Population ageing and prices in an OLG model with money created by credits

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Abstract

This paper provides an explanation of why population ageing is associated with deflationary processes. For this reason, we create an overlapping-generations model (OLG) with money created by credits (inside money) and intergenerational trade. In other words, we combine a neoclassical OLG model, with post-Keynesian monetary theory. The model links demographic factors, such as fertility rates and longevity, to prices. We show that lower fertility rates lead to a smaller demand for credits, and lower money creation, which causes a decline in prices. Changes in longevity affect prices via real savings and capital market. Furthermore, we address a few links between interest rates and inflation, which arise in the general equilibrium, and are not thoroughly discussed in the literature. Long-run results are derived analytically; short-run dynamics is simulated numerically.

JEL Classification: E12, E31, J10

Keywords: Population ageing, inflation, OLG model, inside money, credits.

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1 Introduction

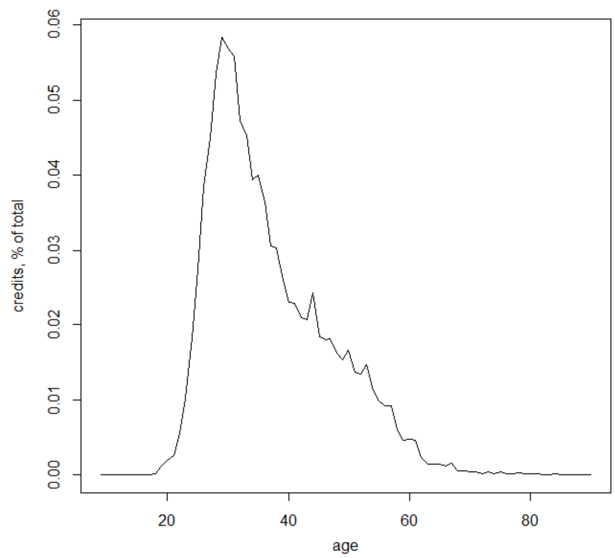
Deflation is usually supposed to be harmful, because people expecting a decline in prices have incentives to cut their spending, reducing economic activity and leading to an economic stagnation. Economic stagnation, reduces incomes, and induces a further decline in spending. During the recent economic crises, many central banks around the world implemented a number of measures to increase inflation and to stimulate economic activity with a different degree of success. The problem of deflation has also increased an interest of researchers in this topic, who discovered that population ageing is one of the main structural factors, which reduces inflation (Yoon, Kim, and Lee 2014; Gajewski 2015). In this paper, we create a macroeconomic model with money created by credits in order to explain why population ageing leads to a decline in prices. We also show that a reduction of interest rates, which is sometimes performed by central banks in order to stimulate crediting, may also have a reverse effect in the medium and long run.

Up to 97% of money in the UK are created by commercial banks whenever they make loans, only 3% of money being created by the government.[2] Giving a loan, banks create new deposits, to which newly created money are transferred. Agents make purchases, and the owners of these deposits change. When a debtor returns the credit, the money is destroyed. These processes of money creation and destruction are explained in detail by McLeay et al. (2014), Werner (2014b) and Jakab and Kumhof (2015), and confirmed empirically (Werner 2014a). We include them into an overlapping generations model (OLG). Loosely speaking, in our model, the deflationary effects of population ageing come from the fact, that young agents are usually liquidity-restricted, they take credits, and return them when they become older. Consequently, population ageing reduces the stock of money in the economy, having a negative impact on prices. If demographic transition stops - price level stabilizes at its new equilibrium level in several periods. To our best knowledge, this is the first paper, which introduces money creation by credits into the OLG framework.

Our model exploits the fact that agents take credits when young. Figure 1 presents a distribution of credit margins in Lithuania in the end of 2014 by age. The data was taken from the PRDB database, maintained by the Bank of Lithuania. The figure presents credits issued to natural entities in 2014 by commercial banks and does not account for credits provided by credit unions, leasing companies, etc. It also excludes loans taken and paid back in 2014. If two or more agents took a loan together, the loan value was divided by the corresponding number of agents. Agents' age corresponds to the end of 2014, and not to the exact date when the credit was received. The figure indicates that most loans were taken by young agents, the maximum corresponding to the age of 31 years. After the age of 31, the volume of credit margins sharply declines. The total amount of credit margins accumulated by agents not older than 40 constitutes 71.7% of the total.

[2] The source: http://www.positivemoney.org/how-money-works/how-banks-create-money/

Figure 1: Credits by age, in Lithuania, 2014



1.1 Literature review

The link between demographic factors and inflation was studied in a number of empirical and theoretical works; the recent evidence showing that population ageing makes a deflationary pressure. Yoon et al. (2014) analysed IMF and World Bank data for 30 OECD countries over the period 1960-2013. They found that the population growth affects inflation in a positive way, the share of agents in the population older than 65 years having a negative impact on inflation. Gajewski (2015) focused on the data for 34 OECD countries over the period 1970-2013, and confirmed the result, finding that older societies are indeed associated with a lower inflation.

The deflationary effects of population ageing in Japan where found by Anderson et al. (2014) and Carvalho and Ferrero (2014), with Faik (2012) making similar findings for Germany. However, Lindh and Malmberg (1998) and Lindh and Malmberg (2000) argued that only the share of agents 75+ in population affects inflation in a negative way, the young retirees (65-74) creating a positive impact. They have also found that young adults (15-29) have a positive impact on inflation. The positive effect of young retires on inflation is explained as follows: as middle-aged agents are suppliers of savings, older population has a negative impact on them. They assume that demand for investment funds and nominal interest rates do not change; therefore, lower savings result in an increase in prices. Similar results were also received by Juselius and Takáts (2015), who restricted their analysis to 22 OECD countries, but took a period of 1955-2010. Juselius and Takáts controlled for a large number of specifications, and reported that the share of young pensioners affects inflation in a positive way, the share of older pensioners having a negative impact on inflation. Indeed, the inflationary effect of young pensioners is understood rather well: agents stop producing goods and start supplying money when retire, leading to an increase in prices. However, the deflationary effect of older pensioners is not so clear. This question is at the focus of our paper.

Doepke and Schneider (2006), Bullard et al. (2012) and Katagiri et al. (2014) studied political aspects of inflation as a tool for wealth redistribution. They argued that inflation redistributes wealth between lenders and borrowers. As young agents are usually borrowers and old are savers, lower inflation is beneficiary for the old. Consequently, population ageing increases the voting power of the old generation, pressing the government to keep inflation at a low rate. However, Katagiri et al. also argued that population aging arising from a decline in birth rate shrinks a tax base and increases government expenditures having a positive influence on inflation. The paper of Bullard et al. is also related to our work because they employed an overlapping generation model with money. Our model differs from this article mainly in the way, how the money market is constructed. In their paper, the stock of money is exogenous and determined by the government, in our model, money are created by credits. This allows us to study a macroeconomic link between demography and inflation instead of the political one.

Regarding the link between demographics and inflation, it shall also be mentioned that Imam (2013) showed that population ageing makes monetary policies less effective in the five developed economies: U.S., Canada, Japan, U.K., and Germany. He also showed that credit channel of monetary policy transmission, which is the focus of our paper, outweigh the wealth channel. These empirical findings are well in line with the predictions of our model.

Developing our model with money created by credits, we take an OLG model introduced by Samuelson as a starting point (Samuelson 1958). Samuelson analysed an intergenerational trade in a three-periods OLG model. He argued that the generation which is in the second period of its life may "bribe" the younger generation to be supported when they become old. He showed that the socially optimal interest rate in this case is equal to the population growth. We add a number of modifications into the Samuelson's model. First of all, incentives for the intergenerational trade in our models arise not only from the will of the second generation to get a support in future, but also from the needs of the first generation: young agents are often liquidity constrained, and they need to borrow resources from the older cohort. Second, there is no direct "bribing" in our model. We introduce a bank, which serves as an intermediary between the generations. The bank issues credits, creating fiat inside money. Money can be used for trade between generations, and they are stored on the bank account giving a positive nominal interest rate in the next period. Such a setting allows analysing the interconnection between monetary and real factors, such as demography and price level.

Overlapping generations with money were discussed in detail by Champ and Freeman (1994). Money in their models serve for savings, which are stored for the next period, as a numeraire and as a mean of exchange, one of the results being that a switch from commodity money to fiat money can be welfare-improving, since instead of storage, commodity can be consumed (page 46). The other example of an OLG model with money is Crettez et al. (2002), who looked for optimal monetary and fiscal policies in a two-overlapping-generations model and found that one of the policies is redundant. Hiraguchi (2014) re-examined their results in a three-overlapping-generations model, finding that optimal monetary policy shall follow the Friedman rule. We follow Hiraguchi by assuming three periods in the model, because such a setting assures different attitudes of agents towards money in different periods, and makes money demand and supply conditions nontrivial; however, our definition of periods is different: for simplification, instead of two working periods, we assume one period of childhood and another of adulthood.

Our model also relates to the literature, which studies the role of banking in OLG models. Qi (1994) analysed bank liquidity and stability in OLG models. He showed that the government shall provide an insurance for deposits, because there can be bank runs due to either the shortage of new deposits or excessive withdrawals. Amable et al. (2002) showed that deposit insurance and banks' entry restriction do not always maximize welfare, even if positive bankruptcy costs are assumed. Therefore, a careful assessment of costs and benefits is required. Andolfatto and Gervais (2008) created an OLG model with endogenous credit constraints, which emanate directly from a life-cycle assumption. They show that an ability of creditors to garnish increases the amount of credits received by agents, however, it may also reduce the capital stock, leading to lower welfare. Our work differs from this literature in two main aspects: First, the topic is different; we focus on the link between demographic factors and prices. Second, in our model, the interaction between bank and agents is performed in terms of inside money, which have a form of bank's records; the other models using commodity money or goods for this reason.

Money creation by loans, which is the key feature of our model, was discussed in detail by Post-Keynesian economists (Kaldor 1970; Moore 1979; Cottrell 1986; Cottrell 1994; Lavoie 2011). However, the post-Keynesian monetary theory is not worldwide accepted, largely because of its non-mathematical nature. An ignorance of mathematical equations makes verifying the theory and making quantitative predictions difficult. Moreover, the non-mathematical nature makes it difficult to see limitations of assumptions. Our paper aims to overcome this barrier. We introduce a post-Keynesian money creation into the neoclassical OLG framework. This allows us to study effects of intergenerational trade - the process, which is usually disregarded by post-Keynesians. We derive long run results analytically, and simulate the dynamics of the model in the short run.

In fact, a few attempts to introduce mathematics into the post-Keynesian monetary theory were made in the past. Cavalcanti and Wallace (1999) created a model with a post-Keynesian money creation by credits. They showed that inside money (money created by the commercial banks) can mimic the outside money (money issued by someone else, central bank, for example), because they are used in a similar way. However, with outside money the purchasing capability of a banker depends on the banker's previous trades, with inside money this is not being so, because the banker may create new additional money at any time. As a result, outside money creates a larger variety of outcomes. Andolfatto and Nosal (2001) argued that money created by banks is efficient because they allow exchange of goods at no costs, and the direct billing between agents is impossible because not all individuals commit to their promises. Disyatat (2011) emphasized the role of the banks as transmission mechanisms for monetary policies via their balance sheets and risk perception. Jakab and Kumhof (2015) created a DSGE model with "financing throw money creation" (FMC), and compared its performance with a model with the standard intermediation of loanable funds (ILF). They showed that, under identical shocks, FMC models predict much larger and faster effects on the bank's balance sheets and greater effects on the real economy than ILF models. They argued that predictions of FMC models are more realistic. In our paper we assume a similar role of credits, and the same money creation. But, in contrast to this literature, our model also features money demand by agents for saving reasons, as it is common in the overlapping generations models (Champ and Freeman 1994).

Apart from the link between the demographic factors and price level, we also study the effects of interest rates on it. We enumerate four different effects, which affect prices in the steady state. First, interest rates affect demand for money via the intertemporal allocation of consumption, the sign of the effect depending on the elasticity of substitution. The second effect comes from the fact, that agents, who need to return their credits, enhance their demand for money, leading to money appreciation and a decline in prices. The third effect is due to savings: having returned credits at the higher interest rate, agents get smaller net incomes and they are able to make fewer savings. This reduces their demand for money, and leads to an increase in prices. The fourth effect comes from the fact, that higher interest rates increase monetary incomes of the old generation, they increase expenditures, affecting prices in the positive way. Given a bank's zero-profit condition, second and fourth effects eliminate each other, the total effect depending on the parameter values. But, as we argue in the text, it is likely that there is a positive effect of increasing interest rates on the prices in the long run. This resembles the Gibson's paradox (Gibson 1923; Keynes 1930), which was observed in the short run. In our model, dependence between interest rates and price level in the short run depends on parameter values, and it may correspond or not to correspond to the Gibson's paradox.

The paper is organised as follows: In the next section, the model is developed and long-run results are derived; the third section presents a numerical example to show the dynamics of the model in the short run. The fourth section discusses the robustness of the results, limitations of the model and its possible extensions. Finally, the fith section concludes.

2 The model

We employ a discrete-time overlapping-generations (OLG) model of a Diamond-Samuelson style (Samuelson 1958; Diamond 1965) with agents, who live for three periods: childhood, working period, and retirement. Agents have children when they enter the second period of life.

We denote the size of a generation born at the period t by N_t. In the beginning of the working period, they have N_{t+1} children, $N_{t+1} = N_t(1+n_{t+1})$.

Period t is populated by children (N_t), adult (N_{t-1}) and senior generation (N_{t-2}). During childhood agents do not get any income, and they are supported by their parents, if they get positive incomes. At the end of the childhood, children take a credit from a bank to buy physical goods, which are invested. Agents at the working age inelastically supply one unit of labour. They get incomes, if they have a positive productivity shock, and make their savings-consumption decisions (they share consumption with children). In the beginning of the last period, agents may die with a probability $1-\psi$, hence, ψ is a probability that agents survive and live during the whole third period of life. Under such a construction of overlapping generations, one period lasts for approximately 25-30 years.

2.1 Banking sector

The bank in the model has features of a commercial bank, pension fund and insurance company, because it gives risky credits to the young agents of size M at the lending interest rate $1 + r_t^l$; its liabilities circulate in the economy in the form of account records and are used as money (bank service); it holds safe deposits at the interest rate $1+r_t^d$, which are consumed during the last period of agent's life (funded pension), and it redistributes funds of people passed away before the third period of life to the survived agents of the same generation (insurance company).

In the end of the first period of life, agents get credits of size M, and create accounts with deposits in the bank of the same size. The deposits have a form of bank records. Then young agents use these deposits to buy physical goods from the middle-aged generation, which is in the end of the second period of life, as a result, the owner of the deposit changes. The same happens to the money stored by the old generation on their deposits. The elderly agents buy physical goods for consumption from the agents in the second period of life, and, as a result, the owner of the deposits changes as well. The generation, which is in the second period of its life, becomes the unique owner of the deposits in the bank. It uses money on these deposits for two purposes: First, to return the credit, which they took a period ago. The return of these credits destroys a part of the money in the economy. Second, they continue keeping money in the bank deposit in order to consume them when old. Hence, the relation between the interest rate on savings depends not only on the interest rate on deposits, but also on the price at which real goods are sold and bought:[3]

$$1 + r^s_{t+1} = \frac{(1 + r^d_{t+1})\pi_t}{\pi_{t+1}}. \tag{1}$$

r^s denotes returns on savings, r^d is a return on deposits, π is a price of physical goods. Following Disyatat (2011), we make an assumption that the bank does not make profits, i.e, the returns to loans are allocated to the deposit owners.

$$p(1 + r^l_t) = 1 + r^d_t. \tag{2}$$

Where p is a probability of default. The logic of this nonprofit condition is the following: Suppose that at a certain point of time the bank is introduced into the model. Having issued a credit, the bank automatically creates a deposit of the same size. Credit is an asset for the bank, and deposit is a liability. At the next period, assets raise by $p(1+r^l_t)$ ($1-p$ agents do not return the credit) and the value of liabilities increases by $1 + r^d_t$. The nonprofit condition implies that bank's assets and liabilities are equal to each other also in the beginning of the next period. More formally, this logic is explained in the appendix.

Bank's liabilities have the form of inside money kept by agents (monetary assets), bank's assets (credits), can be supposed as agents' liabilities. Therefore, the zero-profit condition also implies that agents' monetary assets and liabilities are also equal to each other. As a result, there is always enough money in the economy for agents to return their credits.

2.2 Production

All agents in our model are self-employed. In the end of their childhoods, agents take credits from a bank of size M, buy physical capital for these money, and invest it in their own business, which may be successful or not. Namely:

$$k_t = \frac{M}{\pi_{t-1}}, \tag{3}$$

where π_{t-1} denotes the price of the goods in terms of money in the period $t-1$. (Agents working in period t, need to buy physical capital a period in advance.) Agents supply one unit of labour during their life-time. Furthermore, an agent i working at the period t faces a productivity shock $A_{i,t}$. Therefore, agents produce:

$$y_{i,t} = A_{i,t} f(k_t), \quad i = 1...N_{t-1}, \tag{4}$$

where $f(k_t)$ is an increasing concave production function, $f(0) = 0$. Capital depreciates in one period. Hence, agents' gross income is equal to their production.

There are two possible outcomes of the productivity shock: $A_{i,t} = 1$ with the probability p, and 0 with the probability $1 - p$. We suppose that these shocks

[3] We do not need to assume annuity markets for now, they will be assumed later.

are independent in time and between agents. Therefore, if the number of agents is sufficiently large, the aggregate output is equal to

$$Y_t = \sum_{i=1}^{N_{t-1}} A_{i,t} f(k_t) = pN_{t-1} f(k_t). \tag{5}$$

Having received outputs, those agents, whose productivity shock was positive, need to return credits to the bank. Therefore, their net income expressed in real goods is equal to

$$w_{i,t} = f(k_t) - \frac{M}{\pi_t}(1 + r_t^l). \tag{6}$$

We need to make an assumption that the interest rate is not too high, so that agents' real incomes would be nonnegative. Those agents, whose productivity shock was bad, cannot return the credit. They do not save or consume, and as a result, they do not participate in the economic life anymore. However, their children survive, and may take credits in order to start their own business.

Our aggregate production function Y_t resembles the standard neoclassical one with constant returns to scale. Indeed, post-Keynesian economists often use their own approach for the production modeling. They assume that firms have reserves of real capital and they can easily find labour in the market, implying that supply of goods is completely determined by demand (Dutt 2011). Such an approach has an advantage in certain cases. For example, imagine an economy recovering after an economic crises, which caused a decline of production and an increase in unemployment. It is very possible, that, in such a case, the firms should have some excessive amounts of real capital, and they can easily employ labour. But, in a three-periods OLG model, one period corresponds approximately to 25-30 years. It is very unlikely that someone wants to keep excessive amounts of real capital for such a long period of time. Moreover, everyone can be unemployed for a year or two, but, during such a long time period, most (non-disabled) agents at the working age work if they wish. As a result, the neoclassical-type production function, which assumes full factor employment, seems to be more reasonable in our case. It would be even more realistic to assume an endogenous labour supply, coming from a utility function, but we will leave it for further extensions.

2.3 Saving-consumption behavior

We suppose that only agents with a positive productivity shock consume. The agents, who have a bad shock, get zero incomes and do not participate in the economic life anymore. In childhood, agents do not make savings-consumption decisions. Their consumption is entirely determined by their parents. When agents enter the second period of life, they maximize a CES-type utility function:

$$u_{i,t} = \left[c_{i,t}^{1-\frac{1}{\sigma}} + \frac{\psi}{1+\rho} z_{i,t+1}^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

where $c_{i,t}$ stands for the consumption when middle-aged, $z_{i,t+1}$- consumption when old, ψ- probability to survive before the next period, σ- the intertemporal elasticity of substitution, and ρ- discount factor. The budget constraints are:

$$c_{i,t} = (w_{i,t} - s_{i,t})Q(n); \tag{8}$$

$$z_{i,t+1} = \frac{(1 + r^s_{t+1})s_{i,t}}{\psi}. \tag{9}$$

$Q(n)$ is a function decreasing in population growth n, $0 \leq Q(n) \leq 1$. $Q(n)$ accounts for the expenses made for children. If population growth n is higher - higher number of children reduce consumption per person in families. Agents' income is denoted by $w_{i,t}$. They make savings $s_{i,t}$ and invest them at the interest rate $1+r^s_{t+1}$. Furthermore, we assume perfect annuity markets: savings of agents died in the beginning of the third period of their lives are allocated between the survived agents of the same generation, increasing their total consumption. This is a technical assumption, which ensures that savings of the died agents do not drop out from the model.

Substituting budget constraints to the utility function, its maximization with respect to $s_{i,t}$ gives

$$\left(\frac{z_{i,t+1}}{c_{i,t}}\right)^{\frac{1}{\sigma}} = \frac{1 + r^s_{t+1}}{(1 + \rho)Q(n)}. \tag{10}$$

Now, using the budget constraints again, we solve for savings:

$$s_{i,t} = \frac{w_{i,t}}{1 + \psi^{-1}\left(\frac{Q(n)}{1+r^s_{t+1}}\right)^{\sigma-1}(1 + \rho)^{\sigma}}. \tag{11}$$

As only successful entrepreneurs make savings, the total savings in the economy are equal to $S_t = pN_{t-1}s_t$, or

$$S_t = \sum_{i=1}^{N_{t-1}} s_{i,t} = pN_{t-1}\frac{f(k_t) - \frac{M}{\pi_t}(1 + r^l_t)}{1 + \psi^{-1}\left(\frac{Q(n)}{1+r^s_{t+1}}\right)^{\sigma-1}(1 + \rho)^{\sigma}}. \tag{12}$$

2.4 Steady state

The bank supplies money to the economy in two ways: by issuing credits to the young generations, and by generating returns on deposits for the old generation. In both cases, money supplied by the bank is inside money. The total amount of money received by the young generation at time t is N_tM. The total amount of deposits returned to the old generation is equal to the $S_{t-1}\pi_{t-1}(1 + r^d_t)$. Therefore, the total money supply is equal to the sum of these two values.

The demand for money is created by the middle-age agents, who have a positive technological shock. They need to return their loans: $pMN_{t-1}(1 + r^l_t)$ in total. Moreover, they make savings for future consumption: $S_t\pi_t$ in total.

The multiplier π_t is needed to convert real savings into monetary terms. Total demand for money is equal to the total supply:

$$pN_{t-1}M(1+r_t^l) + S_t\pi_t = S_{t-1}\pi_{t-1}(1+r_t^d) + N_tM. \tag{13}$$

In the appendix we show a stronger result: the second terms on both sides of equation (13) are equal, implying that also the first terms are equal. Inserting equation (12) into (13), dividing it by N_{t-1}, and removing the time indexes we get an expression for the equilibrium values:

$$ps\pi[n - r^d] = M(1+n)[1+n-p(1+r^l)], \tag{14}$$

Now, with the use of equation (2), we see that the terms in square brackets in the both sides of equation (14) are equal to each other and can be canceled out if $r^d \neq n$.

$$\pi = \frac{(1+n)M}{ps}, \tag{15}$$

The equation implies that price level is proportional to the size of the credits M and population growth $1+n$, and depends negatively on savings and probability of a good technological shock. This is rather natural, because higher savings and probability of a good technological shock imply a larger supply of goods to the market, which leads to a decline of its prices. In fact, under one additional assumption, it is possible to receive equation (15) not only for the steady state, but also for each period t. This result is shown in the appendix. Now, inserting equations (3), (6) and (11) into (15) we get:

$$\frac{\pi}{M} f\left(\frac{M}{\pi}\right) = \frac{1+n}{p}\left[1+\psi^{-1}\left(\frac{Q(n)}{1+r^s}\right)^{\sigma-1}(1+\rho)^{\sigma}\right] + 1 + r^l \tag{16}$$

It is easy to see that properties of the function $f(k)$ ensure that the left side of equation (16) is increasing in π.[4] Thus, if p increases (probability of default declines), a higher number of successful firms will supply more goods (with the amount of money unchanged in the economy), leading to a decline in prices.

An increasing longevity ψ has a negative effect on the price level in the long run. This result is due to the fact, that middle-aged agents, who know that they are going to live longer, increase their savings. As a result, supply of goods (demand for money) raises leading to a fall in prices.

A decline in fertility rate n has a double effect on the price level in the long run: first, the multiplier $(1+n)$ has a negative impact on the price level, because smaller number of young agents take fewer credits and, hence, increase money supply. Second, the effect of sharing consumption with children $Q(n)$ depends on the elasticity of intertemporal substitution σ. If consumptions in different periods of life-time are complements a decline of fertility rates increases (real)

[4] Differentiation of the function $f(k)/k$ with respect to k, gives $(kf'(k) - f(k))/k^2$, which is negative due to concavity of the function $f(k)$: $f(k) + (0-k)f'(k) > f(0) = 0$.

savings, resulting in an increase of goods supply into the market and a decline in prices. If $\sigma > 1$, a decline in n leads to an increase in prices, mitigating the negative effect of the term $1 + n$.

Usually the elasticity of intertemporal substitution is estimated to be lower than one (Weber 1970; Weber 1975; Skinner 1985; Hall 1988; Dynan 1993; Yogo 2004; Gomes and Paz 2013). Blundell et al. (1994) estimated σ equal to 0.75-0.77, the robustness check giving a wider range: 0.64-1.17, and Gomes and Ribeiro (2015) estimated it in the range 0.4-1.8. Moreover, the estimates received by Hansen and Singleton (1982), using monthly data, are equivalent to an elasticity of substitution higher than unity. This implies, that the case of $\sigma < 1$ is more consistent with the literature, but we shall discuss the case $\sigma > 1$ as well.

An increase in interest rates affects prices in the long run in four different ways: First, higher interest rates increase money demand, because they force middle-aged agents to sell more goods in order to pay for their loans, having a negative effect on the price. Second, higher payments for the interest rate reduce savings of the middle-aged agents, because having paid for the interest rate, the agents' net income decreases. As a result, demand for money declines, leading to a devaluation of money (this effect is captured by the last term $1 + r^l$ of equation 16). Third, higher interest rates change savings due to the intertemporal preferences of the agents. If $\sigma < 1$, this effect has a negative impact on supply of goods and drives prices up. The opposite happens if $\sigma > 1$. This effect is also visible from the equation (16). Fourth, higher interest rate implies a higher money supply by the old generation, having a positive effect on prices.

The first and the fourth effects are of the opposite signs, and, under the assumption that banks make zero profits, used in the model, they completely eliminate each other; this is visible from the derivation of equation (15) from (14). As a result, the exact effect of interest rates on price level depends on the parameter values, but given the empirical evidence discussed a few paragraphs earlier, it is very likely that higher interest rates increase prices in the long run.

This finding is in line with the famous Gibson's paradox, which states that there is a positive relation between prices and interest rates (Gibson 1923; Keynes 1930). This paradox is also relevant nowadays for some countries (Cheng, Kesselring, and Brown 2013; Škare and Mošnja-Škare 2015); however, the direction of this dependence is disputable (Chen and Lee 1990). In our model, we received that higher interest rates may indeed lead to an increase in prices in the long run. The short run will be analysed in the next section.

3 Short run

In this section we perform simulations to illustrate the short-run dynamics of the model under the population ageing. But, before making simulations, we need to assume specific functional forms and parameter values. The values of variables in the initial steady state are summarized in the appendix.

3.1 Parameters

First of all, we assume a Cobb-Douglass production per worker function: $f(k_t) := k_t^\alpha$, with $\alpha = 0.4$. According to the OECD data on labour shares, this parameter for capital intensity is between UK (0.341) and US (0.485) in 2010. Next, we assume the following function adjusting consumption $Q(n) := (2+n)^{-1/2}$, where $2+n$ stands for one adult plus $1+n$ children. Such a function is in line with the equivalence scales used in OECD (2011) (square root of a household size).

Discount rate ρ is set at the 0.3478 level. As the period in the model is approximately equal to 30 years, this value of ρ corresponds to the annual discount rate of 1%. Such a value is similar to that used by Börsch-Supan et al. (2006), but larger than the discount factor estimated by Giglio et al. (2015) for long periods of time.

In the benchmark case, we use the longevity parameter $\psi = 0.9$. For comparison, according to the data of the World Health Organization, probability to live longer than 65 years was equal to 0.89 in Germany and UK and 0.9 in Ireland in 2012. Also, we assume population growth $n = 0.2$, in annual terms this corresponds approximately to 0.6% population growth due to fertility. Further we will simulate an increase of ψ from 0.9 to 0.95, and a permanent decline in n from 0.2 to 0.

We assume a 2% annual interest rate on deposits; it corresponds to 81.14% in 30 years. In the benchmark case, probability of firms' success p equals to 0.5. From the first glance, such a low probability of success seems to be unrealistic. However, if each year probability of the bad shock is the same, and does not depend on these probabilities at the other periods, the yearly probability of default is equal to 0.023. Such a value is even smaller than most probabilities of default calculated by commercial banks for credit risk management. This gives an interest rate on loans equal to 262.27% per 30 years or 4.38% in annual terms. We will present simulations for an increase in p from 0.5 to 0.6, which leads to a decline in annual interest rate on loans till 3.22% per year.

We normalize credit size M to unity. As dynamics may depend on the intertemporal elasticity of substitution, we perform simulations for three parameter values of σ: $\sigma = 0.5, 1, 2$.

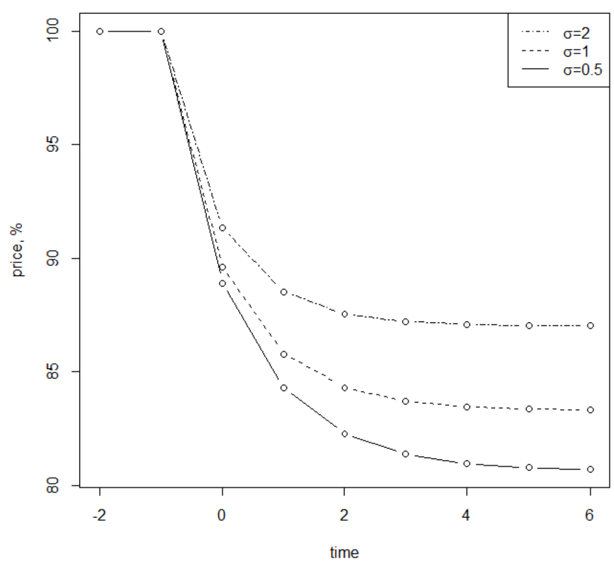
3.2 Simulations

Decline in fertility

First, we simulate a decline in fertility. We assume that, before the period 0, the system is in its equilibrium. At the period $t = 0$, there is a permanent decline of n from 0.2 to 0. The effect of such a decline is shown in Fig. 2.

A smaller number of young, results in that they take fewer credits and less money are supplied to the market. This immediately reduces the price level. The further dynamics is mainly determined by the developments in savings and capital-labour ratios. Smaller fertility rate induces an increase in capital-labour ratio, because the smaller number of middle-aged agents shares the amount of the real capital accumulated by the previous generation. Higher capital-labour ratio raises production, and, as a result, contributes to a further decline in prices.

Figure 2: Decline in fertility



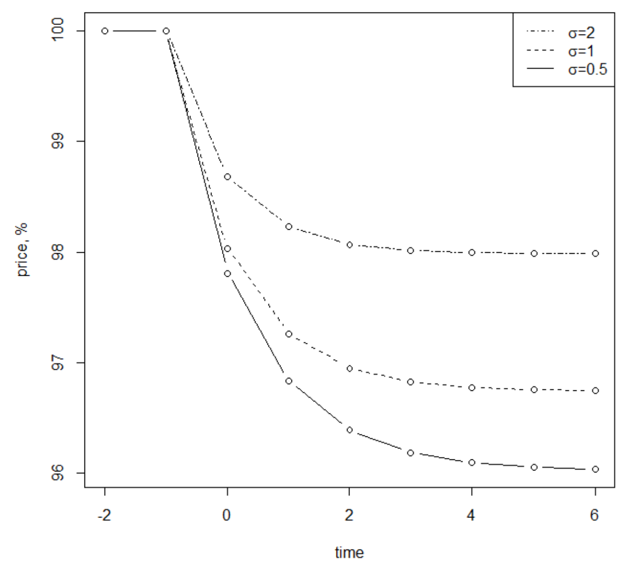
When the new equilibrium is achieved, decline in prices stops, implying that, according to our model, deflation caused by population ageing is temporal. However, the transition period in case of $\sigma = 0.5$ is rather long: keeping in mind that one period in our model is of 25-30 years long, 6 periods of transition correspond to 150-180 years. With a larger σ, the visual transitional period is shorter.

Increase in longevity

The short-run effects of increased longevity are shown in Fig. 3. We suppose that middle-aged agents at the period 0 find out that they are going to live longer, and they adjust their savings to account for this development. Their probability to survive before the third period of their lives increases from 0.9 to 0.95. We suppose that this increase is expected, because in practice population ageing is a rather slow process, thus, agents have opportunities to make necessary adjustments in their savings-consumption behaviour. However, we suppose that increasing longevity of middle-aged agents was not taken into account by the previous generations.

Qualitatively effects of an increased longevity are similar to those of a declined fertility rate, however, the reasons are slightly different: Increasing longevity of the middle-aged agents at the period $t = 0$ leads to their increase in savings. This raises supply of goods to the market, leading to a decline in prices. Then, higher savings increase capital-labour ratios, what leads to a further increase of incomes and savings, and a decline in prices.

Figure 3: Increase in longevity

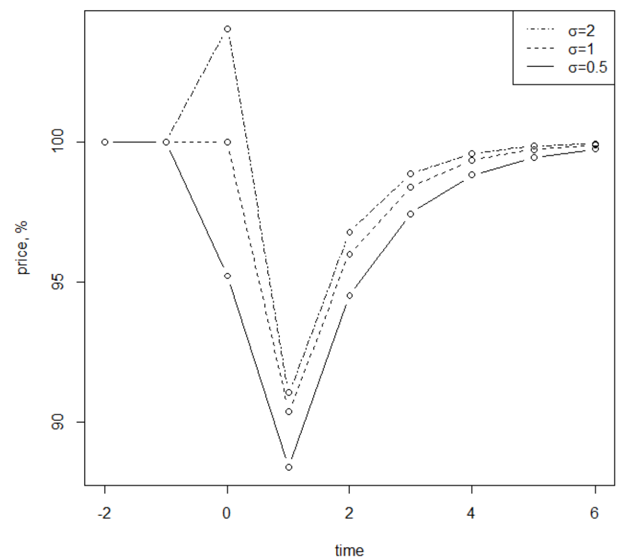


Decline in interest rates

As population ageing is a rather permanent process, there is no need to study a temporal increase in longevity or fertility rate. However, interest rates are rather volatile. Therefore, we consider two cases: first, a decline in interest rates is temporal, second - the decline is permanent. In both cases, we suppose that at time $t = 0$ agents sign contracts for loans and deposits with the bank, at the interest rate different from the previous generations. This means, that the actual decline of interest rates happens at period $t = 1$. We simulate a decline in the deposit interest rate from 2% per year to 1%, which implies a decline of r_d from 81.14% to 34.78% per 30 years. Consequently, according to the bank's zero-profit condition r_l changes from 262.27% per 30 years to 169.57%. In the case of a temporal decline, at the period $t = 2$, interest rates return to their previous values.

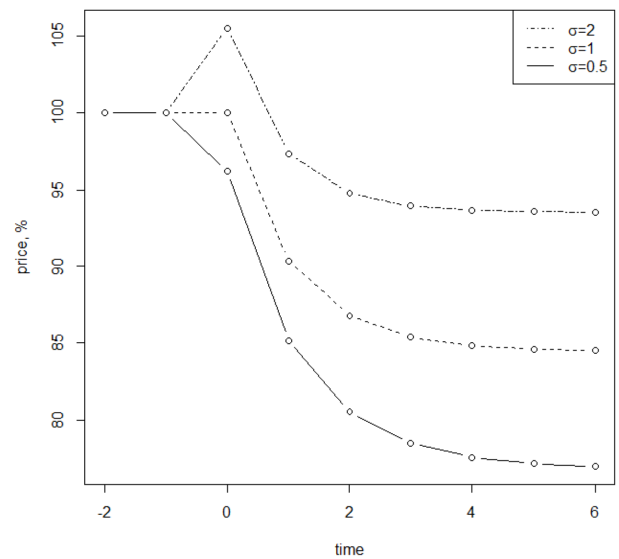
The effects of a temporal decline in interest rates are shown in Fig.4. The effect on prices at the period $t = 0$ highly depends on the elasticity of intertemporal substitution. If consumptions in the second and third periods of agents' life are substitutes, agents prefer to consume more at the first period of life, leading to a decline in savings and smaller supply of goods. This raises prices. If $\sigma < 1$, agents savings/consumption behavior at $t = 0$ is the opposite: they prefer to save more. This enhances supply of the goods to the market, and leads to a decline in prices. If $\sigma = 1$, expected changes in interest rate do not affect prices at the period $t = 0$. Therefore, depending on σ, the model may correspond or not to correspond to the Gibson's paradox in the very short run.

Figure 4: Temporal decline in interest rate



Lower interest rates at $t = 1$ imply, that agents returning their credits need less money to pay for their credits. However, this effect is eliminated by the smaller supply of money by the old generation due smaller interest rates. The main effect which plays a role is that having paid for the credits, agents which are in the second period of their lives at time $t = 1$ obtain more goods, and they may afford higher savings, leading to a higher supply of goods and a decline in their prices. Indeed, the extent to which they increase their savings also depends on the elasticity of substitution. They know that in the next period the interest rate is going to be high again, therefore, if $\sigma < 1$ they consume more at the period $t = 1$, and if $\sigma > 1$ they prefer to increase their savings relatively to the case of $\sigma = 1$. At the following periods the system returns to its initial

Figure 5: Permanent decline in interest rate



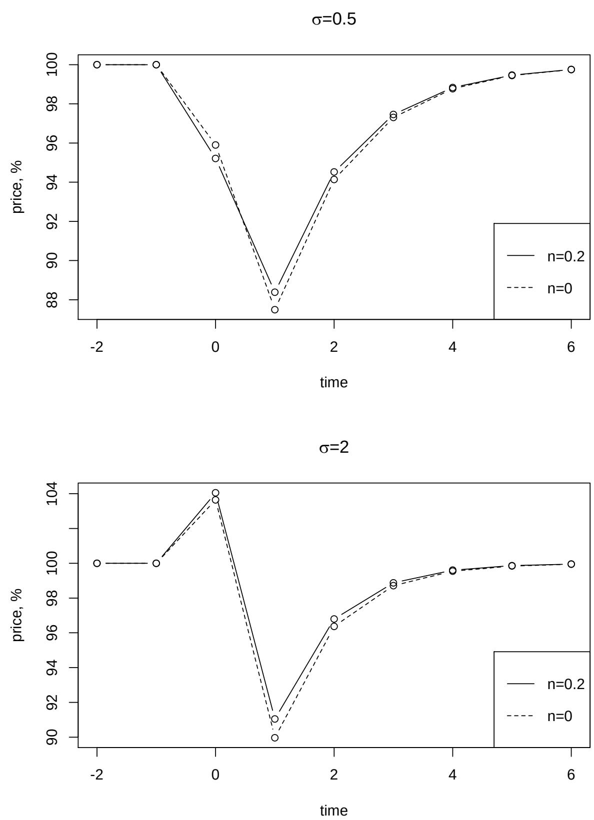
equilibrium.

Fig. 5 presents the effects of the permanent decline in interest rates. In fact, the effects at $t = 0$ and $t = 1$ are rather similar to the temporal decline. The important difference at $t = 1$ is that savings/consumption decisions of the adult agents at this period expect to have lower interest rates as well. This affects their intertemporal allocation of resources, implying that in case of $\sigma > 1$ the decline in prices is smaller, and in $\sigma < 1$ it is deeper than in the case of the temporal shock. At the later periods the system continues converging to its new equilibrium.

It is also interesting to get a look if a decline in interest rates affects prices in two economies with different demographic factors in the same way. In fact, decisions of the European central bank affect interest rates in the whole euro area, but countries have different demographic structure; therefore, we may expect different effects on inflation across the EU.

In Fig. 6 we present the effects of a temporal decline in interest rates when fertility rates are different ($n = 0.2$ and $n = 0$). The experiment designed as earlier, but, now it is performed for two cases $\sigma = 0.5$ and $\sigma = 2$. In both cases, lower fertility rate results in lower changes in price levels at $t = 0$, implying, that changes in interest rates affect prices at the smaller degree, when society is old. This is so, because smaller share of agents populating the economy take credits. This finding is in line with Imam (2013), who showed that monetary

Figure 6: Temporal decline in interest rate

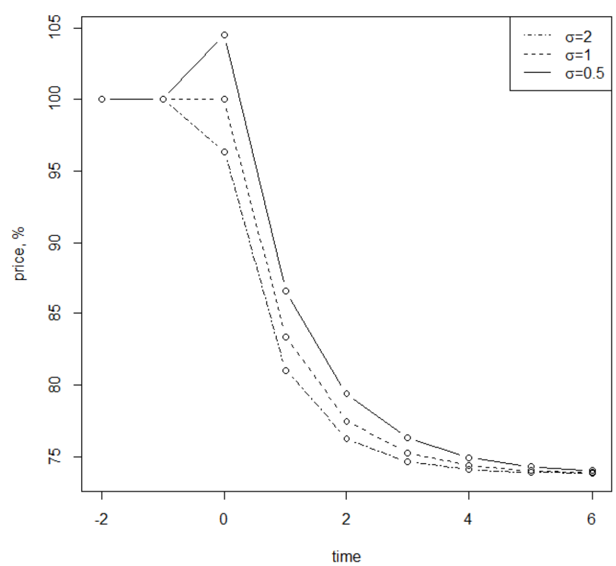


policy is less efficient when an economy has older society. In the next periods older society leads to a deeper decline in prices. However, the difference between the two profiles is rather small.

Increase in the probability of a good productivity shock

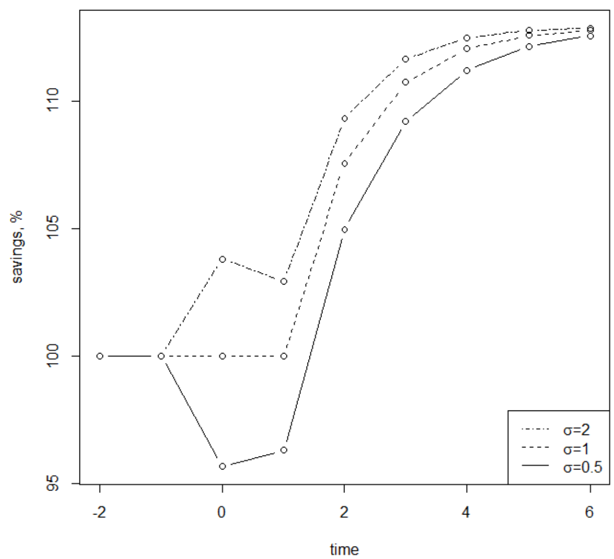
It is also interesting to find out the effects of a change in p - probability that an individual entrepreneur faces a positive productivity shock. Fig. 7 shows the effects of an increase in p from 0.5 to 0.6 at time $t = 1$, which is expected at $t = 0$. We suppose that this shock is expected, because otherwise bank would receive positive profits, violating the non-profit condition. It is also possible to assume that the shock is unexpected, and the interest rate on loans adjusts after the shock is realised to satisfy the bank's zero-profit condition. We will also briefly discuss this case.

Figure 7: Increase in probability of success: Prices



In order to understand the dynamics of prices in Fig. 7, we also plot the dynamics of savings, s_t, of agents, who have a positive technological shock in Fig. 8. The complication here is that the interest rate on savings depends on price, and price level depend on savings, because they determine supply of goods to the market. The simplest dynamics is when $\sigma = 1$. In this case, changes in interest rates do not affect savings. As a result, the fact, that at $t = 1$ probability of a good shock increases does not affect savings at $t = 0$. Thus,

Figure 8: Increase in probability of success: Savings



capital-labour ratios at time $t = 1$ do not change, and incomes of agents with a good technological shock remain the same. As equation (11) implies, unchanged incomes determine that savings of agents, who have a good technological shock, do not change again. However, the number of agents with a good technological shock increases in this case; therefore, the total amount of savings, and the total supply of goods to the market increases as well. As a result, the price declines at $t = 1$. Higher total savings lead to a higher amount of capital, enhancement of income, and higher incomes give rise to higher savings, and cheaper goods. The process continues until the system converges to its new steady state.

In case of $\sigma \neq 1$ the short-run dynamics is rather different. First, consider periods $t \geq 2$. As agents expect a decline in prices, this increases their interest rates on savings, because, in the next period, agents will be able to buy more goods (equation (1)). This raises savings when $\sigma > 1$, and reduces, when $\sigma < 1$, relatively to $\sigma = 1$. As a result, in case $\sigma > 1$, more goods are supplied to the market, and their price is lower than in the case $\sigma = 1$. The opposite happens when $\sigma < 1$.

At time $t = 1$, the decline in price happens because of the same reason as in case with $\sigma = 1$: as the number of agents with a good technological shock increases, they create higher total savings, and more goods are supplied to the market, having a negative effect on their price. Because of equation (1), this decline in price increases interest rates on savings in the period $t = 1$. Higher returns raise savings at $t = 0$, if $\sigma > 1$, and reduce them if $\sigma < 1$. This affects the supply of goods at $t = 0$, reducing the price, when $\sigma > 1$ and increasing it when $\sigma < 1$.

Because of equation (1) decline in price at $t = 1$ affects not only r_1^s, but also r_2^s: agents may sell goods at the lower prices at $t = 1$, they get fewer money, and this reduces their returns to savings. Consequently, agents reduce their individual savings relative to the previous period if $\sigma > 1$ and increase them if $\sigma < 1$. Indeed, returns to savings are also affected by π_2. Its decline has the opposite effect on interest rates and savings than the decline of π_1. The fact that $\pi_2 < \pi_1$ ensures that individual savings are higher than in the initial equilibrium when $\sigma > 1$ and smaller when $\sigma < 1$ at $t = 1$.

On the Fig. 7-8 the effects of an expected increase in the probability of a good technological shock are presented. If the shock is unexpected, the general view remains the same, with the difference that prices and savings do not change at $t = 0$. The further dynamics remains qualitatively the same.

4 Robustness and possible extensions

The model developed in this paper relies on a number of assumptions. Some of them are rather realistic, the others are not. For example, in this model we assumed that the bank gives credits for investments purposes only. Indeed, this assumption is not very restrictive, and the main message of the paper does not change if we introduce consumption credits as long as the demand for credits is created mainly by the young generation. This is a rather realistic condition, because older agents accumulate more savings, and, as a result, their demand for credits declines, while young agents often enter the market being liquidity constrained. Consequently, if we add consumption credits, the results should not change qualitatively: higher share of young agents leads to a higher amount of credits, money supply increases, and this leads to higher prices. Moreover, in the long run, inflationary effect of consumption credits may be even more pronounced, because investment credits directly increase production in the next period, while the effects of consumption credits are not so clear.

The most unrealistic assumption is that all the agents in the economy are self-employed. According to the OECD data only 6.6% of labour force was self-employed in the United States in 2013, but in other countries this percentage was higher: 25.0% in Italy, 27.4% in South Korea, 52.6% in Colombia. If an assumption that all agents are self-employed is changed to a more realistic that only an exogenous portion of agents are entrepreneurs, and the others work in their firms, the main message of the paper does not change: younger population implies higher number of entrepreneurs, this enlarges a number of credits given by the bank, and, as a result, more money are created leading to a higher price level. But, in reality, it is likely that the number of entrepreneurs is endogenous. An even more realistic assumption would be that there is a certain minimal amount of real capital that the entrepreneurs shall obtain in order to create their own firms. If the share of entrepreneurs in population positively depends on capital-labour ratios, this would weaken the results of the model, because population ageing would lead to capital deepening, and would increase the share of entrepreneurs in the population.

In addition, an endogenous share of entrepreneurs may depend on the interest rate on loans. If credits are costly, and entrepreneurs are risk-averse, high interest rates may disgust potential entrepreneurs from investment, and have a negative effect on the amount of money in the economy. As a result, the link between interest rates and price level may not be as pronounced as we predict. However, modeling endogenous choice of being or not being an entrepreneur requires a certain degree of heterogeneity of agents. Otherwise all agents would become entrepreneurs or not. This requires a more sophisticated model, which would be a good extension of the current paper.

5 Conclusions

In this paper we combined a neoclassical overlapping-generations model with the post-Keynesian endogenous money creation via credit markets. We showed that population ageing in terms of an increase in longevity and decline in fertility leads to a decline in prices. The main deflationary impact of declined fertility rates follows from the fact that a smaller number of young agents take fewer credits, this leads to a decline in the stock of money created by the banking sector. The secondary effect comes from the fact, that, if intertemporal elasticity of substitution is smaller than unity, smaller number of children increases consumption in families per person and, as a result, agents afford to create higher savings, having a positive effect on the goods supply to the market. This drives prices down. The effect of an increase in longevity on prices is the following: as agents know that they will live longer, they need to create higher savings. They supply more goods to the market, leading to deflation.

In addition we found a couple of other interesting features disregarded in previous models. For example, higher interest rates may have a positive effect on inflation, in the medium and long run, because they increase spending of the old generation. Moreover, higher interest rates reduce savings of agents, who need to return the credits, having a negative impact on the supply of goods, leading to an increase in prices, and also affect savings (and, therefore, prices) via an intertemporal allocation of consumption. However, the model is rather simplistic; thus, there is a need for further research in order to reveal a more elaborate link between interest rates and inflation.

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Appendix 1

Proposition. Under the assumptions of the model, and assuming that at time t_0, the bank was introduced and it didn't exist before,

$$\pi_t = \frac{(1+n_t)M}{ps_t} \tag{17}$$

holds.

Proof As we suppose that the bank was introduced at the period t_0, $N_{t_0}M$ credits were issued for the young generation, simultaneously creating a deposit of the same size. They bought physical capital, and transferred these money to the generation born at $t = t_0 - 1$, therefore $N_{t_0}M = S_{t_0}\pi_{t_0}$. Multiplying this equality by the bank's nonprofit condition (2) at time $t_0 + 1$, we receive $pN_{t_0}M(1 + r^l_{t_0+1}) = S_{t_0}\pi_{t_0}(1 + r^d_{t_0+1})$. Inserting this condition to equation (13) we obtain $N_{t_0+1}M = S_{t_0+1}\pi_{t_0+1}$. Continuing this process telescopically, we get that $N_tM = S_t\pi_t$ for any t, $t > t_0$. Equation (17) is received from $S_t = pN_{t-1}s_t$ and $N_t = N_{t-1}(1 + n_t)$. ■

Appendix 2

Table 1: Steady state values in the benchmark case

	variable	value
Capital per worker	k	0.0230
Output per worker	y	0.2211
Income per worker	w	0.1378
Price of goods	π	43.491
Interest rate on deposits	$1 + r_d$	1.8114
Interest rate on savings	$1 + r_s$	1.8114
Interest rate on loans	$1 + r_l$	3.6227
Savings per worker	s	0.0552