Tailoring Negligence Standards to Accident Records

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Abstract

Traditional economic models of accident law are static and assume homogeneous individuals under perfect information. This paper relaxes these assumptions and presents a dynamic unilateral accident model in which potential injurers differ in their probability of accident. Information about individual risk-type is hidden from the social planner and from each potential injurer. We ask how negligence standards should be optimally tailored to individual risk-type when this is imperfectly observable. We argue that information about past accident experiences helps to efficiently define negligence standards, narrowing the distance between first-best standards perfectly tailored to individual risk-type and third-best averaged standards. We finally show that negligence standards refined on the basis of past accident experiences and of individual risk-type do not undermine private incentives to undertake due care.

Keywords: accident law, individualized negligence standards, negligence, bayesian updating rule

JEL Codes: K10, K13

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1 Introduction

The standard unilateral accident model is premised on at least three assumptions that cannot be said to hold for many different accidents. First, it is assumed that the relationship between the precautions taken and the probability of an accident is the same for all potential injurers. Second, it is assumed that both the social planner and each potential injurer know what the relationship is between the precautions taken and the probability of an accident. Third, it is assumed that no records of past accidents are kept by either the social planner or by the potential injurers. Consequently, the negligence standard that is set to minimize the cost of accidents is time invariant and is the same for all potential injurers (Shavell, 1980, 1987). In this paper we relax these assumption and define a new standard of due care that can be used to minimize the cost of accidents for those accidents where these three assumptions are not likely to hold.

To do so we add three extensions to the standard model. The first extension adds to the model two types of potential injurers \( \theta \in \{ \bar{\theta}, \tilde{\theta} \} \) who for any given level of precaution \( x \) differ in their probability of an accident \( \bar{\theta} p(x) > \tilde{\theta} p(x) \). The second extension adds to this that neither the potential injurers nor the social planner can observe of which type a potential injurer is. Potential injurers cannot observe their own type and the social planner cannot observe the type of each individual potential injurer. What is observable, however, is the (expected) proportion of each type in the population. Finally, as a third extension an additional period is added to the model and it is assumed that whilst each type is not observable, it is observable to both the social planner and the potential injurer whether a potential injurer was involved in accident in the first period. Starting from a common prior – the proportion of each type in the population – both the social planner and each potential injurer can update their beliefs about \( \theta \) based on whether or not an accident occurred in the first period.

The contribution of this paper to the literature is that whilst the implications of individual heterogeneities and information problems have been considered before in the economic analysis of accident law, these have not been considered before in an inter-temporal setting where records are kept of past accidents (per injurer). The implications of individual heterogeneities for incentives that have been considered before are those with respect to the costs of taking care and of the wealth of potential injurers (cf. Rubinfeld, 1987; Arlen, 1992; Miceli and Segerson, 1995; Schmitz, 2000), the level of harm suffered by the victims and the ability and cost of taking care (cf. Landes and Posner, 1987; Kaplow and Shavell, 1996; Miceli, 1997; Ganuza and Gomez, 2005), and with respect to the difference in relative gain from potentially harmful activities (cf. Emons, 1990a,b; Emons and Sobel, 1991). The implication for the appropriate standard of due care when these type of heterogeneities are not observable has been to set an averaged negligence standard as a second best
solution even though it is acknowledged that this substantially alters parties’ incentives to take care (cf. Landes and Posner, 1987; Shavell, 1987; Schwartz, 1989; Parisi, 1992; Gauza and Gomez, 2005; Miceli, 2006; Bajtelsmit and Thistle, 2009; Endres and Friehe, 2011).

By recasting this problem within an inter-temporal accident model, we show that information about past accidents conveys useful information about a potential injurer’s type. This information allows the social planner to tailor negligence standards to past accidents and thus to better approximate the first best solution. The standards of due-care, therefore, set (optimal) precautionary incentives at the same time as these help to convey information about an individual’s type. We thus suggest and justify a revised definition of negligence standards, by showing that when it is not possible to set the first best solution, due care standards tailored to past accidents implement the second best solution whereas averaged negligence standards implement the third-best solution.

Closely related to our paper are Crocker and Doherty (2000), Bajtelsmit and Thistle (2008) and Bajtelsmit and Thistle (2009), where the focus is on the incentives to purchase liability insurance. In Crocker and Doherty (2000) there are two types of potential injurers who differ in their probability of an accident. The standards of due care are tailored to the type of the potential injurer and the potential injurers do not know their own type. The model in Crocker and Doherty (2000) differs from our model in that there it is assumed that potential injurers can choose to learn their type at zero costs, in which case precaution levels are optimal, or to remain ignorant about their type, in which case liability insurance is purchased. Bajtelsmit and Thistle (2009) extends this analysis and investigates what the incentives of the potential injurers in Crocker and Doherty (2000) are when (i) courts apply a uniform standard of negligence, (ii) courts apply an individualized standard of care, (iii) insurance companies can and (iv) cannot distinguish between informed and uninformed injurers. Bajtelsmit and Thistle (2009) concludes that in equilibrium potential injurers decide to become informed about their own risk-type, undertake the due-care standard and do not demand insurance. Our paper adds to this literature by demonstrating that, in the absence of liability insurance and the possibility to become informed, negligence standards help to convey information about individual risk-type to the social planner as well as to potential injurers.

The paper proceeds as follows. Section 2 presents the basic model; Sections 3 and 4 characterize the social and private objectives. Section 5 examines the implications of the revised model and concludes the paper. The proofs of the propositions can be found in the Appendix A.

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1The previous literature has recognized the inefficiency of applying the reasonable person standard in the presence of heterogeneous parties because it requires different individuals to invest in the same level of care, but defended the application of a uniform standard when parties’ heterogeneities are costly to assess (cf. Diamond, 1974; Landes and Posner, 1987; Shavell, 1987).

2 The Basic Model

The basic model considered here is a model of those unilateral accidents that can be said to occur very often and where a potential injurer (T) can choose to take precaution \( x \) to reduce the probability of an accident \( p(x) \) at a decreasing rate \( (p'<0; p''>0) \) where \( p(x) \) is assumed to be continuously differentiable. The harm suffered by the victim and the damages to be paid by an injurer in an accident are denoted by \( h \) and \( d \).

![Figure 1: Extensive form representation of the timing of the model](image)

The timing of the model is as followed (see Figure 1). At time \( t = 0 \) Nature allocates potential injurers to one of two types \( \theta \in \{ \bar{\theta}, \hat{\theta} \} \) with probability \( \bar{q} \) and \( q \) where \( \bar{\theta} > \hat{\theta} \) (First Extension). This information is hidden from potential injurers and the social planner, and only the probabilities are known as indicated by the dashed line in Figure 1 (Second Extension). At time \( t = 1 \) potential injurers not knowing their own type have to choose how much precaution \( x \) to take. The probability that potential injurers will be involved in an accident is given by \( \bar{\theta} p(x) \) and \( \hat{\theta} p(x) \) with \( \bar{\theta} p(x) \leq 1 \) and \( \hat{\theta} p(x) \geq 0 \) where \( \bar{\theta} p(x) > \hat{\theta} p(x) \) for any given level of precaution taken \( x \). At time \( t = 2 \) potential injurers are divided into two groups: \( A \) and \( \neg A \). The group whose members had an accident before \( t = 2 \) is denoted by \( A \) and the group whose members did not have an accident is denoted by \( \neg A \) (Third Extension). The precaution that the members of each group take at time \( t = 2 \) can now be denoted by \( x_A \) and \( x_{\neg A} \).
At time $t = 1$ the (expected) proportion of potential injurers of type $\bar{\theta}$ and of type $\theta$ in the population are given by $\bar{\sigma}$ and $\sigma$ with $\bar{\sigma} = \bar{q}$ and $\sigma = q$. At time $t = 2$ the proportions are $\bar{\sigma}_A$ and $\sigma_A$ for the group that had an accident and $\bar{\sigma}_{\neg A}$ and $\sigma_{\neg A}$ for the group that did not have an accident. The relationship between these parameters and those mentioned previously can be seen in Figure 1 and can be written down as:

$$\bar{\sigma}_A = \frac{\bar{\sigma} \bar{\theta} p(\bar{x})}{\bar{\sigma} \bar{\theta} p(\bar{x}) + \bar{\sigma} \theta p(x)} \quad (2.1)$$

$$\sigma_A = \frac{\sigma \theta p(x)}{\bar{\sigma} \bar{\theta} p(\bar{x}) + \sigma \theta p(x)} \quad (2.2)$$

$$\bar{\sigma}_{\neg A} = \frac{\bar{\sigma} (1 - \bar{\theta} p(\bar{x}))}{\bar{\sigma} (1 - \bar{\theta} p(\bar{x})) + \sigma (1 - \theta p(x))} \quad (2.3)$$

$$\sigma_{\neg A} = \frac{\sigma (1 - \theta p(x))}{\bar{\sigma} (1 - \bar{\theta} p(\bar{x})) + \sigma (1 - \theta p(x))} \quad (2.4)$$

These are thus the conditional probabilities that a member of group $A$ or $\neg A$ is of type $\bar{\theta}$ or $\theta$ where the probability of an accident or not is given by $q_A$ and $q_{\neg A}$ (see the denominator):

$$q_A = \bar{\sigma} \bar{\theta} p(\bar{x}) + \bar{\sigma} \theta p(x) \quad (2.5)$$

$$q_{\neg A} = \bar{\sigma} (1 - \bar{\theta} p(\bar{x})) + \sigma (1 - \theta p(x)) \quad (2.6)$$

It follows from the difference in the probability of an accident between the types ($\forall x \ {\bar{\theta} p(x)} > \theta p(x)$)) that the ratio of the proportion of type $\bar{\theta}$ to the proportion of type $\theta$ is the highest for the group the members of which were involved in an accident and the lowest for the group the members of which were not involved in an accident: $^3$

$$\frac{\bar{\sigma}_A}{\sigma_A} > \frac{\bar{\sigma}}{\sigma} > \frac{\bar{\sigma}_{\neg A}}{\sigma_{\neg A}} \quad (2.7)$$

In other words: the (low) high risk types are (under-) overrepresented in the group that had an accident at time $t = 1$ and are (over-) underrepresented in the group that did not have an accident at $t = 2$.

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$^3$Equation (2.7) can be re-written as: $\frac{\bar{\sigma} \bar{\theta} p(\bar{x})}{\bar{\sigma} \bar{\theta} p(\bar{x}) + \bar{\sigma} \theta p(x)} > \frac{\sigma (1 - \theta p(x))}{\bar{\sigma} (1 - \bar{\theta} p(\bar{x}))}$ it holds here that $\bar{x} = x$ as the information about the type of the potential injurer is hidden. For $\frac{\bar{\sigma} \bar{\theta} p(\bar{x})}{\bar{\sigma} \bar{\theta} p(\bar{x}) + \bar{\sigma} \theta p(x)} > \frac{\sigma (1 - \theta p(x))}{\bar{\sigma} (1 - \bar{\theta} p(\bar{x}))}$ it holds because $\bar{\theta} > \theta$ meaning that the number the numerator is multiplied with is larger than the number the denominator is multiplied with. For $\frac{\bar{\sigma} \bar{\theta} p(\bar{x})}{\bar{\sigma} \bar{\theta} p(\bar{x}) + \bar{\sigma} \theta p(x)} > \frac{\sigma (1 - \theta p(x))}{\bar{\sigma} (1 - \bar{\theta} p(\bar{x}))}$ it holds because the numerator is multiplied by a smaller number than the number the denominator is multiplied with: $1 - \bar{\theta} p(\bar{x}) > 1 - \bar{\theta} p(\bar{x})$ which simplifies to $\bar{\theta} > \bar{\theta}$. 

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time \( t = 1 \) compared to their proportion (\( \bar{\sigma} \)) \( \bar{\sigma} \) in the population. This is relevant when information about which potential injurer belongs to which type is hidden (\( \bar{x} = \bar{x} \)) from both the social planner and each potential injurer. The social planner can use this information to improve social welfare by tailoring the negligence standard to past accidents. Similarly potential injurers can use this information to adjust their care levels to improve their own private welfare. The extension to Shavell (1980, 1987) that we thus consider is a dynamic unilateral accident model with hidden information, where both the social planner and the injurer are Bayesian updaters.

3 The Social Planner’s Problem

Following Section 2 the social planner’s problem can now be written down as the weighted sum – weighted by the probability of having an accident or not – of three social cost function (omitting arguments):

\[
\min S = S_1 + q_A S_A + q_{\neg A} S_{\neg A}
\] (3.1)

These three social cost functions are defined in equations (3.2), (3.3), and (3.4) as followed:

\[
S_1 = \bar{\sigma} (\bar{\theta} p(\bar{x}) h + \bar{x}) + \sigma (\theta p(\bar{x}) h + \bar{x})
\] (3.2)

\[
S_A = \bar{\sigma}_A (\bar{x}, \bar{x}) (\bar{\theta} p(\bar{x}_A) h + \bar{x}_A) + \sigma_A (\bar{x}, \bar{x}) (\theta p(\bar{x}_A) h + \bar{x}_A)
\] (3.3)

\[
S_{\neg A} = \bar{\sigma}_{\neg A} (\bar{x}, \bar{x}) (\bar{\theta} p(\bar{x}_{\neg A}) h + \bar{x}_{\neg A}) + \sigma_{\neg A} (\bar{x}, \bar{x}) (\theta p(\bar{x}_{\neg A}) h + \bar{x}_{\neg A})
\] (3.4)

The first function is the sum of the total social cost due to the expected accident costs over the two types weighed by their share at time \( t = 1 \). The second function is the sum of the total social cost at time \( t = 2 \) for those potential injurers who were involved in an accident at time \( t = 1 \). The third function is the same as the second function but then for those potential injurers who were not involved in an accident at time \( t = 1 \). These last two are weighted by the probability of an accident and the total of all three adds up to the total social cost of an accident over two periods.

By writing down the social planner’s problem, the welfare implications of the social incentives derived under perfect information with tailoring to potential injurer type, hidden information with tailoring to past accidents, and hidden information without tailoring to past accidents can be compared to one another. The following theorems hold for these comparisons under the assumptions that we’ve made:

Lemma 3.1. The social incentives set by tailoring the (per period) standard of due care to the type...
of potential injurer, and only to the type of the potential injurer, are the first best social incentives.

Proof. see Appendix A

Proposition 3.2. The social incentives under hidden information can be improved upon by tailoring the (per period) standard of due care to records containing information on past accidents.

Proof. see Appendix A

Corollary 3.3. Tailoring the standard of due care to the type of the potential injurer implements the first best solution to the social planner’s problem, tailoring the standard of due care to past accidents implements the second best solution to the social planner’s problem, and not tailoring the standard of due care implements the third best solution to the social planner’s problem.

These comparisons can be said to depend on to which variables the social planner is constrained. When the social planner tailors the standards of due care to the type of the potential injurer, the constraint faced by the social planner is that the standard of due care is the same for each type and does not depend on whether or not the potential injurer was involved in an accident (i.e. \( \bar{x}_A = \bar{x}_{\neg A} \) and \( \bar{x}_A = \bar{x}_{\neg A} \)). Reformulating the social planner’s problem as a Lagrangian we find that the shadow prices of these two constraints are equal to zero proving the first part of the lemma 3.1. The proof demonstrates that under the assumptions that we have made no information about past accidents is required to attain the global minimum of the total social cost function in equation (3.1). The proof of the second part of the lemma 3.1 follows then by imposing constraints such that it is not possible to tailor the standards of due care to the type, but only to past accidents (i.e. \( \bar{x} = \bar{x}_A = \bar{x}_{\neg A} \), and \( \bar{x}_{\neg A} = \bar{x}_{\neg A} \)). The shadow prices associated with these constraints are nonzero which completes the proof of lemma 3.1. For the comparison between the social incentives when information is hidden the relevant constraint is that it is not possible to tailor standards of due care to past accidents in addition to that it is not possible to differentiate the standards of due care by the type of potential injurer. The latter is substituted into the problem leaving us with \( x_A = x_{\neg A} \). The shadow price of this constraint is nonzero indicating that the constraint is binding and constrains the minimum that would otherwise be attained and thus proving proposition 3.2. The corollary 3.3 then follows from lemma 3.1 and proposition 3.2.

The implications of corollary 3.3 are that when the first best is not available (i) that the standard of due care at \( t = 2 \) should be higher (lower) for potential injurers who were (not) involved in an accident at time \( t = 1 \) compared to the situation where information about past accidents (per injurer) is hidden (lemma 3.4); and (ii) that the standard of due care should be lower, higher or the same at \( t = 1 \) compared to the situation where information about past accidents (per injurer) is hidden (lemma 3.5). This provided that social costs of an accident should be minimized.
Lemma 3.4. The social incentives under hidden information with tailoring to past accidents imply a higher (lower) standard of due care for potential injurers at $t = 2$ who were (not) involved in an accident at $t = 1$ compared to standard of due care implied by the social incentives under hidden information without tailoring to past accidents at $t = 2$.

Proof. See Appendix A.

Lemma 3.5. The social incentives under hidden information with tailoring to past accidents at $t = 2$ imply a lower, higher or equal standard of due care at $t = 1$ compared to the standard of due care at $t = 1$ implied by the social incentives under hidden information without tailoring to past accidents at $t = 2$.

Proof. omitted.

The intuition for lemma 3.4 is that when conditioning on past accidents the group that did have an accident at $t = 1$ consists of a higher (lower) proportion of potential injurers that are of the type that have a high (low) probability of an accident than the population of potential injurers at $t = 1$. On average this group, therefore, has a higher probability of being involved in accident and this means that the standard of due care should go up. The same can be said to hold for the group that did not have an accident at $t = 1$. On average this group has a lower probability of being involved in accident and the standard of due care, therefore, should go down.

The intuition for lemma 3.5 is that when conditioning on past accidents the total social costs of an accident are reduced at $t = 2$ compared to the situation when it is not possible to condition on past accidents. This means that the standard of due care at $t = 1$ can now be used to produce information to further reduce the cost of accidents at $t = 2$ by lowering this standard or to reduce the cost of accidents at $t = 1$ by raising this standard. If there is a large difference in the probability of accidents between the two types then the production of information is relatively more valuable, whereas if there is hardly any difference this information has relatively little value for reducing the costs of accidents at $t = 2$. At the same time the standard of due care at $t = 1$ can be used to re-allocate the costs of accidents between $t = 1$ and $t = 2$. This is relatively more valuable if the costs of accidents are high at $t = 1$ and low at $t = 2$. Which of these two effects dominates – the production of information or the redistribution of accident losses – depends on the parameters of the problem.

Any parameterized example can show this. Example available upon request from the authors.
4 Private Objective

Having answered whether negligence standards should be tailored to past accidents in the affirmative, we need to solve for the private minimization problem of each potential injurer before we are able to implement the socially efficient solution. The timeline is the same as above. At $t = 0$ Nature moves and assigns each potential injurer her type. At $t = 1$ each potential injurer chooses how much precaution to take. At $t = 2$ each potential injurer again choose how much precaution to take.

The potential injurer’s objective function under imperfect information is equivalent to that of the social planner, and can be written as (omitting arguments):

$$
\min_{x, x_A, x_{\sim A}} T = T_1 + P_A T_A + P_{\sim A} T_{\sim A}
$$

The three private cost functions are $T_1$ for the first period, $T_A$ for the second period if an accident did occur, and $T_{\sim A}$ for the second period if an accident did not occur, and are defined as follows:

$$
T_1 (x) = \bar{\sigma} \left( \bar{\theta} p (x) d + x \right) + \sigma \left( \theta p (x) d + x \right)
$$

$$
T_A (x, x_A) = \bar{\sigma}_A \left( \bar{\theta} p (x_A) d + x_A \right) + \sigma_A \left( \theta p (x_A) d + x_A \right)
$$

$$
T_{\sim A} (x, x_{\sim A}) = \bar{\sigma}_{\sim A} \left( \bar{\theta} p (x_{\sim A}) d + x_{\sim A} \right) + \sigma_{\sim A} \left( \theta p (x_{\sim A}) d + x_{\sim A} \right)
$$

The private optimization problem is the same as the social optimization problem except with one difference. Whilst from the social planner’s perspective, $\bar{\sigma}_A$, $\bar{\sigma}_{\sim A}$, $\sigma_A$, and $\sigma_{\sim A}$ are functions of the precaution taken in the first period, for the potential injurer these are parameters when there are (infinitely) many pairs of potential injurers and victims.

The following corollary and proposition can be said to hold for the private problem.

**Corollary 4.1.** The lemmas 3.1, 3.4, and 3.5, the proposition 3.2, and the corollary 3.3 from the social problem carry over to the private problem.

**Proposition 4.2.** To align private and social incentives a negligence standard is better than a rule of strict liability.

**Proof.** omitted

The intuition for corollary 4.1 is that none of the proofs above depend on $\bar{\sigma}_A$, $\bar{\sigma}_{\sim A}$, $\sigma_A$, and $\sigma_{\sim A}$ being parameters or not. Qualitatively the results, therefore, carry over to the private problem as the private problem is formally similar to the social problem.
The intuition for the proposition 4.2 is that because $\sigma_A$, $\sigma_{\neg A}$, $\sigma_{B}$, and $\sigma_{\neg B}$ are parameters for the private problem, the potential injurers do not fully internalize the benefits of the production of information and, therefore, will do less of it. If a potential injurer is involved in accident this reveals information to her about her type, but also about the type of all the other potential injurers. The social planner can take all of this information into account, whereas the potential injurer cannot if there is more than one potential injurer. What this means is that under a rule of strict liability the amount of precaution taken at $t = 1$ will not be socially optimal. From the literature, we, however, know that the rule of strict liability can be improved upon here by setting a negligence standard at $t = 1$ that deviates from the amount of precaution taken under a rule of strict liability towards the socially efficient level of precaution. If the socially efficient level of precaution is higher the negligence standard can be set a little bit higher than the amount of precaution taken under strict liability, whereas if the socially efficient level of precaution is lower the negligence standard can be set lower than the amount of precaution taken under strict liability. In both instances taking the same amount of precaution as is required not to be negligent is privately optimal at $t = 1$. For $t = 2$ a rule of strict liability is equivalent to a negligence standard. The policy implication that flows from this is that the negligence standard can not only be used to solve a coordination problem between potential injurers and victims, but also between all potential injurers.

5 Conclusion

This paper revisits the standard unilateral accidents model by relaxing three conventional assumptions that do not correspond with many real-life accident situations: (1) potential injurers are rarely identical in their probability of accident; (2) individuals and social planners do not necessarily have perfect information about individual risk-type; (3) records about past accidents can convey information about individual risk-type and can help defining optimal negligence standards. Each of these observations allowed us to build and analyze a more general and realistic accident model and to derive valuable instruments for policymakers and courts in defining negligence in a large number of real-world accident cases.

Our analysis starts from the premise that in most real-life situations potential injurers differ in their probability of accident and that this heterogeneity is hidden information for both individuals and the social planner. Since individuals constantly face the probability of being involved in an accident, we recast the traditional static model within an inter-temporal accident model where records about previous accident experiences play the crucial role of conveying information about individual risk-type. The heterogeneity of potential injures and the inter-temporal accident frame-
work are at the heart of our analysis and add important aspects of reality to the standard model. Our results shows that when it is not possible to set the first best solution where the standard of due care is perfectly tailored to the risk-type of the potential injurer, due care standards tailored to past accidents implement the second best solution whereas averaged negligence standards implement the third-best solution. We argue that policymakers and courts could optimally exploit information about past accident to better tailor negligence standards to individual risk type.

Lastly, this paper offers several insights for future research. A natural extension of our model would be to analyze the demand of liability insurance in the presence of due-care standards tailored to past accident records. In this case, other factors beyond the hidden information on individual risk-type —as for example the uncertain operation of the legal system— might be necessary to create a demand of insurance (Shavell, 2000). Our framework could be also exploited to identify the conditions under which liability insurance is socially desirable, offering a unified framework to study the socially optimal combination of negligence standards and liability insurance.

References


Appendix A

Proof of Lemma 3.1. This statement is equivalent to – having formulated the Lagrangian \( \Lambda \) – that (i) the shadow prices associated with the constraint \( \bar{x}_A = \bar{x}_{-A} \) and \( \bar{\chi}_A = \bar{\chi}_{-A} \), here denoted by \( \bar{\lambda} \) and \( \bar{\lambda} \), do not differ from zero, and (ii) that having relaxed these constraints that the shadow prices associated with the constraints \( \bar{x} = \bar{\chi}, \bar{x}_A = \bar{\chi}_A \), and \( \bar{x}_{-A} = \bar{\chi}_{-A} \), here denoted by \( \lambda, \lambda_A, \) and \( \lambda_{-A} \) are nonzero. The first means that the minimum cannot be improved upon by tailoring to past accidents when it is already possible to tailor to the type, whereas the second means that the minimum can be improved upon by tailoring to the type of the potential injurer even when already tailoring to past accidents.\(^5\)

Step 1 To start with (i) the Lagrangian \( \Lambda \) can be formulated as:

\[
\Lambda = \bar{\sigma} \left( \bar{\theta} p(\bar{x}) h + \bar{x} \right) + \sigma \left( \theta p(x) h + x \right) + q_A \left( \bar{\sigma}_A (\bar{x}, \bar{x}_A) \left( \bar{\theta} p(\bar{x}_A) h + \bar{x}_A \right) + \sigma_A (\bar{x}, \bar{x}_A) \left( \theta p(\bar{x}_A) h + \bar{x}_A \right) \right) + q_{-A} \left( \bar{\sigma}_{-A} (\bar{x}, \bar{x}_{-A}) \left( \bar{\theta} p(\bar{x}_{-A}) h + \bar{x}_{-A} \right) + \sigma_{-A} (\bar{x}, \bar{x}_{-A}) \left( \theta p(\bar{x}_{-A}) h + \bar{x}_{-A} \right) \right) + \bar{\lambda} (\bar{x}_A - \bar{x}_{-A}) + \bar{\lambda} (\bar{x}_A - \bar{x}_{-A})
\]

(A.1)

This can be simplified using the fact that \( q_A \) and \( q_{-A} \) are the same as the denominator for \( \sigma_A \) and \( \sigma_{-A} \) for each type. The expression to minimize, therefore, becomes:

\[
\Lambda = \bar{\sigma} \left( \bar{\theta} p(\bar{x}) h + \bar{x} \right) + \sigma \left( \theta p(x) h + x \right) + q \left( \bar{\sigma} (\bar{x}, \bar{x}_A) \left( \bar{\theta} p(\bar{x}_A) h + \bar{x}_A \right) + q \left( \theta p(\bar{x}_A) h + \bar{x}_A \right) \right) + q (1 - \bar{\theta} p(\bar{x})) (\theta p(\bar{x}_{-A}) h + \bar{x}_{-A}) + q (1 - \theta p(x)) (\theta p(x_{-A}) h + x_{-A}) + \bar{\lambda} (\bar{x}_A - \bar{x}_{-A}) + \bar{\lambda} (\bar{x}_A - \bar{x}_{-A})
\]

(A.2)

The FOCs for this problem are:

\[
\begin{align*}
\bar{x} : \sigma \left( \theta p'(\bar{x}) + 1 \right) + q \bar{\theta} p'(\bar{x}) \left( \theta p(\bar{x}_A) h + \bar{x}_A \right) - q \bar{\theta} p'(\bar{x}) \left( \theta p(\bar{x}_{-A}) h + \bar{x}_{-A} \right) &= 0 \quad \text{(A.3)} \\
\bar{x} : \sigma \left( \theta p'(\bar{x}) + 1 \right) + q \theta p'(\bar{x}) \left( \theta p(\bar{x}_A) h + \bar{x}_A \right) - q \theta p'(\bar{x}) \left( \theta p(\bar{x}_{-A}) h + \bar{x}_{-A} \right) &= 0 \quad \text{(A.4)} \\
\bar{x}_A : q \bar{\theta} p(\bar{x}) \left( \bar{\theta} p'(\bar{x}_A) h + 1 \right) + \bar{\lambda} &= 0 \quad \text{(A.5)} \\
\bar{x}_A : q \theta p(\bar{x}) \left( \theta p'(\bar{x}_A) h + 1 \right) + \bar{\lambda} &= 0 \quad \text{(A.6)} \\
\bar{x}_{-A} : q (1 - \bar{\theta} p(\bar{x})) \left( \bar{\theta} p'(\bar{x}_{-A}) h + 1 \right) - \bar{\lambda} &= 0 \quad \text{(A.7)} \\
\bar{x}_{-A} : q (1 - \theta p(x)) \left( \theta p'(\bar{x}_{-A}) h + 1 \right) - \bar{\lambda} &= 0 \quad \text{(A.8)} \\
\bar{\lambda} : \bar{x}_A - \bar{x}_{-A} &= 0 \quad \text{(A.9)} \\
\bar{\lambda} : \bar{x}_A - \bar{x}_{-A} &= 0 \quad \text{(A.10)}
\end{align*}
\]

\(^5\)This follows essentially from that, \( \frac{\partial A^*}{\partial b} = \bar{\lambda}^* \) where \( b = \bar{x} - \bar{\chi} \).
By eliminating $\bar{\lambda}$ from equations (A.5) and (A.7) we find that:

$$-\bar{q} \dot{\bar{\theta}} p(\bar{x}) (\dot{\bar{\theta}} p'(\bar{x}_A) h + 1) = \bar{q} (1 - \dot{\bar{\theta}} p(\bar{x})) (\dot{\bar{\theta}} p'(\bar{x}_{-A}) h + 1)$$ (A.11)

By substituting the constraint in equation (A.9) into (A.11) we can re-write it as:

$$\dot{\bar{\theta}} p(\bar{x}) (\dot{\bar{\theta}} p'(\bar{x}_A) h + 1) = 0$$ (A.12)

It follows that $\bar{\lambda}^{**} = 0$ (see equations (A.5) and (A.7) above) the same holds for $\bar{\lambda}^{**}$ proving (i).

**Step 2** To prove (ii) the Lagrangian to be minimized is re-formulated to be the following by:

$$\Lambda = \bar{\sigma} (\dot{\bar{\theta}} p(\bar{x}) h + \bar{x}) + \sigma (\theta p(\bar{x}) h + \bar{x})$$

$$+ \bar{q} \dot{\bar{\theta}} p(\bar{x}) (\dot{\bar{\theta}} p(\bar{x}_A) h + \bar{x}_A) + q \theta p(\bar{x}) (\theta p(\bar{x}_A) h + \bar{x}_A)$$

$$+ \bar{q} (1 - \dot{\bar{\theta}} p(\bar{x})) (\dot{\bar{\theta}} p(\bar{x}_{-A}) h + \bar{x}_{-A}) + q (1 - \theta p(\bar{x})) (\theta p(\bar{x}_{-A}) h + \bar{x}_{-A})$$

$$+ \lambda (\bar{x} - \bar{x}) + \lambda_A (\bar{x}_A - \bar{x}_A) + \lambda_{-A} (\bar{x}_{-A} - \bar{x}_{-A})$$

(A.13)

The FOCs for this problem are:

$$\bar{x}: 0 = \bar{q} \dot{\bar{\theta}} p(\bar{x}) (\dot{\bar{\theta}} p(\bar{x}_A) h + \bar{x}_A) - \bar{q} \dot{\bar{\theta}} p'(\bar{x}) (\dot{\bar{\theta}} p(\bar{x}_{-A}) h + \bar{x}_{-A}) + \lambda = 0$$ (A.14)

$$\bar{x}: 0 = \bar{q} \dot{\bar{\theta}} p'(\bar{x}) (\dot{\bar{\theta}} p(\bar{x}_A) h + \bar{x}_A) + q \theta p'(\bar{x}) (\theta p(\bar{x}_A) h + \bar{x}_A) - \lambda = 0$$ (A.15)

$$\bar{x}_A : \bar{q} \dot{\bar{\theta}} p(\bar{x}) (\dot{\bar{\theta}} p'(\bar{x}_A) h + 1) + \lambda_A = 0$$ (A.16)

$$\bar{x}_A : q \theta p'(\bar{x}_A) (\theta p(\bar{x}_A) h + 1) - \lambda_A = 0$$ (A.17)

$$\bar{x}_{-A} : \bar{q} (1 - \dot{\bar{\theta}} p(\bar{x})) (\dot{\bar{\theta}} p'(\bar{x}_{-A}) h + 1) + \lambda_{-A} = 0$$ (A.18)

$$\bar{x}_{-A} : q (1 - \theta p(\bar{x})) (\theta p'(\bar{x}_{-A}) h + 1) - \lambda_{-A} = 0$$ (A.19)

$$\lambda : \bar{x} - \bar{x} = 0$$ (A.20)

$$\lambda_A : \bar{x}_A - \bar{x}_A = 0$$ (A.21)

$$\lambda_{-A} : \bar{x}_{-A} - \bar{x}_{-A} = 0$$ (A.22)

By combining equations (A.16) and (A.17) $\lambda_A$ can be eliminated resulting in:

$$-\bar{q} \dot{\bar{\theta}} p(\bar{x}) (\dot{\bar{\theta}} p'(\bar{x}_A) h + 1) = q \theta p(\bar{x}) (\theta p'(\bar{x}_A) h + 1)$$ (A.23)

Substituting $\alpha \bar{q}$ for $\bar{q}$ and $\beta \theta$ for $\dot{\bar{\theta}}$ the equation above can be re-arranged into:

$$-\alpha \beta = \frac{\theta p'(\bar{x}_A) h + 1}{\beta \theta p'(\bar{x}_A) h + 1}$$ (A.24)

For $\bar{x}_A = \bar{x}_A$ it follows that either the denominator is positive and the numerator negative or vice-versa:

$$\theta p'(\bar{x}_A) h + 1 \geq \beta \theta p'(\bar{x}_A) h + 1$$ (A.25)
For $\tilde{\theta} > \theta$ or $\beta > 0$ it holds that the numerator is negative and the denominator positive. This means that for the potential injurer of type $\theta$ the standard of due care implied by the social incentives is excessive, $\theta p'(\bar{x}_A) h + 1 < 0$ and that for the potential injurer of type $\tilde{\theta}$ this standard of due care is inadequate $\tilde{\theta} p'(\bar{x}_A) h + 1 > 0$. It follows that in equilibrium $\lambda^{**}_A > 0$. This is sufficient to prove (ii).

Proof of Proposition 3.2. This proposition is equivalent to the statement that the shadow price $\lambda^{**}$ is nonzero for minimization of the following the Lagrangian:

$$\Lambda = \bar{\sigma} \left( \tilde{\theta} p(x) h + x \right) + \sigma \left( \theta p(x) h + x \right) + \bar{q} \tilde{\theta} p'(x) \left( \tilde{\theta} p(x) h + x_A \right) + q \theta p'(x) \left( \theta p(x) h + x_A \right) - \bar{q} \tilde{\theta} p'(x) \left( \tilde{\theta} p(x) h + x_A \right) + q \theta p'(x) \left( \theta p(x) h + x_A \right) = 0 \tag{A.26}$$

The FOCs for this problem are:

$$x : \bar{\sigma} \left( \tilde{\theta} p'(x) h + 1 \right) + \sigma \left( \theta p'(x) h + 1 \right) + \bar{q} \tilde{\theta} p'(x) \left( \tilde{\theta} p(x) h + x_A \right) + q \theta p'(x) \left( \theta p(x) h + x_A \right) - \bar{q} \tilde{\theta} p'(x) \left( \tilde{\theta} p(x) h + x_A \right) + q \theta p'(x) \left( \theta p(x) h + x_A \right) = 0 \tag{A.27}$$

$$x_A : \bar{q} \tilde{\theta} p(x) \left( \tilde{\theta} p'(x) h + 1 \right) + q \theta p(x) \left( \theta p'(x) h + 1 \right) + \lambda = 0 \tag{A.28}$$

$$\lambda : x_A - x_{\bar{\lambda}} = 0 \tag{A.30}$$

Combining equations (A.28) and (A.29) $\lambda$ can be eliminated resulting in:

$$-\bar{q} \tilde{\theta} p(x) \left( \tilde{\theta} p'(x) h + 1 \right) - q \theta p(x) \left( \theta p'(x) h + 1 \right) = \bar{q} \left( 1 - \tilde{\theta} p(x) \right) \left( \tilde{\theta} p'(x_{\bar{\lambda}}) h + 1 \right) + q \left( 1 - \theta p(x) \right) \left( \theta p'(x_{\bar{\lambda}}) h + 1 \right) \tag{A.31}$$

By substituting in the constraint this can be re-arranged into:

$$\bar{q} \left( \tilde{\theta} p'(x_{\bar{\lambda}}) h + 1 \right) + q \left( \theta p'(x_{\bar{\lambda}}) h + 1 \right) = 0 \tag{A.32}$$

Comparing this with the FOC in equation (A.28) it follows that if $\lambda^{**} = 0$ then $\tilde{\theta} = \theta$. This is a contradiction with that $\tilde{\theta} > \theta$ proving the proposition.

Proof of Lemma 3.4. This lemma is equivalent to the statement that:

$$x^{**}_A > x^{**}_A > x^{**}_{\bar{\lambda}} \tag{A.33}$$

where $x^{**}_A$ denotes the standard of due care under hidden information without tailoring. The equilibrium values for the variables $x_A$ and $x_{\bar{\lambda}}$ are the same as implied by the FOCs (A.28) and (A.29) of the problem in equation (A.26) without the constraint (A.30), whereas for $x_{\bar{\lambda}}$ it is the same as
implied by the FOC (A.27) with the constraint (A.30) substituted into the problem:

\[
x_{\Delta} : 2 \bar{\sigma} \left( \tilde{\theta} p' (x_{\Delta}) h + 1 \right) + 2 \sigma (\theta p' (x_{\Delta}) h + 1) = 0 \tag{A.34}
\]

\[
x_A : \bar{q} \tilde{\theta} p(x) \left( \tilde{\theta} p' (x_A) h + 1 \right) + q \theta p(x) \left( \theta p' (x_A) h + 1 \right) = 0 \tag{A.35}
\]

\[
x_{\neg A} : \bar{q} \left( 1 - \tilde{\theta} p(x) \right) \left( \tilde{\theta} p' (x_{\neg A}) h + 1 \right) + q \left( 1 - \theta p(x) \right) \left( \theta p' (x_{\neg A}) h + 1 \right) = 0 \tag{A.36}
\]

These three are equivalent to where \( \bar{\gamma} \in [0, 1] \):

\[
y : \bar{\gamma} \left( \tilde{\theta} p' (y) h + 1 \right) + \left( 1 - \bar{\gamma} \right) \left( \theta p' (y) h + 1 \right) = 0 \tag{A.37}
\]

For this it holds, using the implicit function theorem, that:

\[
\frac{\partial y}{\partial \bar{\gamma}} = -\frac{\tilde{\theta} p' (y) - \theta p' (y)}{\bar{\gamma} \theta p'' (y) + (1 - \bar{\gamma}) \theta p'' (y)} \tag{A.38}
\]

It follows from \( p' < 0 \) and \( \tilde{\theta} > \theta \) that the numerator is negative, whereas the denominator is unambiguously positive given that \( p'' > 0 \). This means that:

\[
\frac{\partial y}{\partial \bar{\gamma}} > 0 \tag{A.39}
\]

Together with the inequality in equation (2.7),\(^6\)

\[
\frac{\bar{\sigma}_A}{\bar{\sigma}} > \frac{\bar{\sigma}}{\bar{\sigma}_{\neg A}}
\]

this is sufficient to prove the lemma. \( \square \)

---

\(^6\)Recall that equation (2.7) can be re-written as: \( \frac{\bar{\sigma} \tilde{\theta} p(\bar{x})}{\bar{\sigma} \theta p(\bar{x})} > \frac{\bar{\sigma}}{\bar{\sigma}_{\neg A}} > \frac{\bar{\sigma} (1 - \tilde{\theta} p(\bar{x}))}{\bar{\sigma} (1 - \theta p(\bar{x}))} \), where \( \bar{\sigma} = \bar{q} \).