An Algorithm for Solving Simple Sticky Information New Keynesian DSGE Model

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Abstract

This paper describes a new algorithm for solving a simple Sticky Information New Keynesian model using the methodology of Wang and Wen (2006). Impulse responses for demand and supply shock have been generated and analyzed intuitively. The strength of our algorithm lies in its analytical solution, which allow to uncover better intuition from the model.

JEL Classification: C22, C61, C63, E63, E52

Keywords: New-Keynesian Model, Algorithm

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1 Introduction

Sticky price New Keynesian DSGE model is the work horse of modern monetary policy analysis. However, the model suffers from several criticisms (Mankiw and Reis, 2002). First, the model fails to produce hump in inflation rate and output to monetary policy shock as observed in the data. Second, the model does not have any endogenous persistence of its own. It simply borrows the persistence of demand and supply shock. Third, the model does not follow the Natural Rate Hypothesis (McCallum, 1998). Fourth, credible disinflation causes booms rather than recessions (Ball, 1994). Mankiw and Reis (2002, 2006) has developed a sticky information New Keynesian model that survives all the criticisms mentioned above.[1] This paper develops an algorithm to solve a simple sticky information model developed by Mankiw and Reis (2002) using the methodology of Wang and Wen (2006).

Mankiw and Reis (2002, 2006) has developed the sticky information model by assuming that information is costly to acquire and process. As a result, information diffuses slowly in population. Such slow diffusion of information causes an information asymmetry among economic agents. This information asymmetry causes fluctuations in inflation and output in the short run. Based on this assumption, Mankiw and Reis (2002) derives a backward looking sticky information Phillips curve. Using such a backward looking Phillips curve representing the supply side of the economy, and demand side represented by a log linearized Quantity Theory of Money, Mankiw and Reis (2002) shows that even such a simple model of sticky information performs better than a sticky price model to match *stylized facts.*

Later, Mankiw and Reis (2006) has developed a full blown Sticky Information New Keynesian model DSGE model, where not only firms but even households suffer from information asymmetry. This is known as the model of pervasive stickiness. The demand side of the pervasive stickiness model is represented by expectational IS schedule which produces a backward looking aggregate demand curve when combined with the information asymmetry of households. On the other hand, the supply side of the model is represented by a combination of wage curve and sticky information Phillips curve. The Phillips curve and wage curve is derived by assuming that households who supply labor and firms who produce goods have information asymmetry. The model is closed by specifying a Taylor rule that determines the nominal interest rate.

The differences between the simple and pervasive stickiness model are, (i) money

[1] See Woodford (2003) for the development and analysis of sticky price New Keynesian model.

supply is the instrument of the monetary authority in the simple model but nominal interest rate is the instrument of the monetary authority in the pervasive stickiness model. (ii) while information asymmetry enters only through the supply side in the simple model, information asymmetry enters both through the supply and demand side in the pervasive stickiness model and (ii) the solution of the simple model was complicated due to its infinite state space but the solution of the pervasive stickiness model becomes even more complicated as the state space of the pervasive stickiness model is doubly infinite. Mankiw and Reis (2002, 2006) have solved both models using their own algorithm.

Wang and Wen (2006) has devised an ingenious methodology that can solve a wide range of sticky information model very easily. Applying Wang and Wen (2006) methodology to solve sticky information model is simple. First, we have to write the model in forecast error form. Then the model is solved using the method of undetermined coefficients. Writing the model in forecast error forms effectively reduces the numbers of parameters to deal with while solving the model. This greatly reduces the possibility of incurring human errors. Moreover, we show that more intuition can be uncovered from the model when solved using the methodology of Wang and Wen (2006). Mankiw and Reis (2006) also appreciated the methodology of Wang and Wen (2006) and one of the future research agenda was to solve the pervasive sticky information model using the methodology of Wang and Wen (2006) and compare its efficiency with their own algorithm.[2]

We have solved a simple sticky information model developed by Mankiw and Reis (2002) using the methodology of Wang and Wen (2006). Moreover, we have extended the derivation of sticky information Phillips curve of Mankiw and Reis (2002) by introducing a supply shock in the model.[3] Beside this, while Mankiw and Reis (2002) have only solved the model under demand shock, we have solved the model under both demand and supply shock. The rest of the paper proceeds as follows. Section 2 briefly describes the model. Section 3 gives the algorithm to solve the model. Section 4 analyzes the impulse response separately for demand and supply shock and section 5 concludes.

2 The Model

We briefly describe the model of Mankiw and Reis (2002) in this section. The demand side of the model is represented by the log linearized Quantity Theory of Money. The

[2] Also see, Verona and Wolter (2013) to solve pervasive stickiness model in dynare.

[3] See appendix for the derivation

demand curve is given in equation (1).

$$m_t = p_t + y_t \tag{1}$$

where, m_t, p_t and y_t are respectively the nominal money supply, price level and output at time t. The supply side of the model is represented by a backward looking sticky information Phillips curve. The supply curve is given in equation (2). The derivation of the curve is given in the appendix. We have assumed that $(1-\theta)$ is the fraction of firm having completely updated information, $\theta \in (0,1)$. $\Delta y_t = (y_t - y_{t-1})$ is the growth rate of output. π_t is inflation rate at time t and E is the expectation operator. $\alpha \in (0,1)$ is a measure of degree of nominal rigidity or strategic complementarity (Mankiw and Reis, 2002).[4]

$$\begin{aligned} y_t &= \theta y_{t-1} + \left(\frac{1-\theta}{\alpha}\right) \sum_{j=1}^{\infty} \theta^j \left[(\pi_t + \alpha \Delta y_t + \gamma \Delta e_t) - E_{t-j} (\pi_t + \alpha \Delta y_t + \gamma \Delta e_t) \right] \\ &\quad - \left(\frac{\gamma}{\alpha}\right) (1-\theta L) e_t \end{aligned} \tag{2}$$

We have also assumed that growth rate of nominal money supply follows the $AR(1)$ process given in equation (3). Supply shock also follows an $AR(1)$ process given in equation (3).

$$\Delta m_t = \rho_{\Delta m} \Delta m_{t-1} + \epsilon_t^{\Delta m}, \rho_{\Delta m} \in [0,1] \tag{3}$$

$$e_t = \rho_e e_{t-1} + \epsilon_t^e, \rho_e \in [0,1] \tag{4}$$

where, $\epsilon_t^{\Delta m}$ and ϵ_t^e are white noise process with mean zero and finite variance $\sigma_{\Delta m}^2$ and $\sigma_{\epsilon^e}^2$. Note, m can also be interpreted as nominal GDP or exogenous shifters of demand curve. The supply shock can be interpreted as technology shock, mark-up shock, oil price shock etc. The value of $\gamma \in (0,1)$ changes with the type and characteristics of the shock. L is the lag operator such that, $L^j(x_t) = x_{t-j}$, where, $j \in (-\infty, \infty)$

3 The Algorithm

This section describes the algorithm to solve the model using the methodology of Wang and Wen (2006). The model is solved using the method of undetermined coefficients after writing the model is forecast error form. This allows us to deal with smaller number of

[4] Alo see, Ball and Romer (1990) and Cooper and John (1988).

parameters while solving the model and greatly reduces the possibility of incurring human error. We first solve the supply equation. The next subsection describes how to solve the demand equation.

3.1 The Supply Curve

Note, we have already written the sticky information supply curve in forecast error form (equation, (2)) so that we can apply the methodology of Wang and Wen (2006). The Wang and Wen (2006) solves the model based on the method of undetermined coefficients. To solve the model we assume we assume that output and inflation follows an $MA(\infty)$ process as given respectively in equation (5) and (6) below using the principle of Wold representation Theorem.

$$y_t = \sum_{j=0}^{\infty} a_{yj}^{\Delta m} \epsilon_{t-j}^{\Delta m} + \sum_{j=0}^{\infty} a_{yj}^{e} \epsilon_{t-j}^{e} \tag{5}$$

and,

$$\pi_t = \sum_{j=0}^{\infty} a_{\pi j}^{\Delta m} \epsilon_{t-j}^{\Delta m} + \sum_{j=0}^{\infty} a_{\pi j}^{e} \epsilon_{t-j}^{e} \tag{6}$$

with,

$$\begin{aligned}
\sum_{j=0}^{\infty} \left(a_{yj}^{\Delta m}\right)^2 &< \infty, \ \sum_{j=0}^{\infty} \left(a_{yj}^{e}\right)^2 < \infty \\
\sum_{j=0}^{\infty} \left(a_{\pi j}^{\Delta m}\right)^2 &< \infty, \ \sum_{j=0}^{\infty} \left(a_{\pi j}^{e}\right)^2 < \infty,
\end{aligned}$$

We then rewrite equation (2) as,

$$(1-\theta L) y_t = \left(\frac{1-\theta}{\alpha}\right) \left[s_{1\pi} + \alpha \left(s_{1y} - s_{2y}\right) + \gamma \left(s_{1e} - s_{2e}\right)\right] - \frac{\gamma}{\alpha} (1-\theta L) e_t \tag{7}$$

where,

$$
\begin{aligned}
s_{1\pi} &= \sum_{j=1}^{\infty} \theta^j \left[\pi_t - E_{t-j}\left(\pi_t\right)\right], \\
s_{1y} &= \sum_{j=1}^{\infty} \theta^j \left[y_t - E_{t-j}\left(y_t\right)\right] \\
s_{2y} &= \sum_{j=1}^{\infty} \theta^j \left[y_{t-1} - E_{t-j}\left(y_{t-1}\right)\right] \\
s_{1e} &= \sum_{j=1}^{\infty} \theta^j \left[e_t - E_{t-j}\left(e_t\right)\right] \\
s_{2e} &= \sum_{i=1}^{\infty} \theta^j \left[e_{t-1} - E_{t-j}\left(e_{t-1}\right)\right]
\end{aligned}
$$

Note, using equation (6) we have,

$$
\begin{aligned}
\pi_t - E_{t-1}(\pi_t) &= a_{y0}^{\Delta m}\epsilon_t^{\Delta m} + a_{y0}^{e}\epsilon_t^{e} \\
\pi_t - E_{t-2}(\pi_t) &= \left(a_{y0}^{\Delta m} + a_{y1}^{\Delta m}L\right)\epsilon_t^{\Delta m} + \left(a_{y0}^{e} + a_{y1}^{e}L\right)\epsilon_t^{e} \\
\pi_t - E_{t-3}(\pi_t) &= \left(a_{y0}^{\Delta m} + a_{y1}^{\Delta m}L + a_{y2}^{\Delta m}L^2\right)\epsilon_t^{\Delta m} + \left(a_{y0}^{e} + a_{y1}^{e}L + a_{y2}^{e}L^2\right)\epsilon_t^{e} \\
&\quad ..... \\
&\quad ......
\end{aligned}
$$

This implies,

$$
\pi_t - E_{t-j}(\pi_t) = \sum_{k=0}^{j-1}\left[a_{\pi k}^{\Delta m}L^k\left(\epsilon_t^{\Delta m}\right)\right] + \sum_{k=0}^{j-1}\left[a_{\pi k}^{e}L^k\left(\epsilon_t^{e}\right)\right], \text{ for } j = 1, 2, 3, ... \tag{8}
$$

Similarly using equation (5) we have,

$$
y_t - E_{t-j}(y_t) = \sum_{k=0}^{j-1}\left[a_{yk}^{\Delta m}L^k\left(\epsilon_t^{\Delta m}\right)\right] + \sum_{k=0}^{j-1}\left[a_{yk}^{e}L^k\left(\epsilon_t^{e}\right)\right], \text{ for } j = 1, 2, 3, ... \tag{9}
$$

Also note that equation (5) gives,

$$\begin{aligned} y_{t-1} - E_{t-1}\left(y_{t-1}\right) &= 0 \\ y_{t-1} - E_{t-2}\left(y_{t-1}\right) &= a_{y0}^{e} L \epsilon_t^e \\ y_{t-1} - E_{t-3}\left(y_{t-1}\right) &= \left(a_{y0}^{e} L + a_{y1}^{e} L^2\right) \epsilon_t^e \\ &\quad ...... \\ &\quad ..... \end{aligned}$$

This implies,

$$\begin{aligned} y_{t-1} - E_{t-1}\left(y_{t-1}\right) &= 0, \\ y_{t-1} - E_{t-j}\left(y_{t-1}\right) &= \sum_{k=0}^{j-1} \left[a_{yk}^{\Delta m} L^{k+1}\left(\epsilon_t^{\Delta m}\right)\right] + \sum_{k=0}^{j-1} \left[a_{yk}^{e} L^{k+1}\left(\epsilon_t^{e}\right)\right], \text{ for } j = 2, 3, 4, ... \end{aligned} \tag{10}$$

$$e_t - E_{t-j}(e_t) = \sum_{k=0}^{j-1} \left[\rho_e^k L^k \left(\epsilon_t^e\right)\right], \text{ for } j = 0, 1, 2, 3, ... \tag{11}$$

$$\begin{aligned} e_{t-1} - E_{t-1}\left(e_{t-1}\right) &= 0, \\ e_{t-1} - E_{t-j}\left(e_{t-1}\right) &= \sum_{k=0}^{j-1} \left[\rho_e^k L^{k+1}\left(\epsilon_t^e\right)\right], \text{ for } j = 2, 3, 4, ... \end{aligned} \tag{12}$$

Now, using equation (8) we can calculate,

$$\begin{aligned} s_{1\pi} &= \sum_{j=1}^{\infty} \theta^j \left[\pi_t - E_{t-j}\left(\pi_t\right)\right] \\ &= \frac{\theta}{1-\theta}\left[a_{\pi 0}^{\Delta m} + \theta a_{\pi 1}^{\Delta m} L + \theta^2 a_{\pi 2}^{\Delta m} L^2 + ...\right] \epsilon_t^{\Delta m} \\ &\quad + \frac{\theta}{1-\theta}\left[a_{\pi 0}^{e} + \theta a_{\pi 1}^{e} L + \theta^2 a_{\pi 2}^{e} L^2 + ...\right] \epsilon_t^{e} \end{aligned} \tag{13}$$

Similarly, by using equation (9) we can calculate,

$$\begin{aligned} s_{1y} &= \sum_{j=1}^{\infty} \theta^j \left[y_t - E_{t-j}\left(y_t\right) \right] \\ &= \frac{\theta}{1-\theta}\left[a_{y0}^{\Delta m} + \theta a_{y1}^{\Delta m} L + \theta^2 a_{y2}^{\Delta m} L^2 + ...\right] \epsilon_t^{\Delta m} \\ &\quad + \frac{\theta}{1-\theta}\left[a_{y0}^{e} + \theta a_{y1}^{e} L + \theta^2 a_{y2}^{e} L^2 + ...\right] \epsilon_t^{e} \end{aligned} \tag{14}$$

and by using equation (10) we can calculate,

$$\begin{aligned} s_{2y} &= \sum_{j=1}^{\infty} \theta^j \left[y_{t-1} - E_{t-j}\left(y_{t-1}\right) \right] \\ &= \frac{\theta}{1-\theta}\left[\theta a_{y0}^{\Delta m} L + \theta^2 a_{y1}^{\Delta m} L^2 + \theta^3 a_{y2}^{\Delta m} L^3 + ...\right] \epsilon_t^{\Delta m} \\ &\quad + \frac{\theta}{1-\theta}\left[\theta a_{y0}^{e} L + \theta^2 a_{y1}^{e} L^2 + \theta^3 a^{e} L^3 + ...\right] \epsilon_t^{e} \end{aligned} \tag{15}$$

Similarly, by using equation (11) we have,

$$\begin{aligned} s_{1e} &= \sum_{j=1}^{\infty} \theta^j \left[e_t - E_{t-j}\left(e_t\right) \right] \\ &= \frac{\theta}{1-\theta}\left[1 + \theta \rho_e L + \theta^2 \rho_e^2 L^2 + ...\right] \epsilon_t^{e} \end{aligned} \tag{16}$$

and by using equation (12) we have,

$$\begin{aligned} s_{2e} &= \sum_{j=1}^{\infty} \theta^j \left[e_{t-1} - E_{t-j}\left(e_{t-1}\right) \right] \\ &= \frac{\theta}{1-\theta}\left[\theta L + \theta^2 \rho_e L^2 + \theta^3 \rho_e^2 L^3 + ...\right] \epsilon_t^{e} \end{aligned} \tag{17}$$

Substracting (14) from (15) we have,

$$\begin{aligned} \alpha\left(s_{1y} - s_{2y}\right) &= \alpha \sum_{j=1}^{\infty} \theta^j \left\{ \left[y_t - E_{t-j}\left(y_t\right) \right] - \left[y_{t-1} - E_{t-j}\left(y_{t-1}\right) \right] \right\} \\ &= \frac{\alpha\theta\left(1-\theta L\right)}{1-\theta}\left[a_{y0}^{\Delta m} + \theta a_{y1}^{\Delta m} L + \theta^2 a_{y2}^{\Delta m} L^2 + ...\right] \epsilon_t^{\Delta m} \\ &= \frac{\alpha\theta\left(1-\theta L\right)}{1-\theta}\left[a_{y0}^{e} + \theta a_{y1}^{e} L + \theta^2 a_{y2}^{e} L^2 + ...\right] \epsilon_t^{e} \end{aligned} \tag{18}$$

and by substracting (16) from (17) we have,

$$\begin{aligned}\gamma\left(s_{1e}-s_{2e}\right) &= \gamma\sum_{j=1}^{\infty}\theta^{j}\left\{\left[e_{t}-E_{t-j}\left(e_{t}\right)\right]-\left[e_{t-1}-E_{t-j}\left(e_{t-1}\right)\right]\right\} \\ &= \frac{\gamma\theta\left(1-\theta L\right)}{1-\theta}\left[1+\theta\rho_{e}L+\theta^{2}\rho_{e}^{2}L^{2}+...\right]\epsilon_{t}^{e} \qquad (19)\end{aligned}$$

Now, by substituting equation (13), (18) and (19) to equation (7) and simplifying gives me the following expression of the sticky information Phillips curve,

$$\begin{aligned}y_{t} &= \theta\sum_{j=0}^{\infty}\theta^{j}\left[\alpha^{-1}\left(1-\theta L\right)^{-1}a_{\pi j}^{\Delta m}+a_{yj}^{\Delta m}\right]L^{j}\epsilon_{t}^{\Delta m} \\ &+\theta\sum_{j=0}^{\infty}\theta^{j}\left[\alpha^{-1}\left(1-\theta L\right)^{-1}a_{\pi j}^{e}+a_{yj}^{e}+\alpha^{-1}\gamma\rho_{e}^{j}\right]L^{j}\epsilon_{t}^{e} \\ &-\frac{\gamma}{\alpha}\left(1-\rho_{e}L\right)^{-1}\epsilon_{t}^{e} \qquad (20)\end{aligned}$$

We can expand (20) to get,

$$\begin{aligned}y_{t} = &\left[\theta\alpha^{-1}a_{\pi 0}^{\Delta m}+a_{y0}^{\Delta m}\right]\epsilon_{t}^{\Delta m}+\left[\theta^{2}\alpha^{-1}a_{\pi 0}^{\Delta m}+\alpha^{-1}a_{\pi 1}^{\Delta m}+a_{y1}^{\Delta m}+\right]\epsilon_{t-1}^{\Delta m} \\ &+\left[\theta^{3}\alpha^{-1}a_{\pi 0}^{\Delta m}+\alpha^{-1}a_{\pi 1}^{\Delta m}+\alpha^{-1}a_{\pi 2}^{\Delta m}+a_{y2}^{\Delta m}\right]\epsilon_{t-2}^{\Delta m}+... \\ &+\left[\theta\left(\alpha^{-1}a_{\pi 0}^{e}+a_{y0}^{e}+\alpha^{-1}\gamma\right)-\frac{\gamma}{\alpha}\right]\epsilon_{t}^{e}+\left[\theta^{2}\left(\alpha^{-1}a_{\pi 0}^{e}+\alpha^{-1}a_{\pi 1}^{e}+a_{y1}^{e}+\alpha^{-1}\gamma\rho_{e}\right)-\frac{\gamma}{\alpha}\rho_{e}\right]\epsilon_{t-1}^{e} \\ &+\left[\theta^{3}\left(\alpha^{-1}a_{\pi 0}^{e}+\alpha^{-1}a_{\pi 1}^{e}+\alpha^{-1}a_{\pi 2}^{e}+a_{y2}^{e}+\alpha^{-1}\gamma\rho_{e}^{2}\right)-\frac{\gamma}{\alpha}\rho_{e}^{2}\right]\epsilon_{t-2}^{e}+.... \qquad (21)\end{aligned}$$

Now, by equating coefficients of (21) and (5) we have,

$$a_{yj}^{\Delta m}=\alpha^{-1}\frac{\theta^{j+1}}{\left(1-\theta^{j+1}\right)}\sum_{k=0}^{j}a_{\pi k}^{\Delta m}\ ,\ \text{for } j=0,1,2,3,... \qquad (22)$$

$$a_{yj}^{e}=\alpha^{-1}\frac{\theta^{j+1}}{\left(1-\theta^{j+1}\right)}\sum_{k=0}^{j}a_{\pi k}^{e}-\frac{\gamma}{\alpha}\rho_{e}^{j},\ \text{for } j=0,1,2,3,... \qquad (23)$$

3.2 The Demand Curve

The demand curve given in equation (1) can be written as,

$$\pi_{t}=\Delta m_{t}-\Delta y_{t} \qquad (24)$$

Note, by using equation (5) and (3), we can write equation (24) as,[5]

$$\begin{aligned}\pi_t &= \left[1-a_{y0}^{\Delta m}\right]\epsilon_t^{\Delta m}+\left[\rho_{\Delta m}-\left(a_{y1}^{\Delta m}-a_{y0}^{\Delta m}\right)\right]\epsilon_{t-1}^{\Delta m}+\left[\rho_{\Delta m}^2-\left(a_{y2}^{\Delta m}-a_{y1}^{\Delta m}\right)\right]\epsilon_{t-2}^{\Delta m}+....\\ &\quad+\left[-a_{y0}^{e}\right]\epsilon_t^{e}+\left[a_{y1}^{e}-a_{y0}^{e}\right]\epsilon_{t-1}^{e}+\left[a_{y2}^{e}-a_{y1}^{e}\right]\epsilon_{t-2}^{e}+....\end{aligned}\tag{25}$$

Now, by equating coefficients of (6) and (25) we can get,

$$a_{\pi 1}^{\Delta m}=1-a_{y0}^{\Delta m}\tag{26}$$

$$a_{\pi j}^{\Delta m}=\rho_{\Delta m}^{j}-\left(a_{yj}^{\Delta m}-a_{y(j-1)}^{\Delta m}\right),\ \text{for } j=1,2,3,....\tag{27}$$

and,

$$a_{\pi 1}^{e}=-a_{y0}^{e}\tag{28}$$

$$a_{\pi j}^{e}=-\left(a_{yj}^{\Delta m}-a_{y(j-1)}^{\Delta m}\right),\text{for } j=1,2,3,....\tag{29}$$

Note from equation (26) and (27) we can calculate,

$$\sum_{k=0}^{j}a_{\pi k}^{\Delta m}=\frac{1-\rho_{\Delta m}^{j+1}}{1-\rho_{\Delta m}}-a_{yj}^{\Delta m}\tag{30}$$

and similarly, from equation (28) and (29) we can calculate,

$$\sum_{k=0}^{j}a_{\pi k}^{e}=-a_{yj}^{e}\tag{31}$$

Now, substituting equation (30) to (22) yields,

$$\begin{aligned}a_{yj}^{\Delta m} &= \left(\frac{A\,(j+1)}{1+A\,(j+1)}\right)\frac{1-\rho_{\Delta m}^{j+1}}{1-\rho_{\Delta m}},\ \text{for } \rho_{\Delta m}\in[0,1)\\ &= \left(\frac{A\,(j+1)}{1+A\,(j+1)}\right)(j+1),\ \text{for } \rho_{\Delta m}=1\end{aligned}\tag{32}$$

and by substituting equation (31) to (23) we have,

$$a_{yj}^{e}=-\left(\frac{1}{1+A\,(j+1)}\right)\frac{\gamma}{\alpha}\rho_e^{j}\tag{33}$$

[5] Note, $a_{yj}=0$ and $a_{\pi j}=0$ for $j=-1,-2,...$

where,

$$A\left(j+1\right)=\alpha^{-1}\frac{\theta^{j+1}}{\left(1-\theta^{j+1}\right)}$$

Note, coefficients of output calculated in equation (32) and (33) enable us to calculate the coefficients of inflation rate from equation (26), (27) and equation (**??**) and (**??**) easily.

4 The Impulse Response

The quarterly impulse response of the model is analyzed separately for demand and supply shock in this section. We have first analyzed the impulse response under demand shock. Impulse response under supply shock is analyzed next. To generate impulse response we have used, $\alpha = 0.2$ and $\theta = 0.8$ following Mankiw and Reis (2002, 2006). Note, $\theta = 0.8$ implies that we have assumed 20% firm has completely updated information. We also set $\gamma = 1$ for our analysis.

4.1 Impulse Response under Demand Shock

We assume, $e_t = 0$ to analyze the impulse response under demand shock. Note when $e_t = 0$ we have, $a^e_{yj} = a^e_{\pi j} = 0$, $\forall j$. Figure 1 and Figure 2 show the impulse response of inflation and output under a 10% positive demand shock with persistence 0.8 and 1 respectively. Figure 1 shows hump shaped response to both output and inflation rate. We also see from Figure 1 that though both output and inflation rate rises in short run due a temporary increase in money growth, they come back to their long run level as the time progresses.

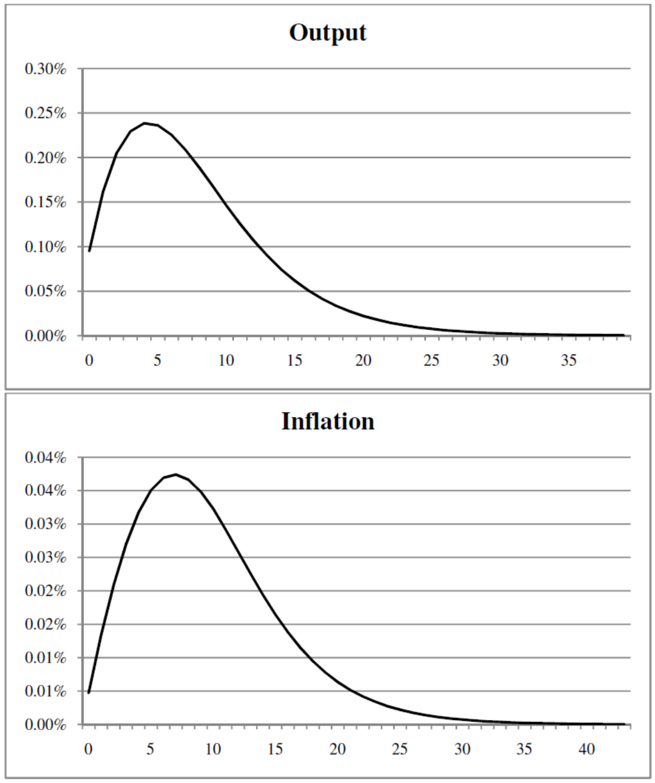

Figure 1: Impulse Response to Temporary Increase in Money Growth

To explain the impulse response portrayed in Figure 1, note when $\rho_{\Delta m} \in (0,1)$, $\lim_{j\to\infty}\left(a_{yj}^{\Delta m}\right) = \lim_{j\to\infty}\left(\frac{A(j+1)}{1+A(j+1)}\right)\frac{1-\rho_{\Delta m}^{j+1}}{1-\rho_{\Delta m}} \to 0$. Moreover, $\lim_{j\to\infty}\left(\frac{A(j+1)}{1+A(j+1)}\right)$ falls and tends 0 but $\lim_{j\to\infty}\left(\frac{1-\rho_{\Delta m}^{j+1}}{1-\rho_{\Delta m}}\right)$ rises and tends to $\frac{1}{1-\rho_{\Delta m}}$. This trade-off between the first and the second component of $a_{yj}^{\Delta m}$ produces the hump in output. We have checked that there is no hump in output when the persistence of demand shock is smaller relative to the degree of information asymmetry, i.e, the trade-off between two terms of $a_{yj}^{\Delta m}$ is smaller.

Figure 2 shows that the simple sticky information model follows the Natural Rate Hypothesis (McCallum, 1998). We see from Figure 2 that a permanent rise in money growth only increases inflation rate permanently but not output. To explain note that, $\lim_{j\to\infty}\left(a_{yj}^{\Delta m}\right) = \lim_{j\to\infty}\left(\left(\frac{A(j+1)}{1+A(j+1)}\right)(j+1)\right) \to 0$ and $\lim_{j\to\infty}\left(a_{\pi j}^{\Delta m}\right) = \lim_{j\to\infty}\left(\rho_{\Delta m}^{j} - \left(a_{yj}^{\Delta m} - a_{y(j-1)}^{\Delta m}\right)\right) \to$

1.

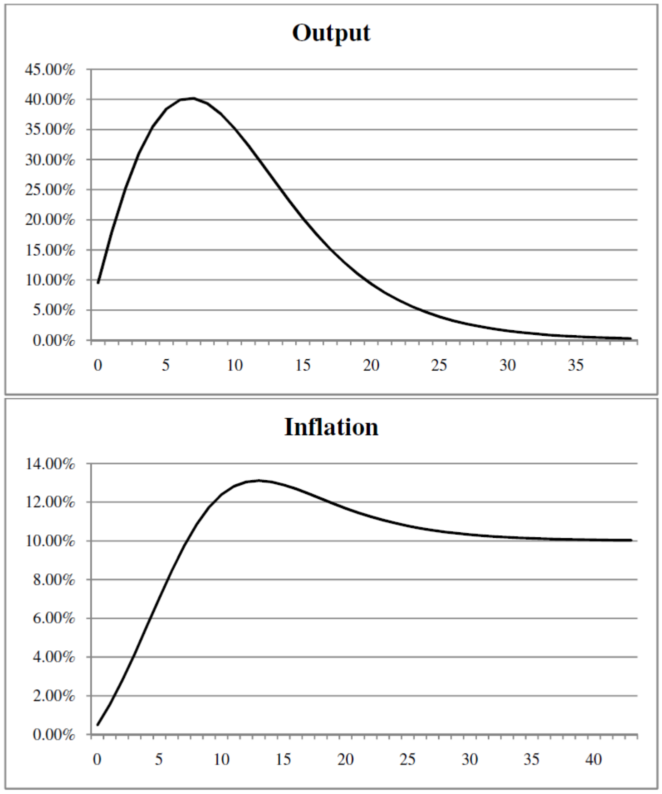

Figure 2: Impulse Response to Permanent
Increase in Money Growth

We see hump shaped response in output even under permanent demand. The reason behind the hump in output is again the trade-off between first and second term of $a_{yj}^{\Delta m}$. Note, while the first term of $a_{yj}^{\Delta m}$ falls and tends to zero, the second term rises and goes to infinity. This produces the hump in output as shown in Figure 2.

4.2 Impulse Response under Supply Shock

We assume, $\Delta m_t = 0$ to analyze the impulse response under supply shock. Note, $\Delta m_t = 0$ implies we have, $a_{yj}^{\Delta m} = a_{\pi j}^{\Delta m} = 0, \forall j$. Figure 3 portrays the impulse response of a 10% contractionary supply shock of persistence 0.8. The supply shock reduces output and increases inflation rate as expected. We see hump shaped response of output and inflation in short run but both go back to their long run level as time progresses. The hump in output obtained under supply shock is also due to the relative strength of the persistence of supply shock and degree of information asymmetry as in under demand shock.

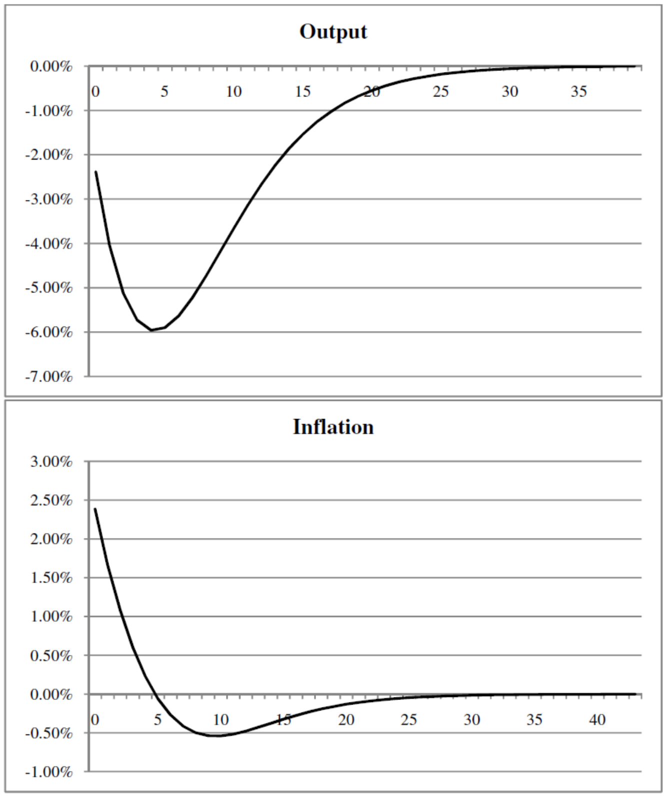

Figure 3: Impulse Response to Temporary Contractionary Supply Shock

Figure 4 shows the impulse response of a permanent supply shock of same magnitude. The figure shows even if there is a permanent shift in output, inflation comes back to its long run level after initial fluctuations as time progresses. The intuition follows directly from equation (29) and (33). Note we have, $\lim_{j\to\infty}\left(a^{e}_{yj}\right) = \lim_{j\to\infty} -\left(\frac{1}{1+A(j+1)}\right)\frac{\gamma}{\alpha}\rho_e^j \to \frac{\gamma}{\alpha}$ and $\lim_{j\to\infty}\left(a^{e}_{\pi j}\right) = \lim_{j\to\infty}\left(-\left(a^{\Delta m}_{yj} - a^{\Delta m}_{y(j-1)}\right)\right) \to 0$

$$\lim_{j\to\infty}\left(a^{e}_{yj}\right) = \lim_{j\to\infty} -\left(\frac{1}{1+A\left(j+1\right)}\right)\frac{\gamma}{\alpha}\rho_e^j \to \frac{\gamma}{\alpha}$$

This explains the impulse response portrayed in Figure 4.

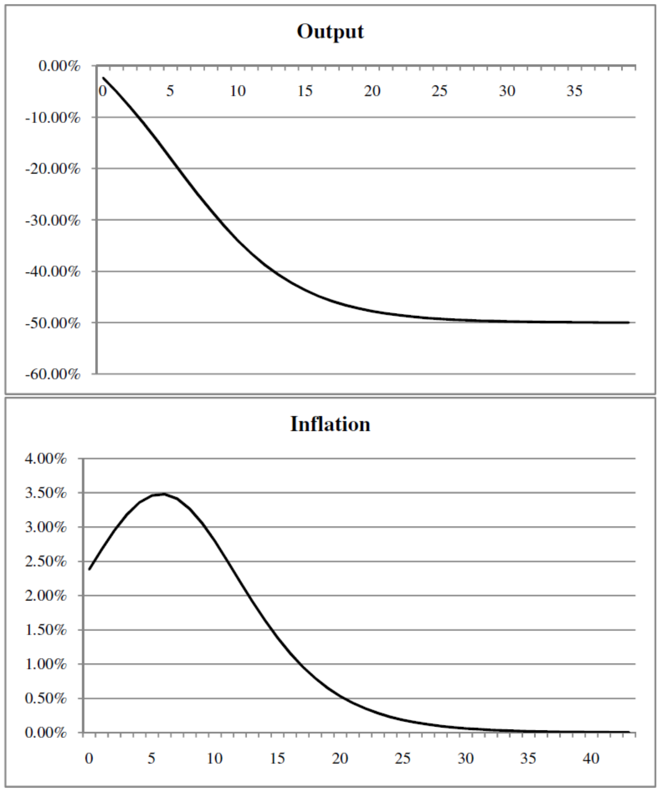

Figure 4: Impulse Response to Permanent Contractionary Supply Shock

5 Conclusion

This paper develops an algorithm to solve a simple sticky information New Keynesian model of Mankiw and Reis (2002). Solution of even the simple sticky information model is not easy as the state space is infinite due to the backward looking sticky information Phillips curve representing the supply side of the model. We have used the methodology of Wang wen (2006) to solve the model. To solve the model, we have to first write the model in terms of forecast error. Then the model is solved using the method of undetermined coefficients. This effectively reduces the number parameters to work with while solving the model which greatly reduces the possibility of committing human error.

The major strength of our algorithm lies in its analytical exposition which allows us to uncover better intuition from the model. For example, we have seen from the impulse responses of both demand and supply shock that the hump shaped response of inflation and output producing by the model depends on the relative strength of exogenous demand and/or supply shock and endogenous persistence determining the degree of information asymmetry preset in the economy. We have seen from the impulse responses that the model does not produce hump in output when exogenous persistence is small enough compared to the endogenous persistence.

Mankiw and Reis (2006) has developed a model of pervasive stickiness where not only firms but even household supplying labor and consuming goods have information asymmetry. The monetary instrument of the model is no longer money supply but nominal interest rate determined by the Taylor rule. The solution of the pervasive stickiness model is more complicated than the simple model as the state space of the pervasive stickiness model is doubly infinite due to expectational IS schedule and sticky information Phillips curve. Our algorithm can be easily extetended to solve the pervasive stickiness model.

6 Appendix: Sticky Information Phillips Curve

Desired/optimal price level of a generic firm j with completely updated information is,

$$p_{t,0}\left(j\right) = p_{t,0} = p_t + \alpha y_t + \gamma e_t \tag{34}$$

Therefore,

$$p_{t,0} - p_{t-1,0} = \pi_t + \alpha \Delta y_t + \gamma \Delta e_t \tag{35}$$

Assuming $(1-\theta)$ as the fraction of firms with updated information and Calvo price setting (Calvo, 1983) the price of firms who have updated their information $(t-j)$ period ahead is,

$$p_{t,j} = (1-\theta)\,\theta^j E_{t-j}\left(p_{t,0}\right) \tag{36}$$

Therefore, the aggregate price level of the economy is,

$$p_t = \sum_{j=0}^{\infty} p_{t,j} = (1-\theta) \sum_{j=0}^{\infty} \theta^j E_{t-j}\left(p_{t,0}\right)$$

Simplifying we get,

$$p_t = \left(\frac{1-\theta}{\theta}\right)\left[\alpha y_t + \gamma e_t\right] + (1-\theta) \sum_{j=0}^{\infty} \theta^j E_{t-j-1}\left(p_{t,0}\right) \tag{37}$$

Similarly,

$$p_{t-1} = (1-\theta)\sum_{j=0}^{\infty}\theta^j E_{t-j-1}\left(p_{t-1,0}\right) \tag{38}$$

Define, $\pi_t = p_t - p_{t-1}$. Now, substracting equation (37) from (38) and using equation (35) we have,

$$\pi_t = \left(\frac{1-\theta}{\theta}\right)\left[\alpha y_t + \gamma e_t\right] + (1-\theta)\sum_{j=0}^{\infty}\theta^j E_{t-j-1}\left(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t\right) \tag{39}$$

Note, by adding and subtracting $(1-\theta)\sum_{j=1}^{\infty}\theta^j\left(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t\right)$ to the R.H.S. of equation (39) and using $\sum_{j=1}^{\infty}\theta^j = \frac{\theta}{1-\theta}$ and $\left(\frac{\theta}{\alpha}\right)\gamma\Delta e_t + \left(\frac{1-\theta}{\alpha}\right)\gamma e_t = \left(\frac{\gamma}{\alpha}\right)(1-\theta L)\, e_t$ gives,

$$\begin{aligned} y_t &= \theta y_{t-1} + \left(\frac{1-\theta}{\alpha}\right)\sum_{j=1}^{\infty}\theta^j\left[\left(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t\right) - E_{t-j}\left(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t\right)\right] \\ &\quad - \left(\frac{\gamma}{\alpha}\right)(1-\theta L)\, e_t \end{aligned} \tag{40}$$

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