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**Abstract**

We present a general model of mixed oligopoly, where competing firms exercise spatial price discrimination. Our findings indicate that the Nash equilibrium locations of firms are always socially optimal irrespective of the number of competitors, the level of privatization, the form of the transportation costs and the number and/or the varieties of the produced goods. An immediate implication of this result is that this form of competition is preferable from a welfare point of view.

*JEL classification: L13; L32; L33; R32*
*Keywords: Mixed oligopoly; Social optimality; Spatial competition; Differentiated goods*

## 1 Introduction

De Fraja and Delbono (1989) initiated a large literature on mixed oligopoly, where public and private firms coexist in the same market.[1] The existing studies can be classified as falling into two groups; one adopting a restricted 'binary' approach where firms are either private or public (e.g., Cremer et al., 1991; Matsumura and Matsushima, 2003; Lu, 2006) and the other allowing for partially privatized firms (e.g., Matsumura, 1998; Fershtman, 1990; Bennett and Maw, 2003; Kumar and Saha, 2008).

Mixed oligopoly, however, has rarely been examined within the context of spatial price discrimination introduced by Hoover (1937) and Lerner and Singer (1937). This type of spatial competition differs from the one introduced by Hotelling (1929) in the fact that the firms do not compete in mill prices but instead bear transportation costs and set delivered price schedules.[2] Notable exceptions, where mixed oligopoly theory accounts for spatial price discrimination, are those of Heywood and Ye (2009a, b) and Beladi et al. (2014). Nevertheless, the aforementioned studies impose too many restrictions in their modeling structure, such as the number of competing firms, the form of the transportation cost function and the attributes /

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[1] For a comprehensive review about mixed oligopoly literature, see De Fraja (2009).

[2] Applications of spatial price discrimination can be found in Lederer and Hurter (1986), Hamilton et al. (1989), Hamilton et al. (1991), McLeod et al. (1992), Braid (2008), and Vogel (2011). Anderson et al. (1992) present an overview of the related literature.

number of goods in the market. It is not at all clear how these restrictions affect the nature of results. For example, d'Aspremont et al. (1979) showed that in the traditional Hotelling's model (Hotelling, 1929), the nature of travel costs is important for the existence of an equilibrium. Surprisingly enough, they showed that an equilibrium exists when transportation costs are proportional to the square of distance while it doesn't when the travel costs are linear. On the other hand, Cremer et al. (1991) highlighted the importance of the number of firms on the welfare properties of the equilibrium.

The present paper contributes to the existing literature in manifold ways. We show that in a model of mixed oligopoly with spatial price discrimination, where the produced goods have the same reservation value for the buyers, the market outcome will be socially optimal, and this result is independent of the number of firms in the market, the level of privatization of each firm, the form of the transportation cost function and the number and/or the varieties of the goods offered by each competitor. The driving force behind our results is the same as in Lederer and Hurter (1986); a firm can increase its profit by opting for a production location so as to decrease the total cost of all firms in the market.[3] However, in Lederer and Hurter (1986) competition is restricted to only two exclusively privately owned firms, therefore, mixed oligopoly and the ensuing welfare questions are not being considered. It should also be observed that the results in Beladi et al. (2014) can be obtained as a special case of our model, with two firms, three varieties of the same good (or three goods) and linear transportation costs.

The rest of the paper is structured as follows. The next section presents the benchmark model where firms offer a homogeneous good. The mixed oligopoly with multiple goods is analyzed in Section 3. Section 4 concludes.

## 2 The benchmark model

We consider a market consisted of $n$ firms and a continuum of consumers uniformly distributed over the unit interval $[0,1]$ of a linear country.[4] Let $x_i$, $i = 1, ..., n$, denote the location of firm $i$ in the interval $[0,1]$ with $0 \leq x_1 < x_2 < ... < x_n \leq 1$. All firms produce and sell the same homogeneous good. The fraction of consumers buying this good is equal to $c \in (0,1]$. Each consumer buys one unit of the good from the lowest price firm, providing that this price is lower or equal to her reservation price (i.e., the maximum price that the consumers are willing to pay for the good), $m > 0$. The marginal costs of production are constant and, without any loss of generality, are set equal to 0. Spatial price discrimination à la Lerner and

[3] This implies the alignment of the social and private optima.

[4] By uniformly distributed, we mean that the proportion of consumers buying the good remains the same, regardless of the subinterval of $[0,1]$.

Singer (1937) is assumed, where Nash equilibrium in delivered price schedules exists. More specifically, the price charged for the good by the firm that is closer to the consumer is equal to (or infinitesimally less than) the delivered cost of the neighboring firm which is further away.[5] Because marginal production costs have been normalized to 0, delivered costs coincide with transportation costs. Transportation cost is measured through a function $f$ of the shipped distance $d$, with $f$ non-negative, increasing and continuous. The aggregate transportation (shipping) cost[6] for all locations $z$ of consumers who buy from any of the $n$ firms is equal to

$$T(x_1, ..., x_n) = \sum_{i=1}^{n} T_i(x_1, ..., x_n) \tag{1}$$

where

$$T_i(x_1, ..., x_n) = \begin{cases} c\left(\int_0^{x_1} f(x_1 - z)dz + \int_{x_1}^{\frac{x_1+x_2}{2}} f(z - x_1)dz\right) \text{ for } i = 1 \\ \\ c\left(\int_{\frac{x_{i-1}+x_i}{2}}^{x_i} f(x_i - z)dz + \int_{x_i}^{\frac{x_i+x_{i+1}}{2}} f(z - x_i)dz\right) \text{ for } 1 < i < n \\ \\ c\left(\int_{\frac{x_{n-1}+x_n}{2}}^{x_n} f(x_n - z)dz + \int_{x_n}^{1} f(z - x_n)dz\right) \text{ for } i = n \end{cases} \tag{2}$$

is the total transportation cost for those consumers buying from firm $i$.

Firm $i$ is selling its product at a price matching (or which is infinitesimally less than) the delivery cost of its direct competitor[7] which is the firm nearest to its location. Thus, the profit function of firm $i$ is

[5] e.g., the neighboring firms for firm $i$, are firms $i-1$ and $i+1$.

[6] The terms 'delivered cost', 'transportation cost', and 'shipping cost' are used interchangeably hereafter.

[7] We assume that this cost does not exceed the reservation value of the consumer.

$$
\Pi_i(x_1,...,x_n) = \begin{cases}
c\left(\int_0^{x_1}[f(x_2-z)-f(x_1-z)]dz+\int_{x_1}^{\frac{x_1+x_2}{2}}[f(x_2-z)-f(z-x_1)]dz\right) & \text{if } i=1 \\
c\begin{pmatrix} \int_{\frac{x_{i-1}+x_i}{2}}^{x_i}[f(z-x_{i-1})-f(x_i-z)]dz \\ +\int_{x_i}^{\frac{x_{i-1}+x_{i+1}}{2}}[f(z-x_{i-1})-f(z-x_i)]dz \\ +\int_{\frac{x_{i-1}+x_{i+1}}{2}}^{\frac{x_i+x_{i+1}}{2}}[f(x_{i+1}-z)-f(z-x_i)]dz \end{pmatrix} & \text{if } 1<i<n \text{ and } x_i \le \frac{x_{i-1}+x_{i+1}}{2} \\
c\begin{pmatrix} \int_{\frac{x_{i-1}+x_i}{2}}^{\frac{x_{i-1}+x_{i+1}}{2}}[f(z-x_{i-1})-f(x_i-z)]dz \\ +\int_{\frac{x_{i-1}+x_{i+1}}{2}}^{x_i}[f(x_{i+1}-z)-f(x_i-z)]dz \\ +\int_{x_i}^{\frac{x_i+x_{i+1}}{2}}[f(x_{i+1}-z)-f(z-x_i)]dz \end{pmatrix} & \text{if } 1<i<n \text{ and } \frac{x_{i-1}+x_{i+1}}{2} \le x_i \\
c\begin{pmatrix} \int_{\frac{x_{n-1}+x_n}{2}}^{x_n}[f(z-x_{n-1})-f(x_n-z)]dz \\ +\int_{x_n}^{1}[f(z-x_{n-1})-f(z-x_n)]dz \end{pmatrix} & \text{if } i=n
\end{cases} \tag{3}
$$

.

**Lemma 1** *The marginal transportation cost is*

$$
\partial T_i/\partial x_i = \begin{cases}
c\left[f(x_1)-\frac{1}{2}f(\frac{x_2-x_1}{2})\right] & \text{for } i=1 \\
c\left[\frac{1}{2}f(\frac{x_i-x_{i-1}}{2})-\frac{1}{2}f(\frac{x_{i+1}-x_i}{2})\right] & \text{for } 1<i<n \\
c\left[\frac{1}{2}f(\frac{x_n-x_{n-1}}{2})-f(1-x_n)\right] & \text{for } i=n
\end{cases}.
$$

**Proof.** Let $F(y) := \int f(y)dy$, then $T_i(x_1,...,x_n) = c\left([-F(x_i-z)]_{\frac{x_{i-1}+x_i}{2}}^{x_i}+[F(z-x_i)]_{x_i}^{\frac{x_i+x_{i+1}}{2}}\right) = c\left[-2F(0)+F(\frac{x_i-x_{i-1}}{2})+F(\frac{x_{i+1}-x_i}{2})\right]$. Hence, $\partial T_i/\partial x_i = c\left[\frac{1}{2}f(\frac{x_i-x_{i-1}}{2})-\frac{1}{2}f(\frac{x_{i+1}-x_i}{2})\right]$. Similarly, we can show that $\partial T_1/\partial x_1 = c\left[f(x_1)-\frac{1}{2}f(\frac{x_2-x_1}{2})\right]$ and $\partial T_n/\partial x_n = c\left[\frac{1}{2}f(\frac{x_n-x_{n-1}}{2})-f(1-x_n)\right]$. ■

Maintaining the notation as above, we can state the first of the two main results of this section.

**Proposition 1** *The marginal aggregate transportation cost with respect to the location of firm $i$, $i=1,...,n$, is opposite to the marginal profit of firm $i$, i.e.*

$$\partial T/\partial x_i = -\partial \Pi_i/\partial x_i.$$

**Proof.** For $1<i<n$, $x_i$ appears in the expression of the aggregate transportation cost in the following

fashion:

$$
\begin{aligned}
T(x_1,...,x_n) \quad &= \quad T_1(x_1,...,x_n)+...+\underbrace{c\left(\int_{\frac{x_{i-2}+x_{i-1}}{2}}^{x_{i-1}} f(x_{i-1}-z)dz+\int_{x_{i-1}}^{\frac{x_{i-1}+x_i}{2}} f(z-x_{i-1})dz\right)}_{T_{i-1}} \\
&+T_i(x_1,...,x_n)+\underbrace{c\left(\int_{\frac{x_i+x_{i+1}}{2}}^{x_{i+1}} f(x_{i+1}-z)dz+\int_{x_{i+1}}^{\frac{x_{i+1}+x_{i+2}}{2}} f(z-x_{i+1})dz\right)}_{T_{i+1}}+ \\
&...+T_n(x_1,...,x_n)
\end{aligned}
$$

From the above expression and Lemma 1, we get

$$\frac{\partial T}{\partial x_i}=c\left[f(\frac{x_i-x_{i-1}}{2})-f(\frac{x_{i+1}-x_i}{2})\right]$$

for $1<i<n$. In view of the notation $F(y):=\int f(y)dy$, we rewrite (3), for $\frac{x_{i-1}+x_{i+1}}{2}\leq x_i$ and $1<i<n$,

$$\Pi_i(x_1,...,x_n)=c\left[2F(\frac{x_{i+1}-x_{i-1}}{2})-2F(\frac{x_i-x_{i-1}}{2})-2F(\frac{x_{i+1}-x_i}{2})+2F(0)\right]$$

Differentiating the above expression, we get

$$\frac{\partial \Pi_i}{\partial x_i}=c\left[-f(\frac{x_i-x_{i-1}}{2})+f(\frac{x_{i+1}-x_i}{2})\right]$$

■

Proposition 1 together with the fact that $\partial T/\partial x_i=\partial T_i/\partial x_i$ imply

**Corollary 1** *The marginal profit of firm $i$, $i=1,...,n$, with respect to its location $x_i$ ($\partial\Pi_i/\partial x_i$) is opposite to its marginal transportation cost ($\partial T_i/\partial x_i$).*

The next step is to compare the Nash equilibrium locations with the socially optimal ones. To derive the socially optimal locations we have to minimize (1) with respect to each firm's location. Hence, the socially optimal locations satisfy the system:

$$\partial T/\partial x_i=0,\ \ i=1,...,n. \tag{4}$$

On the other hand, the Nash equilibrium locations are given by the solution of the following system:

$$\partial \Pi_i/\partial x_i=0,\ \ i=1,...,n. \tag{5}$$

From Proposition 1, we get that systems (4) and (5) are equivalent and therefore have the same solution. This leads us to the following Proposition:

**Proposition 2** *In models of spatial price discrimination à la Lerner and Singer, where firms have constant marginal production costs, produce the same homogeneous good and consumers are distributed uniformly along a linear country of unit length, the Nash equilibrium locations of firms are socially optimal.*

In our analysis so far, all firms are privately owned. Let us now assume that single firm $l$, $l = \{1, ..., n\}$ is partly privately owned and partly publicly owned in proportions $a_l$ and $1 - a_l$ (in other words $a_l$ can be considered as the degree of privatization), respectively with $a_l \in [0, 1]$. In such a case, firm $l$ will decide about its optimal location by maximizing the weighted average of its own profits and social welfare with weights $a_l$ and $1 - a_l$, respectively. Social welfare is equal to the sum of the aggregate profits (the profit of all firms) and consumers' surplus. The consumers' surplus is given by

$$CS(x_1, ..., x_n) = \sum_{i=1}^{n} CS_i(x_1, ..., x_n) \tag{6}$$

where $CS_i(x_1, ..., x_n)$ is the consumer surplus generated for the consumers buying from firm $i$, therefore,

$$CS_i(x_1, ..., x_n) = \begin{cases} c\left(\int_0^{x_1} [m - f(x_2 - z)]dz + \int_{x_1}^{\frac{x_1+x_2}{2}} [m - f(x_2 - z)]dz\right) & \text{for } i = 1 \\ c\begin{pmatrix} \int_{\frac{x_{i-1}+x_i}{2}}^{x_i} [m - f(z - x_{i-1})]dz \\ +\int_{x_i}^{\frac{x_{i-1}+x_{i+1}}{2}} [m - f(z - x_{i-1})]dz \\ +\int_{\frac{x_{i-1}+x_{i+1}}{2}}^{\frac{x_i+x_{i+1}}{2}} [m - f(x_{i+1} - z)]dz \end{pmatrix} & \text{for } x_i \leq \frac{x_{i-1}+x_{i+1}}{2} \text{ and } 1 < i < n \\ c\begin{pmatrix} \int_{\frac{x_{i-1}+x_i}{2}}^{\frac{x_{i-1}+x_{i+1}}{2}} [m - f(z - x_{i-1})]dz \\ +\int_{\frac{x_{i-1}+x_{i+1}}{2}}^{x_i} [m - f(x_{i+1} - z)]dz \\ +\int_{x_i}^{\frac{x_i+x_{i+1}}{2}} [m - f(x_{i+1} - z)]dz \end{pmatrix} & \text{for } \frac{x_{i-1}+x_{i+1}}{2} \leq x_i \text{ and } 1 < i < n \\ c\left(\int_{\frac{x_{n-1}+x_n}{2}}^{x_n} [m - f(z - x_{n-1})]dz + \int_{x_n}^{1} [m - f(z - x_{n-1})]dz\right) & \text{for } i = n \end{cases} \tag{7}$$

Direct calculation proves

**Lemma 2** $\Pi_i(x_1,...,x_n)+ CS_i(x_1,...,x_n) = \begin{cases} \int_0^{\frac{x_1+x_2}{2}} mdz - T_1(x_1,...,x_n) \text{ } for \text{ } i=1 \\ \int_{\frac{x_{i-1}+x_i}{2}}^{\frac{x_i+x_{i+1}}{2}} mdz - T_i(x_1,...,x_n) \text{ } for \text{ } 1<i<n \\ \int_{\frac{x_{n-1}+x_n}{2}}^{1} mdz - T_n(x_1,...,x_n) \text{ } for \text{ } i=n \end{cases}$

**Proof.** Straightforward direct calculations. ■

Summing up over all firms one gets the following Proposition which could be viewed as the second main result of this section.

**Proposition 3**

$$\sum_{i=1}^{n} \Pi_i(x_1,...,x_n) + CS(x_1,...,x_n) = m - T(x_1,...,x_n)$$

**Proof.** Straightforward direct calculations. ■

The profit function of the partly publicly owned firm $l$ will be

$$\hat{\Pi}_l(x_1,...,x_n) = \Pi_l(x_1,...,x_n) + (1-a_l)\left[\sum_{i\neq l} \Pi_i(x_1,...,x_n) + CS(x_1,...,x_n)\right] \tag{8}$$

where $\Pi_l$ would be the profit function of firm $l$ if it were fully privately owned.

**Proposition 4** *Nash equilibria remain socially optimal regardless of the degree of privatization of the individual firms* $i, 1 \leq i \leq n$.

**Proof.** Fix a random $i, 1 \leq i \leq n$. Using Proposition 3 and (8), we get

$$\hat{\Pi}_l(x_1,...,x_n) = \Pi_l(x_1,...,x_n) + (1-a_l)\left[m - T(x_1,...,x_n) - \Pi_l(x_1,...,x_n)\right]$$

From Proposition 1

$$\partial T/\partial x_l = -\partial \Pi_l/\partial x_l \Longleftrightarrow -\partial T/\partial x_l - \partial \Pi_l/\partial x_l = 0$$

which implies that $\partial \hat{\Pi}_l/\partial x_l = \partial \Pi_l/\partial x_l$. Induction on $i$ completes the proof. ■

# 3 The case of multiple goods

We now assume the existence of $L$ different goods or different varieties of the same good or both. Let $k_j$ denote the number of firms producing good $j$, $j = 1,...,L$ with $1 \leq k_j \leq n$. Let $T^j$ denote the aggregate transportation cost related to the provision of good $j$ and $\Pi_i^j$ the corresponding profit of firm $i$ from selling good $j$ with $\Pi_i^j = 0$ if good $j$ is not produced by firm $i$. Let further $\tilde{T} = \sum_{j=1}^{L} T^j$ be the aggregate shipping cost for all products and $\tilde{\Pi}_i = \sum_{j=1}^{L} \Pi_i^j$ the total profit of firm $i$ for all products it produces. The fraction of consumers buying product $j$ is $c_j \in (0,1]$ uniformly spread over $[0,1]$ with $\sum_{j=1}^{L} c_j = 1$. In the case where good $j$ is produced by only one firm, then this firm enjoys monopoly privileges and charges a price equal to, or infinitesimally smaller than, the reservation price $m_j$, i.e. the maximum price the consumer is willing to pay for good $j$. A fundamental assumption in this multi-good setting is that $m_1 = ... = m_L = m$ (i.e. the reservation price is common for all goods).[8]

**Proposition 5** *The marginal aggregate shipping cost for all products with respect to the location of firm i is opposite to the marginal profit of firm i for all products it produces, namely* $\partial\tilde{T}/\partial x_i = -\ \partial\tilde{\Pi}_i/\partial x_i$.

**Proof.** From Proposition 1, we get that for every product $j$ offered by more than one firm

$$\partial T^j/\partial x_i = -\partial \Pi_i^j/\partial x_i$$

It can be easily shown that the above condition holds even if product $j$ is produced by only one firm (i.e. the firm is a monopoly for good $j$). To complete the proof we sum up over all products. ■

**Theorem 1** *In models of spatial price discrimination à la Lerner and Singer, where firms have constant marginal production costs, produce different combination of goods, consumers are distributed uniformly along a linear city of unit length and have the same reservation price for all goods, the Nash equilibrium locations of firms are socially optimal.*

**Proof.** To derive the socially optimal locations we have to minimize $\tilde{T}$ with respect to each firm's location. Hence, the socially optimal locations satisfy the following system of equations:

$$\partial\tilde{T}/\partial x_i = 0,\ \ i = 1, ..., n. \tag{9}$$

On the other hand, the Nash equilibrium locations are given by the solution of the following system:

$$\partial\tilde{\Pi}_i/\partial x_i = 0,\ \ i = 1, ..., n. \tag{10}$$

[8] It should be noted that this assumption is more realistic in the case of the different varieties of the same good and less in the case of different goods.

Because of Proposition 5, systems (9) and (10) are equivalent. It follows that they must have the same set of solutions. ■

Let's now turn to the case where some firms are partly privately owned and partly publicly owned. Keeping the notation the same as in Section 2, $\breve{\Pi}_l = \sum_{j=1}^{L} \hat{\Pi}_l^j$ is the profit of firm $l$ where $\hat{\Pi}_l^j$ is the profit of firm $l$ from selling good $j$ and $\hat{\Pi}_l^j = 0$ if good $j$ is not produced by firm $l$.

**Theorem 2** *The degree of privatization does not affect the socially optimal Nash equilibrium locations.*

**Proof.** From the analysis in the previous section it can be easily shown that $\partial\breve{\Pi}_l/\partial x_l = \sum_{j=1}^{L}(\partial\hat{\Pi}_l^j/\partial x_l) = \sum_{j=1}^{L}(\partial\Pi_l^j/\partial x_l) = \partial\tilde{\Pi}_l/\partial x_l$. ■

# 4 Conclusion

In this paper, we prove that when firms exercise spatial price discrimination, the outcome of the mixed oligopoly is socially optimal and independent from the underlying assumptions about the number of firms, the level of privatization, the nature of transportation costs and the number or the varieties of the provided goods. To the best of our knowledge, our analysis is the first attempt to present an 'holistic' view of the mixed oligopoly theory under spatial price discrimination. While general in nature, our model is restricted by the assumption of linear city/country. Therefore, examining the robustness of our findings under a two-dimensional spatial framework constitutes a topic for future research. In reality, the true task ahead is to relax the framework to that of a two-dimensional Euclidean manifold where the locations of the $n$ firms with $n > 2$ are not on the same geodesic.

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