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Evaluating the effectiveness of Common-Factor Portfolios

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In this paper we use the standard factor models to compose common-factor portfolios by a novel linear transformation extracted from large data sets of asset returns. Although the transformation proposed here retains the basic properties of the usual common factors, some interesting new properties are further included in them. The advantages of using common-factor portfolios in asset pricing are: (i) they produce a dimension reduction in the asset- pricing data-base while preserving the usual restrictions imposed by the asset-pricing equation, and (ii) from the empirical perspective, their performance is better than those of standard factor models. The practical importance is confirmed in two applications: the performance of common-factor portfolios is shown to be superior to those of the asset returns and factors commonly used in the finance literature.

Keywords: Common Factors, Common Features, CCAPM, Stochastic Discount Factor, Linear Multifactor Model.

J.E.L. Codes: C32, C33, E21, E44, G12.

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1. Introduction

In this paper we use the standard factor models to compose common-factor portfolios by a novel linear transformation extracted from large data sets of asset returns. Although the transformation proposed in proposition (1) keeps the basic properties of the usual common factors, some new interesting features are further included in them. In a setup where it is desirable to use the asset-pricing equation, one of the advantages of common-factor portfolios is preservation of the usual restrictions imposed by the asset-pricing equation. On the other hand, it produces a dimension reduction in the set of restrictions involved in the asset-pricing equation on a given set of asset returns, as shown in proposition (2). We present two applications where the usage of common-factor portfolios provides some theoretical and practical advantages.

In the first application, in a generalized method-of-moments (GMM) framework, common-factor portfolios are used to test the implications of the consumption-based asset pricing model (hereafter CCAPM) of Breeden (1979) and Lucas (1978). In this setup, most CCAPM empirical studies have considered only a few asset returns, e.g., a risky and a riskless asset (Hansen and Singleton, 1982; and Epstein and Zin, 1991). One of the reasons for this choice is to ensure the non-singularity of the variance-covariance matrix of the moment restrictions used in GMM estimation, a necessary condition for feasibility of GMM estimates. However, this apparent advantage it is not without drawbacks. As is well known, efficient GMM estimation of preference parameters requires using all the available information on asset returns, $R_{j,t+1}$, $j=1,2....p$, - not just that contained in a handful of returns - to serve as moment restrictions from which to extract preference estimates. In the CCAPM, the necessary first-order conditions are:

$$1 = \mathsf{E}_t\left[\beta \frac{u'(C_{t+1})}{u'(C_t)} R_{j,t+1}\right] \quad \text{for} \quad j=1,2....p,\ t=1,2....T. \qquad (1)$$

where C_t represents aggregate consumption at time t, $\beta \in (0,1)$ is the one-period discount factor, and $u(\cdot)$ is a strictly concave utility function with the usual properties. Define the intertemporal marginal rate of substitution by $M_{t+1} = \beta u'(c_{t+1})/u'(c_t)$. Then equation (1) results in: $1 = \mathsf{E}_t[M_{t+1}R_{j,t+1}]$. This equation is the pricing equation established by Harrison and Kreps (1979), Hansen and Richard (1987) and Hansen and Jagannathan (1991), where the intertemporal marginal rate of substitution M_{t+1} is the stochastic discount factor (SDF). Ideally, we should consider all assets available in the economy in testing the CCAPM model. However, as argued above, this may be infeasible if the number of Euler equations p is large vis-à-vis the number of time observations available (T). In practice, if one resorts to quarterly post-WWII data on consumption, $T \approx 240$, but the number of asset returns that could be used to test the CCAPM is much bigger. Even if one limits the number of returns, but with p relatively large, it may be hard to invert the long-run variance-covariance matrix of moment restrictions. For these reasons, several studies have kept $p=2$, or perhaps a little larger. Thus, by transforming the set of p returns R_t in the set of m common factor portfolios, $\tilde{R}_t$ (shown in Proposition 1), a reduction is produce in the number of original asset-pricing restrictions ($m << p$). So, our starting point is not (1), but rather a new asset pricing equation $\mathsf{E}_t\,[\beta u'(C_{t+1})/u'(C_t)\tilde{R}_{j,t+1}]=1$ for $j=1,2....m$.

Moreover, since the intertemporal marginal rate of substitution in consumption should only respond to systematic changes in real returns, a plausible strategy to identify these changes is using only the pervasive components of asset returns. This point to the use of common-factor methods where theoretical moment restrictions are preserved. This is exactly what common-factor portfolios provide: they keep a low number of moment restrictions while extracting the pervasive information contained in a large number of asset returns.

In a second application, we propose the use of common-factor portfolios as a fundamental risk factor in the multifactor asset-pricing model of Ross (1976). In this arbitrage pricing theory (APT) setup, it is convenient to express a beta pricing model in terms of its factor-mimicking payoffs or factor-mimicking portfolios rather than of the factors themselves. For example, to generate the factor-mimicking returns, f^*, each payoff is divided by its price, $f^*=\frac{proj(f/X)}{p(proj(f/X))}$ where f is the original factor, X is the payoff space, $p(\cdot)$ and $proj(f/X)$ are respectively a pricing function and the projection of the original factors onto the payoff space[3]. By using common-factor portfolios this additional operation can be omitted (dividing by the price). We show how it is done in section 3.

In the two applications, the usefulness of common-factor portfolios in finance studies related to asset pricing is shown. This can be seen from a theoretical perspective (Propositions 1 and 2), where we show they have some desirable properties, especially the dimension reduction in the asset-pricing data-base, while preserving the usual restrictions imposed by the asset-pricing equation. Additionally, from empirical perspective their empirical performance is better than that of standard factor models in an important dimension: while we see little evidence to reject the theory with common-factor portfolios, the opposite is true when we use a small number of assets in testing the CCAPM (risky and riskless) or when we employ standard factor portfolios in testing the APT.

The remainder of this paper is organized as follows. Section 2 presents the construction of common-factor portfolios. Section 3 shows the results of the two applications using the portfolios. The final section concludes.

2. Common-Factor Portfolios

We start with a factor model to construct the underlying common-factor portfolios. Consider a weak stationary and ergodic vector of returns $R_t=(R_{1,t},...,R_{p,t})'$ of p assets in period t, with mean $E(R_t)=\mu$ and covariance matrix $Cov(R_t)=\Omega$, as discussed in Stock and Watson (2002). The linear statistical factor model takes the form:

$$(R_t)_{p\times 1}=(\mu)_{p\times 1}+(\beta)_{p\times m}(F_t)_{m\times 1}+(\varepsilon_t)_{p\times 1} \tag{2}$$

[3]As explained by Cochrane (2001), in the APT model a mimic factor portfolio is commonly used, obtained by projecting the actual factors and the span of the observed returns.

where $F_t = (f_{1t},....,f_{mt})'$ is a vector of a reduced number of m unobservable random variables ($m < p$), the f_{it} 's are the common factors, $\beta = [\beta_{ij}]_{p\times m}$ is the matrix of factor loadings with full rank m, the β_{ij} 's are the loadings of the i th variable on the j th factor, and $\varepsilon_t = (\varepsilon_{1t},...,\varepsilon_{pt},)'$ is formed with p idiosyncratic error terms with zero mean and for which a weak law of large number applies. Here, β and F_t are unobservable. The factor model (1) satisfies the following assumptions:

i) $E(F_t) = 0$ and $Cov(F_t) = I_m$;
ii) $E(\varepsilon_t) = 0$ and $Cov(\varepsilon_t) = D$ where, D is a $p \times p$ diagonal matrix ($D = diag\left(\sigma_1^2,...,\sigma_p^2\right)$);
iii) $Cov(F_t, \varepsilon_t) = E(F_t\varepsilon_t^{'}) = 0$, an $m \times p$ matrix of zeros.

Thus, the information contained in the p assets in R_t is used to obtain the factor-loadings and the m common factors. Given the linear statistical factor model (2), we propose Proposition 1 to construct common-factor portfolios:

Proposition 1 *Let the $p\times m$ matrix β of rank m and F_t, respectively, be the factor loading and the common factors in the factor model. Define the diagonal matrix $A = diag\left(a_{11},...,a_{mm}\right)$, where $a_{ii} = \frac{1}{\sum_{j=1}^{p} b_{i,j}}$ for all $i = 1,...m$, where the $b_{i,j}$'s are the elements of the left-inverse of matrix β, labeled here as $\beta^{-1} = \left(b_{i,j}\right)$. Therefore, the vector $\tilde{R}_t$, defined as $\left(\tilde{R}_t\right)_{m\times 1}$ ≝ $\left(A\beta^{-1}\right)_{m\times p}\left(R_t\right)_{p\times 1} = A\beta^{-1}\mu + AF_t + A\beta^{-1}\varepsilon_t$, is a vector of portfolios. In addition $\tilde{R}_t$ has the property that $Cov(\tilde{R}_t) = \tilde{I}_m + A\beta^{-1}D\left(A\beta^{-1}\right)'$, where $\tilde{I}_m$ is an $m-$ diagonal matrix.*

Proof Since β has full rank m, there exists a left inverse matrix $\beta^{-1} = [b_{i,j}]$ such that $\beta^{-1}_{m\times p}\beta_{p\times m} = I_m$. Pre-multiplying the factor model by $A\beta^{-1}$ produces

$$\tilde{R}_t = A\beta^{-1}R_t = A\beta^{-1}\mu + \tilde{F}_t + A\beta^{-1}\varepsilon_t \tag{3}$$

where $\tilde{F}_t = AF_t = [a_{11}f_{1t},...,a_{mm}f_{mt}]'$ stacks the original factors scaled by constants. In addition, the sum of each row of the matrix $A\beta^{-1}$ is one. This is easy to verify, since $A\beta^{-1}$, with elements $[a_{ii}b_{ij}]$, must have the i th row equal to $\sum_{j=1}^{p} a_{ii}b_{ij} = a_{ii}\sum_{j=1}^{p} b_{i,j} = a_{ii} \times \frac{1}{a_{ii}} = 1$. Therefore, equation (3) defines m portfolios $\tilde{R}_t = A\beta^{-1}R_t$. Finally, given the hypotheses $E(F_t) = 0$ and $Cov(F_t) = I_m$ an important property is shown. The mean and covariance of $\tilde{R}_t$ are, respectively, $E(\tilde{R}_t) = A\beta^{-1}\mu$ and $Cov(\tilde{R}_t) = E\left[(\tilde{F}_t + A\beta^{-1}\varepsilon_t)(\tilde{F}_t + A\beta^{-1}\varepsilon_t)'\right] = E(\tilde{F}_t\tilde{F}_t^{'}) + A\beta^{-1}E(\varepsilon_t\varepsilon_t^{'})\left(A\beta^{-1}\right)' = \tilde{I}_m + A\beta^{-1}D\left(A\beta^{-1}\right)'$, where $\tilde{I}_m$ is an $m-$diagonal matrix.

The next proposition corroborates that the common-factor portfolio satisfied the asset-pricing equation.

Proposition 2 *Let* M_{t+1} *be a stochastic discount factor pricing all assets by means of the asset-pricing equation* $\mathsf{E}_t\left[M_{t+1}R_{i,t+1}\right]=1$, *valid for* $i=1,..,p$ *and* $t=1,..,T$. *Then,* $\mathsf{E}_t\left[M_{t+1}\tilde{R}_{j,t+1}\right]=1$ *also holds for all common-factor portfolios* $j=1,..,m$, *where* $m<<p$.

Proof Define $\tilde{R}_{j,t+1}=\sum_{i=1}^{p}w_{ij}R_{i,t+1}$ portfolios, where $\sum_{i=1}^{p}w_{ij}=1$ for all $j\ =1,..,m$. Pre-multiplying each equation by w_{ij}, and aggregating across i, we have $\mathsf{E}_t\left[M_{t+1}\sum_{i=1}^{p}w_{ij}R_{i,t+1}\right]=\sum_{i=1}^{p}w_{ij}$. Since $\sum_{i=1}^{p}w_{ij}=1$ for all j, $\mathsf{E}_t\left[M_{t+1}\tilde{R}_{j,t+1}\right]=1$, for all $j=1,..,m$. Thus, the asset-pricing equation is valid for common-factor portfolios, since $\sum_{j=1}^{p}a_{ii}b_{ij}=a_{ii}\sum_{j=1}^{p}b_{i,j}=a_{ii}\times\frac{1}{a_{ii}}=1$.

In order to estimate common-factor portfolios $\tilde{R}_t$, we previously estimate the factor loading β. Principal-component analysis can be used for this purpose, as extensively discussed in Stock and Watson (2002) and Forni et al. (2000).

3. Empirical Applications

3.1 Data

In the first application, we use monthly stock returns for the U.S. economy beginning in February 1987 and ending in July 2010, extracted from the Yahoo Finance web site. We consider only companies, for which data are available throughout the period, leading to a sample of $p=263$ individual returns. The U.S. Treasury Bill return is used as a proxy for the risk-free asset return. We estimate the preference parameters for the representative agent and test the implied restrictions by its Euler equation. Person's expenditures on non-durable goods and services are used in measuring per-capita consumption. The latter is extracted from the FRED database. All nominal asset returns are converted to real returns using the respective price deflator for non-durables and services.

In a second application, we use 12 value-weighted returns of industry portfolios and also the three relative factors measuring firm size, book-to-market ratio, and market return of Fama and French (1993). The industry portfolios and the factors of Fama and French were obtained from the Kenneth R. French web site.

3.2 Estimation of the consumption-capital asset pricing model (CCAPM)

In testing the CCAPM, we follow Hansen and Singleton in setting $u\left(C_t\right)=\frac{C_t^{\gamma}}{1-\gamma}$, i.e., a constant relative risk aversion (CRRA) utility function, where γ is the relative risk aversion coefficient. Table 3 shows a descriptive statistics and correlation matrix of the risky asset

R_{wt} - the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) - and three common-factor portfolios, named: $R_{1,t}$, $R_{2,t}$ and $R_{3,t}$, which jointly explain 80% of the variance of the primary asset group of $p = 263$ stocks being studied here. Notice that R_{wt} and the first common factor portfolio $R_{1,t}$ are highly correlated. Table 1 reports the estimation of the CRRA utility using GMM, which supplies a χ^2 test (the J-test) of the overidentifying restrictions implied by the model. It can be seen that when only a risky and a riskless asset are used, as in Hansen and Singleton (1982), the J-test rejects in the majority of cases the validity of the instruments at the 10% level of significance. A similar result holds when we employ just the T-Bill and the first factor $R_{1,t}$. On the other hand, when using more than one common-factor portfolios, the J-test does not rejected the null hypothesis of validity of instruments at the 10% level, strongly supporting the CCAPM. In all cases, the intertemporal discount factor is significant at 5%, positive and close to one. However, the relative risk-aversion coefficient is not significant.

3.3 Testing a Linear Multifactor Model

We can test a multifactor model by running a standard multivariate regression (see Campbell, Lo and MacKinlay, 1997) where returns are a function of factors. Consider in particular the three factor model given by Fama and French (1992, 1993):

$$R_{it} - R_{ft} = c_i + \beta_{im} R_{wt} + \beta_{is} SMB_t + \beta_{ih} HML_t + \varepsilon_{it}. \tag{4}$$

where β_{im}, β_{is} and β_{ih} are the unconditional sensitivities of the i-th asset to the fundamental factors, obtained as the slopes of the empirical counterpart of the model, R_{it} is the rate of return of asset (or portfolio) i at time t, R_{ft} is the risk-free rate of interest at time t, R_{wt} represent the realized excess return of the market portfolio at time t, SMB_t the realized return on the proxy portfolio for size factor and HML_t the realized return on the proxy portfolio for the book-to-market factor at time t.

By taking the expectation of equation (4), the intercept c_i is expected to be zero for all i[4], which can be tested directly by a Wald test of H_0: $c_i = 0$ for the Fama-Franch factors. This restriction implies that the zero-beta expected return should equal the risk-free rate. Rejection of H_0 means that the factors cannot explain the average level of stock returns. In general, finding a positive or negative c_i is evidence of mispricing. The common-factor portfolios can be used in testing by using the excess-returns written in terms of the common-factor portfolios:

$$R_{it} - R_{ft} = c_i + \beta_{i1}\left(R_{1t} - R_{ft}\right) + \beta_{i2}\left(R_{2t} - R_{ft}\right) + \beta_{i3}\left(R_{3t} - R_{ft}\right) + \varepsilon_{it}. \tag{5}$$

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[4] c_i is the average excess return. It is a measure of mispricing with respect to the model.

where R_{1t}, R_{2t}, R_{3t} are, respectively, the first, second and third common-factor portfolios explaining most of the variance of the asset returns being analyzed. Here, three sets of factors are used to test the APT: i) the common-factor portfolios proposed in this paper $\{R_{1t};R_{2t};R_{3t}\}$; ii) the factor-mimicking payoff, $F_{it} = proj(f / X)$, $\{F_{1t};F_{2t};F_{3t}\}$; and iii) the classic Fama and French factors discussed above $\{RM_t;SMB_t;HML_t\}$.

The returns of each of the twelve industrial portfolios are regressed against the selected three factors to determine whether there are excess returns on the risk free rate. Table 3 shows that the test of the zero intercept restriction is rejected less with the common-factor portfolios than with the factor mimicking payoff. Still, the Fama and French factors only reject the model once (category: other), which could be interpreted an improvement over the results with the common-factor portfolios under spherical errors. However, although residual tests for serial correlation (LM) and heteroskedasticity (ARCH) reveal little evidence of problems for the set of common-factor portfolios, this is not true for the other set of factors (Fama-French and factor-mimicking payoffs). In almost all cases, the latter invalidates hypothesis testing with standard methods using Fama-French factors. The same is true for factor-mimicking payoffs, albeit to a lesser degree.

As shown in Table 2, the common-factor portfolios explain the largest fraction of common variation in stock returns for all industry portfolios vis-à-vis the factor-mimicking payoff and the Fama and French portfolios. The adjusted R^2s of the common-factor portfolio range from 42% to 89% with an average of 72% for the 12 portfolios. The Fama and French factors and the factor-mimicking payoff explain on average 66% and 70% of the variation in the dependent variable, respectively.

4. Conclusions

In this paper we propose the use of common-factor portfolios, which is a novel linear transformation of standard factor models using a large dimension data set of gross asset returns. In two propositions, we show that they produce a dimension reduction in the asset-pricing database while preserving the usual restrictions imposed by the asset-pricing equation. From an empirical point of view, we show that their performance in asset-pricing tests is different (and somewhat superior) to that of standard factor models.

Two applications are provided here to test the CCAPM, where we show the empirical usefulness of common-factor portfolios. First, the result of GMM estimation of the representative-agent model with a CRRA utility reveals improvement vis-à-vis the use of two-asset setup (risky and riskless asset). Indeed, common-factor portfolios are not rejected at all in overidentifying restrictions tests, while the use of the two-asset setup rejects the CCAPM in most of the tests performed. This result may be due to the fact that intertemporal substitution is a function of the pervasive component of asset returns, which requires the use of a large data set of assets in testing the theory. Second, the use of common- factor portfolios as fundamental risk factors in testing the arbitrage pricing theory shows that the zero-beta expected return was rejected less often than the set of factor-mimicking portfolios and the classic three-factor model of Fama and French. Moreover, the adjusted R^2 had better performance vis-à-vis that of factor-mimicking portfolios.

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Appendix I: Table 1 - GMM parameter estimates for CRRA function

$$\mathrm{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{j,t+1}-1\right]=0\,,\quad for\ R_{j,t}=\{R_{1t},R_{2t},R_{3t},R_{ft}\}$$

Instrument	T-bill and Risky asset (Rft, Rwt)			T-bill and one Factor (Rft, R1t)			T-bill and two Factors (Rft, R1t, R2t)			T-bill and three Factors (Rft, R1t, R2t, R3t)		
	β	γ	J-test (p-value)	β	γ	J-test (p-value)	β	γ	J-test (p-value)	β	γ	J-test (p-value)
I [Const; cg_t]	0.9986* (0.0006)	0.0502 (0.3277)	0.0571	0.9987* (0.0005)	0.1199 (0.3363)	0.0416	0.9987* (0.0005)	0.0946 (0.3225)	0.1498	0.9988* (0.0005)	0.1446 (0.308)	0.3180
II [Const; R_{wt}; cg_t]	0.9987* (0.0005)	0.0208 (0.2623)	0.0554	0.9987* (0.0005)	0.1213 (0.2587)	0.0247	0.9987* (0.0004)	0.1336 (0.242)	0.1113	0.9988* (0.0004)	0.1647 (0.2216)	0.2486
III [Const; R_{wt}; R_{wt-1}]	0.9986* (0.0008)	-0.2836 (0.4888)	0.0558	0.9983* (0.0009)	-0.1815 (0.4942)	0.0274	0.9983* (0.0008)	-0.1692 (0.4844)	0.1181	0.9984* (0.0008)	-0.1371 (0.4601)	0.2536
IV [Const; cg_t; cg_{t-1}]	0.9986* (0.0003)	0.1034 (0.1428)	0.1730	0.9986* (0.0003)	0.0655 (0.1415)	0.1638	0.9987* (0.0003)	0.0672 (0.1405)	0.3476	0.9988* (0.0003)	0.1351 (0.1303)	0.4946
V [Const; R_{wt}; cg_{t-1}]	0.9988* (0.0005)	0.0725 (0.2708)	0.0330	0.9987* (0.0005)	0.0621 (0.2845)	0.0322	0.9987* (0.0005)	0.105 (0.2742)	0.1269	0.9989* (0.0005)	0.2099 (0.2605)	0.2492
VI [Const; cg_t; R_{wt-1}]	0.9986* (0.0006)	0.0351 (0.3074)	0.1519	0.9987* (0.0005)	0.1311 (0.3094)	0.0958	0.9987* (0.0005)	0.1173 (0.2907)	0.3313	0.9989* (0.0004)	0.2354 (0.2635)	0.5558
VII [Const; cg_{t-1}; R_{wt-1}]	0.9986* (0.0006)	-0.0090 (0.3534)	0.4167	0.9984* (0.0006)	-0.0682 (0.3468)	0.2569	0.9986* (0.0006)	-0.0062 (0.3283)	0.5264	0.9986* (0.0005)	0.0434 (0.2990)	0.5343
VIII [Const; R_{wt}; cg_t cg_{t-1}]	0.9987* (0.00033)	0.1177 (0.1251)	0.0642	0.9987* (0.0003)	0.1170 (0.1268)	0.0583	0.9987* (0.0003)	0.0948 (0.1219)	0.2248	0.9987* (0.0003)	0.1461 (0.1135)	0.3962
IX [Const; cg_t; cg_{t-1} cg_{t-2}]	0.9983* (0.0005)	-0.1728 (0.242)	0.3322	0.9983* (0.0005)	-0.1828 (0.2478)	0.3074	0.9984* (0.0005)	-0.1297 (0.2445)	0.3960	0.9988* (0.0005)	0.0741 (0.2148)	0.4905
X [Const; R_{wt}; R_{wt-1} cg_{t-1}]	0.9988* (0.0004)	-0.0388 (0.2533)	0.0806	0.9985* (0.0005)	-0.0789 (0.2691)	0.0692	0.9986* (0.0005)	-0.0165 (0.2448)	0.2299	0.9985* (0.0004)	-0.0513 (0.2081)	0.4170

Notes: R_{1t}, R_{2t}, R_{3t} are the first three common-factor portfolios, $cg_t = C_t/C_{t-1}$ is the consumption growth rate, and R_{ft} is the risk-free T-Bill. R_{Wt} is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP). The values in parentheses correspond to standard deviation. * denotes significance at 5% levels. GMM estimation uses the Newey-West procedure with a fixed bandwidth and iterate simultaneously over the weighting matrix and the coefficient vector.

Appendix II: Table 2 - Testing different linear multifactor models

Fama and French Factor

$R_{it} - R_{ft} = c_i + m_i (R_{wt} - R_{ft}) + s_i\ HML_t + h_i\ HMB_t + e_{it}$

	NoDur	*Durbl*	*Manuf*	*Enrgy*	*Chems*	*BusEq*	*Telcm*	*Utils*	*Shops*	*Hlth*	*Money*	*Other*
c	0.002	-0.003	0.001	0.003	0.001	0.001	-0.001	0.001	0.001	0.003	-0.001	-0.003
p-value	(0.129)	(0.163)	(0.456)	(0.228)	(0.429)	(0.449)	(0.679)	(0.594)	(0.484)	(0.158)	(0.504)	**(0.014)**
m	0.757	1.300	1.162	0.763	0.877	1.223	0.952	0.533	0.949	0.738	1.188	1.079
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
s	-0.187	0.202	0.059	-0.143	-0.159	0.193	-0.250	-0.177	0.023	-0.276	-0.165	0.123
p-value	(0.000)	(0.005)	(0.131)	(0.066)	(0.002)	(0.001)	(0.000)	(0.004)	(0.667)	(0.000)	(0.000)	(0.001)
h	0.204	0.777	0.315	0.346	0.271	-0.754	-0.060	0.474	0.099	-0.185	0.598	0.197
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.367)	(0.000)	(0.08)	(0.005)	(0.000)	(0.000)
R2-ajust	0.610	0.708	0.861	0.394	0.668	0.843	0.630	0.380	0.702	0.527	0.815	0.874
F-statistic	146.7	226.0	577.2	61.5	188.0	498.9	159.6	57.9	220.5	104.7	411.8	648.7
LM test	0.511	0.209	0.896	0.010	0.830	0.053	0.930	0.429	0.352	0.005	0.231	0.304
ARCH test	0.017	0.004	0.015	0.053	0.003	0.008	0.061	0.000	0.010	0.000	0.000	0.032

Common Factor Porfolio

$R_{it} - R_{ft} = c_i + m_i (R_{1t} - R_{ft}) + s_i (R_{2t} - R_{ft}) + h_i (R_{3t} - R_{ft}) + e_{it}$

	NoDur	*Durbl*	*Manuf*	*Enrgy*	*Chems*	*BusEq*	*Telcm*	*Utils*	*Shops*	*Hlth*	*Money*	*Other*
c	0.002	-0.005	-0.002	0.002	0.000	-0.005	-0.002	0.002	-0.001	0.002	-0.002	-0.005
p-value	(0.158)	**(0.020)**	(0.147)	(0.124)	(0.819)	**(0.005)**	(0.319)	(0.235)	(0.422)	(0.428)	(0.222)	**(0.000)**
m	0.598	1.097	0.984	0.577	0.710	1.080	0.700	0.365	0.825	0.569	0.933	0.908
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
s	0.124	0.034	0.001	0.056	0.092	-0.345	-0.025	0.147	0.005	0.056	0.147	-0.010
p-value	(0.000)	(0.093)	(0.940)	(0.000)	(0.000)	(0.000)	(0.222)	(0.000)	(0.713)	(0.0033)	(0.000)	(0.331)
h	-0.011	-0.015	0.014	0.114	0.003	-0.015	-0.011	0.059	-0.036	-0.012	-0.035	-0.005
p-value	(0.019)	(0.046)	(0.000)	(0.000)	(0.541)	(0.009)	(0.179)	(0.000)	(0.000)	(0.086)	(0.000)	(0.224)
R2-ajust	0.686	0.706	0.891	0.776	0.708	0.848	0.478	0.578	0.781	0.424	0.861	0.849
F-statistic	204.1	224.7	758.8	323.5	226.0	519.0	86.2	128.2	332.2	69.4	578.7	524.8
LM test	0.243	0.237	0.225	0.042	0.766	0.036	0.193	0.059	0.086	0.006	0.022	0.031
ARCH test	0.618	0.051	0.599	0.707	0.416	0.000	0.049	0.000	0.001	0.580	0.070	0.873

Factor-mimicking payoff

$R_{it} - R_{ft} = c_i + m_i\ F_{1t} + s_i\ F_{2t} + h_i\ F_{3t} + e_{it}$

	NoDur	*Durbl*	*Manuf*	*Enrgy*	*Chems*	*BusEq*	*Telcm*	*Utils*	*Shops*	*Hlth*	*Money*	*Other*
c	0.006	0.004	0.007	0.007	0.006	0.006	0.004	0.004	0.006	0.006	0.005	0.002
p-value	**(0.000)**	(0.118)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.001)**	(0.124)	**(0.011)**	**(0.000)**	**(0.005)**	**(0.000)**	(0.065)
m	0.039	0.073	0.065	0.038	0.047	0.071	0.046	0.024	0.054	0.038	0.062	0.060
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
s	-0.036	-0.009	0.000	-0.016	-0.026	0.102	0.008	-0.043	-0.001	-0.016	-0.043	0.004
p-value	(0.000)	(0.114)	(0.885)	(0.000)	(0.000)	(0.000)	(0.196)	(0.000)	(0.882)	(0.005)	(0.000)	(0.262)
h	-0.010	-0.014	0.014	0.110	0.003	-0.015	-0.010	0.057	-0.034	-0.012	-0.033	-0.004
p-value	(0.027)	(0.061)	(0.000)	(0.000)	(0.502)	(0.013)	(0.198)	(0.000)	(0.000)	(0.097)	(0.000)	(0.295)
R2-ajust	0.666	0.701	0.879	0.763	0.691	0.840	0.471	0.564	0.764	0.413	0.850	0.838
F-statistic	186.7	219.2	674.7	300.4	209.0	490.4	83.8	121.3	302.7	66.5	529.1	482.0
LM test	0.203	0.198	0.287	0.022	0.758	0.060	0.278	0.039	0.182	0.014	0.025	0.092
ARCH test	0.575	0.084	0.811	0.783	0.515	0.000	0.048	0.000	0.012	0.639	0.018	0.709

Notes: This table reports the regression coefficients for the selected three factors on twelve industrial portfolios relative to US stock market returns in order to determine whether there are excess returns on the risk free rate. The data have monthly frequency. The p-values are in parentheses. The boldface numbers indicate non-rejection of the null hypothesis at the 5% levels. The ARCH test considers two lags and the LM test is autocorrelated of order 15. The values of the LM test and the ARCH test are the p-values.

Appendix III:

Table 3 – Descriptive Statistics

	$\mathbf{R_{wt}}$	$\mathbf{R_{1t}}$	$\mathbf{R_{2t}}$	$\mathbf{R_{3t}}$
Mean	1.00595	1.00764	0.99313	1.00062
Median	1.01233	1.01083	0.99867	0.99950
Std. Dev.	0.04591	0.05330	0.11336	0.29744
Skewness	-1.07523	-0.79799	-0.75238	-0.26508
Kurtosis	5.96331	6.67174	7.16965	3.40245
Correlation	**Rwt**	$\mathbf{R_{1t}}$	$\mathbf{R_{2t}}$	$\mathbf{R_{3t}}$
$\mathbf{R_{wt}}$	1.0000	0.9338	-0.0853	0.0077
$\mathbf{R_{1t}}$	0.9338	1.0000	0.0045	-0.0023
$\mathbf{R_{2t}}$	-0.0853	0.0045	1.0000	-0.0008
$\mathbf{R_{3t}}$	0.0077	-0.0023	-0.0008	1.0000

Notes: R_{1t}, R_{2t}, R_{3t} are the first three common-factor portfolios. R_{Wt} is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP). The first factor portfolio accounts for 60% of the variance, the first and second factor portfolio for 70% and the three factors for 80%.