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Abstract: In this paper, we examine the daily water demand forecasting performance of double seasonal univariate time series models (Exponential Smoothing, ARIMA and GARCH) based on multi-step ahead forecast mean squared errors. We investigate whether combining forecasts from different methods and from different origins and horizons could improve forecast accuracy. We use daily data for water consumption in Spain from 1 January 2001 to 30 June 2006.

Keywords: ARIMA; Combined forecasts; Double seasonality; Exponential Smoothing; Forecasting; GARCH; Water demand.

1. Introduction

Water demand forecasting is of great economic and environmental importance. Many factors can influence directly or indirectly water consumption. These include rainfall, temperature, demography, pricing and regulation. Weather conditions have been widely used as inputs of multivariate statistical models (regression, transfer function, vector autoregression, and artificial neural networks) for water demand modelling and forecasting. See Maidment and Miaou (1986), Fildes, Randall and Stubbs (1997), Zhou et al. (2000), Jain, Varshney and Joshi (2001), Bougadis, Adamowski and Diduch (2005), and Gato, Jayasuriya and Roberts (2007). These approaches have drawbacks in water demand prediction as a result of weather conditions variability and changes.

Water demand is highly dominated by daily, weekly and yearly seasonal cycles. The univariate time series models based on the historical data series can be quite useful for short-term demand forecasting as we accommodate the various periodic and seasonal cycles in the model specifications and forecasts. To improve forecast accuracy, we may then combine forecasts derived from the various univariate methods and from different forecast horizons. Combining forecasts can reduce errors by averaging of independent forecasts, and is particularly useful when we are uncertain about which forecasting method is better for future prediction. Some relevant empirical studies using combined forecasts are summarized in Clemen (1989) and Armstrong (2001).

In this paper, we examine the daily water demand forecasting performance of double seasonal univariate time series models based on multi-step ahead forecast mean squared errors. We investigate whether combining forecasts from different methods and from different origins and horizons could improve forecast accuracy. The most accuracy forecasting methods are then used for out-of-sample daily and weekly average forecasting of water consumption in Spain. Our interest in this problem arose from time series competition organized by Spanish IEEE Computational Intelligence Society at the SICO'2007 Conference.

The remainder of the paper is organized as follows. Section 2 discusses the methodology used in time series modelling and forecasting. Section 3 describes the dataset used in the study. Section 4 presents the empirical results. Section 5 offers some concluding remarks.

2. Methodology

2.1. Forecast evaluation

Denote the actual observation for time period t by Y_t and the forecasted value for the same period by F_t. The mean squared error (MSE) statistic for the post-sample period $t = m+1, m+2, ..., m+h$ is defined as follows:

$$MSE = \frac{1}{h-1} \sum_{t=m+1}^{m+h} (Y_t - F_t)^2. \quad (1)$$

This statistic is used to evaluate the out-of-sample forecast accuracy using a training sample of observations of size $m < n$ (where n is the sample size) to estimate the model, and then computing recursively the one-step ahead forecasts for time periods $m+1$, $m+2$, ... by increasing the training sample by one. For k-step ahead forecasts, we begin at the start of the training sample and we compute the forecast errors for time periods $t = m+k$, $m+k+1$, ... using the same recursive procedure.

2.2. Random walk

The naïve version of the random walk model is defined as

$$F_{t+1} = Y_t. \quad (2)$$

This purely deterministic method uses the most recent observation as a forecast, and is used as a basis for evaluating of time series models described below.

2.3. Exponential smoothing

Exponential smoothing is a simple but very useful technique of adaptive time series forecasting. Standard seasonal methods of exponential smoothing includes the Holt-Winters' addtive trend, multiplicative trend, damped aditive trend and damped multiplicative trend (see Gardner, 2006). We implemented the double seasonal versions of the Holt-Winters' exponential smoothing (Taylor, 2003) in order to take into account the two seasonal cycle periods in the water consumption (daily and weekly). The double seasonal additive methods outperformed the double seasonal multiplicative methods. Within the double seasonal additive methods, the additive trend was found to be the best for one-step ahead forecasting.

The forecasts for Taylor's exponential smoothing for double seasonal additive method with additive trend are determined by the following expressions:

$$\begin{aligned} L_t &= \alpha(Y_t - S_{t-7} - D_{t-365}) \\ &\quad + (1-\alpha)(L_{t-1} + T_{t-1}) && (3) \\ T_t &= \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} && (4) \\ S_t &= \gamma(Y_t - L_t - D_{t-365}) + (1-\gamma)S_{t-7} && (5) \\ D_t &= \delta(Y_t - L_t - S_{t-7}) + (1-\delta)D_{t-365} && (6) \\ F_{t+h} &= L_t + T_t \times h + S_{t+h-7} + D_{t+h-365} + \phi^h \\ &\quad \times [Y_t - (L_{t-1} - T_{t-1} - S_{t-7} - D_{t-365})] && (7) \end{aligned}$$

where L_t is the smoothed level of the series; T_t is the smoothed additive trend; S_t is the smoothed seasonal index for weekly period $s_1 = 7$; D_t is the smoothed seasonal index for daily period $s_1 = 365$; α and β are the smoothing parameters for the level and trend; γ and δ are the seasonal smoothing parameters; ϕ is an adjustment for first-order autcorrelation; and F_{t+h} is the forecast for h periods ahead, with $h \leq 7$. We initialize the values for the level, trend and seasonal periods as follows:

$$\begin{aligned} L_{365} &= \frac{1}{365} \sum_{t=1}^{365} Y_t \\ T_{365} &= \frac{1}{365^2} \left(\sum_{t=366}^{730} Y_t - \sum_{t=1}^{365} Y_t \right) \\ S_1 &= Y_1 - L_7, ..., S_7 = Y_7 - L_7 \\ D_1 &= Y_1 - L_{365}, ..., D_{365} = Y_{365} - L_{365} \end{aligned}$$

The smoothing parameters α, β, γ, δ and ϕ are chosen by minimizing the MSE statistic for one-step-ahead in-sample forecasting using a linear optimization algorithm.

2.4. ARIMA model

We implemented a double seasonal multiplicative ARIMA model (see Box, Jenkins and Reinsel, 1994) of the form:

$$\phi_p(B)\Phi_{P_1}(B^{s_1})\Pi_{P_2}(B^{s_2})(1-B)^d$$
$$\times(1-B^{s_1})^{D_1}(1-B^{s_2})^{D_2}(Y_t-c)$$
$$=\theta_q(B)\Theta_{Q_1}(B^{s_1})\Psi_{Q_2}(B^{s_2})\varepsilon_t \tag{8}$$

where c is a constant term; B is the lag operator such that $B^kY_t = Y_{t-k}$; $\phi_p(B)$ and $\theta_q(B)$ are regular autoregressive and moving average polynomials of orders p and q; $\Phi_{P_1}(B^{s_1})$, $\Pi_{P_2}(B^{s_2})$, $\Theta_{Q_1}(B^{s_1})$ and $\Psi_{Q_2}(B^{s_2})$ are seasonal autoregressive and moving average polynomials of orders P_1, P_2, Q_1 and Q_2; s_1 and s_2 are the seasonal periods; d, D_1 and D_2 are the orders of integration; and ε_t is a white noise process with zero mean and constant variance. The roots of the polynomials $\phi_p(B) = 0$, $\Phi_{P_1}(B^{s_1}) = 0$, $\Pi_{P_2}(B^{s_2}) = 0$, $\theta_q(B) = 0$, $\Theta_{Q_1}(B^{s_1}) = 0$ and $\Psi_{Q_2}(B^{s_2}) = 0$ should lie outside the unit circle. This model is often denoted as ARIMA(p,d,q)$\times$(P_1,D_1,Q_1)$_{s_1}\times$(P_2,D_2,Q_2)$_{s_2}$. We examine the sample autocorrelations and the partial autocorrelations of the differenced series in order to identify the integer's p, q, P_1, Q_1, P_2 and Q_2. After identifying a tentative ARIMA model, we estimate the parameters by Marquardt nonlinear least squares algorithm (for details, see Davison and MacKinnon, 1993). We check the adequacy of the model by using suitable fitted residuals tests. We use the Schwarz Bayesian Criterion (SBC) for model selection.

2.5. GARCH model

In many practical applications to time series modelling and forecasting, the assumption of nonconstant variance may be not reliable. The models with nonconstant variance are referred to as conditional heteroscedasticity or volatility models. To deal with the problem of heteroscedasticity in the errors, Engle (1982) and Bollerslev (1986) proposed the autoregressive conditional heteroskedasticity (ARCH) and the generalized ARCH (or GARCH) to model and forecast the conditional variance (or volatility). The GARCH(p,q) model assumes the form:

$$\sigma_t^2 = \omega + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2, \tag{9}$$

where p is the order of the GARCH terms and q is the order of the ARCH terms. The necessary conditions for the model (9) to be variance and covariance stationary are: $\omega > 0$; $\beta_j \geq 0$, $j = 1,...,p$; $\alpha_i \geq 0$, $j = 1,...,q$; and $\sum_{j=1}^{p}\beta_j + \sum_{i=1}^{q}\alpha_i < 1$. Last summation quantifies the shock persistence to volatility. A higher persistence indicates that periods of high (slow) volatility in the process will last longer. In most economical and financial applications, the simple GARCH(1,1) model has been found to provide a good representation of a wide variety of volatility processes as discussed in Bollerslev, Chou and Kroner (1992).

In order to capture seasonal and cyclical components in the volatility dynamics, we implemented a seasonal-periodic GARCH model of the form:

$$\begin{aligned}\sigma_t^2 \quad = \quad & \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 + \alpha_7\varepsilon_{t-7}^2 \\ & +\alpha_{365}\varepsilon_{t-365}^2 + \sum_{m=1}^{M}\left[\lambda_m\cos\left(\frac{2\pi mS_t}{7}\right)\right. \\ & +\varphi_m\sin\left(\frac{2\pi mS_t}{7}\right) + \kappa_m\cos\left(\frac{2\pi mD_t}{365}\right) \\ & +\upsilon_m\sin\left(\frac{2\pi mD_t}{365}\right) + \lambda'_m\varepsilon_{t-7}^2\cos\left(\frac{2\pi mS_t}{7}\right) \\ & +\varphi'_m\varepsilon_{t-7}^2\sin\left(\frac{2\pi mS_t}{7}\right) + \kappa'_m\varepsilon_{t-365}^2\cos\left(\frac{2\pi mD_t}{365}\right) \\ & \left. +\ \upsilon'_m\varepsilon_{t-365}^2\sin\left(\frac{2\pi mD_t}{365}\right)\right],\end{aligned} \tag{10}$$

where S_t and D_t are repeating step functions with the days numerated from 1 to 7 within each week, and from 1 to 365 within each year, respectively. This approach was used by Campbell and Diebold (2005) to model conditional variance in daily average temperature data, and by Taylor (2006) to forecast electricity consumption. We set $M = 3$ for the Fourier series. We estimate the model by the method of maximum likelihood, assuming a generalized error distribution (GED) for the innovations series (see Nelson, 1991).

2.6. Combining forecasts

We examine whether combining forecasts from the various univariate methods and from different forecast origins and horizons could provide more accurate forecasts than the individual methods being combined. We consider all possible combinations of the forecast methods Holt-Winters (HW), ARIMA (A) and GARCH (G), and we compute the simple (unweighted) average of the forecasts,

$$F_t = \frac{F_t^{(HW)} + F_t^{(A)} + F_t^{(G)}}{3}, \tag{11}$$

where $F_t^{(\cdot)}$ is the forecasted value of method $(\cdot)$ in time period t. We drop the random walk (the worst method) of the combination.

3. Data

We analyze the daily water consumption series in Spain from 1 January 2001 to 30 June 2006 (2006 observations). We have drop February 29 in the leap year 2004 in order to maintain 365 days in each year. This series is plotted in Figure 1. The dataset was obtained from the Spanish IEEE Computational Intelligence Society (http://www.congresocedi.es/2007/).

We use the first 1976 observations from 1 January 2001 to 31 May 2006 as training sample for model estimation, and the remaining 30 observations from 1 June 2006 to 30 June 2006 as post-sample for forecast evaluation.

4. Empirical study

4.1. Estimation results

The implementation of the double seasonal Holt-Winters method to the water demand series Y_t gives the values: $\alpha = 0.000$, $\beta = 0.755$, $\gamma = 0.303$, $\delta = 0.294$ and $\phi = 0.607$.

After evaluating different ARIMA formulations, we apply the following multiplicative double seasonal ARIMA model:

$$\begin{aligned}&(1-\phi_1 B-\phi_2 B^2-\phi_4 B^4)(1-\Phi_1 B^7-\Phi_2 B^{14})\\&\times(1-B^7)(1-B^{365})(Y_t-c)\\&=(1-\theta_9 B^9)(1-\Theta_3 B^{21})(1-\Psi_1 B^{365})\varepsilon_t\end{aligned}$$

This model can be represented as ARIMA$(4,0,9)\times(2,1,3)_7\times(0,1,1)_{365}$, with $\phi_3 = 0$, $\theta_1 = \cdots = \theta_8 = 0$, and $\Theta_1 = \Theta_2 = 0$. The estimated results are shown in Table 1. We fitted a significant parameter ARIMA-GARCH model of the form:

$$\begin{aligned}&(1-\phi_1 B-\phi_2 B^2-\phi_4 B^4)(1-\Phi_1 B^7-\Phi_2 B^{14})\\&\times(1-B^7)(1-B^{365})(Y_t-c)\\&=(1-\theta_9 B^9)(1-\Theta_3 B^{21})(1-\Psi_1 B^{365})\varepsilon_t\end{aligned}$$

and

$$\begin{aligned}\sigma_t^2 &= \omega+\beta_1\sigma_{t-1}^2+\alpha_1\varepsilon_{t-1}^2+\alpha_{365}\varepsilon_{t-365}^2\\&\quad+\varphi_1\sin\left(\frac{2\pi D_t}{365}\right)+\varphi_3'\varepsilon_{t-365}^2\sin\left(\frac{6\pi D_t}{365}\right).\end{aligned}$$

The model estimates are given in Table 2.

4.2. Forecasting results

The performance of the estimated univariate methods were evaluated by computing MSE statistics for multi-step forecasts from 1 to 7 days ahead. Table 3 gives the forecasts results for the post-sample period from 1 June 2006 to 30 June 2006. Table 4 gives the forecast results for the weekly 7-days of the same post-sample period.

The ARIMA and GARCH models appear to have the same forecast performance for all the forecast horizons. The Holt-Winters outperformed the ARIMA and GARCH models in long horizons. In contrast, for one to four steps ahead forecasting the ARIMA and GARCH models performed better than the Holt-Winters. The random walk model ranked last for any of the forecast horizons considered.

For the 7-days of week, the ARIMA appear to perform well for Monday and Tuesday forecasting, the simple combinations Holt-ARIMA and Holt-GARCH appear to be most useful for Wednesday forecasting, the Holt appears to be the most appropriate method for Thursday, Friday and Sunday forecasting, and the GARCH appears to be the best method for Sunday forecasting.

In Table 5 we present the out-of-sample forecasts for water demand series. Our forecasts are based on the most accuracy forecasting method used for multi-step ahead average forecasting for the 7-days cycle. We consider the periods from 1 July to 31 July 2006 (31 daily forecasts), from

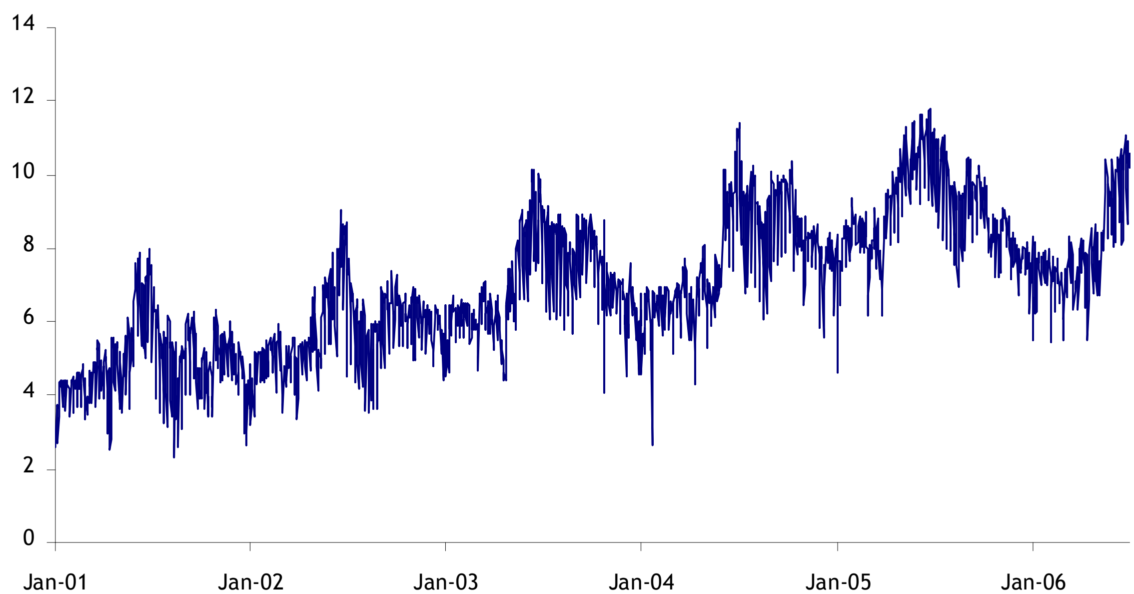

Figure 1. Daily water demand in Spain for the period 1 January 2001 to 30 June 2006

1 July to 29 December 2006 (26 weekly average forecasts) and from 3 July to 31 December 2006 (26 weekly average forecasts).

5. Concluding remarks

In this paper, we compared the forecast accuracy of individual and combined univariate time series models for multi-step-ahead water demand forecasting. We implemented double seasonal versions of the Holt-Winters, ARIMA and GARCH models in order to accommodate the two seasonal cycles periods in water consumption (daily and weekly).

The empirical results suggest that all the univariate time series models can be quite useful for short-term forecasting. Moreover, the examination of the multi-step-ahead forecasting performance for each day of week suggest the use of different methods and different combined forecasts to improve forecasting accuracy.

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Table 1
Seasonal ARIMA model estimates for water demand series

Model: ARIMA(4,0,9)×(2,1,3)$_7$×(0,1,1)$_{365}$				Residual ACF		Residual PACF	
Parameter	Lag	Estimate	Standard error	Lag	Estimate	Lag	Estimate
c		-0.004	0.007	1	0.004	1	0.004
ϕ_1	1	0.592	0.025	2	0.009	2	0.009
ϕ_2	2	0.134	0.027	3	-0.020	3	-0.020
ϕ_4	4	0.061	0.023	4	0.001	4	0.001
θ_9	9	-0.053	0.024	5	-0.026	5	-0.025
Φ_1	7	-0.757	0.023	6	0.015	6	0.015
Φ_2	14	-0.561	0.029	7	-0.010	7	-0.010
Θ_3	21	-0.366	0.032				
Ψ_1	365	-0.644	0.023				R^2 adjusted = 0.662
							$Q(20) = 18.31$ (0.11)
							$Q^2(20) = 71.62$ (0.00)

Notes: Q(20) ($Q^2(20)$) is the Ljung-Box statistic for serial correlation in the residuals (squared residuals) up to order 20; p-value in parentheses.

Table 2
Seasonal-periodic GARCH model estimates for water demand series

Model: ARIMA(4,0,9)×(2,1,3)$_7$×(0,1,1)$_{365}$-GARCH(1,1)×(0,1)$_{365}$				Residual ACF		Residual PACF	
Parameter	Lag	Estimate	Standard error	Lag	Estimate	Lag	Estimate
c		-0.011	0.008	1	-0.007	1	0.007
ϕ_1	1	0.502	0.029	2	0.023	2	0.023
ϕ_2	2	0.137	0.030	3	-0.028	3	-0.028
ϕ_4	4	0.075	0.024	4	-0.026	4	-0.026
θ_9	9	-0.064	0.023	5	-0.042	5	-0.040
Φ_1	7	-0.747	0.023	6	0.026	6	0.027
Φ_2	14	-0.534	0.028	7	-0.006	7	-0.006
Θ_3	21	-0.346	0.031				
Ψ_1	365	-0.640	0.025	Squared residual ACF		Squared residual PACF	
				Lag	Estimate	Lag	Estimate
ω		0.107	0.028	1	0.012	1	0.012
α_1	1	0.103	0.037	2	-0.030	2	-0.031
β_1	1	0.483	0.108	3	0.028	3	0.029
α_{365}	365	0.109	0.032	4	0.018	4	0.016
φ_1		0.026	0.011	5	0.008	5	0.009
φ_3'	365	0.062	0.035	6	-0.023	6	-0.023
GED		1.361	0.055	7	0.015	7	0.015
							R^2 adjusted = 0.657
							Q(20)=19.20 (0.08)
							$Q^2(20)$=13.61 (0.33)

Notes: As Table 1.

Table 3
MSE for multi-step-ahead forecasts for post-sample period

Forecast horizon	RW	HW	ARIMA	GARCH	Combined forecasts HW-A	HW-G	A-G	HW-A-G
1-step	0.96	0.38	0.35	0.35	0.35	0.35	0.35	0.35
2-step	1.55	0.51	0.45	0.45	0.46	0.45	0.45	0.45
3-step	1.82	0.49	0.47	0.45	0.45	0.45	0.45	0.45
4-step	2.09	0.48	0.45	0.46	0.46	0.46	0.46	0.46
5-step	2.23	0.43	0.44	0.46	0.43	0.43	0.45	0.44
6-step	1.91	0.42	0.45	0.47	0.43	0.43	0.46	0.44
7-step	1.33	0.40	0.44	0.46	0.41	0.42	0.45	0.43
Average	1.70	0.44	0.44	0.44	0.43	0.43	0.44	0.43

Table 4
MSE for multi-step ahead forecasts for weekly 7-days in post-sample period

Forecast horizon	Days of week	RW	HW	ARIMA	GARCH	Combined forecasts HW-A	HW-G	A-G	HW-A-G
1-step	Monday	16.18	2.33	1.18	1.25	1.71	1.75	1.21	1.55
	Tuesday	0.28	0.53	0.20	0.19	0.34	0.34	0.19	0.29
	Wednesday	0.18	0.14	0.25	0.26	0.19	0.20	0.26	0.21
	Thursday	3.15	4.19	5.26	5.40	4.71	4.78	5.33	4.93
	Friday	0.47	0.37	0.54	0.54	0.45	0.45	0.54	0.48
	Saturday	3.00	0.23	0.64	0.58	0.39	0.37	0.61	0.45
	Sunday	1.20	1.26	0.40	0.33	0.70	0.61	0.36	0.53
4-step	Monday	3.86	0.42	0.43	0.54	0.42	0.48	0.48	0.46
	Tuesday	2.66	0.15	0.16	0.17	0.15	0.15	0.16	0.16
	Wednesday	8.39	0.48	0.69	0.77	0.58	0.62	0.73	0.64
	Thursday	11.27	3.63	3.79	4.14	3.71	3.88	3.96	3.85
	Friday	1.83	1.78	1.88	1.94	1.83	1.86	1.91	1.87
	Saturday	4.14	1.29	1.21	1.26	1.25	1.28	1.24	1.25
	Sunday	10.23	3.23	1.10	0.81	2.03	1.82	0.95	1.56
7-step	Monday	0.30	0.19	0.24	0.38	0.21	0.28	0.30	0.26
	Tuesday	0.15	0.07	0.06	0.08	0.06	0.06	0.06	0.06
	Wednesday	1.09	0.27	0.39	0.29	0.33	0.28	0.34	0.31
	Thursday	13.60	2.54	3.33	3.42	2.92	2.96	3.38	3.08
	Friday	7.91	2.14	2.25	2.38	2.19	2.26	2.32	2.26
	Saturday	4.19	1.43	1.48	1.59	1.46	1.51	1.54	1.50
	Sunday	0.70	1.14	0.29	0.22	0.63	0.51	0.26	0.42
Average	Monday	4.79	0.61	0.55	0.65	0.57	0.62	0.59	0.59
	Tuesday	4.13	0.44	0.16	0.17	0.25	0.26	0.16	0.21
	Wednesday	4.89	0.43	0.46	0.48	0.41	0.41	0.47	0.42
	Thursday	7.52	3.01	3.71	3.91	3.33	3.43	3.80	3.51
	Friday	5.21	1.99	2.25	2.32	2.12	2.15	2.28	2.18
	Saturday	4.02	1.06	1.24	1.26	1.13	1.14	1.25	1.17
	Sunday	6.10	2.35	0.68	0.50	1.38	1.22	0.59	1.02

Table 5
Out-of-sample daily and weekly average forecasts for water demand series

Forecast period	Daily forecasts	Forecast period	Average forecasts	Forecast period	Average forecasts
1 July 2006	9.3393	1 Jul -7 Jul 2006	9.5410	3 Jul -9 Jul 2006	9.2173
2 July 2006	9.4614	8 Jul-14 Jul 2006	8.7871	10 Jul-16 Jul 2006	8.7170
3 July 2006	8.6222	15 Jul -21 Jul 2006	8.5721	17 Jul -23 Jul 2006	8.5171
4 July 2006	10.3095	22 Jul -28 Jul 2006	8.4674	24 Jul -30 Jul 2006	8.4175
5 July 2006	9.9691	29 Jul - 4 Aug 2006	8.1969	31 Jul - 6 Aug 2006	8.0443
6 July 2006	9.6744	5 Aug - 11 Aug 2006	7.8059	7 Aug - 13 Aug 2006	7.7455
7 July 2006	9.4109	12 Aug - 18 Aug 2006	7.3662	14 Aug - 20 Aug 2006	7.3417
8 July 2006	8.0849	19 Aug - 25 Aug 2006	7.6512	21 Aug - 27 Aug 2006	7.7199
9 July 2006	8.4503	26 Aug - 1 Sep 2006	8.0755	28 Aug - 3 Sep 2006	8.2972
10 July 2006	7.6937	2 Sep - 8 Sep 2006	8.3625	4 Sep - 10 Sep 2006	8.2373
11 July 2006	9.5814	9 Sep - 15 Sep 2006	8.4238	11 Sep - 17 Sep 2006	8.4191
12 July 2006	9.3338	16 Sep - 22 Sep 2006	8.4734	18 Sep - 24 Sep 2006	8.5735
13 July 2006	9.2385	23 Sep - 29 Sep 2006	8.3959	25 Sep - 1 Oct 2006	8.3159
14 July 2006	9.1268	30 Sep - 6 Oct 2006	8.3155	2 Oct - 8 Oct 2006	8.3421
15 July 2006	7.7251	7 Oct - 13 Oct 2006	7.8275	11 Oct - 15 Oct 2006	7.5624
16 July 2006	8.3192	14 Oct - 20 Oct 2006	7.5310	16 Oct - 22 Oct 2006	7.5278
17 July 2006	7.7194	21 Oct - 27 Oct 2006	7.6197	21 Oct - 29 Oct 2006	7.6764
18 July 2006	9.2434	28 Oct - 3 Nov 2006	7.5537	30 Oct - 5 Nov 2006	7.6166
19 July 2006	9.1702	4 Nov - 10 Nov 2006	7.9359	6 Nov - 12 Nov 2006	7.9908
20 July 2006	9.1609	11 Nov - 17 Nov 2006	7.9195	13 Nov - 19 Nov 2006	7.8022
21 July 2006	8.6665	18 Nov - 24 Nov 2006	7.7142	20 Nov - 26 Nov 2006	7.8061
22 July 2006	7.4185	25 Nov - 1 Dec 2006	7.6954	27 Nov - 3 Dec 2006	7.5147
23 July 2006	8.2409	2 Dec - 8 Dec 2006	7.2456	4 Dec - 10 Dec 2006	7.3100
24 July 2006	7.7286	9 Dec- 15 Dec 2006	7.6863	11 Dec- 17 Dec 2006	7.7958
25 July 2006	9.2441	16 Dec - 22 Dec 2006	7.5248	18 Dec - 24 Dec 2006	7.3708
26 July 2006	9.0913	23 Dec - 29 Dec 2006	7.0362	25 Dec - 31 Dec 2006	6.9743
27 July 2006	8.9348				
28 July 2006	8.6137				
29 July 2006	7.2653				
30 July 2006	8.0448				
31 July 2006	7.5470				