Estimation of International Financial Integration: Evidence from European Countries

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8 August 2015

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Abstract

In the current state of the economy, securities and trade flows between countries exist fluently, however, such channels of flow do not completely map one-to-one without some attenuation, thereby preventing the notion of complete financial markets. This paper develops the econometric framework to identify the parameter which measures the degree of (imperfect) international risk-sharing, and employs nonlinear econometric methods to estimate for the values of the parameter across European countries. Our findings show how simple econometric methods can give a sensible measure of this risk-sharing, which can be used as a basis for economic model calibrations when solving DSGE models. Moreover, this paper lays the groundwork for the possibility of implementing further sophisticated nonlinear estimations to improve upon the measures already computed.

JEL Classification Numbers: **C5, E00**
Keywords: Macroeconomics, monetary economics, nonlinear econometrics

*This paper was prepared for the Master of Arts research requirement at the *Vancouver School of Economics*. We would like to thank Professor Michael B. Devereux for his supervision and helpful guidance with this paper, and Professor Vadim Marmer for assisting with the basis of the econometric theory behind the research.

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1 Introduction

With the rapid way the world economy has progressed in the last half-century, we expect that the degree of security trade and international financial integration between countries will have increased. Simulations and estimations of most economic models under dynamic stochastic general equilibria assume *perfect* risk-sharing across countries, or just calibrate for that parameter within their more complicated models. This paper is written by first assuming that there exists *imperfect* risk-sharing, and consequently estimate this degree of international risk-sharing using modern and sophisticated econometric methods with data from the European countries. We expect that the degree of risk-sharing between the United States and each European country will be very similar across all of the European countries because of the existence of a common currency and policies. We have used and extended the works of Devereux and Yetman [7] and Matsumoto, Flood, and Marion [15], both of which laid down the groundwork for all of the estimation used in this paper.

This paper is organized as follows. Section 2 describes the detailed economic model to derive the principal equation from which our estimation methods are derived, including the mapping properties of our necessary parameter. Section 3 briefly overviews the data sources and macroeconomic variables used in our estimation. Section 4 introduces the first class of estimation using a new method of *correlation measures* with a log-linearized approach; section 5 uses the previously computed measures as the initial equilibria and directly estimates for our parameter with nonlinear econometric methods, and the derivations are detailed out in Appendix A. Section 6 combined with Appendix B describes the results of our findings. Section 7 lays down some feasible extensions which can be feasibly added to our paper in the future. Section 8 concludes.

2 Economic Model

2.1 Household Setup

The open economy model used in this paper is a standard New Keynesian DSGE framework, with the differentiation of consumption and price level into home and foreign states; home variables are defined with the regular notations, while those representing the variables within the foreign country are noted with an $(*)$ superscript. Using the subscript t to represent period t variables, let the lifetime utility of a representative home household be defined as:

$$U_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(U(C_t) - V(N_t)\right) \tag{1}$$

where the function U represents the utility of the home consumption bundle C_t with the (dis)utility of labour N_t represented by function V. U is assumed to be differentiable and concave in C, while V is differentiable and convex in N. The home and foreign consumptions are respectively defined as:

$$C_t = \phi C_{Ht}^{\nu/2} C_{Ft}^{1-\nu/2}, \quad C_t^* = \phi C_{Ht}^{*\nu/2} C_{Ft}^{*1-\nu/2} \tag{2}$$

where the exogenous parameter ν represents the home bias in consumption with $\nu \geq 1, \phi = (\nu/2)^{\frac{\nu}{2}}(1-\nu/2)^{1-\frac{\nu}{2}}$. C_{Ht} represents the consumption of the home composite good and C_{Ft} the consumption of the foreign composite good in the home country. Analogously, C^* is the consumption of goods in the foreign economy: C_H^* is the consumption of home composite and C_F^* is the consumption of foreign composite in the foreign economy. With η representing the elasticity of substitution between goods, the consumption composites over the range of differentiated goods i are defined through the Dixit-Stiglitz aggregator form such that:

$$C_H = \left[\int_0^1 C_H(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}, \; C_F = \left[\int_0^1 C_F(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} \tag{3}$$

Price indices for home and foreign consumptions are:

$$P_H = \left[\int_0^1 P_H(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} \;,\; P_F = \left[\int_0^1 P_F(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} \tag{4}$$

Analogously, the aggregate price index for the home economy, represented by the CPI, is: $P = P_H^{\nu/2} P_F^{1-\nu/2}$.

We also assume that the Law of One Price holds, and so for all time periods t, the price of good i in domestic currency $P_t(i)$ equals its price of the good in foreign currency $P_t^*(i)$, multiplied by the nominal exchange rate S_t:

$$P_t(i) = S_t P_t^*(i) \tag{5}$$

2.2 Optimality Conditions

For analysis of the first-order necessary conditions, we can assume that both the households have the same utility functional forms U satisfying differentiability and concavity. Therefore, each of their first-order conditions with respect to aggregate consumptions and asset returns will be of the same functional form within each of the economy. However, while we assumed that there is a full set of Arrow-Debreu securities traded between home and foreign residents, there will also be a state-contingent cost in the asset returns such that the marginal utilities between households in the two economies will not be exactly equalised. Therefore, applying the no arbitrage condition using the law of one price, there exists a cost Ω_t in real exchange rate terms $\left(\frac{P_t}{S_t P_t^*}\right)$, such that:

$$U_C(C_t) = U_C(C_t^*) \left(\frac{P_t}{S_t P_t^*}\right) \Omega_t \tag{6}$$

$U_C(C_t)$ is the marginal utility of consumption of the home economy, $U_C(C_t^*)$ is the marginal utility of consumption in the foreign economy, and $S_t,\; P_t$ and P_t^* are as defined previously.

Devereux and Yetman [7] assumed that this wedge in risk-sharing is governed by the functional relationship:

$$\Omega_t = \left(\frac{P_t C_t}{\overline{P_t} Y_t - \Delta(FR_t)} \right)^{\frac{1-\lambda}{\lambda}} \tag{7}$$

where Y_t represents the home economy GDP (an average of the output of all home firms), $\overline{P_t}$ is the average selling price of all goods produced by home firms and $\Delta(FR_t)$ is the change in the stock of foreign exchange reserves. The exponent of Ω contains the parameter λ, which maps to a function of a measure of the international financial integration $\tilde{\lambda}$. We assume that the utility function U is isoelastic, such that it has the constant relative risk aversion (CRRA) functional form:

$$U(C) = \begin{cases} \dfrac{C^{1-\sigma}}{1-\sigma} & \sigma \neq 1, \sigma > 0 \\ \\ \ln(C) & \sigma = 1 \end{cases}$$

where the parameter σ is a measure of risk aversion, in this case represented by the inverse of elasticity of inter-temporal substitution.

Plugging equation (7) into (6), we get the following equation:

$$\left[\left(\frac{C_t^{-\sigma}}{C_t^{*-\sigma}} \right) \left(\frac{S_t P_t^*}{P_t} \right) \right]^{\lambda} = \left[\frac{P_t C_t}{\overline{P_t} Y_t - \Delta(FR_t)} \right]^{1-\lambda} \tag{8}$$

where $\lambda \equiv f(\tilde{\lambda})$ for a continuous function $f : \mathbb{R} \to \mathbb{R}$, and $\tilde{\lambda}$ is the estimated value of the parameter using our data. The process of the transformation is detailed in the next section.

2.3 Assumptions and Properties of the Parameter

2.3.1 Properties of the Parameter Function

As defined in the previous section, the estimated $\tilde{\lambda}$ from data maps on to a $[0,1]$ measure λ, which represents the level of international risk-sharing, using a continuously differentiable function. First, we assume the prior of our computed estimate:

Assumption: The parameter $\tilde{\lambda}$ estimated from the data has a prior of *standard logistic distribution.*

Under that prior, we propose the following transformation of the data-estimated $\tilde{\lambda}$ into our desired measure λ with the following theorem:

Theorem 1. *Suppose that the random variable X follows a standard logistic distribution, then for the following continuously differentiable function to define a random variable Y:*

$$Y = \frac{\exp(X)}{1 + \exp(X)}$$

the transformed random variable Y follows a standard uniform distribution: $Y \sim U(0,1)$.

Proof. We start by assuming that the random variable X has a prior of standard logistic distribution. So, for each of the realizations $x \in X$, the *pdf* of X is given by the function g:

$$g(x) = \frac{exp(x)}{(1 + exp(x))^2} \tag{9}$$

We define the transformation of X into a new random variable Y:

$$Y = \frac{\exp(X)}{1 + \exp(X)} \tag{10}$$

Therefore, we need to show that Y follows a standard uniform distribution, i.e. the marginal *pdf* of Y for all realizations $y \in Y$ is given by:

$$f_Y(y) = 1 \tag{11}$$

We start by taking equation (10) and isolating for X to get:

$$X = \ln\left(\frac{Y}{1-Y}\right) \tag{12}$$

Now, take the derivative of equation (12) with respect to Y:

$$\frac{dx}{dy} = \frac{1}{Y(1-Y)} \tag{13}$$

Before we compute for the marginal *pdf* of Y, we need to compute the marginal *pdf* of X evaluted at the inverse function from equation (12). Therefore, we plug in equation (12) into equation (9):

$$\begin{aligned} f_X(g^{-1}(y)) \equiv g(X) &= \frac{exp\left(\ln\left(\frac{Y}{1-Y}\right)\right)}{\left(1+exp\left(\ln\left(\frac{Y}{1-Y}\right)\right)\right)^2} \\ &= \frac{Y}{1-Y} \cdot \left(1+\frac{Y}{1-Y}\right)^{-2} \\ \therefore f_X(g^{-1}(y)) &= Y(1-Y) \end{aligned} \tag{14}$$

Finally, we use the "change of variables" formula (derived using the chain rule and the Fundamental Theorem of Calculus) to compute the marginal *pdf* of the transformed variable:

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \cdot \left|\frac{dx}{dy}\right| \\ &= Y(1-Y) \cdot \frac{1}{Y(1-Y)} \\ \therefore f_Y(y) &= 1 \end{aligned} \tag{15}$$

This completes the proof, that the new transformed random variable Y will be distributed uniformly on the (0,1) scale, when we use equation (10) for the transformation function. $\square$

Therefore, we can convert all of our estimated parameter with equation (10), and without loss of generality, we can say that, for all nice behaving open economy models, λ is in the parameter space such that:

$$\lambda_i = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\} \ \forall i, \text{ where } i = \text{each country}$$

For all of the direct identification techniques mentioned in the following sections, this parameter space restriction will be imposed in order to implement a proper economic interpretation, but the indirect identification using correlation measures will impose a different methodology, and so will **not** be using Theorem 1.

2.3.2 Limiting Cases

The two limiting cases for λ are as follows:

- If $\lambda \to 1$, then equation (8) collapses to a no-cost marginal utilities tradeoff between the two economies with the only weighting being the real exchange rate. This is the case of complete financial markets (perfect international risk-sharing) and so, neither the GDP nor the (changes of) foreign exchange reserve stocks have any effects on real exchange rates.

- If $\lambda \to 0$, then this is a state of financial autarky: there are no private financial markets across economies at all and each economy consumes its income period-by-period, adjusted for the change in the stock of foreign exchange reserves.

The appealing part of this model is that there is no need to make any assumptions on the firm's side of the economy, since the parameter of interest λ can be identified solely from the household's problem[1].

[1] Most small open economy models assume that the firm produces differentiated goods, where the pricing friction is implemented by the Calvo pricing strategy (Calvo [4]).

3 Data Source and Variables

The primary source of data used here are the Penn World Tables, version 8.0 [17]. The following variables have been used in the paper for all of the data:

- *rgdpe* : Expenditure-side real GDP at chained PPPs (in mil. 2005US$)
- *pop* : Population (in millions)
- *cgdpe* : Expenditure-side real GDP at current PPPs (in mil. 2005US$)
- *pl_gdpe* : Price level of CGDPe (PPP/XR), price level of USA GDPo in 2005=1
- *csh_c* : Share of household consumption at current PPPs
- *csh_x* : Share of merchandise exports at current PPPs
- *csh_m* : Share of merchandise imports at current PPPs

The time period used is from 1950 to 2014, and the data used is of yearly frequency.

The next few sections describe the different estimation methods derived and used in this paper, and the numerical results follow in the succeeding sections.

4 Indirect Identification: Correlation Measure

The slope coefficients we are looking to compute for indirect identification (β) with ordinary least squares will only be used purely as an *absolute measure* to observe the deviations in the dependant and independent variable: a lot of movement in Y for relatively stable X will show the lack of risk-sharing, while the greater is the variation in X for a given level of variation in Y, the greater is the level of financial integration. Therefore, only the *magnitude* of the slope coefficient is of importance in equation (17) which gives us the following coefficient measure ranges of λ for all countries i:

$$\lambda = \begin{cases} 0^+ & \text{if } |\beta| \to \infty \\ 1 & \text{if } \beta = 0 \\ (0,1) & \text{if } 0 < |\beta| < \infty \end{cases}$$

where we restrict the parameter space of the slope estimate β, such that the following restriction holds for all countries i:

$$|\beta_i| = \{\beta_i \in \mathbb{R} \mid \beta_i \geq 0\}$$

This measure of β (or $|\beta|$, in fact) is sufficient to estimate for λ as a correlation of international risk-sharing, because it gives a measure of how the raw data will vary between each other. Since we are not looking for measures of fits, the slope coefficient will provide more accurate interpretation compared to correlation statistics such as the R^2 or Pearson's coefficient ρ^2.

Given that the estimated λ from this section is already a $[0,1]$ measure of correlation, we will use this value directly instead of transforming using equation (10).

4.1 Ordinary Least Squares Estimation: First Differences

The basic form of estimation of λ comes from log-linearizing equation (8) and estimating the resulting equation using ordinary least squares (OLS) technique, where the slope coefficient estimate is a function of our required parameter λ. Starting with the original Euler equation (8), Appendix A.1 shows the derivation of the OLS estimator in equation (23), which involves log-linearizing equation (23), followed by taking first differences (to account for any possible non-stationarity) and ending up with the following identity:

$$\triangle W_t = \left(\frac{1-\lambda}{\lambda}\right) \triangle Z_t + \nu_t$$

[2] Matsumoto, Flood, and Marion [15] argued that the correlation coefficient is not necessarily a good way of interpreting the risk-sharing extent.

where W_t and Z_t each represents the logs of the left and right-hand sides of the Euler equation respectively, and ν_t is the error term from this model.

In economic terms, W_t represents the logged differences of the foreign and US consumptions, controlled for the real exchange rate. Z_t represents the logged sum of net exports and GDP, less the relative price for each foreign country.

The main assumption we have made here is that the changes in foreign reserves are exogenous to the model and so it belongs in the error term ν_t. So, estimating $\hat{\beta}$ from this model, the estimated coefficient λ^1_{OLS} from equation (23) is given by:

$$\lambda^1_{OLS} = \frac{1}{1 + |\hat{\beta}|} \tag{16}$$

where the absolute value of $|\hat{\beta}|$ is imposed by the assumption stated at the beginning of this section.

4.2 Ordinary Least Squares Estimation: Proxy Variables

The estimation procedure for λ originally suggested by Devereux and Yetman is shown in Appendix A.2. The dependant variable is the same as in 3.1, denoted by Y_t. The regressor is a logarithmic function of exports, imports and foreign reserves, all as shares of GDP. The independent variable is a combined logarithmic function of exports and imports each measures as a share of GDP. The following is the estimation equation using the substituted proxy variables, as discussed in Appendix A.2:

$$Y_t = [-\sigma (\ln(C_t) - \ln(C_t^*)) - \ln(RER_t)]$$

$$X_t = \ln [1 + x_t - m_t]$$

So, the OLS estimation is given by:

$$Y_t = X_t'\beta + u_t \tag{17}$$

Keeping in tandem with the data variables from the Penn World Table, the components of the estimation is given by the following:

$$\begin{aligned}
C_t &= rgdpe \times \frac{csh_c}{pop} \qquad \text{(for all countries)} \\
C_t^* &= rgdpe^* \times \frac{csh_c^*}{pop^*} \qquad \text{(for US)} \\
RER_t &= \frac{pl_gdpe}{pl_gdpe^*} \\
x_t &= csh_x \\
m_t &= csh_m
\end{aligned}$$

The addition of "1" in the expression for X_t is to ensure that the natural log of this variable is defined, since the combination of the other terms is negative for some economies in some periods. Therefore, the financial integration λ^2_{OLS} here is given by equation (16), using the estimated β from equation (17) instead, using the absolute value of estimated β.

5 Direct Identification

In this section, we will be using nonlinear econometric methods to uniquely and directly identify and estimate for σ and $\tilde{\lambda}$. However, we only report for the values of $\tilde{\lambda}$ since that is the only necessary parameter we are interested in. The two main methods used in this section are the generalized method of moments with instrumental variables and nonlinear least squares.

5.1 Generalized Method of Moments: Instrumental Variables

A very common approach to estimating macroeconomic parameters directly from nonlinear equations is the use of Generalized Method of Moments. The methodology set up for this section uses the estimation techniques derived by Hansen and Singleton [11, 1982] and reviewed in Hayashi [13, pgs. 454-455], where we can use the lagged variables' data as instruments, and apply the orthogonality of endogeneity to formulate unconditional moment conditions.

A general GMM estimation setup would have the following form: given a nonlinear function $g(V_t, \theta)$ where V_t is a sequence of known random variables and θ is the set of parameter(s) to be estimated, the following structure and assumptions hold:

1. *Random Sample:* V_t is an *i.i.d.* sequence of random variables
2. *Compactness:* $\theta \in \Theta$, where Θ is compact.
3. *Regularity:* $g(V_t, \theta)$ and $\mathbb{E}\left[g(V_t, \theta)\right]$ are continuous and finite valued on Θ for each V_t
4. *Moment Condition:* $\mathbb{E}\left[g(V_t, \theta)\right] = 0$
5. *Domination:* $\mathbb{E}\left[\sup_{\theta\in\Theta} ||g(V_t, \theta)||\right] < \infty$
6. *Identification:*
    - *Global identification:* $\mathbb{E}\left[g(V_t, \theta_0)\right] \neq 0$ for all $\theta_0 \neq \theta$ in Θ
    - *Local identification:* Given that $g(V_t, \theta)$ is continuously differentiable in a small neighbourhood of θ_0 (which is the true parameter value), then the matrix $\mathbb{E}\left[\dfrac{\delta g(V_t, \theta)}{\delta \theta'}\right]$ has full column rank

Further necessary assumptions are stated along the model setup.

Appendix A.3 derives the necessary moment conditions and the criterion function to minimize for estimation of our parameters respectively in equations (27) and (28) for lags up to period $t - p$. For this paper, the estimation will be done with only the one-period lag as the instrument, therefore containing the vector of $t-1$ data: V_{t-1}. Therefore, the nonlinear moment condition in this estimation is given by:

$$\mathbb{E}_t\left[g(V_t;\theta)\cdot V_{t-1}\right]=0 \tag{18}$$

where $\theta=\left(\sigma,\tilde{\lambda}\right)$ which are simulatenously estimated. Our sample analogue of the moment condition is given by:

$$\hat{m_n}(\theta)=\left[g(V_t;\theta)\right]\cdot\left[V_{t-1}\right]' \tag{19}$$

and we try to minimize the criterion function as in equation (28) and estimate for σ and $\tilde{\lambda}$.

The GMM estimation technique used here is with the **two-step feasible GMM**:

First stage: Taking the criterion function in (28) and setting $W=I$, a *consistent* estimator $\theta_{(1)}$ is computed, which is not necessarily efficient. The initial values chosen to start off the first step iteration are the values of the correlation measures λ^1_{OLS} and λ^2_{OLS} previously computed from the indirect identification methods.

Second stage: In the second iteration, a new weight matrix $\hat{W}$ is computed:

$$\hat{W}=\left(\frac{1}{T}\sum_{t=1}^{T}\hat{m_n}\left(\theta_{(1)}\right)'\cdot\hat{m_n}\left(\theta_{(1)}\right)\right)^{-1}\equiv\left(V\hat{a}r\left[\hat{m_n}\left(\theta_{(1)}\right)\right]\right)^{-1}$$

$$\text{where } \operatorname*{plim}_{n\to\infty}\hat{W}=\Omega^{-1}$$

and $V\hat{a}r\left[\hat{m_n}\left(\theta_{(1)}\right)\right]$ is the estimated variance-covariance matrix from the first iteration with the estimated parameters $\theta_{(1)}$, with Ω being the population variance-covariance matrix. We use the estimates of the first stage with the new weight matrix, and compute consistent and efficient estimates.

Using the initial values of λ^1_{OLS} and λ^2_{OLS}, we respectively estimate $\tilde{\lambda}^1_{GMM,1}$ and $\tilde{\lambda}^2_{GMM,1}$

(with the initial $\sigma = 2$ as in the standard case). The set of estimators $\theta^1_{GMM,1}$ and $\theta^2_{GMM,1}$ will be asymptotically consistent and efficient.

5.2 Generalized Method of Moments with First-Differences

Due to existence of possible non-stationarity[3] (as is the case with most time-series macroeconomic variables), taking first-differences help to get rid of possible serial correlations pertaining to macroeconomic variables. Blundell and Bond [3] showed that the GMM estimator defined upon instrumenting with first-differences corrects for non-stationarity and produces consistent and efficient estimators. Keeping the objective nonlinear objective the same, we can instrument on the first-differences of the lagged variables ϕ_t, such that:

$$\phi_t = V_t - V_{t-1}$$

Proposition 1 and its proof in Appendix A.4 is used to show that instrumenting on the first-differences ϕ_t instead of the lagged data V_{t-1} gives a valid moment condition and subsequently, a consistent and efficient estimator.

Therefore, the sample analogue of the moment condition here is given by:

$$\hat{m_n}(\theta) = [g(V_t;\theta)] \cdot [V_t, V_{t-1}]' \cdot [1, -1] \tag{20}$$

and the criterion function to simultaneously estimate for σ and λ is similar to equation (28), but using equation (20) instead.

Once again, we use the two-step feasible GMM algorithm as summarized in the previous section and estimate for a set of consistent and efficient estimators: $\theta_{GMM,2} = \left(\sigma_{GMM,2}, \tilde{\lambda}_{GMM,2}\right)$. Analogous to the previous method, using the initial values of λ^1_{OLS} and λ^2_{OLS}, we respectively estimate $\tilde{\lambda}^1_{GMM,2}$ and $\tilde{\lambda}^2_{GMM,2}$, with the starting value of σ being equal to 2 again.

[3] It is assumed that these data generating processes do not necessarily have unit roots under the presence of non-stationarity.

5.3 Nonlinear Least Squares

The next type of direct estimation can be done by taking the Euler equation directly, without using any instrumental variables. Using the same notation used for denoting the Euler equation, estimating for the function $g(V_t, \theta)$ as defined in equation (26) will produce the residuals $U(\theta)$ such that:

$$\begin{aligned} U(\theta) &= g(V_t, \theta) - 1 \\ \Rightarrow U(\sigma, \lambda) &= \left[\left(\frac{C_t^{-\sigma}}{C_t^{*-\sigma}} \right) \left(\frac{S_t P_t^*}{P_t} \right) \right]^{\lambda} \left[\frac{\overline{P}_t Y_t - \Delta(FR_t)}{P_t C_t} \right]^{1-\lambda} - 1 \end{aligned}$$

Taking the square of the residuals and summing up over all time periods t:

$$\Rightarrow \sum_{t=1}^{T} \left(U(\sigma, \lambda) \right)^2 = \sum_{t=1}^{T} \left(\left[\left(\frac{C_t^{-\sigma}}{C_t^{*-\sigma}} \right) \left(\frac{S_t P_t^*}{P_t} \right) \right]^{\lambda} \left[\frac{\overline{P}_t Y_t - \Delta(FR_t)}{P_t C_t} \right]^{1-\lambda} - 1 \right)^2 \tag{21}$$

Using the method of least-squares estimation, we try to minimize the sum of the squared residuals U such that:

$$\begin{aligned} &\min_{\sigma, \lambda} \| U(\sigma, \lambda) \|_2^2 \\ \Rightarrow &\frac{\delta}{\delta \sigma} \left[\sum_{t=1}^{T} \left(U(\sigma, \lambda) \right)^2 \right] = 0, \; \frac{\delta}{\delta \lambda} \left[\sum_{t=1}^{T} \left(U(\sigma, \lambda) \right)^2 \right] = 0 \end{aligned}$$

Since the parameters σ and λ do not exist linearly in the original estimation equation, this method is called the **nonlinear least squares** method of estimation. Appendix A.5 shows the derivations of the above first-order necessary conditions to attempt to find a closed form solution. We get the following set of *nonlinear normal equations* of the estimators from eq. (30) and eq. (31):

$$[\lambda] : \sum_{t=1}^{T} \left(\{-\sigma \ln A + \ln B\} \left[\left(A^{-\sigma} \cdot B\right)^{2\lambda} (C)^{2-2\lambda} - \left(A^{-\sigma} \cdot B\right)^{\lambda} (C)^{1-\lambda} \right] \right)$$
$$= \sum_{t=1}^{T} \left(\{\ln C\} \left[\left(A^{-\sigma} \cdot B\right)^{2\lambda} (C)^{2-2\lambda} - \left(A^{-\sigma} \cdot B\right)^{\lambda} (C)^{1-\lambda} \right] \right)$$
$$[\sigma] : \sum_{t=1}^{T} \left[\left(A^{-2\sigma\lambda}\right) (\lambda \ln A) \left(B^{2\lambda}\right) \left(C^{2-2\lambda}\right) \right] = \sum_{t=1}^{T} \left[\left(A^{-\sigma\lambda}\right) (\lambda \ln A) \left(B^{\lambda}\right) \left(C^{1-\lambda}\right) \right]$$

From the above two equations, numerical optimization with the Gauss-Newton algorithm (as developed in Hartley [12]) is used to solve for local minima for each country's λ. The advantage of using this methodology is that, because this estimation uses least squares, the data is efficiently used without requiring any exogenous instruments and it produces "good" estimates of the unknown parameters in the model with relatively small datasets, such as in this case. The local minima are computed using computational methods, and the initial values used to start off the consecutive iterations are the correlation measures λ^1_{OLS} and λ^2_{OLS}, respectively estimating $\tilde{\lambda}^1_{NLS}$ and $\tilde{\lambda}^2_{NLS}$.

6 Results

Tables 1 and 2 in Appendix B show the risk-sharing parameter measures computed in this paper. Columns 1 and 2 contain the correlation measures from the OLS regressions in section 3. Columns 3 and 4 contain the levelled instrumental variable estimation of the parameters used in section 5.1, and columns 7 and 8 contain the first-differenced instrumental variable estimation used in section 5.2. Finally, columns 5 and 6 contain the nonlinear least squares estimates computed in section 5.3. Columns 3, 5 and 7 contain the estimates which used λ^1_{OLS} as the starting point, and columns 2, 6 and 8 are the estimates which used λ^2_{OLS} as the starting point. Table 1 shows the raw computed parameters, and Table 2 shows the parameters transformed with Theorem 1 into a (0,1) scale.

We started off by assuming that the risk-sharing values of European countries are all approximately close to a similar measure: due to the presence of a common currency, the presence of the state-contingent cost across Europe forms a similar level of wedge away from achieving a full set of Arrow-Debreu securities. Our hypothesis is seen to be very accurate, as most of the parameters in table 2 are varying in 2 decimal places between 0.73 and 0.74.

As far as the econometric methods are concerned, the anomalous estimation exist mostly in column 3, where most of the estimated parameters did not converge to the necessary solution, which could be perhaps because of using the lesser efficient correlation measure to start the iteration off with. The most notable finding is that the nonlinear least squares, which did not use any exogenous instruments, **converged to the same solution** regardless of which initial values were used for the optimization algorithm. Most of the anomalous estimates are actually present in columns 7 and 8, which used the first-differenced instruments. This gives us the intuition that perhaps first-differencing on either the instruments or the data (to start off as the initial parameter values) could lead to only a local and **not** an efficient (or even consistent) solution.

7 Extensions

We have only dealt with the basics of estimating the risk-sharing, however we can implement a lot more advanced econometrics and extensions to this paper in the future. The following list shows the possible extension that can be used :

1. Using the *DataStream* data terminal, we can get a higher frequency data (weekly or daily frequency). This will provide more accurate estimation of our parameters.

2. We can estimate the level of international financial integration for a bigger set, e.g. South American countries.

3. We can use the following expression for foreign exchange reserves as defined in Devereux

and Yetman [7]:

$$\triangle FR_t = \left(\frac{S_{t-1}}{S_t}\right)^{\chi}, \ \chi \in [0, \infty)$$

where S_t represents the nominal exchange rate at time t, and χ the response of foreign exchange reserves to nominal exchange rate reserves.

4. As an extension to the Blundell-Bond estimation done in section 5.2, we can estimate for a functional form of the optimally efficient instrument to use in the GMM estimation. Ai and Chen [1], Dominguez and Lobato [8] and Hsu and Kuan [14] proposed various classes of efficient instruments which use Fourier transforms, (non)linear splines and minimum distance estimators.

5. A recent approach to estimating parameters in macroeconomic models is to use Bayesian maximum likelihood estimation. We will be assuming a prior distribution for $\tilde{\lambda}$, and using the computed raw estimates to form a likelihood function to pass through the Bayesian filter, we can estimate for the posterior distribution of λ for each country. The parameter estimation problem is computed through:

$$\lambda_{MLE} = \arg\max_{\lambda} \left[\ln \underbrace{(p(V_t|\lambda))}_{\substack{\text{Likelihood} \\ \text{function}}} + \sum_{i=1}^{N} \ln \underbrace{(p_i(\theta_i))}_{\text{Prior } pdf} \right]$$

As explained in Roberts and Smith [16], the posterior distribution can be estimated with Metropolis-Hastings algorithm (Markov Chain Monte Carlo methods), which is rather time consuming. An alternative and equivalent (free) approach to the Bayesian estimation would be to use a Laplace approximation to the posterior distribution of λ.

6. Finally, given that we have a robust estimate of the distribution of λ for each country, we can look at the approximation of the welfare function. We can observe how much

would a country gain in terms of welfare by going from the estimated value of λ to $\lambda = 1$, which is the representation of complete financial markets.

8 Conclusion

This paper has taken an open economy model and applied the assumption of a state-contingent wedge which exists between the trading of Arrow-Debreu securities. Under the presence of this wedge, this paper used an econometric method based estimation to measure for the degree of international financial integration between European countries and the United States, and applied those methods with macroeconomic data.

The two major classes of estimation used where indirect and direct identifications, where the indirect method measuring 'correlation' used OLS, while the direct estimation used nonlinear econometric methods. We found that, on average, most of the European countries have the same degree of risk-sharing (about 74%), and this applies to all of the direct identification methods.

This paper has shown that the initial hypothesis of European countries having a similar level of international risk-sharing holds due to similar policies and common currency. Our measures might not be the most cutting edge or sophisticated yet, but in the future extensions of this paper, our plan is to formulate more sophisticated econometric techniques, which will help us to refine our estimation and come up with more robust techniques.

A Appendix

A.1 OLS Estimation

Revisiting equation (8) and taking natural logarithms on both sides give:

$$\left[\left(\frac{C_t^{-\sigma}}{C_t^{*-\sigma}}\right)\left(\frac{S_tP_t^*}{P_t}\right)\right]^{\lambda} = \left[\frac{P_tC_t}{\overline{P_t}Y_t - \Delta(FR_t)}\right]^{1-\lambda}$$

$$\Rightarrow \lambda\left[-\sigma\ln\left(\frac{C_t}{C_t^*}\right) + \ln\left(\frac{S_tP_t^*}{P_t}\right)\right] = (1-\lambda)\left[\ln(P_tC_t) - \ln\left(\overline{P_t}Y_t - \Delta FR_t\right)\right]$$

Introducing the following notations for all t:

$c = \ln C;\ c^* = \ln C^*;\ RER = \left(\frac{SP^*}{P}\right)$

where RER denotes the real exchange rate, so $rer = \ln(RER)$

$$\Rightarrow \lambda\left[-\sigma\ln\left(\frac{C_t}{C_t^*}\right) + \ln(RER_t)\right] = (1-\lambda)\left[\ln(P_tC_t) - \ln\left(\overline{P_t}Y_t - \Delta FR_t\right)\right]$$

$$\Rightarrow \lambda\left[\sigma(c_t - c_t^*) - rer_t\right] = (1-\lambda)\left[-\ln(P_tC_t) + \ln\left(\overline{P_t}Y_t - \Delta FR_t\right)\right]$$

$$\Rightarrow \lambda\left[\sigma(c_t - c_t^*) - rer_t\right] = (1-\lambda)\left[-\ln P_t - \ln C_t + \ln\left(\overline{P_t}Y_t\left(1 - \frac{\Delta FR_t}{\overline{P_t}Y_t}\right)\right)\right]$$

Working with the right hand side of the equation:

$$\begin{aligned}
&= (1-\lambda)\left[\ln Y_t - \ln C_t - \ln P_t + \ln\overline{P_t} + \ln\left(1 - \frac{\Delta FR_t}{\overline{P_t}Y_t}\right)\right] \\
&= (1-\lambda)\left[\ln Y_t - \ln C_t - \ln P_t + \ln\overline{P_t} + \ln\left(1 - \frac{\Delta FR_t}{\overline{P_t}Y_t}\right)\right] \\
&= (1-\lambda)\left[\ln\left(Y_t\left(1 - \frac{C_t}{Y_t}\right)\right) - \ln\left(\frac{P_t}{\overline{P_t}}\right) + \ln\left(1 - \frac{\Delta FR_t}{\overline{P_t}Y_t}\right)\right] \\
&= (1-\lambda)\left[\ln\left(1 - \frac{C_t}{Y_t}\right) - \ln\left(\frac{P_t}{\overline{P_t}}\right) + \ln Y_t + \ln\left(1 - \frac{\Delta FR_t}{\overline{P_t}Y_t}\right)\right] \qquad (22)
\end{aligned}$$

According to the aggregate expenditures equation, we have:

$$Y = C + I + G + (X - M)$$

The firm side of the economy assumes that the production process is undertaken with only labour, and so: $I = 0$. Additionally, we assume that there is no fiscal policy in this small open economy, and so $G = 0$. Therefore, we have:

$$\begin{aligned} Y &= C + (X - M) \\ \Rightarrow 1 &= \left(\frac{C}{Y}\right) + \left(\frac{X - M}{Y}\right) \\ \therefore 1 - \left(\frac{C}{Y}\right) &= \left(\frac{X}{Y}\right) - \left(\frac{M}{Y}\right) \end{aligned}$$

Plugging this into the expression (22) gives:

$$\begin{aligned} &= (1-\lambda)\left[\ln\left(\frac{X_t}{Y_t} - \frac{M_t}{Y_t}\right) - \ln\left(\frac{P_t}{\overline{\overline{P_t}}}\right) + \ln Y_t + \ln\left(1 - \frac{\Delta FR_t}{\overline{P_t}Y_t}\right)\right] \\ &= (1-\lambda)\left[\ln\left(\frac{X_t}{Y_t} - \frac{M_t}{Y_t}\right) - \ln\left(\frac{P_t}{\overline{\overline{P_t}}}\right) + \ln\left(Y_t - \frac{\Delta FR_t}{\overline{P_t}}\right)\right] \end{aligned}$$

So, representing the left hand side expression as λW_t and the right hand side expression as $(1-\lambda)Z_t$, let $x = \frac{X}{Y}$ and $m = \frac{M}{Y}$:

$$\begin{aligned} \Rightarrow (1-\lambda)Z_t &= (1-\lambda)\left[\ln(x_t - m_t) - \ln\left(\frac{P_t}{\overline{\overline{P_t}}}\right) + \ln\left(Y_t - \frac{\Delta FR_t}{\overline{P_t}}\right)\right] \\ \Rightarrow (1-\lambda)Z_{t-1} &= (1-\lambda)\left[\ln(x_{t-1} - m_{t-1}) - \ln\left(\frac{P_{t-1}}{\overline{\overline{P_{t-1}}}}\right) + \ln\left(Y_{t-1} - \frac{\Delta FR_{t-1}}{\overline{P_{t-1}}}\right)\right] \end{aligned}$$

Subtracting the above two equations and dividing by $(1-\lambda)$:

$$\Rightarrow (Z_t - Z_{t-1}) = \left[\ln\left(\frac{x_t - m_t}{x_{t-1} - m_{t-1}}\right) - \ln\left(\frac{P_t}{P_{t-1}}\right) + \ln\left(\frac{\overline{P_t}}{\overline{P_{t-1}}}\right)\right] + \left[\ln\left(Y_t - \frac{\Delta FR_t}{\overline{P_t}}\right) - \ln\left(Y_{t-1} - \frac{\Delta FR_{t-1}}{\overline{P_{t-1}}}\right)\right]$$

If we assume that the foreign exchange reserves are exogenous, then $\Delta FR_t \in \nu_t$

$$\therefore \Delta Z_t = \left[\ln\left(\frac{x_t - m_t}{x_{t-1} - m_{t-1}}\right) - \ln\left(\frac{P_t}{P_{t-1}}\right) + \ln\left(\frac{\overline{P_t}}{\overline{P_{t-1}}}\right) + \ln\left(\frac{Y_t}{Y_{t-1}}\right)\right] + \nu_t$$

Similarly, we take the first differences of the dependant variable W_t:

$$\Rightarrow \quad \lambda W_t = \lambda\left[\sigma\left(\ln C_t - \ln C_t^*\right) - \ln(RER_t)\right]$$

$$\Rightarrow \lambda W_{t-1} = \lambda\left[\sigma\left(\ln C_{t-1} - \ln C_{t-1}^*\right) - \ln(RER_{t-1})\right]$$

$$\therefore \lambda \Delta W_t = \lambda\left[\sigma\left(\ln\left(\frac{C_t}{C_{t-1}}\right) - \ln\left(\frac{C_t^*}{C_{t-1}^*}\right)\right) - \ln\left(\frac{RER_t}{RER_{t-1}}\right)\right]$$

Therefore, the OLS estimation is in the linear form as follows:

$$\Delta W_t = \left(\frac{1-\lambda}{\lambda}\right)\Delta Z_t + \nu_t \tag{23}$$

A.2 Devereux and Yetman Estimation Approach

Devereux and Yetman took equation (8), and used log-linearization to the following OLS relationship, using similar notations as in section 3.1. The dependant variable is represented by the same functional form and variables as in W_t:

$$Y_t = \left[\sigma\left(\ln(C_t) - \ln(C_t^*\right) - \ln(RER_t)\right]$$

The regressor is represented by X_t where we substitute the expression for the relative subtracted from the income with net export from the expenditure equation:

$$y_t - \ln\left(\frac{P_t}{\overline{P}_t}\right) \equiv x_t - m_t$$

$$\therefore X_t = \ln\left[1 + x_t - m_t - (\Delta res_t)\right]$$

where x_t and m_t are exports and imports each measured as a share of GDP respectively, and $\Delta(res_t)$ is the change in foreign reserves, where we assume again that the foreign exchange reserves are part of the error terms u_t. The estimation equation they suggested is:

$$Y_t = X_t'\beta + u_t \tag{24}$$

Therefore, estimating the slope coefficient gives us a measurable function of λ such that:
$\therefore \ \beta = \dfrac{1-\lambda}{\lambda} \ \Rightarrow \lambda = \dfrac{1}{1+\beta}$.
Under absolute values of β, $\lambda = \dfrac{1}{1+|\beta|}$

A.3 GMM Estimation on Levels

Taking the Euler equation (8) and rearranging it gives the form:

$$\left[\left(\frac{C_t^{-\sigma}}{C_t^{*-\sigma}}\right)\left(\frac{S_t P_t^*}{P_t}\right)\right]^{\lambda}\left[\frac{\overline{P}_t Y_t - \Delta(FR_t)}{P_t C_t}\right]^{1-\lambda} - 1 = 0$$

Now, taking the conditional expectation of this equation at time t, we have a simplified form of the above expression:

$$\mathbb{E}_t\left[g(V_t;\theta)|I_t\right] = 0 \tag{25}$$

$$\text{where}:\ V_t = (P_t, C_t, Y_t, \Delta FR_t)$$

$$\theta = (\sigma, \lambda)'$$

$$g(V_t;\theta) = \left[\left(\frac{C_t^{-\sigma}}{C_t^{*-\sigma}}\right)\left(\frac{S_t P_t^*}{P_t}\right)\right]^{\lambda}\left[\frac{\overline{P_t}Y_t - \Delta(FR_t)}{P_t C_t}\right]^{1-\lambda} - 1 \tag{26}$$

and $I_t =$ the information set at time t

If Z_t is a vector of variables whose values are known at time t and is orthogonal to $g(V_t;\theta)$, then $z_t \in I_t$ and it follows from (25) that:

$$\mathbb{E}_t\left[g(V_t;\theta)\cdot Z_t|I_t\right] = 0$$

Taking the unconditional expectations on both sides and using the Law of Iterated Expectations (that $\mathbb{E}\left[\mathbb{E}(Z_t|I_t)\right] = \mathbb{E}(x)$), we obtain the necessary *nonlinear moment condition* required for GMM estimation:

$$\mathbb{E}_t\left[g(V_t;\theta)\cdot Z_t\right] = 0 \tag{27}$$

where Z_t will be a vector of lagged variables $\{V_{t-1}, V_{t-2}, \ldots, V_{t-p}\}$, which is a common strategy of estimating Euler equations containing variables at times t and $t+1$. Z_t will be referred to as the set of instrumental variables which are assumed to be orthogonal to the time t variables in V_t.

The sample analogue of (27) is given by $\hat{m_n}(\theta)$, such that:

$$\hat{m_n}(\theta) = \left[g(V_t;\theta)\right]\cdot\left[V_{t-1}, \ldots, V_{t-p}\right]'$$

Therefore, the GMM objective function with the sample analogue is:

$$Q_n(\theta) = (\hat{m_n}(\theta))' W_n (\hat{m_n}(\theta)) \tag{28}$$

and the GMM estimator which solves for the necessary parameters $\theta = (\sigma, \lambda)'$ is defined as:

$$\hat{\theta}_n = \arg\min_{\theta \in \Theta} Q_n(\theta) \tag{29}$$

Q_n is the criterion function which consists of the moment condition for sample of size n, a positive definite weight matrix W_n which converges in probability to a positive definite matrix W, and θ belonging to the compact parameter space Θ such that:

$$\theta_i = \{(\sigma_i, \lambda_i)' \in \mathbb{R}^2_+ \; |\sigma_i > 0, \; 0 \leq \lambda_i \leq 1\} \; \forall i$$

A.4 GMM Estimation on First-Differences

Let ϕ_t represent the first-differences of our data V_t with one-period lag, such that:

$$\phi_t = V_t - V_{t-1}$$

Under this setting, the following proposition can be presented:

Proposition 1. $\mathbb{E}_t \left[g(V_t, \theta) \cdot V_{t-1}\right] = 0 \Rightarrow \mathbb{E}_t \left[g(V_t, \theta) \cdot \phi_t\right] = 0$
If the generalized IV moment condition defined in (27) *for a nonlinear function g at time t holds, then the equivalent moment condition taken with first differences of the instruments ϕ also holds under the same assumptions of the GMM.*

Proof. Take the first difference and expand from the left hand side of the moment condition:

$$\begin{aligned} &\mathbb{E}_t\left[g(V_t,\theta)\cdot\phi_t\right] \\ &= \mathbb{E}_t\left[g(V_t,\theta)\cdot V_t - g(V_t,\theta)\cdot V_{t-1}\right] \\ &= \mathbb{E}_t\left[g(V_t,\theta)\cdot V_t\right] - \mathbb{E}_t\left[g(V_t,\theta)\cdot V_{t-1}\right] \end{aligned}$$

$\mathbb{E}_t\left[g(V_t,\theta)\cdot V_{t-1}\right] = 0$, from equation (27)

$$= \mathbb{E}_t\left[g(V_t,\theta)\cdot V_t\right] - 0$$

Using the law of iterated conditional expectations and linearity of expectation:

$$\begin{aligned} &= \mathbb{E}_t\left[\mathbb{E}\left(g(V_t,\theta)\cdot V_t\right)|I_t\right] \\ &= \mathbb{E}_t\left[\mathbb{E}\left(V_t\right)\cdot \mathbb{E}\left(g(V_t,\theta)\right)|I_t\right] \end{aligned}$$

$\mathbb{E}\left[g(V_t,\theta)|I_t\right] = 0$ from equation (25), which is our original moment condition:

$$\begin{aligned} &= \mathbb{E}_t\left[\mathbb{E}\left(V_t\right)\cdot (0)|I_t\right] \\ \therefore\ &\mathbb{E}_t\left[g(V_t,\theta)\cdot\phi_t\right] = 0 \end{aligned}$$

Therefore, instrumenting on the first-differences of current and one-period lagged data gives a valid moment condition. □

A.5 Nonlinear Least Squares

Equation (21) set up the sum of squared residuals to be minimized with respect to the required estimators:

$$\sum_{t=1}^{T}(U(\sigma,\lambda))^2 = \sum_{t=1}^{T}\left(\left[\left(\frac{C_t^{-\sigma}}{C_t^{*-\sigma}}\right)\left(\frac{S_tP_t^*}{P_t}\right)\right]^{\lambda}\left[\frac{\overline{P_t}Y_t - \Delta(FR_t)}{P_tC_t}\right]^{1-\lambda} - 1\right)^2$$

The following notations are used to simplify the derivations and workings:

- $A = \left(\frac{C_t}{C_t^*}\right)$
- $B = \left(\frac{S_tP_t^*}{P_t}\right)$
- $C = \left(\frac{\overline{P_t}Y_t - \Delta(FR_t)}{P_tC_t}\right)$

Therefore, equation (21) can be rewritten as:

$$\begin{aligned}\sum_{t=1}^{T}(U(\sigma,\lambda))^2 &= \sum_{t=1}^{T}\left(\left(A^{-\sigma}\cdot B\right)^{\lambda}(C)^{1-\lambda} - 1\right)^2 \\ &= \sum_{t=1}^{T}\left(\left(A^{-\sigma}\cdot B\right)^{2\lambda}(C)^{2-2\lambda} - 2\left(A^{-\sigma}\cdot B\right)^{\lambda}(C)^{1-\lambda} + 1\right)\end{aligned}$$

The first-order necessary conditions for each of σ and λ respectively are:

$$\frac{\delta}{\delta\lambda}\left[\sum_{t=1}^{T}(U(\sigma,\lambda))^2\right] = 0$$

$$\frac{\delta}{\delta\sigma}\left[\sum_{t=1}^{T}(U(\sigma,\lambda))^2\right] = 0$$

Taking the partial derivative with respect to λ and expanding:

$$\frac{\delta}{\delta\lambda}\left[\sum_{t=1}^{T}\left(U(\sigma,\lambda)\right)^{2}\right]=0$$
$$\Rightarrow\sum_{t=1}^{T}\left[2\left(\left(A^{-\sigma}\cdot B\right)^{2\lambda}(C)^{2-2\lambda}(-\sigma\ln A+\ln B)\right)-2\left(A^{-\sigma}\cdot B\right)^{2\lambda}(C)^{2-2\lambda}(\ln C)\right]$$
$$=2\sum_{t=1}^{T}\left[\left(A^{-\sigma}\cdot B\right)^{\lambda}(-\sigma\ln A+\ln B)(C)^{1-\lambda}-\left(A^{-\sigma}\cdot B\right)^{\lambda}(C)^{1-\lambda}(\ln C)\right]$$

Simplifying it gives the first normal equation:

$$\boxed{\begin{aligned}&\sum_{t=1}^{T}\left(\{-\sigma\ln A+\ln B\}\left[\left(A^{-\sigma}\cdot B\right)^{2\lambda}(C)^{2-2\lambda}-\left(A^{-\sigma}\cdot B\right)^{\lambda}(C)^{1-\lambda}\right]\right)\\&=\sum_{t=1}^{T}\left(\{\ln C\}\left[\left(A^{-\sigma}\cdot B\right)^{2\lambda}(C)^{2-2\lambda}-\left(A^{-\sigma}\cdot B\right)^{\lambda}(C)^{1-\lambda}\right]\right)\end{aligned}} \tag{30}$$

Now, taking the partial derivative with respect to σ:

$$\frac{\delta}{\delta\sigma}\left[\sum_{t=1}^{T}\left(U(\sigma,\lambda)\right)^{2}\right]=0$$
$$\Rightarrow\sum_{t=1}^{T}\left[\left(A^{-2\sigma\lambda}\right)(-2\lambda\ln A)\left(B^{2\lambda}\right)\left(C^{2-2\lambda}\right)-2\left(A^{-\sigma\lambda}\right)(-\lambda\ln A)\left(B^{\lambda}\right)\left(C^{1-\lambda}\right)\right]=0$$

Simplifying it gives the second normal equation:

$$\boxed{\sum_{t=1}^{T}\left[\left(A^{-2\sigma\lambda}\right)(\lambda\ln A)\left(B^{2\lambda}\right)\left(C^{2-2\lambda}\right)\right]=\sum_{t=1}^{T}\left[\left(A^{-\sigma\lambda}\right)(\lambda\ln A)\left(B^{\lambda}\right)\left(C^{1-\lambda}\right)\right]} \tag{31}$$

B Tables

Table 1: Estimated measure of risk-sharing

	λ^1_{OLS} (1)	λ^2_{OLS} (2)	$\tilde{\lambda}^1_{GMM,1}$ (3)	$\tilde{\lambda}^2_{GMM,1}$ (4)	$\tilde{\lambda}^1_{NLS}$ (5)	$\tilde{\lambda}^2_{NLS}$ (6)	$\tilde{\lambda}^1_{GMM,2}$ (7)	$\tilde{\lambda}^2_{GMM,2}$ (8)
Austria	1.00	0.82	1.03	1.82	1.02	1.02	2.00	1.82
Belgium	1.00	0.71	2.00	1.05	1.02	1.02	1.04	1.04
Switzerland	0.98	0.25	1.98	0.99	0.99	0.99	1.01	1.01
Cyprus	0.96	0.10	1.00	1.00	0.99	0.99	0.99	2.10
Germany	0.95	0.54	1.95	1.05	1.02	1.02	1.95	1.05
Denmark	0.98	0.27	1.98	0.98	0.99	0.99	1.98	1.04
Spain	0.65	0.91	1.06	1.91	1.01	1.01	1.06	1.91
Finland	0.97	0.90	1.97	1.90	1.04	1.04	1.97	1.90
France	0.70	0.61	1.70	1.04	1.01	1.01	1.70	1.04
United Kingdom	0.77	0.59	1.77	1.04	1.02	1.02	1.77	1.03
Ireland	0.98	0.53	1.98	1.04	1.03	1.03	1.98	1.11
Iceland	0.99	0.67	1.09	1.02	1.02	1.02	1.08	1.02
Israel	0.91	0.16	1.18	0.98	0.98	0.98	1.16	0.99
Italy	0.69	0.35	1.69	1.01	1.01	1.01	1.05	1.02
Luxembourg	1.00	0.56	1.00	1.00	1.00	1.00	1.00	1.01
Netherlands	1.00	0.55	2.00	1.05	1.03	1.03	2.00	1.05
Norway	1.00	0.96	2.00	1.96	1.00	1.00	2.00	1.96
Portugal	0.85	0.44	1.08	1.02	1.01	1.01	1.09	1.01
Sweden	0.97	0.29	1.97	1.00	0.99	0.99	1.97	1.12

Table 2: Transformed measure of risk-sharing

	λ^1_{OLS} (1)	λ^2_{OLS} (2)	$\lambda^1_{GMM,1}$ (3)	$\lambda^2_{GMM,1}$ (4)	λ^1_{NLS} (5)	λ^2_{NLS} (6)	$\lambda^1_{GMM,2}$ (7)	$\lambda^2_{GMM,2}$ (8)
Austria	1.00	0.82	0.74	0.86	0.74	0.74	0.88	0.86
Belgium	1.00	0.71	0.88	0.74	0.73	0.73	0.74	0.74
Switzerland	0.98	0.25	0.88	0.73	0.73	0.73	0.73	0.73
Cyprus	0.96	0.10	0.73	0.73	0.73	0.73	0.73	0.89
Germany	0.95	0.54	0.88	0.74	0.73	0.73	0.88	0.74
Denmark	0.98	0.27	0.88	0.73	0.73	0.73	0.88	0.74
Spain	0.65	0.91	0.74	0.87	0.73	0.73	0.74	0.87
Finland	0.97	0.90	0.88	0.87	0.74	0.74	0.88	0.87
France	0.70	0.61	0.85	0.74	0.73	0.73	0.85	0.74
United Kingdom	0.77	0.59	0.85	0.74	0.73	0.73	0.85	0.74
Ireland	0.98	0.53	0.88	0.74	0.74	0.74	0.88	0.75
Iceland	0.99	0.67	0.75	0.73	0.73	0.73	0.75	0.73
Israel	0.91	0.16	0.77	0.73	0.73	0.73	0.76	0.73
Italy	0.69	0.35	0.84	0.73	0.73	0.73	0.74	0.73
Luxembourg	1.00	0.56	0.73	0.73	0.73	0.73	0.73	0.73
Netherlands	1.00	0.55	0.88	0.74	0.74	0.74	0.88	0.74
Norway	1.00	0.96	0.88	0.88	0.73	0.73	0.88	0.88
Portugal	0.85	0.44	0.75	0.73	0.73	0.73	0.75	0.73
Sweden	0.97	0.29	0.88	0.73	0.73	0.73	0.88	0.75

C References

[1] Ai, C., and X. Chen (2003): "Efficient estimation of models with conditional moment restrictions containing unknown functions," *Econometrica*, 71(6), 1795–1843.

[2] Alan, S., O. Attanasio, and M. Browning (2009): "Estimating Euler equations with noisy data: two exact GMM estimators," *Journal of Applied Econometrics*, 24(2), 309–324.

[3] Blundell, R., and S. Bond (2000): "GMM estimation with persistent panel data: an application to production functions," *Econometric reviews*, 19(3), 321–340.

[4] Calvo, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12(3), 383–398.

[5] Choi, W. G., and Y. Jung (2003): "Optimal monetary policy in a small open economy with habit formation and nominal rigidities," *IMF Working Paper*.

[6] Devereux, M. B., and J. Yetman (2014a): "Capital Controls, Global Liquidity Traps, and the International Policy Trilemma," *The Scandinavian Journal of Economics*, 116(1), 158–189.

[7] ——— (2014b): "Globalisation, pass-through and the optimal policy response to exchange rates," BIS Working Papers 450, Bank for International Settlements.

[8] Dominguez, M. A., and I. N. Lobato (2004): "Consistent estimation of models defined by conditional moment restrictions," *Econometrica*, 72(5), 1601–1615.

[9] Gali, J., and T. Monacelli (2005): "Monetary policy and exchange rate volatility in a small open economy," *The Review of Economic Studies*, 72(3), 707–734.

[10] Gertler, M., J. Gali, and R. Clarida (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37(4), 1661–1707.

[11] Hansen, L. P., and K. J. Singleton (1982): "Generalized instrumental variables estimation of nonlinear rational expectations models," *Econometrica: Journal of the Econometric Society*, pp. 1269–1286.

[12] Hartley, H. O. (1961): "The modified Gauss-Newton method for the fitting of nonlinear regression functions by least squares," *Technometrics*, 3(2), 269–280.

[13] Hayashi, F. (2000): *Econometrics*. Princeton Univ. Press, Princeton, NJ [u.a.].

[14] Hsu, S.-H., and C.-M. Kuan (2011): "Estimation of conditional moment restrictions without assuming parameter identifiability in the implied unconditional moments," *Journal of Econometrics*, 165(1), 87–99.

[15] Matsumoto, A., M. R. P. Flood, and M. N. P. Marion (2009): *International risk sharing during the globalization era*, no. 9-209. International Monetary Fund.

[16] Roberts, G. O., and A. F. Smith (1994): "Simple conditions for the convergence of the Gibbs sampler and Metropolis-Hastings algorithms," *Stochastic processes and their applications*, 49(2), 207–216.

[17] Zeileis, A. (2015): *pwt8: Penn World Table (Version 8.0).*