# Grade Inflation and Education Quality

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**Abstract**

We consider a game in which schools compete to place graduates in two distinct ways: by investing in the quality of education, and by strategically designing grading policies. In equilibrium, schools issue grades that do not perfectly reveal graduate abilities. This leads evaluators to have less-accurate information when hiring or admitting graduates. However, compared to fully-revealing grading, strategic grading motivates greater investment in educating students, increasing average graduate ability. Allowing grade inflation and related grading strategies can increase the probability evaluators select high-ability graduates.

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Grades A and B are sometimes given too readily – Grade A for work of no very high merit, and Grade B for work not far above mediocrity. ... One of the chief obstacles to raising the standards of the degree is the readiness with which insincere students gain passable grades by sham work.

–*Report of the Committee on Raising the Standard, 1894*[1]

I don't give C's anymore, and neither do most of my colleagues. And I can easily imagine a time when I'll say the same thing about B's.

–*Stuart Rojstaczer, January 28, 2003*[2]

[1]We found this quotation in Kohn (2002).
[2]Article available at http://today.duke.edu/2003/01/20030128.html

## 1 INTRODUCTION

Since the 1980s, the mean grade point average at American colleges and universities has risen at a rate between 0.1 and 0.15 points per decade. Most of this increase can be attributed to an increase in the share of As assigned, and a decrease in the share of grades assigned at the low end of the scale.[3] The central concern is that these effects are driven by a drop in grading standards, rather than enhanced student performance or ability. Changes in grading standards can have important consequences: when high grades are assigned liberally, they convey less information to employers, graduate schools, and other evaluators about a student's true ability and lead to less-informed placement decisions.

The majority of research on grade inflation in the economics literature (surveyed below) establishes that noisy grading policies are a natural consequence of strategic interactions between schools, arising in equilibrium as schools compete to place their graduates. Grade inflation may therefore be a more fundamental phenomenon than popular wisdom suggests. In addition, the literature has consistently documented the welfare costs resulting from grade inflation and other noisy grading strategies. In many of these analyses, however, schools only reveal information about student ability, doing nothing to improve it.[4] This is in contrast to much of the education literature, which shows that certain school investments—for example, recruiting more-effective teachers—improve graduate ability.[5]

Our analysis contributes to the literature by considering the interaction between a school's investment in education quality and its choice of grading policy. We show that the negative welfare implications established by the economics literature (and generally taken for granted) are often reversed when a school's investment in education is accounted for. Allowing schools the freedom to strategically choose grading policies changes the incentives for schools to invest in developing student ability. In equilibrium, strategic grading leads to greater investments by schools. Although transcripts are less-informative, the average ability of graduates is higher.

We consider a three-stage model of school competition. In the first stage, schools simultaneously invest in education quality, which determines the probability that they produce a high-ability graduate. In the second stage, schools simultaneously design grading policies. These grading policies determine how transcripts are assigned to students of different abilities and affect the inferences that employers, graduate schools, and other evaluators make when observing a student's transcript. In the third stage, each school produces a single graduate who is then evaluated by a third party.[6] The evaluator benefits if she selects a high-ability graduate to receive a prize

[3]See data compiled by Rojstaczer (2011).

[4]Zubrickas (2010) and Dubey and Geanakoplos (2010) are notable exceptions, which we discuss below.

[5]Rivkin, Hanushek and Kain (2005) demonstrate that teachers have powerful effects on student achievement and test scores. At the same time, easily observable teacher characteristics like education and experience explain very little of the variation in teacher quality. Thus, identifying, recruiting, and retaining high-quality teachers is a costly investment, with the potential to significantly improve student outcomes. Other types of investments include caps on class sizes, providing opportunities for teacher professional development, increasing the allocation of school resources, and better monitoring and incentives for student effort.

[6]We discuss the one-graduate assumption in Section 9

(e.g., job, promotion, school admission or fellowship). Observing investments, grading policies, and transcripts, she assigns the prize to the graduate she believes is more likely to be high ability.

Throughout the paper, we compare equilibrium outcomes when schools grade strategically with outcomes arising in a "fully-revealing" benchmark. In this benchmark, all uncertainty about graduate ability is resolved during evaluation; this is often viewed as the ideal in policy discussion. We consider means by which the fully-revealing benchmark may be approximated in Section 5.

In the fully-revealing benchmark, the only way for a school to increase the probability the evaluator selects its graduate is to provide him a better education. It does this by investing more in school quality. The same incentives exist when schools choose grades strategically; however, an additional incentive for investment emerges. When no restrictions are placed on grading, both schools choose equilibrium grading policies that distort grades of both low- and high-ability graduates. By assigning a particular transcript to both types of students, the school makes a low-ability graduate with this transcript appear to be better than he truly is; at the same time, a high-ability graduate with this transcript appears worse than he truly is. The higher the likelihood that a particular transcript is assigned to a high-ability student, the better the evaluator's inference about any graduate receiving this transcript. By changing the way in which it distributes transcripts to graduates of different abilities, a school alters the distribution of the evaluator's posterior beliefs about graduate ability. However, the extent to which a school can influence the evaluator's beliefs is determined by its quality. The school that invested less in education quality in the first stage is less likely to produce a high-ability graduate, and is therefore limited in its ability to improve evaluator beliefs about a low-ability graduate using its grading policy. While both schools are able to control evaluator beliefs by adjusting the way they assign grades, the school with greater first-stage investment has an advantage in the grading process. When grading is strategic, investment not only improves the likely ability of graduates, it also gives a school greater flexibility in designing its grading policy.

The next section surveys the relevant literature. In Section 3, we describe the model and solve for the Perfect Bayesian Equilibrium of the three-stage game. We show that schools strategically choose grading policies that are less-than-fully informative about graduate ability. These policies exhibit grade "compression," the hallmark of grade inflation. We consider the fully-revealing benchmark in Section 5. Section 6 compares the equilibrium outcome under strategic grading with the fully-revealing benchmark, showing that school investment, graduate ability, and evaluator welfare are often higher when grading is strategic. In Section 7 we consider uninformative grading. We show that an environment in which grades are so inflated as to be uninformative (or no grades are assigned at all) can also be better for the evaluator than the fully-revealing benchmark. In Section 8, we consider a variation of the model in which schools may only assign two grades, and are only able to inflate the grades of low-ability graduates. In the two grade, inflation-only model, the equilibrium exhibits a number of novel aspects not present in the general framework, but the benefits of grade inflation are also present in this more-specialized setting. Our model is stylized, designed to best capture the essential aspects of the interaction between school investment and grading policy. We discuss some of the assumptions of the model in more detail in Section 9. The conclusion in Section 10 gives a brief discussion of alternative interpretations of the model, and summarizes its policy implications. Proofs of all results are in the Appendix.

## 2 RELATED LITERATURE

A significant portion of the economics literature on grade inflation argues that inflation is a robust equilibrium phenomenon that often imposes a welfare cost on employers or other evaluators; none of these papers consider the interaction between grading policies and incentives to invest in education quality. Ostrovsky and Schwarz (2010) consider an assortative stable matching in a labor market. Vacancies are distinguished by desirability, and graduates are distinguished by their expected ability. These authors argue that, under certain circumstances, in equilibrium schools do not to completely reveal the ability of their graduates to potential employers, assigning transcripts to students in a way that confounds employer beliefs about graduate ability. Popov and Bernhardt (2012) consider a model of strategic grade assignment with a continuum of student abilities and two grades. They show that universities with better distributions of student abilities set lower grading standards; whereas a social planner would set a higher grading standard at a better university. Chan, Li and Suen (2007) take a different perspective: whereas in the analyses previously described (as well as our own) employers know the distribution of student abilities for each school, these authors consider what would happen if a school knew the distribution of its own students' abilities, but an employer did not. In this paper, the proportion of high ability students at a given school can assume one of two values. A school can assign grades in a manner consistent with true student abilities, or it can exploit its private information, assigning grades as if a high proportion of students at the school is high ability, even though this proportion is actually low. These authors show that in equilibrium, schools will inflate grades by (sometimes) assigning a higher proportion of good grades than there are good students at the school.[7] If they could do so, schools would benefit from a commitment to honest grading. Bar, Kadiyali and Zussman (2012) consider the impact of disclosing information about grading policies to students or evaluators. Much of the grade inflation literature assumes (as we do) that evaluators can observe the grading policy utilized by a school prior to evaluating the graduates.[8] These authors consider a model of grade inflation without this assumption. In their model, students strategically choose courses with different difficulties and different degrees of grade inflation in order to affect employer perceptions about their abilities. They then contrast the impact of disclosing grading information to students prior to course selection and to employers along with transcripts. They find that disclosure of grading policies to students affects course selection decisions. Disclosing information to employers benefits students who elect to enter strictly graded courses, and hurts those who select lenient courses. Overall, this can damage ag-

[7]The model is designed in such a way that it is a school with a worse distribution of student abilities that sometimes inflates grades. This result somewhat contradicts empirical evidence and conventional wisdom, that grade inflation is more extreme at *ex ante* better schools.

[8]We discuss alternatives to this assumption in Section 9.

gregate student welfare.[9] All of these papers treat the distribution of student ability at each school as exogenous. In this case, our analysis supports both of the main conclusions of the literature. We show that grade inflation is a robust equilibrium phenomenon. We also show that if the ability distribution at each school is *exogenous*, then grade inflation imposes a welfare cost, as evaluators are less-able to identify high-ability graduates.

In contrast to this literature, however, the primary focus of our analysis is on the interaction between grading policies and investment in ability. We show that allowing grade inflation motivates schools to invest in the ability of graduates and thus has benefits. In fact, when schools endogenously invest in developing graduate ability, in equilibrium these educational benefits frequently dominate the costs. Unlike the case in which graduate ability is exogenous, when schools invest to develop ability, graduates may be better-educated, and the evaluator's welfare may be higher, if noisy grading is permitted.

Several authors consider the teacher-student relationship in the principal-agent framework. In this interaction, the teacher's goal is to incentivize student effort through the design of grading policies. By considering exam performance as a *game of status*,[10] Dubey and Geanakoplos (2010) demonstrate that in certain circumstances students are best motivated to exert effort when their exact exam performance is not revealed. Instead, it is more effective to publicly reveal performance information in broad categories (like letter grades). Related results are found in Zubrickas (2010) who shows that if the market (or other subsequent evaluator) cannot observe an individual teacher's grading practices, the teacher responds by grading leniently. Our analysis complements this strand of the literature. In both Dubey and Geanakoplos (2010) and Zubrickas (2010) the teacher initially commits to a grading policy in order to incentivize subsequent student effort. Meanwhile, in our work, a school's initial investment decision determines its subsequent advantage in the grading stage game. The interaction of student effort and grading policy is closely related to the interaction of school investment and grading policy. While we don't explicitly address the issue of commitment to grading policy as a means of incentivizing student effort, we address the issue of student effort (observed or not) in Section 9.

A number of other authors consider issues to do with evaluation, grading, and certification but do not focus on the issue of grade inflation. Taylor and Yildirim (2011) consider the interaction of an evaluator's performance standard and an agent's unobservable effort. They find that the evaluator often benefits by committing to ignore information about agent attributes during the assessment process. In their framework, agent effort is endogenous, but the evaluator's signal structure is exogenous; this is in contrast to our framework which includes both endogenous agent effort (i.e., school investment) and signal structures (grading policies), as well as competition between agents. Daley and Green (2011) embed grades into a market signaling model. They show that when

[9]An important distinction between this paper and the rest of the literature, including our own paper, is that grading policies in different courses are (for most of the paper) exogenous. The key question is whether these policies are disclosed to different parties, but the bulk of the analysis is done with exogenous grading policies. They also consider endogenous grading policies, derived from faculty preferences for lenience or strictness.

[10]See Moldovanu, Sela and Shi (2007).

exogenous information about ability is available (e.g., grades), high-ability agents' incentives to signal their type by acquiring education are reduced. They argue that informative grades decrease the effort of high-ability agents, but may increase welfare. Because grade inflation undermines informativeness, it may decrease welfare. In the context of industrial organization, Lizzeri (1999) studies the incentives of a rating agency to disclose information about product quality. A seller with private information about quality, has an opportunity to visit a rating agency with the capability to determine and certify quality. This rating agency commits to a disclosure rule (a stochastic mapping from qualities into reports) and to a price for this certification. The author shows that in a variety of important cases, the rating agency reveals a minimal amount of information to the market but appropriates a large share of the surplus.

## 3 THE MODEL

We consider a three-stage game between two schools and an evaluator. First, each school invests in quality, which determines the likely ability of a graduate. Schools observe qualities and then simultaneously choose grading policies, which determine how transcripts are assigned to graduates of different abilities. Finally, each school produces a single graduate.[11] The evaluator observes each school's investment, the graduate's transcript, and each school's grading policy, and awards a prize to one of the two graduates. The prize could be a desirable job, admission to a prestigious law school or university, or an elite scholarship; the evaluator could be a recruiter, an admissions officer, or a representative of a scholastic trust. The evaluator prefers to assign the prize to a "high-ability" student. Our preferred interpretation is that high-ability graduates are those who are likely to excel in the most-challenging post-graduation environments, whether they involve attending a top graduate school, undergraduate institution, or joining a prestigious company. Meanwhile, a school benefits whenever its graduate receives the prize, independent of his true ability.

In the first stage of the game, each school $i \in \{\alpha, \beta\}$ simultaneously chooses its quality, $q_i \in [0, 1]$. School $i$'s graduate is "high-ability" with probability $q_i$ and "low-ability" with probability $1 - q_i$. Since evaluators want to select high-ability students and schools want their students selected, schools benefit from increasing their quality. However, improving school quality is costly in terms of resources or effort.[12] To achieve quality $q_i$, school $i$ must pay a convex cost $C_i(q_i)$ where

$$C_i(q_i) = \frac{q_i^2}{\rho_i^2}.$$

Parameter $\rho_i$ determines the marginal cost of quality, with higher values representing lower marginal

[11] We discuss this assumption in Section 9.

[12] Greenwald, Hedges and Laine (1996) perform a meta-analysis, demonstrating a substantial link between school resources and student achievement, while Hanushek (2006) provides evidence that increased resources at schools do not necessarily translate into better educational outcomes. However, even those finding little evidence of the link between spending and performance do not necessarily claim that additional resources could not be beneficial. They claim that this link may not be observed strongly in the data because schools do not *allocate* their resources in the most effective way possible. For our purposes, we can interpret school quality as a proxy for all choices made by schools that influence true educational outcomes.

costs. The value $\rho_i$ may represent the availability of resources for the school, for example its infrastructure, endowment, or donor base. Alternatively, it may represent characteristics of the student body (e.g. past preparation or test scores). We assume that $0 < \rho_\beta \leq \rho_\alpha < 2$. Setting $\rho_\beta \leq \rho_\alpha$ reflects differences between schools along these dimensions. Because its marginal cost of quality is smaller, school $\alpha$ is "advantaged" and $\beta$ is "disadvantaged." Limiting $\rho_i$ to values less than two focuses the analysis on the interesting case where marginal costs are high enough that schools do not guarantee that their graduates are high quality with probability one. Each school's investment $q_i$ is made public at the end of the first stage.

In the second stage of the game, the schools simultaneously select grading policies. When the schools make this choice, they know the probability that graduates of each school are high ability (i.e., they know school quality, $q_\alpha, q_\beta$), but they do not know the true ability of either graduate. A *grading policy* at school $i$ is represented by a pair of conditional random variables $(H_i, L_i)$. The transcript of a high-ability student is an independent realization of $H_i$ and the transcript of a low-ability student is an independent realization of $L_i$. For technical reasons, we focus on random variables $(H_i, L_i)$ for which the cumulative distribution function has a finite number of discontinuities or mass points. Except at mass points, $H_i$ and $L_i$ admit differentiable densities with support over an interval. We refer to random variables with this structure as *valid.* Any pair of valid random variables that satisfies the monotone likelihood ratio property is an admissible grading policy.[13] If an evaluator observes a transcript which is in the support of $H_i$ but not in the support of $L_i$, then the evaluator can correctly infer that the graduate receiving that transcript is high ability. Similarly, if an evaluator observes a transcript in the support of $L_i$ but not in the support of $H_i$, then it infers that the graduate must be low-ability. If the evaluator observes a transcript in the support of both $H_i$ and $L_i$, then some uncertainty remains about whether the graduate is high or low ability. Given the prior beliefs, the school grading policy, and the transcript realization, the evaluator's posterior belief about ability is determined by Bayes' rule.

This representation of a school's grading policy is quite general, and includes any possible system of grading that utilizes a finite number of letter grades; it also allows for more complex grading schemes such as assigning students a numerical value in the interval [0,100] or [0,4]. Frequently, actual transcripts are not limited to a numeric or letter score, as they typically provide a list of classes taken by semester, grades by class, and overall grade point average. Some schools also include class/grade distribution on transcripts (see Bar, Kadiyali and Zussman (2012)). This is perfectly consistent with our model, with the random variables $H_i$ and $L_i$ together with the prior determining the likelihood that each possible transcript is owned by a high-ability graduate. We can interpret the process of assigning grades as a process of disclosure, similar to the one in Lizzeri (1999). Following this interpretation, schools choose a grading policy (as described above) prior to learning the student's ability. They then observe student ability, and generate a transcript

[13] In this context the monotone likelihood ratio implies that transcripts are ordered in such a way that a greater transcript realization is associated with a greater posterior belief that the graduate is high ability; that is, higher transcripts always brings "good news" about graduate ability.

for their student in accordance with their promised grading policies.[14] Alternatively, we may interpret a school's grading policy as a signal of student ability, designed by the school. Following this interpretation, schools do not directly observe the ability of their students; rather, they subject their students to a "test" of their own choosing. The verifiable outcome of the test provides a signal about student ability. Choosing the design of the test, the schools control the informativeness of the signals they produce about their students. In this way, grading can be viewed as a process of Bayesian persuasion, as described by Kamenica and Gentzkow (2011).

In the third stage of the game, each school's graduate is evaluated, and the evaluator awards the prize. She makes this decision after observing the quality of each school, the grading policy at each school, and each graduate's realized transcript. If she awards the prize to a high-ability graduate, she receives payoff one. Otherwise, her payoff is zero.[15] A school would like its graduate to receive the prize, independent of ability. Successfully placing a graduate in graduate school or a prestigious firm gives a school a strong immediate benefit whether or not the graduate succeeds in the long term. Once the prize is awarded, the true type of the recipient is revealed and payoffs are realized.

# 4 EQUILIBRIUM WITH STRATEGIC GRADING

We solve for Perfect Bayesian Equilibria of the three stage game in which schools strategically invest in quality and then choose grading policies.

## 4.1 STAGE THREE: EVALUATION

In the third stage of the game, the evaluator's expected payoff of offering the prize to each graduate is equal to the posterior probability that the graduate is high ability. It is therefore sequentially rational for her to offer the position to the graduate who she believes is more-likely to be high-ability. If she holds the same beliefs about each graduate then she randomizes fairly between them, offering the prize to each with probability $\frac{1}{2}$.

## 4.2 STAGE TWO: GRADING

In the second stage of the game, each school designs a grading policy in order to maximize the probability that the evaluator offers its graduate the prize.

We first describe a representation of grading policies that considerably simplifies the analysis

[14] In this interpretation, student's true abilities are verifiable. In this way, an outsider, perhaps an accreditation agency, has the ability to monitor the school's grade assignments to ensure that it adheres to its stated disclosure rule. While the evaluator would like to demand that the school release graduate's *true* abilities in the second stage, we assume that a school's choice of a grading policy is binding.

[15] We assume that awarding the prize to a low-ability graduate dominates assigning the prize to no one. This assumption is not necessary for our results, but streamlines the exposition drastically. We also believe that this assumption is realistic in a variety of settings.

of the second stage of the game.[16] The evaluator's behavior is determined by her posterior belief about graduate ability. For a school, the only payoff-relevant aspect of a grading policy is the probability distribution over the evaluator's posterior belief, generated by the grading policy. We therefore represent any feasible grading policy $(H_i, L_i)$ by a single random variable $\Gamma_i$ from which the evaluator's posterior belief about a graduate's ability is drawn. To elaborate, suppose that given the prior belief about student ability, transcript realization $x$ from grading policy $(H_i, L_i)$ results in the evaluator having posterior belief $\gamma = Pr(t = h|x)$. Along with the prior, grading policy $(H_i, L_i)$ also determines the probability distribution of the graduate's transcript, which is itself a random variable $X$. Thus the prior belief and grading policy determine the *ex ante* distribution of the evaluator's posterior belief: $\Gamma_i = Pr(t = h|X)$, and this random variable captures all payoff-relevant aspects of the underlying grading policy. Random variable $\Gamma_i$ is valid, has support confined to the unit interval, and, according to the law of total expectation, has expectation equal to the prior, $q_i$. In the next Lemma, we show that these are the only restrictions on the *ex ante* posterior beliefs that can be generated by a grading policy.

**Lemma 4.1** *Consider any valid random variable $\Gamma_i$ with support confined to the unit interval and expectation $q_i$. If the prior belief that a student is high ability is $q_i$ then there exists a grading policy $(H_i, L_i)$ for which the ex ante posterior belief is $\Gamma_i$.*

This Lemma considerably simplifies the analysis. All payoff-relevant aspects of a grading policy are summarized by a single random variable, representing the *ex ante* distribution of the evaluator's posterior belief. The Lemma shows that any random variable with support inside $[0, 1]$ and mean equal to the prior is the *ex ante* posterior belief for some grading policy.[17] The analysis can therefore focus on an alternative version of our original game in which each school chooses $\Gamma_i$ rather than $(H_i, L_i)$, as long as $\Gamma_i$ is valid, has support in the unit interval, and expectation $q_i$. We refer to the choice of $\Gamma_i$ as a choice of a grading policy, although $\Gamma_i$ technically represents an entire payoff-equivalent class of grading policies.

In the third stage of the game, each school produces one graduate. The evaluator observes school investments, the realization of each graduate's transcript, and the grading policy at his school, and rationally updates her beliefs about the graduate's ability. This process generates realizations $(\gamma_\alpha, \gamma_\beta)$ of the posterior belief random variables $(\Gamma_\alpha, \Gamma_\beta)$. The evaluator then awards the prize to the graduate she believes is most qualified. That is, she selects the graduate with the higher realized $\gamma_i$. When $\gamma_\alpha = \gamma_\beta$, both graduates are selected with equal probability. Schools receive payoff 1 when their graduate is selected. The expected payoff to school $i$ when schools choose grading policies $(\Gamma_i, \Gamma_j)$ is therefore $Pr(\Gamma_i > \Gamma_j) + \frac{1}{2}Pr(\Gamma_i = \Gamma_j)$.[18]

Schools benefit when their graduates look good, generating high realized values of the posterior belief. If possible, a school would like $\Gamma_i$ to only result in high realizations of the posterior belief.

[16] We also use this type of representation in Boleslavsky and Cotton (2011) and present it again here for completeness. Kamenica and Gentzkow (2011) use a similar representation to study a general class of persuasion games.

[17] In fact, large class of grading policies.

[18] The second stage game is a special case of the one analyzed in Boleslavsky and Cotton (2011), and is also closely related to one analyzed in Conitzer and Wagman (2011).

However, the school is constrained by the expected ability of its student, determined by its investment in education, $q_i$. Because the evaluator rationally updates her beliefs about student ability, the law of iterated expectation requires that the expected value of the posterior belief generated by any feasible grading policy must be equal to the prior probability $q_i$. The freedom to choose a grading policy allows a school to strategically reveal or conceal information about the ability of its graduates, but it can not use its grading policy to make its graduates appear to be better, on average, than they truly are. In order to satisfy the constraint on the mean of the distribution of posterior beliefs imposed by the law of total probability (and Bayesian rationality), any probability mass on realizations above the prior belief must be offset by probability mass on realizations below the prior belief. If a school would like to reveal that its graduate is likely to be high ability some of the time, it must also reveal that its graduate is likely to be low ability some of the time.

Both *fully-revealing* and *uninformative* grading policies are always available to the school. A fully-revealing policy always assigns different sets of transcripts to high- and low-ability graduates (i.e., there is no overlap in the support of $H_i$ and $L_i$). Any fully-revealing grading policy is associated with a Bernoulli distribution for the posterior belief, $\Gamma_i$:

$$Pr(\Gamma_i = 1) = q_i$$
$$Pr(\Gamma_i = 0) = 1 - q_i$$

At the opposite extreme, a school may assign transcripts independent of ability. Such a grading policy is consistent with rampant grade inflation in which all students receive As.[19] In this case, the evaluator learns nothing about student ability by observing their transcripts, and her beliefs about ability are based only on the quality of the graduate's alma mater, $q_i$. Any uninformative grading policy is associated with a constant value of $\Gamma_i$

$$Pr(\Gamma_i = q_i) = 1$$

Schools also have the ability to choose grading policies between these extremes. For example, suppose that a school begins with a fully-revealing grading policy. With probability $q_i$ the evaluator will draw posterior belief $\gamma = 1$, and will be convinced that the graduate is high-ability for certain; with complementary probability, the employer will learn that the graduate is low ability for certain. Suppose that the school decides that it is revealing its graduate to be low-ability too often. The school therefore reduces the probability mass on realization $\gamma = 0$, spreading some of the probability mass from $\gamma = 0$ (the worst realization possible), toward higher realizations in the interior of the unit interval that have the potential to win more often. Of course in order to preserve the ex ante mean of the distribution (and be consistent with Bayes' rule) this upward shift in probability mass must also be offset by a corresponding downward shift in mass elsewhere in the distribution.[20] The

[19] It is also consistent with an outright ban on grades, as transcripts contain no information about graduate ability.

[20] A school accomplishes these shifts in probability mass throughout the distribution of posterior beliefs by sometimes assigning certain transcripts to both high and low ability graduates, and controlling the mix of high and low ability graduates to which this transcript is assigned. Increasing the likelihood that a high ability student receives

net result is a new distribution for the posterior belief that is more concentrated around its ex ante mean, $q_i$. The evaluator's posterior belief is therefore less-likely to be significantly different than her prior belief, and the grading policy is less Blackwell-informative.[21]

Suppose one of the schools, which we refer to as $A$, invests more in school quality than the other school, which we refer to as $B$. That is, $q_b \leq q_a$. Because school $A$ has invested more, we refer to $A$ as the high-quality school.[22] The following Lemma characterizes the Nash equilibrium of the second stage game, for each possible combination $(q_a, q_b)$.

**Lemma 4.2** *Strategic Grading Equilibrium. The unique Nash equilibrium of the grading stage is given by the following combination of grading policies* $(\Gamma_a, \Gamma_b)$*:*

- *When* $q_a \leq \frac{1}{2}$*:*

$$\Gamma_a = U[0, 2q_a]$$

$$\Gamma_b = \begin{cases} 0 & \textit{with probability } 1 - \frac{q_b}{q_a} \\ U[0, 2q_a] & \textit{with probability } \frac{q_b}{q_a} \end{cases}$$

- *When* $q_a > \frac{1}{2}$*:*

$$\Gamma_a = \begin{cases} U[0, 2(1-q_a)] & \textit{with probability } \frac{1}{q_a} - 1 \\ 1 & \textit{with probability } 2 - \frac{1}{q_a} \end{cases}$$

$$\Gamma_b = \begin{cases} 0 & \textit{with probability } 1 - \frac{q_b}{q_a} \\ U[0, 2(1-q_a)] & \textit{with probability } \frac{q_b}{q_a}(\frac{1}{q_a} - 1) \\ 1 & \textit{with probability } \frac{q_b}{q_a}(2 - \frac{1}{q_a}). \end{cases}$$

When $q_a \leq \frac{1}{2}$, both schools are more-likely to produce a low-ability graduate than a high-ability graduate. In this situation, the school with the quality advantage chooses a grading policy that leaves the evaluator less-than-fully informed about the quality of its graduate, employing a grading policy that generates a uniform distribution over posterior beliefs centered on the prior. In equilibrium, the lower quality school adopts a grading policy that mimics the grading policy of the advantaged school with one exception: in order to maintain $E[\Gamma_b] = q_b$, school $B$ sometimes issues a transcript that reveals (for certain) that its graduate is low ability. Any transcript issued by the high-quality school provides a noisy signal about student ability, never completely revealing whether a graduate is high or low quality. The low-quality school would like to neutralize school $A$ initial advantage by mimicking its grading policy exactly, but can not because Bayesian rationality on the part of the evaluator means that the *ex ante* expected quality at school $B$ must be lower.

the transcript improves evaluator beliefs about all students who receive it.

[21]See Ganuza and Penalva (2010).

[22]As one would expect, when we derive the equilibrium of the investment stage, the advantaged school $\alpha$ will choose to invest more in education, and will play the role of school $A$ on the equilibrium path.

The low-quality school responds to this by sometimes issuing a low-ability student a transcript that fully reveals his type, but otherwise utilizing the same grading policy as the high quality school.[23]

When $q_a > \frac{1}{2}$, the higher-quality school is more likely to produce a high-ability graduate than a low-ability graduate. In this case, the higher-quality school sometimes reveals that its graduate is high-ability, but never reveals that its graduate is low-ability. To do this, the school reserves some transcripts (i.e., 4.0 GPA, Honors Program, etc.) for only high-ability graduates. All other transcripts are assigned to a mix of both high- and low-ability graduates. The lower-quality school again responds by sometimes revealing when its graduate is low ability, but otherwise mimicking the posterior belief distribution (and underlying grading policy) of the other school.

Both schools' grading policies exhibit "compression" at the top of the transcript distribution. It is argued (see Cizek (1996)) that grade compression is a natural consequence of grade inflation. Because no grade higher than an A exists, as schools assign higher grades to their graduates, "As remain As, but Bs become As, Cs become Bs, and so on. The result is that it takes less to achieve an A." Thus, when schools inflate their grades, the inference that the evaluator draws from good transcripts becomes worse; when lots of low ability students are assigned good transcripts, seeing a good transcript does not leave the evaluator as optimistic. However, because it is low ability students being assigned higher grades, relatively low grades still convey the same negative information as they did before, though they are less-likely to be assigned. The equilibrium we find conforms to this pattern. In each equilibrium of the game, the worst transcript possible conveys to evaluators that the student is low-ability. The school with a lower initial investment assigns this transcript with a positive probability. While this realization is in the support of the high-investment school's assignment policy, it does not assign this transcript with positive probability. Thus in all equilibria the worst grade still conveys that the graduate is low ability, but it is assigned less often by both schools than under fully-revealing grades. At the top of the distribution, things are a bit more subtle. In the first type of equilibrium, the best belief in the support of each school's posterior belief distribution is less than one. This means that the best transcript possible might be assigned to the low ability student.[24] In the second type of equilibrium, both schools reserve the best-possible transcripts for the high-ability students. The best transcripts are therefore "uncompressed." However, the inference that an evaluator draws from the "second-best transcript"[25] becomes worse; while the best transcript is uncompressed in this equilibrium, compression is magnified for the "second-best" transcript.

The main properties of the second stage equilibrium are summarized in the following proposition.

**Proposition 4.3** *If school A invests more than school B (i.e., $q_b \leq q_a$), then in equilibrium of the second stage:*

[23] In the appendix we argue that this mimicry exists not only on the level of the posterior distribution, but also exists on the underlying grading policy. In order to achieve the equilibrium posterior belief distribution school $B$ can use a grading policy $(H_b, L_b)$ that is identical to school $A$'s, $(H_a, L_a)$ except that $L_b$ contains a special realization (or set of realizations) that is issued only to low ability students.

[24] Because even the best transcript is in the support of $L_i$, the evaluator can not be sure that a student with the best possible transcript is really high ability.

[25] The best transcript that does not reveal that the student is high ability for certain

- *No equilibrium exists in which either school uses a fully-revealing grading policy.*
- *School $A$ does not reserve any transcripts realizations for low-ability students. All transcripts assigned to low ability students could also be assigned to high-ability students.*
- *School $B$ reserves certain transcript realizations for low-ability students, but otherwise uses a grading policy that generates the same distribution over posterior beliefs as school $A$'s.*
- *The grading policy at school $B$ is more Blackwell informative than the grading policy at school $A$.*

Consistent with previous literature on grade assignment, we find that fully-revealing grades are not part of an equilibrium. Furthermore, consistent with previous literature and conventional wisdom, we find that schools whose students are less-likely to be high ability use grading policies that are more informative.

In the next section, we consider the interaction between grading policy and a school's initial investment in student ability. In order to facilitate this analysis, we summarize and discuss the equilibrium payoffs for the players.

Given $q_a, q_b$, the expected payoff to the evaluator is

$$u_e(q_a, q_b) = \begin{cases} q_a + \frac{1}{3}q_b & \text{when } q_a \leq \frac{1}{2} \\ (3q_a^4 - 7q_a^3 q_b + 12q_a^2 q_b - 6q_a q_b + q_b)/(3q_a^3) & \text{when } q_a > \frac{1}{2}, \end{cases}$$

and the expected payoffs of the two schools are

$$u_a(q_a, q_b) = 1 - \frac{q_b}{2q_a} \quad \text{and} \quad u_b(q_a, q_b) = \frac{q_b}{2q_a}.$$

As expected, school $A$ has a payoff advantage in equilibrium. The structure of the equilibrium reveals the source of this advantage. If school $B$ were able to perfectly mimic school $A$'s posterior belief distribution, it could completely neutralize school $A$'s initial advantage, guaranteeing a payoff of $\frac{1}{2}$ for both schools. However, because the prior belief about ability is lower at school $B$, school $A$'s posterior belief distribution is not feasible. In order to stay competitive, school $B$ sometimes reveals that its graduate is low ability, which allows $B$ to plausibly mimic $A$'s posterior belief distribution the rest of the time. Conditional on $B$ not revealing its graduate to be bad, each school acts in an identical way, and therefore both schools expect the same payoff. However, if school $B$ reveals that its graduate is bad, graduate $A$ is assigned the prize for certain.[26] This happens with probability $1 - \frac{q_b}{q_a}$. Therefore, the greater the investment gap between schools, the more often $B$ is forced to reveal that its graduate is low-ability. School $B$ has an incentive to close this gap in order to keep from having to reveal its graduate to be bad; meanwhile school $A$ has an incentive to widen the gap, forcing school $B$ to reveal that its graduate is low ability more often.

[26] This effect can be clearly seen by decomposing the payoff functions for the schools in the following way$u_a(q_a, q_b) = \frac{q_b}{q_a}(\frac{1}{2}) + (1 - \frac{q_b}{q_a})$ and $u_b(q_a, q_b) = \frac{q_b}{q_a}(\frac{1}{2})$

### 4.3 STAGE ONE: INVESTMENT IN SCHOOL QUALITY

In the first stage of the game, schools simultaneously invest in education quality, which affects the expected ability of their graduate. Remember, the $\alpha$ school has a lower marginal cost of improving education quality compared to the $\beta$ school; that is, $0 < \rho_\beta \leq \rho_\alpha$.

Anticipating the second period equilibrium grading policies, in the first stage of the game each school $i \in \{\alpha, \beta\}$ expects payoff $u_i(q_\alpha, q_\beta)$ from quality investments $q_\alpha$ and $q_\beta$, where

$$u_i(q_\alpha, q_\beta) = \begin{cases} \frac{q_i}{2q_j} - \frac{q_i^2}{\rho_i^2} & \text{if} \quad q_i \leq q_j \\ 1 - \frac{q_j}{2q_i} - \frac{q_i^2}{\rho_i^2} & \text{if} \quad q_i > q_j. \end{cases}$$

This function is differentiable, continuous, and concave in $q_i$. The schools simultaneously choose $q_i$ and $q_j$ to maximize their expected payoff. In equilibrium, they choose

$$q_\alpha = \frac{\sqrt{\rho_\alpha \rho_\beta}}{2} \quad \text{and} \quad q_\beta = \frac{\rho_\beta^2}{2\sqrt{\rho_\alpha \rho_\beta}}.$$

If $\rho_\beta$ increases, then the difference between $\rho_\alpha$ and $\rho_\beta$ decreases, the competition between the two schools becomes more intense, and the schools both respond by increasing investment in education quality. A decrease in $\rho_\alpha$ has a similar effect, causing the disadvantaged school to investment more in education quality. However, the increase in the costs of investment for the advantaged school more than offsets the increased competitive pressure, and the advantaged school decreases investment in education quality as $\rho_\alpha$ decreases.

Given these choices, the expected payoff for the evaluator is

$$u_e = \begin{cases} (3\rho_\alpha + \rho_\beta)\sqrt{\rho_\alpha \rho_\beta}/(6\rho_\alpha) & \text{if} \quad \rho_\alpha \rho_\beta \leq 1 \\ ((3\rho_\alpha^2 - 7\rho_\alpha \rho_\beta - 24)\sqrt{\rho_\alpha \rho_\beta} + 24\rho_\alpha \rho_\beta + 8)/(6\rho_\alpha^2) & \text{if} \quad \rho_\alpha \rho_\beta > 1. \end{cases}$$

Expected aggregate graduate ability is

$$q_\alpha + q_\beta = \frac{\rho_\alpha \rho_\beta + \rho_\beta^2}{2\sqrt{\rho_\alpha \rho_\beta}}.$$

We will return to these results when we compare the outcome under strategic grading with the fully-revealing benchmark, to which we turn in the next section.

## 5 FULLY-REVEALING BENCHMARK

In this section we consider a benchmark in which uncertainty about graduate ability is completely resolved during evaluation. This corresponds to an environment in which grade inflation has been eliminated. The fully-revealing benchmark may result from a rule put into place by university accreditation agencies, requiring that member institutions adhere to certain grading standards. It may also be a consequence of entrance or licensing exams given to all graduates applying for a position. Entrance exams for undergraduate and graduate study, as well as industry licenses and board certifications, all provide evaluators with independent assessments about graduate ability. The more comprehensive the exam, the greater its potential to distinguish students' abilities. In principle, such exams may be sufficiently informative to fully reveal student types.

If the evaluator knows his ability, then each graduate can win in one of two ways. When the realized abilities of the graduates are the same, each graduate wins the prize half the time. When a graduate is high ability and his competitor is low ability, he wins the prize for certain. School $i$'s expected payoff given school quality investments $q_\alpha$ and $q_\beta$ is

$$u_i(q_\alpha, q_\beta) = q_i(1 - q_j) + \frac{1}{2}(q_i q_j + (1 - q_i)(1 - q_j)) - \frac{q_i^2}{\rho_i^2}.$$

$$u_i(q_\alpha, q_\beta) = \frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho_i^2}.$$

As is evident from the above expression, the marginal benefit of improving quality for either school is $\frac{1}{2}$, independent of the other school's investment. By marginally improving its quality, school $i$ slightly increases the probability of generating a high ability graduate and reduces the probability of generating a low ability graduate. If the other school's graduate turns out to have low ability, this increase reduces the probability of ties (worth $\frac{1}{2}$), and increases the probability of winning the prize (worth 1), resulting in a net benefit of $1 - \frac{1}{2} = \frac{1}{2}$. If the other school's graduate turns out to be high ability, then the increased investment reduces the probability of losses (worth 0), and increases the probability of a tie (worth $\frac{1}{2}$), resulting in a net benefit of $\frac{1}{2} - 0 = \frac{1}{2}$. Thus, under fully-revealing grading, the marginal benefit of improving school quality is fixed. It is therefore not surprising that when schools choose investment in stage one, each school has a dominant strategy:

$$q_\alpha = \frac{\rho_\alpha^2}{4} \text{ and } q_\beta = \frac{\rho_\beta^2}{4}.$$

Given the investment in quality by both schools, the equilibrium expected payoff is

$$u_i(q_\alpha, q_\beta) = \frac{1}{2}(1 + \frac{\rho_i^2 - \rho_j^2}{4})$$

for school $i \in \{\alpha, \beta\}$ and

$$u_e(\rho_\alpha, \rho_\beta) = 1 - (1 - q_\alpha)(1 - q_\beta) = \frac{\rho_\alpha^2 + \rho_\beta^2}{4} - \frac{\rho_\alpha^2 \rho_\beta^2}{16}$$

for the evaluator. Expected aggregate student ability equals

$$q_\alpha + q_\beta = \frac{\rho_\alpha^2 + \rho_\beta^2}{4}.$$

In the next section we use these results to compare educational outcome and payoffs under strategic grading and the fully-revealing benchmark.

# 6 THE BENEFITS OF STRATEGIC GRADING

In this section, we compare outcomes in the game when schools strategically choose grading policies to the outcomes in the fully-revealing benchmark. First, we make this comparison while treating school quality as exogenous. This is a standard assumption in the literature on grade inflation, and in this case we find results consistent with the literature. Requiring fully-revealing grades makes the evaluator better off compared to strategic grading.

We then compare outcomes when school quality is endogenous. Here, we find that allowing strategic grading brings surprising benefits: when schools grade strategically they often invest more to improve education quality. This benefit also may reverse the evaluator's welfare ranking; with endogenous investment the evaluator can prefer strategic grading to fully-revealing grading. Even if she had the capability to eliminate grade inflation (and other noisy grading policies), it may not be in her interest to do so.

## 6.1 EXOGENOUS SCHOOL QUALITY

The first comparison takes quality differences between the schools as fixed.

**Proposition 6.1** *For exogenous school qualities, $q_b \leq q_a$, requiring fully-revealing grading policies rather than strategic grading*

- *Benefits the evaluator and school $B$.*
- *Hurts school $A$.*

Proposition 6.1 shows that when qualities are exogenous, the higher-quality school is made worse off by a requirement that schools adhere to fully-informative grading policies, while the lower-quality school is made better off. Under fully-revealing grading, school $B$ is always competitive with school $A$. If both schools produce low ability graduates, for example, each graduate is equally-likely to be awarded the prize. In the strategic grading equilibrium, however, school $B$ is forced to reveal that it has a low-ability graduate some of the time, while school $A$ never reveals that its graduate is low ability. If school $B$ reveals a low-ability graduate, then school $A$'s graduate is selected for certain. Thus, under strategic grading school $B$ effectively concedes the prize to school $A$ some of the time, which never happens when transcripts are fully-revealing. Therefore, school $A$ loses and school $B$ benefits from fully-revealing transcripts.

The evaluator clearly benefits from fully-revealing grading policies for fixed school investments. The evaluator cares only about assigning the prize to a high-ability graduate. There are therefore two ways in which the evaluator can be made better off: (i) if schools invest more, improving the probability of generating high-ability graduates, and (ii) if schools select more-informative grading policies, improving the evaluator's ability to identify and select high-ability graduates. Because the first means of improving evaluator welfare is absent (i.e. school quality is fixed) the evaluator benefits whenever grading policies are more-informative.

### 6.2 ENDOGENOUS SCHOOL QUALITY

A link exists between strategic grading and investment in education quality. Allowing strategic grading with endogenous investment brings confounding effects: investment increases as grading informativeness decreases. Often the benefits of educational investment more than offset the detrimental effects associated with less-informative grading policies.

Previously we determined equilibrium school investment in two different situations: when schools are allowed to design any grading policy that they choose, and when schools are required to use fully-revealing grading policies. We are now ready to compare the equilibrium outcomes of the three-stage game in these two circumstances. Our first main result concerns school investment:

**Proposition 6.2** *Requiring schools to use fully-revealing grading policies rather than strategic grading*

- *always decreases investment in school quality by the disadvantaged school*
- *decreases investment in school quality by the advantaged school if and only if* $\rho_\beta \in (\frac{1}{4}\rho_\alpha^3, \rho_\alpha)$,
- *decreases average school quality (and expected aggregate student ability) if and only if* $\rho_\beta \in (\bar{r}, \rho_\alpha)$, *where* $\bar{r} \leq \frac{1}{4}\rho_\alpha^3$.

Schools often invest more in quality when they grade strategically. To develop intuition for this result, recall that with fully-revealing grading the marginal benefit of increased investment for either school is fixed and equal to $\frac{1}{2}$.[27] With strategic grading, however, the marginal benefit of investment is not fixed at $\frac{1}{2}$ for either school. As discussed previously, by closing the investment gap, the lower quality school, $B$, reduces the probability of revealing that its graduate is low ability, contesting the prize more-often. In equilibrium the prize is contested with probability equal to $\frac{q_b}{q_a}$. If it does not reveal a low ability graduate, school $B$ mimics the grading policy of school $A$, giving it an expected payoff of $\frac{1}{2}$ in this circumstance. Thus for school $B$ the marginal benefit of increased investment in quality is $\frac{1}{2q_a}$. This marginal benefit of investment is fixed (because the probability of contesting the allocation of the prize is linear in $q_b$), and is greater than the marginal benefit of investment with fully-revealing grades. The disadvantaged school, $\beta$, therefore always invests more with unrestricted strategic grading than with fully-revealing grading, in order to avoid conceding the prize to $\alpha$ as often. Conversely, school $A$ benefits from widening the investment gap by forcing school $B$ to concede the prize more-often in equilibrium. However, the probability that school $B$ concedes the prize, $1-\frac{q_b}{q_a}$ depends on $q_a$ in a concave way. The marginal benefit of increasing quality for school $A$, $\frac{q_b}{2q_a^2}$ is diminishing. Thus, depending on the marginal costs of increasing quality, school

[27] As discussed in Section 5, marginally increasing investment results causes fewer ties in which both schools produce low-ability graduates and allows the investing school to win more often, and also causes more ties in which both schools produce high ability graduates, creating ties in situations in which the school would have lost for certain.

$\alpha$ may choose either higher or lower investment when grading is unrestricted. If the schools are similar *ex ante*, then competition is most-fierce, and both schools invest more in education quality.

We have already shown that for fixed levels of school quality, the evaluator is worse off when schools choose strategic rather than fully-informative grading policies since she is less-able to determine a graduate's ability from observing his transcript. When schools invest in quality, however, strategic grading may motivate schools to invest more in education quality, increasing the probability that graduates are high-ability. Although strategic grading makes the evaluator less able to determine each graduate's ability, because of the increased investment, the evaluator may, overall, be better off. It is a simple matter to check that whenever $\rho_\alpha = \rho_\beta$ the evaluator prefers strategic grading to the fully-revealing benchmark. By continuity, a region exists around the diagonal in which this result holds. This brings us to our second main result:

**Proposition 6.3** *Requiring schools to assign fully-revealing grades rather than engage in strategic grading hurts the evaluator whenever the initial asymmetry between schools is not too large. For each value of $\rho_\alpha$ there exists a value $\bar{r} < \rho_\alpha$ such that, if $\rho_\beta \in (\bar{r}, \rho_\alpha)$, then the evaluator equilibrium payoff is lower when grading policies are required to be fully-revealing.*

Allowing schools the freedom to grade strategically can benefit the evaluator. Eliminating grade inflation can decrease both the average ability of graduating students, and evaluator payoffs.

## 7 UNINFORMATIVE GRADING

In equilibrium of the grading game, schools strategically choose grading policies which are neither fully-revealing nor uninformative. The previous analysis shows that allowing strategic, rather than fully-revealing, grading may lead to better outcomes, improving aggregate student ability and evaluator welfare. Here, we compare the two most-extreme grading policies, showing that uninformative grades (or equivalently, committing to ignore or banning grades) can lead to better outcomes than fully-revealing grades.[28]

In the absence of grades, the evaluator assigns the job to the graduate who is more-likely to be high-ability given her prior belief. Thus, the evaluator assigns the job to the graduate of the school that invests more in education. The game between schools is therefore a full-information all-pay auction with asymmetric convex costs. In the absence of grades, competition over investment in education quality is most significant. Fierce competition can lead to the highest expected ability of graduates. Although the evaluator cannot observe any information about realized graduate ability, the fierce quality competition between schools causes them to produce high-ability graduates more often, frequently giving the evaluator a higher expected payoff.

We formally solve for the equilibrium in the appendix. The following proposition summarizes the results.

[28] This analysis also applies to a situation in which grades have become so inflated as to be completely uninformative about student ability. These results do show that even in the worst-case scenario, rampant grade inflation and uninformative grades, combatting grade inflation may still be worse than permitting it.

**Proposition 7.1** *When $\rho_\beta$ is sufficiently large, uninformative grading results in higher expected aggregate student ability and higher evaluator payoffs than fully-revealing grading.*

## 8 TWO GRADE, INFLATION-ONLY MODEL

In equilibrium of our general game, schools do not choose fully-revealing grading policies. At least some transcripts that schools issue go to both high- and low-ability graduates, preventing the evaluator from perfectly inferring the type of graduate if she observes such a transcript. The analysis shows that the less-than-fully-informative equilibrium grading policies lead to potentially suboptimal selection decisions by the evaluator who is no longer certain of graduate types; however, these grading policies also lead to increased school investment in education quality, making it more likely that graduates are high quality. Furthermore, the equilibrium grading policy exhibits grade compression, frequently associated with grade inflation. Because of the generality of the model, the equilibrium that we derive in the previous sections can be associated with a variety of underlying grading policies $(H_i, L_i)$, including both inflationary and deflationary policies. The policy discussion focuses primarily on issues of grade inflation and compression; here we present a variant of our model in which all noise in grading is due exclusively to the inflation of low-ability student grades.

We show that our qualitative results all continue to hold if we focus on a narrow class of grading policies in which schools assign only two grades and explicitly choose their level of grade inflation. Additionally, the more-coarse grading structure leads to some results that were not present in the general analysis, including identifying instances in which uninformative grading takes place in equilibrium.

### 8.1 A SIMPLE MODEL OF EXPLICIT GRADE INFLATION

Here, we present a version of our model in which schools issue only two grades: a "good" grade $G$, and a "bad" grade $B$. To focus on grade inflation exclusively, the analysis assumes that a high-ability student always receives a good grade. A school can award all low-ability students bad grades (i.e. use a fully-revealing policy). However, it can also choose to inflate grades by awarding some portion of low-ability students good grades. We refer to this as the "two grade, inflation only game" and summarize the analysis in this section. Details are provided in the appendix.

If the prior belief about student ability at school $i$ is $q_i$ and the school assigns a good grade to low ability students with probability $\theta_i$, then the overall probability that a graduate receives a good transcript is

$$p_i = q_i + \theta_i(1 - q_i).$$

If a graduate of school $i$ receives grade $G$, then the evaluator believes that he is high-ability with probability

$$g_i = \frac{q_i}{q_i + \theta_i(1 - q_i)} = \frac{q_i}{p_i}.$$

Because the bad grade is only assigned to low ability graduates, observing grade $B$ the evaluator always concludes that the graduate is low ability.

Regardless of how many low ability students are assigned the good grade, observing the bad grade reveals that a student is low ability for certain. By assigning the good grade to low ability students more-often, the school increases the probability its graduate receives the good grade but reduces the evaluator's posterior belief about those students that receive the good grade. Thus, the decision to inflate grades involves a tradeoff between making graduates "look good" more often, and *how good* these graduates look to the evaluator.

As in the previous section, we represent a grading policy as a random variable associated with the evaluator's ex ante posterior belief. Following our previous approach, each grading policy is associated with a random variable $\Gamma$, the ex ante distribution of the posterior belief about student quality. However, unlike the previous section in which schools could choose any random variable (i.e., any grading policy) that was consistent with Bayesian rationality, here the random variable must be selected from a particular parametric class:

$$Pr(\Gamma_i = g_i) = p_i \;\text{ and }\; Pr(\Gamma = 0) = 1 - p_i.$$

Given school quality $q_i$, the random variable for the posterior belief is completely determined by the choice of $\theta_i$. We treat the choice of $\theta_i \in [0,1]$ as the school's grading strategy, with $\theta_i = 0$ representing a fully-revealing grading policy and $\theta_i = 1$ representing a completely-inflated (uninformative) grading policy.

The timing is unchanged from the earlier sections: in stage one schools simultaneously invest in educating students. This investment determines $q_i$, the prior probability that the school's graduate is high ability. Next, schools simultaneously design inflationary grading policies, by each choosing $\theta_i \in [0,1]$. This choice induces a random variable for the posterior belief about quality at each school, as described above. As before, grading policies are observed by the evaluator at the evaluation stage. In this stage, the posterior beliefs are realized, and the evaluator offers the position to the graduate with the higher posterior belief realization (breaking ties with equal probability). In the second stage, each school's goal is therefore to choose $\theta_i$ to maximize the probability that its graduate generates the higher realization. The payoff function for each school is therefore

$$u_i(\theta_i, \theta_j | q_i, q_j) = \begin{cases} p_i(1-p_j) + \frac{1}{2}(1-p_i)(1-p_j) & \text{if} \quad g_i < g_j \\ p_i + \frac{1}{2}(1-p_i)(1-p_j) & \text{if} \quad g_i > g_j \\ p_i(1-\frac{1}{2}p_j) + \frac{1}{2}(1-p_i)(1-p_j) & \text{if} \quad g_i = g_j \end{cases}$$

Both schools are equally likely to have their graduate selected when both generate bad transcript realizations. However, by choosing a lower value of $\theta_i$ (that is, by inflating grades less than the other school), a school "takes priority" when the evaluator allocates the position: if its graduate is assigned a $G$, he will be awarded the position. The cost of this priority is a reduction in the probability of generating a good transcript.

Below, we describe the structure of the equilibrium of the second stage subgame for any combination of initial investment in school quality, $q_a \geq q_b$. For some parameter cases, one pure strategy equilibrium exists, and this equilibrium is fully-revealing. Otherwise, each school plays a mixed strategy; at the time it chooses its grading policy, a school cannot perfectly anticipate the other school's level of grade inflation. It is important to point out, however, that once each school chooses its level of grade inflation, these are observed by the evaluator prior to awarding the prize.

In the two-grade, inflation-only game, the unique Nash equilibrium of the second stage game depends on the difference between $q_a$ and $q_b$. There are three cases to consider:

**Case I**: $q_b \leq \frac{1}{2}(1 - q_a)$. The lower quality school chooses $\theta_b$ according to a mixed strategy. The support for $\theta_b$ is always within $(0, 1)$. This means that the grading policy at the lower-quality school always inflates the grades of some—but not all—of its low-ability students. Grade inflation is more extreme at the higher-quality school. With positive probability, the higher-quality school chooses $\theta_a = 1$, the completely inflated (uninformative) grading policy. With complementary probability, it chooses $\theta_a$ utilizing the same mixed strategy as the lower-quality school.

**Case II**: $\frac{1}{2}(1 - q_a) \leq q_b \leq 1 - q_a$. Here, the lower-quality school chooses a mixed strategy with a mass point at $\theta_b = 0$, and support over an interval inside $(0, 1)$. As was also true in the previous case, the higher-quality school sometimes chooses $\theta_a = 1$, but otherwise mixes according to the same distribution as the lower-quality school. The primary difference in the grading policies here as opposed to in Case I is that each school sometimes implements a fully-revealing grading policy.

**Case III**: $q_b \geq 1 - q_a$. This case requires $1/2 < q_a$ and for the differences in quality between the two schools to be relatively small. In this case, neither school inflates grades. Both schools choose fully revealing grading policies, $\theta_a = \theta_b = 0$.

Consistent with the general results derived earlier the paper, we find that equilibrium grading policies at both schools often involve grade inflation. Whenever $q_b < 1 - q_a$ each school utilizes an inflationary grading policy with positive probability; if $q_b \leq \frac{1}{2}(1 - q_a)$ each school utilizes an inflationary grading policy with probability 1. Furthermore, whenever grade inflation may take place in equilibrium ($q_b < 1 - q_a$) grade inflation is more-prevalent at the high-quality school. The high-quality school completely inflates grades with positive probability, issuing all low ability students grade $G$. Otherwise it uses the same mixed strategy over inflation levels as school $B$. Compared to the game with general grading policies, two novel phenomena emerge. First, if investment at both schools is relatively high, then in equilibrium both schools use fully-revealing grading policies. Second, when the fully-revealing equilibrium does not exist, the high quality school may completely inflate grades, setting $\theta_a = 1$.

In order to analyze the period one school investment choices, we present the expected payoff of each school in this equilibrium. If $1 - q_a < q_b$ the payoff functions are the fully-revealing payoff function

$$u_a(q_a, q_b) = \frac{1}{2}(1 + q_a - q_b) \text{ and } u_b(q_a, q_b) = \frac{1}{2}(1 - q_a + q_b).$$

If $q_b \leq 1 - q_a$ then the equilibrium payoff functions are simply

$$u_a(q_a, q_b) = \frac{q_a}{q_a + q_b} \text{ and } u_b(q_a, q_b) = \frac{q_b}{q_a + q_b}.$$

Although it is not the focus of this paper, it is remarkable to note that our model provides a micro-foundation for the often-used contest success functions proposed by Tullock (1980).

We now turn to the investment decision in stage one. To streamline the exposition we focus on the case of schools with identical, and relatively high marginal costs of education, $\rho_\alpha = \rho_\beta = \rho \leq \sqrt{2}$. At the investment stage, anticipating the stage two equilibrium, the payoff function for school $i$ is

$$u_i(q_i, q_j) = \begin{cases} \frac{q_i}{q_i+q_j} - \frac{q_i^2}{\rho^2} & \text{if} \quad q_i \leq 1 - q_j \\ \frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho^2} & \text{if} \quad q_i > 1 - q_j \end{cases}$$

This payoff function is continuous, and is differentiable, everywhere except for possibly $q_i = q_j$. The Nash equilibrium of the investment stage is given in the following lemma.

**Lemma 8.1** *The Nash equilibrium of the first stage game is* $q_i = q_j = \frac{\sqrt{2}}{4}\rho$. *On the equilibrium path, grade inflation takes place at both schools with positive probability.*

As in the general framework, investment is higher when grade inflation is permitted. Furthermore, even though for any fixed investment levels the evaluator always prefers a fully-revealing signal at stage two, when effort is endogenous, the increased effort reverses this ranking. The evaluator is always weakly better off when grade inflation is permitted than when it is banned.

**Proposition 8.2** *In the two-grade, inflation-only game, allowing grade inflation can increase investment in school quality, evaluator payoffs and aggregate graduate ability. Eliminating grade inflation can decrease school quality, evaluator payoffs and aggregate graduate ability.*

## 9 ON THE MODEL'S ASSUMPTIONS

In this section we discuss some of the assumptions of the model in more detail.

### 9.1 COMPETITIVE STRUCTURE

Our model includes a single evaluator assigning a single prize, and one student per school. The framework incorporates the simplest assortative matching of graduates to prizes, allowing us to maintain tractability as we generalize other aspects of the analysis. While the interaction of matching and grade inflation is the primary focus of other papers in the literature, our focus is different. We simplify the matching aspect of the problem, in order to focus on the interaction of grading and investment.

While it may be possible to incorporate assortative matching of $N$ students and $M$ prizes into the analysis, doing so is beyond the scope of this analysis. However, if we consider *random* matching between graduates of each school and evaluators, then our results apply directly to situations with any number of graduates and evaluators.[29] We find random matching to be an intuitive assumption in certain situations. For example, imagine that a number of equally prestigious law firms operate in different cities throughout the country. If the substantive dimensions of potential job offers are more-or-less equivalent at each firm, a graduate's application and employment preference may be strongly influenced by their location preference. If these preferences are uncorrelated with the graduate's transcript and ability, then matching between graduates and evaluators is essentially random. Similarly, it is not difficult to imagine that (for a certain subset of) high school graduates, college preference is strongly influenced by idiosyncratic tastes. Fraternity and sorority cultures, the characteristics of student bodies, location, and variety of courses or majors offered all influence the graduate's college application decision. To the extent that these factors are independent of ability and transcript, matching is effectively random.

### 9.2 OBSERVABLE GRADING POLICY

Our analysis focuses on a situation in which an evaluator is able to observe a school's grading policy directly. Equivalent results follow if a school's grading policy is verifiable, but not observable.[30] We believe that this assumption is reasonable because evaluators frequently have the capability to learn about grading policies at schools in a variety of ways. By collecting information about course content and "rigor," as well as grade distributions, the evaluator can learn about a school's grading policy. In some cases, schools directly supply information about grading policies to potential evaluators, including statistics on graduate's transcripts.[31]

Without this assumption, the grading game unravels. This is easiest to see in the context of the binary, inflation-only model analyzed in Section 8. If the evaluator is unable to observe the probability that a low ability student is assigned grade $G$, she must infer it from the school's equilibrium behavior. Suppose that the evaluator anticipates that a school assigns grade $G$ to a low-ability student with probability $\theta$, but can not observe the value of $\theta$ directly. If she sees grade $G$, the value of her posterior belief $g$ is determined by her prior belief, and the value of $\theta$

[29] If students from each school are matched randomly to compete for a prize, then the school's goal is to maximize the probability that a randomly selected graduate generates a posterior belief, higher than the posterior beief generted by the other graduate. Thus, the single graduate in the model presented in the paper, could be interpreted as a school's randomly chosen graduate.

[30] If a school's grading policy is verifiable, but not observable, the school could potentially try to avoid disclosing its grading policy to the evaluator. If the evaluator believes that whenever a school does withold its grading policy, the graduate's transcript would reveal him to be low ability for certain, then a school would never withold its grading policy from the evaluator (except perhaps in this instance).

[31] As part of an assessment of its grading policies, the reviewers recommended that The University of North Carolina, Chapel Hill put the following clarifying quotation on its transcripts "The University of North Carolina at Chapel Hill strictly monitors its grading system in order to insure fairness and consistency both across units and over time. Therefore, the grades on this transcript reflect an overall grade average of 2.6-2.7. Special care should be taken in comparing grades on this transcript with grades from colleges and universities that have not controlled grade inflation. See the distribution of grades on the back of this transcript."

she anticipates the school will use. However, given the evaluator's beliefs, a school could simply assign the grade of $G$ to all of its graduates, guaranteeing that the evaluator's posterior belief about graduate ability is *always* $g$, and never zero. This is only consistent with Perfect Bayesian equilibrium if this is the behavior the evaluator initially expects. Thus, if the grading policy at the school is unobservable, grade inflation runs rampant: grades are maximally inflated and minimally informative. Nonetheless, as we show in Section 7, even this extreme situation may be better than the fully-revealing benchmark.

### 9.3 QUADRATIC COSTS

In order to derive relatively simple closed-form solutions for all variables of interest, we have focused on the case in which the cost of improving expected student ability is quadratic. In the body of the paper, we have described the intuition, as much as possible, only in terms of marginal benefit; the reasoning and intuition in the paper therefore also apply for other strictly convex cost functions. While strict convexity is a standard assumption, we also point out that if costs were linear, the three stage game would be essentially outcome-equivalent to a setting with uninformative grades.

### 9.4 STUDENT EFFORT

In the analysis we present, the student is a passive participants in the learning process: only a school's level of investment affects his likely educational outcome. It is natural to think that students actively participate in their own education by exerting effort. Student effort is consistent with the model we have presented thus far, as long as it is observable. In this case, part of a school's investment in quality may include costly programs to directly monitor student effort and apply pressure to shirking students. However, if student effort is unobservable, monitoring and enforcing effort targets is difficult. In this case, a student's effort choice is determined strategically, and is influenced by the school's grading policy. Because the strategic grading equilibrium exhibits grade inflation, allowing strategic grading may undermine student effort.

To address this concern, we have solved a version of the model that features unobservable student effort. Here, we describe the model and the main results.[32]

We augment the model by incorporating two strategic students, one for each school. In the first stage of the game, school investments and student effort are chosen simultaneously. These choices determine each student's likely ability. Let $q_i$ represent the probability that the graduate of school $i$ is high ability (as in the models given so far), $s_i$ represents school $i$'s investment, and $t_i$ represent student $i$'s effort. We focus on the following specification:

$$q_i = \omega s_i + (1-\omega)t_i$$

Thus, the probability that graduate $i$ is high ability is a weighted sum of school investment and student effort, where the weight on school investment is given by $\omega \in [0,1]$. Each choice of school

[32] Details are available upon request.

investment and student effort is associated with a quadratic cost function, with a potentially different marginal cost. As in the previous model, school investment is revealed publicly at the end of the first stage. However, each student's effort is not observed by any other party. Both schools and the evaluator forecast each student's effort during the grading and evaluation stages, and in equilibrium these forecasts are correct. Denote the anticipated effort of student $i$ by $\hat{t}_i$ and the anticipated probability that $i$'s graduate is high ability by $\hat{q}_i = \omega s_i + (1-\omega)\hat{t}_i$.

In the second stage of the game, schools simultaneously choose grading policies. In previous sections, each school observed the probability that each student is high ability; here, each school designs its grading policy given its *forecast* about each student's likely ability, $(\hat{q}_i, \hat{q}_j)$.[33] The evaluator therefore assigns the prize to the student she believes is most-likely to be high-ability on the basis of her forecasts $(\hat{q}_i, \hat{q}_j)$, each school's grading policy, and the transcript realizations.

Because behavior in the grading and evaluation stages is determined by the *anticipated* probability that each student is high ability, a fundamental asymmetry exists between school investment and student effort. Because it becomes public at the end of stage one, by changing investment a school alters the grading stage equilibrium leading to the same effects described in Section 4. However, because equilibrium grading policies and evaluator beliefs depend on forecasted effort (but not on chosen effort), a student deviating from forecasted effort does not change either the evaluator's inference from any particular transcript realization, nor does it change a school's choice of *underlying* grading policy $(H_i, L_i)$. It does however, change the distribution of transcript realizations. By exerting greater effort than is expected, for example, the student increases the probability that his transcript is the realization of $H_i$. Although neither $(H_i, L_i)$ or the posterior belief associated with each transcript realization are sensitive to student effort, the distribution of the evaluator's posterior belief *is* sensitive to student effort. This leads to moral hazard: the student can secretly alter the distribution of the evaluator's posterior belief. In equilibrium, the student should not have an incentive to (secretly) deviate from the forecasted level of effort.

Unsurprisingly, when $\omega = 0$ (so that educational outcomes are determined solely by unobserved student effort), noisy grading is associated with lower student effort and achievement compared to the fully-informative benchmark. In this case, allowing noisy grading is unambiguously bad for welfare, because it reduces both effort and grading policy informativeness. As we have already shown, however, when $\omega = 1$, the equilibrium with strategic grading may exhibit greater investment by schools than in the fully-revealing benchmark, leaving the evaluator better off. The equilibrium is continuous in $\omega$, so the qualitative implications hold for values of $\omega$ near the extremes.[34] Therefore, if educational outcomes are primarily determined by (unobservable) student effort, educational outcomes and evaluator welfare are reduced when noisy grading is permitted.[35] If educational outcomes are primarily determined by (observable) school investment in education quality, the main qualitative results of our analysis continue to hold, even when students contribute unobserved

[33] Because schools also cannot observe student effort, their grading policies do not signal private information to the evaluator.

[34] The solution cannot be written in closed form unless $\omega = 0$ or $\omega = 1$.

[35] This effect may still tempered by increased school investment.

effort into the education process.

## 10 CONCLUSION

We use a sequential model of school competition, featuring endogenous investment, grading, and evaluation, to study the interaction between a school's investment decision and its choice of grading policy. When schools control information about student ability, they choose grading strategies that do not perfectly reveal student types to evaluators. However, they also invest more to increase education quality than in a benchmark case where all information about ability is observed by evaluators. The increased investment has the potential to offset the cost of noisy grading.

Viewed from a broad perspective, our analysis considers competing institutions that make (real) investments to improve outcomes and also exert influence over the release of information about these outcomes. While our focus is on competition between schools, similar considerations are important in a variety of other settings. For example, a division within a firm may exert effort to develop a prototype, and simultaneously control the way in which the prototype is tested prior to undertaking a development decision. Competing for promotion, a number of employees may exert effort to improve their output, while simultaneously controlling the flow of information to superiors within the firm. In finance, firms make investments and exert effort to generate profits for shareholders, but have the capability to influence the information available to investors about their performance by controlling the way in which earnings are disclosed. Producers of consumer products invest to develop new product lines or models, but they may limit the availability of information about new product attributes prior to their release.

In the context of education, our results have novel implications for both the policy discussion and common industry practices. We highlight a potential benefit of grade inflation—that it leads to greater investment in education quality—and show that the benefit can dominate the costs associated with information loss. This suggests that grade inflation may not be as bad for welfare as popular discussion suggests. In fact, it may lead to better outcomes than would arise if student ability is perfectly observed. By implication, the results suggest that policies or practices that convey information about graduate ability can reduce investment in the quality of education, decreasing student ability. Industry licensing exams and university or graduate school entrance exams provide employers and universities with information about applicant ability. To the extent that performance on these tests is an indicator of ability, graduate school entrance exams (such as the LSAT, GRE, MCAT and GMAT) and state licensing exams in various industries may undermine the incentives of colleges and universities to invest in quality. College entrance exams like the SAT and ACT as well as state assessment exams may may have similar unintended consequences for primary and secondary education. Paradoxically, the use of such exams may *reduce* investment in education quality and graduate ability.

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# A APPENDIX

## A.1 FULLY REVEALING BENCHMARK

In the fully revealing benchmark, schools are required to use grading policies that completely reveal the ability of the graduate. As described in text, a fully-revealing grading policy generates a Bernoulli posterior: $Pr(\Gamma_i = 1) = q_i$ and $Pr(\Gamma_i = 0) = 1 - q_i$. Thus, at the grading stage, for fixed investment levels $(q_a, q_b)$ the schools and evaluator expected payoffs are

$$u_i(q_i, q_j) = q_i(1 - \frac{q_j}{2}) + \frac{1}{2}(1 - q_i)(1 - q_j) = \frac{1}{2}(1 + q_i - q_j)$$

$$u_e(q_a, q_b) = 1 - (1 - q_a)(1 - q_b)$$

If a school generates a high ability graduate, its graduate is assigned the job, unless the other graduate is also high ability, in which case they tie. It also ties if both schools generate low ability graduates. The evaluator receives a payoff of 1, unless both graduates are low-ability.

The marginal benefit of investment is constant at each school, equal to $\frac{1}{2}$. Each school's investment level satisfies its first order condition: $\frac{1}{2} = \frac{2}{\rho_i^2}$ Thus, each school has a dominant strategy to invest $q_i = \frac{1}{4}\rho_i^2$. The associated expected payoff for each school and evaluator is given by

$$u_i = \frac{1}{2}(1 + \frac{\rho_i^2 - \rho_j^2}{4})$$

$$u_e = 1 - (1 - \frac{\rho_\alpha^2}{4})(1 - \frac{\rho_\beta^2}{4})$$

## A.2 GENERAL GRADING POLICY REPRESENTATION

In the text we represent a grading policy by the distribution of the posterior belief that will be induced by the grading policy. Here we explain this construction in detail.

Recall that for $(H, L)$, to be an admissible grading policy, each random variable $H, L$ can have a countable number of mass points, and otherwise admits a differentiable density function $f_H(x), f_L(x)$. Any random variable that satisfies these conditions is *valid*. In addition, the likelihood ratio of a grading policy must be monotone. Let $T \in \{H, L\}$. Also $m_T^j$ represent a mass point of random variable $T$ and $\mu_T^j = Pr(H = m_T^j)$. The density function of random variable $T$ is given by

$$p_T(x) = f_T(x) + \delta(x - m_T^j)\mu_T^j$$

where $\delta(x)$ represents the Dirac $\delta$-function. Of course, because the density contains the Dirac delta function, it is not a proper function. This statement should be interpreted to mean that the cumulative distribution function of $T$ is $P_T(x) = F_T(x) + \sum_j S(x - m_T^j)\mu_T^j$, where $F_T(x)$ is the integral of $f_T(x)$, and $S(x)$ is the step (or Heaviside) function. A graduate's transcript is also a random variable, $X$, whose distribution and density depends on the prior $q$ and on the grading policy at the school:

$$F_X(x) = Pr(X \leq x) = qP_H(x) + (1-q)P_L(x) \tag{1}$$

$$f_X(x) = qp_H(x) + (1-q)p_L(x)$$

Transcripts convey information about student quality. Because the true ability of the graduate is unknown, each possible realization of his transcript induces a posterior belief that the graduate is high ability. This posterior belief depends on the prior belief $q$, (determined by school investment), the realization of the graduate's transcript $x$, and the school's grading policy $(H, L)$. the evaluator's posterior belief that the graduate is high ability (derived from Bayes' rule) is given by

$$Pr(t = h|x) = \frac{qp_H(x)}{qp_H(x) + (1-q)p_L(x)}$$

At the time the school designs its grading policy, the realization of the transcript is uncertain. The transcript $X$ is a random variable, whose distribution is determined by the school grading policy. This also implies that the posterior belief is a random variable:

$$\Gamma = Pr(t = h|X)$$

Once a school designs its grading policy, but before the student's transcript is realized, the value of the posterior belief is a random variable $\Gamma$. This random variable $\Gamma$ is thus the *ex ante* value of the posterior belief. The random variable $\Gamma$ is valid, has support confined to the unit interval, and (from the law of total expectation) has expected value equal to the prior: $E[\Gamma] = q$.[36] In the text we claim that, these are the only substantive restrictions on random variable $\Gamma$. Here we provide the proof:

**Lemma A.1** *Let $\Gamma$ be a valid random variable with support confined to the unit interval, a countable number of mass points, and expectation $q$. If the prior belief that student quality is high is $q$, then there exists an admissible grading policy $(H, L)$ such that the ex ante posterior belief is $\Gamma$.*

**Proof.** Let $\Gamma$ be a random variable that satisfies the assumptions of the Lemma. $\Gamma$ therefore has some density function $g(x)$ of the type given in equation (1). Consider two random variables $H$ and $L$ with densities $h(x), l(x)$ given by the following:

$$h(x) = \frac{xg(x)}{q} \text{ and } l(x) = \frac{(1-x)g(x)}{1-q}$$

[36] To see that $\Gamma$ is valid, consider the following. A mass point of $\Gamma$ must be in $M_G \cap M_B \cup \{0, 1\}$, a finite set. Any realization outside of $I_G \cap I_B$ leads to posterior belief 0 or 1. All non-mass points inside of $\Gamma$ are generated by signal realizations inside $I_G \cap I_B$. Because $f_G(x)$ and $f_B(x)$ are continuous on this interval, the posterior belief is also continuous on this interval. By the intermediate value theorem, if two posteriors are generated by a realization inside the support, then all posteriors in between are also generated by a signal realization inside $I_G \cap I_B$. Continuity of the density over the interval of support follows from differentiability and monotonicity of the likelihood ratio.

Observe that the supports of $H$ and $L$ are identical to the support of $\Gamma$. Also observe that because the expected value of $\Gamma$ is equal to $q$, both $h(x)$ and $l(x)$ are proper density functions. Consider next the posterior belief associated with realization $x$

$$\begin{aligned} Pr(t = h|x) =& \frac{qh(x)}{qh(x) + (1-q)l(x)} \\ =& \frac{xg(x)}{xg(x) + (1-x)g(x)} \\ =& x \end{aligned}$$

Thus the posterior belief associated with any transcript realization is equal to the realization of the transcript itself. The likelihood ratio is monotone, as the posterior belief is monotonic in the realization. The density of the transcript realization is given by

$$qh(x) + (1-q)l(x) = g(x)$$

For the constructed grading policy, the posterior belief is equal to the realization of the transcript, and the transcript has density $g(x)$. Thus for the grading policy constructed, $\Gamma$ is the ex ante posterior belief. ■

### A.3 STAGE TWO EQUILIBRIUM

The next two Lemmas characterize the Nash equilibrium of the second stage game. In the text, both Lemmas are combined into (). Suppose school $A$ invests $q_a$, while school $B$ invests $q_b$ and $q_a \geq q_b$.

**Lemma A.2** *If $q_a \leq \frac{1}{2}$ then the unique Nash equilibrium of the second stage game is given by the following:*

$$\Gamma_a = U[0, 2q_a]$$

$$\Gamma_b = \begin{cases} 0 & \textit{with probability} \quad 1 - \frac{q_b}{q_a} \\ U[0, 2q_a] & \textit{with probability} \quad \frac{q_b}{q_a} \end{cases}$$

**Proof.** Here, we verify that the proposed strategies are an equilibrium. Uniqueness is proved separately in Appendix II. Consider the best response of school $B$ to the strategy of school $A$. Any admissible strategy on the part of school $B$ is a random variable $G_b$ that can be represented in the following way:

$$G_b = \begin{cases} G_1 & \text{with probability} \quad p \\ G_2 & \text{with probability} \quad 1-p \end{cases}$$

Here $G_1$ is any valid random variable over support $[0, 2q_a]$ density $g_1(x)$ and expectation $\bar{g}_1$, and $G_2$ is any valid random variable over support $[2q_a, 1]$ density $g_2(x)$ and expectation $\bar{g}_2$. If a mass point exists at $2q_a$ then we include the mass point in random variable $G_1$. The constraint on the mean of $G_b$ implies that

$$\bar{g}_1 p + (1-p)\bar{g}_2 = q_b \rightarrow \bar{g}_1 = \frac{q_b - (1-p)\bar{g}_2}{p}$$

The expected payoff of school $B$ from any random variable $G$ against $\Gamma_a$ is given by

$$p\int_0^{2q_a} g_1(x)\frac{x}{2q_a}dx + (1-p)$$

$$\begin{aligned}
&= p\frac{\bar{g}_1}{2q_a} + (1-p) \\
&= \frac{q_b - (1-p)\bar{g}_2}{2q_a} + (1-p) \\
&= \frac{q_b}{2q_a} - (1-p)(\frac{\bar{g}_2}{2q_a} - 1)
\end{aligned}$$

If any probability mass exists in the interval $(2q_a, 1]$ then $\bar{g}_2 > 2q_a$, and thus in any best response $p = 1$. Furthermore, any random variable $G_b$ for which $p = 1$ gives the same expected payoff $u_b = \frac{q_b}{2q_a}$, and is therefore a best response. Because the support of $\Gamma_b$ is $[0, 2q_a]$, it is a best response.

Next, we show that $\Gamma_a$ is a best response to $\Gamma_b$. Any admissible best response on the part of school $A$ is a random variable $G_a$ that can be represented in the following way:

$$G_a = \begin{cases} G_1 & \text{with probability} \quad p \\ G_2 & \text{with probability} \quad 1-p \end{cases}$$

Here $G_1$ is any valid random variable over support $[0, 2q_a]$ density $g_1(x)$ and expectation $\bar{g}_1$, and $G_2$ is any valid random variable over support $[2q_a, 1]$ density $g_2(x)$ and expectation $\bar{g}_2$. If a mass point exists at $2q_a$ then we include the mass point in random variable $G_1$. The constraint on the mean of $G_a$ implies that

$$\bar{g}_1 p + (1-p)\bar{g}_2 = q_a \rightarrow \bar{g}_1 = \frac{q_a - (1-p)\bar{g}_2}{p}$$

The expected payoff of school $B$ from any random variable $G$ against $\Gamma_a$ is given by

$$p(1 - \frac{q_b}{q_a} + \frac{q_b}{q_a}\int_0^{2q_a} g_1(x)\frac{x}{2q_a}dx) + (1-p)$$

$$= p(1 - \frac{q_b}{q_a} + \frac{q_b}{q_a}\frac{\bar{g}_1}{2q_a}) + (1-p)$$
$$= p(1 - \frac{q_b}{q_a} + \frac{q_b}{q_a}\frac{q_a - (1-p)\bar{g}_2}{2pq_a}) + (1-p)$$
$$= 1 - \frac{q_b}{2q_a} - (1-p)\frac{q_b}{2q_a^2}(\bar{g}_2 - 2q_a)$$

If any probability mass exists in the interval $(2q_a, 1]$ then $\bar{g}_2 > 2q_a$, and thus in any best response $p = 1$. Furthermore, any random variable $G$ for which $p = 1$ gives the same expected payoff $u_a = 1 - \frac{q_b}{2q_a}$, and is therefore a best response. Because the support of $\Gamma_a$ is $[0, 2q_a]$, it is a best response.

■

**Lemma A.3** *If $q_a > \frac{1}{2}$ then the unique Nash equilibrium of the second stage game is given by the following:*

$$\Gamma_a = \begin{cases} U[0, 2(1-q_a)] & \textit{with probability} \ \ \frac{1}{q_a} - 1 \\ 1 & \textit{with probability} \ \ 2 - \frac{1}{q_a} \end{cases}$$

$$\Gamma_b = \begin{cases} 0 & \textit{with probability} \ \ 1 - \frac{q_b}{q_a} \\ U[0, 2(1-q_a)] & \textit{with probability} \ \ \frac{q_b}{q_a}(\frac{1}{q_a} - 1) \\ 1 & \textit{with probability} \ \ \frac{q_b}{q_a}(2 - \frac{1}{q_a}) \end{cases}$$

**Proof.** Here, we verify that the proposed strategies are an equilibrium. Uniqueness is proved separately in Appendix II. Consider the best response of school $B$ to the strategy of school $A$. Any admissible strategy on the part of school $B$ is a random variable $G_b$ that can be represented in the following way:

$$G_b = \begin{cases} G_1 & \text{with probability} \ \ p_1 \\ G_2 & \text{with probability} \ \ p_2 \\ 1 & \text{with probability} \ \ p_3 \end{cases}$$

$$p_1 + p_2 + p_3 = 1$$

Here $G_1$ is any valid random variable over support $[0, 2(1-q_a)]$ density $g_1(x)$ and expectation $\bar{g}_1$, and $G_2$ is any valid random variable over support $[2(1-q_a), 1]$ density $g_2(x)$ and expectation $\bar{g}_2$. If a mass point exists at $2(1-q_a)$ then we include the mass point in random variable $G_1$. The constraint on the mean of $G_b$ implies that

$$\bar{g}_1 p_1 + p_2 \bar{g}_2 + p_3 = q_b \rightarrow \bar{g}_1 = \frac{q_b - p_2\bar{g}_2 - p_3}{p_1}$$

The expected payoff of school $B$ from any random variable $G_b$ against $\Gamma_a$ is given by

$$p_1(\frac{1}{q_a} - 1)\int_0^{2(1-q_a)} g_1(x)\frac{x}{2(1-q_a)}dx + p_2(\frac{1}{q_a} - 1) + p_3((\frac{1}{q_a} - 1) + (2 - \frac{1}{q_a})\frac{1}{2})$$

$$= p_1(\frac{1}{q_a} - 1)\frac{\bar{g}_1}{2(1-q_a)} + p_2(\frac{1}{q_a} - 1) + p_3\frac{1}{2q_a}$$
$$= \frac{q_b}{2q_a} - \frac{1}{2q_a}(\bar{g}_2 - 2(1-q_a))p_2$$

If any probability mass exists in the interval $(2(1-q_a), 1]$ then $\bar{g}_2 > 2(1-q_a)$. The coefficient on $p_2$ is therefore negative. Thus in any best response $p_2 = 0$. Furthermore, any random variable $G_b$ for which $p_2 = 0$ gives the same expected payoff $u_b = \frac{q_b}{2q_a}$, and is therefore a best response. Because the support of $\Gamma_b$ is $[0, 2(1-q_a)] \cup 1$, it is a best response.

Next, we show that $\Gamma_a$ is a best response to $\Gamma_b$. Any admissible best response on the part of school $A$ is a random variable $G_a$ that can be represented in the following way:

$$G_a = \begin{cases} G_1 & \text{with probability} \quad p_1 \\ G_2 & \text{with probability} \quad p_2 \\ 1 & \text{with probability} \quad p_3 \end{cases}$$

$$p_1 + p_2 + p_3 = 1$$

Here $G_1$ is any valid random variable over support $[0, 2(1-q_a)]$ density $g_1(x)$ and expectation $\bar{g}_1$, and $G_2$ is any valid random variable over support $[2(1-q_a), 1]$ density $g_2(x)$ and expectation $\bar{g}_2$. If a mass point exists at $2(1-q_a)$ then we include the mass point in random variable $G_1$. The constraint on the mean of $G_a$ implies that

$$\bar{g}_1 p_1 + p_2\bar{g}_2 + p_3 = q_b \to \bar{g}_1 = \frac{q_a - p_2\bar{g}_2 - p_3}{p_1}$$

The expected payoff of school $A$ from any random variable $G_a$ against $\Gamma_b$ is given by

$$(1 - \frac{q_b}{q_a}) + \frac{q_b}{q_a}[p_1(\frac{1}{q_a} - 1)\int_0^{2(1-q_a)} g_1(x)\frac{x}{2(1-q_a)}dx + p_2(\frac{1}{q_a} - 1) + p_3((\frac{1}{q_a} - 1) + (2 - \frac{1}{q_a})\frac{1}{2})]$$

$$= (1 - \frac{q_b}{q_a}) + \frac{q_b}{q_a}[p_1(\frac{1}{q_a} - 1)\frac{\bar{g}_1}{2(1-q_a)} + p_2(\frac{1}{q_a} - 1) + p_3\frac{1}{2q_a}]$$
$$= (1 - \frac{q_b}{q_a}) + \frac{q_b}{q_a}[\frac{1}{2} - \frac{1}{2q_a}(\bar{g}_2 - 2(1-q_a))p_2]$$

If any probability mass exists in the interval $(2(1-q_a), 1]$ then $\bar{g}_2 > 2q_a$. The coefficient on $p_2$ is therefore negative. Thus in any best response $p_2 = 0$. Furthermore, any random variable $G_a$ for which $p_2 = 0$ gives the same expected payoff $u_a = (1 - \frac{q_b}{q_a}) + \frac{q_b}{q_a}(\frac{1}{2}) = 1 - \frac{q_b}{2q_a}$, and is therefore a best response. Because the support of $\Gamma_a$ is $[0, 2(1-q_a)] \cup 1$, it is a best response. ■

**Equilibrium mimicry**. We briefly show that in equilibrium, school $B$ underlying grading policy is identical to that of school $A$, except for the inclusion of a transcript or set of transcripts that are assigned to low ability students only.

Suppose that school $A$ utilizes an equilibrium grading policy $(H_a, L_a)$, where the density of $H_a$ is $h_a(x)$ and the density of $L_a$ is $l_a(x)$. Thus, posterior belief about ability at school $A$ given transcript $x$ is $\frac{q_a h_a(x)}{q_a h_a(x)+(1-q_a)l_a(x)}$ and the density of the posterior is $q_a h_a(x) + (1-q_a)l_a(x)$.

In equilibrium, the distribution of posterior beliefs at both schools is identical, except that school $B$ reveals a probability mass of $1-\frac{q_b}{q_a}$ students to be low ability. In order to do so, school $B$ assigns an additional grade (or set of grades) that we call $F$. This grade is assigned only to low ability students; thus observing $F$ reveals that the student is low ability. In equilibrium the probability that school $B$ assigns $F$ is $1-\frac{q_b}{q_a}$. Therefore, the conditional probability that a student gets an $F$ given that he is low ability is $\phi = \frac{1-\frac{q_b}{q_a}}{1-q_b} = \frac{q_a-q_b}{q_a(1-q_b)}$.

Suppose that school $B$ grading policy is identical to school $A$, except for the inclusion of the $F$ grade, assigned to low ability students with conditional probability $\phi$. The posterior belief about $B$ graduate if the transcript is not $F$ is given by $\frac{q_b h_a(x)}{q_b h_a(x)+(1-q_b)(1-\phi)l_a(x)} = \frac{q_a h_a(x)}{q_a h_a(x)+(1-q_a)l_a(x)}$, and the density of this posterior is $q_a h_a(x) + (1-q_a)l_a(x)$. Thus, school $B$ equilibrium grading policy assigns an $F$ to low ability students with conditional probability $\phi$, and otherwise is identical to the grading policy of school $A$.

**Equilibrium payoffs**. We calculate the equilibrium payoffs for the evaluator. Equilibrium payoffs for the schools have been calculated in the body of the proof of equilibrium.

**Corollary A.4** *evaluator payoff*

- *If* $q_a \leq \frac{1}{2}$ *then evaluator expected payoff is* $u_e = q_a + \frac{1}{3}q_b$.
- *If* $q_a > \frac{1}{2}$ *then evaluator expected payoff is* $u_e = \frac{3q_a^4 - 7q_a^3 q_b + 12q_a^2 q_b - 6q_a q_b + q_b}{3q_a^3}$

**Proof.** In either case, the evaluator's payoff is given by the following expression:

$$\frac{q_b}{q_a}E[\Gamma_a^{(2)}] + (1-\frac{q_b}{q_a})q_a$$

Here, $\Gamma_a^{(2)}$ represents the maximum order statistic from two draws of random variable $\Gamma_a$. If school $b$ does not reveal the student to be low ability for sure, then it uses the school $A$ grading policy. In this case the evaluator payoff is the maximum draw. If, however, school $b$ reveals its graduate to be low ability, the evaluator only receives the expected quality of a graduate from school $A$.

**Case I**: $q_a \leq \frac{1}{2}$

In this case, $\Gamma_a = U[0, 2q_a]$ and therefore, $E[\Gamma_a^{(2)}] = \frac{4}{3}q_a$. Evaluating the above expression gives:

$$\frac{q_b}{q_a}(\frac{4}{3}q_a) + (1-\frac{q_b}{q_a})q_a = q_a + \frac{1}{3}q_b$$

**Case II**: $q_a > \frac{1}{2}$

$$\Gamma_a = \begin{cases} U[0, 2(1-q_a)] & \text{with probability} \quad \frac{1}{q_a} - 1 \\ 1 & \text{with probability} \quad 2 - \frac{1}{q_a} \end{cases}$$

Evaluating $E[\Gamma_a^{(2)}]$ gives:

$$E[\Gamma_a^{(2)}] = 1 - (\frac{1}{q_a} - 1)^2 + (\frac{1}{q_a} - 1)^2(\frac{4}{3})(1 - q_a)$$

$$E[\Gamma_a^{(2)}] = \frac{1 - 6q_a + 12q_a^2 - 4q_a^3}{3q_a^2}$$

Substituting and simplifying gives the expression given above. ■

### A.4 STAGE ONE EQUILIBRIUM

Here we derive the stage one equilibrium effort levels for both schools. The payoff function for each school $i \in \{\alpha, \beta\}$ is given by

$$u_i(q_i, q_j) = \begin{cases} \frac{q_i}{2q_j} - \frac{q_i^2}{\rho_i^2} & \text{if} \quad q_i \leq q_j \\ 1 - \frac{q_j}{2q_i} - \frac{q_i^2}{\rho_i^2} & \text{if} \quad q_i > q_j \end{cases}$$

This function is differentiable, continuous, (weakly) concave in $q_i$. Therefore the unique maximum occurs wherever the derivative of the payoff function is equal to zero:

$$\frac{du_i(q_i, q_j)}{dq_i} = \begin{cases} \frac{1}{2q_j} - 2\frac{q_i}{\rho_i^2} & \text{if} \quad q_i \leq q_j \\ \frac{q_j}{2q_i^2} - 2\frac{q_i}{\rho_i^2} & \text{if} \quad q_i > q_j \end{cases}$$

Suppose that in equilibrium $q_a \geq q_b$. Such an equilibrium is described by the following first order conditions:

$$\frac{1}{2q_a} - \frac{2q_b}{\rho_b^2} = 0 \text{ and } \frac{q_b}{2q_a^2} - 2\frac{q_a}{\rho_a^2} = 0$$

Therefore we find that

$$q_a = \frac{\sqrt{\rho_a \rho_b}}{2} \text{ and } q_b = \frac{\rho_b^2}{2\sqrt{\rho_a \rho_b}}$$

The difference $q_\alpha - q_\beta = \frac{\sqrt{\rho_a \rho_b}}{2\rho_a}(\rho_a - \rho_b)$. Hence, for this combination to be an equilibrium it must be that $\rho_a \geq \rho_b$, that is, school with the higher resource, $\alpha$, invests more in equilibrium (thereby playing the role of school $A$). Thus

$$q_\alpha = \frac{\sqrt{\rho_\alpha \rho_\beta}}{2} \text{ and } q_\beta = \frac{\rho_\beta^2}{2\sqrt{\rho_\alpha \rho_\beta}}$$

$$u_\alpha = \frac{\rho_b}{4\rho_a} \text{ and } u_\beta = 1 - \frac{3\rho_b}{4\rho_a}$$

We find the evaluator payoff by substituting the school effort levels into the evaluator payoff function. The two cases arise because $\rho_\alpha \rho_\beta \leq 1$ implies $q_\alpha \leq \frac{1}{2}$ which influences the nature of the equilibrium that arises in the second stage.

$$u_e = \begin{cases} \frac{(3\rho_\alpha+\rho_\beta)\sqrt{\rho_\alpha\rho_\beta}}{6\rho_\alpha} & \text{if} \quad \rho_\alpha\rho_\beta \leq 1 \\ \frac{(3\rho_\alpha^2-7\rho_\alpha\rho_\beta-24)\sqrt{\rho_\alpha\rho_\beta}+24\rho_\alpha\rho_\beta+8}{6\rho_\alpha^2} & \text{if} \quad \rho_\alpha\rho_\beta > 1 \end{cases}$$

## A.5 COMPARISON OF FULLY-REVEALING BENCHMARK AND STRATEGIC GRADING EQUILIBRIUM

**Investment Comparison**. Here we compare the investment of schools in the strategic grading benchmark to investment under fully-informative grading. The following inequalities define the conditions under which investment under strategic grading is higher than under fully revealing grading.

$$\text{School } \alpha\text{: } \frac{\sqrt{\rho_\alpha\rho_\beta}}{2} \geq \frac{\rho_\alpha^2}{4} \leftrightarrow \rho_\beta \geq \frac{\rho_\alpha^3}{4}$$

$$\text{School } \beta\text{: } \frac{\rho_\beta^2}{2\sqrt{\rho_\alpha\rho_\beta}} \geq \frac{\rho_\beta^2}{4} \leftrightarrow 2 \geq \sqrt{\rho_\alpha\rho_\beta}$$

The last inequality is always satisfied as long as $\rho_i \leq 2$, an assumption we maintain.

**Payoff comparison with fixed investment**. Here we show that for fixed investments the evaluator and school $\beta$ expect higher payoff when grades must be fully-revealing, while school $\alpha$ prefers the equilibrium without grading restrictions. We begin with the schools.

$$\frac{1}{2}(1+q_b-q_a) = \frac{q_b}{2q_a} + \frac{1}{2}(1-\frac{q_b}{q_a})(1-q_a)$$

$$\frac{1}{2}(1+q_a-q_b) = 1 - \frac{q_b}{2q_a} - \frac{1}{2}(1-\frac{q_b}{q_a})(1-q_a)$$

On the left hand sides we have the school payoffs under full-information, while on the right we have the payoffs in equilibrium without restriction. The school with less investment does better under fully informative transcripts while the school that invests more does worse. For the evaluator,

$$1-(1-q_a)(1-q_b) - \frac{q_b}{3}(2-3q_a) = q_a + \frac{1}{3}q_b$$

$$1-(1-q_a)(1-q_b) - \frac{q_b(1-q_a)^3}{3q_a^3}(3q_a-1) = \frac{3q_a^4 - 7q_a^3q_b + 12q_a^2q_b - 6q_aq_b + q_b}{3q_a^3}$$

The right hand of the top equation is the evaluator payoff whenever $q_a \leq \frac{1}{2}$. Thus, the full information payoff is higher. Similarly, the right hand side of the bottom equation is the evaluator expected payoff when $q_a > \frac{1}{2}$. Again the full-information expected payoff is higher.

**evaluator payoff comparison with endogenous investment** Here we compare the evaluator's payoff under fully revealing and strategic grading, when investment is endogenous. The following inequalities define the region of $(\rho_\alpha, \rho_\beta)$ for which the evaluator payoff is higher with strategic grading, assuming endogenous investment.

$$\begin{array}{lll} \frac{(3\rho_\alpha+\rho_\beta)\sqrt{\rho_\alpha\rho_\beta}}{6\rho_\alpha} - (1-(1-\frac{\rho_\alpha^2}{4})(1-\frac{\rho_\alpha^2}{4})) \geq 0 & \text{if} & \rho_\alpha\rho_\beta \leq 1 \\ \frac{(3\rho_\alpha^2-7\rho_\alpha\rho_\beta-24)\sqrt{\rho_\alpha\rho_\beta}+24\rho_\alpha\rho_\beta+8}{6\rho_\alpha^2} - (1-(1-\frac{\rho_\alpha^2}{4})(1-\frac{\rho_\alpha^2}{4})) \geq 0 & \text{if} & \rho_\alpha\rho_\beta > 1 \end{array}$$

This function is continuous. We establish that these inequalities are satisfied along the diagonal $\rho_\alpha = \rho_\beta$, and therefore hold in a region around the diagonal. Let $\rho_\alpha = \rho_\beta = \rho$. These inequalities become:

$$\begin{array}{lll} \frac{1}{16}\rho^4 - \frac{1}{2}\rho^2 + \frac{2}{3}\rho \geq 0 & \text{if} & \rho \leq 1 \\ \frac{1}{48\rho^2}(2-\rho)^2((2-\rho)^4 + 2\rho^2(\rho-\sqrt{31}+5)(\rho+\sqrt{31}+5) \geq 0 & \text{if} & 1 \leq \rho \leq 2 \end{array}$$

The first inequality is satisfied because, $\frac{2}{3}\rho - \frac{1}{2}\rho^2 \geq$ for all $0 \leq \rho \leq \frac{4}{3}$. The second inequality is also satisfied because in the region of interest, all terms in the sum are positive. Thus, around the diagonal, the evaluator payoff is higher under strategic grading than under fully-informative grading. Plotting these regions using a software package capable of implicit plots shows that the inequalities hold in an arc-shaped region around the diagonal.

# B APPENDIX II: INTENDED FOR ONLINE PUBLICATION

## B.1 NO GRADES/RAMPANT GRADE INFLATION

If grades or transcripts are banned, then the evaluator assigns the job based solely on the investment level of the school. An identical outcome would follow if for some exogenous reason grades become so inflated that they convey no information about student ability. This game is essentially a full-information all-pay auction with ann asymmetric convex cost of bidding, and a bid cap. Each school simultaneously chooses $q_i \in [0,1]$. The school with the higher value of $q$ receives a payoff of one, but both schools lose their investments, $C_i(q_i) = \frac{q_i^2}{\rho_i^2}$. We describe the equilibrium of this game in a series of Lemmas.

**Lemma B.1** *If $\rho_\beta \geq \sqrt{2}$ then $q_\alpha = q_\beta = 1$ is the unique equilibrium.*

**Proof.** Suppose that school $j$ chooses $q_j = 1$. All pure strategies $q_i \in (0,1)$ lead to payoff zero. Because investment is costly, and given school $j$ strategy, the best deviation from $q_i = 1$ is therefore $q_i = 0$. Thus, provided

$$\frac{1}{2} - \frac{1}{\rho_i^2} \geq 0 \leftrightarrow \rho_i \geq \sqrt{2}$$

$q_i = 1$ is a best response to $q_j = 1$. Because $\rho_\beta \leq \rho_\alpha$, if $\rho_\beta \geq \sqrt{2}$ then $q_\alpha = q_\beta = 1$ is the unique Nash equilibrium. ■

**Lemma B.2** *If $1 < \rho_\beta < \sqrt{2}$ then the mixed strategy Nash equilibrium is as follows:*

$$q_\alpha = \begin{cases} Q \sim F(x) = \frac{x^2}{2-\rho_\beta^2} & \textit{with probability} \quad \frac{2}{\rho_\beta^2} - 1 \\ 1 & \textit{with probability} \quad 2(1 - \frac{1}{\rho_\beta^2}) \end{cases}$$

$$q_\beta = \begin{cases} 0 & \textit{with probability} \quad 1 - (\frac{\rho_\beta}{\rho_\alpha})^2 \\ Q \sim F(x) = \frac{x^2}{2-\rho_\beta^2} & \textit{with probability} \quad (\frac{\rho_\beta}{\rho_\alpha})^2(\frac{2}{\rho_\beta^2} - 1) \\ 1 & \textit{with probability} \quad 2(\frac{\rho_\beta}{\rho_\alpha})^2(1 - \frac{1}{\rho_\beta^2}) \end{cases}$$

**Proof.** Under the parameter range in the proposition, all values we claim are probabilities are in $[0,1]$ and sum to one. Also, $F(x)$ is increasing on the support of $Q$ which is $[0, \sqrt{2-\rho_\beta^2}]$. To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each school is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player. A derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available upon request.

Consider school $\beta$'s expected payoff from a pure strategy $p$ in the support of its mixed strategy:

$$u_\beta = \begin{cases} 0 & \text{if} \quad p = 0 \\ (\frac{2}{\rho_\beta^2} - 1)F(p) - \frac{p^2}{\rho_\beta^2} & \text{if} \quad p \in [0, \sqrt{2 - \rho_\beta^2}] \\ 1 - \frac{2(1 - \frac{1}{\rho_\beta^2})}{2} - \frac{1}{\rho_\beta^2} & \text{if} \quad p = 1 \end{cases}$$

Substituting and simplifying gives:

$$u_\beta = \begin{cases} 0 & \text{if} \quad p = 0 \\ (\frac{2}{\rho_\beta^2} - 1)\frac{p^2}{2-\rho_\beta^2} - \frac{p^2}{\rho_\beta^2} = 0 & \text{if} \quad p \in [0, \sqrt{2 - \rho_\beta^2}] \\ 1 - \frac{2(1 - \frac{1}{\rho_\beta^2})}{2} - \frac{1}{\rho_\beta^2} = 0 & \text{if} \quad p = 1 \end{cases}$$

Thus all pure strategies in the support of school $\beta$ mixed strategy give the school expected payoff zero. Choosing any pure strategy $\hat{q} \in (\sqrt{2 - \rho_\beta^2}, 1)$ is dominated by choosing $q = \sqrt{2 - \rho_\beta^2}$, because the probability of winning is the same for both pure strategies, but $q$ is less costly. Thus all pure strategies in the support of $\beta$ mixed strategy give the same expected payoff against $\alpha$ mixed strategy, and no strategy outside the support gives $\beta$ a higher payoff. Thus, $\beta$ mixed strategy is a best response to $\alpha$'s mixed strategy.

Consider school $\alpha$'s expected payoff from a pure strategy $p$ in the support of its mixed strategy:

$$u_\alpha = \begin{cases} 1 - (\frac{\rho_\beta}{\rho_\alpha})^2 + (\frac{\rho_\beta}{\rho_\alpha})^2(\frac{2}{\rho_\beta^2} - 1)F(p) - \frac{p^2}{\rho_\alpha^2} & \text{if} \quad p \in [0, \sqrt{2 - \rho_\beta^2}] \\ 1 - \frac{2(\frac{\rho_\beta}{\rho_\alpha})^2(1 - \frac{1}{\rho_\beta^2})}{2} - \frac{1}{\rho_\alpha^2} & \text{if} \quad p = 1 \end{cases}$$

Substituting and simplifying gives:

$$u_\alpha = \begin{cases} 1 - (\frac{\rho_\beta}{\rho_\alpha})^2 + (\frac{\rho_\beta}{\rho_\alpha})^2(\frac{2}{\rho_\beta^2} - 1)\frac{p^2}{2-\rho_\beta^2} - \frac{p^2}{\rho_\alpha^2} = 1 - (\frac{\rho_\beta}{\rho_\alpha})^2 & \text{if} \quad p \in [0, \sqrt{2 - \rho_\beta^2}] \\ 1 - \frac{2(\frac{\rho_\beta}{\rho_\alpha})^2(1 - \frac{1}{\rho_\beta^2})}{2} - \frac{1}{\rho_\alpha^2} = 1 - (\frac{\rho_\beta}{\rho_\alpha})^2 & \text{if} \quad p = 1 \end{cases}$$

Thus all pure strategies in the support of school $\alpha$ mixed strategy give the school expected payoff $1 - (\frac{\rho_\beta}{\rho_\alpha})^2$, equal to the probability that school $\beta$ plays $q = 0$. Choosing any pure strategy $\hat{q} \in (\sqrt{2 - \rho_\beta^2}, 1)$ is dominated by choosing $q = \sqrt{2 - \rho_\beta^2}$, because the probability of winning is the same for both pure strategies, but $q$ is less costly. Thus all pure strategies in the support of $\alpha$ mixed strategy give the same expected payoff against $\beta$ mixed strategy, and no strategy outside the support gives $\alpha$ a higher payoff. Thus, $\alpha$ mixed strategy is a best response to $\beta$'s mixed strategy. ■

**Lemma B.3** *If $\rho_\beta \leq 1$ then the mixed strategy Nash equilibrium is as follows:*

$$q_\alpha = Q \sim F(x) = \frac{x^2}{\rho_\beta^2}$$

$$q_\beta = \begin{cases} 0 & \textit{with probability} \quad 1 - (\frac{\rho_\beta}{\rho_\alpha})^2 \\ Q \sim F(x) = \frac{x^2}{\rho_\beta^2} & \textit{with probability} \quad (\frac{\rho_\beta}{\rho_\alpha})^2 \end{cases}$$

**Proof.** To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each school is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player. A derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available upon request.

Consider school $\beta$'s expected payoff from a pure strategy $p$ in the support of its mixed strategy:

$$u_\beta = \begin{cases} 0 & \text{if} \quad p = 0 \\ F(p) - \frac{p^2}{\rho_\beta^2} & \text{if} \quad p \in [0, \rho_\beta] \end{cases}$$

Substituting and simplifying gives that in both cases $u_\beta = 0$. Thus all pure strategies in the support of school $\beta$ mixed strategy give the school expected payoff zero. Choosing any pure strategy $\hat{q} \in (\rho_\beta, 1)$ is dominated by choosing $q = \rho_\beta$, because the probability of winning is the same for both pure strategies, but $q$ is less costly. Thus all pure strategies in the support of $\beta$ mixed strategy give the same expected payoff against $\alpha$ mixed strategy, and no strategy outside the support gives $\beta$ a higher payoff. Thus, $\beta$ mixed strategy is a best response to $\alpha$'s mixed strategy.

Consider school $\alpha$'s expected payoff from a pure strategy $p$ in the support of its mixed strategy:

$$u_\alpha = \ 1 - (\tfrac{\rho_\beta}{\rho_\alpha})^2 + (\tfrac{\rho_\beta}{\rho_\alpha})^2 F(p) - \frac{p^2}{\rho_\alpha^2} \quad \text{if} \quad p \in [0, \rho_\beta]$$

Simplifying gives that all pure strategies in the support of school $\alpha$ mixed strategy give the school expected payoff $1 - (\frac{\rho_\beta}{\rho_\alpha})^2$, equal to the probability that school $\beta$ plays $q = 0$. Choosing any pure strategy $\hat{q} \in (\rho_\beta, 1)$ is dominated by choosing $q = \rho_\beta$, because the probability of winning is the same for both pure strategies, but $q$ is less costly. Thus all pure strategies in the support of $\alpha$ mixed strategy give the same expected payoff against $\beta$ mixed strategy, and no strategy outside the support gives $\alpha$ a higher payoff. Thus, $\alpha$ mixed strategy is a best response to $\beta$'s mixed strategy.
■

**Banning Grades.** Here we show that banning grades outright may be preferred to either strategic grading or fully-revealing grading. In order to make this calculation, we need to know the evaluator payoff when grades are banned. In the interest of space, we make the calculation along the diagonal, $\rho_\alpha = \rho_\beta = \rho$. If the results hold on the diagonal, by continuity, they also hold in a region off-diagonal.

Case I: $1 \leq \rho \leq \sqrt{2}$ Observe that

$$E[Q^{(2)}] = 2\int_0^{\sqrt{2-\rho^2}} (\frac{2x}{2-\rho^2})(\frac{x^2}{2-\rho^2})(x)dx = \frac{4}{5}\sqrt{2-\rho^2}$$

Where $Q^{(2)}$ represents the maximum of two draws of $Q$. The evaluator expected payoff is therefore

$$1 - (\frac{2}{\rho^2} - 1)^2 + (\frac{2}{\rho^2} - 1)^2 E[Q^{(2)}] = \frac{4}{5\rho^4}((2-\rho^2)^{\frac{5}{2}} - 5(1-\rho^2))$$

Case II: $\rho \leq 1$ Observe that the evaluator expected payoff is given by:

$$E[Q^{(2)}] = 2\int_0^{\rho} (\frac{2x}{\rho^2})(\frac{x^2}{\rho^2})(x)dx = \frac{4}{5}\rho$$

A simple plot reveals that these payoffs dominate both fully-revealing and strategic grading along the diagonal.

### B.2 GRADE INFLATION ONLY

We consider a variant of the model with only two grades, and only inflationary strategies: low ability students are assigned the high grade with a certain probability. The posterior belief that the graduate is high ability associated with a good grade $G$ is therefore less than 1, but greater than or equal to $q_i$, the prior probability. The posterior associated with a bad grade realization $B$ is therefore 0. If a school designs a grading policy that induces posterior belief $g$ when the transcript is $G$, then the law of total probability implies that the probability of assigning transcript $G$ must be $\frac{q_i}{g}$. Thus in the second stage, each school's strategy is represented by a choice of $g_i \in [q_i, 1]$. Once each school makes its choice, the posteriors are realized, and the school with a higher realized posterior receives payoff 1. In the text, we assume that the school picks the degree of grade inflation $\theta$, directly. If schools are confined to inflationary strategies only, then given a prior about graduate ability $q$, Bayes rule provides a one-one mapping between $\theta$ and posterior $g$

$$g = \frac{q}{q + (1-q)\theta}$$

The choice of $\theta = 1$ corresponds to $g = q$ (completely inflated grades, while, $\theta = 0$ corresponds to $g = 1$, uninflated or fully-revealing grades. A higher value of $g$ corresponds to a smaller value of $\theta$, and less inflation. We work with $g$ directly for convenience.

The expected payoff of school $i$ is equal to the following:

$$u_i(g_i, g_j) = \begin{cases} \frac{q_i}{g_i}(1-\frac{q_j}{g_j}) + \frac{1}{2}(1-\frac{q_i}{g_i})(1-\frac{q_j}{g_j}) & \text{if} \quad g_i < g_j \\ \frac{q_i}{g_i} + \frac{1}{2}(1-\frac{q_i}{g_i})(1-\frac{q_j}{g_j}) & \text{if} \quad g_i > g_j \\ \frac{q_i}{g_i}(1-\frac{1}{2}\frac{q_j}{g_j}) + \frac{1}{2}(1-\frac{q_i}{g_i})(1-\frac{q_j}{g_j}) & \text{if} \quad g_i = g_j \end{cases}$$

The next series of Lemmas describes the structure of the second stage equilibrium following for combination of investments $q_a \geq q_b$.

**Lemma B.4** *If $q_b \geq 1 - q_a$ then the unique Nash equilibrium of the second stage game with inflation only is $g_a = g_b = 1$.*

**Proof.** If school $j$ chooses $g_j$=1, then the best possible deviation from $g_i = 1$ is $g_i = q_i$. By choosing this deviation, school $i$ assures that if the other school's graduate receives a $B$ and is thus revealed to be low ability, school $i$ payoff is 1. Thus, $g_a = g_b = 1$ is a Nash equilibrium if and only if for each school $i$

$$q_i(1-\frac{q_j}{2}) + \frac{1}{2}(1-q_i)(1-q_j) \geq 1 - q_j \leftrightarrow q_i + q_j \geq 1$$

Thus, the equilibrium of the second stage is fully revealing if and only if $q_b \geq 1 - q_j$ ■

**Lemma B.5** *If $\frac{1}{2}(1-q_a) < q_b < 1 - q_a$ then the mixed strategy Nash equilibrium is as follows:*

$$g_a = \begin{cases} q_a & \textit{with probability} \quad 1-\phi_1-\phi_2 = 1-\frac{2q_b}{q_b+q_a} \\ G \sim F(x) = \frac{x^2-q_a^2}{4(1-q_b)(1-q_a-q_b)} & \textit{with probability} \quad \phi_1 = (\frac{2q_b}{q_b+q_a})(1-\frac{q_a+2q_b-1}{q_b(q_a+q_b)}) \\ 1 & \textit{with probability} \quad \phi_2 = (\frac{2q_b}{q_b+q_a})\frac{q_a+2q_b-1}{q_b(q_a+q_b)} \end{cases}$$

$$g_b = \begin{cases} G \sim F(x) = \frac{x^2-q_a^2}{4(1-q_b)(1-q_a-q_b)} & \textit{with probability} \quad \lambda = 1-\frac{q_a+2q_b-1}{q_b(q_a+q_b)} \\ 1 & \textit{with probability} \quad 1-\lambda = \frac{q_a+2q_b-1}{q_b(q_a+q_b)} \end{cases}$$

**Proof.** Casual inspection reveals that for the parameters defined in the proposition, all numbers we claim to be probabilities are positive and sum to one. We also point out that the density of $G$ is given by:

$$f(x) = \frac{x}{2(1-q_b)(1-q_a-q_b)}$$

The support of $G$ is $[q_a, 2-q_a-2q_b]$, and for the parameters of the proposition, the top of the support is in $[0,1]$. To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each school is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player. A derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available upon request.

Consider school $B$'s expected payoff from a pure strategy $p$ in the support of its mixed strategy:

$$u_b = \begin{cases} \frac{q_b}{p}(1-\phi_1\int_p^{2-q_a-2q_b} f(s)\frac{q_a}{s}ds - \phi_2 q_a) + \frac{1}{2}(1-E[\frac{q_a}{g_a}])(1-\frac{q_b}{p}) & \text{if} \quad p \in [q_a, 2-q_a-2q_b] \\ q_b(1-\frac{\phi_2 q_a}{2}) + \frac{1}{2}(1-E[\frac{q_a}{g_a}])(1-q_b) & \text{if} \quad p = 1 \end{cases}$$

Substitution and simplification gives:

$$u_b = \begin{cases} \frac{q_b}{p}\frac{q_b^2+pq_a}{(q_a+q_b)^2} + \frac{1}{2}(\frac{2q_b^2}{(q_a+q_b)^2})(1-\frac{q_b}{p}) & \text{if} \quad p \in [q_a, 2-q_a-2q_b] \\ q_b(1-\frac{q_a}{2}(\frac{2q_b}{q_b+q_a})\frac{q_a+2q_b-1}{q_b(q_a+q_b)}) + \frac{1}{2}(\frac{2q_b^2}{(q_a+q_b)^2})(1-q_b) & \text{if} \quad p=1 \end{cases}$$

Further simplification gives $u_b = \frac{q_b}{q_a+q_b}$ in both cases. Thus all pure strategies in the support of school $B$ mixed strategy give the school the same expected payoff. Choosing any pure strategy $\hat{g} \in (2-q_a-2q_b, 1)$ is dominated by choosing $g = 2-q_a-2q_b$, because the probability of winning is the same for both pure strategies, but $g$ is more likely to generate a good realization. It is also straightforward to verify that choosing $q_b$ is dominated by the equilibrium mixed strategy. Thus all pure strategies in the support of $B$ mixed strategy give the same expected payoff against $A$ mixed strategy, and no strategy outside the support gives $B$ a higher payoff. Thus, $B$ mixed strategy is a best response to $A$'s mixed strategy.

Consider school $A$'s expected payoff from a pure strategy $p$ in the support of its mixed strategy:

$$u_a = \begin{cases} \frac{q_a}{p}(1-\lambda\int_p^{2-q_a-2q_b} f(s)\frac{q_b}{s}ds - (1-\lambda)q_b) + \frac{1}{2}(1-E[\frac{q_b}{g_b}])(1-\frac{q_a}{p}) & \text{if} \quad p \in [q_a, 2-q_a-2q_b] \\ q_a(1-\frac{(1-\lambda)q_b}{2}) + \frac{1}{2}(1-E[\frac{q_b}{g_b}])(1-q_a) & \text{if} \quad p=1 \end{cases}$$

Substitution and simplification gives:

$$u_a = \begin{cases} \frac{q_a}{p}(\frac{p+q_a}{2(q_a+q_b)}) + \frac{1}{2}\frac{q_a}{q_a+q_b}(1-\frac{q_a}{p}) & \text{if} \quad p \in [q_a, 2-q_a-2q_b] \\ q_a(1-\frac{q_b}{2}\frac{q_a+2q_b-1}{q_b(q_a+q_b)}) + \frac{1}{2}(\frac{q_a}{(q_a+q_b)})(1-q_a) & \text{if} \quad p=1 \end{cases}$$

Further simplification gives $u_a = \frac{q_a}{q_a+q_b}$ in both cases. Thus all pure strategies in the support of school $A$ mixed strategy give the school the same expected payoff. Choosing any pure strategy $\hat{g} \in (2-q_a-2q_b, 1)$ is dominated by choosing $g = 2-q_a-2q_b$, because the probability of winning is the same for both pure strategies, but $g$ is more likely to generate a good realization. Thus all pure strategies in the support of $A$ mixed strategy give the same expected payoff against $B$ mixed strategy, and no strategy outside the support gives $A$ a higher payoff. Thus, $A$ mixed strategy is a best response to $B$'s mixed strategy. ■

**Lemma B.6** *If $q_b \leq \frac{1}{2}(1-q_a)$ then the mixed strategy Nash equilibrium is as follows:*

$$g_a = \begin{cases} q_a & \textit{with probability} \quad 1-\phi = 1-\frac{2q_b}{q_b+q_a} \\ G \sim F(x) = \frac{x^2-q_a^2}{4q_b(q_a+q_b)} & \textit{with probability} \quad \phi = \frac{2q_b}{q_b+q_a} \end{cases}$$

$$g_b = \quad G \sim F(x) = \frac{x^2-q_a^2}{4q_b(q_a+q_b)} \quad \textit{with probability} \quad \frac{2q_b}{q_b+q_a}$$

**Proof.** Casual inspection reveals that for the parameters defined in the proposition, all numbers we claim to be probabilities are positive and sum to one. We also point out that the density of $G$ is given by:

$$f(x) = \frac{x}{2q_b(q_a+q_b)}$$

The support of $G$ is $[q_a, q_a + 2q_b]$, and for the parameters of the proposition, the top of the support is in $[0, 1]$. To show that the proposed strategies constitute a mixed strategy Nash equilibrium, we verify that each school is indifferent among all pure strategies inside the support and that no pure strategy outside the support delivers a better expected payoff against the mixed strategy of the other player. A derivation of the equilibrium from the indifference conditions, which also shows uniqueness, is available upon request.

Consider school $B$'s expected payoff from a pure strategy $p$ in the support of its mixed strategy:

$$u_b = \frac{q_b}{p}(1 - \phi \int_p^{q_a+2q_b} f(s)\frac{q_a}{s}ds) + \frac{1}{2}(1 - E[\frac{q_a}{g_a}])(1 - \frac{q_b}{p}) \quad \text{if} \quad p \in [q_a, q_a + 2q_b]$$

Substitution and simplification gives:

$$u_b = \frac{q_b}{p}(1 - \frac{2q_b}{q_a+q_b}q_a\frac{q_a+2q_b-p}{2q_b(q_a+q_b)}) + \frac{1}{2}(1 - \frac{2q_b}{q_a+q_b}\frac{q_a}{(q_a+q_b)} - (1 - \frac{2q_a}{q_a+q_b}))(1 - \frac{q_b}{p}) \quad \text{if} \quad p \in [q_a, q_a + 2q_b]$$

Further simplification gives $u_b = \frac{q_b}{q_a+q_b}$. Thus all pure strategies in the support of school $B$ mixed strategy give the school the same expected payoff. Choosing any pure strategy $\hat{g} \in (q_a + 2q_b, 1)$ is dominated by choosing $g = q_a + 2q_b$, because the probability of winning is the same for both pure strategies, but $g$ is more likely to generate a good realization. It is also straightforward to verify that choosing $q_b$ is dominated by the equilibrium mixed strategy. Thus all pure strategies in the support of $B$ mixed strategy give the same expected payoff against $A$ mixed strategy, and no strategy outside the support gives $B$ a higher payoff. Thus, $B$ mixed strategy is a best response to $A$'s mixed strategy.

Consider school $A$'s expected payoff from a pure strategy $p$ in the support of its mixed strategy:

$$u_a = \frac{q_a}{p}(1 - \int_p^{q_a+2q_b} f(s)\frac{q_b}{s}ds) + \frac{1}{2}(1 - E[\frac{q_b}{g_a}])(1 - \frac{q_a}{p}) \quad \text{if} \quad p \in [q_a, q_a + 2q_b]$$

Substitution and simplification gives:

$$u_a = \frac{q_b}{p}(1 - q_b\frac{q_a+2q_b-p}{2q_b(q_a+q_b)}) + \frac{1}{2}(1 - \frac{q_b}{(q_a+q_b)})(1 - \frac{q_a}{p}) \quad \text{if} \quad p \in [q_a, q_a + 2q_b]$$

Further simplification gives $u_a = \frac{q_b}{q_a+q_b}$. Thus all pure strategies in the support of school $A$ mixed strategy give the school the same expected payoff. Choosing any pure strategy $\hat{g} \in (q_a + 2q_b, 1)$ is dominated by choosing $g = q_a + 2q_b$, because the probability of winning is the same for both pure strategies, but $g$ is more likely to generate a good realization. Thus all pure strategies in the support of $A$ mixed strategy give the same expected payoff against $B$ mixed strategy, and no strategy outside the support gives $A$ a higher payoff. Thus, $A$ mixed strategy is a best response to $B$'s mixed strategy. ∎

**Evaluator payoff with fixed investment**. Here we calculate the evaluator payoff for symmetric investment levels for symmetric fixed investments $q_a = q_b = q$. Because we consider symmetric schools, the equilibrium of the investment stage will also be symmetric. This calculation will therefore facilitate the comparison of evaluator payoff under inflation to the evaluator payoff in the fully-revealing benchmark. Given the realizations of $(g_a, g_b)$ generates by the schools' equilibrium mixed strategies, define $g_m = max[g_a, g_b]$ and $g_n = min[g_a, g_b]$. The evaluator's expected payoff for a particular combination of $(g_m, g_n)$ is given by

$$g_m \frac{q}{g_m} + (1 - \frac{q}{g_m}) g_n \frac{q}{g_n} = q(2 - \frac{q}{g_m})$$

If the graduate of the school with less inflated grading policy $g_m$ generates a good transcript, an event that happens with probability $\frac{q}{g_m}$, then the evaluator will accept that graduate, giving the evaluator payoff $g_m$. If the graduate generates a bad transcript, then the evaluator knows for sure that he is low ability. If the other graduate generates a good transcript realization, (probability $\frac{q}{g_n}$) the evaluator accepts the graduate of the school with the more-inflated grading policy, giving the school expected payoff $g_n$.

In order to evaluate the evaluator payoff, we need to determine the expected value of the inverse of the maximum order statistic from the equilibrium mixed strategies.

Case I: $q \le \frac{1}{3}$. When $q_a = q_b = q \le \frac{1}{3}$, the equilibrium mixed strategy of each school is to randomize over support $[q, 3q]$ using distribution function $F(x) = \frac{x^2 - q^2}{8q^2}$ with corresponding density $f(x) = \frac{x}{4q^2}$. The expectation in question, $E[\frac{1}{g_m}]$ is therefore[37]

$$\int_q^{3q} 2(\frac{1}{x})(\frac{x^2 - q^2}{8q^2})\frac{x}{4q^2} dx = \frac{5}{12q}$$

Thus the evaluator expected payoff is $q(2 - q\frac{5}{12q}) = \frac{19}{12}q$.

Case II: $\frac{1}{3} \le q \le \frac{1}{2}$. When $\frac{1}{3} \le q \le \frac{1}{2}$, the equilibrium mixed strategy of each school is as follows. With probability $\phi = \frac{3q-1}{2q^2}$ choose $g = 1$. With probability $1 - \phi$ randomize over support $[q, 2 - 3q]$ using distribution function $F(x) = \frac{x^2 - q^2}{4(1-q)(1-2q)}$ with corresponding density $f(x) = \frac{x}{2(1-q)(1-2q)}$.

The expectation in question, $E[\frac{1}{g_m}]$ is therefore

$$1 - (1-\phi)^2 + (1-\phi)^2 \int_q^{2-3q} 2(\frac{1}{x})(\frac{x^2 - q^2}{4(1-q)(1-2q)})(\frac{x}{2(1-q)(1-2q)}) dx = \frac{1}{12q^4}(32q^3 - 27q^2 + 9q - 1)$$

Thus the evaluator expected payoff is $q(2 - q\frac{1}{12q^4}(32q^3 - 27q^2 + 9q - 1)) = \frac{1}{12q^2}(1 - 9q + 27q^2 - 8q^3)$.

Case III: $q \ge \frac{1}{2}$. In this case schools use fully revealing grading strategies in the second stage.

**Equilibrium investment with inflation only**. We now turn to the investment decision in stage one. To streamline the exposition we focus on the case of schools with identical marginal costs of education, $\rho_i = \rho$. At the investment stage, anticipating the stage two equilibrium, the

[37] The density of the maximum order statistic $g_m$ is $2f(x)F(x)$

payoff function for school $i$ is

$$u_i(q_i, q_j) = \begin{cases} \frac{q_i}{q_i+q_j} - \frac{q_i^2}{\rho^2} & \text{if} \quad q_i \leq 1 - q_j \\ \frac{1}{2}(1 + q_i - q_j) - \frac{q_i^2}{\rho^2} & \text{if} \quad q_i > 1 - q_j \end{cases}$$

This payoff function is continuous, and is differentiable, everywhere except for possibly $q_i = q_j$. The Nash equilibrium of the investment stage is given in the following Lemma

**Lemma B.7** *If $\rho \leq \sqrt{2}$ then the Nash equilibrium of the first stage game is $q_i = q_j = \frac{\sqrt{2}}{4}\rho$.*

**Proof.** Suppose that $\rho \leq \sqrt{2}$, and $q_j = \frac{\sqrt{2}}{4}\rho$. The best deviation greater that $1 - \frac{\sqrt{2}}{4}\rho$ is given by the solution to the following maximization problem:

$$max \ \tfrac{1}{2}(1 + q_i - \tfrac{\sqrt{2}}{4}\rho) - \tfrac{q_i^2}{\rho^2} \quad \text{subject to} \quad q_i > 1 - \tfrac{\sqrt{2}}{4}\rho$$

This function is differentiable and concave. The unique critical point of function $\frac{1}{2}(1+q_i-\frac{\sqrt{2}}{4}\rho)-\frac{q_i^2}{\rho^2}$ is equal to $\frac{1}{4}\rho^2$. However, because $\rho \leq \sqrt{2}$, this point is infeasible:

$$\rho < \sqrt{2} \rightarrow (\rho - \sqrt{2})(\rho + 2\sqrt{2}) < 0$$

$$\rho^2 + \sqrt{2}\rho - 4 < 0$$

$$\rho^2 < 4 - \sqrt{2}\rho$$

$$\frac{1}{4}\rho^2 < 1 - \frac{\sqrt{2}}{4}\rho$$

Because the critical point is less than the smallest feasible value, the maximum occurs at the left endpoint, which gives a payoff of

$$\frac{1}{2}(1 + (1 - \frac{\sqrt{2}}{4}\rho) - \frac{\sqrt{2}}{4}\rho) - \frac{(1 - \frac{\sqrt{2}}{4}\rho)^2}{\rho^2} = \frac{\sqrt{2}}{4\rho^2}(2\sqrt{2} - \rho)(\rho^2 + \frac{\sqrt{2}}{4}\rho - 1)$$

If no deviation in the region $q_i \leq 1 - \frac{\sqrt{2}}{4}\rho$ dominates $q_i = \frac{\sqrt{2}}{4}\rho$, and the payoff associated with playing $q_i = \frac{\sqrt{2}}{4}\rho$ as a response to $q_j = \frac{\sqrt{2}}{4}\rho$ gives a higher payoff than the above expression, then $q_i = q_j = \frac{\sqrt{2}}{4}\rho$ is the unique Nash equilibrium. Consider now the best deviation in the region $q_i \leq 1 - \sqrt{2}4\rho$, defined by the following maximization problem:

$$max \ \frac{q_i}{q_i + \frac{\sqrt{2}}{4}\rho} - \frac{q_i^2}{\rho^2} \quad \text{subject to} \quad q_i \leq 1 - \tfrac{\sqrt{2}}{4}\rho$$

This function is differentiable and concave. The max therefore occurs at the critical point, provided this point is feasible. The first order condition is given by:

$$\frac{1}{\rho^2(q_i + \frac{\sqrt{2}}{4}\rho)^2}(\frac{\sqrt{2}}{4}\rho^3 - \frac{1}{4}\rho^2 q_i - \sqrt{2}\rho q_i^2 - 2q_i^3)$$

substituting $q_i = \frac{\sqrt{2}}{4}\rho$ reveals that the derivative is zero, and therefore that, if feasible, this level of $q_i$ is the best value less than $1 - \frac{\sqrt{2}}{4}\rho$. In fact, because $\rho \leq \sqrt{2}$, $\frac{\sqrt{2}}{4}\rho \leq \frac{1}{2}$, and thus $\frac{\sqrt{2}}{4}\rho \leq 1 - \frac{\sqrt{2}}{4}\rho$, so this value is feasible. Finally, we compare the payoff of choosing $q_i = \frac{\sqrt{2}}{4}\rho$ to the best deviation in the other interval. The payoff of choosing $q_i = \frac{\sqrt{2}}{4}\rho$ is given by $\frac{1}{2} - \frac{(\frac{\sqrt{2}}{4}\rho)^2}{\rho^2} = \frac{3}{8}$. The difference in the payoffs is therefore

$$\frac{3}{8} - \frac{\sqrt{2}}{4\rho^2}(2\sqrt{2} - \rho)(\rho^2 + \frac{\sqrt{2}}{4}\rho - 1) = \frac{\sqrt{2}}{4\rho^2}(\rho + \sqrt{2})(\rho - \sqrt{2})^2 \geq 0$$

Thus, whenever $\rho \leq \sqrt{2}$, choosing $q_i = q_j = \frac{\sqrt{2}}{4}\rho$ is a Nash equilibrium. ■

### B.3 COMPARISON OF INFLATION-ONLY EQUILIBRIUM WITH FULLY-REVEALING EQUILIBRIUM

**Investment comparison.** In the equilibrium with symmetric abilities ($\rho_\alpha = \rho_\beta = \rho \leq \sqrt{2}$) above, schools invest $\frac{\sqrt{2}}{4}\rho$. For these parameters, school investment in the fully-revealing benchmark is $\frac{\rho^2}{4}$. Whenever $\rho \leq \sqrt{2}$, investment is greater in the inflationary equilibrium.

**evaluator payoff comparison with fixed investment.** Here we show that, as in the general model, when investments are fixed, the evaluator receives a higher payoff when schools are constrained to use fully-revealing grading policies.

The difference between the evaluator expected payoff in the fully-revealing benchmark and the inflation equilibrium is given by

$$\begin{array}{ll} 1 - (1-q)^2 - \frac{19}{12}q = \frac{1}{12}q(12q - 5) & \text{if} \quad q \leq \frac{1}{3} \\ 1 - (1-q)^2 - \frac{1}{12q^2}(1 - 9q + 27q^2 - 8q^3) = \frac{1}{4q^2}(1-2q)^2(q - \frac{\sqrt{13}}{6} - \frac{5}{6})(q + \frac{\sqrt{13}}{6} - \frac{5}{6}) & \text{if} \quad \frac{1}{3} \leq q \leq \frac{1}{2} \\ 0 & \text{if} \quad q > \frac{1}{2} \end{array}$$

Casual inspection reveals that the payoff from the revealing benchmark $1 - (1-q)^2$ is greater than the equilibrium payoff, whenever the equilibrium is not itself fully-revealing, $q \leq \frac{1}{2}$.

**evaluator payoff comparison with endogenous investment.** Here we show that, as in the general model, when investments are endogenous, the evaluator receives a (weakly) higher payoff when schools are permitted to inflate grades. We consider $\rho_\alpha = \rho_\beta = \rho \leq \sqrt{2}$. In this case equilibrium investment with inflation is given by $\frac{\sqrt{2}}{4}\rho$, while fully revealing investment is $\frac{\rho^2}{4}$.

The difference between the evaluator expected payoff in the fully-revealing benchmark and the inflation equilibrium is given by

$$\begin{array}{ll} 1 - (1 - \frac{\rho^2}{4})^2 - \frac{19}{12}\frac{\sqrt{2}}{4}\rho = \frac{1}{12}q(12q - 5) & \text{if} \quad \rho \leq \frac{2}{3}\sqrt{2} \\ 1 - (1 - \frac{\rho^2}{4})^2 - \frac{1}{12(\frac{\sqrt{2}}{4}\rho)^2}(1 - 9\frac{\sqrt{2}}{4}\rho + 27(\frac{\sqrt{2}}{4}\rho)^2 - 8(\frac{\sqrt{2}}{4}\rho)^3) & \text{if} \quad \frac{2}{3}\sqrt{2} \leq \rho \leq \sqrt{2} \end{array}$$

Plotting these payoff functions clearly shows that when investment is endogenous, evaluator payoff is higher when grade inflation is allowed.

## B.4 UNIQUENESS IN LEMMA 4.2 (STRATEGIC GRADING EQUILIBRIUM)

In this section we derive the equilibrium of the stage 2 game with general grading policies from first principles and demonstrate that the equilibrium is unique.

A strategy for player $j$, is a random variable $\Gamma_j$ with support contained in the unit interval, and expectation $q_j$. Furthermore, (as discussed above) because the underlying signal structure is valid, $\Gamma_j$ has a finite number of mass points $m_k^j$, contained in set $M_j$. Let $\mu_k^j = Pr(\Gamma_j = m_k^j)$. Ignoring mass points, random variable $\Gamma_j$ has support on a (closed) interval $I_j$. Denote $support(\Gamma_j) = I_j \cup M_j$ as $S_j$.

Let $W_j(x)$ represent the probability that graduate $i$ is selected when $i$ posterior belief realization is equal to $x$.

$$W_j(x) = Pr(\Gamma_j < x) + \frac{1}{2}Pr(\Gamma_j = x)$$

Thus, for any point inside $x \in S_j$ this function i given by the following expression:

$$W_j(x) = \begin{cases} P_j(x) & \text{if} \quad x \in I_j \cap M_j^C \\ P_j(x) - \frac{1}{2}\mu_k^j & \text{if} \quad x = m_k^j \end{cases} \tag{2}$$

Here we use $X^C$ to represent the complement of set $X$. Also note that function $W_j(x)$ maintains a constant value in any interval that does not intersect $S_j$. This is because $P_j(x)$ is neither increases nor decreases outside of set $S_j$.

As is the case for $\Gamma_j$ the support of a strategy for player $i$, is $S_i = I_i \cup M_i$, where $I_i$ represents an interval and $M_i$ represents a finite set of mass points; $m_k^i$ represents mass point $k$ and $\mu_k^i$ represents the mass on point $m_k^i$.The best response of player $i$ to $\Gamma_j$ is a choice of random variable $\Gamma_i$ with generalized density $p_i(x)$ to solve the following maximization:

$$max \int_0^1 p_i(x)W_j(x)dx$$

subject to

$$\int_0^1 p_i(x)xdx = q_i$$

Consider the Lagrangian for this problem:

$$L = \int_0^1 p_i(x)(W_j(x) - \lambda_i(x - \gamma_i))dx$$

Standard maximization principles require that the value of the integrand of the Lagrangian is the same at any value of $x$ inside the support of $\Gamma_i$ and no value outside of the support of $\Gamma_i$ gives a higher value. Therefore:

$$L_i(x) \equiv W_j(x) - \lambda_i(x - q_i) \tag{3}$$

$$x \in S_i \Rightarrow L_i(x) = v_i \tag{4}$$

$$x \notin S_i \Rightarrow L_i(x) \leq v_i$$

Equation 4 implies several properties of best-responses.

**Properties of best responses**

1. If $\Gamma_i$ is a best response to $\Gamma_j$, its interval support $I_i$ is a weak subset of the interval support of $\Gamma_j$: $I_i \subseteq I_j$

   Suppose $I_i \cap I_j^C$, is non-empty. If so, it contains an interval. Outside $I_j$, only support of $\Gamma_j$ is a finite set of mass points. Hence, exists a subinterval in $I_i \cap I_j^C$ that does not intersect $S_j$. On this interval, however, $W_j(x)$ is constant, while $\lambda_i(x - q_i)$ is increasing, and $L_i(.)$ is non-constant. Contradicts equation (4).

2. For any best response $\Gamma_i$, if $\Gamma_j$ has a mass point on $m_j \in M_j$, then there exists $\epsilon$ such that $(m_j - \epsilon, m_j)$ does not intersect $I_i$.

   Because $W_j(x)$ jumps up at $m_j$ but $\lambda_i x$ does not, $L_i(x)$ jumps up at $m_j$. Hence, any value of $x \geq m_j$ can not give the same value of $L(.)$ as a value of $x \in (m_j - \epsilon, m_j)$.

3. If $\Gamma_j$ does not have a mass point on 1 and the right endpoint of $I_j$ is strictly less than 1, then player $i$ best response $\Gamma_i$ does not have a mass point on 1.

   If $\Gamma_j$ does not have a mass point on 1 and the right endpoint of $I_j$ is strictly less than 1, then for sufficiently small $\epsilon$, $L_i(1 - \epsilon) > L_i(1)$.

These properties have significant implications for the structure of possible equilibria.

**Properties of Equilibrium**

1. In equilibrium no mass point inside $[0, 1)$ can be common to both $\Gamma_i$ and $\Gamma_j$. Suppose a mass point exists at $m$, and $Pr(\Gamma_j = m) = \mu$. $W_j(.)$, the probability of winning for player $i$, jumps up at $m$ by $\mu/2$. The second component of the Lagrangian, $\lambda_i(x - q_i)$, is continuous. Hence, $L_i(.)$ jumps up at $m$ by $\mu/2$ i.e. for any $\epsilon > 0$, $L_i(m + \epsilon) > L(m)$. If exists $\epsilon$ for which $m + \epsilon < 1$, then this contradicts condition (4). Hence, only possible common mass point is $m = 1$.

2. In equilibrium $I_i = I_j$.

   Direct consequence of point 1. in properties of best responses.

3. In equilibrium, the smallest element of the (identical) interval support $I$, is 0.

   In equilibrium, both random variables are supported on the same interval $I$. Suppose the smallest element of $I$, denoted $\underline{x}$ is strictly above 0. Because no common mass point exists in $[0, 1)$ at most one of $\Gamma_i$ and $\Gamma_j$ can have a mass point on $\underline{x}$. If exactly one has a mass point on $\underline{x}$, let $\Gamma_i$ be the random variable with no mass point on $\underline{x}$. Besides interval $I$, $\Gamma_i$ is supported on a set of mass points. This implies that for sufficiently small $\epsilon$, no mass point exists between $\underline{x} - \epsilon$ and $\underline{x}$. For $\Gamma_i$, no mass point exists on $\underline{x}$, and, because $\underline{x}$ is the smallest element of $I$, $F_j(\underline{x}) = 0$. Hence $W_j(\underline{x}) = W_j(\underline{x} - \epsilon)$. However, the second component of the Lagrangian is decreasing. Thus $L(\underline{x} - \epsilon) > L(\underline{x})$. This contradicts condition 4.

4. In equilibrium, no mass point exists in $(0, 1)$ for either player.

   In equilibrium each player's strategy has the same interval support $I = [0, r]$. By point 2. of properties of best responses, no mass point can exist in interval $(0, r]$. Otherwise, a gap must exist in the interior of $I$. However, if a player has mass point above $r$, then it must be shared with the other player. If it is not shared with the other player, then a point just below generates the same winning probability, but lower $\lambda_i(x - q_i)$ and hence a greater value of $L(.)$. However, according to property of equilibrium 1, only possible common mass point is 1.

5. In equilibrium, if $\Gamma_i$ has a mass point on 1, then $\Gamma_j$ also has a mass point on 1.

   f $i$ has a mass point on 1, then exists an interval $(1 - \epsilon, 1)$ outside of the support of $\Gamma_j$. If no mass point on 1 is part of $j$'s strategy, then, because the probability of $i$ winning is constant outside of $S_j$, then there exists $\epsilon$ for which $L_i(1 - \epsilon) > L_i(1)$, contradicting condition (4).

These conditions partially characterize equilibrium strategies: Conditions 1.-4. characterize the structure of equilibrium strategies. An equilibrium strategy for player $k \in \{a, b\}$ must have the following structure:

$$\Gamma_k = \begin{cases} \Phi_k & \text{with probability} \quad m_k \\ 1 & \text{with probability} \quad n_k \end{cases}$$

where $m_k + n_k = 1$ and $\Phi_k$ is a random variable with support over an interval $I = [0, r]$ and no mass points, except possibly at 0. Note, however, that at most one equilibrium strategy can have a mass point at zero. The CDF of $\Phi_k$ is given by $F_k(x)$, where $F_k(x)$ is continuous (and differentiable) and $F_k(\theta) \geq 0$ and $F_k(r) = 1$. In this case the win-probability for player $i$ has the following structure:

$$W_j(x) = \begin{cases} m_k F_j(x) & \text{if} \quad x \in [0, r] \\ 1 - \frac{n_j}{2} & \text{if} \quad x = 1 \end{cases}$$

The above structure allows us to simplify condition (4). For $i, j \in \{a, b\}$ and $i \neq j$, condition (4) reduces to the following:

$$m_j F_j(x) - \lambda_i(x - q_i) = v_i \text{ for all } x \in [0, r] \tag{5}$$

$$n_i > 0 \Rightarrow 1 - \frac{n_j}{2} - \lambda_i(1 - q_i) = v_i$$

The first part of this condition therefore implies that:

$$F_j(x) = \frac{v_i - \lambda_i q_i}{m_j} + \frac{\lambda_i}{m_j} x$$

$$F_i(x) = \frac{v_j - \lambda_j q_j}{m_i} + \frac{\lambda_j}{m_i} x$$

Thus, random variable $\Phi_k$ must be uniformly distributed with a possible mass point on 0. However, because it can not be that both $i, j$ have a mass point on 0 (No common mass points in $[0, 1)$), at most one player has a mass point on 0. Let this be player $j$.

$$F_i(0) = 0 \text{ and } F_j(0) \geq 0 \Leftrightarrow$$

$$\frac{v_j - \lambda_j q_j}{m_i} = 0 \text{ and } \frac{v_i - \lambda_i q_i}{m_j} \geq 0.$$

Neither player can have a mass point at $r$. Therefore,

$$F_i(r) = F_j(r) = 1 \Leftrightarrow$$

$$\frac{v_j - \lambda_j q_j}{m_i} + \frac{\lambda_j}{m_i} r = 1 \text{ and } \frac{v_i - \lambda_i q_i}{m_j} + \frac{\lambda_i}{m_j} r = 1$$

In addition each strategy must satisfy the appropriate mean constraint. Given the above conditions,

$$E[\Phi_i] = \frac{\lambda_j r^2}{2m_i} \text{ and } E[\Phi_j] = \frac{\lambda_i r^2}{2m_j}$$

Therefore the mean constraints are:

$$m_i E[\Phi_i] + n_i = q_i \Leftrightarrow \lambda_j(\frac{r^2}{2}) + n_i = q_i$$

$$m_j E[\Phi_j] + n_j = q_j \Leftrightarrow \lambda_i(\frac{r^2}{2}) + n_j = q_j$$

Thus every equilibrium must satisfy the following conditions (SC) (collected from above):

$$\frac{v_j - \lambda_j q_j}{m_i} = 0 \text{ and } \frac{v_i - \lambda_i q_i}{m_j} \geq 0$$

$$\frac{v_j - \lambda_j q_j}{m_i} + \frac{\lambda_j}{m_i} r = 1 \text{ and } \frac{v_i - \lambda_i q_i}{m_j} + \frac{\lambda_i}{m_j} r = 1$$

$$\lambda_j(\frac{r^2}{2}) + n_i = q_i \text{ and } \lambda_i(\frac{r^2}{2}) + n_j = q_j$$

$$m_i + n_i = 1 \text{ and } m_j + n_j = 1$$

Next, observe that the equilibrium properties imply that only two equilibrium structures are possible. In the first, $n_i = n_j = 0$, and in the other $n_i > 0$ and $n_j > 0$.

**Case I**: $n_i = n_j = 0 \Rightarrow m_i = m_j = 1$. In this case, system (SC) reduces to the following:

$$v_j = \lambda_j q_j \text{ and } v_i \geq \lambda_i q_i$$

$$\lambda_j r = 1 \text{ and } v_i - \lambda_i q_i + \lambda_i r = 1$$

$$\lambda_j (\frac{r^2}{2}) = q_i \text{ and } \lambda_i (\frac{r^2}{2}) = q_j$$

The unique solution of this system is

$$r = 2q_i \text{ and } \lambda_i = \frac{q_j}{2q_i^2} \text{ and } \lambda_j = \frac{1}{2q_i} \text{ and } v_i = \frac{q_j - 2q_i}{2q_i} \text{ and } v_j = \frac{q_j}{2q_i}$$

Finally, $v_i - \lambda_i q_i = 1 - q_j/q_i$. In order for the required inequality to hold, it must be that $q_j \leq q_i$; hence, school $A$ plays the role of school $i$. This is exactly the equilibrium described in the first part of Lemma 4.2, and this equilibrium is therefore unique.

**Case II**: $n_i > 0, n_j > 0$. In addition to conditions (SC), we also obtain the following indifference conditions:

$$1 - \frac{n_j}{2} - \lambda_i (1 - q_i) = v_i \text{ and } 1 - \frac{n_i}{2} - \lambda_j (1 - q_j) = v_j$$

The solution of the system of equations is given by:

$$r = 2(1 - q_i) \text{ and } \lambda_i = \frac{1}{2q_j^2} \text{ and } \lambda_j = \frac{1}{2q_j} \text{ and } m_i = \frac{1}{q_i} - 1 \text{ and } m_j = \frac{q_j + q_i^2 - 2q_i q_j}{q_i^2}$$

$$n_i = 2 - \frac{1}{q_i} \text{ and } n_j = \frac{q_j}{q_i}(2 - \frac{1}{q_i})$$

Finally, $v_i \geq \lambda_i q_i \Rightarrow q_i \geq q_j$. Hence, school A plays the role of school $i$. This is exactly the equilibrium described in the second part of the Lemma 4.2, and this equilibrium is therefore unique.