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Abstract. A firm's incentive to invest in product safety is affected by both the market environment and the liability when its product causes consumer harm. A long-standing question in law and economics is whether competition can (partially) substitute for product liability in motivating firms to improve product safety. We investigate this issue in a spatial model of oligopoly with product differentiation, where reputation provides a market incentive for product safety and higher product liability may distort consumers' incentive for proper product care. We find that partial liability, together with reputation concerns, can motivate firms to make socially desirable safety investment. Increased competition due to less product differentiation lowers equilibrium market price, which diminishes a firm's gain from maintaining reputation and *raises* the socially desirable product liability. On the other hand, an increase in the number of competitors reduces both the benefit from maintaining reputation and the potential cost savings from cutting back safety investment; consequently, the optimal liability may vary non-monotonically with the number of competitors in the market. In general, therefore, the relationship between competition and product liability is subtle, depending on how competition is measured.

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1. INTRODUCTION

Market competition and product liability are two major mechanisms that affect firms' incentives to increase product safety and prevent product harm to consumers. Extensive studies in law and economics have examined the effects of product liability and derived optimal liability rules under a given market structure. However, there has been little formal analysis of how competition and liability rules may interact to incentivize firms.[1] This is rather surprising, given the importance of product safety in many consumer markets. What is the relationship between competition and the socially desired liability rules? Are competition and product liability substitutes or complements in increasing product safety and social welfare? This paper provides an economic analysis that aims to shed light on these questions.

We consider a two-period spatial model with $N \geq 2$ firms selling differentiated products to heterogenous consumers. The products may malfunction and cause consumer harm with some probability. At the beginning of the first period, each firm can decide whether to invest in product safety. Investment leads to a safer product that causes less damage when it fails. Consumers cannot observe firms' safety investments, but in the second period they can observe the damages to the harmed consumers in the first period and update their beliefs about product safety. That is, there are reputation concerns for firms. If a firm's product causes consumer harm, it may need to compensate consumers according to product liability rules. In particular, under partial liability the firm is required to compensate only a proportion of consumer loss, whereas under full liability the firm is liable for the full damage. While liability can motivate firms to invest in product safety, it is not without undesirable incentive consequences due to the presence of two-sided moral hazard: After purchase, consumers can take precaution to reduce the potential harm from product failure; and high product liability lowers the consumers' precaution efforts.

We consider situations where investment in product safety is socially desirable. The

[1] A notable exception is Polinsky and Shavell (2010), who argue that market mechanisms and product liability are substitutes as they both can increase product safety.

choice of product liability is thus concerned with how to sustain the equilibrium where firms make safety investment. If a firm chooses a low- instead of high-safety product, it faces high (expected) liability costs and reputation loss. When reputation loss is high enough, the firm will make the safety investment even without product liability, in which case the socially optimal liability is zero in order to encourage consumer precaution. However, if reputation concern is not sufficient, then product liability needs to be increased in order to sustain the firm's investment incentive. For a given number of competitors in the market, less horizontal product differentiation will reduce equilibrium market price so that the expected reputation loss from offering a low-safety product becomes smaller. Consequently, the socially optimal liability is larger when competition becomes more fierce due to reduced product differentiation. In this sense, competition and product liability are complements in improving product safety and social welfare.

However, the socially optimal liability may vary non-monotonically with another measure of competition intensity, the number of competitors. If a firm deviates from a high-safety to a low-safety product, it benefits from saving the fixed cost of investment and the variable production cost in period 1, but suffers from the potential extra liability cost in period 1 and the reputation loss in period 2. In period 1, a deviating firm's net cost saving is the sum of fixed and variable production costs, minus the extra liability cost. A firm would sell a high-quality product only when the reputation loss is larger than the net cost saving from deviation. In our model, an increase in the number of competitors always reduces the reputation loss from deviation, but has a non-monotonic impact on the net cost saving from deviating to low safety in period 1. As a result, the optimal liability may vary non-monotonically with the number of competitors, possibly first decreasing and then increasing.

Thus, in general, the relationship between competition and product liability is subtle, depending on the measure of competition intensity. While they can often be complements, the relationship may also be non-monotonic when competition is measured by the number of competitors in the market. Our results can shed light on the mixed empirical evidence concerning the effects of competition on firms' investment incentives for product safety. For example, a 2008 survey among product development managers in the US revealed that companies were more likely to reduce safety investment and to speed up new product introductions, possibly with lower safety, when facing more competition (Lynn and Reiley, 2008). In the automobile industry, when more companies entered the market of SUVs, many products had low quality and later caused substantial consumer harm (Los Angeles Times, March 14, 2010). However, there have also been empirical studies showing that competition can increase product quality, though most of the studies do not focus on safety issues.[2] Furthermore, there is empirical evidence suggesting that product liability can have non-monotonic effects on firms' innovation incentives (e.g., Kip Viscusi and Moore, 1993).

Our paper contributes to the literature on product liability. Studies with a single firm analyze, for example, the effects of liability rules on a firm's precaution to ensure product safety (Simon, 1981), on its quality choice (Polinsky and Rogerson, 1983; Chen and Hua, 2012), or on its incentive to disclose quality information through price and other devices (Daughety and Reinganum, 1995, 2008b). Studies with competition include Epple and Raviv (1978), Cooper and Ross (1985), and Daughety and Reinganum (2006, 2008a). Polinsky and Shavell (2010) argue that market mechanisms and product liability are substitutes as they both can increase product safety. Our study provides a formal analysis on the relationship between product liability and competition, and we find that product liability and competition can be either complements or subsititutes under alternative measures of competition.

Our paper is also related to the literature in industrial organization, where market reputation can be an effective mechanism to improve product quality (e.g., Shapiro, 1983; Allen, 1984; Bagwell and Riordan, 1991; Kranton, 2003), and where market competition may have either positive or negative impacts on product quality (e.g., Shaked and Sutton, 1987; Riordan, 1986; Horner, 2002; Dana and Fong, 2011). We depart from the literature by focusing on safety investments and product liability.

The rest of the paper is organized as follows. Section 2 presents our model and derives consumers' precaution effort in equilibrium. Section 3 analyzes firms' investment incentives,

[2] For example, Mazzeo (2003) shows that airline companies had better on-time performance when there was more competition. Matsa (2011) finds a positive relationship between product quality and competition in the supermarket industry.

and how changes in competition, measured alternatively by product differentiation and the number of competitors, affect the socially optimal liability. Section 4 discusses some modeling issues and concludes. Proofs are gathered in the appendix.

2. THE MODEL

A market has $N \geq 2$ firms and a unit mass of consumers. Consumers are uniformly distributed on a network of $\frac{N(N-1)}{2}$ Hotelling lines of length 1, and the density of consumers on each line is thus $\frac{2}{N(N-1)}$. Each firm is uniquely located at one end of each of $N-1$ Hotelling lines. In the static form of the model, firms choose prices simultaneously, with firm i competing with every other firm j on a separate Hotelling line l_{ij}, for $j \neq i$ and $i, j = 1, ..., N$. Each consumer, who values the product at V and demands at most one unit, must travel to a firm in order to make a purchase, with unit transportation cost $t > 0$. A consumer on l_{ij} is uniquely denoted by $x_{ij} \in [0, 1]$, whose distance is x_{ij} from firm i and $1 - x_{ij}$ from firm j. Consumer x_{ij} will purchase the product if her net surplus from the product — V minus price and transportation cost — is non-negative, and she patronizes the firm with the highest net surplus between the two firms at the two ends of the Hotelling line to which she belongs, i and j. Adapted from Chen and Riordan's (2007) spokes model, this model provides a tractable formulation of oligopoly price competition that extends the Hotelling analysis to any number of firms with non-localized competition, where effectively each firm competes with every firm else for different segments of consumers.[3] Notice that it reduces to the standard Hotelling model when $N = 2$.

The static model described above is then embedded into a simple two-period dynamic game with safety investment and product liability. Specifically, each firm's product may cause consumer harm with probability θ. At the beginning of Period 1, a firm can choose to invest k, which enables it to produce a high-safety product in both periods at variable cost $c \geq 0$. Without the investment, the product will be of low safety and zero variable cost. After purchasing a product, a consumer can take precaution effort. Without such effort,

[3] For other recent applications of the spokes model, see, for example, Caminal (2010), Caminal and Granero (2012), Germano and Meier (2013), Rhodes (2011), and Reggiani (2014).

if a consumer is harmed, her damage is d from a high-safety product and $D > d$ from a low-safety product.[4] We define $z = D - d$, and assume $c < \theta z$. Then, if we ignore the fixed cost of investment, it is socially efficient for firms to produce and sell the high-safety product.

With precaution effort, a consumer can reduce the damage by $\delta \in [0, d)$.[5] Each consumer's precaution cost is $\phi(\delta)$, which is strictly increasing and convex, with $\phi(0) = 0$, $\phi'(0) = 0$, and $\phi'(d) > \theta$. With consumer precaution, the expected damage level from a high-safety product is $\theta(d - \delta)$, and the expected damage level from a low-safety product is $\theta(D - \delta)$.

If a consumer is harmed, the firm is required to compensate the consumer α fraction of the damage according to its product liability. The firm bears "partial liability" if $\alpha < 1$, "full liability" if $\alpha = 1$ and punitive damage compensation if $\alpha > 1$. For simplicity, we focus on the scenarios with $\alpha \leq 1$.

In neither periods can consumers directly observe the firms' investments or product safety. In Period 2, however, firms and consumers observe the damage levels suffered by harmed consumers in Period 1. Consequently, they can update their beliefs about product safety. In particular, if product j causes larger damage than all other products in Period 1, then consumers in Period 2 will believe that product j has low safety while the other products have high safety. We denote firm j's total profit in two periods as $\Pi_j \left(I_j, I_{-j} | \, B\right)$, where (I_j, I_{-j}) is a vector of investments by all N firms and B is consumers' belief in Period 1 about product safety. In our simple setting, consumers in Period 2 will always have the correct belief about product quality, because with a continuum of consumers, fraction θ of consumers will be harmed in period 1, and the damage levels suffered by them, which reveal product safety levels, are observed by all consumers at the beginning of Period 2. Hence beliefs by all consumers in period 2 are denoted simply by B.

[4] For example, a firm can develop and install a high-safety device on the product (such as an airbag in a car), which can reduce consumer damage if the product fails (e.g., if the car is in an accident possibly due to break malfunction). We could alternatively assume that product safety affects the likelihood for consumers to be harmed. Our formulation with product safety affecting the damage level is more convenient for analysis.

[5] As shown later in this section, given this "additive" nature on potential damage, consumers' optimal precaution effort does not depend on the safety level of the product. This simplifies our analysis. Our main insight from the analysis could still hold if consumer precaution effort were to depend on the safety level.

To summarize, the timing of the model is as follows:

- Period 1:
  - Stage 1: Each firm independently decides whether to invest in product safety, $I_j = 0$ or k, $j = 1, 2, ..., N$.
  - Stage 2: Firms simultaneously choose their prices p_j, $j = 1, 2..., N$, followed by consumers' possible purchases.
  - Stage 3: Each consumer chooses her precaution effort δ after purchase.
  - State 4: If any consumer is harmed by a firm's product, the firm bears liability $\alpha \leq 1$.
- Period 2: Consumers observe the damage levels to the harmed consumers. Stages 2-4 in period 1 are then repeated, with firm $j's$ price in period 2 denoted by q_j.

Throughout the paper, we make the following assumption to ensure the full coverage of the market:

A1: $V \geq \frac{3}{2}t + c + [\theta D + \phi(d)]$.

Before analyzing the firms' strategies, we first examine consumers' precaution effort and the efficient safety investment. If all N firms make the safety investment and all consumers purchase, in two periods the total costs to produce the high-safety product is $Nk + 2c$, with social benefit $2\theta(D - d) = 2\theta z$. Hence, without consumer precaution, it is efficient for all the firms to invest in safety if and only if $2\theta z \geq Nk + 2c$.

Regardless of whether the purchased product has high or low safety, each consumer will choose δ to minimize her expected loss from product malfunction (excluding liability compensation):

$$\max_{\delta} \{(1 - \alpha)\theta\delta - \phi(\delta)\}, \tag{1}$$

and the optimal $\delta^* \equiv \delta(\alpha)$ satisfies the first order condition:

$$(1-\alpha)\theta - \phi'(\delta^*) = 0. \tag{2}$$

Since $\phi(\delta)$ is convex and $\phi'(d) > \theta$, we have $\delta(\alpha) < d$ for any α, and $\delta(\alpha)$ decreases in α. Intuitively, when product liability (α) is larger, consumers expect to receive more compensation from the firm if they are harmed, which reduces their incentive to take precaution. However, efficiency requires that every consumer takes precaution to maximize $\theta\delta - \phi(\delta)$. Therefore, as the result below states, given the firms' investment decisions, consumers' precaution effort becomes more efficient, or total welfare improves, when liability is smaller.[6]

Lemma 1 *Given the firms' safety investments, consumers' precaution effort $\delta(\alpha)$ and total welfare are higher when product liability α is lower.*

The observation above that consumers' precaution incentives decrease in product liability will play an important role in our analysis to follow, where firms' safety investment incentives will be shown to increase in product liability. This conflict will lead to some unique (possibly interior) liability that maximizes total welfare.

We will be interested in situations where $2\theta z - Nk - 2c > \max\{\theta\delta - \phi(\delta)\}$, so that investment in high safety is always efficient. For this purpose our analysis will further assume

A2: $N \leq \bar{N}$, with $\bar{N} \equiv \frac{2\theta z - 2c - \max_\delta\{\theta\delta - \phi(\delta)\}}{k} > 2$.

3. COMPETITION AND PRODUCT LIABILITY

In this section, we will derive the firms' equilibrium strategies and then address two questions. First, given the number of competitors, how will changes in the degree of product differentiation, a measure of competition intensity, impact the socially optimal liability?

[6] This result is consistent with findings in the literature on strict liability rule and negligence rules. Brown (1973) shows that strict liability does not induce victims to take care, while negligence rules may lead both injurers and victims to take optimal care. Rubinfeld (1987) and Bar-Gill and Ben-Shahar (2003) also examine comparative negligence rules with bilateral due care.

Second, how will the change in market structure (i.e., the number of competitors) affect the optimal liability? In addressing these questions, we also investigate whether market competition and product liability are complements or substitutes in increasing product safety and welfare.

3.1 High-Safety Equilibrium

We focus on the symmetric equilibrium where all N firms invest in product safety (i.e., choose high safety) and charge the same price. On the equilibrium path, consumers will have the correct belief about product safety. For any consumer $x_{ij} \in [0,1]$ on l_{ij} , $i \neq j$, and $i, j \in \{1, 2, ..., N\}$, she is indifferent between products i and j if

$$V - x_{ij}t - p_i - [(1-\alpha)\theta(d-\delta^*) + \phi(\delta^*)] = V - (1-x_{ij})t - p_j - [(1-\alpha)\theta(d-\delta^*) + \phi(\delta^*)], \quad (3)$$

where $[(1-\alpha)\theta(d-\delta^*)+\phi(\delta^*)]$, which we shall call a consumer's expected damage, consists of her expected loss when a product malfunctions and her cost of exerting precaution effort. Given the same product safety for both firms, a consumer's expected damages from the two products cancel each other in (3). Hence, given that all other firms charge p^*, the per-period demand for product j is

$$\sum_{i \neq j, i \in \{1,..N\}} \frac{2}{N(N-1)} \frac{t - p_j + p^*}{2t} = \frac{2}{N} \frac{t - p_j + p^*}{2t}. \quad (4)$$

In period 1, along the equilibrium path, firm j chooses price p_j to maximize its profit:

$$\max_{p_j} [p_j - c - \alpha\theta(d - \delta^*)] \frac{2}{N} \frac{t - p_j + p^*}{2t}, \quad (5)$$

where $\alpha\theta(d - \delta^*)$ is the firm's expected liability cost per unit of sales. Similarly, it chooses q_j to maximize its second-period profit. It is straightforward to establish the following:

Lemma 2 *Suppose that all firms choose high safety, i.e., $I = k$. Then, there is a unique* *symmetric equilibrium where each firm sets price* $p^* = q^* = t + c + \alpha\theta(d - \delta^*)$ *in each period, sells* $\frac{1}{N}$ *units of output in each period, and earns* $\Pi = 2\frac{t}{N} - k$ *total profit in two periods.*

Each firm's effective marginal cost in each period is $\hat{c} \equiv [c + \alpha\theta(d - \delta^*)]$, and the equilibrium price is $t + \hat{c}$, same as in the standard Hotelling model. Intuitively, each firm's profit decreases when there are more competitors or when there is less product differentiation. Note that liability does not affect the firms' profits on the equilibrium path, because at the symmetric equilibrium where all the firms have the same product safety, consumers face the same expected damage from all firms, so that the liability level merely shifts the equilibrium price (p^*) without affecting the equilibirum markup ($p^* - \hat{c}$).

We next turn to the investment decisions by firms. At the proposed equilibrium where all firms choose $I = k$, suppose that one firm, say firm 1, deviates to $I = 0$. In period 1, if firm 1 sets $p_1 = p^* = t + c + \alpha\theta(d - \delta^*)$, the demand for its product is still $\frac{1}{N}$ but its expected liability cost is higher, so that firm 1's expected profit in Period 1 is

$$[p^* - \alpha\theta(D - \delta^*)]\frac{1}{N} = \frac{t + c - \alpha\theta z}{N}.$$

If firm 1 instead chooses a price different from p^*, consumers' belief is that firm 1's product has low quality while the other products still have high quality. Given that all the other firms charge p^*, the demand for product 1 in period 1 would be

$$\frac{2}{N}\max\left\{\frac{t - p_1 + p^* - (1-\alpha)\theta z}{2t}, 0\right\}, \tag{6}$$

provided $\frac{t - p_1 + p^* - (1-\alpha)\theta z}{2t} < 1$.

Firm 1's optimal price after deviation in period 1 therefore solves:

$$\max_{p_1}[p_1 - \alpha\theta(D - \delta^*)]\frac{2}{N}\max\left\{\frac{t - p_1 + p^* - (1-\alpha)\theta z}{2t}, 0\right\}. \tag{7}$$

When $t > \frac{\theta z - c}{2}$, the optimal deviating price under $p_1 \neq p^*$ is $\tilde{p}_1 = t +\ \alpha\theta(D - \delta^*) - \frac{\theta z - c}{2}$, with firm 1's deviating profit in period 1 as

$$[\tilde{p}_1 - \alpha\theta(D-\delta^*)]\frac{2}{N}\frac{t-\frac{\theta z-c}{2}}{2t} = \frac{(t-\frac{\theta z-c}{2})^2}{Nt} = \frac{t}{N} - \left(\frac{\theta z-c}{N} - \frac{(\theta z-c)^2}{4Nt}\right). \tag{8}$$

Thus, when $t > \frac{\theta z-c}{2}$, firm 1's optimal deviating profit in period 1 is

$$\max\left\{\frac{t+c-\alpha\theta z}{N}, \frac{t+c}{N} - \frac{1}{N}\left[\theta z - \frac{(\theta z-c)^2}{4t}\right]\right\} = \frac{1}{N}\left(t+c-\min\left\{\alpha\theta z, (t+c) - \frac{(t-\frac{\theta z-c}{2})^2}{t}\right\}\right),$$

which can be shown to be positive.

When $t \leq \frac{\theta z-c}{2}$, firm 1 would sell zero if it charges $p_1 \neq p^*$ and is thus known to have low quality. If firm 1 instead charges p^*, its profit in period 1 would be $\frac{t+c-\alpha\theta z}{N}$. Thus firm 1's optimal deviating profit in period 1 would be

$$\max\left\{\frac{t+c-\alpha\theta z}{N}, 0\right\} = \frac{1}{N}\left(t+c-\min\left\{\alpha\theta z, t+c\right\}\right).$$

The result below follows straightforwardly from the discussion above:

Lemma 3 *Suppose that firm 1 deviates to low safety while the other firms choose high safety. Then in period 1: the deviating firm will choose* $p_1 = p^*$ *when* $t > \frac{\theta z-c}{2}$ *and* $\alpha \leq 1-\frac{(\theta z-c)^2}{4t\theta z}$ *or when* $t \leq \frac{\theta z-c}{2}$ *and* $\alpha \leq \frac{t+c}{\theta z}$*; but will choose* $p_1 = \tilde{p}_1 \equiv t + \alpha\theta(D-\delta^*) - \frac{\theta z-c}{2}$ *when* $t > \frac{\theta z-c}{2}$ *and* $\alpha > 1-\frac{(\theta z-c)^2}{4t\theta z}$ *or when* $t \leq \frac{\theta z-c}{2}$ *and* $\alpha > \frac{t+c}{\theta z}$*. Firm 1's deviating profit in period 1 is* $\frac{1}{N}\left[t+c-\min\left\{\alpha\theta z, (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\}\right]$*, where* $t_1 \equiv \frac{\theta z-c}{2}$*, and* $\lambda_{t>t_1}$ *is an indicator function that equals* 1 *if* $t > t_1$ *and* 0 *otherwise.*

Firm 1's deviating profit in period 1 can be understood intuitively. Recall that, without deviation, firm 1's profit in period 1 (excluding investment cost k) would be $\frac{t}{N}$. By deviating to low safety, firm 1 saves costs c but suffers from additional liability costs or revenue reduction $\min\left\{\alpha\theta z, (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\}$, which depends on the level of horizontal product differentiation.

The deviating firm desires to charge price higher than p^* to partly cover the higher expected liability cost, but this lowers consumer demand for its product. The latter effect does not depend on product liability α, since the firm's pricing decision fully incorporates consumers' expected damage, whereas the former effect increases in product liability. Therefore, when liability is small, the latter effect dominates and the deviating firm would charge p^*.

When liability is large enough, however, the former effect dominates and the deviating firm would charge a price different from p^*. In this case, the deviating firm is known to have a low safety product. If $t > t_1$, the deviating firm can still make positive profit due to large horizontal product differentiation; but if $t \leq t_1$, or product differentiation is small, the deviating firm cannot have positive sales at a profitable price, and hence it has zero sales.

In Period 2, along the equilibrium path, each firm sets price $q^* = p^*$. If firm 1 has deviated to $I = 0$, it will be known as the less safer firm in period 2, because with a continuum of consumers, a positive proportion of consumers will experience product malfunction from each firm and the larger damage of firm 1's product when that happens. Suppose that the prices are q_j, $j = 1, ..., N$ following firm 1's deviation. Then consumer x_{1j} on l_{1j}, for $j \neq 1$, is indifferent between products 1 and j if

$$V - x_{1j}t - q_1 - [(1-\alpha)\theta(D-\delta^*) + \phi(\delta^*)] = V - (1-x_{1j})t - q_j - [(1-\alpha)\theta(d-\delta^*) + \phi(\delta^*)]. \quad (9)$$

The demand for product 1 in Period 2 is thus

$$F_1\left(q_1, ..., q_N\right) = \frac{2}{N\left(N-1\right)} \sum_{j \in \{2,..N\}} \max\left\{\min\left\{\frac{t - q_1 + q_j - (1-\alpha)\theta z}{2t}, 1\right\}, 0\right\}, \quad (10)$$

where we recall $z \equiv D - d$. Firm 1 chooses q_1 to maximize its profit in Period 2:

$$\max_{q_1}[q_1 - \alpha\theta(D - \delta^*)]F_1\left(q_1, ..., q_N\right). \quad (11)$$

For any firm $j \neq 1$, it competes for two types of consumers: consumers located on l_{j1}, and consumers located on l_{jm}, for $m \neq j$ and $m \neq 1$. For consumers located on l_{j1}, their demand for product j is $\frac{2}{N(N-1)} \max\left\{\min\left\{\frac{t+q_1-q_j+(1-\alpha)\theta z}{2t}, 1\right\}, 0\right\}$. For consumers located on l_{jm}, given that products j and m have the same quality, their total demand for product j is $\frac{2}{N(N-1)}\max[\min(\frac{t+q_m-q_j}{2t},1),0]$. Hence, the total demand for product $j\neq 1$ in Period 2 is

$$F_j\left(q_1,...,q_N\right)=\frac{2\max\left\{\min\left\{\frac{t+q_1-q_j+(1-\alpha)\theta z}{2t},1\right\},0\right\}}{N\left(N-1\right)}+\sum_{m\neq j,m\neq 1,m\in\{1,..N\}}\max\left\{\min\left\{\frac{t+q_m-q_j}{2t},1\right\},0\right\}. \tag{12}$$

Firm $j\neq 1$ chooses q_j to maximize its profit in Period 2:

$$\max_{q_j}\left[q_j-c-\alpha\theta(d-\delta^*)\right]F_j\left(q_1,...,q_N\right). \tag{13}$$

We have:

Lemma 4 *Suppose that firm 1 has low safety while the other firms have high safety. Then, firm 1's profit in Period 2, with firms choosing Nash equilibrium prices in this subgame, is*

$$\frac{1}{N}\left[\lambda_{t>t_2}\frac{(t-t_2)^2}{t}\right], \tag{14}$$

where $t_2\equiv t_2(N)\equiv\frac{N-1}{2N-1}(\theta z-c)$*, and* $\lambda_{t>t_2}$ *equals* 1 *if* $t>t_2$ *and* 0 *otherwise.*

As we show in the appendix, with firms choosing Nash equilibrium prices in the subgame of period 2, $F_1\left(q_1,...,q_N\right)>0$ if $t>\frac{N-1}{2N-1}(\theta z-c)$, while $F_1\left(q_1,...,q_N\right)=0$ if $t\leq\frac{N-1}{2N-1}(\theta z-c)$. Firm 1's profit in period 2 is positive when $t>\frac{N-1}{2N-1}(\theta z-c)$, whereas its profit is zero when t is small so that firm 1 has zero output in period 2 after the deviation.

According to Lemma 4, liability does not affect firms' profits in period 2 even under the scenario where firm 1 has low safety and the other firms have high safety. Consider competition between firm 1 and firm j, $j\neq 1$, in period 2. On one hand, firm 1 and firm j face different expected liability costs ($\alpha\theta z$), so that they may charge different prices in the subgame. On the other hand, in period 2, consumers have the correct belief about the difference in safety between firm 1 and firm j, and correspondingly the difference in their expected loss ($(1-\alpha)\theta z$), which affects demand levels for the two firms' products. As shown in the appendix, by affecting both differences in firms' liability costs and consumer preferences, liability level (α) only impacts firms' equilibrium prices in this subgame but does not influence their markups or output levels.

From Lemmas 3 and 4, firm 1's total profit in two periods following the deviation is

$$\Pi^d = \frac{2t+c}{N} - \frac{1}{N}\min\left\{\alpha\theta z, (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\} - \frac{1}{N}\left[t - \lambda_{t>t_2}\frac{(t-t_2)^2}{t}\right]. \quad (15)$$

Therefore, a firm's potential gain (or loss) from deviating to low safety (i.e., the change in the firm's profit due to the deviation) is

$$\Pi - \Pi^d = \frac{1}{N}\min\left\{\alpha\theta z, (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\} - \frac{1}{N}\left[t - \lambda_{t>t_2}\frac{(t-t_2)^2}{t}\right] - \left(k + \frac{c}{N}\right), \quad (16)$$

where the first term on the right hand side is the extra liability cost or revenue reduction in period 1 if the firm deviates, the third term $(k + \frac{c}{N})$ is the cost saving in Period 1 if the firm deviates to no investment, and the second term, defined as

$$\Delta(N) \equiv \frac{1}{N}\left[t - \lambda_{t>t_2}\frac{(t-t_2)^2}{t}\right],$$

is the "reputation loss" in period 2. We thus have:

Proposition 1 *There exists an equilibrium where all firms produce the high-safety product if and only if*

$$\frac{1}{N}\min\left\{\alpha\theta z, (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\} + \Delta(N) \geq k + \frac{c}{N}. \quad (17)$$

It can be verified that $\Delta(N) < \frac{1}{N}(\theta z - c)$ and $\frac{1}{N}\min\left\{\alpha\theta z, (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\} - \frac{c}{N} < \frac{1}{N}(\theta z - c)$. Therefore, condition (17) is consistent with (A2). Intuitively, if the reputation loss in Period 2 is large enough, firms would make safety investments even without product liability. If the reputation loss is small, then product liability need to be increased to motivate investments. However, there is an upper bound of the effect from increasing liability. As shown in Lemma 3, if product liability is large enough, when a firm deviates to low safety, it would adjust its price in period 1 so that its deviating profit would not depend on liability. In this scenario, increasing liability further would not increase the firm's investment incentives.

Defining the socially optimal liability as α^N that ensures condition (17) to hold, we have the following Corollary from Proposition 1:

Corollary 1 *There exist two cut-off values* $k_1 = \Delta(N) - \frac{c}{N}$ *and* $k_2 = \Delta(N) + \frac{1}{N}[t - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}] > k_1$. *If* $k \leq k_2$, *the high-safety equilibrium exists where all firms choose* $I = k$; $\alpha^N = 0$ *when* $k \leq k_1$, *while* $\alpha^N = \frac{Nk+c-N\Delta(N)}{\theta z} > 0$ *and* α^N *decreases in* z *when* $k_1 < k \leq k_2$. *If* $k > k_2$, *the socially optimal liability is zero and the high-safety equilibrium does not exist.*

Corollary 1 characterizes the socially optimal liability that ensures the existence of the high-safety equilibrium. We next examine how the optimal product liability depends on competition, considering in turn two alternative measures of competition intensity: product differentiation between firms and the number of firms in the market.

3.2 Product Liability and Product Differentiation

In our spatial model of oligopoly, consumers' unit transportation cost (t), which indicates their preference heterogeneity or the degree of product differentiation, is a natural measure of the intensity of competition. When t decreases, consumers are less heterogenous, which reduces product differentiation and lowers equilibrium market prices. The result below shows that the optimal liability generally increases when competition is more severe in the sense that t decreases.

Proposition 2 *Holding all other parameter values constant, there exist two cut-off values* $t_L < t_H$ *such that: (i) when* $t \leq t_L$ *or* $t \geq t_H$, *the socially optimal liability* α^N *is zero, and (ii) when* $t_L < t < t_H$, α^N *is positive and strictly decreases in* t.

Thus, product liability and market competition tends to be complements, when competition intensity is measured by the degree of product differentiation. When there is less consumer heterogeneity or less product differentiation, the firms compete more aggressively, resulting in less profit from being known as a high-safety producer in period 2. Then, if a firm deviates to no investment, its "reputation loss" in period 2 would be smaller. This increases the firm's incentive to deviate. Consequently, to sustain the high-safety equilibrium, product liability should be increased to raise the deviation cost.

3.3 Product Liability and the Number of Competitors

We next examine how the optimal product liability may vary with the number of competitors. Different from the degree of product differentiation, the number of competitors affects not only the reputation loss from deviation, but also each firm's output level which in turn changes a firm's net cost savings from deviation. The net effect of a change in the number of competitors on the optimal liability can thus be ambiguous. For convenience, we should treat N as a continuous variable in our analysis.

Lemma 5 *At the high-safety equilibrium, a firm's reputation loss from deviating to low safety,* $\Delta(N)$, *strictly decreases in* N.

As stated in Corollary 1, when $\alpha^N > 0$, it satisfies

$$\Delta(N) = k + \frac{1}{N}(c - \alpha^N \theta z).$$

The term on the right-hand side above is the net cost saving for the deviating firm in period 1. If $\Delta(N) \geq k$, then $c - \alpha^N \theta z \geq 0$; and if $\Delta(N) < k$, then $c - \alpha^N \theta z < 0$. Therefore, the net cost saving for the deviating firm in period 1 may decrease or increase in N, depending on the sign of $c - \alpha^N \theta z$. In the appendix, we show that $N\Delta(N)$ is a concave function in N. Define $\Psi(N) = \frac{d[N\Delta(N)]}{dN}$. The result below shows that, holding all other parameters constant, the optimal liability may vary non-monotonically with the number of competitors.

Proposition 3 *Suppose that* $\alpha^N > 0$ *for* $N \in [N_1, N_2]$, *with* $2 \leq N_1 < N_2 \leq \overline{N}$. *(i) If* $\Psi(N_2) < k < \Psi(N_1)$, *then there exists some* $\widehat{N} \in (N_1, N_2)$ *such that, for any* $N \in [N_1, N_2]$, *the optimal liability α^N decreases in N for $N < \widehat{N}$ and increases in N for $N > \widehat{N}$. (ii) If $k \leq \Psi(N_2)$, then α^N decreases in N for any $N \in [N_1, N_2]$. (iii) If $k \geq \Psi(N_1)$, then α^N increases in N for any $N \in [N_1, N_2]$.*

The intuition behind Proposition 3 is as follows. If a firm deviates from high safety to low safety, it benefits from saving the fixed cost of investment and the variable production cost in period 1, but suffers from the extra liability cost in period 1 and the reputation loss in period 2. In period 1, a deviating firm's net cost saving is the sum of fixed and variable production costs, minus the extra liability cost. A firm would sell a high-safety product only when the reputation loss is larger than the net cost savings from deviation. An increase in the number of competitors always reduces the reputation loss from deviation (due to each firm's lower output), while the net cost savings from deviation, as we argued earlier, may vary non-monotonically with the number of competitors. Now consider two cases.

First, if the number of competitors (N) is relatively small, reputation loss from deviation is large, so that the optimal liability cost sustaining the high-safety equilibrium is smaller than the variable production cost (i.e., $\alpha^N \theta z < c$). In this case, a deviating firm's net cost saving in period 1 ($k + \frac{1}{N}(c - \alpha^N \theta z)$) decreases in N. Thus, when N increases, while the decreased reputation loss raises the firms' incentive for deviation, the decreased net cost saving reduces it. For small enough N, the latter effect dominates, and hence the optimal liability decreases in N. In this case, competition and product liability are substitutes to achieve high product safety and also efficiency.

Second, if the number of competitors is relatively large, the reputation loss from deviation is small, so that the optimal liability cost sustaining the high-safety equilibrium is larger than the variable production cost. In this case, a deviating firm's net cost saving in period 1 increases in the number of competitors. When N increases, both the decreased reputation loss and the increased net cost saving raises the firms' incentive for deviation. Therefore, the optimal liability must be increased to maintain the firms' investment incentive. In this case, competition and product liability become complements to achieve high product safety and also efficiency.

When $\bar{N}$ is not too small, it is likely that there exist some $N_1 < N_2$ such that $\alpha^N > 0$ for $N \in [N_1, N_2]$. To illustrate Proposition 3, consider the following numeric example.

Example 1 *Let* $\theta z = 2, t = 1, c = 1$ *and* $k = 0.01$.[7] *In addition, let* $\max_\delta\{\theta\delta - \phi(\delta)\} < 1$. *Then, we find that* $\alpha^N > 0$ *for any* $N \in [2, \overline{N}]$, *and* $\widehat{N} = 6$. *That is, the socially optimal liability first decreases in* N *when* $N < 6$ *and then increases in* N *when* $6 < N < \overline{N}$, *as shown in Figure 1 below:*

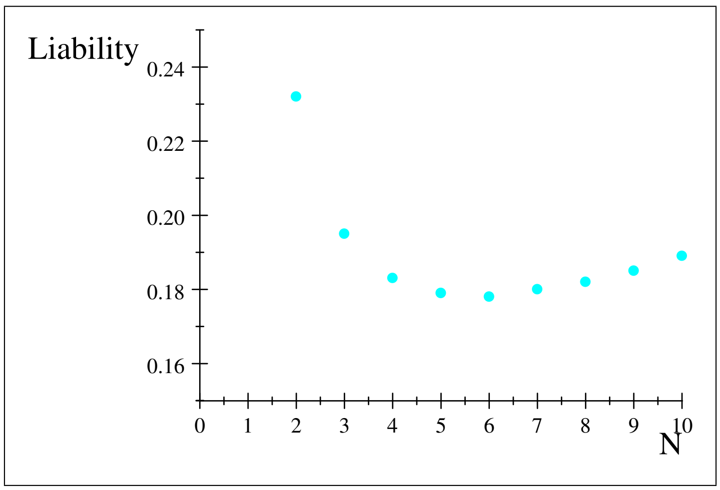

Figure 1: Optimal Liability

The following corollary considers some special cases of Proposition 3. When product differentiation is sufficiently small or the variable cost of providing high safety becomes zero, the socially optimal liability always increases with the number of competitors.

Corollary 2 *(1) If* $t \leq \frac{\theta z - c}{3}$ *and* $\alpha^N > 0$ *for* $N \in [N_1, N_2]$, *with* $2 \leq N_1 < N_2 \leq \overline{N}$., *then* α^N *strictly increases in* N *for any* $N \in [N_1, N_2]$. *(2) If* $c = 0$ *and* $\alpha^N > 0$ *for* $N \in [N_1, N_2]$, *with* $2 \leq N_1 < N_2 \leq \overline{N}$, *then* α^N *strictly increases in* N *for any* $N \in [N_1, N_2]$.

In practice, firms can adopt various technologies or methods to improve product safety and reduce consumer damage. For example, if firms can take R&D projects to improve product

[7] For illustration, we choose small k so that Nk is not too large when N becomes large. As discussed in Section 2, if Nk is too large, it is socially efficient not to make investments in product safety. Notice that for convenience we have normalized the size of consumer population to 1. If consumer population size is large, then k can also be large for our numerical example.

safety and warn consumers about the potential harm, then the relevant costs for firms are mainly fixed costs instead of variable costs. In such cases, as shown in Corollary 2, product liability and competition as measured by the number of competitors are complements. In industries with more competitors, liability should be increased to motivate firms' R&D effort. In contrast, if firms not only incur fixed R&D costs but also add safety devices with variable costs to increase product safety, then product liability and competition can be either substitutes or complements, as shown in Proposition 3.

4. DISCUSSION AND CONCLUSION

This paper has studied the relationship between competition and product liability in their roles to improve product safety and efficiency. We find that this relationship is subtle, depending importantly on what causes the change in competition intensity. Under a given market structure, when competition increases due to less product differentiation, the socially optimal product liability generally increases. In this sense, competition and product liability are complements. On the other hand, as competition increases because the number of competitors rises, the optimal product liability may vary non-monotonically, first decreasing and then increasing. We further explain why the alternative measures of competition affect product liability differently: more competition under these two measures will both reduce a firm's future profit and hence the reputation loss from producing a low-safety product, which calls for higher product liability; but more competitors have the additional effect of decreasing each firm's output in the market, potentially lowering the cost savings from deviating to a low-cost/low-safety product, and this effect can dominate when the number of competitors starts to increase from a relatively low level, leading to initial decreases in the optimal product liability.

We have conducted our analysis in a variant of the spokes model that extends the classic Hotelling duopoly. We wish to allow product differentiation between firms in the market, for which the Hotelling model is known to have very desirable features. To extend Hotelling to an oligopoly with any number of firms, one motivation to use the spokes model in stead of, say, the circle model (Salop, 1979), is that when only one firm deviates from safety investment, in the subgame of period 2 there is a two-price equilibrium that is easy to characterize analytically in the spokes model, because all $N-1$ firms remain symmetric to each other and with respect to the deviating firm. By contrast, in the circle model, firms are not symmetric as they are farther away from the deviating firm in each direction, and hence in the equilibrium of period 2 following a firm's deviation, there will be at least $\frac{N}{2}$ distinctive prices, which could be extremely difficult, if not impossible, to characterize for an arbitrary N.

Our results are derived under several strong assumptions, and should thus be interpreted with caution. In addition to postulating a specific model of oligopoly competition, our assumption on the safety investment, with only two possible levels and with its only effect as reducing consumer damages when the product malfunctions, is obviously very crude. Our highly-stylized setting in which reputation works, with only two periods and with consumers' perfect ability to detect a low-safety product in period 2, is also rather restrictive. While these modeling choices are motivated mainly by analytical tractability, it would be desirable for future research to extend our analysis in other and more general settings.

APPENDIX

Proof of Lemma 2:

Consider firm $j's$ pricing decision in period 1. Suppose that all other firms set their prices as p^*. As long as $-t \leq p_j - p^* < t$, the per-period demand for product j becomes $\frac{2}{N(N-1)}(N-1)\frac{t-p_j+p^*}{2t}$. Correspondingly, firm j chooses its price to maximize its per-period profit

$$\max_{p_j}[p_j - c - \alpha\theta(d-\delta^*)]\frac{2}{N}\frac{t-p_j+p^*}{2t}. \tag{18}$$

The first order condition leads to $p_j = p^* = t + c + \alpha\theta(d-\delta^*)$. It can be verified that the second order condition holds. Correspondingly, the per-period demand for each firm's product is $\frac{1}{N}$. The analysis for period 2 is the same. Therefore, each firm's total profit in two periods is $2[p^* - \alpha\theta(d - \delta^*)]\frac{1}{N} - k = \frac{2t}{N} - k$.

Proof of Lemma 4:

Suppose that firm 1 sells a low safety product while all the other firms sell the high safety product. Given that all products except for firm 1's product have the same quality, we focus on the symmetric pricing decision for any firm $j \neq 1$. First, assume that $F_1(q_1, ..., q_N) \in (0,1)$ and $F_j(q_1, ..., q_N) \in (0,1)$. That is, every firm has positive output. Thus, firm 1's maximization problem is

$$\max_{q_1} \left\{ [q_1 - \alpha\theta(D - \delta^*)] \frac{2}{N(N-1)} \sum_{j \in \{2,..N\}} \frac{t - q_1 + q_j - (1-\alpha)\theta z}{2t} \right\}. \tag{19}$$

The first order condition is

$$t - 2q_1 + q_j - (1-\alpha)\theta(D - d) + \alpha\theta(D - \delta^*) = 0. \tag{20}$$

For any $j \neq 1$,its maximization problem is

$$\max_{q_j} \left\{ [q_j - c - \alpha\theta(d - \delta^*)] \frac{2}{N(N-1)} \left[\frac{t + q_1 - q_j + (1-\alpha)\theta z}{2t} + \sum_{m \neq j, m \neq 1, m \in \{1,..N\}} \frac{t + q_m - q_j}{2t} \right] \right\}.$$

In equilibrium, $q_m = q_j$ for $m \neq j, m \neq 1$. The first order condition leads to

$$(N-1)t - Nq_j + q_1 + (1-\alpha)\theta(D - d) + (N-1)\alpha\theta(d - \delta^*) - (N-1)c = 0. \tag{21}$$

Solving (20) and (21), we have the optimal prices as

$$\begin{aligned} \tilde{q}_1 &= t + \alpha\theta(D - \delta^*) - \frac{N-1}{2N-1}(\theta z - c), \\ \tilde{q}_j &= t + c + \alpha\theta(d - \delta^*) + \frac{1}{2N-1}(\theta z - c) \text{ for any } j \neq 1. \end{aligned}$$

If $t > \frac{N-1}{2N-1}(\theta z - c)$,

$$\frac{t - \tilde{q}_1 + \tilde{q}_j - (1-\alpha)\theta z}{2t} = \frac{t - \frac{N-1}{2N-1}(\theta z - c)}{2t} \in (0,1).$$

Correspondingly, $F_1(\tilde{q}_1, ..., \tilde{q}_N) \in (0,1)$ and $F_j(\tilde{q}_1, ..., \tilde{q}_N) \in (0,1)$. And firm 1's profit in period 2 is

$$[\tilde{q}_1 - \alpha\theta(D - \delta^*)]\frac{2}{N}\frac{1}{N-1}(N-1)\frac{t - \frac{N-1}{2N-1}(\theta z - c)}{2t} = \frac{[t - \frac{N-1}{2N-1}(\theta z - c)]^2}{Nt}.$$

If $t \leq \frac{N-1}{2N-1}(\theta z - c)$, however, $F_1(\tilde{q}_1, ..., \tilde{q}_N) = 0$. And firm 1's profit in period 2 is zero.

Proof of Corollary 1:

Define $k_1 \equiv \Delta(N) - \frac{c}{N}$ and

$$k_2 \equiv \Delta(N) + \frac{1}{N}[(t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}] - \frac{c}{N} = \Delta(N) + \frac{1}{N}[t - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}].$$

Given $t - \lambda_{t>t_1}\frac{(t-t_1)^2}{t} > 0$, we have $k_2 > k_1$.

First, when $k \leq k_1$,

$$\frac{1}{N}\min\left\{\alpha\theta z, (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\} + \Delta(N) > k + \frac{c}{N}$$

for any α. That is, the firms make investments given any liability level. According to Lemma 1, consumer precaution effort is largest when $\alpha = 0$. Therefore, $\alpha^N = 0$.

Second, when $k_1 < k \leq k_2$, consider small α such that $\min\left\{\alpha\theta z, (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\} = \alpha\theta z$. Then each firm makes investment if and only if $\frac{1}{N}\alpha\theta z + \Delta(N) \geq k + \frac{c}{N}$. The lowest liability motivating each firm to make investment satisfies $\frac{1}{N}\alpha\theta z + \Delta(N) = k + \frac{c}{N}$, i.e., $\alpha = \frac{Nk+c-N\Delta(N)}{\theta z}$. If $\alpha = \frac{Nk+c-N\Delta(N)}{\theta z}$, we have

$$\alpha\theta z = Nk + c - N\Delta(N) \leq Nk_2 + c - N\Delta(N) = (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t},$$

and therefore, $\min\left\{\alpha\theta z, (t+c) - \lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\} = \alpha\theta z$. According to Lemma 1, $\alpha^N =$

$\frac{Nk+c-N\Delta(N)}{\theta z}$.

Finally, when $k > k_2$,

$$\begin{aligned} k+\frac{c}{N} &> k_2+\frac{c}{N}=\Delta(N)+\frac{1}{N}[(t+c)-\lambda_{t>t_1}\frac{(t-t_1)^2}{t}] \\ &\geq \frac{1}{N}\min\left\{\alpha\theta z,(t+c)-\lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right\}+\Delta(N). \end{aligned}$$

Therefore, firms have no incentive to make investments. From Lemma 1, the optimal liability should thus be $\alpha^N = 0$.

Proof of Proposition 2:

Note that if $t > t_2$,

$$\Delta(N)=\frac{1}{N}\left[t-\lambda_{t>t_2}\frac{(t-t_2)^2}{t}\right]=\frac{1}{N}\left\{2\frac{N-1}{2N-1}(\theta z-c)-\frac{1}{t}\left[\frac{N-1}{2N-1}(\theta z-c)\right]^2\right\};$$

and if $t \leq t_2$, $\Delta(N)=\frac{t}{N}$. Therefore, given the other parameters, $\Delta(N)=\Delta(N,t)$ strictly increases in t. Correspondingly, $k_1=\Delta(N)-\frac{c}{N}$ and $k_2=\Delta(N)+\frac{1}{N}[t-\lambda_{t>t_1}\frac{(t-t_1)^2}{t}]$ also strictly increase in t. Given any k, let t_L satisfy $k=\Delta(N,t_L)+\frac{1}{N}[t_L-\lambda_{t_L>t_1}\frac{(t_L-t_1)^2}{t_L}]$ and t_H satisfy $k=\Delta(N,t_H)-\frac{c}{N}$. It can be verified that $t_L < t_H$.

When $t \leq t_L$, we have

$$k=\Delta(N,t_L)+\frac{1}{N}\left[t_L-\lambda_{t_L>t_1}\frac{(t_L-t_1)^2}{t_L}\right]\geq\Delta(N,t)+\frac{1}{N}\left[t-\lambda_{t>t_1}\frac{(t-t_1)^2}{t}\right]=k_2.$$

Hence, from Corollary 1, $\alpha^N = 0$.

When $t \geq t_H$, we have

$$k=\Delta(N,t_H)-\frac{c}{N}\leq\Delta(N,t)-\frac{c}{N}=k_1.$$

Hence, from Corollary 1, $\alpha^N = 0$.

When $t_L < t < t_H$, we have $k_1 < k < k_2$. Hence, from Corollary 1, $\alpha^N = \frac{Nk+c-N\Delta(N)}{\theta z} > 0$. Since $\Delta(N)$ strictly increases in t, α^N strictly decreases in t.

Proof of Lemma 5:

Recall that $\Delta(N) = \frac{1}{N}\left[t - \lambda_{t>t_2}\frac{(t-t_2)^2}{t}\right]$, where $t_2 = \frac{N-1}{2N-1}(\theta z - c)$, and t_2 strictly increases in N.

If $t \leq t_2$, then $\Delta(N) = \frac{t}{N}$, which strictly decreases in N.

If $t > t_2$, we have

$$\Delta(N) = \frac{1}{N}\left\{2\frac{N-1}{2N-1}(\theta z - c) - \frac{1}{t}\left[\frac{N-1}{2N-1}(\theta z - c)\right]^2\right\}.$$

Differentiating $\Delta(N)$, we have, for $N \geq 2$:

$$\begin{aligned}
\Delta'(N) &= \frac{\theta z - c}{N^2}\left\{\frac{N}{(2N-1)^2}\left[2 - 2\frac{\theta z - c}{t}\frac{N-1}{2N-1}\right] - \frac{N-1}{2N-1}\left[2 - \frac{\theta z - c}{t}\frac{N-1}{2N-1}\right]\right\} \\
&< \frac{\theta z - c}{N^2}\left\{\frac{N}{(2N-1)^2}\left[2 - 2\frac{\theta z - c}{t}\frac{N-1}{2N-1}\right] - \frac{N}{2N-1}\left[2 - \frac{\theta z - c}{t}\frac{N-1}{2N-1}\right]\right\} \\
&= \frac{(\theta z - c)N}{N^2(2N-1)}\left\{\frac{1}{(2N-1)}\left[2 - 2\frac{\theta z - c}{t}\frac{N-1}{2N-1}\right] - 2 + \frac{\theta z - c}{t}\frac{N-1}{2N-1}\right\}.
\end{aligned}$$

Since $2 - 2\frac{\theta z - c}{t}\frac{N-1}{2N-1} < 2N - 1$ for $N \geq 2$, and $\frac{\theta z - c}{t}\frac{N-1}{2N-1} < 1$ when $t > t_2 = \frac{N-1}{2N-1}(\theta z - c)$, we have $\Delta'(N) < 0$. That is, $\Delta(N)$ strictly decreases in N.

Proof of Proposition 3:

Corollary 1 implies that $\alpha^N = \frac{Nk + c - N\Delta(N)}{\theta z} > 0$ if and only if $k_1 < k < k_2$. In the following, we will consider three different scenarios.

Scenario A: Suppose that $t > t_1 \equiv \frac{\theta z - c}{2}$. Then we always have $t > \frac{\theta z - c}{2} > \frac{N-1}{2N-1}(\theta z - c) \equiv t_2$. Correspondingly, $\Delta(N) = \frac{1}{N}[t - \frac{(t-t_2)^2}{t}]$. Thus,

$$N\Delta(N) = 2\frac{N-1}{2N-1}(\theta z - c) - \frac{1}{t}[\frac{N-1}{2N-1}(\theta z - c)]^2.$$

Let $\gamma(N) \equiv N\Delta(N)$ for $N \geq 2$. We have

$$\gamma'(N) = [2 - 2\frac{\theta z - c}{t}\frac{N-1}{2N-1}]\frac{\theta z - c}{(2N-1)^2} > 0.$$

Furthermore, note that $\frac{N-1}{2N-1}$ increases in N and therefore $[2 - 2\frac{\theta z - c}{t}\frac{N-1}{2N-1}]$ decreases in N. In addition, $\frac{\theta z - c}{(2N-1)^2}$ decreases in N. Hence, the differentiation $\gamma'(N)$ is positive but decreases in N. That is, $N\Delta(N)$ is strictly increasing and concave in N for $N \geq 2$. Define $\Psi(N) \equiv \gamma'(N) = \frac{d[N\Delta(N)]}{dN}$.

(1) If $\Psi(N_2) < k < \Psi(N_1)$, then define $\widehat{N} = \min\{N : N \in [N_1, N_2] \mid \Psi(N) \leq k\}$. Because $\Psi(N)$ decreases in N, $\widehat{N}$ is well-defined and unique. Based on Corollary 1, for any $N > M$ such that $\alpha^M > 0$ and $\alpha^N > 0$, we have $\alpha^N \theta z + N\Delta(N) = Nk + c$ and $\alpha^M \theta z + M\Delta(M) = Mk + c$.

Note that, for any $N \in [N_1, \widehat{N})$, $\Psi(N) > k$; and for any $N \in (\widehat{N}, N_2]$, $\Psi(N) < k$. Correspondingly, for any given N and $M \in [N_1, \widehat{N})$ such that $N > M$, we have $N\Delta(N) - M\Delta(M) > (N - M)k$. Thus,

$$(\alpha^N - \alpha^M)\theta z = (N - M)k - [N\Delta(N) - M\Delta(M)] < 0.$$

That is, α^N decreases in N for $N \in (N_1, \widehat{N})$.

Similarly, for any given N and $M \in [N_1, \widehat{N})$ such that $N > M$, we have $N\Delta(N) - M\Delta(M) < (N - M)k$. Thus,

$$(\alpha^N - \alpha^M)\theta z = (N - M)k - [N\Delta(N) - M\Delta(M)] > 0.$$

That is, α^N increases in N for $N \in [\widehat{N}, N_2]$.

(2) If $k \leq \Psi(N_2) < \Psi(N_1)$, then $\Psi(N) > k$ for any $N \in [N_1, N_2)$. Similar to the above analysis, for any $N \in [N_1, N_2]$, α^N decreases in N.

(3) If $\Psi(N_2) < \Psi(N_1) \leq k$, then $\Psi(N) < k$ for any $N \in (N_1, N_2]$. Similar to the above analysis, for any $N \in [N_1, N_2]$, α^N increases in N.

Scenario B: Suppose that $\frac{\theta z - c}{3} < t \leq t_1 \equiv \frac{\theta z - c}{2}$. Then given t, there exists a unique y^* such that $t = t_2(y^*) = \frac{y^* - 1}{2y^* - 1}(\theta z - c)$. Note that y^* may not be an integer. We have $t > t_2(N)$ for $N \in [N_1, y^*)$ and $t < t_2(N)$ for $N \in (y^*, N_2]$. Correspondingly, for $N \in (y^*, N_2]$, we have $N\Delta(N) = t$ and $\alpha^N = \frac{Nk + c - t}{\theta z}$. Also, similar to the proof under Scenario A, it can be verified that $\Psi(N) \equiv \frac{d[N\Delta(N)]}{dN}$ strictly decreases for $N \in [N_1, y^*)$ and becomes 0 for $N \in (y^*, N_2]$.

(1) If $\Psi(y^*) < k < \Psi(N_1)$, then define $\widehat{N} = \min\{N \in [N_1, y^*] \mid \Psi(N) \le k\}$. Similar to the proof under Scenario A, it can be shown that α^N decreases in N for $N \in (N_1, \widehat{N})$ and increases in N for $N \in [\widehat{N}, y^*]$. Also note that, for $N \in (y^*, N_2]$, $\alpha^N = \frac{Nk+c-t}{\theta z}$ increases in N. In sum, α^N decreases in N for $N \in (N_1, \widehat{N})$ and increases in N for $N \in [\widehat{N}, N_2]$.

(2) If $k \le \Psi(y^*) < \Psi(N_1)$, similar to the proof under Scenario A, it can be shown that α^N decreases in N for any $N \in [N_1, y^*]$. Then define $\widehat{N}$ as the smallest integer within $[y^*, N_2]$. Thus α^N decreases in N for $N \in (N_1, \widehat{N})$ and increases in N for $N \in [\widehat{N}, N_2]$.

(3) If $\Psi(y^*) < \Psi(N_1) \le k$, similar to the proof under Scenario A, it can be shown that α^N increases in N for any $N \in [N_1, y^*]$. Thus α^N increases in N for any $N \in [N_1, N_2]$.

Scenario C: Suppose that $t \le \frac{\theta z - c}{3}$. Then for any $N \ge 2$, $t < t_2 \equiv \frac{N-1}{2N-1}(\theta z - c)$. Thus, we always have $N\Delta(N) = t$. Thus, whenever $\alpha^N > 0$, $\alpha^N = \frac{Nk+c-t}{\theta z}$, which strictly increases in N. Also note that

$$\Psi(N_2) = \Psi(N_1) = \frac{d[N\Delta(N)]}{dN} = 0 < k.$$

That is, when $k \ge \Psi(N_1)$, α^N increases in N for any $N \in [N_1, N_2]$.

Proof of Corollary 2:

(1) Suppose that $t \le \frac{\theta z - c}{3}$. Then the proof of Proposition 3 directly implies that α^N increases in N.

(2) Suppose that $c = 0$. As shown in Lemma 5, $\Delta(N)$ strictly decreases in N. Thus we have $(N+1)\Delta(N+1) - N\Delta(N) < \Delta(N)$.

Based on Corollary 1, $\alpha^N > 0$ only when

$$\Delta(N) < k \le \Delta(N) + \frac{1}{N}[(t - 1_{(t>t_1)}\frac{(t-t_1)^2}{t}],$$

given $c = 0$. We can show that α^N strictly increases in N. Suppose, to the contrary, there exists a particular $N \in [N_1, N_2)$ such that $0 < \alpha^{N+1} \le \alpha^N$. Given the proof of Proposition 3, it must be true that $(N+1)\Delta(N+1) - N\Delta(N) \ge k$. Then we have

$$k \le (N+1)\Delta(N+1) - N\Delta(N) < \Delta(N),$$

which implies $\alpha^N = 0$, based on Corollary 1 and the fact that $c = 0$. This is a contradition to the assumption $\alpha^N > 0$. Therefore, α^N strictly increases in N for any $N \in [N_1, N_2]$.

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