Modeling and Applied Research in Sustainable Development

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Abstract: We develop an algebraic polynomial model to measure and compare the sustainability in 4 countries after studying existing sustainable development index systems. The model consists of three facets of indicators: natural resources reserve, environment carrying capacity and social welfare level. We use recursive least-squares method (RLS) to determine the parameters of the fitted model and apply this model to design a sustainable development plan for Tanzania. Considering the country profile and model testing results, the plan comprises of five programs: producing clean water, generating electricity, improving transport conditions, developing tourism industry and advancing medical and health services. Finally we predict the change of each indicator in the next two decades and compare the results under natural state, finding that the sustainability of Tanzania will increase.

1. Research Overview

1.1 Methodology

There are various methods to assess a country′s sustainability [1-8], mainly including Environmental Sustainability Index (ESI), Ecological Footprint (EF), Emergy Analysis (EmSI), Sustainability Evaluation using Indicators (SEI), etc. However, all of the above methods have their pros and cons in the following aspects.

(1) *Calculation principles:* ESI bases its criteria on a series of complicated concept descriptions, each of which has different unit and carries too much information. Thus the comparability of data is limited. EmSI requires knowledge of emergy based on thermodynamics in an open system. EF has too simple principles that production and consumption are converted into acreage. SEI has no concrete theoretical support and the assessment criteria are also determined by evaluation object.

(2) *Computation process:* ESI uses multiple algorithms to replace missing data, statistical tool and hypothesis, and is the most strenuous method; EF use a sole measurement of acreage but the calculation of acreage itself is very complicated; EmSI converts all variable data into solar emergy joules (sej), so it requires many thermodynamic coefficients such as eddy diffusivity and vertical diffusion coefficient and thus is also very complicated. SEI, with no specific measure, consists of different units of indexes and is comparatively easy to compute.

(3) *Time scale:* The listed methods can only be used to assess the current status of evaluation objectives so they are often seen as static methods. However, it is clear that the ecosystem and economic system are both dynamic. With the change of technology and social system, those indicators will also change and need dynamic interpretation. ESI and SEI use the data within current range so they can only estimate the current system. The builders of EF consider it an″ ecology camera″ [9], which means that every analysis is like a photo of current needs of nature, economy and society. EmSI is the only one found to develop a set of dynamic model to explain the world [10-11] because this theory considers the changes of internal reserves pushed by external force, making it possible to build an extrapolation model of ecosystem and economic system.

(4) *Spatial scale:* EF an EmSI can be applied in all different spatial scales (e.g. global

scale), while ESI is only applicative in national scale, and SEI the regional scale.

(5) *Distinguishment of resource sustainability:* Only EmSI has a clear system of distinguishing between renewable and non-renewable resources. EF has such distinction when considering ecological carrying capacity but it does not reflect that distinction in computation process. So eventually EF fails to clearly express the degree of sustainability. In ESI and SEI, many of their indicators represent the utilization of renewable resources, the concept of which, however, are not directly stated.

(6) EF and EmSI suppose that residents in a region are not isolated. They consume goods imported, and also goods exported. The difference is that EmSI uses emergy of import and export as the variable of system assessment and as well an indicator of it. EF, on the other hand, is the indirect mode in consumption calculation and is not reflected among the ultimate indicators.

(7) ESI is prone to explain the ability of a country when facing sustainability crisis, which represents the nation′s strength to cooperate with other countries when managing environmental problems, both locally and globally. Whereas EF and EmSI can be applied to individual, local, regional and global levels, and they focus on explaining the effects on global sustainability.

1.2 Sustainable development index system

Since the establishment of the UN Conference on Environment and Development in 1992, governments of all countries have gradually recognized sustainable development pattern. Although people understand sustainable development in many different ways, it is generally defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs." We see sustainable development as the realization of economic, social and environmental goals, and as the harmonious interaction of three related systems. It covers the achievement of economic development and efficiency, effective allocation of natural resources, improvement of environment and realization of social equity, etc. OECD proposed in 1991 the sustainable development indicators system and its diagram of pressure - status – response can be seen in Figure 1.

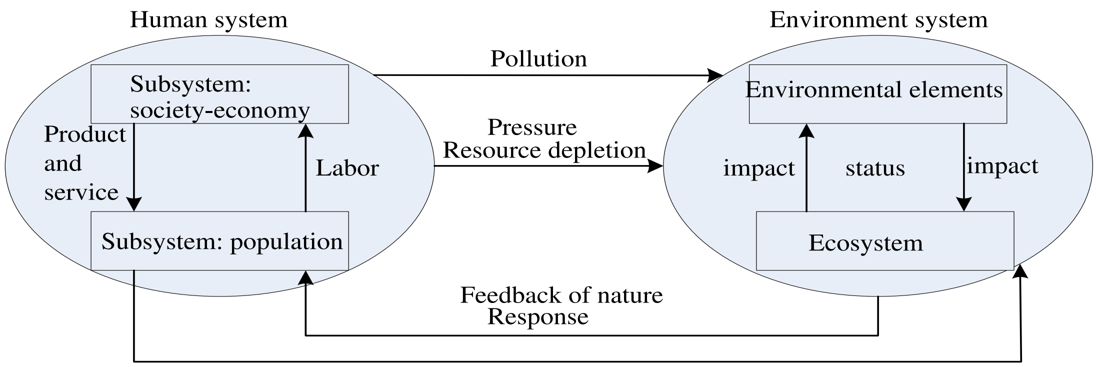

Figure 1: Concept framework: Index system of sustainable development

Here the status indicator refers to physical and ecological status and the subsequent social and economic development. The pressure indicator refers to human activities that affect environment, mainly linking to the cause of environmental issues. The response indicator refers to countermeasures of those issues, showing the efforts our society makes to deal with pollution and resources destruction. From the three facets of indicators, we can see the indicator system should not only provide quantifying information about environment changes, but reveal how we are going to handle it. Therefore, we conclude that a complete sustainable development indicators system should have the following features.

(1) User orientation: the indicators should serve the need of public and decision makers.
(2) Policy relevance: the indicators must be able to reflex what the authority concerns and can show to what degree the policies work in improving sustainability.
(3) Integration of indicators and the quantification: facilitating their own evaluation.
(4) Assessment practice: requiring corresponding indicators to assess different facets.

According to the above features and the understanding of sustainability, sustainable development indicators system should also serve the following functions (see Figure 2).
(1) Able to describe current status of various facets in development at some point.
(2) Able to reflex the changing trend of various facets in development at some point.
(3) Able to reveal the coordination degree of various facets in development at some point.

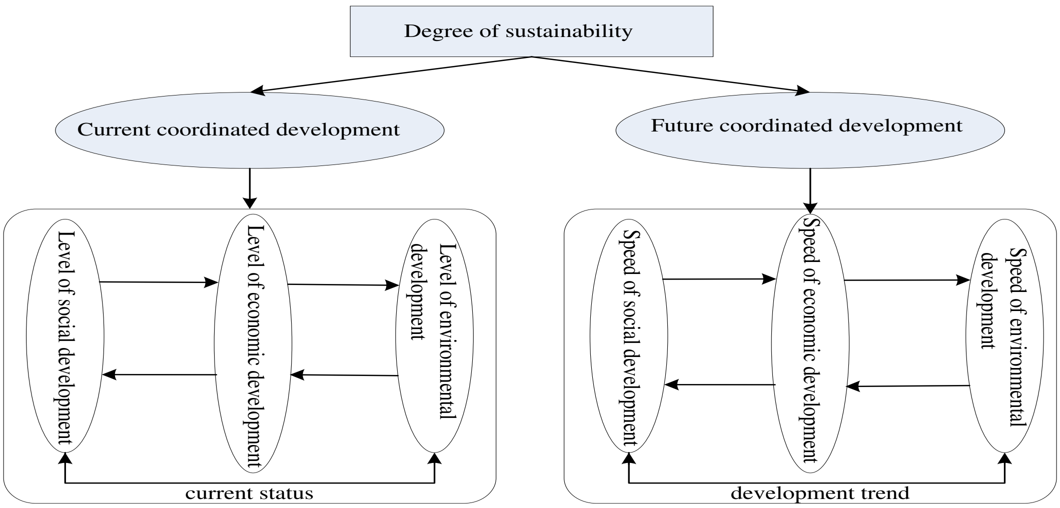

Figure 2: Function of sustainable development indicators

Sustainable development indicator system includes resources reserves, environment carrying capacity, socio-economic development status and the superstructure (see Figure 3).

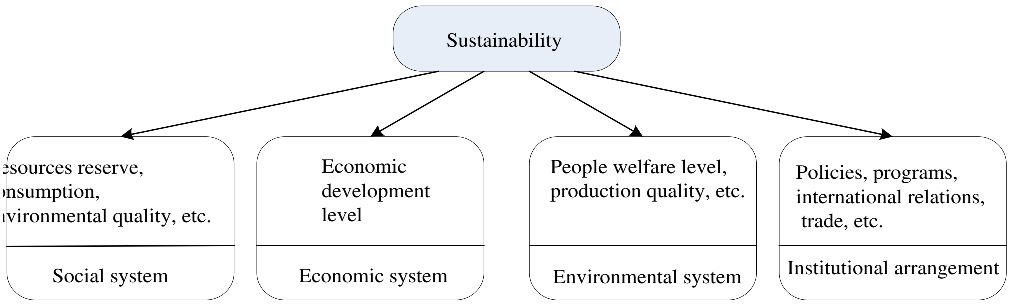

Figure 3: Content of sustainable development indicators system

1.3 Selection of sustainable development indicators

The connotation of sustainable development has at least two points. First, the natural resources reserve and environmental carrying capacity are limited, thus combining to the restrictive condition of economic development. Second, development should meet the needs of the present while not compromising the ability of future generations to meet their own needs and hence the social welfare should increase as time changes. Based on that understanding, we see the following three facets as criteria of whether the development is sustainable or not and to what degree.

(1) If the quantity of emission, effluent or other waste exceeds the environmental carrying capacity, then the development concerned is not sustainable.

(2) If the exploitation rate of renewable resources exceeds their regrowth rate (or replacement rate), or the utilization of non-renewable resources exceeds their replacement rate, then we cannot achieve sustainable development in a sense of" increase total capital stock while time changes".

(3) If the social welfare level does not increase, then the development is also not sustainable.

In order to facilitate analysis, we use natural resources reserve, environmental carrying capacity and social welfare level as three facets for rating sustainability. According to historical data, we build a fitting model of algebraic polynomial and use recursive least-squares method (RLS) to calculate model parameter. Then we use the model to predict the changing tendency of these three indicators in 20 years and to determine sustainability levels according to parameter variation and compare among four different countries.

2. Model Construction

We carry out our research mainly from three indexes: natural resources reserve per capita, environmental carrying capacity per capita and social welfare level, each of which has three indicators:

Table 1: Index hierarchy

Upper level	**Lower level**
Natural resources reserve per capita index	Agricultural acreage per capita
	Forest area per capita
	Renewable water resource per capita
	Corp production index
Environment carrying capacity per capita index	Carbon dioxide emissions per capita
	Organic matter emissions per capita
	Energy consumption per capita
Social welfare level index	Health expenditure per capita
	GDP per capita
	Income Gini coefficient

Table 2: Symbol definition

Symbols	**Definition**
m	No. m Index
i	No. i Indicator
t^j	power function
$g_{mi}(t)$	the function(value) sample data in m index and I indicator
t_k	the sample data selected
$p_{mi}(t_k)$	the polynomial in the function of $g_{mi}(t)$
$r_{mi}(t_k)$	the remainder error associated with $p_{mi}(t_k)$
w_{jmi}	the weight vector

A	the $(n+1)\times(n+1)$ *Vandermonde* matrix
$e_{mi}(k)$	the error function
J_{mi}	the objective function
Q	the *Kalman* gain vector
R	the correlation matrix
I	the identity matrix of degree $(n+1)$
λ	forgetting factors

2.1 Model Assumptions

- No global natural disasters or world war will happen in the following 20 years, thus the development environment remains largely stable.
- The selected indicators of different countries share the same caliber and precision.
- The factors that have low regularity and comparability are neglected (e.g. political factors).
- Factors influencing a country's sustainability level can be represented by 9 indicators.

Table 3: Factors categorization

Factors	**Substitute Indicators**
Human health	Average expenditure on medical care and public health
Food security	Average agricultural acreage and crop production index
Clean water access	Renewable water resource
Local environmental quality	Average forestland
	Average carbon dioxide emission
	Average organic matter emission
Energy access	Average energy consumption
Living standard	GDP per capita
Social equity	Gini Coefficient

2.2 Algebraic Polynomial model based on the power function basis

Assume sample data in the i indicators of m index as $\{g_{mi}(t_k), t_k\}$. Considering the power function basis $t^j \in C^\infty[0,1]$, hence, let

$$t = (x-a)/(b-a) \tag{1}$$

or

$$x = (b-a)t + a \tag{2}$$

then $x \in [a,b] \Leftrightarrow t \in [0,1]$.

Let us observe that: without the loss of generality, we can consider interval $\{t\}$ ordered and belonging to $[0,1]$. So we will solve the following problem:

If $t_0, t_1, \cdots t_n$ are $n+1$ distinct numbers and $g_{mi}(t)$ is a function whose values are given at those numbers, i.e. $g_{mi}(t_k)$ for each $k = 0,1,\cdots,n$, then a unique polynomial $p_{mi}(t)$ of degree at most n exists with

$$g_{mi}(t_k) = p_{mi}(t_k) + r_{mi}(t_k)\text{, for each } k = 0,1,\cdots,n$$

where, $r_{mi}(t_k)$ is called the remainder error associated with $p_{mi}(t_k)$, and the nth algebraic polynomial was given by

$$p_{mi}(t) = \sum_{j=0}^{n} w_{j\ m} t_i^j \tag{3}$$

We know from the Eq. (3) that such an algebraic polynomial is a least-square fit to the data if it minimizes the sum of the squares of the deviations from the data,

$$\sum_{k=0}^{n} \left| g_{mi}(t_k) - p_{mi}(t_k) \right|^2 \tag{4}$$

Solving this problem is equivalent to solve in the least squares sense the linear system $\mathbf{A}\mathbf{w}_{mi} \approx \mathbf{g}_{mi}$, where, $\mathbf{g}_{mi} = [g_{mi}(t_0), g_{mi}(t_1), \cdots, g_{mi}(t_n)]^T$ is called the data vector, and $\mathbf{w}_{mi} = [w_{0mi}, w_{1mi}, \cdots, w_{nmi}]^T$ is called the weight vector containing the weighted coefficients of the power function basis that we want to compute on $[0,1]$, and the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & t_0 & \cdots & t_0^n \\ 1 & t_1 & \cdots & t_1^n \\ \vdots & \vdots & \cdots & \vdots \\ 1 & t_n & \cdots & t_n^n \end{bmatrix} \tag{5}$$

is called the $(n+1) \times (n+1)$ *Vandermonde* matrix associated with the power function basis t_k^j, for $j = 0,1,\cdots,n$ and $k = 0,1,\cdots,n$. If $\mathbf{A}$ has full rank $n+1$, then the problem has a unique solution $\mathbf{w}_{mi}^* \approx \mathbf{A}^{-1}\mathbf{g}_{mi}$ given by the following normal linear system

$$\mathbf{A}\mathbf{w}_{mi} \approx \mathbf{g}_{mi} \tag{6}$$

Since $\mathbf{A}$ is usually an ill-condition matrix, it was early recognized that solving the normal equations was not an adequate method [12-13]. In order to solve the problem, we proposed the adaptive method based on RLS algorithm in the next section.

2.3 Adaptive method to solve weighted coefficients

The recursive least square (RLS) algorithm has been extensively used in adaptive filtering, self-tuning control systems, prediction and interference cancellation, system identification, etc. [14-16]. In order to compute the weighted coefficients of nth order algebraic polynomial $p_{mi}(t)$ by RLS algorithm, the error function $e_{mi}(k)$ and the objective function J_{mi} were given respectively, as follows

$$e_{mi}(k) = g_{mi}(t_k) - \mathbf{A}(k,:)\mathbf{w}_{mi} \tag{7}$$

where, $p_{mi}(t_k) = \mathbf{A}(k,:)\mathbf{w}_{mi}$.

and

$$J_{mi} = 0.5\sum_{k=0}^{n} e_{mi}^{2}(k) \tag{8}$$

To minimize the J_{mi}, the weight vector $\mathbf{w}_{mi}$ is recursively calculated via using a Recursive Least Squares (RLS) rule [14-16]. In the standard RLS algorithm, the update of the vector $\mathbf{w}_{mi}$, the *Kalman* gain vector $\mathbf{Q}$, and the correlation matrix $\mathbf{R}$, is described as

$$\mathbf{Q}^{k} = \frac{\mathbf{R}^{k}\mathbf{A}^{T}(k,:)}{\lambda + \mathbf{A}(k,:)\mathbf{R}^{k}\mathbf{A}^{T}(k,:)} \tag{9}$$

$$\mathbf{w}_{mi}^{k+1} = \mathbf{w}_{mi}^{k} + \mathbf{Q}^{k} e_{mi}(k) \tag{10}$$

$$\mathbf{R}^{k+1} = \frac{1}{\lambda}[\mathbf{I} - \mathbf{Q}^{k}\mathbf{A}(k,:)]\mathbf{R}^{k} \tag{11}$$

where, $k = 0,1,\cdots,n$, $\mathbf{R}^{0} = \quad \mathbf{I} \quad \mathbf{R}^{(n+1)\ (n+1)}$. Generally, set $\alpha = 10^{6} \sim 10^{12}$, $\mathbf{I} \in \mathbf{R}^{(n+1)\times(n+1)}$ is called the identity matrix of degree $(n+1)$, and forgetting factor $0 < \lambda \le 1$.

Theorem1. Suppose $t_0, t_1, \cdots, t_n$ are distinct numbers in $[0,1]$ and $g_{mi}(t) \in C^{n+1}[0,1]$. Then, a number ξ in $(0,1)$ exists with truncation error that

$$r_{mi}(t) = \frac{g_{mi}^{(n+1)}(\xi)}{(n+1)!} t^{n+1} = o\left[\frac{1}{(n+1)!}\right] \tag{12}$$

Proof. Note first that if $t = t_k$, for each $k = 0,1,\cdots,n$, let

$$g_{mi}(t_k) = p_{mi}(t_k) + r_{mi}(t_k), \text{ for each } k = 0,1,\cdots,n \tag{13}$$

If $t \ne t_k$, for all $k = 0,1,\cdots,n$, define the function q_{mi} for τ in $[0,1]$ by

$$q_{mi}(\tau) = g_{mi}(\tau) - p_{mi}(\tau) - r_{mi}(t) \tag{14}$$

Here, $r_{mi}(t) = g_{mi}(t) - p_{mi}(t)$.

Suppose $g_{mi}(t) \in C^{n+1}[0,1]$. Since $p_{mi} \in C^{\infty}[0,1]$ and $r_{mi} \in C^{n+1}[0,1]$, hence, it follows that $q_{mi} \in C^{n+1}[0,1]$. If $\tau = t = t_k$, for each $k = 0,1,\cdots,n$, we have

$$q_{mi}(t_k) = g_{mi}(t_k) - p_{mi}(t_k) - r_{mi}(t_k) = 0$$

If $\tau = t \ne t_k$ for each $k = 0,1,\cdots,n$, let

$$r_{mi}(t) = K_{mi} t^{n+1} \tag{15}$$

We have

$$q_{mi}(t) = g_{mi}(t) - p_{mi}(t) - K_{mi}t^{n+1} \tag{16}$$

Moreover, there must exist $t_\varsigma \neq t_k$, for all $k = 0,1,\cdots,n$, with the property that

$$q_{mi}(t_\varsigma) = g_{mi}(t_\varsigma) - p_{mi}(t_\varsigma) - K_{mi}t_\varsigma^{n+1} = 0 \text{, for } t_\varsigma \in (0,1)$$

Thus, $q_{mi}(t)$ is zero at $(n+2)$ distinct numbers $t_\varsigma, t_0, t_1, \cdots, t_n$. By the Generalized Rolle's Theorem, there exists a number ξ in (0,1) for which $q_{mi}^{(n+1)}(\xi) = 0$. So

$$0 = q_{mi}^{(n+1)}(\xi) = g_{mi}^{(n+1)}(\xi) - p_{mi}^{(n+1)}(\xi) - K_{mi}(n+1)! \tag{17}$$

Since $p_{mi}(t)$ is a polynomial of degree at most n, the $(n+1)st$ derivative, $p_{mi}^{(n+1)}(t)$, is identically zero. Equation (17) now becomes

$$0 = g_{mi}^{(n+1)}(\xi) - 0 - K_{mi}(n+1)!$$

Considering the Eq. (15), we have

$$r_{mi}(t) = \frac{g_{mi}^{(n+1)}(\xi)}{(n+1)!} t^{n+1} = o\left[\frac{1}{(n+1)!}\right]$$

The error formula in *Theorem 1* is an important theoretical result. Error bounds for the technique are obtained from the error formula of algebraic polynomial.

3. Model Application

We select America, Sweden, China and Tanzania as objects and do the fitting and forecasting of 10 indicators among 3 indexes. Set $\lambda = 1$, $\alpha = 10^6$. The study results are as follows:

3.1 Index of natural resources reserve per capita

(1) Agricultural Acreage per capita

The data of four countries are shown in Table 4 (We leave out the data tables in the following analysis in order to save space).

Table 4: Agricultural Acreage per capita/unit: hectare/person

	America	Sweden	China	Tanzania
2004	0.57	0.30	0.09	0.25
2005	0.56	0.30	0.09	0.25
2006	0.54	0.29	0.09	0.24
2007	0.54	0.29	0.08	0.24
2008	0.54	0.28	0.08	0.27
2009	0.52	0.28	0.08	0.26

2010	0.52	0.28	0.08	0.26
2011	0.51	0.28	0.08	0.25
2012	0.49	0.27	0.08	0.30
2013	0.48	0.27	0.08	0.25

Data source: http://data.worldbank.org (missing values are supplemented by interpolation method)

Assume $a = 2004, b = 2033, n = 1$, from formula (6) we use RLS algorithm to fit and predict the data above. The results are shown in Figure 4.

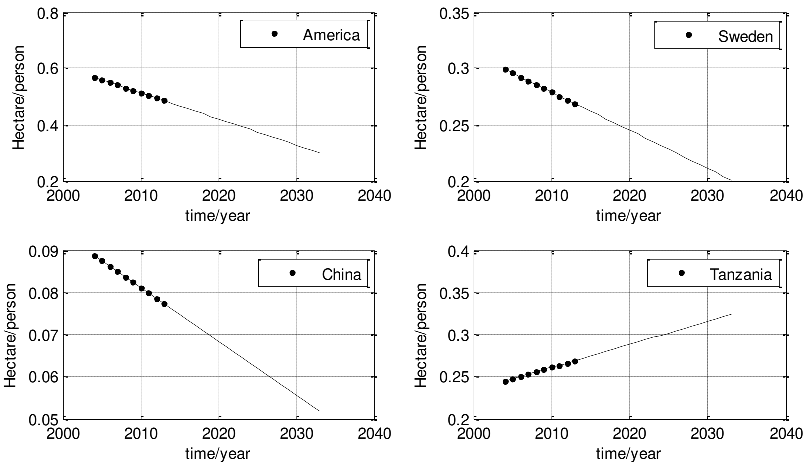

Figure 4: Fitting and forecasting results of agricultural acreage per capita

(2) Forest Area per capita

Assume $a = 2004, b = 2035, n = 2$, from formula (6) we use RLS algorithm to fit and predict the data above. The results are shown in Figure 5.

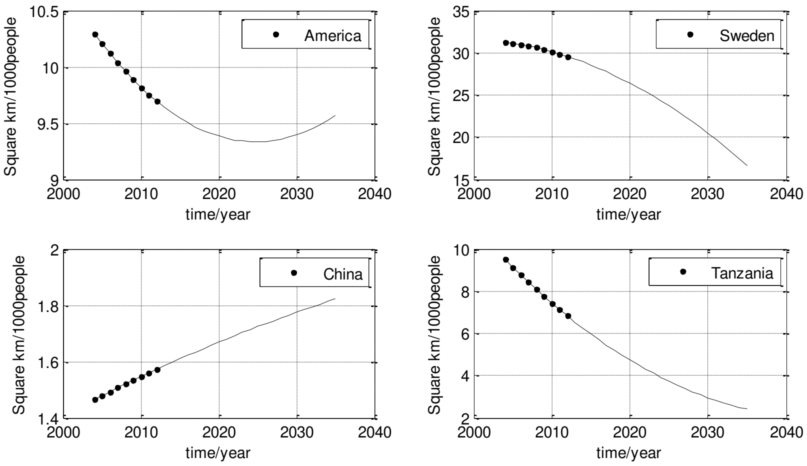

Figure 5: Fitting and forecasting results of forest area per capita

(3) Renewable Water Resource per capita

Assume $a = 1992, b = 2035, n = 2$, from formula (6) we use RLS algorithm to fit and predict the data above. The results are shown in Figure 6.

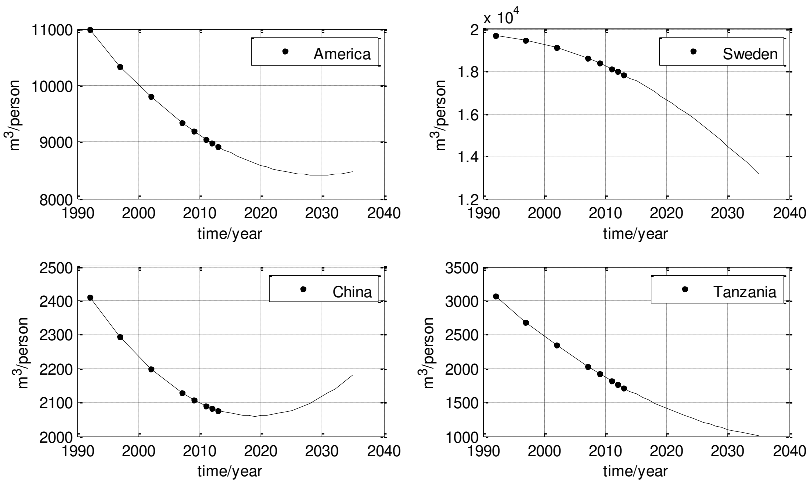

Figure 6: Fitting and forecasting results of renewable water resource per capita

(4) Crop production index

We find that there are no significant differences among those four countries hence we dismiss that indicator from consideration.

3.2 Index of Environmental carrying capacity per capita

(1) Energy Consumption per capita

Assume $a = 2004$, $b = 2035$ $n = 1$ from formula (6) we use RLS algorithm to fit and predict the data above. The results are shown in Figure7.

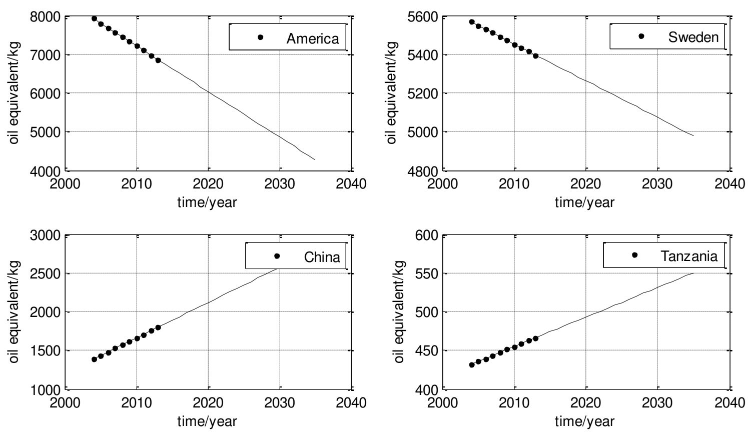

Figure 7: Fitting and forecasting results of energy consumption per capita

(2) co_2 Emissions per capita

Assume $a = 2004$, $b = 2035$, $n = 1$ from formula (6) we use RLS algorithm to fit and predict the data above. The results are shown in Figure 8.

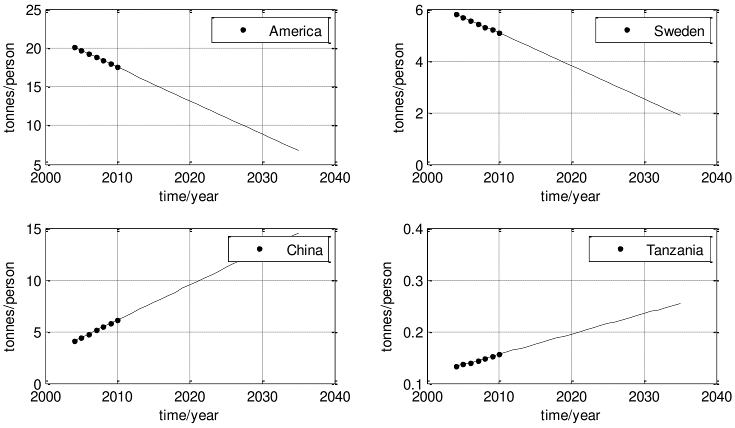

Figure 8: Fitting and forecasting results of co_2 emissions per capita

(3) Organic Matter Emissions per capita
We find that there are no significant differences among those four countries hence we dismiss that indicator from consideration.

3.3 Index of social welfare level

(1) GDP per capita
Assume $a = 2004$, $b = 2035$, $n = 1$ from formula (6) we use RLS algorithm to fit and predict the data above. The results are shown in Figure 9.

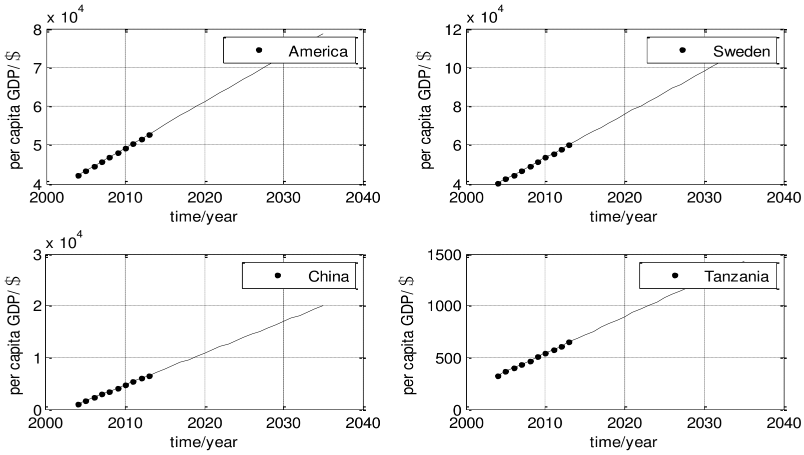

Figure 9: Fitting and forecasting results of GDP per capita

(2) Health expenditure per capita
Assume $a = 2004$, $b = 2035$, $n = 1$ from formula (6) we use RLS algorithm to fit and predict the data above. The results are shown in Figure 10.

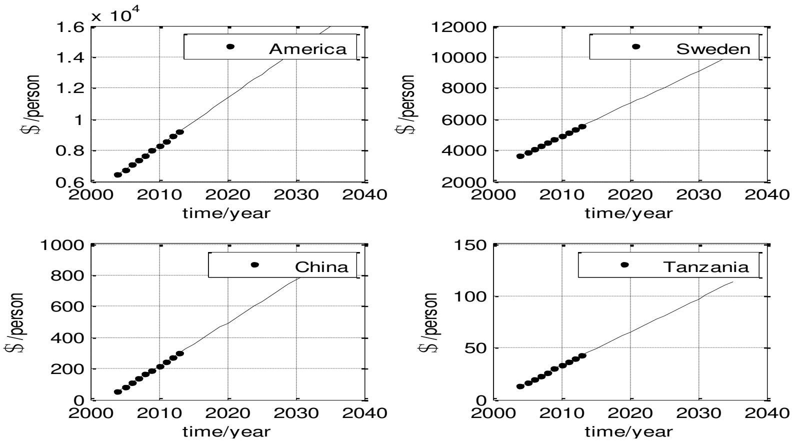

Figure 10: Fitting and forecasting results of Health expenditure per capita

(3) Income Gini Coefficient

Assume $a = 2004$, $b = 2035$, $n = 1$ from formula (6) we use RLS algorithm to fit and predict the data above. The results are shown in Figure 11.

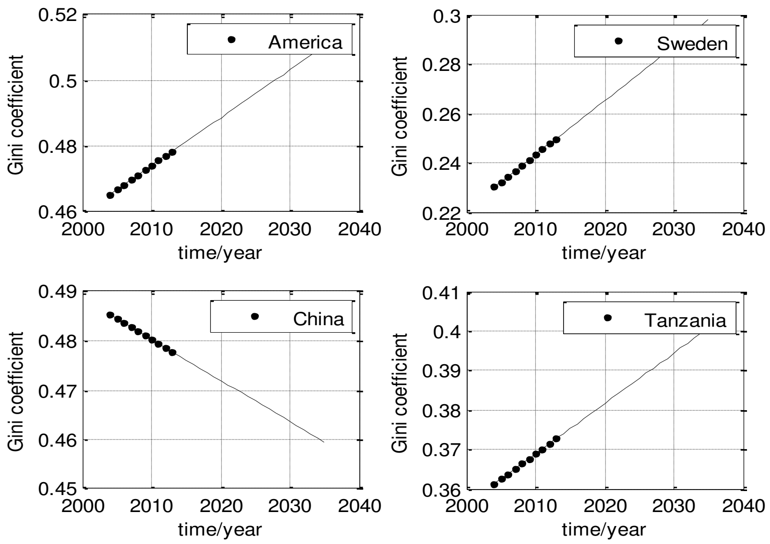

Figure 11 Fitting and forecasting results of Income Gini Coefficient

3.4 Study of aggregative indicators

In order to facilitate analysis, we arrange the three index systems as follows:

(1) Natural Resources Reserves per capita

- agricultural acreage per capita I_{11}
- forest area per capita I_{12}
- renewable water resource per capita I_{13}

(2) Environmental Carrying Capacity per capita

- energy consumption per capita I_{21}
- carbon dioxide emissions per capita I_{22}

(3) Social welfare level

- health expenditure per capita I_{31}
- GDP per capita I_{32}
- income Gini coefficient I_{33}

The interaction of the above three index systems is mainly reflected in the following facets:

(1) First, the effect energy consumption per capita has on natural resources reserves per capita. Suppose the influence weight is the same, so impact factor (IF) can be described as:

$$J_{21} = \frac{I_{11} + I_{12} + I_{13}}{3I_{21}} \tag{18}$$

Clearly, the bigger J_{21} is, the more sustainable a country will be.

(2) Second, the effect energy consumption per capita has on social welfare level. Suppose the influence weight is the same, so IF can be described as:

$$J_{23} = \frac{I_{31} + I_{32}}{2I_{21}} \tag{19}$$

Clearly, the bigger J_{23} is, the more sustainable a country will be.

(3) Third, the effect GDP per capita has on health expenditure per capita. The IF can be described as:

$$J_{33} = I_{31} / I_{32} \tag{20}$$

Clearly, the bigger J_{33} is, the more sustainable a country will be.

(4) Fourth, the effect energy consumption per capita has on carbon dioxide emissions per capita. The IF can be described as:

$$J_{22} = I_{22} / I_{21} \tag{21}$$

Clearly, if I_{21} remains unchanged, the smaller J_{22} is, the smaller I_{22} will be, which has less influence on environment and which is advantageous to sustainable development.

Suppose the impact factors above have the same weight, then the composite index of sustainable development is:

$$J = \frac{J_{21} + J_{23} + J_{33} - J_{22}}{4} \tag{22}$$

If we include social equity indicator (i.e. Gini coefficient), then the ultimate composite index could be defined as the following:

$$J_s = J / I_{33} \tag{23}$$

Similarly, the greater the indicator is, the more sustainable a country will be.

Here we grade the sustainability on the basis of J_s and the ratings are as follows:

Table 5: Ratings of countries' sustainability

$J_s \geq 5$	$5J_s < 5$	$2 \leq J_s < 3$	$1 \leq J_s < 2$	$0 < J_s < 1$
A+	**A**	**B+**	**B**	**C**
Sweden(6.2708)		America(2.3939)	Tanzania(1.4265)	China(0.8183)

Here it is clear that the greater the value of index is, the more sustainable a country will be. From the result, Sweden is the most sustainable country, followed by America, Tanzania and China. In order to judge the level of sustainability, we rate the value from highest to lowest as,

A+: very sustainable
A : sustainable
B+: acceptable in sustainability
B : unsustainable
C : very unsustainable

3.5 Model Evaluation

Strengths: The RLS model has three major advantages. First, it considers the measurement of sustainable development in three facets: natural resources reserves (how much we have), environmental carrying capacity (how much we can consume) and social welfare level (to what extent our standards of lives improve). These indicators chosen cover the field of natural resources, pollutant discharge, sanitation, economy and social equity, etc. Hence, the model is built on an extensive and structured indicator system in comparison with the current UNSCD frameworks and OECD system. Second, the model is not static but a dynamic one in determining the future trends of the selected indicators. The model considers the changes of internal reserves (e.g. renewable water resource per capita) pushed by external forces (e.g. organic matter emissions per capita). Therefore, it is possible to build an extrapolation model of ecosystem and economic system. Third, from the model construction (see *Theorem* 1) the error bounds decrease as the polynomial factorial n increase, which means the accuracy goes higher when n increases.

Weaknesses: The model also has some weaknesses due to lack of data in some critical indicators. In order to assess social welfare level, we should consider the Green GDP (GGDP) which was adopted by the World Bank since 1997. However, there exists insurmountable technical difficulty as to measure the value of GGDP, and there are no sufficient data about it. Thus we use health expenditure per capita, GDP per capita and income Gini coefficient to represent social welfare level. Another flaw is that the data from the least development countries (LDCs, e.g. Tanzania) are insufficient, which results in greater simulation error. In fact, we still have a long way to go before optimizing our RLS model.

4. Twenty-year Plan for Tanzania

4.1 Country Profile

We look at the profile from five different perspectives: demographics, natural resources, economics, social status, and political environment. We find that Tanzania has great potential in development in that it has rich resources; however, it also has lots of constraints. The high fertility rate, poor technological level and health conditions largely prevent Tanzania from development.

The figure below shows the ongoing projects and operations in 2014. Most of the projects are carried out in urban areas and the structure is unbalanced. Remote areas are badly in need of clean water, affordable electricity, and convenient transportation, etc.

Table 6: Profile of Tanzania in 2013

<table>
<tr><td rowspan="14">Profile of Tanzania (2013)</td><td rowspan="3">Demographics</td><td rowspan="2">Population</td><td colspan="2">Gender Structure</td><td colspan="3">Age Structure</td><td rowspan="2">Growth Rate</td></tr>
<tr><td>Male</td><td>Female</td><td>0-14</td><td>15-64</td><td>65 and above</td></tr>
<tr><td>49.25 million</td><td>50%</td><td>50%</td><td>45%</td><td>52%</td><td>3%</td><td>3.00%</td></tr>
<tr><td rowspan="2">Natural Resources</td><td>Forest Area (sq. km)</td><td colspan="2">Natural Gas (cubic feet)</td><td colspan="3">Coal (ton)</td><td>Gold (ounce)</td></tr>
<tr><td>326,212</td><td colspan="2">43 trillion</td><td colspan="3">10 million</td><td>18 million</td></tr>
<tr><td rowspan="2">Economics</td><td>GDP (current US$)</td><td colspan="2">GDP Growth (annual %)</td><td colspan="4">GDP per capita (current US$)</td></tr>
<tr><td>33.23 billion</td><td colspan="2">7.00%</td><td colspan="4">694.8</td></tr>
<tr><td rowspan="3">Social Status Quo</td><td rowspan="2">Technological Level</td><td colspan="2" rowspan="2">Health Expenditure per capita</td><td colspan="4">Education</td></tr>
<tr><td colspan="2">Literacy Rate</td><td colspan="2">Compulsory Education</td></tr>
<tr><td>extremely low</td><td colspan="2">$73</td><td colspan="2">90.40%</td><td colspan="2">10 years</td></tr>
<tr><td rowspan="2">Political Conditions</td><td colspan="3">Key Areas</td><td colspan="2">Stability</td><td colspan="2">Diplomatic Relations</td></tr>
<tr><td colspan="3">education/sanitation/transportation</td><td colspan="2">relatively high</td><td colspan="2">good</td></tr>
</table>

*Source: http://data.worldbank.org/country/tanzania

4.2 Comparison and Predicting Results

In order to better design our development plan, we first compare the country's sustainability factors with those of other countries (America, Sweden and China). Here we use the 2009 statistics as an example (see Figure 18).

Table 7: Comparison of four countries in 2009

	America	Sweden	China	Tanzania
I_{11}	0.52	0.28	0.08	0.26
I_{12}	9.882802	30.329068	1.533113	7.752427
I_{13}	9185.99	18390.03	2112.85	1929.94
I_{21}	7055.60	4883.20	1717.27	443.29
I_{22}	17.32	4.70	5.78	0.15
I_{31}	8008.67	4357.07	188.98	27.52
I_{32}	46998.82	43639.55	3748.50	504.20
I_{33}	0.47	0.25	0.490	0.37
J_S	**2.3939**	**6.2708**	**0.8183**	**1.4265**

We can infer from the result (see the bottom line) that Sweden is the most sustainable among four countries, followed by United States, Tanzania and China. From the respective indicator, we can see Tanzania is extremely insufficient in renewable water resource (I_{13}), energy consumption (I_{21}), carbon dioxide emissions (I_{22}), health expenditure (I_{31}) and GDP per capita (I_{32}), leading to the relatively low rate in composite index (J_S). Therefore, the major bottlenecks to Tanzania's sustainable development are water resource, electricity and other relevant industrial materials, healthcare, and sustainable economic structure.

According to all kinds of data of Tanzania, we come out of the fitted results of all indicators in the next two decades, and then we calculate the composite index J_S in respective year (see Figure 12).

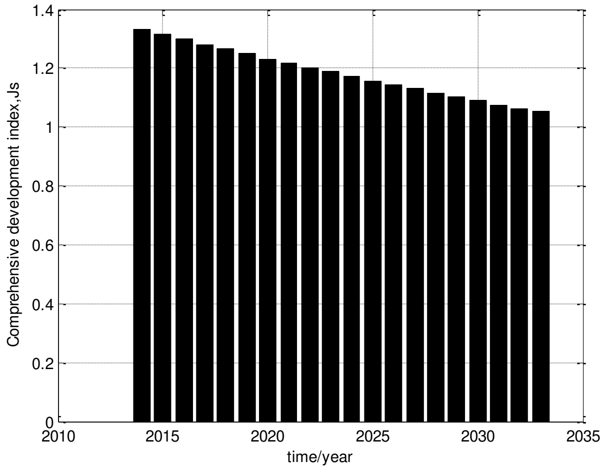

Figure 12: Predicted trend of composite index (20 years)

From the figure it is clear that the composite index of Tanzania shows a down trend, reflecting the decrease of its future sustainability.

4.3 Plan Content

To promote Tanzania's abilities in sustainable development, we sketch a plan based on the model and the country's profile. Tanzania has expansive arable land and forest area, but the shortage in water largely hinders the agriculture from developing. Therefore, agricultural development cannot be the major driving force in Tanzania's overall development. Instead, we believe Tanzania has to strive in industrial development and make progress in technology, education, medical and health services. The plan consists of five major development programs suitable for Tanzania status quo. They are producing clean water, generating electricity, developing tourism industry, improving transport conditions and advancing medical and health services. We believe these five are essential and fundamental to Tanzania's sustainable development. Also, these five are interactive and as integral parts of the whole plan. Among these, clean water production, electricity generation and transportation improvement are of significant importance because those above facilitate the development of the other two programs.

(1) Producing Clean Water

In Tanzania, the access to clean water is quite difficult. The high price of clean water makes it hard for many people to use, so they use unclean water instead, which may carry several kinds of diseases such as schistomiasis, dysentery and malaria.

In order to improve the quality of water consumption, our program aims at producing more clean water. The production is consists of two parts. One is to establish seawater desalination plants near the sea; the other is to help people in the rural areas to purify water through chemical or physical ways.

Seawater Desalination Project: establish large desalinators in coastal area within five years to increase the renewable water resource per capita from 1705 m^3 to 2746 m^3.

Water Purification Project: send volunteer teams every year to inland counties to impart simplified water purification methods, and bring necessary materials and equipment.

(2) Generating Electricity

Only 30% of rural area has access to stable electricity in Tanzania, and with the price going up to 12 cents/KWH, which is 2-3 time that of Kenya and Uganda(data in 2009)[17], only 10% of population have access to electricity and can afford to its consumption. Local companies are frequently affected due to blackouts, making electricity one of the major bottlenecks of Tanzania's sustainable development. Considering the situation, we plan to increase the percentage of natural gas in the electricity generation structure in the short term so as to replace the more polluting petroleum and coal. In the long run, solar energy (or other clean power) is expected to play a more important role in total power generation.

- Short-term (y. 1~5): build large gas power plants to ensure the stable and sufficient supply of major cities; lay micro-electric grids powered by solar energy in remote areas.
- Mid- and long- term (y. 6~20): build three large solar power plants (1 GW scale) to replace fossil fuel in power generation. Massive foreign direct investment (FDI) is needed in the long run.

(3) Improving Transport Conditions

Transportation has for long been vital to development because it generates economic benefits as every road is built. This is especially true for undeveloped areas, where roads not only build connections with the outside world of the region, but bring many opportunities as well.

This program aims to improve transportation conditions and constructing more new highways and country roads, as we believe better transportation network would benefit the country's sustainable development in multiple ways.

Current land transportation conditions in Tanzania are unfavorable. In raining reasons, muddy roads make it extremely difficult for driving and riding: goods cannot be delivered; travel cannot be made. So we consider improving transport conditions are of great importance.

We would choose bitumen instead of cement to pave the road, as bitumen provides better conditions and is convenient to mend while cement does not. We have assured that Tanzania has asphalt roads that we believe it has satisfactory conditions in building more of them. We would start with the main traffic routes, then spread it over.

(4) Developing Tourism Industry

Tourism has become one of the pillar industries in Tanzania, contributing 14% to its total GDP, creating more than thirty thousand jobs[1]. Tanzania government has also listed it as its development focus in the following years. Most of the tourist attractions are located in southeast part of Tanzania, which causes unbalanced development of regional economy. Furthermore, the authority has frequently received complaints from unsatisfied tourists due to terrible facilities and services.

Therefore, we design the following two programs to develop tourism industry and increase GDP per capita.

First, open up new tourist attractions in some relative rural and least developed regions. New tourist attractions not only create economic benefits in the region, but also drive the regional economic development. More importantly, the preparation work for the site enables the local citizens to have better infrastructure and live a better life. It also creates many jobs and attracts investments to animate business in this area.

Second, build more hotels. The maintenance fee during off-season might be the reason many investors reluctant to build more hotels. But we can apply many marketing and promotion strategies to raise occupancy. For example, we could corporate with government and investment firms to overcome off-season impact. Therefore, we can maximize our profits in building more hotels at local areas.

(5) Advancing Medical and Health Services

According to the country profile, we find that Tanzania has terrible health conditions. There are 175 hospitals, 3014 clinics, 276 medical stations in total that only 3 doctors and 1.1 sickbeds are shared among 1,000 people[2]. We plan to establish 5 more hospitals in five years,

[1] http://www.ce.cn/culture/gd/201412/15/t20141215_4120524.shtml
[2] http://www.fmprc.gov.cn/zflt/chn/ltda/ltjj/cyg/tsny/t931909.htm

and build 30 clinics and medical stations every year. In that case, we would be able to achieve 15% increase per year in average health expenditure indicator.
In addition, we aim to increase contraceptive prevalence (of women ages 15-49) from 34% to 50% by educating them of birth control importance because we find that the high birth rate (40‰, 2009 data, world databank) has impede Tanzania's development.

4.4 Plan Evaluation

Here we revise the mathematical model previously established. The three indexes in the model, natural resource reserves, environmental carrying capacity and average social welfare level are modified in the following ways:

(1). Natural Resource Reserves indicator system:

- agricultural acreage per capita I_{11}: decreasing by 1% each year
- forest area per capita I_{12}: increasing by 10% each year (protect environment)
- renewable water resource per capita I_{13}: increasing by 10% each year (protect environment)

(2) Environmental Carrying Capacity indicator system

- energy consumption per capita I_{21} : increasing by 8% each year
- carbon dioxide emissions per capita I_{22}: increasing by 5% each year (mainly due to the improvement of living standards)

(3) Social welfare level indicator system:

- health expenditure per capita I_{31}: increasing by 15% each year (focus on improvement of Medical and health conditions)
- GDP per capita I_{32}: increasing by 10% each year
- Income Gini Coefficient I_{33}: increasing by 0.2% each year.

According to the modified index, the composite index of Tanzania can be seen in Figure 13.

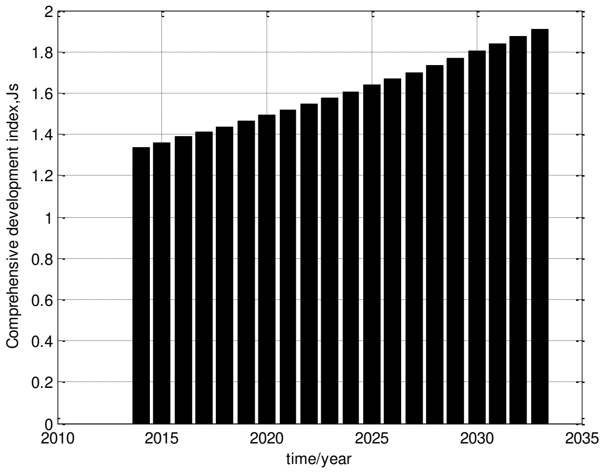

Figure 13: Modified trend of composite index (20 years)

Therefore, after implementing the 20-year plan, we are glad to see the rising trend of its composite index, showing the improvement of the country's sustainability and further verifying the rationality of our model and plan as well.

5. References

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