Are Deep Parameters Policy-Invariant?

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Abstract

To see the variance of the intertemporal elasticity of substitution, we ran an experiment described by the following steps. First, we calibrate several models with heterogeneous agents and generate aggregate time series data. Then, we estimate the elasticity in the New Keynesian Model using the data from former models. Finally, we check and compare the estimated parameters. Our main finding is that there is some possiblity of misestimating the parameter due to the differences of fiscal policy regimes and heterogeneous agents.

1 Introduction

One of the main advantages of modern DSGE models is that the models have so-called deep parameters which are thought to be invariant against policy change, and because of the stability of these parameters, economists could evaluate economic policy. In other words, when we evaluate an economic policy using the DSGE model, we implicitly assume that the model does not have to be subject to the critique of Lucas (1976).

However, several studies indicate the instability of deep parameters. Fernandez-Villaverde, Rubin-Ramirez, Conley, and Schorfheide (2007) estimated a DSGE model which allowed for parameters drifting. They found evidence that deep parameters changed in response to monetary policy within the sample periods in the U.S. Furthermore, they showed that the movements in the pricing parameters are correlated with inflation.

Similarly, Inoue and Rossi (2008) investigated the instability of structural parameters within the DSGE and VARs models, and they found that such instabilities are not only a concern for the monetary policy function but also for the IS equation and Euler equation. Conley and Yagihashi (2010) generated simulated data using a model with state-dependent pricing, and then used the data to estimate a model with time-dependent pricing. They found that the private-sector parameters are unstable against the monetary policy.

Chang, Kim and Schorfheide (2013) also ran an experiment similar to Conley and Yagihashi (2010). First, they simulated a model with heterogeneous agents and generated the

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aggregate time series data. After that, they estimated a representative agent model using the data from the heterogeneous agents economy and found that several important parameters, especially the labor supply elasticity, are not invariant against the fiscal policy change.

This paper aims to examine the elasticity of intertemporal substitution in consumption (which is one of the most important preference parameters for fiscal policy because it formulates the shape of the Euler equation) to determine if it is policy variant or not, and if so, to understand what kind of fiscal policy we could evaluate using the New Keynesian Model.

To do that, we ran an experiment similar to Chang, Kim and Schorfheide (2013) and checked the variance of the parameter. As pointed out in the empirical study of Inoue and Rossi (2008) empirical study, the parameters of the Euler equation are not stable in the postwar U.S. However, we do not know the theoretical reason of the instability. Different from Chang, Kim and Schorfheide (2013), we reduced the differences between the data-generating model and the estimation model, and tried to simplify those models as much as possible. In this way, we could identify the misestimation that comes from the imperfect aggregation of households. Our experiment is described by the following steps: Calibrate and simulate several heterogeneous agent models whose differences are types of redistributive regimes.

We use four types of regimes in our experiment, so we run four true models for data generation. Then, we gain the time series data of aggregate production, consumption, capital stock, and productivity, according to the four regimes. Using the data from the heterogeneous agents economy, estimate the elasticity of the intertemporal substitution of the New Keynesian Model. Compare the estimated values between the policy regimes. If the value is different from that of another economy, the parameter is misestimated.

The rest of this paper is organized as follows. In Section 2, I introduce the data-generating model with heterogeneous agents that feature an incomplete capital market and the agents' motive for precautionary saving. After that, we calibrate the economy and obtain the aggregate time series data. In section 3, I describe the estimation model, which features rational and nonrational agents and a complete capital market. In section 4, we estimate the estimation model using data generated from the preceding model and check the policy variance of the elasticity of intertemporal substitution. Section 5 concludes with some suggestions for future work.

2 The Data Generating Model

The model is a version of the stochastic-growth model with heterogeneous agents. In the economy, there is a large (measure one) population of finitely lived agents whose preferences and ex-ante wealth levels are identical. They maximize their intertemporal utility[1].

[1]Krusell and Smith(2006) and Heathcote, Storesletten and Violante(2009) survey the model with heterogeneous agents. Ljungqvist and Sargent(2012), Guvenen(2009), and Heer and Maussner(2009) describe the model in detail.

2.1 Model Specification

Households.

The model is a version of the stochastic-growth model with heterogeneous agents. In the economy, there is a large (measure one) population of finitely lived agents whose preferences and ex-ante wealth levels are identical. They maximize their intertemporal utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2.1}$$

with

$$u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}. \tag{2.2}$$

where c_t is their consumption and θ denotes the coefficient of relative risk-aversion which is the inverse of the elasticity of intertemporal substitution in consumption. Households could be employed or unemployed. If the agent is employed, he earns wage w_t, while if unemployed, he receives unemployment compensation b_t from the government. Then, the household's budget constraint is described as

$$a_{t+1} = \begin{cases} (1+(1-\tau_K)r_t)a_t + (1-\tau_L)w_t - c_t, & if \quad \epsilon_t = e, \\ (1+(1-\tau_K)r_t)a_t + b_t - c_t, & if \quad \epsilon_t = u, \end{cases} \tag{2.3}$$

where a_t is his asset level which yields the real interest rate r_t . The income from the real interest and wage are taxed at fixed rates τ_K and τ_L. The idiosyncratic labor risks ϵ_t ($e = employd, u = unemployed$) vary exogeneously, following a first order Markov process with the transition matrix

$$\pi(\epsilon' \mid \epsilon) = p\left\{\epsilon_{t+1} = \epsilon' \mid \epsilon_t = \epsilon\right\} = \begin{pmatrix} p_{uu} & p_{ue} \\ p_{eu} & p_{ee} \end{pmatrix}, \tag{2.4}$$

Additionally, we assume that $a_{t+1} \geq a_{min}$ where a_{min} is th*e natural borrowing limit* introduced by Aiyagari(1994).

Firms.

There is only one good per period, and production Y_t is determined by the Cobb-Douglas function of capital input K_t and labor input L_t and the aggregate productivityZ_t: $Y_t = Z_t L_t^{1-\alpha} K^{\alpha}$. We assume that Z_t could take only two values Z_g and Z_b whose probability follows a first order Markov process

$$\pi(Z'|Z) = p\left\{Z_{t+1} = Z'|Z_t = Z\right\} = \begin{pmatrix} p_{Z_g Z_g} & p_{Z_g Z_b} \\ p_{Z_b Z_g} & p_{Z_b Z_b} \end{pmatrix}. \tag{2.5}$$

In a market equilibrium, the factor prices satisfy the first order conditions of the firm

$$w_t = Z_t(1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha}, \tag{2.6}$$

$$r_t = Z_t\alpha\left(\frac{N_t}{L_t}\right)^{1-\alpha} - \delta. \tag{2.7}$$

Government.

Redistributive fiscal policies are characterized by labor tax rates τ_L, capital tax rates τ_K, and the level of lump-sum transfersb_t. The revenue of the government T_t is

$$T_t = \tau_L w_t L_t + \tau_K r_t K_t, \tag{2.8}$$

and the total amount of transfers B_t is

$$B_t = \int b_t f_t(u_t, a_t) da_t, \tag{2.9}$$

and balanced government budget

$$T_t = B_t. \tag{2.10}$$

Market Clearing Conditions.

The sum of households' asset and consumption are equal to aggregate capital and consumption

$$K_t = \sum_{\epsilon_t \in \{e,u\}} \int a_t f_t(\epsilon_t, a_t) da_t, \tag{2.11}$$

$$C_t = \sum_{\epsilon_t \in \{\epsilon.u\}} \int c_t f_t(\epsilon_t, a_t) da_t, \tag{2.12}$$

and the sum of employed households' equal to aggregate labor supply

$$L_t = \int f_t(e_t, a_t) da_t, \tag{2.13}$$

where f_t is the density function associated with the distribution function F_t.

Dynamics of the Distribution.

From (2.4) and (2.5), the employment transition probability follows

$$\Gamma(Z',\epsilon'|Z,\epsilon) = p\left\{Z_{t+1} = Z',\, \epsilon_{t+1} = \epsilon'|Z_t = Z,\, \epsilon_t = \epsilon\right\} = \begin{pmatrix} p_{Z_geZ_ge} & p_{Z_geZ_gu} & p_{Z_geZ_be} & p_{Z_geZ_bu} \\ p_{Z_guZ_ge} & p_{Z_guZ_gu} & p_{Z_guZ_be} & p_{Z_guZ_bu} \\ p_{Z_beZ_ge} & p_{Z_beZ_gu} & p_{Z_beZ_be} & p_{Z_beZ_bu} \\ p_{Z_buZ_be} & p_{Z_buZ_gu} & p_{Z_buZ_be} & p_{Z_buZ_bu} \end{pmatrix}, \tag{2.14}$$

and then, the dynamics of the distribution are described by

$$F_{t+1}(\epsilon_{t+1}, a_{t+1}; Z_{t+1}, K_{t+1}) = \sum_{\epsilon_t \in \{e,u\}} \Gamma(Z_{t+1}\epsilon_{t+1} \mid Z_t, \epsilon_t) F_t(\epsilon_t, a_t; Z_t, K_t). \tag{2.15}$$

Approximate Aggregation.

From (2.1) to (2.15), we could formulate the recursive problem. The households maximize their value function

$$V(\epsilon, a, Z, F) = \max_{c,a'} \left[u(c) + \beta E\left\{V(\epsilon', a', Z', F'|\epsilon, Z, F\right\}\right], \tag{2.16}$$

subject to the budget constraint (2.4), the employment transition probability (2.14), and the dynamics of the distribution (2.15). However, we could not solve this problem because the distribution F and the employment transition probability Γ are high-dimensional object, and it is well known that finding a numerical solution for dynamic programming problems becomes increasingly difficult as the size of the state space increases. To solve the problem, we follow Krusell and Smith(1998) and assumed that agents are boundedly rational[2]. We suppose that agents perceive current or future distribution depending on only I moments of Γ. Then, the law of motion for I 's moments m

$$m' = H_I(m) \tag{2.17}$$

where H_I is a dynamics of I's moments m. We rather follow Krusell and Smith(1998), and assume (2.17) is a log linear law of motion

$$lnK' = \gamma_0 + \gamma_1 lnK. \tag{2.18}$$

To simplify the dynamics of aggregate employment, we also assume that the transition matrix $\Gamma$is restricted

$$u_z \frac{p_{ZuZ'u}}{p_{ZZ'}} + (1 - uz_Z)\frac{p_{ZeZ'u}}{p_{ZZ'}} = uz' \tag{2.19}$$

for $Z, Z' \in \left\{Z_g, Z_b\right\}$. The condition (2.19) implies that we do not have to consider employment L as a state variable. Then, the value function (2.16) could be rewrite as

[2]There are other methods to solve the problem. For example, Algan, Yann, and Den Haan(2008) and Reiter(2009).

$$V(\epsilon, a, Z, m) = \max_{c,a'} \left[u(c) + \beta E \left\{ V(\epsilon', a', Z', m' | \epsilon, Z, m \right\} \right]. \tag{2.20}$$

Now we could solve the problem (2.20) using numerical methods.

2.2 Calibration

TABLE 1. HOUSEHOLDS' PARAMETERS

α	Labor share	0.36
β	Discount rate	0.96
δ	Capital depreciate rate	0.10
θ	Coefficient of relative risk aversion	1.50

We now specify the parameter values of the heterogeneous agents economy. The parameters are selected on the basis of empirical and theoretical knowledge, and the models used to generate quantitative statements. Table 1. illustrate the calibrated parameter values. The parameters are selected for a model period equal to one year. The preference parameters α,β,δ,θ are consistent with postwar U.S. which taken from Castaneda, Dıaz-Giménez, and Rıos-Rull(1998). We assume that the natural borrowing limit $a_{min} = -2.00$, and the aggregate productivity takes only two values $z_g = 1.10$ and $Z_b = 0.90$, and the average duration of a boom and recession are 5 years. Then, the transition matrix of Z (2.5) is equal to

$$\pi(Z'|Z) = prob \left\{ Z_{t+1} = Z' | Z_t = Z \right\} = \begin{pmatrix} 0.80 & 0.20 \\ 0.20 & 0.80 \end{pmatrix}. \tag{2.21}$$

The conditional employment probabilities are also taken from Castaneda, Dıaz-Giménez, and Rıos-Rull(1998)

$$\pi(\epsilon' \mid \epsilon, Z' = Z_g, Z = Z_g) = p \left\{ \epsilon_{t+1} = \epsilon' \mid \epsilon_t = \epsilon \right\} = \begin{pmatrix} 0.9615 & 0.0385 \\ 0.9581 & 0.0492 \end{pmatrix}, \tag{2.22}$$

$$\pi(\epsilon' \mid \epsilon, Z' = Z_b, Z = Z_b) = p \left\{ \epsilon_{t+1} = \epsilon' \mid \epsilon_t = \epsilon \right\} = \begin{pmatrix} 0.9625 & 0.0475 \\ 0.3952 & 0.6048 \end{pmatrix}, \tag{2.23}$$

then, $u_g = 3.86\%$ and $u_b = 10.73\%$. The conditional employment probabilities for the transition from good to bad $\pi(\epsilon'|\epsilon, Z' = Z_g, Z = Z_b)$ and bad to good $\pi(\epsilon'|\epsilon, Z' = Z_b, Z = Z_g)$ are calibrated using the condition (2.19). Finally, we set the redistributive fiscal policy parameters. Following Chang, Kim Schorfheide(2013), we set the benchmark tax rates as the same as the ones in the U.S. in 2004. Table 2. illustrates the value of fiscal policy regimes. We calibrate and simulate four kinds of redgimes, which differ in the value of tax rates, and generate four kinds of aggregate time series data.

TABLE 2. POLICY PARAMETERS

	Benchmark	Labor tax cut	Capital tax cut	No tax
τ_L	0.269	0.000		0.000
τ_K	0.327		0.000	0.000

2.3 Algorithm

We compute the dynamics of the heterogeneous agents model using the algorithm below.[3].

1. Compute aggregate employment N' as a function of Z.
2. Choose $F_0(K_0)$, I, and initial parameters of H_I.
3. Compute $v(\epsilon, a, Z, m)$ using the value function iteration method and endogenous gridpoints method.
4. Simulate the dynamics of the distribution.
5. Estimate the law of motion for m using the path of the distribution.
6. Iterate until the parameters of H_I converge.
7. Test the goodness of fit for H_I, if fit is satisfactory, stop. Otherwise change the form of H_I or choose another moments.

3 The Estimation Model

The Estimation model is almost the same as the previous model except for the abstracted households' heterogeneity. We followed Gali, Lopez-Salido and Valles (2007) and added the simple Neoclassical Growth Model to the rule-of-thumb agents who spent all of their income in every period. In other words, they have a Keynesian Consumption Function. Therefore, it could be said that Gali, Lopez-Salido and Valles (2007) included both the Neoclassical style rational households and the Keynesian style nonrational households. Recently, this assumption has often been used for the model for fiscal policy evaluation, because if there are only rational agents, there is no impact on the fiscal policy due to the Ricardian Equivalence Theorem.

3.1 Model Specification

In the economy, there are two types of agents: one is the rational agent, and the other is a *rule of thumb* agent who consumes all of their income. The government has an incentive to use the redistributive policy because its could be increase y_t. The rule-of-thumb agent got a lump-sum transfer from the government and they spend it all in time t

$$c_t^{'} = B_t \tag{3.1}$$

[3]For more detail, see Krusell and Smith(1998), Heer and Maussner(2009) for coumptation of the heterogeneous agent model, Miranda, Mario and Fackler(2002), Judd(1991) for the value function iteration method and the endogenous gridpoints method.

where c_t' denotes the rule-of-thumb agent's consumption. Different from the heterogeneous agents economy, agents do not face idiosyncratic earning shocks, so neither agents has precautionary saving motives.

The aggregate consumption C_t is given by

$$C_t = \omega c_t + (1-\omega)c_t^{'} \tag{3.2}$$

where ω is a portion of agents and $\omega \in [0,1]$. The rational agent save and consume with his budget constraint

$$K_{t+1} = (1 + (1-\tau_K)r_t)K_t + (1-\tau_L)w_t - c_t. \tag{3.3}$$

Because the rule-of-thumb agent never save, the rational agent's asset level is the same as the aggregate capital stock.

Other parts of the economy are the same as the last one. The CRRA utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}, \tag{3.4}$$

the factor prices satisfies the first order conditions of the firm,

$$w_t = Z_t(1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha}, \tag{3.5}$$

$$r_t = Z_t\alpha\left(\frac{N_t}{K_t}\right)^{1-\alpha} - \delta, \tag{3.6}$$

the government budget constraint

$$T_t = \tau_L w_t L_t + \tau_K r_t K_t, \tag{3.7}$$

and balanced government budget

$$B_t = T_t. \tag{3.8}$$

3.2 Econometric Analysis

We now estimate the intertemporal elasticity of substitution of the estimation model using the Bayesian techniques. We assume that the *estimatian* knows the true value of some parameters(α,β,δ) and tax rates (τ_K, τ_K) when estimating the model. Table 3. shows the prior density, prior mean and standard deviation of the coefficient of relative risk aversion. We use the true value of θ as prior mean, and the prior density and the standard deviation are followed Sugo and Ueda(2008).

TABLE 3. PRIOR DISTRIBUTION

	Density	Mean	S.D.
θ	Gamma	1.50	0.50

4 Estimation Results

TABLE 4. ESTIMATION RESULTS

	Benchmark	Labor tax cut	Capital tax cut	No tax
Posterior mean	1.3118	1.3145	1.4248	1.4263
90% Intv.	[0.7966, 2.0224]	[0.8560,1.9276]	[0.7274,2.1264]	[0.7212, 2.1316]

Table 4. show the results of the estimation. The calibrated parameter of data-generating model $\theta = 1.50$ is the true value for the estimation model because if the estimation model is perfectly same as the data-generating model, the parameter values coincidewith each other except for the estimation error.

Although all 90 percentile intervals include the true value ϑ =1.50, the results show little policy variance of the parameter. The parameter is largest in the economy with No Tax, whose wealth inequality level is largest in the true model. On the other hand, the benchmark economy, which is the most redistributive regime, was estimated to have the smallest value of the parameter. Therefore, the results suggest that if the regime in the true economy is redistributive, the parameter estimated is much less than the real value, and vice versa.

One of the possible explanations of this fact is that if the government will increase the transfers, the poor households will consume much more, so the aggregate consumption will increase and the aggregate capital stock will decrease. For an estimatian who estimates using the New Keynesian Model, the change of variables are thought to be due to the rational agent beginning to prefer consuming rather than saving, that is, she preferred to consume today rather than tomorrow. That means the intertemporal elasticity of substitution will increase, and the coefficient of relative risk aversion θ will decrease.

The results also suggest that the estimated values are smaller than the true value due to the absence of the precautionary saving motive. In the true economy, households save against the idiosyncratic earning shocks. Therefore, they save more than the rational agent, who only saves to maximize their intertemporal utility. Therefore, an agent with no uncertainty prefers to consume today rather than agents who face idiosyncratic shocks, if other circumstances are the same. When an estimatian estimates, he indirectly assumes that the observable data are from the agent's rational behavior, so if the aggregate consumption increases, it is thought there will be a decrease of the elasticity of substitution in consumption. However, in our experiments, we fixed the value of the elasticity of substitution and only changed the redistributive regimes. The policy differences vary the aggregate consumption levels, and the estimatian thought the differences were coming from the elasticity of substitution. These misestimations are attributed to the lack of households' precautionary saving motives in the estimation model.

5 Conclusion

This paper has studied the possibility of misestimation of the elasticity of substitution due to the abstracted households. The quantitative results suggest that the parameter could be seen differently according to the differences in the redistributive policy regimes, even if the true parameter value is the same. This is because of the differences between the two models: whether agents face the idiosyncratic income shock or not. In other words, the results imply we should care not only about the presence of non-rational households but also about the precautionary saving motives. Furthermore, the results also suggest that such estimation is larger when the regime is redistributive. However, we only experimented with the simplest model, so much remains to be done. For one thing, we do not consider the labor supply elasticity whose instability is indicated in Chang, Kim and Schorfheide (2013). Furthermore, we make a strong assumption that the rule-of-thumb agent is unemployed. In addition to that, we suppose that the ratio of the two agents is given exogenously. As we denoted, these assumptions are made to simplify the model, and our main findings are not denied by these assumptions because the base of the New Keynesian Model is similar to our estimation model, so the possibility of misestimation remains even if the model is more complex. Of course, I do not mean to criticize the quantitative DSGE models, just trying to find out the individual roles of the models. These kinds of studies contribute to the fields of model selection and make the future policy evaluation more accurate.

References

1. Aiyagari, S. Rao. "*Uninsured idiosyncratic risk and aggregate saving." The Quarterly Journal of Economics* 109.3 (1994): 659-684.

2. Algan, Yann, Olivier Allais, and Wouter J. Den Haan. "*Solving heterogeneous-agent models with parameterized cross-sectional distributions." Journal of Economic Dynamics and Control* 32.3 (2008): 875-908.

3. Calvo, Guillermo A. *"Staggered prices in a utility-maximizing framework." Journal of monetary Economics* 12.3 (1983): 383-398.

4. Castaneda, Ana, Javier Dıaz-Giménez, and José-Victor Rıos-Rull. *"Exploring the income distribution business cycle dynamics." Journal of Monetary economics* 42.1 (1998): 93-130.

5. Chang, Yongsung, Sun-Bin Kim, and Frank Schorfheide. *"Labor-Market Heterogeneity, Aggregation, and Policy (In) Varianve of DSGE Model Parameters." Journal of the European Economic Association* 11.s1 (2013): 193-220.

6. Fernandez-Villaverde, Jesús, Juan F. Rubio-Ramírez, Timothy Conley, and Frank Schorfheide. *How Structural Are Structural Parameters?[with Comments and Discussion]. NBER Macroeconomics Annual*, (2007): 83-167.

7. Gali, Jordi, J. David López-Salido, and Javier Vallés. "*Understanding the effects of government spending on consumption." Journal of the European Economic Association* 5.1 (2007): 227-270.

8. Guvenen, Fatih. *Macroeconomics with heterogeneity: A practical guide.* No. w17622. *National Bureau of Economic Research,* 2011.

9. Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. *The macroeconomic implications of rising wage inequality in the United States. No. w14052. National Bureau of Economic Research,* (2008).

10. Heer, Burkhard, and Alfred Maussner. *Dynamic general equilibrium modeling: computational methods and applications. Springer,* 2009.

11. Judd, Kenneth L. *Numerical methods in economics. Hoover Inst. on War Revolution and Peace, Stanford Univ.,* (1991).

12. Krusell, Per, and Anthony A. Smith, Jr. *"Income and wealth heterogeneity in the macroeconomy." Journal of Political Economy* 106.5 (1998): 867-896.

13. Krusell, Per, and Anthony A. Smith. *"Quantitative Macroeconomic Models With Heterogeneous Agents." Econometric Society Monographs* 41 (2006): 298.

14. Ljungqvist, Lars, and Thomas J. Sargent. *Recursive macroeconomic theory. MIT press,* (2012).

15. Lucas Jr, Robert E. "Econometric policy evaluation: A critique*." Carnegie-Rochester conference series on public policy. Vol. 1. North-Holland*, (1976).

16. Miranda, Mario J., and Fackler, Paul L. *Applied computational economics and finance. The MIT Press*, (2002).

17. Reiter, Michael. "*Solving heterogeneous-agent models by projection and perturbation." Journal of Economic Dynamics and Control* 33.3 (2009): 649-665.

18. Sugo, Tomohiro, and Kozo Ueda. "*Estimating a dynamic stochastic general equilibrium model for Japan." Journal of the Japanese and International Economies 22.4* (2008): 476-502.

19. Yun, Tack. "*Nominal price rigidity, money supply endogeneity, and business cycles." Journal of Monetary Economics* 37.2 (1996): 345-370.