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ABSTRACT

This paper examines the effect of fertility and official pension age on long-run pay-as-you-go (PAYG) pensions based on an overlapping generations model. We find that increasing the fertility rate or official pension age does not necessarily raise pensions. When the output elasticity of capital is low, an increase in the fertility rate or official pension age may raise pensions, but such a change reduces pensions if the output elasticity of capital and the tax rate are high.

Keywords: Fertility; Official pension age; OLG, PAYG pensions.

JEL Classification: J13, J26, H55.

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1. INTRODUCTION

Over the past century, many developed countries have witnessed steady declines in their fertility rate and mortality rate, leading to concerns about the sustainability of social security systems, particularly the pay-as-you-go (PAYG) pension system. There is a huge amount of literature devoted to possible solutions to the problem of the increasing pension burden caused by aging populations. One possible solution is to reform the pension system from unfunded schemes to funded schemes. Since such a reform would cause a dramatic effect on individuals during the transition,[1] the provision for child allowances and the postponement of the official pension age have become popular governmental policies in recent decades, with the common belief that raising the fertility rate or delaying the official pension age would lead to a situation where more workers support fewer old people, thus reducing the burden placed upon government budgets. Providing child allowances reduces the cost of raising children, and parents may then have stronger incentives to have more children. This policy can be found in many developed countries such as Australia, Ireland, Japan and Sweden. By postponing the official pension age, workers work longer and benefit from pensions for a shorter length of time. In Australia, men and women currently aged 65 are eligible for the age pension. The Australian government announced in 2009 that the Age Pension age will start increasing from 2017 and reach 76 years of age by 2023.

While most people believe that raising the fertility rate and official pension age can increase pensions, Cigno (2007) has a different opinion: "The combined effect of fewer births, longer lives and sluggish retirement age is putting public pension system, all essentially pay-as-you-go, under increasing strain." This calls for a need to analytically examine the effect of fertility and official pension age on pensions. Although policies of increasing the fertility rate and official pension age are usually implemented together in developed countries, previous studies tend to examine their effects separately. For example, Fanti and Gori (2012) study the effect of fertility on pensions based on a model without the consideration of official pension age. Miyazaki (2014) analyzes the influence of official pension age on pensions by assuming that the population is constant. These two studies find that increasing the fertility rate or official pension age does not necessarily raise pensions.

This paper studies how fertility and the official pension age impact long-run pension benefits based on an overlapping generations model that incorporates both the fertility rate and official pension age. Since our focus is the influence of fertility and official pension age on pensions, we follow Fanti and Gori (2012) and Miyazaki (2014) and assume that the fertility rate and official pension age are exogenously given. Adults need to work in order to consume and to raise children. Old agents need to work for a fraction of time in old age. After they retire, they enjoy pension benefits. Government

[1] See Zhang and Zhang (2003) for a comparison of unfunded and funded social security systems.

implements the PAYG social security system. All workers need to pay income tax to support the pensions.

We find that increasing the fertility rate or official pension age does not necessarily raise pensions. Such a change in fertility or official pension age causes a positive direct effect due to more labor contributing to pensions and a negative effect due to decreases in the equilibrium wage and savings. We show that the output elasticity of capital plays an important role in determining the effects of fertility and official pension age on pension benefits since it governs the general equilibrium feedback of the wage rate. When the output elasticity of capital is low, an increase in the fertility rate or official pension age may raise pensions, but such a change reduces pensions if the output elasticity of capital and the tax rate are high. Our results also indicate that raising fertility tends to reduce pensions for developing countries, while postponing the official pension age tends to increase pensions for developed countries.

The remainder of this paper is organized as follows. Section 2 develops the model. Section 3 analyzes the effects of fertility and the official pension age on long-run pension benefits. Section 4 concludes.

2. THE MODEL

We consider an overlapping generations model where agents live for three periods - childhood, adulthood (parenthood), and old age. Adults in period *t* care about their consumption in adulthood (c_t) and in old age (c_{t+1}). The utility function, which is identical for all agents, is defined as:

$$u_t = \ln c_t + \beta \ln c_{t+1}, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor.

All decisions are made in adulthood, with adults deciding how much to consume, and how much they should save for their old age (s_t). In each period, agents are endowed with one unit of time. Adults with population (N_t) spend all their time working to earn wages (w_t). Following Fanti and Gori (2012), we assume that the number of children (*n*) is exogenous. It takes a fixed proportion ($q \in (0,1)$) of each adult's wage to raise a child (see, e.g., Wigger 1999; Boldrin and Jones 2002; Fanti and Gori 2012). In order to pay for pensions for the elderly, the government levies an income tax with the rate $\tau \in (0,1)$. The budget constraint for an adult is:

$$c_t + s_t + nqw_t = (1-\tau)w_t. \tag{2}$$

Following Miyazaki (2014), we assume that old agents are eligible for pensions at age $(1+l)$ as regulated by law. Since we do not distinguish between official pension age and retirement age, this implies that agents work at a fraction of time $l \in (0,1)$ in their old age. The budget constraint for an old agent is:

$$c_{t+1} = R_{t+1}s_t + (1-\tau)\theta l w_{t+1} + (1-l)P_{t+1}, \tag{3}$$

where R_{t+1} is the gross rate of return of savings, P_{t+1} is the pensions per unit of time and $\theta > 0$ is the productivity of an old agent.[2]

The government runs a balanced budget. Given that the tax revenue is used to provide pensions for the old, this implies that:

$$(1-l)N_t P_{t+1} = \tau w_{t+1}(N_{t+1} + \theta l N_t). \tag{4}$$

Output is produced by using capital (K_t) and effective labor ($L_t = N_t + \theta l N_{t-1}$) and is based on the Cobb-Douglas production function $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where $A > 0$ and $\alpha \in (0,1)$ is the output elasticity of capital. We define $k_t = K_t / L_t$. The gross rate of the return on capital and the real wage rate are respectively:

$$R_t = \alpha A k_t^{\alpha-1}, \tag{5}$$

$$w_t = (1-\alpha)A k_t^{\alpha}. \tag{6}$$

2.1 Equilibrium and steady state

The optimal decision of s_t is:

$$s_t = \frac{\beta(1-\tau-qn)w_t}{1+\beta} - \frac{(1-\tau)\theta l w_{t+1} + (1-l)P_{t+1}}{(1+\beta)R_{t+1}}. \tag{7}$$

Market clearing in the capital market indicates that $(n+\theta l)k_t = s_t$. Using (4)-(7), the law of motion of k_t is:

$$k_{t+1} = \frac{\alpha(1-\alpha)A\beta(1-\tau-qn)}{(n+\theta l)\alpha(1+\beta) + (1-\alpha)(\tau n + \theta l)} k_t^{\alpha}. \tag{8}$$

Equation (8) implies that there exists a unique, globally stable steady state k^*:

$$k^* = \left[\frac{\alpha(1-\alpha)A\beta(1-\tau-qn)}{\alpha(1+\beta)(n+\theta l) + (1-\alpha)(\tau n + \theta l)}\right]^{\frac{1}{1-\alpha}}. \tag{9}$$

To ensure $k^* > 0$, we assume that $1-\tau < qn$.

3. PENSIONS

Equation (4) indicates that:

$$p^* = p^*(n, l, k^*(n,l)). \tag{10}$$

From (10), we derive:

[2] Sala-i-Martin (1996) argues that old workers are less productive than young workers due to the depreciation of human capital.

$$\frac{dp^*}{dx} = \underbrace{\frac{\partial p^*}{\partial x}}_{+} + \underbrace{\underbrace{\frac{\partial p^*}{\partial w^*}}_{+}\underbrace{\frac{\partial w^*}{\partial k^*}}_{+}\underbrace{\frac{\partial k^*}{\partial x}}_{-}}_{-}, \tag{11}$$

where $x = n, l$. Although a higher fertility rate (retirement age) means more labor (direct effect) contributing to pensions, the lower equilibrium wage rate reduces savings and capital accumulation, causing per efficiency unit of pensions to decrease (indirect effect). Fertility or retirement age and pensions are positively (negatively) correlated if the direct (indirect) effect dominates.

Combining (4), (7) and (9), we can derive long-run PAYG pensions as:

$$p^* = \frac{\tau(n+\theta l)}{1-l}\left\{A(1-\alpha)\left[\frac{\alpha\beta(1-\tau-qn)}{\alpha(1+\beta)(n+\theta l)+(1-\alpha)(\tau n+\theta l)}\right]^{\alpha}\right\}^{\frac{1}{1-\alpha}}. \tag{12}$$

Proposition 1. (1) For $\alpha \in (0, \frac{1}{2}]$, there exists a unique $q_n > 0$ such that $\frac{dp^*}{dn} > 0$ if $q < q_n$ and vice versa. (2) For $\alpha \in (\frac{1}{2}, 1)$, there exists a unique τ_n such that if $\tau < \tau_n$, then $\frac{dp^*}{dn} > 0$ if $q < q_n$ and vice versa. If $\tau > \tau_n$, then $\frac{dp^*}{dn} < 0$.

Proof: From (12), we have:

$$\frac{dp^*}{dn} = \frac{\tau[-q\xi_1 + (1-\tau)\xi_2]}{1-l}\left\{\frac{A[(1-\alpha)\alpha\beta]^{\alpha}(1-\tau-qn)^{2\alpha-1}}{\alpha(1+\beta)(n+\theta l)+(1-\alpha)(\tau n+\theta l)}\right\}^{\frac{1}{1-\alpha}},$$

where

$$\xi_1 = (1-\alpha)n^2[\alpha(1+\beta)+\tau(1-\alpha)] + (n+\alpha\theta l)(1+\alpha\beta)\theta l > 0,$$

$$\xi_2 = (1-2\alpha)n[\alpha(1+\beta)+\tau(1-\alpha)] + (1-\alpha)(1+\alpha\beta)\theta l.$$

Define $q_n = \frac{(1-\tau)\xi_2}{\xi_1}$. Note that $\xi_2 > 0$ if $\alpha \in (0, \frac{1}{2}]$. Thus, $\frac{dp^*}{dn} > 0$ if $q < q_n$ and vice versa.

If $\alpha \in (\frac{1}{2}, 1)$, then $\xi_2 > 0$ if $\tau < \tau_n = \frac{(1-\alpha)(1+\alpha\beta)\theta l - (2\alpha-1)n\alpha(1+\beta)}{(2\alpha-1)n(1-\alpha)}$. This implies that $\frac{dp^*}{dn} < 0$ if $q < q_n$ and vice versa. If $\tau > \tau_n$, then $\xi_2 < 0$, implying $\frac{dp^*}{dn} < 0$.

QED.

Recall that an increase in fertility produces two opposite effects on p^*. When the output elasticity of capital is lower, an increase in fertility causes a smaller reduction in wages. Moreover, the lower cost of rearing children (q) induces higher savings and capital accumulation, resulting in a higher p^*. Proposition 1 shows that if $\alpha \le \frac{1}{2}$, then an increase in fertility raises p^* if $q < q_n$ and vice versa.

When $\alpha > \frac{1}{2}$, the result also depends on the tax rate. On the one hand, a higher tax rate directly increases contributions to p^*. On the other hand, it reduces p^* indirectly due to lower after-tax income and savings. We find that if the tax rate is sufficiently low such that $\tau < \tau_n$, then the effect of an increase in fertility on pensions is similar to the case of $\alpha \le \frac{1}{2}$ and that an increase in n may increase or decrease p^*, depending on the value of q. However, if the tax rate is sufficiently high such that $\tau > \tau_n$, then an increase in fertility reduces p^*, regardless of q. Notice that $\frac{\partial \tau_n}{\partial l} > 0$ and $\frac{\partial \tau_n}{\partial \theta} > 0$. Since developing countries tend to have a higher α and a lower τ_n (due to early retirement age and low productivity of old workers), then an increase in fertility is very likely to reduce p^* in these countries.

The following proposition concerns the effect of retirement age on pensions.

Proposition 2. There exists a unique $\alpha_l \in (0,1)$ such that: (1) for $\alpha \in (0, \alpha_l]$, then $\frac{dp^*}{dl} > 0$; (2) for $\alpha \in (\alpha_l, 1)$, then there exists a unique τ_l such that $\frac{dp^*}{dl} < 0$ if $\tau < \tau_l$ and vice versa.

Proof: From (12), we have:

$$\frac{dp^*}{dl} = \frac{\tau(\xi_3 + \xi_4)}{(1-l)^2(1-\alpha)}\left[\frac{A(1-\alpha)[\alpha\beta(1-\tau-qn)]^\alpha}{\alpha(1+\beta)(n+\theta l)+(1-\alpha)(\tau n+\theta l)}\right]^{\frac{1}{1-\alpha}},$$

where

$$\xi_3 = (1-\alpha)n(\theta+n)[\alpha(1+\beta)+\tau(1-\alpha)] > 0,$$

$$\xi_4 = (1+\alpha\beta)\theta\{l[\theta(1+l)+n] - \alpha(2\theta l+n)\}.$$

Define $\alpha_l = \frac{l[\theta(1+l)+n]}{2\theta l+n}$. Note that $0 < \alpha_l < 1$. If $\alpha \le \alpha_l$, then $\xi_4 \ge 0$, implying $\frac{dp^*}{dl} > 0$.

If $\alpha \in (\alpha_l, 1)$, then $\xi_4 < 0$. Define $\tau_l = \frac{(1+\alpha\beta)\theta\{l[\theta(1+l)+n]-\alpha(2\theta l+n)\}-(1-\alpha)n(\theta+n)\alpha(1+\beta)}{n(\theta+n)(1-\alpha)^2}$. Thus, $\frac{dp^*}{dl} < 0$ if $\tau < \tau_l$ and vice versa.

QED.

Because the lower the output elasticity of capital is, the smaller the reduction in wages caused by the postponement of retirement age, then $\frac{dp^*}{dl} > 0$ if $\alpha < \alpha_l$. If $\alpha > \alpha_l$, then the postponement of retirement age will cause a larger reduction in wages and the results will depend on τ. Note that α_l depends on n, l, and θ with $\frac{\partial \alpha_l}{\partial n} < 0$, $\frac{\partial \alpha_l}{\partial l} > 0$, and $\frac{\partial \alpha_l}{\partial \theta} > 0$. Since developed countries tend to have a higher α_l due to lower fertility, higher retirement age and higher productivity of old workers , it is very likely that postponing the retirement age would increase p^* in developed countries. Our results are different from Miyazaki (2014) who finds that p^* will increase in l if l is sufficiently high, regardless of the value of α.

4. CONCLUSIONS

This paper studies how the fertility rate and official pension age affect PAYG pensions based on an overlapping generations model. Although most people believe that increasing fertility or delaying retirement age would raise pensions, this paper's findings suggest that governments should also take the output elasticity of capital and the tax rate into consideration when looking into the problem of a pension crisis.

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