The law of one price revisited: How do goods market frictions generate large and volatile price deviations?

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The Law of One Price Revisited: How Do Goods Market Frictions Generate Large and Volatile Price Deviations?

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Abstract

This paper analyzes the role of goods market frictions in accounting for the large and volatile deviations from the Law of One Price in a framework of flexible prices. We draw a distinction between goods market frictions that are required to consume tradable goods (e.g., distribution costs) and those that are necessary for international transactions (e.g., trade costs). We find that trade costs generate LOP deviations by introducing a no-arbitrage band, while distribution costs cause the price to deviate from the LOP by affecting the probability that trade will occur, given the band. We then conduct a Monte Carlo simulation to show that real exchange rate volatility is positively associated with trade costs, but negatively related to distribution costs. This effect depends on the interplay of trade costs and distribution costs, as they work in opposite directions when creating arbitrage opportunities.

Keywords: Distribution costs, trade costs, law of one price, real exchange rate volatility

JEL classification: F31; F37

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1 Introduction

One of the most enduring puzzles in international macroeconomics, observed since the beginning of the post-Bretton Woods era, is that deviations from the Law of One Price (LOP) and its generalization, Purchasing Power Parity (PPP), are not only large but also highly volatile and persistent. Rogoff (1996) refers to the juxtaposition of these observations and the predictions of structural models as the PPP puzzle. Traditionally, the attempts to address this puzzle have been based on the distinction between tradable and non-tradable goods. However, since the influential work by Engel (1999), most studies have shed light on the LOP deviations of tradable goods as the empirically relevant foundation of the current theoretical approaches.[1] Two main branches of the literature explore the LOP deviations of tradable goods. The first branch introduces nominal rigidities into dynamic equilibrium models (see, for example, Betts and Devereux (2000), Bergin and Feenstra (2001), Chari et al. (2002), Kehoe and Midrigan (2007), and Carvalho and Nechio (2011)). While useful in addressing monetary policy, such models lack the ability to provide a plausible mechanism for sustaining deviations from the LOP of the magnitude observed in the data. A second strand of the literature emphasizes the importance of transaction costs. These models predict that real exchange rates are bounded by the fixed limits of arbitrage costs, which are treated broadly to include transportation and other costs of bringing goods to final consumption markets (see, for example, Sercu et al. (1995), Obstfeld and Rogoff (2000), Burstein et al. (2003), Sercu and Uppal (2003), Crucini et al. (2005), and Corsetti et al. (2007)). This paper fits into the second strand of the literature.

Although the importance of international transaction costs in generating LOP deviations has been well documented, only a few papers have studied the role played by domestic transaction costs and their interplay with international costs in shaping the behavior of the real exchange rate. In this paper, we offer a new approach to explain the PPP puzzle in a framework of flexible prices. We specify a two-country world economy, allowing for two types of goods market frictions, namely, international trade costs and domestic distribution services. The inclusion of these frictions allows us to endogenously drive a natural wedge between the

[1] Engel (1999) shows that almost all the real exchange rate fluctuations are attributable to fluctuations in the relative prices of tradable goods between the U.S. and other industrialzed countries.

prices in different locations and thus account for the deviations from the LOP, even though all goods are tradable. Further, the distinction between domestic and international costs guides us to highlight two different channels through which these costs affect LOP deviations.

Our model thus encompasses the main elements of the standard models that study the role of transaction costs in explaining real exchange rate dynamics. However, our approach has three notable distinctions. First, we distinguish between domestic and international goods market frictions. Specifically, in addition to iceberg-type international trade costs, we incorporate domestic distribution costs by assuming that consuming a tradable good requires certain units of distribution services. Second, we elaborate on channels through which these costs and their interactions can affect the magnitude and volatility of LOP deviations. Third, we do not model nominal rigidities and deliberately focus on the role of goods market frictions. Our approach is meant to offer a framework to help understand the long-run real exchange rate by placing an emphasis on real frictions that drive large and volatile deviations from the LOP.

Our main findings are as follows. As widely known, trade costs appear to introduce the no-arbitrage band in which trade does not occur and hence directly generate the LOP deviations. We find that distribution costs also contribute to these deviations by affecting the direction of trade and the probability that trade will occur given the no-arbitrage band. By doing so, an increase in trade costs enlarges the deviations from the LOP by widening the no-arbitrage band, whereas a unilateral rise in distribution costs makes the real exchange rate more likely to move toward the boundary of the band generating the LOP deviations. The Monte Carlo Simulation shows that the volatility of LOP deviations is positively associated with trade costs, but negatively related to distribution costs. This effect depends on the interplay of trade costs and distribution costs, as they work in opposite directions when creating arbitrage opportunities.

The paper is organized as follows. Section 2 presents the model setup. Section 3 solves for the equilibrium real exchange rate and discusses how distribution costs and trade costs affect the deviations from the LOP. Section 4 carries out a Monte Carlo simulation to examine the effects of goods market frictions on the real exchange rate volatility. Section 5 concludes.

2 The Model

Our framework builds on the model proposed by Sercu and Uppal's (2003) model, which we generalize in two respects.[2] First, we introduce non-tradable goods. Second, we incorporate distribution services that are made up of non-tradable goods. The main differences arising from the existence of non-tradable goods and distribution services appear in the preference and resource constraints. The world economy consists of two countries of identical size, a home (HC) and a foreign country (FC). We use an asterisk (*) to denote variables associated with the foreign country. Each country is populated by a large number of infinitely-lived consumers who have utility defined over sequences of consumption of tradable (C_t^T) and non-tradable goods (C_t^{NT}),

$$U = \sum_{t=0}^{\infty} \beta^t \frac{\left[\left(C_t^T \right)^{\alpha} \left(C_t^{NT} \right)^{1-\beta} \right]^{1-\gamma}}{1-\gamma} \tag{1}$$

in the home country, and

$$U^* = \sum_{t=0}^{\infty} \beta^t \frac{\left[\left(C_t^{T*} \right)^{\alpha} \left(C_t^{NT*} \right)^{1-\beta} \right]^{1-\gamma}}{1-\gamma} \tag{2}$$

in the foreign country, where $0<\alpha<1$ is the expenditure share of tradable goods, $0<\beta<1$ is the discount factor, and $\gamma>1$ is the inverse of the intertemporal elasticity of substitution. In every period, each economy is exogenously endowed with tradable (Y_t^T) and non-tradable (Y_t^{NT}) goods that are non-storable. We assume that financial markets are perfectly integrated and complete such that financial claims are traded freely. Following Burstein et al. (2003), we introduce a distribution sector by assuming that consuming a tradable good requires θ units of distribution services, which consist of non-tradable goods.[3] Distribution sectors are assumed to be heterogeneous across countries because wholesaling, retailing, and local transportation tend to be isolated from other countries and hence exhibit a wide range of

[2] Sercu and Uppal (2003) analyze the relationship between exchange rate volatility and volume of trade, pointing out that a drop in the shipment costs implies a decrease in exchange rate volatility.

[3] Noting that wholesaling, retailing and transportation do not play a significant role in the most important non-tradable sectors (i.e., housing, health, and education expenditures), we assume that consumptions of non-tradables do not require distribution services, as suggested by Burstein et al. (2003).

distribution costs across countries. Since the endowment of non-tradable goods can be either consumed or used for distribution services in each country, we have the following:

$$Y_t^{NT} = C_t^{NT} + \theta C_t^T \tag{3}$$

$$Y_t^{NT*} = C_t^{NT*} + \theta^* C_t^{T*} \tag{4}$$

We next introduce international trade costs of the iceberg type by assuming that a proportion of the physical shipment of the tradable good is lost in transit. We use τ to denote the trade costs faced by home and foreign individuals, which are symmetric across countries. In this setting, when one unit is shipped, only $\frac{1}{1+\tau}$ units actually arrive. Given the presence of trade costs, the resource constraints for the home and foreign country are given by

$$C_t^T = Y_t^T - X_t + \frac{X_t^*}{1+\tau} \tag{5}$$

$$C_t^{T*} = Y_t^{T*} - X_t^* + \frac{X_t}{1+\tau} \tag{6}$$

$$0 \leq X_t \leq Y_t^T \tag{7}$$

$$0 \leq X_t^* \leq Y_t^{T*} \tag{8}$$

where X_t is the amount of exports from the home country (measured before trade costs) and $\frac{X_t^*}{1+\tau}$ is the amount of imports from the foreign country (measured after trade costs).

Given our assumption of complete financial markets, the model is solved as a central planner problem whose objective is to maximize the aggregate utility by choosing the amount of trade:

$$\underset{\{X_t, X_t^*\}}{Max}\, U\left(C_t^T, C_t^{NT}\right) + U\left(C_t^{T*}, C_t^{NT*}\right) \tag{9}$$

subject to constraints (3)-(8)

Because this problem is essentially static, we drop the time subscript t henceforth to simplify the notations.

3 LOP deviations

When financial markets are complete, the ratio of home to foreign marginal utility of consumption is linked to real exchange rates. From the optimality condition of consumption, it follows that the relative price of any pair of goods can be seen as a marginal rate of substitution in the optimum,

$$Q = \frac{\partial U\left(C^{T*}, C^{NT*}\right) \big/ \partial C^{T*}}{\partial U\left(C^{T}, C^{NT}\right) \big/ \partial C^{T}} \tag{10}$$

where Q denotes the real exchange rate of tradable goods between the home and foreign countries. From a standard Lagrangian problem of a central planner, the real exchange rate of tradable goods is then given as follows:[4]

$$Q = \begin{cases} 1+\tau & \text{if } K > 1+\tau \quad : \text{ HC exports} \\ K = \frac{\left(\frac{Y^T}{Y^{T*}}\right)^{\gamma}\left(\frac{Y^{NT*}}{Y^{T*}}-\theta^*\right)^{\omega}\left(\alpha\frac{Y^{NT*}}{Y^{T*}}-\theta^*\right)}{\left(\frac{Y^{NT}}{Y^{T}}-\theta\right)^{\omega}\left(\alpha\frac{Y^{NT}}{Y^{T}}-\theta\right)} & \text{if } \frac{1}{1+\tau} \leq K \leq 1+\tau \quad : \text{ No-trade} \\ \frac{1}{1+\tau} & \text{if } K < \frac{1}{1+\tau} \quad : \text{ FC exports} \end{cases} \tag{11}$$

where $\omega = -\alpha + \alpha\gamma - \gamma$.[5]

Equation (11) shows that the behavior of the real exchange rate is determined by three key elements: first, the output ratios of tradable and non-tradable goods between countries; second, the trade costs; and third, the distribution services. In the absence of non-tradable goods and trade costs, the central planner always equalizes the marginal utilities of consumption of tradable goods across countries, leading the real exchange rate equal to unity. However, in an economy with goods market frictions, the tradable goods will be moved across countries only if the gains from trade are sufficiently large to cover these frictions. To see how our model bears this out, consider three possible trade and price pairings, as shown in Equation (11). First, if the home output of tradable goods is sufficiently large relative to the foreign output that the gains from trade are large enough to cover the trade costs, the goods flow from the home country to the foreign country, and the price in the foreign country is τ

[4] The derivation of equation (11) is available upon request

[5] Following Burstein et al. (2003), we assume $\frac{Y^{NT}}{Y^T} > \frac{\theta}{\alpha}$ and $\frac{Y^{NT*}}{Y^{T*}} > \frac{\theta^*}{\alpha}$ throughout our theoretical and simulation studies

greater than the home price. Second, if the foreign output is sufficiently large relative to the home output, goods flow from the foreign country to the home country and the price in the home country is τ greater than the foreign price. Third, when the output ratio of tradable goods is sufficiently close across countries, a non-trade equilibrium exists. In this case, the implied price differential is not sufficient at these endowment levels to justify paying the trade costs. Therefore, the implicit relative price is a matter of reading off the appropriate marginal valuations, expressed as the ratio of home and foreign marginal utility evaluated at autarkic output points.

It is worthwhile to note that, because of distribution costs, the real exchange rate (Q) does not move in tandem with the output ratios within the no-arbitrage band, driving a natural wedge between relative prices in different locations. To see why distribution costs generate the deviations from the LOP, note that Equation ((11) implies that the response of K to a change in Y^T is

$$\frac{dK}{dY^T} = K\left(\frac{-\alpha + \alpha\gamma + 1}{Y^T} + \frac{\theta}{\alpha Y^{NT} - \theta Y^T}\right) > 0 \tag{12}$$

The change in K is increasing in Y^T and, more importantly, this effect is magnified by the home distribution cost (θ). This is intuitive, since increasing the distribution cost has similar effects to decreasing home consumptions of the tradable good. The combination of the first condition of Equation (11) and Equation (12) implies that a given increase in Y^T produces a larger probability that the home country can export the larger is the home domestic distribution cost. As such, an increase in home distribution costs makes the price in the foreign country more likely to be τ greater than the home price, thus causing deviations from the LOP. Therefore, trade costs generate the LOP deviations by introducing the no-arbitrage band, while distribution costs cause the price to deviate from the LOP by affecting the probability that trade occurs given the no-arbitrage band.

Lastly, from Equation (11), one can easily see how the LOP deviations of tradable goods decay as we move from an economy with goods market frictions to an economy without these frictions. When distribution services do not exist (i.e., $\theta = \theta^* = 0$), trade costs still drive a natural wedge between relative prices in different locations, however the real exchange rates now move in tandem with output ratios within the threshold. When both distribution services and non-tradable goods are absent (i.e., $\theta = \theta^* = 0$, $\alpha = 1$), the real exchange rate within the threshold does not depend on the output ratio of non-tradable goods. In an extreme case where all goods market frictions are eliminated ($\tau = \theta = \theta^* = 0$, $\alpha = 1$), the central planner sets the optimal relative consumption of the tradable good equal to unity and corrects any deviations from unity by re-allocating goods. As a result, the LOP will unambiguously hold for tradable goods such that the real exchange rate equals to unity.

4 Volatility of LOP deviations

Our results discussed in Section 3 support the argument that unless price differentials exceed the no-arbitrage band, trade does not take place and price deviates freely; however, when the price differentials are large enough to offset international trade costs, trade occurs and price differentials in excess of the band are eventually arbitraged away. Therefore, the no-arbitrage band generated by goods market frictions is equivalent to a "band of inaction", such that LOP deviations decay slowly within the no-arbitrage band but rapidly outside the band.

Motivated by the fact that the arbitrage limits depend on the goods market frictions, in the following discussion we explore the effect of these frictions on the real exchange rate volatility by simulating our model. Since the volatility of LOP deviations, measured as the time series variance of $\ln Q$ in Equation (11), does not have closed form analytical solutions, we resort to a numerical experiment by conducting a Monte Carlo simulation. To gain insights into how changes in trade costs and distribution costs can affect the volatility of LOP deviations, we first analyze the approximation of the volatility by focusing on the probability that trade does not occur. Then we conduct numerical experiments to quantify the importance of these costs.[6]

[6] We aim to provide meaningful insights into how goods market frictions can affect real exchange rate volatility rather than evaluate the performance of the model in matching the time series properties observed in the data. We leave the latter work for our future study.

4.1 Simulating the model

Since the two economies considered in this model are symmetric in their preferences, all of the parameters governing the curvature properties of the utility function have the same value in both economies. We set γ to 2, as suggested by Backus et al. (1994), and α to 0.3 which is obtained from the US Bureau of Labor Statistics.[7] Following Burstein et al. (2003), we set $\theta = \theta^* = 1.0$ as a baseline value and vary θ from 1.0 to 2.0 to see the responses of the real exchange rate volatility to an increase in home distribution costs.[8] As is widely known, it is difficult to compute the trade costs due to the inconsistency across countries in the bilateral value and quantity data for trade, the cross-hauling of goods, and aggregation bias. For this reason, we choose a reasonable value for τ $(= 0.1)$ and then evaluate the behavior of the real exchange rate for different values of τ $(0.1 \sim 1.0)$.[9]

The Monte Carlo simulation is based on a data generation process of which algorithms depend on the distributional assumptions of output ratios. Our distributional assumptions are as follows. First, $\ln \frac{Y^T}{Y^{T*}} \overset{iid}{\sim} N\left(0, \left(\sigma^T\right)^2\right)$.[10] A set of sufficient conditions for this assumption is $Y^T \sim N\left(\mu^T, \frac{1}{2}\left(\sigma^T\right)^2\right), Y^{T*} \sim N\left(\mu^T, \frac{1}{2}\left(\sigma^T\right)^2\right)$ for $Y^T \perp Y^{T*}$. Second, $\ln \frac{Y^{NT}}{Y^T}$ and $\ln \frac{Y^{NT*}}{Y^{T*}}$ are identically distributed. Third, from the recognition that $\frac{Y^{NT}}{Y^T} > \frac{\theta}{\alpha}$, $\frac{Y^{NT*}}{Y^{T*}} > \frac{\theta^*}{\alpha}$,

[7] The expenditure share of the non-tradable sector in the U.S. ranges between 69.0% and 72.1% for the period 1990-2010.

[8] Setting $\theta = 1$ implies that the distribution margin is 50%, which is defined as the excess of retail price over producer price and measured as a percentage of the retail price. Burstein et al. (2003) report that distribution margins of consumption goods for six OECD countries (Canada, France, Germany Italy, Japan, the UK, and the U.S.) range between 35% and 50%.

[9] Hummels (2001) estimates trade costs based on the direct measurement of the freight rate, which is defined as the ratio of transportation expenditure to the value of imports exclusive of freight and insurance charges. The all-commodities trade-weighted average freight rate ranges from 3.8% for the U.S. to 13.3% for Paraguay. Across commodities in the U.S., the freight rate ranges from a low of 0.9% for transport equipment to a high of 27% for crude fertilizer. In their extensive survey of the measurement of trade costs, Anderson and van Wincoop (2004) show that 170% of 'representative' trade costs in industrialized countries breaks down into 21% of transportation costs, 44% of border-related trade barriers, and 55% of retail and wholesale distribution costs.

[10] One might also assume that the difference of the log of the output ratio is normally distributed, considering the fact that the real exchange rate tends to exhibit random walk behavior within the band.

we assume that $\ln \frac{Y^{NT}}{Y^{T}} \sim TruncN\left(\mu, \sigma^2, \ln R_{\min}, \infty\right)$, $\ln \frac{Y^{NT*}}{Y^{T*}} \sim TruncN\left(\mu, \sigma^2, \ln R_{\min}, \infty\right)$ where $TruncN\left(\mu, \sigma^2, \ln R_{\min}, \infty\right)$ is a normal distribution with mean μ and variance σ^2 truncated from below $\ln R_{\min}$. Lastly, we assume $\frac{Y^{T}}{Y^{T*}}$, $\frac{Y^{NT}}{Y^{T}}$, and $\frac{Y^{NT*}}{Y^{T*}}$ are mutually independent of each other.

The basic idea of the Monte Carlo simulation is simple. Given the parameters μ, σ, σ^T, and $R_{\min}$ as well as τ, γ, α, θ^*, and θ, we can generate a multiple of triples $\left(\frac{Y^{T}}{Y^{T*}}, \frac{Y^{NT}}{Y^{T}}, \frac{Y^{NT*}}{Y^{T*}}\right)$ from the assumed distributions, and then from each of which we can compute $\ln K$ and $\ln Q$. Suppose we generate n of the triples, namely $\left\{\left(\left(\frac{Y^{T}}{Y^{T*}}\right)_i, \left(\frac{Y^{NT}}{Y^{T}}\right)_i, \left(\frac{Y^{NT*}}{Y^{T*}}\right)_i\right), i = 1, ..., n\right\}$. Then we can compute $\left\{(\ln K)_i, i = 1, ..., n\right\}$ and $\left\{(\ln Q)_i, i = 1, ..., n\right\}$ using Equation (11). The $\frac{1}{n}\sum_{i=1}^{n}\left((\ln Q)_i - \overline{\ln Q}\right)^2$ converges to $V[\ln Q]$ in probability as $n \to \infty$ due to the law of large numbers for iid observations, where $\overline{\ln Q} = \frac{1}{n}\sum_{i=1}^{n}(\ln Q)_i$ and $V[\ln Q]$ is the volatility of LOP deviations measured by the time-series variance of $\ln Q$. The effect of changes in τ and θ can be visualized by showing $V[\ln Q]$ for different τ and θ.

Let us explain how we set the parameters μ, σ, σ^T, and $R_{\min}$ for our simulations. While we vary τ from 0.1 to 1 to see the effects of τ on $V[\ln Q]$, the choice of τ does not affect the generation of $\left\{\left(\left(\frac{Y^{T}}{Y^{T*}}\right)_i, \left(\frac{Y^{NT}}{Y^{T}}\right)_i, \left(\frac{Y^{NT*}}{Y^{T*}}\right)_i\right), i = 1, ..., n\right\}$. $R_{\min}$ needs to be greater than $\frac{\max\{\theta^*, \theta\}}{\alpha}$ for the values of (θ^*, θ). Since we vary θ from 1.0 to 2.0 while holding the θ^* at 1.0 in order to see the effect of changes in θ, $R_{\min}$ must be greater than $\frac{2.0}{\alpha} = \frac{2.0}{0.3}$. We choose $R_{\min} = \frac{2.01}{0.3} = 6.7$. As far as we know, there is no consensus on the values for μ, σ, and σ^T. We choose $\mu \approx 0.138$ and $\sigma^2 = \left(\sigma^T\right)^2 \approx 0.078$ so that, along with $R_{\min} = 6.7$, $E\left[\frac{Y^{NT}}{Y^{T}}\right] = E\left[\frac{Y^{NT*}}{Y^{T*}}\right] \approx 7.5$ which we believe is a reasonable value. Lastly, we choose $n = 500,000$.

Before proceeding to simulations, it is useful to note that, from Equation (11), the volatility of the LOP deviations is given by

$$\begin{aligned} V[\ln Q] &= E\left[\ln Q^2\right] - (E[\ln Q])^2 \\ &= (P_l + P_u)(\ln(1+\tau))^2 + (1 - P_l - P_u)E\left[(\ln K)^2\right] \\ &\quad - ((P_u - P_l)\ln(1+\tau) + (1 - P_l - P_u)E[\ln K])^2 \\ &= (P_l(E[\ln K] + 2\ln(1+\tau)) + P_u(E[\ln K] - 2\ln(1+\tau)))(1 - P_l - P_u)E[\ln K] \\ &\quad + \left((P_l + P_u) - (P_u - P_l)^2\right)(\ln(1+\tau))^2 + (1 - P_l - P_u)V[\ln K] \qquad (13) \end{aligned}$$

where $P_l \equiv \Pr[\ln K \leq -\ln(1+\tau)]$ and $P_u \equiv \Pr[\ln K \geq \ln(1+\tau)]$. Although this expression can be hardly approximated, it provides meaningful insights into how goods market frictions can affect the volatility of LOP deviations. Consider a rise in trade costs between countries, *i.e.*, an increase in τ for unchanged levels of θ and θ^*. From Equation (11), we know that a higher τ unambiguously widens the no-arbitrage band within which price differentials can fluctuate before arbitrage begins, while K remains unaffected. Thus, an increase in τ is expected to produce a larger $V[\ln Q]$ by raising $(1 - P_l - P_u)$. A rise in home distribution cost (θ) has two effects: a direct effect on $(1 - P_l - P_u)$ and an indirect effect *via* the probability mass of K being on the boundary point of the no-arbitrage band. These effects are captured by looking at the impact of θ on $E[\ln K]$ which is approximated as follows:

$$E[\ln K] \approx (\theta^* - \theta)\frac{e^{-\mu+\frac{1}{2}\sigma^2}\Phi_{(\rho-\sigma)}}{\Phi_\rho}\left(\delta_2 + (\theta^* + \theta)\,\delta_3 e^{-\mu+\frac{3}{2}\sigma^2}\frac{\Phi_{(\rho-2\sigma)}}{\Phi_{(\rho-\sigma)}}\right) \tag{14}$$

where $\delta_2 = (1-\alpha)\left(\gamma - 1 - \frac{1}{\alpha}\right)$, $\delta_3 = \frac{1}{2}(1-\alpha)\left(\gamma - 1 - \alpha - \frac{1}{\alpha^2}\right)$, $\rho = -\frac{\ln R_{\min} - \mu}{\sigma}$, $\Phi_s = \int_{-\infty}^{s}\phi(t)\,dt$ and $\phi_s = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}s^2}$. Given our parameter values, we have $\delta_2 \approx -1.63$, $\delta_3 \approx -3.64$ and hence Equation(14) implies

$$\frac{\partial}{\partial\theta}E[\ln K] \approx -\delta_2\frac{e^{-\mu+\frac{1}{2}\sigma^2}}{\Phi_\rho}\Phi_{(\rho-\sigma)} - 2\theta\delta_3\frac{e^{-2\mu+2\sigma^2}}{\Phi_\rho}\Phi_{(\rho-2\sigma)} > 0 \tag{15}$$

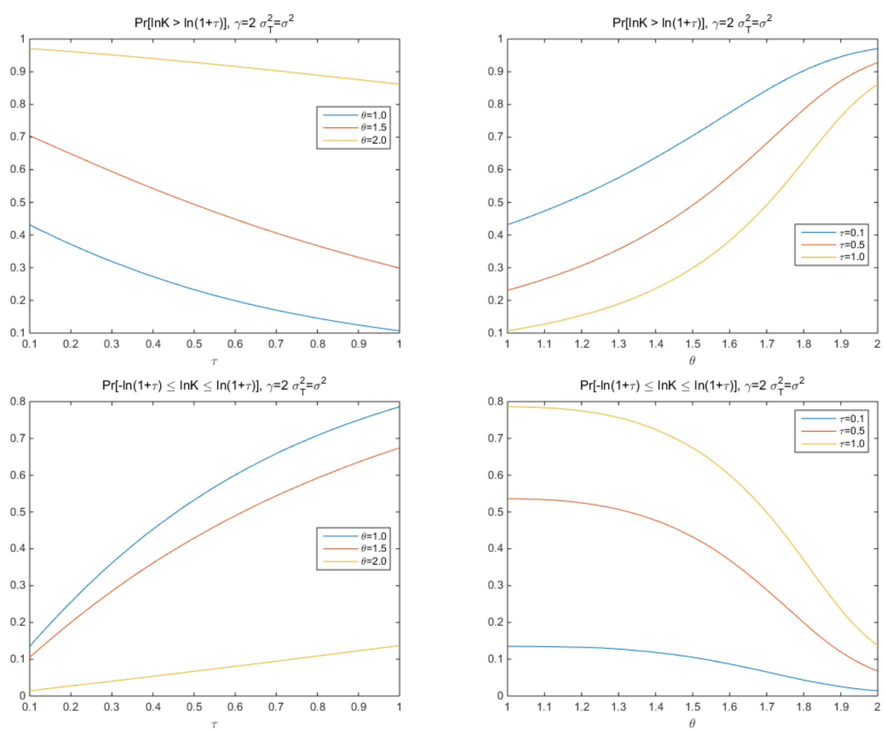
Although not provided here, our simulation results show that $V[\ln K]$ is a positive function of θ, indicating $\frac{\partial}{\partial\theta}V[\ln K] > 0$.[11]

Equation(15) and $\frac{\partial}{\partial\theta}V[\ln K] > 0$ imply that an increase in θ drives up the mean and the variance of $\ln K$ and hence increases the probability that the home country will export to the foreign country (*i.e.*, a rise in P_u and a fall in $(1 - P_l - P_u)$), which is consistent with the discussion in the previous section. As the chance that $\ln K$ falls outside the no-arbitrage band rises, there will be more probability mass of $\ln K$ at the boundary point $\ln(1+\tau)$, which will then reduce the real exchange rate volatility. Therefore, we can infer from Equation (13) that a rise in τ increases $V[\ln Q]$ by broadening the extent to which real exchange rates fluctuate freely, while a larger value of θ will decrease $V[\ln Q]$ by placing more probability mass of $\ln K$ on the upper boundary of the no-arbitrage band.

[11] The derivation of $E[\ln K_t]$ and $V[\ln K_t]$ and the simulation result of $\frac{\partial}{\partial\theta}V[\ln K_t]$ are offered in Appendix A and Appendix B, respectively.

4.2 Simulation results

We provide simulation results for the volatility of LOP deviations in which initially $\tau = 0.1$, $\theta = \theta^* = 1$. Figure 1 illustrates the responses of arbitrage opportunities to changes in trade costs and home distribution costs. As expected, one can see that the probability that trade does not occur increases with trade costs and decreases with distribution costs. More specifically, a rise in τ monotonically increases $\Pr\left[-\ln(1+\tau) \leq \ln K \leq \ln(1+\tau)\right]$, and this effect is magnified as θ falls. In contrast, $\Pr\left[-\ln(1+\tau) \leq \ln K_t \leq \ln(1+\tau)\right]$ decreases in θ, with the effect being magnified as τ rises. This result suggests that the extent to which goods market frictions affect arbitrage opportunity depends on the interplay of trade costs and distribution costs. As a higher θ is associated with higher $\Pr\left[\ln K_t \geq \ln(1+\tau)\right]$, a given increase in τ produces a lower $\Pr\left[-\ln(1+\tau) \leq \ln K \leq \ln(1+\tau)\right]$ the larger is θ. Similarly, a given increase in θ produces a lower $\Pr\left[\ln K \geq \ln(1+\tau)\right]$ the larger is τ.



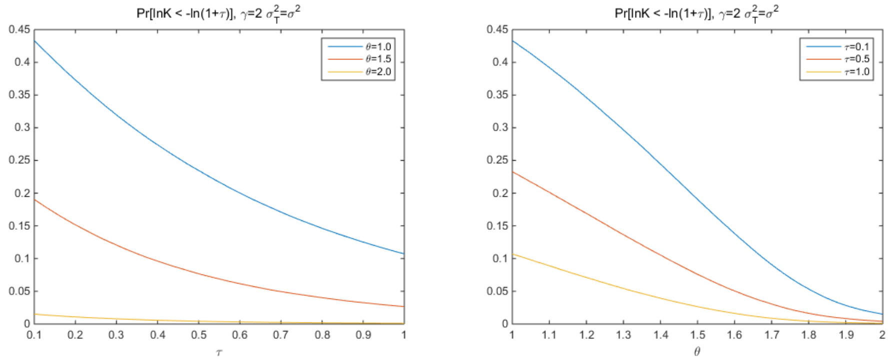


Figure 1. Responses of arbitrage opportunities to τ and θ

Figure 2 plots the variance of LOP deviations against trade costs and distribution costs where τ and θ range from 0.1 to 1 and 1 to 2, respectively. Apparently, the volatility of LOP deviations is positively associated with the size of trade costs, while it is negatively related to distribution costs. The economic mechanisms at work can be summarized as follows. There are two effects at work. The first effect stems from the expansion of the no-arbitrage band caused by the increase in trade costs. The second is the boundary effect caused by the increase in distribution costs. Higher trade costs make goods less likely to be traded, and hence widens the no-arbitrage band within which LOP deviations fluctuate, leading to an increase in the real exchange rate volatility. Because trade costs and distribution costs work in opposite directions when creating arbitrage opportunities, the positive relationship between trade costs and real exchange rate volatility becomes more evident when home distribution costs decline.

On the other hand, higher home distribution costs make goods more likely to flow from the home country to the foreign country, causing the real exchange rate to lie on the upper boundary of the band. This means that an increase in home distribution costs reduces real exchange rate volatility by placing more probability mass of $\ln K$ on the upper boundary of the no-arbitrage band. Note that, when trade costs are high, an increase in θ in an early stage (say, from $\theta = 1$ to $\theta = 1.5$) may not be enough to lead LOP deviations to lie on the boundary of the band, making little changes in real exchange rate volatility. However, the negative relationship between home distribution costs and real exchange rate volatility becomes stronger as θ increases further (say, from $\theta = 1.5$ to $\theta = 2.0$), which induces LOP deviations to be more likely to lie on the boundary.

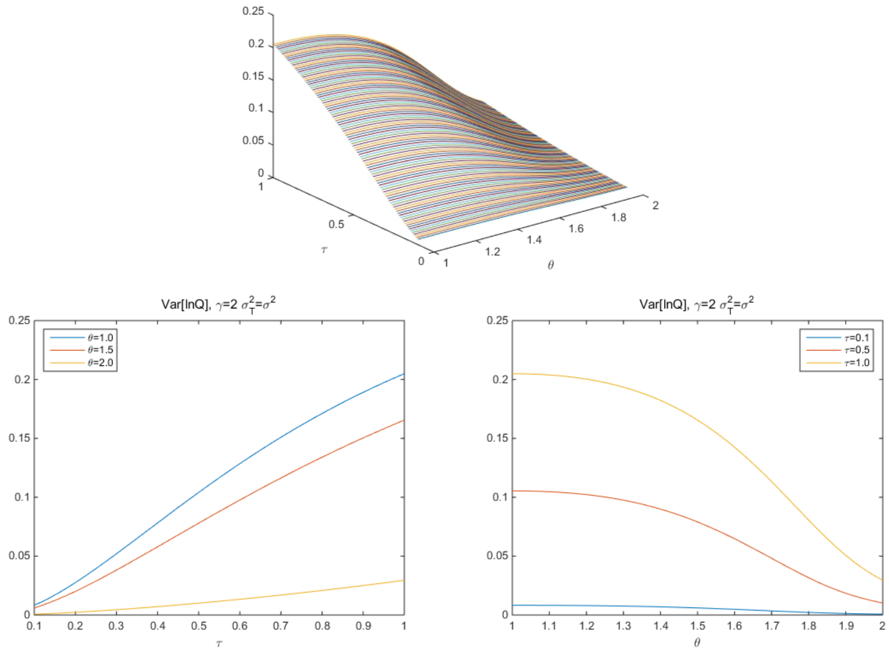


Figure 2. Responses of real exchange rate volatility to τ and θ

We next vary the value of the intertemporal substitution parameter as a sensitivity check of the benchmark model. In Panel B of Table 1, we provide the volatility of LOP deviations for $\gamma = 5$. The main findings in the benchmark case still hold when γ gets large: real exchange rate volatility responds positively to trade costs, but negatively to distribution costs. A notable difference arises with respect to the magnitude of the volatility itself. Compared to the benchmark case, the LOP deviations are evidently more volatile at every level of goods market frictions. For example, the variance rises from 0.2049 to 0.3051 for the case where $\tau = 1.0$ and $\theta = 1.0$. The parameter γ measures the extent to which individuals are willing to substitute consumptions over time. Therefore, as γ gets large, it takes larger changes in relative prices to get individuals to alter their consumption plans over time, leading LOP deviations to become more volatile.

Table 1. Summary of simulation results: $V[\ln Q_t]$

Panel A: $\gamma = 2$	$\tau = 0.1$	$\tau = 0.5$	$\tau = 1.0$
$\theta = 1.0$	.0083	.1031	.2049
$\theta = 1.5$	.0060	.0772	.1655
$\theta = 2.0$	.0007	.0100	.0295
Panel B: $\gamma = 5$	**$\tau = 0.1$**	**$\tau = 0.5$**	**$\tau = 1.0$**
$\theta = 1.0$	.0086	.1266	.3051
$\theta = 1.5$	.0083	.1217	.2931
$\theta = 2.0$	.0032	.0482	.1244

5 Conclusions

Understanding the determinants of LOP deviations and their links to real exchange rate volatility is a key challenge in international macroeconomics. Although a substantial body of empirical and theoretical work has documented this question, the channel through which real and nominal frictions affect the puzzling behavior of real exchange rates remains unclear. In this paper, we offer an alternative approach to address the issue. By modeling the distribution costs and trade costs in a framework of flexible prices, we explicitly analyze the role of goods market frictions in driving large and volatile deviations from the LOP. We first show that trade costs in conjunction with output ratios generate the no-arbitrage band in which trade does not occur, while distribution costs cause the price to deviate from the LOP by affecting the probability that trade will occur, given the band. Therefore, arbitrage opportunities and LOP deviations are directly affected by trade costs through the introduction of the band of inaction, whereas distribution costs influence them through a direction of trade and the resulting movements of LOP deviations toward the boundary of the band.

We next show that the real exchange rate volatility is positively associated with trade costs, but negatively related to distribution costs. Because these costs work in opposite directions when creating arbitrage opportunities, the extent to which a cost affects the real exchange rate volatility hinges on its interaction with the other cost. Our work contributes to the literature by providing a tractable theoretical framework that can be used to carry out explicit analyses of the long-run real exchange rate through the lens of two types of goods market frictions: domestic transaction costs and international transaction costs. We view our framework as complementary to those that emphasize the role of sticky prices.

Our finding implies that goods market frictions generate a rigid threshold around which real exchange rates behave in a non-linear manner. A promising avenue for future work involves the development of the tractable framework considered here to explicitly examine the role of distribution services and trade costs in the non-linear mean reversion of the real exchange rate.

Appendix A. Approximation of $E\left[\ln K_t\right]$ and $V\left[\ln K_t\right]$

Let $\frac{Y^T}{Y^{T*}} \equiv R_t^T$, $\frac{Y^{NT}}{Y^T} \equiv R_t^D$, $\frac{Y^{NT*}}{Y^{T*}} \equiv R_t^F$. Then $\ln K$ can be rewritten as

$$\begin{aligned}\ln K_t &= \gamma \ln R_t^T + \ln\left(R_t^F - \frac{\theta^*}{\alpha}\right) - \left(\alpha + (1-\alpha)\gamma\right)\ln\left(R_t^F - \theta^*\right) \\ &\quad + \left(\alpha + (1-\alpha)\gamma\right)\ln\left(R_t^D - \theta\right) - \ln\left(R_t^D - \frac{\theta}{\alpha}\right)\end{aligned}$$

Provided $R_t^D > \frac{\theta}{\alpha}$ and $R_t^D > \theta$,it follows that

$$\begin{aligned}\ln\left(R_t^D - \theta\right) &= \ln\left(R_t^D\left(1 - \frac{\theta}{R_t^D}\right)\right) = \ln\left(R_t^D\right) + \ln\left(1 - \frac{\theta}{R_t^D}\right) \\ &\approx \ln R_t^D - \theta\frac{1}{R_t^D} - \frac{\theta^2}{2}\frac{1}{\left(R_t^D\right)^2} \\ \ln\left(R_t^D - \frac{\theta}{\alpha}\right) &\approx \ln R_t^D - \frac{\theta}{\alpha}\frac{1}{\alpha R_t^D} - \frac{\theta^2}{2\alpha^2}\frac{1}{\left(R_t^D\right)^2}\end{aligned}$$

Similarly, assuming $R_t^F > \frac{\theta^*}{\alpha}$ and $R_t^F > \theta^*$, we have

$$\begin{aligned}\ln\left(R_t^F - \theta^*\right) &\approx \ln R_t^F - \theta^*\frac{1}{R_t^F} - \frac{\left(\theta^*\right)^2}{2}\frac{1}{\left(R_t^F\right)^2} \\ \ln\left(R_t^F - \frac{\theta^*}{\alpha}\right) &\approx \ln R_t^F - \frac{\theta^*}{\alpha}\frac{1}{R_t^F} - \frac{\left(\theta^*\right)^2}{2\alpha^2}\frac{1}{\left(R_t^F\right)^2}\end{aligned}$$

Therefore, $\ln K_t$ is approximately given by

$$\begin{aligned}\ln K_t &\approx \gamma \ln R_t^T - \delta_1 \ln R_t^F + \theta^*\delta_2\frac{1}{R_t^F} + \left(\theta^*\right)^2\delta_3\frac{1}{\left(R_t^F\right)^2} \\ &\quad + \delta_1 \ln R_t^D - \theta\delta_2\frac{1}{R_t^D} - \theta^2\delta_3\frac{1}{\left(R_t^D\right)^2}\end{aligned}$$

where $\delta_1 \equiv (1-\alpha)(\gamma-1)$, $\delta_2 \equiv (1-\alpha)\left(\gamma - 1 - \frac{1}{\alpha}\right)$, and $\delta_3 \equiv \frac{1}{2}(1-\alpha)\left(\gamma - 1 - \alpha - \frac{1}{\alpha^2}\right)$. It then follows that

$$\begin{aligned}E\left[\ln K_t\right] &\approx \gamma E\left[\ln R_t^T\right] - \delta_1 E\left[\ln R_t^F\right] + \theta^*\delta_2 E\left[\frac{1}{R_t^F}\right] + \left(\theta^*\right)^2\delta_3 E\left[\frac{1}{\left(R_t^F\right)^2}\right] \\ &\quad + \delta_1 E\left[\ln R_t^D\right] - \theta\delta_2 E\left[\frac{1}{R_t^D}\right] - \theta^2\delta_3 E\left[\frac{1}{\left(R_t^D\right)^2}\right]\end{aligned}$$

and, with a further assumption that $\ln R_t^T \perp \ln R_t^D \perp \ln R_t^F$,

$$
\begin{aligned}
V[\ln K_t] \;\; \approx \;\; & \gamma V\left[\ln R_t^T\right] + \delta_1^2 V\left[\ln R_t^F\right] - 2\delta_1\delta_2\theta^* Cov\left[\ln R_t^F, \frac{1}{R_t^F}\right] \\
& + (\theta^*)^2 \left(\delta_2^2 V\left[\frac{1}{R_t^F}\right] - 2\delta_1\delta_3 Cov\left[\ln R_t^F, \frac{1}{\left(R_t^F\right)^2}\right]\right) \\
& + 2\delta_2\delta_3 (\theta^*)^3 Cov\left[\frac{1}{R_t^F}, \frac{1}{\left(R_t^F\right)^2}\right] + (\theta^*)^4 \delta_3^2 V\left[\frac{1}{\left(R_t^F\right)^2}\right] \\
& + \delta_1^2 V\left[\ln R_t^D\right] - 2\delta_1\delta_2\theta Cov\left[\ln R_t^D, \frac{1}{R_t^D}\right] \\
& + \theta^2 \left(\delta_2^2 V\left[\frac{1}{R_t^D}\right] - 2\delta_1\delta_3 Cov\left[\ln R_t^D, \frac{1}{\left(R_t^D\right)^2}\right]\right) \\
& + 2\delta_2\delta_3\theta^3 Cov\left[\frac{1}{R_t^D}, \frac{1}{\left(R_t^D\right)^2}\right] + \theta^4\delta_3^2 V\left[\frac{1}{\left(R_t^D\right)^2}\right].
\end{aligned}
$$

With the assumptions of $\ln R_t^T \sim N\left(0, \sigma_T^2\right)$, $\ln R_t^D \sim TruncN\left(\mu, \sigma^2, \ln R_{\min}, \infty\right)$,and $\ln R_t^F \sim TruncN\left(\mu, \sigma^2, \ln R_{\min}, \infty\right)$,we have

$$
E[\ln K_t] \approx \gamma E\left[\ln R_t^T\right] + (\theta^* - \theta)\,\delta_2 E\left[\frac{1}{R_t}\right] + \left((\theta^*)^2 - \theta^2\right)\delta_3 E\left[\frac{1}{R_t^2}\right]
$$

and

$$
\begin{aligned}
V[\ln K_t] \;\; \approx \;\; & \gamma^2 V\left[\ln R_t^T\right] + 2\delta_1^2 V[\ln R_t] - 2\delta_1\delta_2(\theta^* + \theta)\, Cov\left[\ln R_t, \frac{1}{R_t}\right] \\
& + \left((\theta^*)^2 + \theta^2\right)\left(\delta_2^2 V\left[\frac{1}{R_t}\right] - 2\delta_1\delta_3 Cov\left[\ln R_t, \frac{1}{R_t^2}\right]\right) \\
& + 2\delta_2\delta_3\left((\theta^*)^3 + \theta^3\right) Cov\left[\frac{1}{R_t}, \frac{1}{R_t^2}\right] \\
& + \left((\theta^*)^4 + \theta^4\right)\delta_3^2 V\left[\frac{1}{R_t^2}\right],
\end{aligned}
$$

Under the distributional assumptions, it can be shown that, for $\rho \equiv -\frac{\ln R_{\min} - \mu}{\sigma}$, $\phi_s =$

$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}s^2}$, $\Phi_s = \int_{-\infty}^{s} \phi(t)\, dt$, and $\lambda_s \equiv \frac{\phi_s}{\Phi_s}$,

$$\begin{aligned}
E\left[R^r\right] &= e^{r\mu+\frac{1}{2}r^2\sigma^2}\frac{\Phi_{(\rho+r\sigma)}}{\Phi_\rho} \text{ for } r \in \mathbb{R}, \\
E\left[\ln R \cdot R^r\right] &= e^{r\mu+\frac{1}{2}r^2\sigma^2}\frac{\Phi_{(\rho+r\sigma)}}{\Phi_\rho}\left(\sigma\lambda_{(\rho+r\sigma)} + \mu + r\sigma^2\right), \\
E\left[\ln R\right] &= \mu + \sigma\lambda_\rho, \\
E\left[(\ln R)^2\right] &= \sigma^2\beta\lambda_\rho + 2\mu\sigma\lambda_\rho + \sigma^2 + \mu^2.
\end{aligned}$$

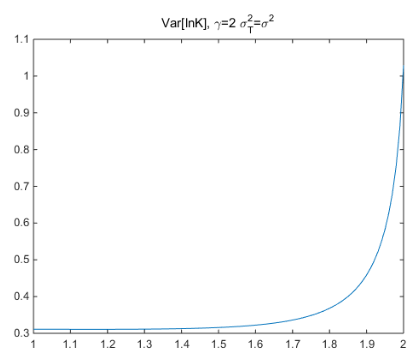
Therefore under the log-normality and truncated log-normality assumptions, we have

$$\begin{aligned}
E\left[\ln K_t\right] &\approx (\theta^* - \theta)\,\delta_2\frac{e^{-\mu+\frac{1}{2}\sigma^2}}{\Phi_\rho}\Phi_{(\rho-\sigma)} + \left((\theta^*)^2 - \theta^2\right)\delta_3\frac{e^{-2\mu+2\sigma^2}}{\Phi_\rho}\Phi_{(\rho-2\sigma)} \\
&= (\theta^* - \theta)\frac{e^{-\mu+\frac{1}{2}\sigma^2}\Phi_{(\rho-\sigma)}}{\Phi_\rho}\left(\delta_2 + (\theta^* + \theta)\,\delta_3 e^{-\mu+\frac{3}{2}\sigma^2}\frac{\Phi_{(\rho-2\sigma)}}{\Phi_{(\rho-\sigma)}}\right)
\end{aligned}$$

and

$$\begin{aligned}
V\left[\ln K_t\right] \approx\ & \gamma^2\sigma_T^2 + 2\delta_1^2\sigma^2\left(1 + \beta\lambda_\rho - \lambda_\rho^2\right) \\
&+ (\theta^* + \theta)\,2\delta_1\delta_2\sigma e^{-\mu+\frac{1}{2}\sigma^2}\frac{\Phi_{(\rho-\sigma)}}{\Phi_\rho}\left(\sigma + \lambda_\rho - \lambda_{(\rho-\sigma)}\right) \\
&+ \left((\theta^*)^2 + \theta^2\right)e^{-2\mu+2\sigma^2}\frac{\Phi_{(\rho-2\sigma)}}{\Phi_\rho} \\
&\times\left[2\delta_1\delta_3\sigma\left(2\sigma + \lambda_\rho - \lambda_{(\rho-2\sigma)}\right) + \delta_2^2\left(1 - e^{-\sigma^2}\frac{\Phi_{(\rho-\sigma)}}{\Phi_\rho}\frac{\Phi_{(\rho-\sigma)}}{\Phi_{(\rho-2\sigma)}}\right)\right] \\
&+ \left((\theta^*)^3 + \theta^3\right)2\delta_2\delta_3 e^{-3\mu+\frac{9}{2}\sigma^2}\frac{\Phi_{(\rho-3\sigma)}}{\Phi_\rho}\left(1 - e^{-2\sigma^2}\frac{\Phi_{(\rho-\sigma)}}{\Phi_\rho}\frac{\Phi_{(\rho-2\sigma)}}{\Phi_{(\rho-3\sigma)}}\right) \\
&+ \left((\theta^*)^4 + \theta^4\right)\delta_3^2 e^{-4\mu+8\sigma^2}\frac{\Phi_{(\rho-4\sigma)}}{\Phi_\rho}\left(1 - e^{-4\sigma^2}\frac{\Phi_{(\rho-2\sigma)}}{\Phi_\rho}\frac{\Phi_{(\rho-2\sigma)}}{\Phi_{(\rho-4\sigma)}}\right).
\end{aligned}$$

Appendix B. Simulation result for $\frac{\partial}{\partial\theta}V\left[\ln K_t\right]$



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