods to solve the above equations: matricial calculus or by substitution.

If we adopt the method of substitution, we will have:

$p_1 = [(1+r)(a_{22}wL_1 - a_{21}wL_2) - wL_1]/[(a_{12}\, a_{21} - a_{22}\, a_{11})(1+r)^2 +(a_{11}+a_{22})(1+r) - 1]$

$p_2 = \{p_1[1- a_{11}(1+r)] - wL_1\}/[a_{21}(1+r)]$

In p_1 all the variables are previously known and the result is 4.31559363

With the value of p_1, p_2 is easily found and equals 3.34207995

Worth mentioning the price of a commodity depends not only on the conditions of production (technology and labour), but also on distribution of income between wages and profits, as can be seen above in the equation of p_1.

Remember that the wage rate is determined as a residue, since the distribution of net income was determined outside the system of production. In our numerical example, w = 1/3. If such level of the wage rate is or not is in accordance with both the labourer's standard of living and expectations is another question to a future discussion.

On the other hand, if w = 1, that is, all the net income goes to wages, the price of a commodity is exactly to its correspondent labour -embodied value, proving Adam Smith's dictum, that is:

p_1.= 4.35210374 and p_2.= 3.30672430

B] Labour values

$$a_{11}v_1+a_{21}v_2+L_1 = v_1$$

$$a_{12}v_1+a_{22}v_2+L_2 = v_2$$

With the same figures above for both the coefficients of production and direct labour in each industry, the labour-embodied value of each commodity is

v_1 = 4.35210374

v_2 = 3.30672430

Marx suggested the identity net income *equals* surplus value and the wage as a previously known basket of consumption goods.

In other words, it means that $p_i = v_i$ (i = 1, 2, ..., n) and $w = p_1\, c_1+ p_2\, c_2+ \ldots + p_k\, c_k$

Based on Marx's premise, the rate of profits in each industry is

$r_1 = 10.6607\%$

$r_2 = 9.1741\%$

The two postulates of invariance above suggested by Marx do not provide the same magnitude or result for the rate of profits in the two industries, which is contrary to the premise of free competition.

Using a software spreadsheet, for example, it is also possible to make numerical simulations with the distribution of income and also to make comparisons between the results obtained from the Sraffa's price system with that obtained from Marx's.

Appendix III

Technical progress and the distribution of income

Let's see now how technical progress affects the distribution of income in a very simplified situation.

In this general numerical example there are only two industries and two periods of production.

The prevailing technical progress could be of three types, as follow:

- *Capital-saving*: there is a decrease in the quantity of inputs or in the coefficients of production (a_{ij}) in order to produce one unity of commodity i (i = 1, 2, ..., n). The output to capital (Y/K) increases.

- *Capital–using*: there is an increase in the quantity of inputs or in the coefficients of production (a_{ij}) in order to produce one unity of commodity i (i = 1, 2, ..., n). The output to capital ratio (Y/K) decreases.

- *Neutral*: there is no change in the coefficients of production from a period to other. The output to capital ratio (Y/K) keeps flat.

The general price equations at end of Period 1:

$$(50p_1 + 30p_2)(1+r) + 50w = 100p_1$$

$$(30p_1 + 20p_2)(1+r) + 30w = 70p_2$$

The first industry needs 50 tons of commodity 1 and 30 tons of commodity 2 plus 50 units of direct labour to produce 100 tons of commodity 1.

The second industry needs 30 tons of commodity 1 and 20 tons of commodity 2 plus 30 units of direct labour to produce 70 tons of commodity 1.

The surplus is given by:

$$20p_1 + 20p_2$$

The two above general price equations can be transformed into Sraffa's price equations:

$$[(50/100)p_1 + (30/70)p_2](1+r) + (50/80)w = (100/100)p_1$$

$$[(30/100)p_1 + (20/70)p_2](1+r) + (30/80)w = (70/70)\ p_2$$

Which can be reduced to

$$[0.5p_1 + (3/7)p_2](1+r) + (5/8)w = p_1$$

$$[0.3p_1 + (2/7)p_2](1+r) + (3/8)w = p_2$$

Surplus: $0.2p_1 + 0.2p_2$

Note that the each of direct quantity of labour was divided by 80, the sum of L1 with L2.

With the general formulas on page 3, 4 and 8, we obtain the following results:

$Q_1 = 1.042901$ $Q_2 = 0.928498$

R = 0.303626 or 30.3626%

Let's suppose the wage rate is 0.5

$p_1 = 2.652030$ $p_2 = 1.645365$

r = 0.151815 or 15.1815%

Net income (Y): 1.000510

Capital (K): 3.296885

Therefore, the ratio Y/K = 0.303471

Total profits (P) = 0.500510

Total wages (W) = 0.500000

Output to capital ratio (Y/K) = 0.303471

In the next period, the entrepreneur of industry 1 uses a new technique, which reduces losses of raw materials during the production. The quantity of commodity 1 necessary to produce 100 tons of commodity 1 was reduced in 10%, from 50 tons to 45 tons.

This is a *capital-saving* technique.

Therefore, the new price equations in Period 2 are given below:

$$(45p_1 + 30p_2)(1+r) + 50w = 100p_1$$

$$(30p_1 + 20p_2)(1+r) + 30w = 70p_2$$

Now the first industry needs 45 tons of commodity 1 and 30 tons of commodity 2 plus 50 units of direct labour in order to produce 100 tons of commodity 1.

The second industry needs 30 tons of commodity 1 and 20 tons of commodity 2 plus 30 units of direct labour to produce 70 tons of commodity 1.

The surplus or Net Income is given by:

$$25p_1 + 20p_2$$

The two above general price equations transformed into Sraffa's price equations:

$$[0.45p_1 + (3/7)p_2)](1+r) + (5/8)w = p_1$$

$$[0.30p_1 + (2/7)p_2)](1+r) + (3/8)w = p_2$$

Surplus: $0.25p_1 + 0.20p_2$

$Q_1 = 1.018182$ $Q_2 = 0.969697$

R = 0.359223 or 35.9223%

Let's suppose again that the wage rate is 0.5. The labourers kept the same share in the National Income.

$p_1 = 2.285161$ $p_2 = 1.502608$

r = 0.179612 or 17.9612%

Net income (Y): 1.000607

Capital (K): 2.787162

Therefore, the ratio Y/K = 0.359006

Total profits (P) = 0.500607

Total wages (W) = 0.500000

Output to capital ratio (Y/K) = 0.359006

Which are the major conclusions when we compare the two periods above?

The new technique in industry 1 is more efficient than the previous one, because the surplus in such industry rose from 20 tons to 25 tons with the same quantity of inputs (direct labour and commodity 2).

As we saw on pages 18 and 19, the determination of the ratio R depends on both the physical quantities of commodities and direct labour. If there is no physical surplus, R will be null.

In the Period 2 the Standard ratio rose from 30.4% to 35.9%, which explains the rise in the value of net income in relation to the value of means of production.

The output to capital ratio, Y/K, also rose from 0.303 to 0.359, since the *capital-saving* technique in industry 1 on the one side provided a significant increase in the net income and on the other reduced the value of the capital employed in the production of commodity 1.

Regarding the *capital-using* technique, the conclusions will be the opposite of the *capital-saving.* If the *capital-using* technique leads to a decrease in the Standard ratio R, why such technique is adopted in some industries? In my humble opinion, it is because the entrepreneurs in order to increase the market-share of their products are more interested to increase the sum of profits even with a fall of the rate of profits. Of course, not below a minimum level that does not offset the business risk.

There is a long discussion in the economic literature about the choice of techniques since Nobuo Okishio (1927-2003), emeritus professor at Kobe University in Japan, published in 1961 his famous paper "Technical Changes and the Rate of Profit", *Kobe University Economic Review* vol 7, pp 85-99.

Appendix IV

Rent and interest

Let's suppose the industrial plants of both industries are located in the properties of third parties and the entrepreneurs pay an annual rent for the *rentiers* or owners of such real state properties.

We suppose that both the entrepreneurs and the *rentiers* signed a long term contract for the use of the real state properties. The contract is important to avoid future demands from both sides regarding eventual oscillations in the economic cycle, which may affect the level of the money interest rate, for instance.

The *price* the *rentier* charges for the use of his land is the money rate of interest, the same we referred to in the Appendix II on page 20.

This annual rent C_j is a fixed cost for entrepreneur and it is given by the following relation

$C_1 = i\lambda_1$

$C_2 = i\lambda_2$

where:

λ_j (j =1,2) can be interpreted as the quantity of the real state property adjusted per unit of commodity j produced.

The annual rent is paid by the entrepreneur to the *rentier* at the end to the production period and it is a deduction of gross profits in each industry.

$r(a_{11}p_1+a_{21}p_2) - C_1$ = remaining profits after the deduction of rent in industry 1

$r(a_{12}p_1+a_{22}p_2) - C_2$ = remaining profits after the deduction of rent in industry 2

Therefore, in this case the effective rate of profits for the entrepreneurs will be lower than that obtained by the relation $r = i+\sigma$ or by $r = R(1-w)$ and it depends on the magnitude of C_j.

The rent paid to owners of land and real state properties is a complex question in any country and particularly in some countries where land and real state properties for residential, commercial and industrial purposes in large cities are highly concentrated in the hands of few people or families due to historical reasons. The rent requested by the owner in several cases is so high that any business can not thrive for a long time. This situation explains partially the high cost of living in the majority of large cities in some countries.

As far I am concerned, in some countries of Europe such question was partially solved many decades ago by the government through a higher taxation on personal income or on the value of land and real state properties. Great part of the additional taxation is reverted to companies and families that pay a high rent.

Likewise the amount of interest paid by the entrepreneur on loans from banks to finance his business must be treated as a deduction of gross profits in Sraffa's price system.

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