Evidence on Common Feature and Business Cycle Synchronization in Mercosur

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Evidence on Common Feature and Business Cycle Synchronization in Mercosur[*]

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Abstract

The aim of this work is to analyze the business cycles of Mercosur member countries in order to investigate their degree of synchronization. The econometric model uses the Beveridge-Nelson-Stock-Watson multivariate trend-cycle decomposition, taking into account the presence of common features such as common trend and common cycle. Once the business cycles are estimated, their degree of synchronization is analyzed by means of linear correlation in time domain and coherence and phase in frequency domain. Despite the evidence of common features, the results suggest that the business cycles are not synchronized. This may generate an enormous difficulty to intensify Mercosur agreements.

Keywords: Mercosur, Business Cycles, Trend-Cycle Decomposition, Common Features, Spectral Analysis.

JEL Codes: C32, E32, F02, F23.

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1. Introduction

The design of economic blocs, such as the European Union and the Mercosur, has the purpose of increasing social welfare through the unification of economic policies and trade agreements. According to Backus and Kehoe (1992) and Christodoulakis et al. (1995), the success of these policies depends on the similarities of the business cycles of the member states. A business cycle is a periodic but irregular up-and-down movement in economic activity, measured by fluctuations in real GDP and other macroeconomic variables. However, according to Lucas Jr. (1977), many authors focus the analysis on GDP, defining business cycles as the difference between the actual GDP and its long-run trend.

The aim of this paper is to analyze the business cycles of Mercosur member countries. The Mercosur, or southern common market, is a regional trade agreement created in 1991 by the Treaty of Asunción. Its members are: Argentina, Brazil, Paraguay, Uruguay, and Venezuela, welcomed as the fifth member in 2006. These countries differ in their institutions, economic policies and industrial structures, creating an enormous internal asymmetry in Mercosur (Flôres Júnior, 2005). Although the bloc was created in 1991, we will analyze a broader period, from 1951 to 2003. Therefore, if we find evidence in favor of similarity we can safely assume that it cannot be attributed only to Mercosur.[1] In fact, an inverse causality is investigated: if the similarities among the countries lead to commercial integration.

In the empirical literature, there is no consensus about how to estimate the trend-cycle components of economic time series and how to analyze the so-called comovements[2] in their business cycles. In the past decades a rich debate on the abilities of different statistical methods to decompose time series in long-run and short-run fluctuations has taken place (Baxter and King, 1995, Guay and St-Amant, 1996). The Hodrick-Prescott (HP) filter and the linear detrending are the usual univariate methodologies applied. However, these methodologies do not take into account the existence of common features among the economic series. In addition, as shown by Harvey and Jaeger (1993), the HP filter can induce spurious cyclicality when applied to integrated data. Therefore, in order to obtain a measure of the business cycles, we employ the Beveridge-Nelson-Stock-Watson (BNSW) multivariate trend-cycle decomposition (Beveridge and Nelson, 1981), considering the occurrence of cointegration and serial correlation common feature among the variables.

In order to investigate the degree of synchronization or comovement of their business cycles an extra effort is necessary. Many authors have used the linear correlation between cycles; however, this analysis gives a static measure of the comovements since it is not a simultaneous analysis of the persistence of comovement

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[1]Besides, there is not a consensus that Mercosur led to an increase in the flow of commerce among its integrated parts.

[2]Two countries present comovements when their real GDP expansions and downturns are simultaneous.

(Engle and Kozicki, 1993). To avoid this critique, the measures of coherence and phase in frequency domain are employed in order to investigate how synchronized the business cycles are (Wang, 2003). These frequency domain techniques constitute a straightforward way to represent economic cycles, because they provide information for all frequencies.

Finally, the results indicate the existence of common trends and common cycles among the economies studied. Thus, we confirm the need to use a multivariate approach, which is our first contribution. Time domain analysis found synchronization in two subgroups: Paraguay-Uruguay and Argentina-Brazil. However, frequency domain findings did not corroborate these results. Thus, in general, Mercosur countries are not synchronized.

Besides this introduction, the paper is organized as follows. Section 2 presents the econometric methodology. Section 3 reports the econometric results while Section 4 analyzes the degree of synchronization of the business cycles. Finally, the conclusions are summarized in the last section.

2.   Econometric Model

Common features may be seen as restrictions upon the dynamics of the countries and, consequently, upon the dynamics of their business cycles. While cointegration refers to long-run relationships, common cyclical restrictions refer to short-run dynamics. Engle and Kozicki (1993) and Vahid and Engle (1993) proposed the serial correlation common feature (SCCF) as a measure of common cyclical feature in the short run, which is applied in many empirical works. For example, Gouriéroux and Peaucelle (1993) analyzed some issues on purchase power parity; Campbell and Mankiw (1989) found a common cycle between consumption and income for most G-7 countries; Engle and Kozicki (1993) found common international cycles in GNP data for OECD countries; Engle and Issler (1993) found common cycles among sectoral output for the US; Candelon and Hecq (2000) tested Okun's law.

To implement the BNSW decomposition, taking into account the common features restrictions, a VAR model is estimated and the existence of long-run and short-run common dynamics is tested. Consider a Gaussian vector autoregression of finite order p, VAR(p):

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t \tag{1}$$

where y_t is a vector of n first-order integrated series, $I(1)$, and ϕ_i, $i = 1, \dots, p$ are matrices of dimension $n \times n$ and $\varepsilon_t \sim Normal\,(0, \Omega)$, $E\,(\varepsilon_t) = 0$ and $E\,(\varepsilon_t \varepsilon_\tau) = \{\Omega$, $if\ t = \tau$ and $0_{n \times n}$, $if\ t \neq \tau$; where Ω is not singular. The model (1) can be written equivalently as:

$$\Pi\,(L)\,y_t = \varepsilon_t \tag{2}$$

where $\Pi\left(L\right) = I_n - \sum_{i=1}^{p} \phi_i L^i$ and L represents the lag operator. Besides, $\Pi\left(1\right) = I_n - \sum_{i=1}^{p} \phi_i$ when $L = 1$.

2.1 Long-run restrictions (Cointegration)

The following hypotheses are assumed:

Proposition 1: *The $(n \times n)$ matrix $\Pi\left(\cdot\right)$ satisfies:*

1. *Rank $(\Pi\left(1\right)) = r$, $0 < r < n$, such that $\Pi\left(1\right)$ can be expressed as $\Pi\left(1\right) = -\alpha\beta'$, where α and β are $(n \times r)$ matrices with full column rank r.*

2. *The characteristic equation $|\Pi\left(L\right)| = 0$ has $n - r$ roots equal to 1 and all others are outside the unit circle.*

Assumption 1 implies that y_t is cointegrated of order $(1, 1)$. The elements of α are the adjustment coefficients and the columns of β span the cointegration space. Decomposing the polynomial matrix $\Pi\left(L\right) = \Pi\left(1\right)L + \Pi^*\left(L\right)\Delta$, where $\Delta \equiv (1 - L)$ is the difference operator, a vector error correction (VEC) model is obtained:

$$\Delta y_t = \alpha\beta' y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \varepsilon_t \tag{3}$$

where $\alpha\beta' = -\Pi\left(1\right), \Gamma_j = -\sum_{k=j+1}^{p} \phi_k$ $(j = 1, ..., p - 1)$ and $\Gamma_0 = I_n$.

2.2 Common cycle restrictions

The VAR(p) model can have short-run restrictions as shown by Vahid and Engle (1993).

Definition 1: *The serial correlation common feature holds in (3) if there is an $(n \times s)$ matrix $\tilde{\beta}$ of rank s, whose columns span the cofeature space, such as $\tilde{\beta}'\Delta y_t = \tilde{\beta}'\varepsilon_t$, where $\tilde{\beta}'\varepsilon_t$ is an s-dimensional vector that constitutes an innovation process with respect to all information prior to period t.*

Consequently, the SCCF restrictions occur if there is a cofeature matrix $\tilde{\beta}$ that satisfies the following assumption:

Proposition 2: $\tilde{\beta}'\, \Gamma_j = 0_{s \times n}$ $\qquad j = 1, ..., p - 1$

Proposition 3: $\tilde{\beta}'\, \alpha\beta' = 0_{s \times n}$

2.3 Trend-Cycle Decomposition

The BNSW trend-cycle decomposition can be introduced by means of the Wold representation of the stationary vector Δy_t given by:

$$\Delta y_t = C(L)\varepsilon_t \tag{4}$$

where $C(L) = \sum_{i=0}^{\infty} C_i L^i$ is a polynomial matrix in the lag operator, $C_0 = I_n$ and $\sum_{i=1}^{\infty} j \, |C_j| < \infty$. Using the following polynomial factorization $C(L) = C(1) + \Delta C^*(L)$, it is possible to decompose Δy_t such that:

$$\Delta y_t = C(1)\,\varepsilon_t + \Delta C^*(L)\,\varepsilon_t \tag{5}$$

where $C_i^* = \sum_{j>i}^{\infty}(-C_j)$, $i \geq 0$, and $C_0^* = I_n - C(1)$. Ignoring the initial value y_0 and integrating both sides of (5), we obtain:

$$y_t = C(1)\sum_{j=1}^{T} \varepsilon_t + C^*(L)\varepsilon_t \ = \tau_t + c_t \tag{6}$$

Equation (6) represents the BNSW decomposition where y_t is decomposed into an "n" random walk process named "stochastic trend" and "n" stationary process named "cycles." Thus, $\tau_t = C(1)\sum_{j=1}^{T}\varepsilon_t$ and $c_t = C^*(L)\varepsilon_t$ represent trend and cycle components, respectively. Assuming that long-run restrictions exist, then r cointegration vectors exist $(r < n)$. These vectors eliminate the trend component which implies that $\beta'C(1) = 0$. Thus, $C(1)$ has dimension $n - r$, which means that there are $n-r$ common trends. Analogously, assuming short-run restrictions, there are s cofeature vectors that eliminate the cycles, $\tilde{\beta}'C^*(L) = 0$, which implies that $C^*(L)$ has dimension $n - s$, which is the number of common cycles. It is worth noting that $r + s \leq n$ and the cointegration and cofeature vectors are linearly independent (Vahid and Engle, 1993). In order to obtain the common trends, it is necessary (and sufficient) to multiply equation (6) by $\tilde{\beta}'$, such that

$$\tilde{\beta}'y_t = \tilde{\beta}'C(1)\sum_{j=1}^{T} \varepsilon_t \ = \tilde{\beta}'\tau_c$$

This linear combination does not contain cycles because cofeature vectors eliminate them. Likewise, to get the common cycles it is necessary to multiply equation (6) by β', and so

$$\beta'y_t = \beta'C^*(L)\varepsilon_t \ = \beta'c_t$$

This linear combination does not contain the stochastic trend because the cointegration vectors eliminate the trend component. A special case emerges when $r + s = n$. In this case, it is extremely easy to estimate the trend and cycle components of y_t. As $\tilde{\beta}'$ and β' are linearly independent matrices, it is possible

to build a matrix A, such that $A_{n \times n} = (\tilde{\beta}', \beta')'$ has full rank and, therefore, is invertible. Notice that the inverse matrix can be partitioned as $A^{-1} = (\tilde{\beta}^- \quad \beta^-)$ and the trend and cycle components can be obtained as follows:

$$y_t = A^{-1} A y_t = \tilde{\beta}^- \left(\tilde{\beta}' y_t \right) + \beta^- \left(\beta' y_t \right) = \tau_t + c_t \qquad (7)$$

This implies that $\tau_t = \tilde{\beta}^- \tilde{\beta}' y_t$ and $c_t = \beta^- \beta' y_t$. Therefore, trend and cycle are linear combinations of y_t. Note that τ_t is generated by a linear combination of y_t using the cofeature vectors, containing the long-run component (because $\tilde{\beta}' y_t$ is a random walk component). On the other hand, c_t is generated by a linear combination of y_t using the cointegration vectors, containing the short-run component (because $\beta' y_t$ is $I(0)$ and serially correlated).

2.4   Estimation and testing

Considering the SCCF and the cointegration restrictions, we can rewrite the vector error correction as a model of reduced-rank structure. In (3) we define a vector $X_{t-1} = [y_{t-1}\beta', \Delta y'_{t-1}, ...\Delta y'_{t-p+1}]'$ of dimension $(n(p-1)+r) \times 1$ and an $n \times (n(p-1)+r)$ matrix $\Phi = [\alpha, \ \Gamma_1, ..., \Gamma_{p-1}]$. Therefore (3) is written as:

$$\Delta y_t = \Phi X_{t-1} + \varepsilon_t \qquad (8)$$

If Assumptions (1), (2) and (3) hold, then the matrices $\Gamma_i, i = 1, ..., p-1$ are all of reduced rank $(n-s)$ and they can be written as $\Phi = A[\Psi_0, \Psi_1, ..., \Psi_{p-1}] = A\Psi$, where A is an $n \times (n-s)$ full column rank matrix and Ψ has dimension $(n-s) \times (n(p-1)+r)$ and $\tilde{\beta}' A \Psi = 0$, that is, $\tilde{\beta} \in sp(A_\perp)$ where $A_\perp$ is the orthogonal complement of A. Therefore, let $A = \tilde{\beta}_\perp$.[3] Hence the model (8) can be expressed as a dynamic factor model with $n-s$ factor, given by ΨX_{t-1}, which are linear combinations of the right hand side variables in (3).

$$\Delta y_t = \tilde{\beta}_\perp \left(\Psi_0, \Psi_1, ..., \Psi_{p-1} \right) X_{t-1} + \varepsilon_t \qquad (9)$$

$$= \tilde{\beta}_\perp \Psi X_{t-1} + \varepsilon_t \qquad (10)$$

To estimate the coefficient matrices $\tilde{\beta}_\perp$ and Ψ in the reduced-rank model (10) we use Anderson's (1951) procedure (see also Johansen (1995)). This procedure is based on canonical analysis, which is a special case of a reduced-rank regression. More specifically, the maximum likelihood estimation of the parameters of the reduced-rank regression model may cause a problem of canonical analysis.[4] Therefore, we can use the expression $CanCorr\{X_t, Z_t | W_t\}$, which denotes the

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[3]The orthogonal complement of the $n \times s$ matrix B, $n > s$ and rank$(B) = s$, is the $n \times (n-s)$ matrix $B_\perp$ such that $B'_\perp B = 0$ and rank$(B : B_\perp) = n$. Hence, $B_\perp$ spans the null space of B and B' spans the left null space of $B_\perp$. The space is denoted by sp.

[4]This estimation is referred to as *Full Information Maximum Likelihood* – FIML.

partial canonical correlations between X_t and Z_t: both sets are concentrated out of the effect of W_t that allows us to obtain canonical correlation, represented by the eigenvalues $\hat{\lambda}_1 > \hat{\lambda}_2 > \hat{\lambda}_3....... > \hat{\lambda}_n$.

By the way, Johansen's cointegration test statistic is also based on canonical correlation. In model (3) we can use the expression $CanCorr\{\Delta y_t, y_{t-1} | W_t\}$ where $W_t = [\Delta y_{t-1}, \Delta y_{t-2}, ..., \Delta y_{t+p-1}]$ summarizes the reduced-rank regression procedure used in Johansen's approach. It means that one extracts the canonical correlations between Δy_t and y_{t-1}: both sets are concentrated out of the effect of lags of W_t.

Moreover, we could also use a canonical correlation approach to determine the rank of the common feature space due to SCCF restrictions. It is a test for the existence of cofeatures in the form of linear combinations of the variables in first differences, which are white noise (i.e., $\tilde{\beta}' \Delta y_t = \tilde{\beta}' \varepsilon_t$ where $\tilde{\beta}' \varepsilon_t$ is a white noise). Based on Tiao and Tsay (1985), Vahid and Engle (1993) proposed a sequential test for SCCF, assuming that the rank of β is known. The sequence of hypotheses to be tested are: $H_0 : rank\left(\tilde{\beta}\right) \geq s$ against $H_a : rank\left(\tilde{\beta}\right) < s$, (Lütkepohl, 1993, Velu et al., 1986) starting with $s = 1$ against the alternative model with $s = 0$ (there is no common cycle). If the null hypothesis is not rejected, we implement the test for $s = 2$, and so on.

In the VEC model the significance of the s smallest eigenvalues is determined through the following statistic:

$$\xi_s = -T \sum_{i=1}^{s} Ln\left(1 - \lambda_i^2\right) \sim \chi_{(v)}^2, \qquad s = 1, ..., n - r \qquad (11)$$

$\lambda_1 < \lambda_2 ....., < \lambda_{n-r} < 1$, with $v = s\left[n\left(p-1\right)+r\right] - s(n-s)$ degrees of freedom, where n is the dimension of the system and p is the lag order of the VAR model.[5] Suppose that the statistical test (11) has found s independent linear combinations of the elements of Δy_t unpredictable. This implies that there is an $n \times s$ matrix $\tilde{\beta}$ of full rank s with s eigenvectors associated with the s smallest eigenvalues. Reinsel and Ahn (1992) propose a correction in statistic (11) in small samples $\xi_s^{corr} = \frac{T-n(p-1)-r}{T}\xi_s$, where T is the real number of observations after the deduction of initial points in regressions containing lags.

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[5]For $p = 1$ the degrees of freedom are $(r + s)^2$. Notice in the model $\Delta y_t = \alpha\beta' y_{t-1} + \varepsilon_t$, the $rank(\alpha\beta') = \tilde{r} = n - s - r$, hence $\nu = (n - \tilde{r}) \times (np - \tilde{r}) = (n - (n - s - r))^2 = (r + s)^2$.

3. Empirical results

3.1 Database

The database used was extracted from the Penn World Table, corresponding to real GDP *per capita* series of Mercosur countries.[6] The frequency is annual, ranging from 1951 to 2003.[7] We consider the model $Y_t = T_t\, C_t$, where C_t is the cycle and T_t is the trend of the series. Define $y_t \equiv \log Y_t$, $\tau_t \equiv \log T_t$ and $c_t \equiv \log C_t$. Then, $y_t = \tau_t + c_t$. Figure 1 reports the GDP expressed in log terms. After 1975, in general, the series turned out to be similar to the behavior that may be generated by a common trend. Figure 2 shows the growth rates of real gross domestic product, i.e., $\ln(Y_t/Y_{t-1})$. It is possible to see the recession in Argentina, in 1989-1990.
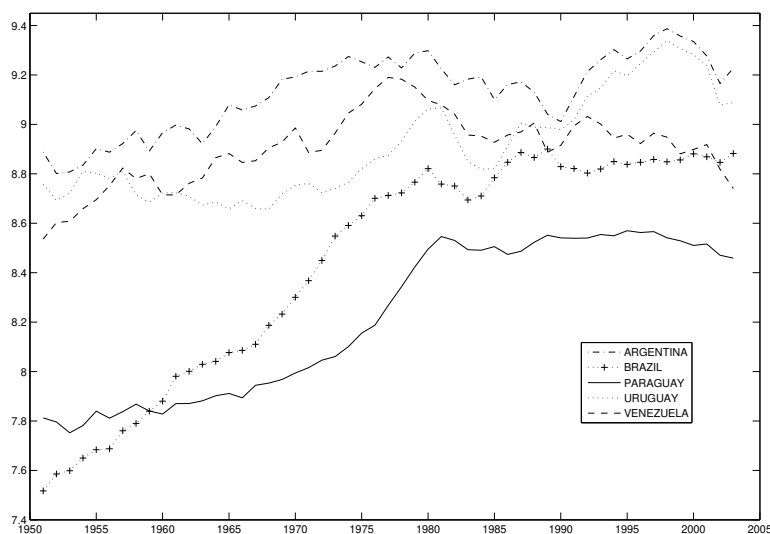

Figure 1
Real GDP (in log) *per capita* series of Mercosur countries (1951-2003)

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[6]Heston, Robert Summers and Bettina Aten, Penn World Table Version 6.2, Center for International Comparisons at the University of Pennsylvania (CICUP). Real GDP per capita (Constant Prices: Chain series).

[7]Penn World Table Version 6.2, Center for International Comparisons at the University of Pennsylvania (CICUP). Version 6.2 contains data from 1950 to 2004. However, some countries present missing data, like Brazil in 2004 and Paraguay in 1950 and 2004. `http://pwt.econ.upenn.edu/php\_site/pwt62/pwt62\_form.php`
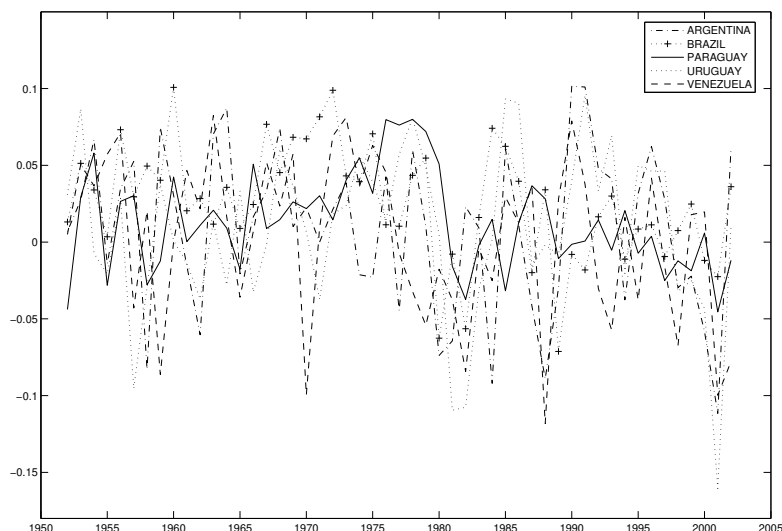
Figure 2
The growth rates of the real GDP *per capita* series of Mercosur countries (1951-2003)

Table 1 displays the descriptive statistics of the real GDP growth rates. In general, the average growth rate is much lower; except that of Brazil (2.63%). Actually, Figure 1 shows that Brazil had the lowest income level in 1951 and became an intermediate country in 2003. Figure 1 also shows a kind of convergence of Paraguay toward rich countries. In fact, Paraguay has the second largest growth rate (1.24%). The other countries are below the 1% rate. While Brazil and Paraguay show an upward trend, the other countries oscillate around a similar level and the standard deviation reflects these behaviors. Argentina, Uruguay and Venezuela are more volatile than Brazil and Paraguay. All countries experienced years of high growth, some of them above 10%, like Argentina and Brazil; but, episodes of sharp decreases are also present.

Table 1
Descriptive statistics

	Argentina	Brazil	Paraguay	Uruguay	Venezuela
Mean	0.65%	2.63%	1.24%	0.64%	0.39%
Standard Dev.	5.40%	3.79%	3.16%	5.38%	5.23%
Maximum	10.15%	10.08%	8.00%	9.56%	8.25%
Minimum	-11.16%	-7.12%	-4.55%	-16.05%	-11.82%

3.2   Common Feature results

To implement the previously stated methodology, a hierarchical procedure is followed to estimate the parameter of the model (Vahid and Engle, 1993). First, the VAR order, p, is estimated via information criteria: Akaike (AIC), Schwarz (SC) and Hannan-Quinn (HQ) (Lütkepohl, 1993). After that, we identify the number of long-run restrictions, r, through Johansen's cointegration test. Then the number of short-run restrictions due to SCCF, s, is estimated using the χ^2 test. Finally, the parameters are estimated in model (3) using the FIML procedure (Vahid and Issler, 2001)).

Since BNSW decomposition assumes that the series are $I(1)$, we begin the analysis using the augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and DF-GLS unit root tests. In addition, we apply the KPSS procedure, which differently from previous tests, has a stationary null hypothesis. The results for all countries are reported in Table 2.[8] The ADF, PP and DF-GLS tests do not reject the unit root null hypothesis at the 5% level of significance for all countries. At the 5% level, the KPSS does not reject the stationarity null hypothesis only for Uruguay. Even in this case, at the 10% level, the null hypothesis is rejected. After all, the results suggest that the series are I(1).

Table 2
Statistics of Unit Root Tests

Country	ADF	PP	DF-GLS	KPSS
Argentina	-1.8691	-1.9276	-1.9447	0.1543**
Brazil	-0.2404	-0.4308	-0.5998	0.2411***
Paraguay	-0.5757	-0.7392	-1.0805	0.1475**
Uruguay	-2.6644	-2.0443	-2.5328	0.1433**
Venezuela	-1.0972	-1.0780	-0.8094	0.2318***

Note: *, **, *** means rejection at 10%, 5% and 1% level of significance.

To estimate the order of the VAR, the AIC, HQ and SC information criteria are used. Table 3 shows the results for $p \in \{1, 2, 3, 4, 5\}$. As the data are annual we consider that an upper bound of 5 lags is sufficient. We observe that the three criteria suggest $p = 1$, indicating a VAR(1) model. Although the p selected by the criteria was one, to check the robustness of the results, we additionally test the model for $p = 2$ and $p = 3$.

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[8]In the case of ADF and DF-GLS tests, the choice of lags of the dependent variable on the right side of the test equation is based on the Schwarz criterion. In the PP and KPSS tests we use the nucleus of Bartlett and the window of Newey-West. All test equations have a constant and a linear trend. In any case, the results are robust to exclude the linear trend.

Table 3
Identification of the VAR order

Criteria	Lag				
	1	2	3	4	5
AIC	-18.4272*	-18.3237	-18.2791	-18.3222	-18.2962
SC	-17.2577*	-16.1797	-15.1605	-14.2289	-13.2284
HQ	-17.9852*	-17.5135	-17.1006	-16.7753	-16.3811

Note: *indicates the lag suggested by the information criteria.

Considering $p = 1, 2, 3$ the usual diagnostic tests are applied in order to verify if these specifications are suitable. For $p = 1$ and $p = 2$ the LM test does not indicate the presence of serial autocorrelation in the residuals at the 5% level of significance.[9] On the other hand, for $p = 3$ the opposite result is obtained. White's heteroskedasticity test (without cross terms) does not find evidence of heteroskedasticity at the 5% level of significance, for $p = 1, 2, 3$. The Jarque-Bera normality test does not reject the null hypothesis of normal distribution of residuals only for $p = 1$, at the 5% level of significance.[10] Consequently, the best specification is obtained when $p = 1$.

To test if the series are cointegrated, Johansen's (1988) procedure is used. We introduced a constant in the cointegration equation. The results for the cointegration test are shown in Table 4. The trace and the maximum eigenvalue test indicate $r = 2$ for $p = 1, 2$ while for $r = 1$ for $p = 3$. Even though, we use $r = 2$ for $p = 3$ to check the robustness of the subsequent analysis.

Table 5 shows the SCCF test for $p = 1, 2, 3$ using the correction given by Reinsel and Ahn (1992). For $p = 1$ the test indicates that $s = 4$, at the 5% level of significance, but as the p value is close to 5% we may assume $s = 3$ without trouble (see Table 5 (a)). For $p = 2, 3$ the test indicates $s = 3$ (see Table 5 (b) e (c)). Therefore, in all cases $s + r = n$. These results confirm the necessity to use a multivariate approach to identify the business cycles. In the next section we analyze the economic cycles obtained from the BNSW decomposition, considering the common cycles and the common trend restrictions. Once $s + r = n$, it is possible to find the trend and cycle components as shown above. Figure 3 shows the common cycles for each value of p. We observe that common cycles are very similar for $p = 1, 2$.

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[9]The null hypothesis of the LM test is the absence of serial correlation up to lag h. We consider h from 1 to 5.

[10]The normality test uses Cholesky orthogonalization.

Table 4
Johansen's cointegration test

a) Johansen's cointegration test for $p = 1$						
		Trace Test		Maximum Eigenvalue Test		
Null hypothesis	Statistic	Critical value	p value	Statistic	Critical value	p value
$r = 0$	103.2097*	69.81889	0.0000	52.74712 *	33.87687	0.0001
$r \leq 1$	50.46258 *	47.85613	0.0279	33.30722 *	27.58434	0.0082
$r \leq 2$	17.15536	29.79707	0.6286	11.76805	21.13162	0.5707
$r \leq 3$	5.387311	15.49471	0.7664	4.090286	14.26460	0.8497
$r \leq 4$	1.297025	3.841466	0.2548	1.297025	3.841466	0.2548
b) Johansen's cointegration test for $p = 2$						
		Trace Test		Maximum Eigenvalue Test		
Null hypothesis	Statistic	Critical value	p value	Statistic	Critical value	p value
$r = 0$	95.68994*	69.81889	0.0001	38.13180 *	33.87687	0.0146
$r \leq 1$	57.55814 *	47.85613	0.0047	36.27881 *	27.58434	0.0030
$r \leq 2$	21.27933	29.79707	0.3404	11.42115	21.13162	0.6053
$r \leq 3$	9.858185	15.49471	0.2918	7.026434	14.26460	0.4860
$r \leq 4$	2.831751	3.841466	0.0924	2.831751	3.841466	0.0924
c) Johansen's cointegration test for $p = 3$						
		Trace Test		Maximum Eigenvalue Test		
Null hypothesis	Statistic	Critical value	p value	Statistic	Critical value	p value
$r = 0$	96.33886 *	69.81889	0.0001	49.69316 *	33.87687	0.0003
$r \leq 1$	46.64570	47.85613	0.0647	24.17728	27.58434	0.1287
$r \leq 2$	22.46842	29.79707	0.2732	13.51631	21.13162	0.4059
$r \leq 3$	8.952108	15.49471	0.3698	7.064437	14.26460	0.4816
$r \leq 4$	1.887672	3.841466	0.1695	1.887672	3.841466	0.1695

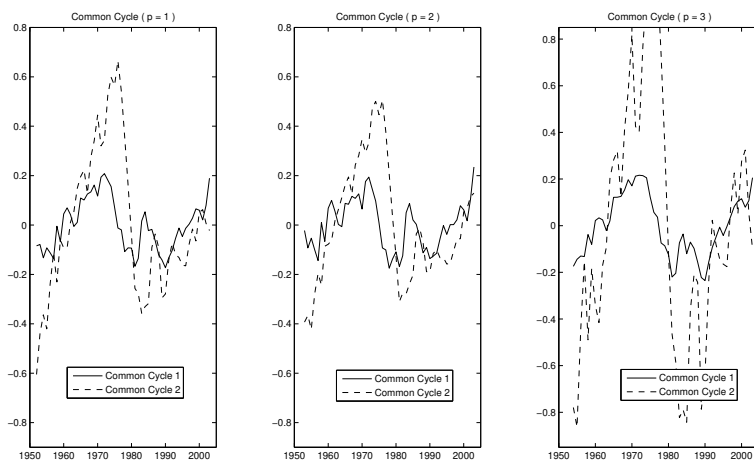Note: *indicates rejection of null hypothesis at the 5% leve l of significance.

Figure 3
Common cycles

Table 5
Common cycle test

a)	$r=2$, $n=5$, $p=1$ (Constant)			
Null hypothesis	λ^2	$\xi_{(p,s)}$	$[r+s]^2$	p value
$s>0$	0.0246	1.2971	9	0.9984
$s>1$	0.0756	5.3875	16	0.9935
$s>2$	0.2025	17.1553	25	0.8761
$s>3$	0.4730	50.4638	36	0.0554
$s>4$ *	0.6373	103.2064	49	0.0000

b)	$r=2$, $n=5$, $p=2$ (Constant)			
Null hypothesis	λ^2	$\xi^{corr}_{(p,s)}$	$s[n(p-1)+r]$ $+s^2-sn$	p value
$s>0$	0.0059	0.2606	3	0.9673
$s>1$	0.1215	5.9605	8	0.6517
$s>2$	0.1856	14.9927	15	0.4519
$s>3$*	0.5996	55.2643	24	0.0003
$s>4$ *	0.6781	105.1426	35	0.0000

c)	$r=2$, $n=5$, $p=3$ (Constant)			
Null hypothesis	λ^2	$\xi^{corr}_{(p,s)}$	$s[n(p-1)+r]$ $+s^2-sn$	p value
$s>0$	0.0644	2.5316	8	0.9602
$s>1$	0.2441	13.1665	18	0.7816
$s>2$	0.4422	35.3461	30	0.2303
$s>3$*	0.6317	73.3036	44	0.0036
$s>4$ *	0.7392	124.3791	60	0.0000

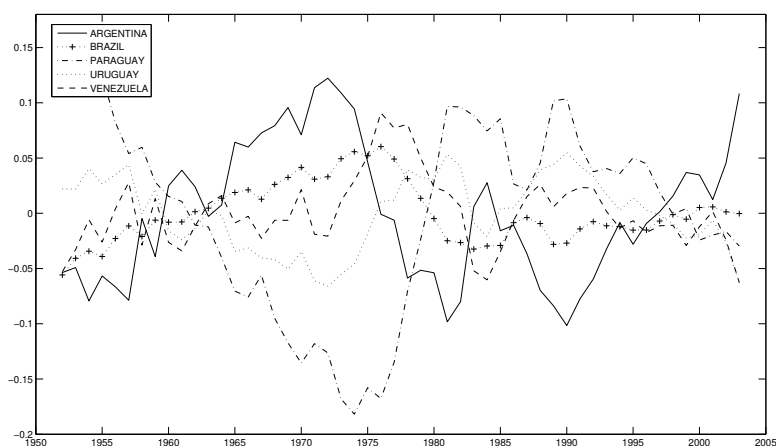Note: *indicates rejection of null hypothesis at the 5% level of significance.

Figure 4
Cyclical components for $p=1$, $s=3$ and $r=2$

Figure 4 shows the business cycle components for our best specification: $p = 1$, $s = 3$ and $r = 2$. We notice an enormous contraction in Argentina in the 1990s, as expected. As to Brazil, the period of the economic miracle is apparent. To analyze the robustness of the results we estimate business cycles for each country for $p = 1, 2, 3$. Figure 5 shows the business cycle for each country. It is possible to see that the business cycles obtained from different p are similar.
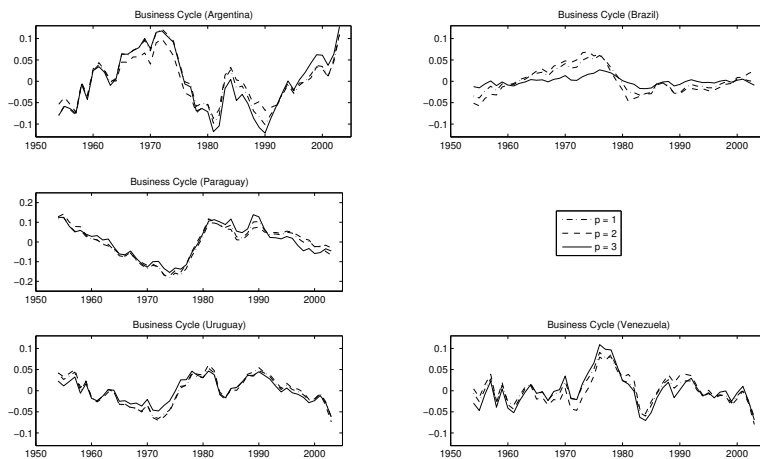

Figure 5

Cyclical components in each country for $p = 1, p = 2$ and $p = 3$

4. Business Cycle Analysis

The degree of association among contemporaneous movements may be measured through the pairwise linear correlation as reported in Table 6 for $p = 1, 2, 3$. We can observe for $p = 1$ that Paraguay and Uruguay have a high positive correlation. The same occurs for Brazil and Argentina. So far, based on correlation analysis there are two pairs of countries with similar patterns. The correlations of each country with cycles 1 and 2 explain these results. Paraguay and Uruguay have a negative correlation with both cycles, while Brazil and Argentina are negatively related to both cycles. Not surprisingly, Venezuela has a different behavior, being negatively correlated with the first cycle and positively correlated with the second one.

For $p = 1$, it is worth mentioning that Argentina is correlated with common cycle 1 near 1 while Uruguay has a correlation near -1, which means that basically Argentina and Paraguay are in opposite directions. When Argentina is booming,

there is a recession in Uruguay. Indeed, the linear correlation between Argentina and Paraguay is almost -1. In addition, Brazil has a correlation with common cycle 2 almost equal to 1, while Paraguay has a correlation with common cycle 2 around -1. Not surprisingly, the linear correlation between Brazil and Paraguay is almost -1. So far, we know that the Argentina-Brazil and Paraguay-Uruguay pairs seem to be closely connected, while the Argentina-Uruguay and Brazil-Paraguay pairs seem to be in opposite direction, which is also relevant information.

Despite the fact that $p = 1$ is our best specification, the association between Paraguay and Uruguay remains high for $p = 2, 3$. However, the association between Brazil and Argentina plunged for $p = 3$. The correlations with the common cycles are robust in the following sense: Paraguay and Uruguay have a negative correlation with both cycles for $p = 1, 2, 3$ while Brazil and Argentina are negatively related to both cycles for $p = 1, 2, 3$. The results for Venezuela are more sensitive to p. Indeed, for $p = 3$, its correlation with the first common cycle becomes positive, although close to zero.

Table 6
Linear correlations in business cycles and in common cycle

Countries	Argentina	Brazil	Paraguay	Uruguay	Venezuela	C.cycle 1	C.cycle 2
			VAR(1)				
Argentina	1.0000					0.9994	0.6165
Brazil	0.6383	1.0000				0.6110	0.9996
Paraguay	-0.7476	-0.9885	1.0000			-0.7239	-0.9838
Uruguay	-0.9944	-0.5533	0.6731	1.0000		-0.9975	-0.5297
Venezuela	-0.2806	0.5597	-0.4277	0.3806	1.0000	-0.3140	0.5827
			VAR(2)				
Argentina	1.0000					0.9910	0.5637
Brazil	0.6306	1.0000				0.5212	0.9965
Paraguay	-0.6536	-0.9995	1.0000			-0.5466	-0.9936
Uruguay	-0.9926	-0.7201	0.7406	1.0000		-0.9675	-0.6597
Venezuela	-0.7706	0.0088	0.0213	0.6876	1.0000	-0.8488	0.0921
			VAR(3)				
Argentina	1.0000					0.9824	0.5526
Brazil	0.2604	1.0000				0.4360	0.9486
Paraguay	-0.7832	-0.8042	1.0000			-0.8855	-0.9510
Uruguay	-0.9547	0.0387	0.5627	1.0000		-0.8824	-0.2796
Venezuela	-0.1772	0.9041	-0.4731	0.4620	1.0000	0.0096	0.7223

Once the analysis through linear correlation gives a static measure of the co-movements, as noted by Engle and Kozicki (1993), we complement this analysis using techniques based on the frequency domain. Two measures are employed in frequency domain: coherence and phase.[11]

The coherence between two time series is a measure of the degree to which the series are jointly influenced by cycles of frequency w. Coherence belongs to the interval $[0, 1]$. If two time series have perfect linear correlation (positive or negative) the coherence is equal to one. It happens because the same cycles of frequency w are present in both time series. The phase spectrum measures phase difference between two cycles at frequency w. Two oscillators that have the same frequency and different phases have a phase difference, and the oscillators are said to be out of phase with each other.

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[11]See Appendix B.

In summary, we can define comovements using information in time and frequency domains. Therefore, two time series are synchronized when they have a positive linear correlation, their coherence is close to one and their phase difference is close to zero at each frequency w. Two cases where one of this condition fails is shown in Figure 7 where the Argentina-Uruguay and Brazil-Paraguay pairs have coherence close to one and phase close to zero, but the first condition is not satisfied, which means that they have a high negative linear correlation (see Table 7). Indeed, we commented that while Argentina is almost identical to common cycle 1, Uruguay is almost minus common cycle 1. The same happens for Brazil and Paraguay, but in relation to common cycle 2.

Given that, we focus our analysis on the subgroups identified by the time domain approach. Two groups have high positive linear correlation; i) Brazil and Argentina (0.6383 for $p = 1$) and ii) Paraguay and Uruguay (0.6731 for $p = 1$). Results for $p = 2, 3$ are also reported.

Figures 6 to 9 show the coherence and phase between pairs of the business cycles of Mercosur members.[12] These pictures show values of coherence varying between zero and one (vertical axis) for each value of frequency (horizontal axis). Values of phase (vertical axis) are calculated for each value of frequency (horizontal axis). At the final point of the horizontal axis, the frequency 0.5 corresponds to the period of two years, the point 0.25 corresponds to four years, and frequency 0.1 corresponds to 10 years, and so on.

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[12]Coherence is estimated using the the *mscohere* function of Matlab and the phase, indirectly, using the *cpsd* function. The function cpsd is used to estimate the cross power spectral density (CPSD) via Welch's method.
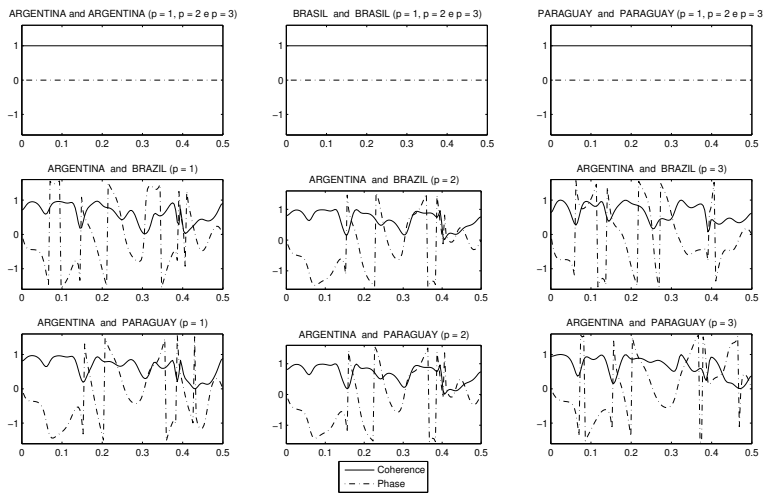
Figure 6
Coherence and phase

The first row in Figure 6 shows the ideal values of coherence and phase, that is, coherence one and phase zero at all frequencies. For example, this picture shows the results for synchronization of business cycle of Argentina with itself at each value p, and, after, the same is done for Brazil and Paraguay.

Figure 6 also shows the results of coherence and phase for the first group: Argentina and Brazil. Focusing on $p = 1$, coherence is close to one for some frequencies; however, the phase is not close to zero in most frequencies. Thus, when we scrutinized the time domain results, using frequency domain tools, the degree of association between Argentina and Brazil was drastically reduced. The results are similar for $p = 2, 3$.

Figure A.1 reports the results of coherence and phase for the second group: Uruguay and Paraguay. For $p = 1$, Uruguay and Paraguay have coherence close to one for some frequencies; however, their phase is, in general, far from zero. Thus, this deeper analysis in frequency domain casts some doubt on the Uruguay-Paraguay association. Qualitatively, the results are the same for $p = 2, 3$. Hence, the findings suggested no association between this pair.
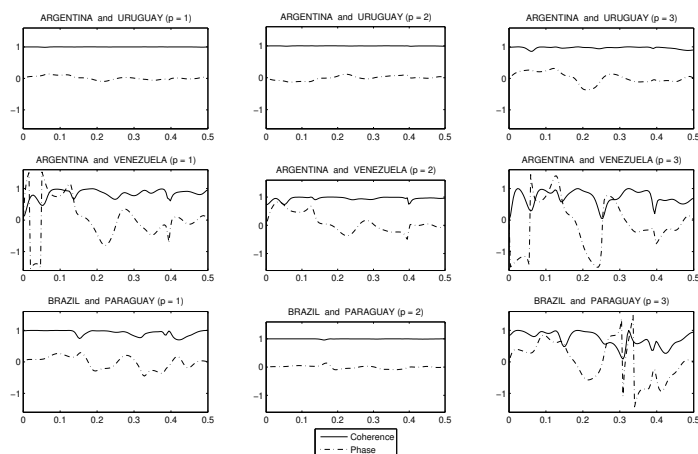
Figure 7
Coherence and phase

Therefore, the lack of synchronization among the business cycles illustrates the importance to conduct this analysis in frequency domain. Lastly, Appendix A presents the results for other pairs of countries.

5.   Conclusion

The design of economic blocks is based on the harmonization of economic and trade policies. However, as argued by Backus and Kehoe (1992) and Christodoulakis et al. (1995), this harmonization is successful when the member states are sufficiently similar. If this is true, it is of utmost importance to analyze the dynamics of the members and investigate the degree of synchronization of their business cycles. Regarding Mercosur, it is common to see in the media discussions on the intensification of this economic bloc. However, it is not usual to argue which the necessary conditions for this intensification are and if they are valid. Considering the members of Mercosur (Argentina, Brazil, Paraguay, Uruguay and Venezuela), this paper analyzes if there are any common dynamics in their economies and if their business cycles are synchronized.

To implement the analysis we estimate a VAR model and test the presence of common trends and common cycles. Using the BNSW trend-cycle decomposition, the business cycles were estimated, taking into account the cointegration and serial correlation common feature restrictions. In addition, beyond the usual correlation analysis, measures of coherence and phase, in the frequency domain, were used to examine the degree of comovements in business cycles.

The results suggest that there are three common trends and two common cycles among the countries. These results confirm the necessity to use a multivariate approach to obtain the business cycles, which is the first contribution of this work. Time domain results identified evidence of comovements in two subgroups: Paraguay-Uruguay and Argentina-Brazil. However, frequency domain tools casts some doubt on the synchronization of these pairs. Hence, the lack of synchronism or symmetry in the business cycle of Mercosur hinders a greater integration into this economic bloc.

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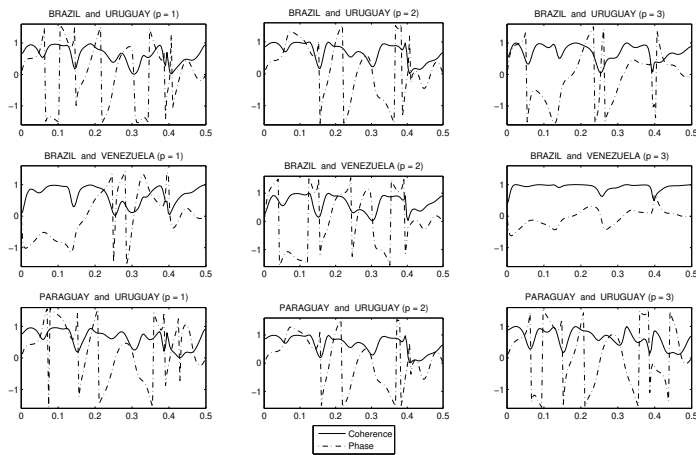# Appendix A: Coherence and Phase Results
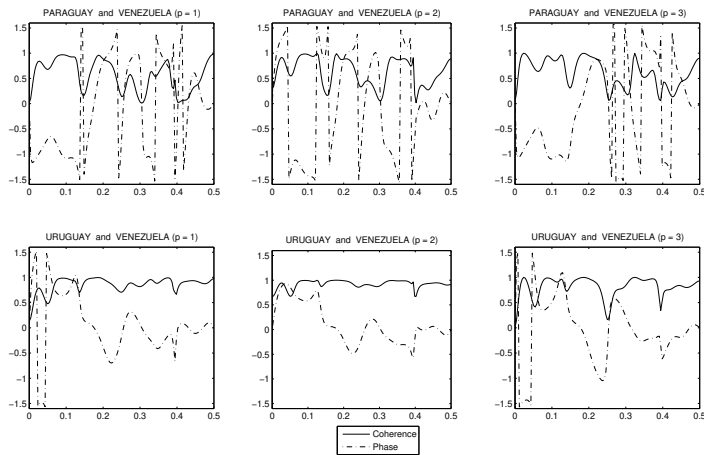

Figure A.1
Coherence and phase

Figure A.2
Coherence and phase

Appendix B: Coherence and Phase

Consider a vector of two stationary variables $y_t = (X_t, Y_t)$. Let $S_{YY}(w)$ represent the *population spectrum* of Y and $S_{YX}(w)$, the *population cross spectrum* between X, Y. The *population cross spectrum* can be written in term of its real and imaginary components as $S_{YX}(w) = C_{YX}(w) + i\, Q_{YX}(w)$, where $C_{YX}(w)$ and $Q_{YX}(w)$ are labeled the *population cospectrum* and *population quadrature spectrum* between X, Y, respectively. The *population coherence* between X and Y is a measure of the degree to which X and Y are jointly influenced by cycles of frequency w.

$$h_{YX}(w) = \frac{[C_{YX}(w)]^2 + [Q_{YX}(w)]^2}{S_{YY}(w)\ S_{XX}(w)}$$

Coherence takes values in $0 \leq h_{YX}(w) \leq 1$. A value of one for coherence at a particular point means the two series are altogether in common at that frequency or cycle; if coherence is one over the whole spectrum then the two series are common at all frequencies or cycles. The *cross spectrum* is in general complex, and may be expressed in its polar form as:

$$S_{YX}(w) = C_{YX}(w) + i\, Q_{YX}(w) = R(w) \exp(i\, \theta(w))$$

where $R(w) = \left\{ [C_{YX}(w)]^2 + [Q_{YX}(w)]^2 \right\}^{\frac{1}{2}}$ and $\theta(w)$ represent the gain and the angle in radians at the frequency w. The angle satisfies $tang(\theta(w)) = \frac{Q_{YX}(w)}{C_{YX}(w)}$. More details in Hamilton (1994).