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Amavilah, Voxi Heinrich

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Productivity and technical change according to Salter – A note

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Voxi Heinrich Amavilah*
REEPS
PO Box 38061
Phoenix, AZ 85069-8061 USA
vhsamavilah@gmail.com

Abstract: Salter's simple and clear explanation of productivity and how it relates to technical change has anchored many elaborate and fancy growth and change analyses. Unfortunately many of these elaborations do not even reference Salter. They should. This note shows that some old ideas are like wine which gets better with age.

Key phrases: Salter and productivity; Salter and technical change; productivity and technical change
JEL Codes: O3, 04

WEG Salter's (1966[1960]) book generated a huge literature on the connection between productivity and technical change. Other area experts, too many to single out, have extended the theoretical and practical aspects of Salter to such things as the decomposition of total factor productivity into technical change, technological change, efficiency, returns to scale, and economies of scale. However, rapid progress has also disoriented understanding of the foundations of productivity analysis. This note outlines Salter's key insights which are essential for thinking about productivity and technical change. First, it links the two concepts in simple and clear way. Second, it outlines the components of productivity. Third, it describes how the components of productivity are measured and how they compare to each other. Fourth, it sketches a simple modification of productivity to demonstrate the staying power of his ideas. The last section makes a concluding remark.

1. **Productivity**

According to Salter the growth of productivity ($\dot{\rho}$) is some function φ of the rate of technical change ($\dot{\tau}$), i.e., $\dot{\rho} = \varphi(\dot{\tau}), \varphi' > 0$. The rate of technical change depends on factor biases (B), factor substitutions (σ), changes in relative factor prices ($\dot{p}$), and the extent of technical change (E). In other words, $\dot{\tau} = \theta(B, \sigma, \dot{p}\ E)$, suggesting that

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*Amavilah is an independent resource and engineering economist for REEPS, and an adjunct professor of economics in the Maricopa County Community Colleges District, Arizona, USA.

$$\dot{\rho} = \varphi(\theta(B, \sigma, \dot{p}, E)), \tag{1}$$

Then Salter argues that factor biases represent factor proportional changes relative to factor costs being constant. Factor substitutions are measured by factor elasticities, while the extent of technical change the relative distance to which unit costs fall holding factor prices constant. Take a time-dependent Solow (1957) economy in which output (Q) depends on labor (L) and capital (K), i.e., $Q(t) = Q(K(t), L(t), t)$. In such an economy the growth rate of Q would be

$$\dot{q} = r\dot{K} + w\dot{L} + \dot{\rho} \Rightarrow \dot{\rho} = \dot{q} - r\dot{K} - w\dot{L}, \tag{2}$$

where $\dot{\rho}$ is a measure of technical change. This specification bunches into one ρ and τ, a good reason for Abramovits (1979) to call it a "measure of our ignorance," or Schmookler (1966) a "terra incognito of modern economics" (p. 13, cf. Mansfield, 1980, Rosenberg, 1972, Choi, 1983, pp. 62-91).

To get the full meaning of (2), assume that $Q = e^{mt}K^{r}L^{w}$, so that three characterizations of $\dot{\rho}$ can be made. First is the Harrod-Robinson neutral technical progress, which is labor-augmenting, i.e., $Q = e^{\lambda t}K^{r}L^{w}$, $\lambda = m(1-r) = mw$, implying that $\partial Q/\partial K$ is constant for constant Q/K. Second is Solow neutrality, which is capital-augmenting, i.e., $Q = e^{\lambda t}K^{r}L^{w}$, $\lambda = m(1-w) = mr$. For Hicks neutrality $Q = e^{\lambda t}K^{r}L^{w}$, $\lambda = m$, such that a Hicks neutral technical change is also a disembodied technical change.

From (2) it is now clear that technical change is determined by economies of scale, improved resource allocation (efficiency), changes in the structure of the production activity, and the degree to which capacity is utilized. Thus, productivity change has at least four components: technical change, efficiency change, scale change, and the input-output mix effect (Nemoto and Goto, 2005, Sabasi and Shumway, 2014). A lot of effort has gone into modeling these components. For example, Nadiri and Schankerman (1981) developed a framework for decomposing total factor productivity when the underlying production function is characterized by economies of scale. Using what they called a 'quasi-Divisia" index they demonstrate total factor productivity $(TF\dot{\rho})$ to be

$$TP\dot{\rho} = \dot{\rho} = \frac{1 + \delta - \eta}{1 + \delta}\dot{q} + \frac{\eta}{1 + \delta}\dot{\tau} + \alpha\dot{D}, \tag{3}$$

where $1 + \delta$ is ratio of the output price (P_q) to output average variable cost (AVC), $\eta = MC/AVC$ is the elasticity of variable cost such that marginal cost (MC), $MC = \eta AVC = \eta[P_q(1 + \delta)^{-1}]$, and D is some indicator of the endogenous component(s) of productivity represented by a research and development indicator. The full version of (3) here is Nadiri and Schankerman's Equation 8 on p.

3 (cf. Kendrick, 1973, 1979, Harris, 1992).[1]

2. Components of $\dot{\rho}$

Present-day decompositions of productivity are sophisticated and elegant, but they nearly all rely on Salter, many without acknowledgment. Following is how Salter expresses the components of productivity. First,

$$E = (w\dot{L} + r\dot{K})(wL + rK)^{-1} = \dot{q}q^{-1}, \tag{4}$$

for w is the wage rate and r is the rental rate. Second, the bias is

$$B = \dot{k}k,\ k = K/L, \tag{5}$$

where $\dot{k}$ is the growth rate of capital-labor ratio (K/L). Third, factor substitution is

$$\sigma_{LK} = \sigma_{KL} = \xi_{KL}/S_K, \tag{6}$$

where $S_K = rK(wL + rK)^{-1} = rKq^{-1}$, $\xi_{KL} = S_K(\sigma_{LK} + \sigma_{KL}) = S_K(2\sigma_{LK}) \Rightarrow [\dot{k}\ell]/[\frac{r/w}{\dot{r}}]$, $\ell = L/K$, $\dot{r}$ is the growth rate of r/w, and $r/w = MP_L/MP_K$, under perfect competition. Therefore, $\dot{\rho} = \varphi(\theta(E, \sigma, B, \dot{p}))$, and p is given as factor suppliers are price-takers.

3. Measuring $\rho \approx \tau$

Often measurement mixes, not only productivity and technical progress ($\rho \approx \tau$), but also technical change and technological change, the distinction between "the progress of know-why" and the "progress of know-how" have been lost entirely (see Amavilah, 1997, Amavilah and Newcomb, 2004).[2] That said, ρ is commonly measured in three different ways: Single factor productivity: $SF\dot{\rho} = (Q_t/X_{it})100$, or $SF\dot{\rho} = (\Delta Q_t/\Delta X_{it})100$; Non-factor specific productivity: $NF\dot{\rho} = (\Delta Q_t/Q_t)100$; and Total factor productivity (TFP): $TF\dot{\rho} = (Q_t/\sum X_{it})100$. However, instead of getting (2) and then $\dot{\rho}$ from it, often researchers use well-known index numbers like: Paasche,

[1] I am here referring to the notes I took when I was DeVerle Harris's student, but his ideas on the implications of technology, productivity, and learning in the minerals industries and sectors are well-known.

[2] However, "know-how" depends on "know-why." As my old professor used to say, "First think it up, then write it up."

Laspeyres, Fisher, and Divisia.[3] Therefore, in empirical work ρ is price-indexed Q, so that., $\rho = Q_t I_t^j$ ***for*** I = ***index number***, j = ***Paasche, Laspeyres, Fisher, Divisia, etc.***, i.e., a Fisher $\rho = QI_t^F$.

Now, since $E = \dot{q}q^{-1} = \Delta Cost/Cost$, one can say that

$$TF\dot{\rho} = \dot{q}/E.Cost, \text{ where } Cost = f(\sigma, \dot{p}, ...). \tag{7}$$

If (7) is a true statement, then $\Delta k \rightharpoonup \Delta\dot{q}$. Alternatively, for $\partial\dot{q}/\partial t \neq 0 \leftrightarrow \Delta k \mapsto \Delta\dot{q}$. To illustrate this let $\dot{q}' = \dot{q} + \Delta\dot{q}, \forall\Delta\dot{q} > 0$, then

$$\dot{q}\dot{q}^{-1} < \dot{q}+\Delta\dot{q}\dot{q}^{-1} = 1 < 1 + \Delta\dot{q}\dot{q}^{-1} \rightarrow TF\dot{\rho}|_{\dot{q}} < TF\dot{\rho}|_{\dot{q}'}. \tag{8}$$

Since $\dot{q} < \dot{q}'$, the residual $R = TF\dot{\rho}|_{\dot{q}'} - TF\dot{\rho}|_{\dot{q}}$. Antonelli and Quantraro (2014) has termed R, the "bias effect," (cf. Hsiao, 1968). Hence, one can show that

$$R = \dot{q}'\dot{q}^{-1} - \dot{q}\dot{q}^{-1} = (\dot{q} + \Delta\dot{q} - \dot{q})\dot{q}^{-1} = \Delta\dot{q}\dot{q}^{-1}. \tag{9}$$

Eq. (9) embodies Solow's (1957) perspective.

4. Comparing and modifying ρ's

How do the ρ's compare? From the preceding we know that $SF\rho = f(\dot{q}, prices)$, $NF\rho = f(\dot{q}, Cost)$, and $TF\rho = f(B, \sigma, E, \dot{p})$. Moreover, since R differs across economies, productivity may be modified in many ways. For example, one can represent it as

$$TF\dot{\rho} = \dot{q} - w\dot{L} - (r + \delta)\dot{K} = TF\dot{\rho}|_{\dot{q}} - \delta\dot{K}. \tag{10}$$

If $\delta\dot{K}$ in (10) happens but $\dot{q}$ is constant, then the system is Solow neutral, i.e., technical change is K-augmenting and R falls as the true $\dot{\rho}$ increases and its technical part falls. For Harrod neutrality technical change is L-augmenting, while for Hicks neutrality technical change is autonomous, or it is at least factor unbiased. Consequently, while they are crucial for Harrod- and

[3] I see no need to write down the formulae for these indexes as they are well-known to most economists.

Solow-neutrality, for Hicks factor biases are less important than other determinants of productivity growth. A common view of (10) is to think of δ as either depreciation, depletion, extenalities, and such. These all affect productivity through different avenues like costs so that one may write the general production function as $Q(t) = \rho Q(K, L)\delta = \Omega Q(K, L)$, $\Omega = \rho \times \delta$ (Newcomb, 1976). From this Amavilah (1997), and Amavilah and Newcomb (2004) argue that technical change can be simultaneously Hicks neutral and Arrow-learning, so that one can write $Q = K^r L^w e^{\lambda t + \delta\tau}$. Since $\tau = \theta(.)$ is cumulative Q in Arrow's (1962, 1969) sense, and it may be transcendental, i.e.,

$$Q = K^r L^w e^{\lambda t + \delta[r'K + w'L]}. \tag{11}$$

Taking the natural logs and time derivatives of (11) leads to:

$$\dot{q} = ([\lambda + \delta[r'\dot{K} + w'\dot{L}]) + r\dot{K} + w\dot{L}. \tag{12}$$

In a way this is the point Solow (1997) makes in "*Learning from 'Learning by doing': Lessons for economic growth.*" [1] There Chapter 1 focuses on technical change as in his 1957 article. Chapter 2 is about technological change, and Chapter 3 describes a policy that allows for the integration of technical change and technological change. Thus, from (12) it is clear that $\dot{\rho}$ can be either or the following:

$$\begin{aligned}
&(a)\ \dot{\rho} \equiv (\lambda + \delta[r'\dot{K} + w'\dot{L}]) = \dot{q} - r\dot{K} - w\dot{L} \\
&(b)\ \dot{\rho} \equiv (\lambda + \delta r'\dot{K}) = \dot{q} - r\dot{K} - (w + \delta w')\dot{L} \\
&(c)\ \dot{\rho} \equiv (\lambda + \delta w'\dot{L}) = \dot{q} - w\dot{L} - (r + \delta r')\dot{K} \\
&(d)\ \dot{\rho} \equiv \lambda = \dot{q} - (r + \delta r')\dot{K} - (w + \delta w')\dot{L}
\end{aligned} \tag{13}$$

Again, (13) suggests that neutrality and learning are not mutually exclusive. This is a new research-intensive area. For example, Acemoglu (2002) start with something like (5) and finds that B is determined by prices and market size, and in turn B influences ρ (directed/biased) technical change. While research like Acemoglu's is clearly novel, its insights are not brand new. In fact, Kindrick (1972, 1979) has suggested that the time-dependent technical change (λt) explains productivity, but it may not, and often does not, imply technological change (δτ) and learning. Technological change, productivity, and learning go hand in hand.

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[1] If my notations look familiar to the reader, it is because I relied on Allen (1968). I am also an admirer of Nick Kaldor's work on technical change, although I have not looked at it while writing this note.

Concluding remark

With all the fancy and elegant productivity math, the meaningful content of productivity analysis remain Salterian. For practical application, especially where limited data availability is the rule than the exception, Salter's approach remains attractive. It is too bad that many no longer give the original idea its due credit – they should as new elaborations do not always mean new knowledge.

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