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Going to the Discounter: Consumer Search with Local Market Heterogeneities

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Abstract

This article proposes a new rationale for consumer search and mixed-strategy pricing: the presence of local market heterogeneities. In the model, two spatially separated markets, each home to an identical local monopolist, differ in size and their consumers' willingness to pay (e.g., as caused by differences in local income). Consumers observe their native market's price and a flexible subset of them may travel to the other market at strictly positive cost, hoping for a bargain. I show that as long as the proportion of flexible consumers in the high-valuation market is not too large, directed search to the low-valuation market will occur in equilibrium. If the high-valuation market is relatively large in size, the opposed firm faces a commitment problem that induces non-trivial mixed-strategy pricing in equilibrium. In particular, low-valuation consumers are excluded from the product market with positive probability. Informative advertising with price-commitment may decrease market performance.

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1 Introduction

It is a well-established empirical finding that “the law of one price is no law at all” (Varian (1980)): prices for homogeneous products are often widely dispersed.¹ A prominent explanation is that consumers cannot freely observe product prices across competing firms. Instead, obtaining additional price information is costly, and consumers have to search actively across sellers in order to find a good deal. This informational imperfection provides firms with market power and enables them to raise prices above marginal cost.

Unfortunately, the famous Diamond paradox (1971) establishes that if *all* (identical) consumers find it costly to compare prices (no matter how small these costs are), consumers search sequentially, and firms as well as the initial consumer distribution across them are symmetric, the unique equilibrium entails monopoly-pricing by all firms, while no consumers search.²

Since Diamond’s seminal contribution, numerous attempts have been made to overcome this counterfactual theoretical prediction. In the here considered framework of sequential consumer search for homogeneous products, the vast majority of models introduce a positive mass of consumers without search costs (“shoppers”) who observe all prices in the market, and thus put a downward pressure on prices.³ Examples include the celebrated paper by Stahl (1989) on costly sequential search in oligopoly, as well as modifications allowing for heterogeneity across consumers with positive search costs (Stahl (1996)), or truly costly sequential search (Janssen et al. (2005)). However, in reality, it is far from clear whether a group of consumers exists that does not face *any* costs of obtaining additional price information (let alone, costs of visiting additional shops or spending more time on search).

A further drawback of most contemporaneous sequential-search models is that no proper search occurs in equilibrium. This is because an endogenous reservation price emerges above which consumers with positive search cost would prefer to visit another firm. But then, unless further ingredients are added, no firm finds it optimal to price above this reservation price in equilibrium, as doing so implies zero demand.⁴

¹See Baye et al. (2006) for a detailed survey of theoretical and empirical studies on price dispersion in homogeneous-goods markets.

²The reason is straightforward: Suppose to the contrary that not all firms price at the monopoly level p^m . Since pricing above the monopoly price is clearly suboptimal, the firm(s) with the lowest price in the market must price strictly below p^m . But this cannot be part of an equilibrium, because slightly increasing this lowest price towards the monopoly level does not lead any consumers to purchase elsewhere, as they face a strictly positive search cost.

³One exception is Reinganum (1979), who generates price dispersion by marginal-cost differences across a continuum of firms, with consumers having downward-sloping demand.

⁴In Stahl (1996) and Chen and Zhang (2011), search-cost heterogeneities across consumers with positive search costs may lead some consumers to search actively in equilibrium. However, both models require an atom of shoppers in order to generate price dispersion.

Finally, a typical feature of standard search models is that search (or hypothetical search, if it never occurs in equilibrium) is undirected. This means that even after rejecting a high price and moving on, the next firm a consumer samples is drawn randomly from a set of firms with identical characteristics. Hence, these models do not allow for directed search towards firms that are perceived to offer particularly good deals in expectation, i.e., discounters.⁵

The present paper addresses the above issues by introducing search across locally separated and heterogeneous submarkets. Particular features of the model are that (i) all consumers have positive search costs, yet equilibria arise where not all firms price at the monopoly level (ii) there may be proper search in equilibrium, and (iii) search is directed towards a firm that is perceived as discounter and does in fact offer lower prices in expectation.

The general mechanism that leads to search in the model is straightforward, robust, and, to the best of my knowledge, has not been pointed out by the theoretical literature. The key idea is to consider locally separated monopolistic markets which, due to either demand-side or supply-side heterogeneities, would give rise to different monopoly prices in isolation. However, there is a link between the markets in the sense that some “flexible” consumers may, given their beliefs about the other market’s unobserved price, find it worthwhile to travel to this market at strictly positive cost.

One can then conjecture that an equilibrium exists in which the local monopolists cater to different consumer groups. While the firm in the market with the higher monopoly price (henceforth called “regular firm”) focuses on exploiting a captive segment of local consumers, its rival in the market with the lower monopoly price (henceforth called “discounter”) charges a lower price which attracts the outside market’s flexible consumers *and* is optimal given its local market’s characteristics (including incoming consumers).

In the main model that I develop below, I formalize this intuition by focusing on the case of demand-side heterogeneities. For tractability, I assume that all consumers have unit demand up to a maximal valuation, but that this valuation is higher in one market than the other. While stylized, there are various real-world market configurations which share similar features. For example, many empirical studies document that income tends to be highly segregated in urban

⁵Arbatskaya (2007) considers the implications of search in a homogeneous-product market where consumers with heterogeneous search costs have to follow an exogenous search order. Under certain conditions, such markets exhibit a unique equilibrium in pure strategies in which prices are strictly declining in the search order, and active search occurs in equilibrium. Besides specific physical constraints (e.g., the line-up of vendors in an Oriental bazaar), it is not clear why consumers should not upset this equilibrium by visiting the last firm in the search order first.

areas – the rich rarely locate door-to-door with the poor.⁶ It is then natural to think that the poor will have a lower maximum willingness to pay for certain products, and that firms located in poor neighborhoods will have to put a lower price tag on these products if they want to avoid excluding their local population from buying.⁷ Alternatively, the setup may reflect a situation of “cross-border shopping” that is induced by differences in income or favorable exchange rates. In continental Europe, cross-border shopping has a long history, and is still observed frequently (see e.g. “Swiss Shoppers Storm German Border Towns,” Spiegel Online, 2011).⁸

Analyzing the described market configuration, the following main results are shown. First, if a large fraction of consumers in the high-valuation market is flexible, paradoxically no search occurs in the unique equilibrium of the game. This is because the regular firm in the high-valuation market finds its local flexible consumers too important to lose, and optimally charges a sufficiently low price that discourages them from leaving.

Second, if the fraction of flexible consumers in the high-valuation market is sufficiently small *and* at the same time the high-valuation market is not too large relative to the low-valuation one, the unique equilibrium of the game follows the intuition from above: the regular firm charges the high monopoly price, the discounter charges the low one, and the high-valuation market’s flexible consumers travel to the low-valuation market and purchase there with certainty. The discounter has no incentive to increase its price, as this would drive out its local consumers with a lower willingness to pay. At the same time, the regular firm has no incentive to discourage its local flexible consumers from search, as it would have to decrease its price by too much.

Third, if the high-valuation market is large relative to the low-valuation market (and the proportion of flexible consumers in the former is not too high), the discounter faces a *commitment problem*. While the discounter would like the flexible high-valuation consumers to believe that it charges a low price and therefore induce search, the expected incoming mass of flexible high-valuation consumers would be so large that the discounter would prefer to maximally exploit these searching consumers by (almost) charging the price of its rival, despite driving out its local consumers. But clearly, this cannot constitute an equilibrium, as then the high-valuation market’s flexible consumers would have no incentive to search in the first place.

⁶See e.g. Bischoff and Reardon (2013) and Florida and Mellander (2015) for recent reports on income segregation in major U.S. metropolitan areas.

⁷Even in the absence of income differences, the population’s composition may vary considerably across regions. In turn, differences in willingness to pay for an identical product may prevail.

⁸A relatively recent survey of the vast economic literature on cross-border shopping is given by Leal et al. (2010).

It turns out that the discounter's commitment problem can only be resolved by mixed-strategy pricing in which the firm sometimes prices above its local consumers' valuation, but also sometimes does not sell at all because it is priced out by its rival. The latter occurs because with positive probability, the regular firm engages in a sale that may beat the discounter's price, at least if the latter tries to exploit the incoming searchers by pricing above its local monopoly price. Moreover, if the discounter's commitment problem is severe enough, in the unique corresponding equilibrium the regular firm sometimes engages in a deep sale, which altogether discourages its local flexible consumers from search (thus further reducing the discounter's incentive to price above its local consumers' valuation).

Following up on these observations, it is worthwhile to point out that, to the best of my knowledge, my model is the first which can generate both spatial *and* temporal price dispersion in equilibrium. The price dispersion is spatial, in the sense of Salop and Stiglitz (1977) and Reinganum (1979), because the discount firm charges prices that are on average lower than the regular firm's. On the other hand, if the discounter faces a commitment problem, the price dispersion is also temporal, in the sense of e.g. Shilony (1977), Varian (1980) and Rosenthal (1980), because in equilibrium, both firms sample prices from overlapping supports. Hence, complex sales patterns arise in which the regular firm sometimes engages in promotions which beat the discounter's price, or altogether discourage its local flexible consumers from shopping around.

After discussing the different types of equilibria that arise in the model, I turn to a social-welfare analysis. I identify two potential sources of welfare loss in the market: wasteful travel expenditures undertaken by searching high-valuation consumers, and deadweight loss created by dropout low-valuation consumers. While the former occurs whenever the fraction of flexible high-valuation consumers is not too large (otherwise, the regular firm fights for its flexible consumers and the social first-best is achieved), the latter only occurs if the discounter faces a commitment problem. In that case, the firm prices above its local consumers' valuation with positive probability in equilibrium.

The latter phenomenon also endogenizes an empirical regularity that has widely been documented, namely that poorer consumer groups tend to find it more difficult to access product markets (see e.g. Somekh (2012, 2015) and the references therein). In my model, I find that, if the high-valuation (high-income) market is relatively large in size, the firm in the low-valuation (low-income) market may consider it optimal to (probabilistically) exclude its local consumers

from purchasing. This is because higher rents can be extracted from less price sensitive (or more wealthy) shoppers coming from outside.

Comparative statics with respect to social welfare (and other equilibrium objects like the firms' pricing strategies and profits) are provided. Often, the sign of marginal effects changes after transitions between the different equilibrium regions. Moreover, in some cases, countervailing effects are at play which may lead to non-monotonicities within equilibrium regions. For example, if the discounter faces a moderate commitment problem, an increase in the fraction of flexible consumers in the high-valuation market unambiguously increases the aggregate search friction that is incurred, but may at the same time decrease the probability that the discounter prices above its local consumers' valuation in equilibrium, reducing expected deadweight loss. Which effect dominates depends on the exact parameter constellation.

Finally, in the most important extension of the baseline model, I study the effects of informative advertising on firms' equilibrium behavior. In particular, I investigate whether the discounter's commitment problem is mitigated if it can perfectly advertise (and thereby, commit to) a deterministic price at small but positive advertising cost. Somewhat surprisingly, it turns out that costly advertising may often cause more harm than good. In particular, conditions are identified under which (i) the discounter advertises a price *lower* than its local consumers' valuation, thereby discouraging the regular firm from pricing aggressively and inducing wasteful search behavior, and (ii) the discounter advertises a price *higher* than its local consumers' valuation, leading all of its local consumers to drop out of the market deterministically. The only case where informative advertising and price-commitment by the discounter is privately optimal and *may* enhance social welfare is if the firm faces a severe commitment problem, but finds it optimal to price-advertise its local consumers' valuation. On the other hand, if the firm faces a moderate commitment problem, it never finds it optimal to engage in costly advertising, but this would be welfare-improving in those cases where the firm's advertising cost is not too high.

The remainder of this article is organized as follows. The paragraph below discusses related literature in more detail. In Section 2, the model setup is introduced. The different equilibria of the baseline game are analyzed in Section 3. Section 4 is concerned with social welfare. Comparative statics with respect to several equilibrium objects are provided in Section 5. An extension to costly advertising and price-commitment is outlined in Section 6. Section 7 demonstrates that the principal mechanism which leads to search and price dispersion also extends to the case of supply-side heterogeneities. Section 8 concludes and points out some potential directions for future research. Technical proofs, mostly related to existence of the characterized equilibria, are relegated to Appendix A. Online Appendix B establishes uniqueness of these equilibria.

Related Literature

This article closely relates to research on price dispersion and consumer search under asymmetric market configurations. An important early contribution was given by Narasimhan (1988), who extends Varian's (1980) classic model of sales (where firms have symmetric loyal consumer bases, and compete in prices for a perfectly price-sensitive mass of "shoppers") to the case of asymmetric shares of loyal consumers across (duopolistic) firms. However, in contrast to the present work, (sequential) search is ruled out, as consumers are either perfectly informed about all prices, or are fully captive to their preferred firm. The rationale for price dispersion thus differs greatly from the showcased model.⁹

More similar in spirit is a recent paper by Astorner-Figari and Yankelevich (2014), who consider a setup in which duopolistic competitors differ in their number of local (captive) consumers.¹⁰ As in my model, these consumers do not directly observe the price of the outside firm, but may obtain this information at positive cost. The authors show that in the unique equilibrium of this game, both firms play mixed strategies, but the price distribution of the firm with the larger mass of local consumers first-order stochastically dominates the one of its rival. The major difference between their model and the present work is that price dispersion is driven by an atom of shoppers, rather than by local market differences. Proper search does not occur in equilibrium, and eliminating the atom of shoppers leads to the Diamond result. Moreover, the firm with lower average prices cannot face a commitment problem, as non-local consumers with positive search cost never visit it.

Other related papers that explicitly account for market asymmetries in a search framework are given by Burdett and Smith (2010) and Kuniavsky (2014). In Burdett and Smith (2010), one dominant firm with a continuum of retail outlets competes with a fringe mass of atomistic sellers, and consumers employ a noisy search technology in the spirit of Burdett and Judd (1983). Kuniavsky (2014) extends the standard sequential search model of Stahl (1989) to allow for heterogeneously sized sellers (where sellers with more outlets have a higher probability of being sampled first). In both of these papers, price dispersion is driven by different forces than in the present model. In particular, directed search to a perceived discount store, which tends to offer lower prices due to local market characteristics, does not occur.

⁹An interesting follow-up paper by Deneckere et al. (1992) contrasts the equilibrium of Narasimhan (1988) with the case of exogenous and endogenous price-leadership by one of the firms in the model. Some of their results under price-leadership resemble those of the present paper's section on informative advertising.

¹⁰See also Astorner-Figari and Yankelevich (2011) for a more detailed, earlier working paper version.

Since all consumers in my model face positive search costs, yet prices are dispersed in equilibrium, the paper also relates to a small literature on resolving the Diamond paradox under strictly positive search costs. Examples include Bagwell and Ramey (1992), who resolve the paradox by consumers making repeat purchases, and Rhodes (2014), who avoids the problem by considering multi-product retailers.

The model extension to informative advertising of Section 6 is connected to a growing literature on the interplay of consumer search and advertising, e.g. as given by Robert and Stahl (1993), Janssen and Non (2008), and Janssen and Non (2009).¹¹ Most closely related is the work by Bester and Petrakis (1995), who analyze costly advertising by locally separated duopolists. However, due to their perfectly symmetric configuration with identical local consumer populations and market sizes, advertising is never undertaken to resolve a firm’s commitment problem of charging low prices.¹²

Finally, an older strand of literature combines location models in the spirit of Hotelling (1929) with imperfectly informed consumers (see, e.g. Gabszewicz and Garella (1986, 1987)). From today’s perspective, the search technology and equilibrium concepts used in these models are non-standard (e.g., consumers initially know the average price in the market, while their beliefs about unobserved prices need not be correct in equilibrium), and an important focus lies on establishing conditions for equilibrium existence in pure pricing-strategies. The main mechanism for search that is portrayed in this paper, the presence of local market heterogeneities, is not explicitly considered.

2 Model Setup

Consider the following market. There are two spatially separated local submarkets H (“high valuation”) and L (“low valuation”) that host one risk-neutral firm each, labeled and indexed by their locations. The firms compete in prices p_H , p_L and sell a single homogeneous product that is offered in their respective market only. The firms’ identical, constant unit costs are normalized to zero.

A total mass $\alpha \in (0, 1)$ of consumers live in H , whereas the remaining mass $1 - \alpha$ live in L . The consumers’ valuations for the homogeneous product are identical within the local markets.

¹¹See also Butters (1977) for a seminal contribution on informative advertising, albeit without allowing for (active) consumer search.

¹²Under different setups, the role of advertising as commitment device to resolve the Diamond paradox has been analyzed. A useful discussion can be found in Anderson and Renault (2006).

That is, all consumers that live in H have unit demand up to a maximum valuation of v_H , whereas all consumers that live in L have unit demand up to a lower maximum valuation of $v_L < v_H$.

In the baseline model, each consumer only observes the price posted by the firm in her home market. However, some consumers are flexible in the sense that they can travel to the other market at positive cost, purchasing there if the observed price is lower. For expositional simplicity, assume that the L -market consumers are fully captive in the sense that they will never visit H .¹³ Given p_L , they either buy directly (if $p_L \leq v_L$), or not at all. In contrast, some consumers in H have the possibility to search. Being heterogeneous with respect to their search behavior, a fraction $1 - \beta$ of H -consumers is captive as well. Given p_H , they either buy directly (if $p_H \leq v_H$), or not at all. On the other hand, a fraction β of H -consumers are (potential) searchers: at a travel cost $s \in (0, v_H - v_L)$,¹⁴ they can visit market L and return, purchasing on the way if the observed price is lower. In all of what follows, I will refer to these potentially searching consumers as *flexible H-consumers*. Note that in the model, searching consumers have to return to their home market after observing the other firm's price. While intuitive, this setup is also consistent with the usual assumption of *free recall* in search models.

For the formal analysis, it is moreover necessary to specify the following tie-breaking rules: (i) if flexible H -consumers observe a price p_H that keeps them indifferent between visiting L , or buying directly (given their beliefs about L 's pricing), they will buy directly at H (ii) if flexible H -consumers who indeed search observe a price p_L that is equal to p_H , they buy on the way at L .

The timing of the game is as follows. First, firms H and L simultaneously choose their prices p_H and p_L which are then fixed for the rest of the game. Second, each consumer observes her home market's price, and all captive consumers buy immediately as long as the observed price does not exceed their valuation. Third, the mass $\alpha\beta$ flexible H -consumers observe p_H and decide whether to visit L or not, given their beliefs and travel cost s . If not, they purchase at H , provided that $p_H \leq v_H$. If they visit the L -market, they incur the travel cost s , observe L 's price, and optimally buy at the cheaper firm (given that its price does not exceed their valuation).

¹³This assumption does not affect any of the results and is only made to streamline the model setup. In Lemma 2 in Appendix A, I show that as long as there is no positive mass of L -consumers with zero travel costs, L -consumers will never search in equilibrium, irrespective of their search-cost distribution.

¹⁴For $s \geq v_H - v_L$, the unique equilibrium of the game is given by the uninteresting case in which H prices at v_H , L prices at v_L , and no consumers search.

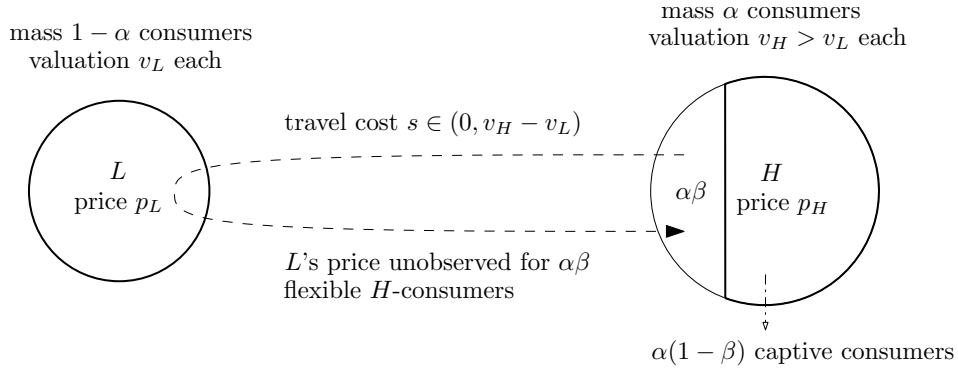


Figure 1: Depiction of the analyzed market.

In the next section, I will solve for the equilibrium of the described game given the parameters v_H , v_L , α , β , and s .¹⁵ Note that since this is a game of imperfect information, the $\alpha\beta$ flexible H -consumers will have to form beliefs about L 's unobserved price in order to make their search decision. I restrict these consumers' out-of-equilibrium beliefs when observing an off-equilibrium price p_H that is never played in equilibrium in such a way that their beliefs about L 's pricing are not affected (passive beliefs). As is usual, the flexible H -consumers' beliefs need to be correct in equilibrium.

Figure 1 provides a graphical summary of the described market structure.

3 Equilibrium Analysis

The game's different types of equilibria are characterized by the following sequence of propositions. Uniqueness is established in Online Appendix B.

Proposition 1. *If $\beta > \bar{\beta} := 1 - \frac{v_L + s}{v_H} \in (0, 1)$, the unique equilibrium of the game is in pure strategies such that $p_H^* = v_L + s \in (v_L, v_H)$, $p_L^* = v_L$, and all $\alpha\beta$ flexible H -consumers purchase in H . H 's equilibrium profit is given by $\Pi_H^* = (v_L + s)\alpha$, whereas L 's equilibrium profit is given by $\Pi_L^* = v_L(1 - \alpha)$.*

Proof. (Existence) The proposed equilibrium implies the above firm profits of Π_H^* and Π_L^* , as can easily be calculated. Clearly, given that H prices at $v_L + s$ and the flexible H -consumers do not search, L can do no better than to price at v_L (as pricing higher than v_L would induce all L -consumers to exit the market, and pricing lower than v_L induces no search, as it is unobserved by

¹⁵Clearly, either v_H , v_L or s can be normalized to some arbitrary constant, e.g., $v_H = 1$ (such that v_L and s can be expressed as fractions of v_H). For expositional reasons (and in order to allow for comparative statics with respect to v_H), I will not do so throughout the paper.

the flexible H -consumers). On the other hand, H 's best possible deviation is to increase its price to v_H , lose all $\alpha\beta$ flexible H -consumers, but fully exploit its captive consumers. This gives rise to a maximal deviation profit of $\Pi_H^{dev^*} = v_H\alpha(1 - \beta)$. It is easy to check that $\beta > \bar{\beta} := 1 - \frac{v_L + s}{v_H}$ implies $\Pi_H^{dev^*} < \Pi_H^*$.

□

Example 1. Let $v_H = 200$, $v_L = 100$, $s = 10$.¹⁶ It immediately follows that $\bar{\beta} = 0.45$. Hence, for $\beta > 0.45$, no matter what α , the unique equilibrium of the game (given the specified v_H , v_L and s) is such that $p_L^* = 100$, $p_H^* = 110$, and no consumers search. This gives rise to deterministic firm profits of $\Pi_L^* = 100(1 - \alpha)$ and $\Pi_H^* = 110\alpha$.

The intuition to Proposition 1 is straightforward: if sufficiently many H -consumers are flexible, H finds it worthwhile to fight for these consumers and discourage them from search. The optimal way for H to achieve this is by charging the maximal markup over L 's price which deters the flexible H -consumers from search: $p_L^* + s$. Note moreover that $p_L^* < v_L$ cannot be part of an equilibrium. If it was, H would either find it optimal to charge $p_L^* + s < v_H$ (if p_L^* is sufficiently close to v_L) or the highest possible price v_H (if p_L^* is small). In either case, L could achieve a higher profit by increasing its price a little, as this would not decrease its demand. Hence, for a large β , the only possible equilibrium is such that $p_L^* = v_L$, $p_H^* = v_L + s$, and no search occurs.

Proposition 2. If $\beta < \bar{\beta}$ and $\alpha \leq \underline{\alpha}(\beta) := \frac{v_L}{\beta(v_H - v_L) + v_L} \in (\alpha_{min}, 1)$, where $\alpha_{min} = \frac{v_H v_L}{v_H^2 - (v_L + s)(v_H - v_L)} \in (0, 1)$, the unique equilibrium of the game is in pure strategies such that $p_H^{**} = v_H$, $p_L^{**} = v_L$, and all $\alpha\beta$ flexible H -consumers search and purchase in L .¹⁷ H 's equilibrium profit is given by $\Pi_H^{**} = v_H\alpha(1 - \beta)$, whereas L 's equilibrium profit is given by $\Pi_L^{**} = v_L(1 - \alpha + \alpha\beta)$.

Proof. (Existence) The proposed equilibrium implies the above firm profits of Π_H^{**} and Π_L^{**} , as can easily be calculated. From each firm's perspective, there is a unique optimal deviation to this. First, H can reduce its price to $v_L + s$, discourage the $\alpha\beta$ flexible H -consumers from leaving, and make an optimal deviation profit of $\Pi_H^{dev^{**}} = (v_L + s)\alpha$. However, by the reverse logic of Proposition 1, this is not profitable if $\beta < \bar{\beta}$. Second, L can increase its price to v_H , lose all

¹⁶In order to allow for a meaningful comparison of equilibria, this parameter combination will be used in most subsequent examples (for varying values of α and β). It should be noted though that as long as $s < v_H - v_L$, for any triplet (v_H, v_L, s) , all different types of equilibria can be found in (α, β) space.

¹⁷In the zero-measure event where $\beta = \bar{\beta}$, given that $\alpha \leq \underline{\alpha}(\bar{\beta}) = \alpha_{min}$, the equilibria of Propositions 1 and 2 coexist (see Figure 2 below for an illustration). This is because for $\beta = \bar{\beta}$, H is indifferent between discouraging its local flexible consumers from search (by pricing at $v_L + s$) or maximally exploiting its captive consumers while letting go of its flexible consumers (by pricing at v_H). Note though that the equilibrium of Proposition 1 is less plausible, as it gives rise to a strictly lower equilibrium profit of L . Hence, L could profitably "bribe" H to play the more favorable (from L 's perspective) equilibrium strategy of Proposition 2.

L -consumers who drop out of the market, but fully exploit the $\alpha\beta$ searching H -consumers, who expect to find a price of $p_L^{**} = v_L$. This gives rise to an optimal deviation profit of $\Pi_L^{dev**} = v_H\alpha\beta$. It is easy to see that this optimal deviation is not profitable if $\alpha \leq \frac{v_L}{\beta(v_H - v_L) + v_L} = \underline{\alpha}(\beta)$. \square

Example 2. Let $v_H = 200$, $v_L = 100$, $s = 10$. Then $\bar{\beta} = 0.45$ and $\underline{\alpha}(\beta) = \frac{1}{1+\beta}$. Hence, if both $\beta < 0.45$ and $\alpha \leq \frac{1}{1+\beta}$, the unique equilibrium of the game is such that $p_L^* = 100$, $p_H^* = 200$, and all $\alpha\beta$ flexible consumers search and buy at L . This gives rise to equilibrium profits of $\Pi_L^{**} = 100(1 - \alpha + \alpha\beta)$ and $\Pi_H^{**} = 200\alpha(1 - \beta)$.

Intuitively, $\beta < \bar{\beta}$ is simply the converse of the condition in Proposition 1: if sufficiently few H -consumers are flexible, H would not even find it worthwhile to fight for them if L priced at v_L deterministically. Instead, H prefers to fully exploit its captive consumers by pricing at v_H , and accepts the fact that all its local flexible consumers will buy at the other firm.

More interesting is the other condition, $\alpha \leq \underline{\alpha}(\beta)$, which rules out that L has a profitable deviation. Clearly, given that H prices at v_H deterministically and does not fight for its flexible consumers, an expectation of $p_L^* = v_L$ by the flexible H -consumers would induce them to search. But then, if the H -market is sufficiently important in size (α is large), L faces a *commitment problem* which destroys the proposed pure-strategy equilibrium. Namely, rather than to also serve its own local consumers at v_L , L would prefer to exploit the flexible consumers' beliefs (of finding $p_L^* = v_L$ in L) and charge them the highest possible price (v_H) for which they do not return to H . This is the case if $\alpha > \underline{\alpha}(\beta)$.

The outlined commitment problem and the tension to resolve it is what generates the mixed-strategy equilibria which will be discussed below. Figure 2 illustrates the different equilibrium regions in (α, β) -space.

Proposition 3. If $\beta < \bar{\beta}$ and $\alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)]$, where $\bar{\alpha}(\beta) := \frac{v_L}{(1-\beta)\left\{v_L + \frac{v_H\beta[v_H(1-\beta) - v_L]}{v_H(1-\beta) - v_L - \beta s}\right\}} \in (\underline{\alpha}(\beta), 1)$,¹⁸ the unique equilibrium of the game is in mixed strategies such that¹⁹

- H prices at v_H with probability $q_H^* := \frac{v_L(1-\alpha+\alpha\beta)}{v_H\alpha\beta} \in (0, 1)$.
- With probability $1 - q_H^*$, H samples prices continuously from the interval $[p, v_H]$, where $p := \frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta} \in (v_L, v_H)$, following the cumulative distribution function $F_H(p) := 1 - \frac{v_L(1-\alpha+\alpha\beta)}{v_H\alpha\beta - v_L(1-\alpha+\alpha\beta)}(v_H/p - 1)$.

¹⁸While $\bar{\alpha}(\beta)$ always falls in this range (with $\bar{\alpha}(0) = 1$ and $\bar{\alpha}(\bar{\beta}) = \underline{\alpha}(\bar{\beta}) = \alpha_{min}$), it can be non-monotonic in β for certain combinations of v_H , v_L and s .

¹⁹Note that unlike the case where $\beta = \bar{\beta}$, there is no multiplicity of equilibria for $\alpha = \underline{\alpha}(\beta)$. This is because the equilibria of Propositions 2 and 3 coincide for $\alpha = \underline{\alpha}(\beta)$. See the subsection on mixed-strategy equilibria in Section 5 for the corresponding calculation.

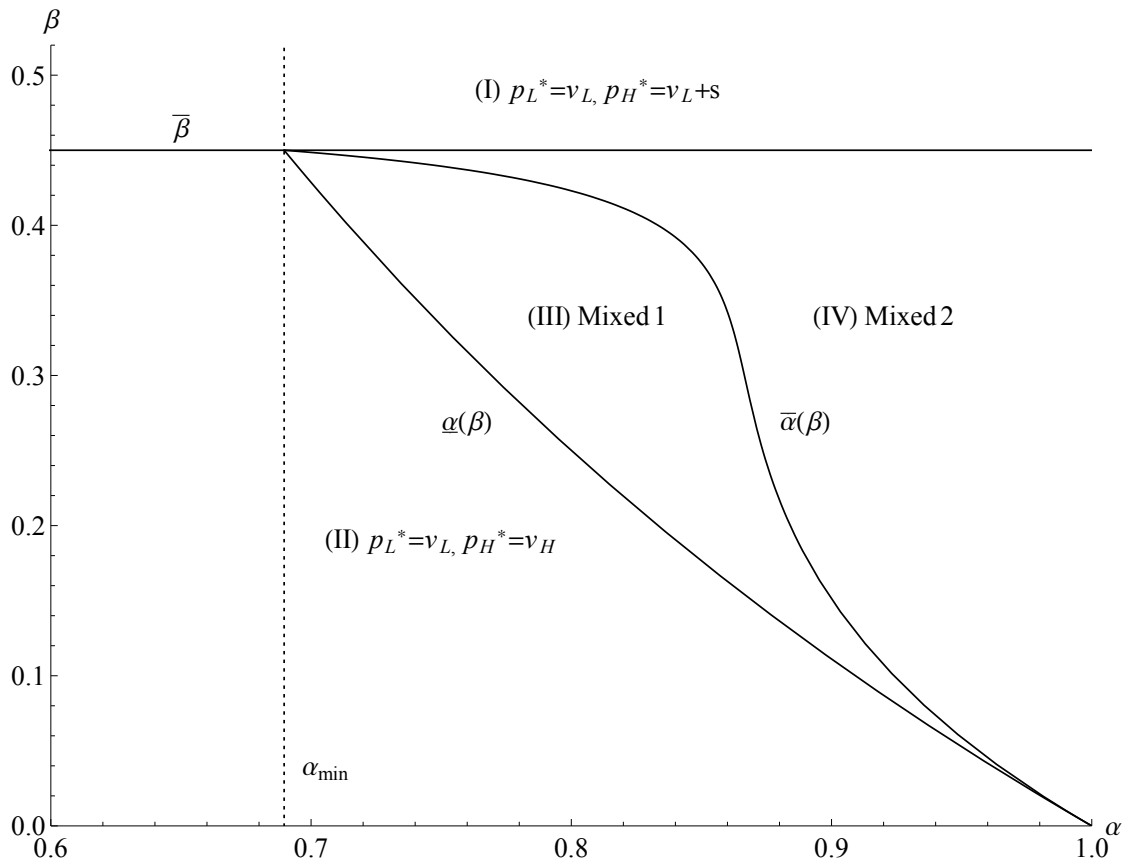


Figure 2: Equilibrium regions for $v_H = 200, v_L = 100, s = 10$.

- L prices at v_L with probability $q_L^* := \frac{1}{\beta} - \frac{v_H \alpha (1 - \beta)}{v_L (1 - \alpha + \alpha \beta)} \in (0, 1)$.
- With probability $1 - q_L^*$, L samples prices continuously from $[\underline{p}, v_H]$ (the same interval as H), following the same cumulative distribution function $F_L(p) := F_H(p)$.
- As $\underline{p} > \rho$, where the flexible H -consumers' reservation price ρ solves $q_L^*(\rho - v_L) = s$, all $\alpha\beta$ flexible H -consumers search initially. However, given that H prices at $p_H \in [\underline{p}, v_H)$, they return with probability $(1 - q_L^*)(1 - F_L(p_H))$, as in those cases L charges a higher price than H .
- As in the case of Proposition 2, H 's equilibrium profit is given by $\Pi_H^{**} = v_H \alpha (1 - \beta)$, whereas L 's equilibrium profit is given by $\Pi_L^{**} = v_L (1 - \alpha + \alpha \beta)$.

Proof. See Appendix A. □

The following example illustrates an equilibrium of the above type.

Example 3. Let $v_H = 200$, $v_L = 100$, $s = 10$, $\alpha = 0.9$, $\beta = 0.14$. Then $\bar{\beta} = 0.45$, $\underline{\alpha} \approx 0.877$ and $\bar{\alpha} \approx 0.905$. As all requirements for Proposition 3 are fulfilled, the unique equilibrium of the game must be characterized by it. Plugging the model parameters into the relevant equations, one finds that $\rho^* = 134.095$, $q_H^* \approx 0.8968$, $q_L^* \approx 0.2933$, $\Pi_H^{**} = 154.8$, and $\Pi_L^{**} = 22.6$. Moreover, $\underline{p} = 179.365$, and the (identical) cumulative distribution functions $F_H(\cdot)$ and $F_L(\cdot)$ can easily be calculated. Figure 3 depicts the described equilibrium graphically.

The intuition to Proposition 3 is as follows. Because the H -market is large compared to L ($\alpha > \underline{\alpha}(\beta)$), firm L cannot commit to charging v_L deterministically if the flexible H -consumers were to search (after facing $p_H = v_H$), as it strictly prefers to fully exploit these consumers' beliefs of finding $p_L = v_L$ by charging v_H . However, this cannot be an equilibrium, because (a) given $p_L = v_H$, the flexible H -consumers would clearly prefer not to search, and (b) even if these consumers were to search, H would have a profitable deviation by marginally undercutting v_H (say, by pricing at $v_H - \epsilon$), which would lead all flexible H -consumers to return to H after observing $p_L = v_H$. Consequently, L would also have a profitable deviation of pricing marginally below $v_H - \epsilon$, and so on. It turns out that this mutual undercutting argument gives rise to the mixed-strategy equilibrium characterized in the proposition: both L and H price at their local consumers' valuation with positive probability mass, but they also "fight" for the flexible H -consumers in those cases where L prices above v_L . In some sense, in order to mitigate L 's incentive to always exploit the searchers, H alters its strategy in such a way that it becomes

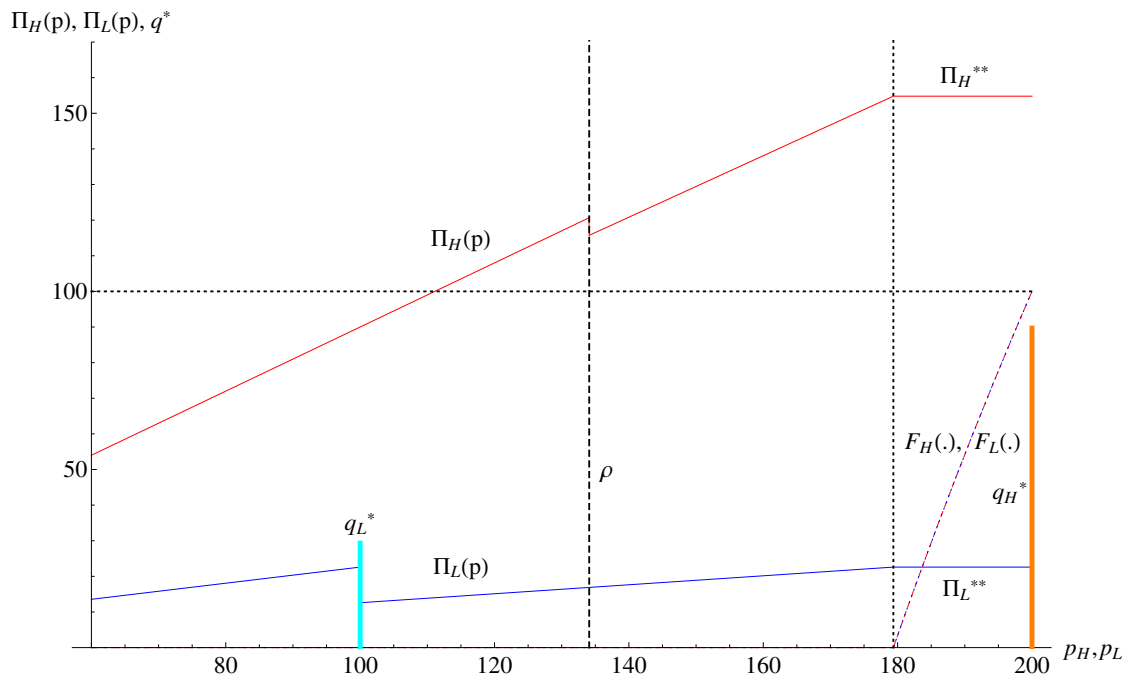


Figure 3: Expected firm profits and equilibrium strategies for $v_H = 200$, $v_L = 100$, $s = 10$, $\alpha = 0.9$, $\beta = 0.14$. The vertical axis can both be interpreted as monetary units (for $\Pi_H(p)$ and $\Pi_L(p)$) and percentage points (for q_H^* , q_L^* , $F_H(\cdot)$, $F_L(\cdot)$).

harder for L to sell to the searching H -consumers if it prices above v_L . H achieves this by spreading positive probability mass on some interval below v_H , implying that L is indifferent between choosing v_L or any price larger than v_L that lies in that interval.

Since firm L charges prices higher than v_L with positive probability in equilibrium, this implies that low-valuation consumers in the discount market are excluded from buying probabilistically. Hence, the characterized equilibrium is in line with the empirical finding that low-income consumers tend to suffer from a poor access to certain product markets, as discussed in the introduction. This continues to hold for the last type of equilibrium to be characterized below.

Proposition 4. *If $\beta < \bar{\beta}$ and $\alpha \in (\bar{\alpha}(\beta), 1)$, the unique equilibrium of the game is in mixed strategies such that²⁰*

- H prices at v_H with probability $q_{H,v_H}^* := \frac{(1-\alpha)(1-\beta)v_L[(1-\beta)v_H-v_L]}{\alpha\beta\{[(1-\beta)v_H-v_L]^2+v_L\beta s\}} \in (0, 1)$ and at $\rho^* := v_H(1-\beta)$ with probability $q_{H,\rho}^* := 1 - \frac{v_L(1-\alpha)}{\alpha\beta\left(\frac{v_H(1-\beta)[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s} - v_L\right)} \in (0, 1)$, where $q_{H,v_H}^* + q_{H,\rho}^* < 1$.
- With probability $1 - q_{H,v_H}^* - q_{H,\rho}^*$, H samples prices continuously from the interval $[\underline{p}, v_H]$, where $\underline{p} := \frac{v_H(1-\beta)[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s}$, following the cumulative distribution function $G_H(p) := 1 - \frac{(1-\beta)[v_H(1-\beta)-v_L]}{\beta[v_H(1-\beta)-v_L-s]}(v_H/p - 1)$.
- L prices at v_L with probability $q_{L,v_L}^* := \frac{s}{v_H(1-\beta)-v_L} \in (0, 1)$.
- With probability $1 - q_{L,v_L}^*$, L samples prices continuously from $[\underline{p}, v_H]$ (the same interval as H), following the same cumulative distribution function $G_L(p) := G_H(p)$.
- As H prices at the flexible H -consumers' reservation price ρ^* with positive probability $q_{H,\rho}^*$, these consumers will only search if H prices at or above $\underline{p} > \rho^*$, which happens with probability $1 - q_{H,\rho}^*$. However, given that H prices at $p_H \in [\underline{p}, v_H]$, they return with probability $(1 - q_{L,v_L}^*)(1 - G_L(p_H))$, as in those cases L charges a higher price than H .
- H 's equilibrium profit is given by $\Pi_H^{**} = v_H\alpha(1-\beta)$, whereas L 's equilibrium profit is given by $\Pi_L^{***} := \frac{(1-\alpha)(1-\beta)v_Hv_L[(1-\beta)v_H-v_L]}{[(1-\beta)v_H-v_L]^2+v_L\beta s}$.

Provided that $\alpha > \alpha_{min}$, the above constitutes an equilibrium whenever β lies sufficiently close below $\bar{\beta}$.

²⁰Note again that unlike the case where $\beta = \bar{\beta}$, there is no multiplicity of equilibria for $\alpha = \bar{\alpha}(\beta)$. This is because the equilibria of Propositions 3 and 4 coincide for $\alpha = \bar{\alpha}(\beta)$. See the subsection on mixed-strategy equilibria in Section 5 for the corresponding calculation. On the other hand, the equilibria of Propositions 1 and 4 coexist if $\beta = \bar{\beta}$ and $\alpha > \bar{\alpha}(\beta) = \alpha_{min}$ (see Figure 2 above for an illustration). As in the case where Propositions 1 and 2 coexist (if $\beta = \bar{\beta}$ and $\alpha \leq \alpha_{min}$), L makes a strictly higher profit if the equilibrium of Proposition 4 is played.

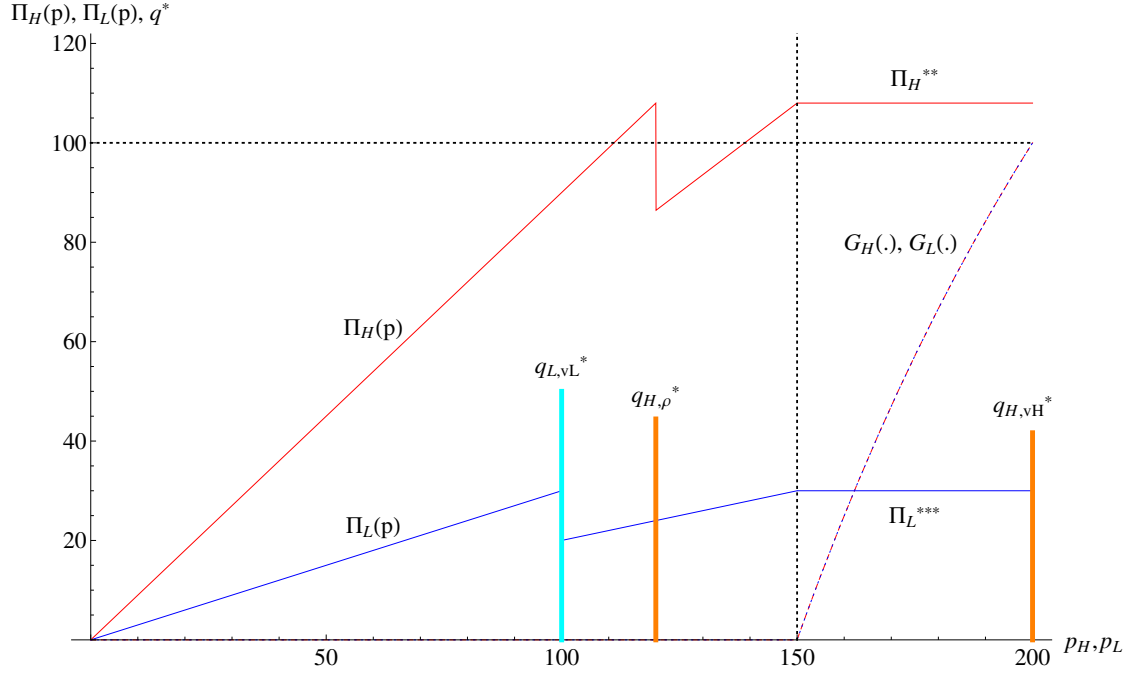


Figure 4: Expected firm profits and equilibrium strategies for $v_H = 200$, $v_L = 100$, $s = 10$, $\alpha = 0.9$, $\beta = 0.4$. The vertical axis can both be interpreted as monetary units (for $\Pi_H(p)$ and $\Pi_L(p)$) and percentage points (for q_{H,v_H}^* , $q_{H,\rho}^*$, q_{L,v_L}^* , $G_H(\cdot)$, $G_L(\cdot)$).

Proof. See Appendix A. □

Again, the example below showcases an equilibrium of the above type.

Example 4. Let $v_H = 200$, $v_L = 100$, $s = 10$, $\alpha = 0.9$, $\beta = 0.4$. Then $\bar{\beta} = 0.45$, $\underline{\alpha} \approx 0.714$ and $\bar{\alpha} \approx 0.833$. Hence, all requirements for Proposition 4 are fulfilled, which implies that the unique equilibrium of the game must be characterized by it. Plugging the model parameters into the relevant equations, one finds that $\rho^* = 120$, $q_{H,v_H}^* = 0.41\dot{6}$, $q_{H,\rho}^* = 0.4$, $q_{L,v_L}^* = 0.5$, $\Pi_H^{**} = 108$, and $\Pi_L^{***} = 30$. Moreover, $\underline{p} = 150$, and the (identical) cumulative distribution functions $G_H(\cdot)$ and $G_L(\cdot)$ can easily be calculated. Figure 4 depicts the described equilibrium graphically.

The intuition to Proposition 4 is similar to the one of Proposition 3. The crucial difference is that for $\alpha > \bar{\alpha}(\beta)$, the H -market is so large relative to L that firm L 's commitment problem becomes severe. This means that in order to reduce L 's incentive to charge prices above v_L , it is not sufficient for H to solely put positive probability mass directly below v_H . Instead, L can only be made indifferent between charging v_L or exploiting the searching H -consumers if the flexible H -consumers do not always search. H achieves this by additionally putting positive

probability mass on the flexible H -consumers' reservation price ρ^* . A direct implication is that L cannot even be certain to exploit the flexible H -consumers if it prices at \underline{p} (the lowest price in its pricing range above v_L), as with positive probability, the flexible H -consumers do not search at all. This reduction in L 's profitability of pricing above v_L is able to resolve the tension that is created by L 's severe commitment problem.

Note that the equilibrium of Proposition 4, particularly the pricing strategy of firm H , is consistent with empirical evidence that retail price distributions tend to be bimodal, with prices fluctuating between a “regular” high price and a low “sales” price, and little mass between (see Hosken and Reiffen (2004), Pesendorfer (2002)). The present model provides a complimentary explanation to that of Garcia et al. (2015), who generate a two-point price distribution by introducing costly retailer search for manufacturers' offers.

4 Welfare

Since the consumers have inelastic demand up to a maximum valuation of v_H in H (where a total mass α of consumers reside) and up to v_L in L (where the remaining $1 - \alpha$ consumers reside), it is obvious that the maximal surplus which can be achieved in the whole market is given by

$$W^{max} := \alpha v_H + (1 - \alpha)v_L. \quad (1)$$

Considering the different equilibria which were outlined in Section 3, there are two possible sources of welfare loss in the market. First, wasteful travel expenditures to the extent of $\alpha\beta s$ can be incurred if the $\alpha\beta$ flexible H -consumers search. And second, the L -market surplus of $(1 - \alpha)v_L$ is lost in those cases where L prices above v_L , as this leads all L -consumers to drop out of the market. The following proposition then follows straightforwardly from Propositions 1 to 4.

Proposition 5. *The total loss of welfare in the market is given by*²¹

$$W_{loss} := \begin{cases} \alpha\beta s & \text{if } \beta < \bar{\beta} \text{ and } \alpha \leq \underline{\alpha}(\beta) \\ \alpha\beta s + (1 - q_L^*)(1 - \alpha)v_L & \text{if } \beta < \bar{\beta} \text{ and } \alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)] \\ \alpha\beta s(1 - q_{H,\rho}^*) + (1 - q_{L,v_L}^*)(1 - \alpha)v_L & \text{if } \beta < \bar{\beta} \text{ and } \alpha \in (\bar{\alpha}(\beta), 1) \\ 0 & \text{if } \beta > \bar{\beta}. \end{cases} \quad (2)$$

Note that the aggregate consumers surplus for each parameter region can easily be calculated as $CS = W^{max} - \Pi_H^{[*]} - \Pi_L^{[*]} - W_{loss}$, where $\Pi_i^{[*]}$ denotes the equilibrium profit of firm $i \in \{H, L\}$ in the respective parameter region. For $\beta > \bar{\beta}$, it holds that $CS = \alpha(v_H - v_L - s)$, whereas for $\beta < \bar{\beta}$ and $\alpha \leq \underline{\alpha}(\beta)$, it holds that $CS = \alpha\beta(v_H - v_L - s)$. The expressions for the aggregate consumer welfare if $\beta < \bar{\beta}$ and $\alpha > \underline{\alpha}(\beta)$ are cumbersome and will not be reported here.²²

Comparative statics with respect to social welfare (and other equilibrium objects) will be provided in the subsequent section.

5 Comparative Statics

Equilibrium Regions in (α, β) -Space

First, note that the pure-strategy regions of the game are those with either $\beta \geq \bar{\beta} = 1 - \frac{v_L + s}{v_H}$ (where H fights for its flexible consumers), or $\beta < \bar{\beta}$ and $\alpha \leq \underline{\alpha}(\beta) = \frac{v_L}{\beta(v_H - v_L) + v_L}$ (where H prices at v_H and lets its flexible consumers purchase at L).

The former region evidently becomes larger in (α, β) -space if either v_L or s increases relative to v_H (v_H decreases relative to v_L and s). The intuition is simple: in order to discourage the flexible H -consumers from search, H cannot price higher than $v_L + s$. But if $v_L + s$ increases (v_H decreases), H 's loss when decreasing its price from v_H to $v_L + s$ decreases. Hence, the region where the firm find it's worthwhile to fight for its flexible consumers becomes larger.

The other pure-strategy region cannot be ranked in (α, β) -space if v_L increases (v_H decreases). This is because there are two countervailing effects at work. First, an increase of v_L (decrease of v_H) makes it more worthwhile for H to fight for its flexible consumers (following the above logic), which cuts down the equilibrium region from above by reducing the admissible set of β 's

²¹If $\beta = \bar{\beta}$, the equilibrium welfare loss depends on which equilibrium is played. It is zero if H plays $v_L + s$, whereas it is $\alpha\beta s$ if H plays v_H (for $\alpha \leq \underline{\alpha}(\bar{\beta})$), or $\alpha\beta s(1 - q_{H,\rho}^*) + (1 - q_{L,v_L}^*)(1 - \alpha)v_L$ if H plays the mixed-strategy equilibrium of Proposition 4 (for $\alpha > \underline{\alpha}(\bar{\beta}) = \bar{\alpha}(\bar{\beta})$).

²²They can be obtained from the author upon request.

($\bar{\beta}$ decreases). But second, a larger v_L (smaller v_H) also reduces firm L 's commitment problem: if v_L is closer to v_H , there is less of an incentive to exploit the searching consumers' beliefs (of finding v_L) and deviating to v_H . Hence, the range of admissible α 's increases ($\underline{\alpha}(\beta)$ increases). Since the latter effect is not present for increases in s , the discussed pure-strategy equilibrium region unambiguously shrinks when s increases.

The combined mixed-strategy region (with either a moderate or severe commitment problem by L) unambiguously shrinks when v_L or s increases relative to v_H (v_H decreases relative to v_L and s). This is because H becomes more willing to fight for its flexible consumers if either v_L or s increases (v_H decreases), and also L 's commitment problem is softened as v_L increases (v_H decreases). Moreover, because $\bar{\alpha}(\beta)$ is strictly decreasing in s (for the relevant region where $\beta < \bar{\beta}$), while $\underline{\alpha}(\beta)$ does not depend on s , for increasing s the range of α 's where L faces a moderate commitment problem becomes unambiguously smaller.

Mixed-Strategy Equilibria

Case (1): $\beta < \bar{\beta}$ and $\alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)]$.

Consider the limit behavior of q_H^* and q_L^* first. Inserting $\underline{\alpha}(\beta)$ easily shows that $\lim_{\alpha \downarrow \underline{\alpha}(\beta)} q_H^* = \lim_{\alpha \downarrow \underline{\alpha}(\beta)} q_L^* = 1$. Moreover, the distribution functions $F_H(\cdot)$ and $F_L(\cdot)$ become degenerate, with $\underline{p} = v_H$. All of this should not be surprising, as for every $\alpha < \underline{\alpha}(\beta)$ (with $\beta < \bar{\beta}$), the same pure-strategy equilibrium is played. Hence, there is no discontinuity of the equilibrium strategies played around $\alpha = \underline{\alpha}(\beta)$.

On the other hand, it is straightforward to derive that $\lim_{\alpha \uparrow \bar{\alpha}(\beta)} q_H^* = \frac{(1-\beta)[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s}$, whereas $\lim_{\alpha \uparrow \bar{\alpha}(\beta)} q_L^* = \frac{s}{v_H(1-\beta)-v_L}$. Also, the distribution functions $F_H(\cdot)$ and $F_L(\cdot)$ do not become degenerate around $\bar{\alpha}(\beta)$.

By properly manipulating q_H^* , q_L^* , \underline{p} , and $F_H(\cdot) = F_L(\cdot)$ such that one parameter gets isolated (e.g., $q_H^* = \frac{v_L(\frac{1}{\alpha}-1+\beta)}{v_H\beta}$), one can see directly that q_H^* , and \underline{p} are strictly decreasing in α and β , while q_L^* is only strictly decreasing in α . The comparative statics of q_L^* with respect to β are ambiguous.²³ At the same time, $F_H(\cdot)$ and $F_L(\cdot)$ are strictly increasing in α and β . Hence, if L 's commitment problem gets more severe (α increases), in equilibrium less probability mass is put on the mass points q_H^* and q_L^* , while the firms' pricing also gets more aggressive (in the sense of first order stochastic dominance) when sampling prices from their continuous range. One interpretation is that L tries to exploit the searching consumers more due to its larger

²³While q_L^* is typically decreasing in β , numerical examples can be provided where q_L^* increases with β for α close to $\bar{\alpha}(\beta)$.

commitment problem, but this is counterbalanced by H , which makes it harder for L to “steal” H ’s flexible consumers while charging a price higher than v_L .

None of the discussed equilibrium objects depends on s , while it is easy to check that q_H^* , \underline{p} , q_L^* ($F_H(\cdot)$, $F_L(\cdot)$) strictly increase (decrease) in v_L . Finally, it is straightforward to establish that q_H^* , q_L^* , $F_H(\cdot)$ and $F_L(\cdot)$ strictly decrease in v_H , while \underline{p} is constant in v_H .

Case (2): $\beta < \bar{\beta}$ and $\alpha > \bar{\alpha}(\beta)$.

Consider the limit behavior of q_{H,v_H}^* , $q_{H,\rho}^*$ and q_{L,v_L}^* first. Inserting $\bar{\alpha}(\beta)$ shows that $\lim_{\alpha \downarrow \bar{\alpha}(\beta)} q_{H,v_H}^* = \frac{(1-\beta)[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s} = \lim_{\alpha \uparrow \bar{\alpha}(\beta)} q_H^*$. Moreover, $\lim_{\alpha \downarrow \bar{\alpha}(\beta)} q_{H,\rho}^* = 0$ and $\lim_{\alpha \downarrow \bar{\alpha}(\beta)} q_{L,v_L}^* = \frac{s}{v_H(1-\beta)-v_L} = \lim_{\alpha \uparrow \bar{\alpha}(\beta)} q_L^*$. As also the distribution functions $G_H(\cdot) = G_L(\cdot)$ coincide with $F_H(\cdot) = F_L(\cdot)$ for $\alpha = \bar{\alpha}(\beta)$, it is established that there is no discontinuity of the equilibrium strategies played around $\alpha = \bar{\alpha}(\beta)$.

Next, while q_{L,v_L}^* is constant in α (and thus stays at $\frac{s}{v_H(1-\beta)-v_L}$), it holds that $\lim_{\alpha \uparrow 1} q_{H,v_H}^* = 0$ and $\lim_{\alpha \uparrow 1} q_{H,\rho}^* = 1$. Hence, for values of α close to one (implying a huge H -market relative to L), H will almost certainly price at ρ and discourage its flexible consumers from search. Only because of that, L has no incentive to always price above v_L in equilibrium.

One can also look at the mass points’ comparative statics as β tends to $\bar{\beta}$. Doing so, I find that $\lim_{\beta \uparrow \bar{\beta}} q_{H,v_H}^* = \frac{(1-\alpha)v_H v_L}{\alpha(v_H-v_L)(v_H-v_L-s)} \in (0, 1)$, $\lim_{\beta \uparrow \bar{\beta}} q_{H,\rho}^* = 1 - \frac{(1-\alpha)v_H v_L}{\alpha(v_H-v_L)(v_H-v_L-s)} \in (0, 1)$, and $\lim_{\beta \uparrow \bar{\beta}} q_{L,v_L}^* = 1$. Thus, there *is* a discontinuity of H ’s (but not L ’s) equilibrium strategy around $\beta = \bar{\beta}$. While for $\beta < \bar{\beta}$, H (almost exclusively) mixes between charging v_H and ρ at a certain ratio, H “jumps” to charging v_H deterministically for $\beta > \bar{\beta}$.²⁴

Furthermore, it can be seen directly from their definitions that q_{H,v_H}^* is strictly decreasing in α whereas $q_{H,\rho}^*$ is strictly increasing in α (as mentioned before, q_{L,v_L}^* is constant in α). Moreover, $G_H(\cdot)$ and the identical $G_L(\cdot)$ (and therefore, \underline{p}) are constant in α . As L ’s commitment problem gets stronger (α increases), H will put more probability mass on ρ in order to counterbalance L ’s larger incentive to exploit the flexible H -consumers. Since it becomes less likely that L is visited by them for increasing α , also L ’s incentive to charge prices above v_L is mitigated.

While q_{L,v_L}^* is strictly increasing in β , the comparative statics of q_{H,v_H}^* , $q_{H,\rho}^*$ and $G_H(\cdot) = G_L(\cdot)$ with respect to β can be ambiguous. In particular, numerical simulations reveal that this ambiguity is typically the case for the distribution functions $G_H(\cdot) = G_L(\cdot)$, whereas it is only the case for the mass points q_{H,v_H}^* and $q_{H,\rho}^*$ if s is relatively small. In contrast, for sufficiently large s , q_{H,v_H}^* is strictly decreasing in β whereas $q_{H,\rho}^*$ is strictly increasing in β .

²⁴For $\beta = \bar{\beta}$, H is indifferent between doing either.

It is also straightforward to check that $q_{H,\rho}^*$, q_{L,v_L}^* and \underline{p} are strictly increasing in s , whereas q_{H,v_H}^* and $G_H(\cdot) = G_L(\cdot)$ are strictly decreasing in s . The comparative statics of q_{H,v_H}^* and $q_{H,\rho}^*$ with respect to v_L are ambiguous (as can easily be shown numerically). On the other hand, q_{L,v_L}^* and \underline{p} are strictly increasing in v_L , whereas $G_H(\cdot) = G_L(\cdot)$ are strictly decreasing in it. Lastly, apart from q_{L,v_L}^* which is strictly decreasing in v_H , all other equilibrium-strategy objects have ambiguous comparative statics with respect to v_H .

Table 1 summarizes the comparative statics of the mixed-strategy price distributions characterized for Cases (1) and (2) above. Hereby, +, -, 0, and +- denote positive, negative, neutral, and ambiguous effects, respectively.

Table 1: Comparative statics of the firms' equilibrium price distributions

	α	β	s	v_L	v_H		α	β	s	v_L	v_H
q_H^*	-	-	0	+	-	q_{H,v_H}^*	-	+-	-	+-	+-
q_L^*	-	+-	0	+	0	$q_{H,\rho}^*$	+	+-	+	+-	+-
\underline{p}	-	-	0	+	-	q_{L,v_L}^*	0	+	+	+	-
$F_H(\cdot)$	+	+	0	-	-	\underline{p}	0	+-	+	+	+-
$F_L(\cdot)$	+	+	0	-	-	$G_H(\cdot)$	0	+-	-	-	+-
						$G_L(\cdot)$	0	+-	-	-	+-

Equilibrium Profits

Case (1): $\beta > \bar{\beta}$. For $\beta > \bar{\beta}$, the unique equilibrium of the game gives rise to equilibrium profits of $\Pi_H^* = (v_L + s)\alpha$ and $\Pi_L^* = v_L(1 - \alpha)$. It is thus apparent that the firms' equilibrium profits in the discussed region are independent of β , strictly increase (decrease) in α for H (L), and strictly increase in the valuation v_L of L -consumers. Moreover, H 's equilibrium profit is strictly increasing in the search cost s .

The intuition to these results is as follows. Since β is large, H fights for its flexible consumers by pricing at their reservation price. Given L 's price v_L , this reservation price is $v_L + s$. Hence, H 's equilibrium profit increases for larger v_L and s . As H serves the whole H -market in the respective equilibrium, its profit strictly increases with this market's size α . The price L charges in equilibrium is v_L , and doing so it can only attract the mass $1 - \alpha$ of its local consumers. Hence, a larger v_L and lower α (that is, a bigger L -market $1 - \alpha$) increases L 's profit.

Case (2): $\beta < \bar{\beta}$. For $\beta < \bar{\beta}$, no matter what α , the respective unique equilibrium of the game implies an (expected) profit of $\Pi_H^{**} = v_H\alpha(1 - \beta)$ for H . That is, for sufficiently few flexible consumers β , H 's equilibrium profit is strictly increasing in its local consumers' valuation v_H

and the H -market size α , while it is strictly decreasing in the fraction of flexible consumers β . The exact intuition depends on the type of equilibrium that is played (Propositions 2 to 4), which is a function of the severity of L 's commitment problem (if any). But generally speaking, $\alpha(1-\beta)$ is the mass of H 's captive consumers (the mass H -consumers who are not flexible), and the maximal price H can charge them is v_H . Hence, the more captive consumers there are in H 's market, and the higher their willingness to pay, the higher is H 's equilibrium profit.

For $\beta < \bar{\beta}$ and $\alpha \leq \bar{\alpha}(\beta)$, L 's (expected) equilibrium profit is given by $v_L(1-\alpha+\alpha\beta)$. It is easy to see that this expression is strictly decreasing in α and v_L , while it is strictly increasing in β . Again, the exact interpretation depends on the type of equilibrium that is played (no commitment problem vs. a moderate commitment problem). A general intuition is that if L 's commitment problem is not too large, the firm's equilibrium profit increases with the fraction β of flexible H -consumers it can attract, its local consumers' valuation v_L , and also the relative size of L 's local market $1-\alpha$. The latter is true because the total mass of consumers L can (potentially) serve is $1-\alpha+\alpha\beta$, which increases in the fraction of L -consumers $1-\alpha$.

For $\beta > \bar{\beta}$ and $\alpha > \bar{\alpha}(\beta)$, L 's equilibrium profit is given by $\Pi_L^{***} = \frac{(1-\alpha)(1-\beta)v_H v_L [(1-\beta)v_H - v_L]}{[(1-\beta)v_H - v_L]^2 + v_L \beta s}$. It is easy to see that this expression is strictly decreasing in α . Similar to the case where L 's commitment problem is less severe (or not there at all), a smaller relative size of the L -market (larger α) leads to lower equilibrium profits of L . Next, the comparative statics of Π_L^{***} with respect to β are generally ambiguous. The intuition is that there are typically two opposing effects at play. Namely, a higher β means that more flexible consumers coming from H can potentially be served, but also that L 's commitment problem becomes more severe. In turn, this leads H to sample its flexible consumers' reservation price ρ more often, which discourages the flexible H -consumers from search.

One can also observe that L 's equilibrium profit strictly decreases with s . This is true because H will price at the flexible H -consumers' reservation price ρ more often for larger s , which directly reduces L 's expected demand. Finally, the comparative statics of Π_L^{***} with respect to v_L and v_H are generally ambiguous.

Table 2 summarizes the (local) comparative statics of the firms' equilibrium profits across the different equilibrium regions. Again, +, -, 0, and +- denote positive, negative, neutral, and ambiguous effects, respectively.

Table 2: Local comparative statics of the firms' equilibrium profits

	α	β	s	v_L	v_H		α	β	s	v_L	v_H
$\Pi_H^*(I)$	+	0	+	+	0	$\Pi_L^*(I)$	-	0	0	+	0
$\Pi_H^*(II)$	+	-	0	0	+	$\Pi_L^*(II)$	-	+	0	+	0
$\Pi_H^*(III)$	+	-	0	0	+	$\Pi_L^*(III)$	-	+	0	+	0
$\Pi_H^*(IV)$	+	-	0	0	+	$\Pi_L^*(IV)$	-	+-	-	+-	+-

Welfare

Since the maximal achievable welfare in the market is given by $\alpha v_H + (1 - \alpha)v_L$ and thus depends on α , v_H and v_L , it makes sense to focus on the relative welfare loss that arises in equilibrium. From equation (2), it is easy to see that this relative welfare loss can be written as

$$W_{loss}^r = \begin{cases} \frac{\alpha\beta s}{\alpha v_H + (1-\alpha)v_L} & \text{if } \beta < \bar{\beta} \text{ and } \alpha \leq \underline{\alpha}(\beta) \\ \frac{\alpha\beta s + (1-q_L^*)(1-\alpha)v_L}{\alpha v_H + (1-\alpha)v_L} & \text{if } \beta < \bar{\beta} \text{ and } \alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)] \\ \frac{\alpha\beta s(1-q_{H,\rho}^*) + (1-q_{L,v_L}^*)(1-\alpha)v_L}{\alpha v_H + (1-\alpha)v_L} & \text{if } \beta < \bar{\beta} \text{ and } \alpha \in (\bar{\alpha}(\beta), 1) \\ 0 & \text{if } \beta > \bar{\beta}. \end{cases} \quad (3)$$

First, note that for $\beta < \bar{\beta}$ and $\alpha \leq \underline{\alpha}(\beta)$, the relative welfare loss strictly increases in α and β . The simple intuition is that a higher α or β increases the mass of flexible H -consumers who incur wasteful travel expenditures in the respective equilibrium (in the case of α , the increase in the total achievable welfare cannot make up for that). Moreover, given that β does not fall short of $\bar{\beta}$, also increases in s unambiguously increase the relative welfare loss in the market. This is because each (searching) flexible H -consumer incurs a larger loss from search if s increases. Increases in v_L or v_H lead to an unambiguous decrease of the relative welfare loss, as the absolute welfare loss of $\alpha\beta s$ stays constant, but the total achievable surplus in the market increases.

Second, the comparative statics of the relative welfare loss with respect to α and β are generally ambiguous if $\beta < \bar{\beta}$ and $\alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)]$.²⁵ Intuitively, this is the case because there can be countervailing effects at work. Clearly, an increase in α or β increases the mass of flexible H -consumers (who all search initially), which implies that larger wasteful travel expenditures to the extent of $\alpha\beta s$ are incurred. Also, increases in α unambiguously reduce the probability that L serves its local consumers by pricing at v_L in equilibrium (see the discussion on the comparative statics of mixed-strategy equilibria above), which increases the absolute welfare loss by giving rise to additional deadweight loss (caused by L -consumers dropping out of the market). However,

²⁵However, it should be noted that the relative welfare loss is typically increasing in α and β in the relevant region. This is true in particular if s is large.

provided that s is sufficiently low, an increase in α can also have a beneficial effect because it may increase the total surplus achievable in the market by more (relatively speaking) than it increases the total welfare loss.²⁶ On the other hand, an increase in β can have a beneficial effect because for some parameter combinations, increases in β lead L to sample v_L more often. In turn, less deadweight loss by dropout L -consumers is created.

Moreover, given that β does not start to exceed $\bar{\beta}$ or α starts to exceed $\bar{\alpha}(\beta)$, increases in s unambiguously increase the relative welfare loss in the market. As above, this is because each (searching) flexible H -consumers incurs a larger loss from search if s increases, while at the same time, the probability that L samples v_L (and serves its local consumers) is unaffected by s . It is also not difficult to show via differentiation that the relative welfare loss in the analyzed region must strictly decrease in v_L . A somewhat more intricate proof reveals that the relative welfare loss in the region is strictly increasing in v_H .²⁷

Third, if $\beta < \bar{\beta}$ and $\alpha > \bar{\alpha}(\beta)$, the comparative statics of the relative welfare loss with respect to α and β are unambiguously *negative*.²⁸ The intuition is that although increases in α and β increase the total search friction created by *searching* flexible H -consumers, this is always more than offset by welfare-increasing changes in the firms' equilibrium strategies. Namely, increases in α unambiguously increase the probability that H samples ρ in equilibrium (avoiding wasteful travel expenditures altogether), while at the same time they do not alter L 's probability of sampling v_L (and thus serving its local consumers). On the other hand, increases in β may have an ambiguous effect on the probability that H prices at ρ , but they unambiguously increase L 's probability of serving its local consumers by pricing at v_L .

It may be interesting to observe that the relative welfare loss in the discussed region unambiguously *decreases* in s .²⁹ Hence, given that L 's commitment problem is severe, an increase in

²⁶This effect is not possible in the case where the only welfare loss stems from wasteful search expenditures (see above). The reason is that in this case, an increase in α implies a one-to-one increase in the absolute welfare loss, whereas it results in a less than one-to-one increase of the maximal achievable welfare.

²⁷For a sketch of the proof, note first that the derivative of the region's relative welfare loss with respect to v_H has the same sign as $\Phi(\alpha) := \alpha^2(1-\beta)(\beta^2s + v_L) - \alpha[2v_L - \beta v_L + \beta^2(s - v_L)] + (1-\beta^2)v_L$, which is strictly convex in α . Since it is required that $\alpha > \bar{\alpha}(\beta)$ in the considered parameter region, it is sufficient to show that $\Phi(\alpha)$ is non-negative for $\alpha = \bar{\alpha}(\beta)$. This is indeed the case for $\beta < \bar{\beta}$.

²⁸The proof for α is simple, as $\alpha\beta(1 - q_{H,\rho}^*)$ reduces to $1 - \alpha$ times a positive (parameter-dependent) factor. For β , a straightforward calculation reveals that $\frac{d}{d\beta} \left(\frac{\alpha\beta s(1 - q_{H,\rho}^*) + (1 - q_{L,v_L}^*)(1 - \alpha)v_L}{\alpha v_H + (1 - \alpha)v_L} \right) < 0$ is equivalent to $\frac{d}{d\beta} \left(\frac{v_H(1-\beta) - v_L - \beta s}{[v_H(1-\beta) - v_L]^2 + v_L\beta s} - \frac{1}{v_H(1-\beta) - v_L} \right) < 0$. After differentiating and simplifying, this condition can be stated as $(1-\beta)^3(1+\beta)v_H^3 - 3(1-\beta)^2v_H^2v_L + 3(1-\beta)v_Hv_L^2 - v_L^3 + v_L^2(\beta^2s + 2\beta v_L) > 0$. Rewriting and setting $s = 0$ yields the sufficient condition $[(1-\beta)v_H - v_L]^3 + \beta[(1-\beta)^3v_H^3 - 3(1-\beta)v_Hv_L^2 + 2v_L^3] > 0$. Here, the left bracket is strictly positive due to $\beta < \bar{\beta}$, while the right bracket reaches its global minimum of 0 (over the range $\beta \in [0, 1]$) at $\beta_{min} = 1 - \frac{v_L}{v_H}$. Hence, the sum is strictly positive.

²⁹This result is straightforward to obtain via differentiation.

the search friction unambiguously improves market performance. Intuitively, this is true because in the relevant region, an increase in s unambiguously increases the probability that L prices at v_L (reducing the deadweight loss from dropout L -consumers) *and* that H prices at ρ (reducing the probability of wasteful travel expenditures). This is always more than enough to offset the adverse effect of higher travel costs on welfare.

Finally, it can be shown numerically that the comparative statics of the relative welfare loss with respect to v_L and v_H are generally ambiguous in the considered parameter region.

Table 3 summarizes the (local) comparative statics of the relative welfare loss in the market across the different equilibrium regions. As above, $+$, $-$, 0 , and $+ -$ denote positive, negative, neutral, and ambiguous effects, respectively.

Table 3: Local comparative statics of the relative welfare loss in the market

	α	β	s	v_L	v_H
$W_{loss}^r(I)$	0	0	0	0	0
$W_{loss}^r(II)$	+	+	+	-	-
$W_{loss}^r(III)$	+ -	+ -	+	-	+
$W_{loss}^r(IV)$	-	-	-	+ -	+ -

An interesting take-away from this table and the preceding discussion is that in several regions in parameter space, more isolated markets (lower β or larger s), as well as a larger difference in willingness to pay (larger v_H or lower v_L ; this could also be interpreted as an increased income inequality), improve market performance. A major driving force of the former result is of course that travel is socially wasteful in the model, such that decreasing the number of traveling consumers, or discouraging consumers from traveling due to higher costs, can improve social welfare. The second result can either stem from a dominant direct effect – increasing the valuation of high-valuation consumers while keeping everything else equal increases the total surplus more than it increases wasteful travel expenditures – or from more subtle changes in firms’ equilibrium pricing behavior. Perhaps surprisingly, decreasing the maximum valuation of low-valuation consumers (“making the poor even poorer”) can still improve market performance, although this has a direct negative effect *and*, at the same time, fosters the exclusion of this consumer group (see Table 1). The reason is that due to firms’ equilibrating strategies, high-valuation consumers may find themselves more often in a situation with a very attractive price offer (ρ^*) that discourages them from engaging in wasteful travel.

6 Advertising

In this section, I will extend the baseline model in order to study the effects of informative advertising on firms' equilibrium pricing. In particular, consider the following extension of the main model outlined in Section 2.

Setup

There is a preliminary stage in which both firms simultaneously decide whether to engage in an advertising campaign or not. For a cost of $A > 0$ (firm L) and $A_H > 0$ (firm H), such a campaign *commits* the engaging firm to charge some (freely chosen) advertised price for the rest of the game (essentially eliminating its price-setting stage), while fully informing all consumers (in particular, the flexible consumers from the other market) and its rival that it charges and advertises the respective price.

The consumers' travel costs in L follow an arbitrary distribution, with no positive mass of L -consumers having zero travel cost. Therefore, let the lowest travel cost in L be given by $s_L \in (0, v_L)$. As in the baseline model, the H -market has a fraction β of flexible consumers with common travel cost s .

Given this, there are three possibilities. First, if both firms advertise, the prices in the market become common knowledge, and the flexible consumers optimally buy at the firm which offers them a lower price (net of travel costs). Second, if only one firm engages in the advertising campaign, I assume that the other firm (which has not invested in advertising) becomes a Stackelberg-follower. That is, the firm observes the other firm's advertised price and may freely choose an arbitrary price as response. Importantly, while in such a scenario it is common knowledge that only one firm has advertised (and which firm that is), consumers *do not* observe the price of the non-advertising firm if it is located in the other market. After the non-advertising firm sets a price in response to the advertised price by its rival, the consumers make their purchase decision (forming beliefs if the other firm's price is unobserved). And third, if both firms do not advertise, the original game outlined in Section 2 is played.

Equilibrium Analysis

As a start, the following lemma is easy to prove.

Lemma 1. *There cannot be an equilibrium where both L and H advertise.*

Proof. Suppose this was the case. Then clearly, there must be at least one firm that does not attract the flexible consumers from the other local submarket (because its advertised price is not lower than its rival's advertised price). Hence, given the rival's advertising strategy, the concerned firm could certainly do better by not advertising, but charging the same price as before, being a Stackelberg-follower. Doing so, the firm will not lose any (additional) local consumers, but can save the (otherwise wasteful) advertising cost. \square

Consequently, there can only exist three types of equilibria: two asymmetric ones in which either L or H advertises, and one "symmetric" one (in the sense of advertising) in which none of the firms advertises.

Note furthermore that an asymmetric equilibrium where only H advertises exists under certain circumstances. The necessary ingredients of such an equilibrium are that H 's advertising cost A_H is low, few consumers live in H , the minimal travel cost of L -consumers s_L is not too high, and the L -market consists of consumers that are heterogeneous with respect to their travel costs (such that L may prefer not to fight for a subset of these consumers after H advertises a very low price).³⁰ As these conditions are somewhat contrived, I will subsequently ignore for equilibria where H advertises. Moreover, in order to avoid tedious checks whether H may have a profitable deviation by advertising, the following assumption is made.

Assumption 1. *Given the model parameters (including the search-cost distribution of L -consumers), H never has an incentive to advertise due to its high advertising cost A_H .*

Clearly, a sufficient condition for this is that $v_H\alpha(1 - \beta) \geq v_L - s_L - A_H$: even if H could attract all consumers in L by advertising $v_L - s_L$ (and thus serving the whole market for a net profit of $v_L - s_L - A_H$), this would not be more profitable than to only serve its captive consumers at price v_H .

Now that H 's advertising decision need not be considered anymore, I will proceed to characterize when advertising by L is an equilibrium outcome. For this, start with a scenario in which L is forced to engage in an advertising campaign. I will first derive which price is optimal to advertise for L under this assumption, calculate the corresponding profit, and finally compare this with the profit L would obtain if it did not advertise (and hence, with the equilibrium profit of the baseline model with no advertising).

³⁰Examples are easy to construct and can be obtained from the author upon request.

First, note that it can never be optimal for L to advertise a price $p_L \geq v_H - s$. Such a price would never attract the flexible H -consumers due to their search cost,³¹ and also L would not be able to serve its local consumers due to $v_H - s > v_L$. Hence, suppose L advertises some price $p_L \in (0, v_H - s)$. In turn, H 's best reply as Stackelberg-follower to such a price must either be v_H (letting the flexible H -consumers move on and fully exploiting its captive consumers), or $p_L + s$ (discouraging the flexible H -consumers from search). The former leads to a profit of $\Pi_H(v_H) = v_H\alpha(1 - \beta)$, the latter to $\Pi_H(p_L + s) = (p_L + s)\alpha$. Comparing these two expressions, it follows that³²

$$BR_H(p_L) = \begin{cases} v_H & \text{if } p_L \leq v_H(1 - \beta) - s \\ p_L + s & \text{if } p_L > v_H(1 - \beta) - s. \end{cases}$$

Which price p_L should L advertise? It is easy to see that there only two alternatives which can be optimal. One possibility is to price at $v_H(1 - \beta) - s$, which is the highest possible price for which H "accommodates" L , allowing L to serve H 's flexible consumers (but possibly lies above L 's local consumers' valuation v_L). The other is to price at v_L . This fully exploits L 's local consumers, but either implies that H will fight for its local consumers (if $v_L > v_H(1 - \beta) - s$, i.e., $\beta > \bar{\beta}$), or that the flexible H -consumers are not fully exploited (if $v_L < v_H(1 - \beta) - s$, i.e., $\beta < \bar{\beta}$). Importantly, in all of the subsequent analysis, I will ignore the zero-measure event where $\beta = \bar{\beta}$. This is because if, and only if $\beta = \bar{\beta}$, there is equilibrium multiplicity in the baseline game, which makes L 's optimal advertising strategy contingent on which equilibrium would be played without advertising.

Now, in the first case where $v_L > v_H(1 - \beta) - s$ ($\beta > \bar{\beta}$), it holds that $\Pi_L(v_H(1 - \beta) - s) = [v_H(1 - \beta) - s](1 - \alpha + \alpha\beta)$, whereas $\Pi_L(v_L) = v_L(1 - \alpha)$. Comparing these expressions, one finds that advertising $v_H(1 - \beta) - s$ (rather than v_L) is strictly better if and only if

$$\alpha > \tilde{\alpha}(\beta) := \frac{v_L - [v_H(1 - \beta) - s]}{v_L - (1 - \beta)[v_H(1 - \beta) - s]} > 0. \quad (4)$$

³¹The weak inequality follows from the original tie-breaking rule according to which the flexible H -consumers will not search if they are indifferent between doing so and purchasing at H directly.

³²As a tie-breaking rule, I assume that H will accommodate L (by pricing at v_H) if it is indifferent between doing so and fighting for its flexible consumers (by charging $p_L + s$).

³³Note that $\tilde{\alpha}(\bar{\beta}) = 0$ and $\frac{d\tilde{\alpha}(\beta)}{d\beta} > 0$ for all $\beta > \bar{\beta}$. One way to show the latter is to rewrite $\tilde{\alpha}(\beta)$ as $\frac{\frac{v_L}{v_H(1-\beta)-s} - 1}{\frac{v_L}{v_H(1-\beta)-s} - 1 + \beta} := \frac{\gamma(\beta)}{\gamma(\beta) + \beta}$. Then $\frac{d(\frac{\gamma(\beta)}{\gamma(\beta) + \beta})}{d\beta}$ has the same sign as $\gamma'(\beta)\beta - \gamma(\beta) = \frac{\beta v_L v_H}{[v_H(1-\beta)-s]^2} - \frac{v_L}{v_H(1-\beta)-s} + 1 \geq \frac{\beta v_L v_H}{[v_H(1-\beta)-s]v_L} - \frac{v_L}{v_H(1-\beta)-s} + 1$, where the last inequality follows from $v_H(1 - \beta) - s \leq v_L$ (due to $\beta \geq \bar{\beta}$). Simplifying $\frac{\beta v_L v_H}{[v_H(1-\beta)-s]v_L} - \frac{v_L}{v_H(1-\beta)-s} + 1 > 0$ leads to $s < v_H - v_L$, as assumed.

The interpretation to this is as follows. If $\beta > \bar{\beta}$ and hence many consumers in H are flexible, it does not suffice to advertise v_L in order for L to attract these flexible consumers. This is because H would respond by charging a price that is sufficiently low to discourage its local consumers from search. Hence, to avoid such an aggressive response by H , L has to advertise a price below the reservation price of its local consumers (namely, it can charge at most $v_H(1 - \beta) - s < v_L$). But this will only be preferred to fully exploiting L 's local consumers (by charging v_L) if the H -market is sufficiently large in size ($\alpha > \tilde{\alpha}(\beta)$), given the fraction of flexible consumers in that market. If β is very large ($\beta > 1 - \frac{s}{v_H}$), it turns out that even $\alpha = 1$ would not be sufficient to induce L to advertise $v_H(1 - \beta) - s < v_L$. The simple reason is that $v_H(1 - \beta) - s < 0$ for $\beta > 1 - \frac{s}{v_H}$: L would have to advertise a price below its marginal cost in order to prevent an aggressive price response by H .

In the second case where $v_L < v_H(1 - \beta) - s$ ($\beta < \bar{\beta}$), pricing at $v_H(1 - \beta) - s$ drives out L 's local consumers. Hence, one finds that $\Pi_L(v_H(1 - \beta) - s) = [v_H(1 - \beta) - s]\alpha\beta$, whereas $\Pi_L(v_L) = v_L(1 - \alpha + \alpha\beta)$. Again comparing these expressions, advertising $v_H(1 - \beta) - s$ (rather than v_L) is strictly preferred if and only if

$$\alpha > \hat{\alpha}(\beta) := \frac{v_L}{\beta[v_H(1 - \beta) - s] + v_L(1 - \beta)} \in (\bar{\alpha}(\beta), 1).^{34} \quad (5)$$

In this case, the interpretation is different. As the fraction of flexible H -consumers is low ($\beta < \bar{\beta}$), H would not even fight for its local consumers if L advertised v_L . But if α is very large ($\alpha > \hat{\alpha}(\beta)$) and hence the L -market is relatively unimportant in size, L prefers to advertise the highest possible price which triggers no aggressive response by H . This price fully exploits the searching flexible H -consumers, but drives out L 's local consumers.

One can now calculate L 's maximal advertising profit in each parameter region and contrast this with L 's equilibrium profit of the baseline game. Combining the above results, H 's maximal advertising profit is given by

$$\Pi_L^{A*} = \begin{cases} v_L(1 - \alpha) - A & \text{if } \beta > \bar{\beta} \text{ and } \alpha \leq \tilde{\alpha}(\beta) \\ [v_H(1 - \beta) - s](1 - \alpha + \alpha\beta) - A & \text{if } \beta > \bar{\beta} \text{ and } \alpha > \tilde{\alpha}(\beta) \\ v_L(1 - \alpha + \alpha\beta) - A & \text{if } \beta < \bar{\beta} \text{ and } \alpha \leq \hat{\alpha}(\beta) \\ [v_H(1 - \beta) - s]\alpha\beta - A & \text{if } \beta < \bar{\beta} \text{ and } \alpha > \hat{\alpha}(\beta). \end{cases}$$

³⁴The inequality $\hat{\alpha}(\beta) < 1$ is equivalent to $\beta < \bar{\beta}$, which has to hold in the considered region. On the other hand, after some straightforward calculation it turns out that $\hat{\alpha}(\beta) > \bar{\alpha}(\beta)$ is equivalent to $v_H(1 - \beta) - v_L + \beta(1 - \beta)v_H - \beta s > 0$. This is true because $v_H(1 - \beta) - v_L - s > 0$ whenever $\beta < \bar{\beta}$.

Comparing these with the equilibrium profits of the game without advertising (see Propositions 1 to 4), it is immediately apparent that L should not advertise if either $\beta > \bar{\beta}$ and $\alpha \leq \tilde{\alpha}(\beta)$, or $\beta < \bar{\beta}$ and $\alpha \leq \bar{\alpha}(\beta)$. The former is true because in the baseline game, H will charge v_L in equilibrium whenever $\beta > \bar{\beta}$. Hence, it cannot pay to engage in costly advertising of v_L , as this price would be expected anyway by the flexible H -consumers.

The latter is true because of two reasons. First, similar to the case of $\beta > \bar{\beta}$, H would charge a deterministic price of v_L anyway if $\beta < \bar{\beta}$ and $\alpha \leq \underline{\alpha}(\beta)$. Therefore, it is again pointless to advertise v_L at positive cost. Second, while L faces a commitment problem if $\alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)]$, the firms' equilibrating strategies still lead to an expected L -profit of $v_L(1 - \alpha + \alpha\beta)$ (this is because L sometimes sells to the searching H -consumers at a price larger than v_L , but sometimes makes no sales at all). As this expected profit is identical to the gross profit (gross of the advertising cost) the firm could achieve by advertising v_L , but does not require costly advertising, L strictly prefers to abstain from advertising.

Next, note that L will advertise $v_H(1 - \beta) - s$ in equilibrium if $\beta > \bar{\beta}$, $\alpha > \tilde{\alpha}(\beta)$, and A is sufficiently small. This is because L 's profit without advertising in this region would be given by $\Pi_L^* = v_L(1 - \alpha)$ (see Proposition 1), whereas L 's (maximal) gross profit when advertising is given by $[v_H(1 - \beta) - s](1 - \alpha + \alpha\beta)$. The latter exceeds the former whenever $\alpha > \tilde{\alpha}(\beta)$, as was already established above. Hence, for a sufficiently small advertising cost A , L 's dominant action in the discussed parameter region is to advertise $v_H(1 - \beta) - s < v_L$.

Similarly, L will advertise v_L in equilibrium if $\beta < \bar{\beta}$, $\alpha \in (\bar{\alpha}(\beta), \hat{\alpha}(\beta))$, and A is sufficiently small. The reason is that L 's profit without advertising in this region would be given by $\Pi_L^{***} = \frac{(1-\alpha)(1-\beta)v_H v_L [(1-\beta)v_H - v_L]}{[(1-\beta)v_H - v_L]^2 + v_L \beta s}$ (see Proposition 4), while L 's (maximal) gross profit when advertising is given by $\Pi_L^{**} = v_L(1 - \alpha + \alpha\beta)$. The latter exceeds the former in the relevant region because (a) $\Pi_L^{***}|_{\alpha=\bar{\alpha}(\beta)} = \Pi_L^{**}|_{\alpha=\bar{\alpha}(\beta)}$ (as is straightforward to check) and (b) $\frac{\partial \Pi_L^{***}}{\partial \alpha} < \frac{\partial \Pi_L^{**}}{\partial \alpha} < 0$ (where the first inequality follows from $s < v_H(1 - \beta) - v_L$, i.e., $\beta < \bar{\beta}$). Consequently, for a sufficiently small advertising cost A , L 's dominant action in the discussed parameter region is to advertise v_L .

Finally, L will advertise $v_H(1 - \beta) - s > v_L$ in equilibrium if $\beta < \bar{\beta}$, $\alpha > \hat{\alpha}(\beta)$, and A is sufficiently small.³⁵ The simple reason is that advertising $v_H(1 - \beta) - s$ (in order to fully exploit the searching consumers from H) outperforms advertising v_L if α exceeds $\hat{\alpha}(\beta)$ (as was already established above). Because advertising v_L is already superior to not advertising (in terms of

³⁵If $\alpha = \hat{\alpha}(\beta)$ (and $\beta < \bar{\beta}$ as well as A small), L is indifferent between advertising v_L and $v_H(1 - \beta) - s$.

gross profit) for $\alpha > \bar{\alpha}(\beta)$ (see the previous paragraph), a sufficiently small advertising cost A will induce L to advertise $v_H(1 - \beta) - s$ in equilibrium, given that $\alpha > \hat{\alpha}(\beta)$.

The above findings are summarized in the following proposition.

Proposition 6. *Let Assumption 1 hold. Then, L 's equilibrium advertising behavior is characterized as follows.³⁶*

- (1) Advertise $v_H(1 - \beta) - s < v_L$ if $\beta > \bar{\beta}$, $\alpha > \tilde{\alpha}(\beta)$, and $A \leq [v_H(1 - \beta) - s](1 - \alpha + \alpha\beta) - v_L(1 - \alpha)$.
- (2) Advertise v_L if $\beta < \bar{\beta}$, $\alpha \in (\bar{\alpha}(\beta), \hat{\alpha}(\beta)]$, and $A \leq v_L(1 - \alpha + \alpha\beta) - \frac{(1 - \alpha)(1 - \beta)v_H v_L [(1 - \beta)v_H - v_L]}{[(1 - \beta)v_H - v_L]^2 + v_L \beta s}$.
- (3) Advertise $v_H(1 - \beta) - s > v_L$ if $\beta < \bar{\beta}$, $\alpha > \hat{\alpha}(\beta)$, and $A \leq [v_H(1 - \beta) - s]\alpha\beta - \frac{(1 - \alpha)(1 - \beta)v_H v_L [(1 - \beta)v_H - v_L]}{[(1 - \beta)v_H - v_L]^2 + v_L \beta s}$.
- (4) Otherwise, do not advertise.

Figure 5 depicts an example of L 's equilibrium advertising regions in (α, β) -space.

In the final part of this section, I will discuss the welfare consequences of advertising by L . First, note that advertising when $\beta > \bar{\beta}$ is clearly wasteful from a social point of view. While without advertising, the social first-best would be achieved (which is characterized by no search and an L -market that is always served), advertising of $v_H(1 - \beta) - s < v_L$ by L leads to a deterministic welfare loss of $\alpha\beta s + A$. In particular, even if A is close to zero, wasteful travel expenditures to the extent of $\alpha\beta s$ would be induced by L 's advertising.

Next, it is also apparent that advertising must have an adverse effect on social welfare if $\beta < \bar{\beta}$ and $\alpha \geq \hat{\alpha}(\beta)$. The reason is that in this parameter region, L will advertise a price that is higher than its local consumers' valuation (namely $v_H(1 - \beta) - s > v_L$), yet induces deterministic search by the flexible H -consumers. Therefore, the total welfare loss that is generated is given by $\alpha\beta s + (1 - \alpha)v_L + A$. In contrast, as $\hat{\alpha}(\beta) > \bar{\alpha}(\beta)$, the welfare loss in the baseline model without advertising would only be given by $\alpha\beta s(1 - q_{H,\rho}^*) + (1 - q_{L,v_L}^*)(1 - \alpha)v_L$. This is because without advertising, H would sometimes price at its flexible consumers' reservation price ρ (reducing wasteful travel expenditures), while L would only sometimes price above v_L (reducing the welfare loss that stems from L -consumers dropping out).

Observe moreover that L does not gain from advertising if $\beta < \bar{\beta}$ and $\alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)]$. However, provided that advertising is not too costly, it would be desirable from a social-welfare perspective. This is because for $\alpha > \underline{\alpha}(\beta)$, L 's commitment problem kicks in, which leads the firm to price above v_L with positive probability. Yet at the same time, the flexible H -consumers always search, as H always prices above their reservation price. If instead firm L advertised v_L ,

³⁶ As tie-breaking rules, I assume that L will advertise if it is indifferent between doing so and not advertising, and that it will advertise v_L if it is indifferent between doing so and advertising $v_H(1 - \beta) - s$.

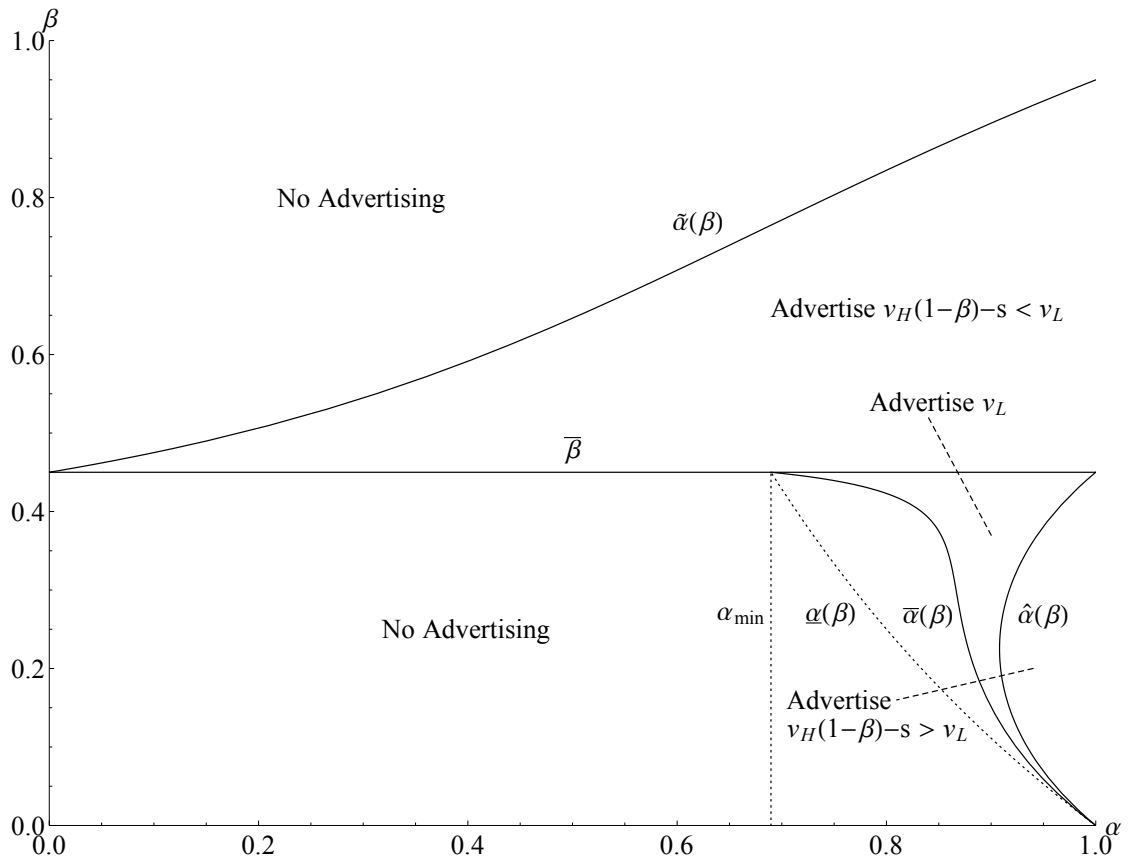


Figure 5: L 's equilibrium advertising regions for $v_H = 200$, $v_L = 100$, $s = 10$, A negligibly small.

the L -market could be served with certainty, while no additional welfare loss would be incurred. Hence, for $\alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)]$, there may be underinvestment in advertising.

Finally, the welfare consequences of advertising when $\beta < \bar{\beta}$ and $\alpha \in (\bar{\alpha}(\beta), \hat{\alpha}(\beta))$ are ambiguous. This is because v_L would be advertised (eliminating the welfare loss which arises from L -consumers dropping out when L prices above v_L), but also more wasteful travel expenditures would be generated, as H ceases to price at its flexible consumers' reservation price with positive probability.

7 Supply-side Heterogeneities

The main model of Sections 2 to 6 focuses on the case of local *demand-side* heterogeneities. In particular, the flexible H -consumers' equilibrium search behavior is caused by a difference in the local monopoly price which stems from heterogeneous consumer product valuations across the local submarkets. For a sufficiently large high-valuation market, the firm in the low-valuation market cannot commit to serving searching H -consumers at the low-valuation monopoly price v_L , which gives rise to non-trivial mixed-strategy equilibria.

The purpose of this section is to show that directed consumer-search behavior to (perceived) discount markets can also be explained by *supply-side* heterogeneities alone. In fact, the only necessary ingredients are that in isolation, the local monopoly prices would differ, and that the flexible consumers' search cost is sufficiently low such that search is profitable. More precisely, I identify two simple supply-side heterogeneities which can induce search in equilibrium: (1) a unit cost difference with downward sloping demand, and (2) a difference in the number of incumbent firms (intensity of competition).

Unit Cost Difference

Consider the following variation of the market setup of Sections 2 to 6. All consumers have a common downward sloping demand schedule $D(p)$, with an associated monopoly price of $p^m(c)$ (monopoly profit of $\Pi_m(c)$) that strictly increases (decreases) with a firm's constant unit cost c .³⁷ A mass $1 - \alpha$ of consumers live in market L , whereas the remaining mass α consumers live in H (this includes the even more symmetric case where $\alpha = \frac{1}{2}$). In each local market, a fraction $\beta \in (0, 1)$ of consumers is flexible and can travel to the other market at strictly positive cost $s > 0$, purchasing on the way according to their schedule $D(p)$ if the observed price is lower.

³⁷It is a straightforward exercise to show that every well-behaved demand function must fulfill the latter two criteria.

Importantly, the incumbent firms are *not* symmetric: whereas the firm in L has a unit cost that is normalized to $c_L = 0$, the other firm has a strictly positive unit cost of $c_H = c > 0$. Moreover, in order to make the problem interesting, assume that $\Delta(CS) := \int_{p^m(0)}^{p^m(c)} D(p)dp > s$: if both firms were to price at their respective market's local monopoly price deterministically, the flexible H -consumers' search cost would be sufficiently low to generate search. Then it is not difficult to prove the following proposition.

Proposition 7. *If $\beta < 1 - \frac{(\rho-c)D(\rho)}{\Pi^m(c)}$, where $\rho \in (p^m(0), p^m(c))$ solves $\int_{p^m(0)}^{\rho} D(p)dp = s$, the unique equilibrium of the game is in pure strategies such that $p_H^* = p^m(c)$, $p_L^* = p^m(0)$, and all $\alpha\beta$ flexible H -consumers search and purchase in L . If $\beta > 1 - \frac{(\rho-c)D(\rho)}{\Pi^m(c)}$, the unique equilibrium of the game is such that $p_H^* = \rho$, $p_L^* = p^m(0)$, and no consumers search in equilibrium.³⁸*

Proof. See Appendix A. □

The intuition to Proposition 7 is simple: if and only if there are sufficiently few flexible consumers in H , firm H prefers to “accommodate” its rival and price at the local monopoly price, rather than to fight for its local flexible consumers by charging such a low price that makes them indifferent between switching to the other market or purchasing directly at H . If H has a major cost disadvantage, it can even be the case that H will never fight for its local flexible consumers (even if all of them were flexible), as it would have to decrease its price below marginal cost.

Differing Number of Firms

Consider the following simple setup. All consumers have unit demand up to a maximum valuation of $v > 0$. A mass $1 - \alpha$ of consumers live in market L , whereas the remaining mass α consumers live in H (this includes the even more symmetric case where $\alpha = \frac{1}{2}$). In each local market, a fraction $\beta \in (0, 1)$ of consumers is flexible and can travel to the other market at strictly positive cost $s \in (0, v)$, purchasing on the way if the observed price is lower (given that it does not exceed v). The asymmetry comes from the number of firms in the market: while there is only one firm in H , there are $N \geq 2$ identical firms in L . All of them have identical, constant unit costs which are normalized to zero. For simplicity, assume that consumers within a given market observe all of the local market's prices and always buy at the cheapest firm. Hence, there is a Bertrand-type of competition within L , but not H . Then the following proposition easily follows.

³⁸In the borderline case where $\beta = 1 - \frac{(\rho-c)D(\rho)}{\Pi^m(c)}$, both constitute an equilibrium, as H is indifferent between exploiting its captive consumers and discouraging search (i.e., charging $p^m(c)$ or ρ).

Proposition 8. *If $\beta < 1 - \frac{s}{v}$, in any equilibrium of the game it holds that $p_H^* = v$, $p_{L,i}^* = 0$ for at least two firms $i \in \{1, \dots, N\}$ in L , and all $\alpha\beta$ flexible H -consumers search and purchase in L at price 0. If $\beta > 1 - \frac{s}{v}$, in any equilibrium it holds that $p_H^* = s$, $p_{L,i}^* = 0$ for at least two firms $i \in \{1, \dots, N\}$ in L , and no consumers search in equilibrium.³⁹*

Proof. Since all prices in L are observed by all consumers in that market (including potentially searching H -consumers), it is well known (see e.g. Harrington (1989)) that at least two firms in L must price competitively in equilibrium. Thus, in every possible equilibrium, the lowest price in L must equal marginal cost: $\min_i(p_{L,i}^*) = 0$. Given that, H will either find it optimal to price at s and fight for its flexible consumers, or fully exploit its captive consumers by pricing at v . The former gives a profit of $s\alpha$, whereas the latter gives a profit of $v\alpha(1 - \beta)$. Comparing these two and solving for β , the proposition immediately follows. \square

The intuition to this result is the same as in the scenario with unit-cost heterogeneity: if and only if there are sufficiently few flexible consumers in H , firm H prefers to fully exploit its captive consumers and let go of its flexible consumers, rather than to fight for the latter by offering such a low price that makes them indifferent between switching to L or purchasing directly at H .

8 Conclusion

I have analyzed a market configuration in which consumers' price-search behavior is driven by local market heterogeneities, rather than by a mass of perfectly informed consumers. In the model, two local monopolists simultaneously set prices, where initially, each firm's price is only observed by its local consumer base. Absent any link between the two markets, the firms would set two different monopoly prices, as consumers' willingness to pay is greater in one market than the other. However, the markets *are* linked in the sense that a subset of consumers is "flexible", allowing them to search the non-local market at strictly positive cost. Inner-city shopping across neighborhoods, or cross-border shopping, can be thought of examples for this.

A main contribution of the paper is that a tractable model of sequential consumer search is introduced in which search is costly for all consumers, yet prices are dispersed in equilibrium, and active consumer search occurs. Furthermore, consumers' search activity is directed such that only high-valuation consumers from the (on average) higher priced high-valuation market may search the opposed "discounter" in the low-valuation market.

³⁹In the borderline case where $\beta = 1 - \frac{s}{v}$, both types of equilibria coexist.

A precise equilibrium characterization reveals that paradoxically, active search only takes place in equilibrium if the fraction of flexible consumers in the high-valuation market is sufficiently *low*. Otherwise, the local incumbent prefers to price aggressively and thereby discourage its flexible consumers from search.

If active search occurs, I show that the relative size of the two markets is crucial for determining the equilibrium outcome. If the mass of flexible high-valuation consumers is sufficiently large, the discounter faces a commitment problem, as it would find it more profitable to overcharge the incoming searchers than to serve its local consumers at a price that is acceptable to them. This commitment problem is resolved by non-trivial mixed-strategy pricing in *both* local markets: the discounter sometimes charges high prices in order to exploit incoming searchers (while at the same time excluding its local consumer base), whereas the high-priced firm sometimes offers discounts which may beat the discounter's exploitative prices. If the market imbalance is severe enough, the high-priced firm offers a deep discount with positive probability, which altogether discourages its local flexible consumers from search.

In an extension, I show that informative advertising by the discounter, even if it fully eliminates its commitment problem, tends to decrease rather than increase market performance. One reason is that socially wasteful search activities can be induced, the other is that the discounter may even find it optimal to advertise a price that is higher than its local consumers' valuation. This excludes the whole submarket deterministically and introduces significant deadweight loss.

The model can be extended further in several dimensions. For example, it would be desirable to allow for a more general (i.e., continuous) search-cost distribution. Preliminary calculations have revealed that the main qualitative features of the characterized equilibrium remain intact: active search can still only occur if not too many high-valuation consumers are tempted to search, and also the same commitment problem is faced by the discounter as in the analyzed model. However, the resulting mixed-strategy equilibria become more complex, and it is difficult to derive firms' equilibrium strategies explicitly.

Alternatively, the simple two-firm setup could be altered. It might be interesting to understand consumers' search behavior and firms' equilibrium pricing in an arbitrary network. In such a network, each node would represent a local submarket hosting a single firm, while each edge would indicate the travel cost of searching consumers going from one of the connected nodes to the other. The local submarkets could again be differentiated by consumers' valuations, firms' unit costs, or any other factors.

Finally, the assumption of homogenous consumer valuations within submarkets could be relaxed. For example, consumers' valuations could be normally distributed with two different means across submarkets. Then, intuitively, similar equilibria as in the baseline model should be expected. However, the analysis would have to be augmented by two additional considerations. First, since consumers' search expenditures are sunk once they arrive at the outside market, there is again an incentive for a perceived discounter to exploit incoming searchers. But depending on the distribution of consumers' valuations, there might be no hard cap on the prices a discount firm can charge while still serving its local consumers. Thus, a pure-strategy equilibrium in which a low-priced discounter serves (part of) its own local consumers *and* incoming searchers may be less likely to exist. Second, if there are flexible consumers with a sufficiently high valuation in the discount market (as would be the case with normally distributed valuations), and if the firms play mixed pricing-strategies due to the described commitment problem, there might also be gains from search for consumers in the discount market. Hence, equilibria where search in both directions takes place may emerge.

Overall, the presented framework lends itself as a flexible way of modeling consumer search across spatial structures. As such, it is hoped that fruitful and diverse future applications will arise.

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Appendix A: Technical Proofs

Lemma 2. *L-market consumers will never search in equilibrium, provided that their search costs are bounded away from zero.*

Proof. Denote the lower support bound of firm i 's $\in \{L, H\}$ pricing strategy by \underline{p}_i and the upper support bound by \bar{p}_i . Without loss of generality, let $\underline{p}_i \leq \underline{p}_j$. Denote the infimum of the low-valuation consumers' search costs by $s_L > 0$.

Then it must hold that $\underline{p}_i \geq v_L$. To see this, assume to the contrary that $\underline{p}_i < v_L$. Given this, note first that consumers who observe a price in the range $[\underline{p}_i, \underline{p}_i + \min\{s, s_L\}]$ (where $\underline{p}_i + \min\{s, s_L\} < v_H$ by our assumption of $\underline{p}_i < v_L$ and the general parameter restriction of $v_L + s < v_H$) will never find it optimal to search. This is because at best, they can hope to

find a price of \underline{p}_i , implying a price reduction of at most $\min\{s, s_L\}$, which does not exceed their search cost. Hence, instead of pricing at or slightly above \underline{p}_i , firm i could profitably deviate by transferring all of this probability mass to $\underline{p}_i + \min\{s, s_L\}$ (if $\underline{p}_i + \min\{s, s_L\} \leq v_L$) or v_L (if $\underline{p}_i + \min\{s, s_L\} > v_L$), as doing so does not decrease its demand. Hence, $\underline{p}_i < v_L$ cannot be part of an equilibrium.

But since it is now established that $\underline{p}_i \geq v_L$, it can never be profitable to search for L -market consumers, as the expected surplus of doing so is negative due to their strictly positive search cost. \square

Proof of Proposition 3. (Existence) In order for the proposed strategy-combination to form an equilibrium, it is necessary that each price that is sampled by the firms with positive probability (probability density) must yield the same, maximal expected profit. Furthermore, all equilibrium objects need to be well-behaved (e.g., mass points must fall in the range $[0, 1]$). In what follows, I will solve for the outlined equilibrium in a constructive manner.

For this, suppose an equilibrium exists in which H prices at v_H with probability q_H and samples prices continuously from an interval $[\underline{p}, v_H]$, following a distribution function $F_H(\cdot)$, with probability $1 - q_H$. At the same time, L prices at v_L with probability q_L and samples prices continuously from the same interval as H , $[\underline{p}, v_H]$, following a distribution function $F_L(\cdot)$, with probability $1 - q_L$. Assume moreover that $q_L(\underline{p} - v_L) > s$ (and thus, $\underline{p} > v_L$), which implies that even if flexible H -consumers observe the lowest (equilibrium) H -market price of \underline{p} , they find it worthwhile to visit L just for the chance to buy at the low price v_L (this condition will be verified later in the proof). Hence, in the proposed equilibrium, H -market consumers will always visit L initially.

Now, given the specified strategies, L 's expected profit when pricing at v_H can be calculated as

$$\Pi_L(v_H) = v_H \alpha \beta q_H. \quad (6)$$

This results from the tie-breaking rule specified in the model setup. Given that L prices at v_H , the firm will only sell to the searching flexible H -consumers when H prices at v_H as well, which happens with probability q_H .

Next, L 's expected profit when pricing at \underline{p} is equal to

$$\Pi_L(\underline{p}) = \underline{p} \alpha \beta. \quad (7)$$

This follows from the fact that \underline{p} always outperforms H 's price, and $\underline{p} > v_L$ (hence, L cannot serve its local low-valuation consumers).

Finally, L 's expected profit when pricing at v_L is equal to

$$\Pi_L(v_L) = v_L(1 - \alpha + \alpha\beta). \quad (8)$$

Clearly, if L prices at v_L , it deterministically serves the searching H -market consumers (as v_L outperforms any of H 's prices), as well as its own low-valuation consumers.

Solving $\Pi_L(v_H) \stackrel{!}{=} \Pi_L(v_L)$ and $\Pi_L(\underline{p}) \stackrel{!}{=} \Pi_L(v_L)$ immediately gives rise to the equilibrium conditions

$$q_H^* = \frac{v_L(1 - \alpha + \alpha\beta)}{v_H\alpha\beta} \quad (9)$$

and

$$\underline{p} = \frac{v_L(1 - \alpha + \alpha\beta)}{\alpha\beta}. \quad (10)$$

While clearly $q_H^* > 0$ and $\underline{p} > v_L$, it remains to check whether $q_H^* < 1$ and $\underline{p} < v_H$. As both of these conditions follow directly from $\alpha > \underline{\alpha}(\beta)$, q_H^* and \underline{p} are well-behaved.

Turning to H , the firm's expected profit when pricing at v_H is given by

$$\Pi_H(v_H) = v_H\alpha(1 - \beta). \quad (11)$$

This is because if H prices at the highest possible price v_H , its flexible consumers will search and find a lower price with certainty. Hence, H can only sell to its captive consumers.

On the other hand, if H prices at \underline{p} , its expected profit is given by

$$\Pi_H(\underline{p}) = \underline{p}\alpha(1 - \beta q_L). \quad (12)$$

This is because if H prices at \underline{p} , all flexible H -consumers will search initially. However, they will return to H unless L prices at v_L (as otherwise, L certainly charges a higher price than \underline{p}), which happens with probability q_L .

Solving $\Pi_H(v_H) \stackrel{!}{=} \Pi_H(\underline{p})$ and inserting \underline{p} from equation (10) leads to the next equilibrium condition of

$$q_L^* = \frac{1}{\beta} - \frac{v_H\alpha(1 - \beta)}{v_L(1 - \alpha + \alpha\beta)}. \quad (13)$$

Note that it is again the case that $q_L^* < 1$ follows immediately from $\alpha > \underline{\alpha}(\beta)$. In order to show that $q_L^* > 0$, note first that q_L^* is strictly decreasing in α . Hence, as Proposition 3 requires that $\alpha \leq \bar{\alpha}(\beta) = \frac{v_L}{(1-\beta)\left\{v_L + \frac{v_H\beta[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s}\right\}}$, q_L^* is bounded below by

$$\frac{1}{\beta} - \frac{v_H(1-\beta)}{v_L\left(\frac{1}{\alpha} - 1 + \beta\right)} \Big|_{\alpha=\bar{\alpha}(\beta)}.$$

Simplifying this expression in a straightforward manner, it follows that $q_L^* \geq \frac{s}{v_H(1-\beta)-v_L}$, which is clearly positive due to $\beta < \bar{\beta}$. Hence, q_L^* is well-behaved.

In order to solve for the equilibrium distribution functions $F_L(\cdot)$ and $F_H(\cdot)$, one now simply needs to calculate the firms' expected profits when setting an arbitrary price in the interval (\underline{p}, v_H) , and set this equal to their equilibrium profit levels which were already found via equations (8) and (11).

For L , this equilibrium condition is given by

$$\Pi_L(p) = p\alpha\beta[q_H^* + (1 - q_H^*)(1 - F_H(p))] \stackrel{!}{=} v_L(1 - \alpha + \alpha\beta). \quad (14)$$

The rationale behind equation (14) is straightforward: L 's expected profit when charging some price $p \in (\underline{p}, v_H)$ is given by this price times the mass of searching H -consumers $\alpha\beta$ times the probability that these consumers will buy at L (after finding out that p is lower than p_H), which is $q_H^* + (1 - q_H^*)(1 - F_H(p))$. For any price $p \in (\underline{p}, v_H)$, this expected profit has to be equal to L 's candidate equilibrium profit of $v_L(1 - \alpha + \alpha\beta)$.

Inserting q_H^* from equation (9) and solving for $F_H(p)$ yields

$$F_H(p) = 1 - \frac{v_L(1 - \alpha + \alpha\beta)}{v_H\alpha\beta - v_L(1 - \alpha + \alpha\beta)} (v_H/p - 1). \quad (15)$$

It is easy to check that $F_H(\underline{p}) = 0$ and $F_H(v_H) = 1$. Moreover, since $v_H\alpha\beta - v_L(1 - \alpha + \alpha\beta) > 0$ because of $\alpha > \underline{\alpha}(\beta)$, it is obvious that $F_H(p)$ is strictly increasing in p . Hence, $F_H(\cdot)$ is well-behaved.

On the other hand, the equilibrium condition for H is given by

$$\Pi_H(p) = p\alpha[1 - \beta(q_L^* + (1 - q_L^*)F_L(p))] \stackrel{!}{=} v_H\alpha(1 - \beta). \quad (16)$$

The rationale behind equation (16) is as follows: H 's expected profit when charging some price $p \in (\underline{p}, v_H)$ is given by this price times the expected mass of H -consumers who do not switch to L . There are $\alpha\beta$ flexible H -consumers who search, but they will only purchase on the way at L with probability $q_L^* + (1 - q_L^*)F_L(p)$ (as with the remaining probability, L charges a higher price than H and the consumers return). Hence, the expected mass of consumers who leave H is $\alpha\beta(q_L^* + (1 - q_L^*)F_L(p))$, implying that $\alpha - \alpha\beta(q_L^* + (1 - q_L^*)F_L(p)) = \alpha[1 - \beta(q_L^* + (1 - q_L^*)F_L(p))]$ consumers will stay at H in expectation, given p . For any price $p \in (\underline{p}, v_H)$, H 's expected profit has to be equal to H 's candidate equilibrium profit level of $v_H\alpha(1 - \beta)$.

Inserting q_L^* from equation (13) and solving for $F_L(p)$, it turns out that

$$F_L(p) = F_H(p). \quad (17)$$

Hence, also $F_L(\cdot)$ is well-behaved.

It still needs to be verified that the flexible H -consumers will always search initially, i.e., $q_L^*(\underline{p} - v_L) > s$. As it is known that $q_L^* \geq \frac{s}{v_H(1-\beta)-v_L}$ due to $\alpha \leq \bar{\alpha}(\beta)$, the above inequality is certainly fulfilled if $\frac{s}{v_H(1-\beta)-v_L}(\underline{p} - v_L) > s$, which implies $\frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta} > v_H(1 - \beta)$. Because the LHS of this inequality strictly decreases in α , it is straightforward to show that this is indeed the case if $\alpha \leq \bar{\alpha}(\beta)$, and $\beta < \bar{\beta}$.

One also has to prove that none of the firms has an incentive to deviate and charge a price outside of their specified range. To see that this is the case, note first that pricing above v_H can never be optimal, as this leads all consumers to drop out of the market and implies zero profits. Moreover, given H 's strategy, pricing below v_L or pricing strictly between v_L and \underline{p} can never be a profitable deviation for L . This is because for both $p_L < v_L$ and $p_L \in (v_L, \underline{p})$, L can increase its price to v_L (respectively, \underline{p}) without losing any demand. A similar logic prevents H from pricing below \underline{p} , however there is one exception. Namely, H could price at the flexible consumers' reservation price ρ , where $\rho < \underline{p}$ solves $q_L^*(\rho - v_L) = s$, and prevent all flexible H -consumers from engaging in search (implying a maximal deviation profit of $\rho\alpha$). This deviation is not profitable if and only if

$$\rho\alpha \leq v_H\alpha(1 - \beta),$$

which implies

$$v_L + \frac{s}{q_L^*} - v_H(1 - \beta) \leq 0. \quad (18)$$

Using the previous observation that $\alpha \leq \bar{\alpha}(\beta)$ implies $q_L^* \geq \frac{s}{v_H(1-\beta)-v_L}$, it is easy to prove that the above inequality is satisfied.

Lastly, it remains to show that $\bar{\alpha}(\beta) \in (\underline{\alpha}(\beta), 1)$ whenever $\beta < \underline{\beta}$, as claimed by the proposition. For $\bar{\alpha}(\beta) = \frac{v_L}{(1-\beta)\left\{v_L + \frac{v_H\beta[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s}\right\}}$ $> \frac{v_L}{\beta(v_H-v_L)+v_L} = \underline{\alpha}(\beta)$, a straightforward manipulation shows that this is indeed the case (for $\beta < \underline{\beta}$). On the other hand, the inequality $\bar{\alpha}(\beta) < 1$ can be reduced to $[(1-\beta)v_H - v_L]^2 + v_L\beta s > 0$ if $\beta > 0$, which is always satisfied. For $\beta = 0$, it holds that $\bar{\alpha}(\beta) = 1$.

All in all, the proposed strategy combination thus forms an equilibrium if $\beta < \bar{\beta}$ and $\alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)]$, where $\bar{\alpha}(\beta) \in (\underline{\alpha}(\beta), 1)$. This concludes the proof of Proposition 3. \square

Proof of Proposition 4. (Existence) Again, in order for the proposed strategy-combination to form an equilibrium, it is necessary that each price that is sampled by the firms with positive probability (probability density) must yield the same, maximal expected profit. Furthermore, all equilibrium objects need to be well-behaved (e.g., mass points must fall in the range $[0, 1]$). As in the proof of Proposition 3, I will solve for the outlined equilibrium in a constructive manner.

For this, suppose an equilibrium exists in which H prices at v_H with probability q_{H,v_H} , charges some lower price $\rho < v_H$ with probability $q_{H,\rho}$, and samples prices continuously from an interval $[\underline{p}, v_H]$, following a distribution function $G_H(\cdot)$, with probability $1 - q_{H,v_H} - q_{H,\rho}$. Assume moreover that $\underline{p} > \rho$ (this will be verified after solving for these equilibrium objects). At the same time, L prices at v_L with probability q_{L,v_L} and samples prices continuously from the same interval as H , $[\underline{p}, v_H]$, following a distribution function $G_L(\cdot)$, with probability $1 - q_{L,v_L}$.⁴⁰ Furthermore, assume that $q_{L,v_L}(\underline{p} - v_L) > s$ (hence, $\underline{p} > v_L$) and $q_{L,v_L}(\rho - v_L) \leq 0$. These two conditions imply that the flexible H -consumers find it worthwhile to search even when H charges \underline{p} (the lowest price in its continuous pricing range), but not when H charges ρ .

Given the specified strategies, H 's expected profit when pricing at v_H can be calculated to be

$$\Pi_H(v_H) = v_H\alpha(1 - \beta). \quad (19)$$

This is because if H prices at the highest possible price v_H , its flexible consumers will search and find a lower price with certainty. Hence, H can only sell to its captive consumers.

⁴⁰Although L has only one mass point at v_L in the equilibrium that will be specified, it is named q_{L,v_L} in order to distinguish it from the equilibrium object q_L^* of Proposition 3.

If H prices at \underline{p} , its expected profit is given by

$$\Pi_H(\underline{p}) = \underline{p}\alpha(1 - \beta q_{L,v_L}). \quad (20)$$

This is because if H prices at \underline{p} , all flexible H -consumers will search initially. However, they will return to H unless L prices at v_L (as otherwise, L certainly charges a higher price than \underline{p}), which happens with probably q_{L,v_L} .

Finally, if H prices at ρ , the flexible consumers will not find it optimal to search, and hence all H -market consumers will purchase in H . This implies that

$$\Pi_H(\rho) = \rho\alpha \quad (21)$$

Solving $\Pi_H(v_H) \stackrel{!}{=} \Pi_H(\rho)$ immediately gives rise to the first equilibrium condition,

$$\rho^* = v_H(1 - \beta). \quad (22)$$

Note next that it cannot be part of an equilibrium strategy that $q_{L,v_L}(\rho - v_L) < s$, implying that the flexible H -consumers strictly prefer to stay at H when they face a price of $p_H = \rho$. This is because H could charge a slightly higher price and still deter all flexible consumers from search, implying a profitable deviation. Hence, using that $q_{L,v_L}(\rho - v_L) \stackrel{!}{=} s$ and inserting ρ^* from equation (22), it follows that

$$q_{L,v_L}^* = \frac{s}{v_H(1 - \beta) - v_L}. \quad (23)$$

Observe that $q_{L,v_L}^* \in (0, 1)$ directly follows from $\beta < \bar{\beta} = 1 - \frac{v_L + s}{v_H}$. In particular, the denominator is positive (and hence, $\rho^* > v_L$), as $v_H(1 - \beta) - v_L > 0$ is equivalent to $\beta < 1 - \frac{v_L}{v_H}$, which is implied by $\beta < \bar{\beta}$.

Inserting q_{L,v_L}^* into equation (20) and solving $\Pi_H(\underline{p}) \stackrel{!}{=} \Pi_H(v_H)$, it is furthermore possible to solve for \underline{p} . Doing so, one finds that

$$\underline{p} = \frac{v_H(1 - \beta)[v_H(1 - \beta) - v_L]}{v_H(1 - \beta) - v_L - \beta s}. \quad (24)$$

It is easy to check that $\underline{p} > \rho^*$ is satisfied for $\beta < \bar{\beta}$. Thus, it is established that $\rho^* \in (v_L, \underline{p})$.

While $q_{L,v_L}^*(\rho^* - v_L) \leq s$ has already been shown (as $q_{L,v_L}^*(\rho^* - v_L) = s$), it still needs to be proven that $q_{L,v_L}^*(\underline{p} - v_L) > s$. Of course, this is trivially the case, as $\underline{p} > \rho^*$.

Next, turn to firm L . Its expected profit when pricing at v_H is given by

$$\Pi_L(v_H) = v_H \alpha \beta q_{H,v_H}. \quad (25)$$

This is a consequence from the tie-breaking rule specified in the model setup. Given that L prices at v_H , the firm will only sell to the flexible H -consumers when H prices at v_H , inducing its flexible consumers to search, which happens with probability q_{H,v_H} .

L 's expected profit when pricing at \underline{p} is equal to

$$\Pi_L(\underline{p}) = \underline{p} \alpha \beta (1 - q_{H,\rho}). \quad (26)$$

The above is true because \underline{p} always outperforms H 's price, but the flexible H -consumers only search when H does not price at ρ , which has a probability of $1 - q_{H,\rho}$. Moreover, since $\underline{p} > v_L$, L cannot serve its local low-valuation consumers.

Finally, L 's expected profit when pricing at v_L is equal to

$$\Pi_L(v_L) = v_L (1 - \alpha + \alpha \beta (1 - q_{H,\rho})). \quad (27)$$

This follows the same logic as equation (26), but at the low price v_L , L can also serve its local low-valuation consumers.

Solving $\Pi_L(\underline{p}) \stackrel{!}{=} \Pi_L(v_L)$ and inserting \underline{p} from equation (24), it is straightforward to establish that

$$q_{H,\rho}^* = 1 - \frac{v_L(1-\alpha)}{\alpha\beta(\underline{p}-v_L)} = 1 - \frac{v_L(1-\alpha)}{\alpha\beta\left(\frac{v_H(1-\beta)[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s} - v_L\right)} < 1. \quad (28)$$

Note that $q_{H,\rho}^* > 0$ is equivalent to $\underline{p} > \frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta}$. As $\underline{p} = \frac{v_H(1-\beta)[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s}$ does not depend on α while $\frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta}$ strictly decreases in α , the inequality is hardest to fulfill for the boundary level $\bar{\alpha}(\beta)$. Indeed, after a straightforward calculation, it turns out that the RHS equals the LHS for $\alpha = \bar{\alpha}(\beta)$. Hence, for every $\alpha > \bar{\alpha}(\beta)$ (as required by the proposition), it is in fact the case that $q_{H,\rho}^* > 0$. Thus, $q_{H,\rho}^*$ is well-behaved.

Next, inserting $q_{H,\rho}^*$ back into equation (26) and simplifying, one can explicitly solve for L 's equilibrium profit level Π_L^{***} . It holds that

$$\Pi_L^{***} = \frac{\underline{p} v_L (1 - \alpha)}{\underline{p} - v_L} = \frac{(1 - \alpha)(1 - \beta)v_H v_L [(1 - \beta)v_H - v_L]}{[(1 - \beta)v_H - v_L]^2 + v_L \beta s} > 0. \quad (29)$$

Using this, one can set Π_L^{***} equal to $\Pi_L(v_H)$ in order to solve for q_{H,v_H}^* . It is found that

$$q_{H,v_H}^* = \frac{\underline{p}v_L(1-\alpha)}{\alpha\beta v_H(\underline{p}-v_L)} = \frac{(1-\alpha)(1-\beta)v_L[(1-\beta)v_H-v_L]}{\alpha\beta\left\{[(1-\beta)v_H-v_L]^2+v_L\beta s\right\}} > 0. \quad (30)$$

Finally, note that $q_{H,\rho}^* + q_{H,v_H}^* = 1 - \frac{v_L(1-\alpha)(v_H-\underline{p})}{\alpha\beta v_H(\underline{p}-v_L)}$, which is clearly less than one. Hence, all of the characterized mass points are well behaved.

It remains to solve for the equilibrium distribution functions $G_H(\cdot)$ and $G_L(\cdot)$. Start with H . In order for H to be indifferent between any price in the interval (\underline{p}, v_H) , it has to hold that

$$\Pi_H(p) = p\alpha[1 - \beta(q_{L,v_L}^* + (1 - q_{L,v_L}^*)G_L(p))] \stackrel{!}{=} v_H\alpha(1 - \beta) = \Pi_H^{**}. \quad (31)$$

The logic behind equation (31) is as follows: H 's expected profit when charging some price $p \in (\underline{p}, v_H)$ is given by this price times the expected mass of H -consumers who do not switch to L . There are $\alpha\beta$ flexible H -consumers who search, but they will only purchase on the way at L with probability $q_{L,v_L}^* + (1 - q_{L,v_L}^*)G_L(p)$ (as with the remaining probability, L charges a higher price than H and the consumers return). Hence, the expected mass of consumers who leave H is $\alpha\beta(q_{L,v_L}^* + (1 - q_{L,v_L}^*)G_L(p))$, implying that $\alpha - \alpha\beta(q_{L,v_L}^* + (1 - q_{L,v_L}^*)G_L(p)) = \alpha[1 - \beta(q_{L,v_L}^* + (1 - q_{L,v_L}^*)G_L(p))]$ consumers will stay at H in expectation, given p . For any price $p \in (\underline{p}, v_H)$, H 's expected profit has to be equal to H 's candidate equilibrium profit level of $v_H\alpha(1 - \beta)$.

Inserting q_{L,v_L}^* from equation (23), solving for $G_L(p)$ and rearranging yields

$$G_L(p) = 1 - \frac{(1-\beta)[v_H(1-\beta)-v_L]}{\beta[v_H(1-\beta)-v_L-s]}(v_H/p-1). \quad (32)$$

It is easy to check that $G_L(\underline{p}) = 0$ and $G_L(v_H) = 1$. Moreover, since $v_H(1-\beta) - v_L - s > 0$ due to $\beta < \bar{\beta}$, it is obvious that $G_L(p)$ is strictly increasing in p . Hence, $G_L(\cdot)$ is well-behaved.

Next, consider L . In order for L to be indifferent between any price in the interval (\underline{p}, v_H) , it has to hold that

$$\Pi_L(p) = p\alpha\beta[q_{H,v_H}^* + (1 - q_{H,v_H}^* - q_{H,\rho}^*)(1 - G_H(p))] \stackrel{!}{=} \Pi_L^{***}. \quad (33)$$

The rationale behind equation (33) is simple: L 's expected profit when charging some price $p \in (\underline{p}, v_H)$ is given by this price times the mass of flexible H -consumers $\alpha\beta$ times the probability

that these consumers will search and buy at L (after finding out that p is lower than p_H), which is $q_{H,v_H}^* + (1 - q_{H,v_H}^* - q_{H,\rho}^*)(1 - G_H(p))$. For any price $p \in (\underline{p}, v_H)$, this expected profit has to be equal to L 's candidate equilibrium profit of Π_L^{***} (see equation (29) for the latter).

Inserting q_{H,v_H}^* and $q_{H,\rho}^*$ from equations (30) and (28) and solving for $G_H(p)$, after some tedious calculation it turns out that

$$G_H(p) = G_L(p). \quad (34)$$

Hence, also $G_H(\cdot)$ is well-behaved.

Lastly, it still needs to be verified that the firms do not have an incentive to price outside their specified ranges. Clearly, pricing above v_H is not a profitable deviation, as this implies zero demand. Also, pricing below ρ^* for H is certainly suboptimal: H can already deter its flexible consumers from leaving the market for $p_H = \rho^*$, so pricing lower than that is pointless. Could H find it worthwhile to set prices in (ρ^*, \underline{p}) ? The answer is no, because this does not generate any additional demand relative to pricing at \underline{p} : since $p_H > \rho^*$, the flexible H -consumers would still search and end up buying at L if $p_L = v_L$. For a similar reason, L cannot find it optimal to price in (v_L, \underline{p}) . This is because none of the prices in this interval will lead L 's local consumers to buy, and \underline{p} already beats all prices set by H which lead to search.

All in all, the proposed strategy combination thus forms an equilibrium if $\beta < \bar{\beta}$ and $\alpha \in (\bar{\alpha}(\beta), 1)$.

The final claim in the proposition states that as long as $\alpha > \alpha_{min} = \frac{v_H v_L}{v_H^2 - (v_L + s)(v_H - v_L)}$ and β lies sufficiently close below $\bar{\beta}$, the discussed strategy-combination constitutes an equilibrium. To see this, it suffices to check that $\bar{\alpha}(\bar{\beta}) = \alpha_{min}$. Hence, since $\bar{\alpha}(\beta)$ is a well-behaved function for all $\beta < \bar{\beta}$, it must “bend away” from $\bar{\beta}$ (when depicted as function of β) for $\beta < \bar{\beta}$. This implies that for $\alpha > \alpha_{min}$ and β sufficiently close to $\bar{\beta}$, it holds that $\alpha > \bar{\alpha}(\beta)$. See also Figure 2 for graphical intuition. This concludes the proof of Proposition 4. \square

Proof of Proposition 7. (Existence) Consider first the hypothetical equilibrium where $p_H^* = p^m(c)$, $p_L^* = p^m(0)$, and the flexible H -consumers always search and purchase in L . The latter is guaranteed by the assumption that $\Delta(CS) := \int_{p^m(0)}^{p^m(c)} D(p)dp > s$. Because of that, the flexible H -consumer find it indeed worthwhile to search given the proposed prices, as their expected gain in consumer surplus outweighs their search cost. It is moreover clear that L can have no profitable deviation: as all searching consumers from H have the same downward sloping demand schedule as the consumers in L , pricing at its monopoly price $p^m(0)$ is certainly optimal for L .

It remains to check that H has no profitable deviation. Given the proposed strategy-combination, its profit is given by $\Pi_H^* = \alpha(1 - \beta)\Pi^m(c)$. On the other hand, the firm's best possible deviation is to price at the flexible H -consumers reservation price ρ , where ρ solves $\int_{p^m(0)}^{\rho} D(p)dp = s$, and discourage them from search. This gives rise to a maximal deviation profit of $\Pi_H^{dev*} = \alpha(\rho - c)D(\rho)$. Solving $\Pi_H^{dev*} \leq \Pi_H^*$ for β , this deviation is not profitable if and only if $\beta \leq 1 - \frac{(\rho - c)D(\rho)}{\Pi^m(c)}$.

Now, an almost identical logic also applies for the proposed equilibrium where $p_H^* = \rho$, $p_L^* = p^m(0)$, and no search occurs in equilibrium. L can again do no better than to charge its monopoly price. As its price choice is unobservable by the flexible H -consumers, it cannot induce search by undercutting. On the other hand, H 's best possible deviation is to ignore its flexible consumers and increase its price to $p^m(c)$ in order to maximize its profit from its loyal consumer base. This is not profitable if $\beta \geq 1 - \frac{(\rho - c)D(\rho)}{\Pi^m(c)}$.

(Uniqueness) Note first that L will never price above $p^m(0)$ in equilibrium. This is because reducing its price towards $p^m(0)$ would generate a higher profit per consumer, while it may additionally win consumers that would otherwise purchase at H . Next, note that the lower bound of H 's pricing support, \underline{p}_H , must lie weakly above the lower bound of L 's pricing support, \underline{p}_L (where $\underline{p}_L \leq p^m(0)$ due to the first observation). Otherwise, if it was the case that $\underline{p}_H < \underline{p}_L$, H could profitably shift up probability mass to \underline{p}_L , which would increase its profit per consumer (as $\underline{p}_L \leq p^m(0) < p^m(c)$) and certainly not lose any demand to L .

Now, if it was the case that $\underline{p}_L < p^m(0)$, L could profitably shift up probability mass from around \underline{p}_L to a slightly higher level because of the following. First, doing so would not induce any flexible L -consumers to search due to their strictly positive search cost, and $\underline{p}_H \geq \underline{p}_L$. Second, the expected demand from searching H -consumers would not decrease either, as those consumers do not search anyway for prices p_H close to \underline{p}_L . Third, the expected profit per consumer would increase. Hence, $\underline{p}_L < p^m(0)$ cannot be part of any equilibrium, which establishes that L must play the pure strategy of $p_L^* = p^m(0)$ in every equilibrium. It is then trivial to see that H 's (generically) unique best response is either $p^m(c)$ or ρ , depending on β . \square

Appendix B: Uniqueness of Equilibria (not for publication)

In what follows, I will prove that the equilibria characterized in Propositions 1 to 4 are unique. This is established by a sequence of claims.

(A) *Pure-strategy equilibria*

Claim 1. *In any pure-strategy equilibrium, $p_L^* = v_L$.*

Proof. From the proof of Lemma 2 as stated above in the appendix, it is known that no firm will ever price below v_L in equilibrium (as a corollary, flexible L -consumers will never search). Hence, it remains to show that $p_L^* > v_L$ cannot be part of any pure-strategy equilibrium. Suppose to the contrary that it does (and hence, that L cannot sell to its local consumers). Then clearly, it must hold that $p_L^* + s < v_H$. Otherwise, the flexible H -consumers would not even search L after observing $p_H = v_H$, implying that L would not attract any consumers (which means L could profitably deviate to charging v_L). Given a pure p_L^* (with $p_L^* + s < v_H$), there is no need for H to randomize, such that one of the following two options must maximize its profit. First, if β is sufficiently large, H finds it optimal to price at $p_L^* + s$ and discourage its local flexible consumers from search. But this cannot be part of an equilibrium, as L would face zero demand (and could again profitably deviate to v_L). Second, if β is not that large, H finds it optimal to charge v_H . This fully exploits its local captive consumers, but due to $p_L^* + s < v_H$, the flexible H -consumers would all search and purchase at L . However, this cannot be part of an equilibrium either, as then L would have a profitable deviation by increasing its price to v_H . As this is unobserved by the flexible H -consumers, they would still search L and purchase there due to the specified tie-breaking rule. Hence, a pure-strategy equilibrium in which L charges a higher price than v_L can be ruled out. \square

Claim 2. *There are only two possible pure-strategy equilibria. First, if and only if $\beta \geq \bar{\beta} = 1 - \frac{v_L + s}{v_H}$, the pair $p_L^* = v_L, p_H^* = v_L + s$ constitutes an equilibrium. Second, if and only if $\beta \leq \bar{\beta}$ and $\alpha \leq \underline{\alpha}(\beta) = \frac{v_L}{\beta(v_H - v_L) + v_L}$, the pair $p_L^* = v_L, p_H^* = v_H$ constitutes an equilibrium.*

Proof. From Claim 1 it is known that $p_L^* = v_L$ in any pure-strategy equilibrium. Given this price and the flexible H -consumers' optimal search behavior, it is easy to check that H finds it strictly optimal to charge $v_L + s$ if $\beta > \bar{\beta}$, while it finds it strictly optimal to charge v_H if $\beta < \bar{\beta}$ (for $\beta = \bar{\beta}$, H is indifferent). In the first case, since the flexible H -consumers are discouraged from search, L cannot have a profitable deviation, as it already maximally exploits its local consumers. In the second case, since $v_H - v_L > s$ by assumption, all $\alpha\beta$ flexible H -consumers find it optimal

to search and buy at L . However, given this, L may have a profitable deviation by increasing its price to v_H , driving out its local consumers, but fully exploiting the incoming searchers. It is easy to check that this is not strictly profitable for L if $\alpha \leq \underline{\alpha}(\beta)$. \square

(B) Mixed-strategy equilibria

In what follows, denote the lower support bound of firm i 's pricing strategy, where $i \in \{L, H\}$, by \underline{p}_i and the upper support bound by \bar{p}_i , with $\bar{p}_i > \underline{p}_i$.

Claim 3. $\underline{p}_L = v_L$.

Proof. Again, from the proof of Lemma 2 as stated further above, it is known that no firm will ever price below v_L in equilibrium, and that the flexible L -consumers will never search. Now, suppose to the contrary of Claim 3 that $\underline{p}_L \in (v_L, v_H)$.⁴¹ Then the L -consumers will never buy in equilibrium. Moreover, it has to hold that $\underline{p}_H \geq \min\{\underline{p}_L + s, v_H\}$. The latter is true because H can already guarantee to discourage its flexible consumers from search for $p_H = \underline{p}_L + s$, so pricing any lower than that cannot be optimal (unless $\underline{p}_L + s > v_H$, in which case v_H should be played with certainty). But in turn, pricing in the interval $[\underline{p}_L, \min\{\underline{p}_L + s, v_H\})$ is dominated for L by pricing at $\min\{\underline{p}_L + s, v_H\}$, which contradicts the assertion that \underline{p}_L can be L 's lower support bound. \square

Claim 4. L must have a mass point at v_L .

Proof. Given $\underline{p}_L = v_L$, it must hold that $\underline{p}_H \geq v_L + s$, where $v_L + s < v_H$ by assumption. The reason is again that H can fully discourage its flexible consumers from search by charging $\underline{p}_L + s$, so pricing any lower than that is pointless. In turn, any price $p_L > v_L$ that L may charge in equilibrium must satisfy $p_L \geq v_L + s$. This implies that there is certainly a hole in L 's equilibrium price distribution over the range $(v_L, v_L + s)$, which shows that L must have a mass point at v_L (without a mass point on v_L , it would hold that $\underline{p}_L \geq v_L + s$, which contradicts Claim 3). \square

Claim 5. $\bar{p}_H = v_H$.

Proof. Suppose to the contrary that $\bar{p}_H \in (v_L + s, v_H)$.⁴² Then there are two possibilities. First, suppose that the flexible H -consumers search if they observe \bar{p}_H . Then, since it must hold that $\bar{p}_L \leq \bar{p}_H$ (otherwise, L would not make any sales for prices where $p_L > \bar{p}_H$, implying zero profits),

⁴¹Clearly, $\underline{p}_L = v_H$ cannot be part of an equilibrium, as this would never generate search from H , implying zero profits by L .

⁴²Since $\underline{p}_L = v_L$, it is clear that $\underline{p}_H \geq v_L + s$. For a mixed-strategy equilibrium, it also cannot hold that $\underline{p}_H = \bar{p}_H = v_L + s$ (such that H plays a pure strategy), as this would induce L to play the pure strategy v_L , giving rise to a pure-strategy equilibrium. Hence $\bar{p}_H > v_L + s$ must hold.

H could make a higher profit by pricing at v_H instead of \bar{p}_H , as this would not lose any additional consumers. Hence this cannot be part of an equilibrium. Second, suppose that the flexible H -consumers do not search at \bar{p}_H . If they strictly prefer not to search, H would have a profitable deviation by pricing slightly higher. If the flexible H -consumers are indifferent between searching and not searching for $p_H = \bar{p}_H$, it follows that H should concentrate all probability mass at \bar{p}_H (as it makes no sense to charge any price lower than \bar{p}_H if the latter already discourages the flexible H -consumers from search). That is, in the respective equilibrium, H would charge some deterministic price $p_H^* \in (v_L + s, v_H)$, and the flexible H -consumers would all stay at H (being indifferent between searching and not searching). But then, L 's dominant action would be to charge v_L with full probability mass in order to maximally exploit its local consumers. In turn, since the hypothesized p_H^* is larger than $v_L + s$, the flexible H -consumers *should* optimally search: a contradiction. \square

From now on, let \underline{p}'_L denote the lower support bound of L 's pricing strategy for prices that strictly exceed v_L . Furthermore, let ρ denote the flexible H consumers' reservation price, i.e. the price which makes them indifferent between visiting L and purchasing directly at H .

Claim 6. $\rho \in (v_L + s, \underline{p}'_L]$.

Proof. First, $\rho > v_L + s$ follows directly from $\underline{p}_L = v_L$ (see Claim 3) and the fact that L does not put full probability mass on v_L (if it did, a pure-strategy equilibrium would result). Hence, in order to make the flexible H -consumers indifferent between searching L and purchasing at H , choosing a price slightly larger than $v_L + s$ is sufficient for H . Second, in order to establish that $\rho \leq \underline{p}'_L$, suppose to the contrary that $\rho > \underline{p}'_L$. But then, the positive probability mass that L puts in the range $[\underline{p}'_L, \rho)$ could profitably be transferred to ρ , as the flexible H -consumers will not search anyway if H samples a price that is weakly lower than ρ (hence, charging $p_L = \rho$ already beats all the prices H may set which induce search). \square

Claim 7. *In any mixed-strategy equilibrium, H 's equilibrium profit is given by $\Pi_H^{**} = v_H\alpha(1-\beta)$.*

Proof. Because $\rho \leq \underline{p}'_L \leq \bar{p}_L \leq v_H$ (where the first inequality follows from Claim 6), the flexible H -consumers will certainly search and purchase at L if H prices at v_H . Hence, H 's expected profit at its equilibrium upper support bound (see Claim 5) is given by $v_H\alpha(1-\beta)$. This profit must obviously be achieved for any (equilibrium) price that H samples with positive probability density (mass). \square

Claim 8. *H must have a mass point at v_H .*

Proof. Suppose to the contrary that $\bar{p}_H = v_H$ (as established by Claim 5), but H has no mass point at v_H . Then there exists some $d > 0$ such that H puts a probability mass of less than $\frac{v_L(1-\alpha)}{v_H} < 1$ in the interval $[v_H - d, v_H]$. This in turn implies that L will not find it optimal at all to sample prices $p_L \geq v_H - d$, i.e. it must hold that $\bar{p}_L \leq v_H - d$. This is because by pricing in that interval, L 's profit is bounded above by $v_H * \Pr\{\tilde{p}_H \geq v_H - d\} < v_H * \frac{v_L(1-\alpha)}{v_H} = v_L(1-\alpha)$, where the latter profit could be guaranteed if L priced at v_L (since, by Lemma 2, L -consumers will never search and leave the L -market). Because of this, also H cannot find it optimal to put any probability mass in the interval $(v_H - d, v_H)$.⁴³ But if H has no mass point at v_H , this leads to a contradiction, as it would then follow that $\bar{p}_H \leq v_H - d$. \square

Claim 9. *If H samples ρ in equilibrium, it must be the case that H has a mass point at ρ , and that there is no probability mass below ρ or immediately above ρ .*

Proof. First, it is clear that H will not put any probability mass below ρ , as already pricing at ρ ensures that all H -consumers will stay in H (recall that the L -consumers will never search H due to Lemma 2). Moreover, since L has a mass point at v_L (see Claim 4), pricing marginally above ρ entails a discrete loss for H (since the flexible H -consumers will search and find a price of v_L with positive probability). Hence, H cannot put any probability mass immediately above ρ . Then, the fact that H can only sample ρ directly (and not any prices very close to ρ) already implies that H must have a mass point at ρ if ρ is sampled at all in equilibrium. \square

Claim 10. *H cannot put any probability mass in (ρ, \underline{p}'_L) .*⁴⁴

Proof. Suppose to the contrary that this was the case. Then any price p_H in that interval would lead to search by the flexible H -consumers. However, because L will only sell to these searching consumers if it prices at v_L (due to $p_H < \underline{p}'_L$), H has a profitable deviation to transfer all of its probability mass in (ρ, \underline{p}'_L) to a price arbitrarily close to \underline{p}'_L . \square

From now on, let \underline{p}'_H denote the lower support bound of H 's pricing strategy for prices that strictly exceed ρ .

Claim 11. $\underline{p}'_L = \underline{p}'_H =: \underline{p}$.

⁴³As $\rho \leq \underline{p}'_L \leq \bar{p}_L$ (where the first inequality follows from Claim 6), any price that H samples in $(v_H - d, v_H)$ would induce search by the flexible H -consumers, which leads them to leave H with certainty (due to $\bar{p}_L \leq v_H - d$). Therefore, H strictly prefers to sample v_H in order to maximally exploits its captive consumers.

⁴⁴Recall that $\rho \leq \underline{p}'_L$ due to Claim 6.

Proof. Claim 10 already established that $\underline{p}'_H \geq \underline{p}'_L$. It remains to show that it cannot be the case that $\underline{p}'_H > \underline{p}'_L$. To see this, suppose that the latter relation holds. But then, due to Claim 6, the flexible H consumers will always search when H does not price at ρ (if it does so at all), and thus L could profitably deviate by transferring all of its probability mass in the interval $[\underline{p}'_L, \underline{p}'_H)$ to \underline{p}'_H . \square

Claim 12. $\bar{p}_L = \bar{p}_H = v_H$. In contrast to H , L can have no mass point at v_H .

Proof. The second equality is given by Claim 5. To show the first equality, suppose to the contrary that $\bar{p}_L < \bar{p}_H = v_H$. Then clearly, because $\rho \leq \bar{p}_L$ (as follows e.g. from Claim 6), H will not find it optimal to put any probability mass in $[\bar{p}_L, v_H)$, as this is strictly dominated by pricing at v_H (and at least fully exploiting its captive consumers). But in turn, it cannot be optimal for L to sample \bar{p}_L , as this will only win H 's flexible consumers if H prices at v_H . Hence, by deviating to v_H , L could unilaterally increase its profit. The last claim is obvious: since H has a mass point at v_H due to Claim 8, L cannot also have a mass point at the same price. If it did, H could choose its mass point at an ϵ lower price and induce the flexible L -consumers to return whenever L samples v_H , which would lead to a discrete increase in H 's profit. \square

Claim 13. Neither H nor L can have a mass point in $[\underline{p}, v_H)$.

Proof. Suppose to the contrary that H (L) does have a mass point at some price $\hat{p} \in [\underline{p}, v_H)$. Then there must exist some $d > 0$ such that L (H) will never find it optimal to price in $(\hat{p}, \hat{p} + d]$, as L 's (H 's) profit drops discontinuously at \hat{p} . But then, H (L) should not have a mass point at \hat{p} in the first place, as pricing closer to $\hat{p} + d$ would give the firm a strictly higher profit. \square

Claim 14. If one firm puts no probability mass in some interval $[a, b] \subset [\underline{p}, v_H)$, the other firm cannot do so either.

Proof. Suppose to the contrary that only one firm puts positive probability mass in $[a, b]$. But then, it must have a mass point at b , as pricing anywhere in (a, b) gives the firm a strictly lower expected profit than pricing at b . However, this contradicts Claim 13. \square

Claim 15. The firms cannot have any holes in their pricing range above \underline{p} .

Proof. Suppose they do. Then, examine the lowest of such holes, and denote its infimum by $z > \underline{p}$. Clearly, as the firms can have no mass points in $[\underline{p}, v_H)$ due to Claim 13, it cannot be optimal for either firm to price at z and close below, as shifting this probability mass towards

the top of the lowest hole yields a strictly higher expected profit. In particular, this is always possible due to Claim 14. \square

To sum up, the above claims establish the following. First, only two pure-strategy equilibria exist. These are given by $p_L^* = v_L$ and $p_H^* = v_L + s$ (for $\beta \geq \bar{\beta}$) and $p_L^* = v_L, p_H^* = v_H$ (for $\beta \leq \bar{\beta}$ and $\alpha \leq \underline{\alpha}(\beta)$). Second, any mixed-strategy equilibrium must satisfy the following: (i) no firm ever prices below v_L , (ii) L has a mass point on v_L , (iii) H has a mass point on v_H , (iv) if H samples the flexible H -consumers' reservation price ρ , it must have a mass point on it (and it holds that $v_L < \rho < \underline{p}'_L$), (v) there can be no other mass points, (vi) both L and H spread probability mass over a common interval $[\underline{p}, v_H]$, and (vii) this interval does not contain any holes.

It is now straightforward to check that each of the four equilibria in Propositions 1 to 4 fulfills these criteria. Moreover, it can be verified that only the specified equilibria in their respective parameter ranges do indeed constitute equilibria.

More specifically, Proposition 1 states that whenever $\beta > \bar{\beta}$, the unique equilibrium of the game is such that $p_L^* = v_L, p_H^* = v_L + s$, and no consumers search. The candidate equilibria of Propositions 2 to 4 must all fail, as in each of these, firm H 's expected profit is given by $v_H \alpha (1 - \beta)$, which, for $\beta > \bar{\beta}$, is strictly worse than what H could achieve by deviating to $v_L + s$ (and thereby, discouraging its local consumers from search).

Proposition 2 states that whenever $\beta < \bar{\beta}$ and $\alpha \leq \underline{\alpha}(\beta)$, the unique equilibrium of the game is such that $p_L^* = v_L$ and $p_H^* = v_H$. Clearly, the equilibrium of Proposition 1 must fail, because for $\beta < \bar{\beta}$, H finds discouraging its local consumers from search by pricing at $v_L + s$ to be dominated by pricing at v_H . The equilibrium of Proposition 3 must fail, for example because $\alpha \leq \underline{\alpha}(\beta)$ implies $q_H^* \geq 1$ (with strict equality for $\alpha < \underline{\alpha}(\beta)$). The equilibrium of Proposition 4 must also fail, as $\alpha \leq \underline{\alpha}(\beta)$ together with $\beta < \bar{\beta}$ implies that $q_{H,\rho}^* < 0$.

Proposition 3 claims that there is a unique mixed-strategy equilibrium with mass points on v_H (by H) and v_L (by L) if $\beta < \bar{\beta}$ and $\alpha \in (\underline{\alpha}(\beta), \bar{\alpha}(\beta)]$. Again, the pure-strategy equilibrium of Proposition 1 must fail because discouraging its local consumers from search by pricing at $v_L + s$ is not profitable for H when $\beta > \bar{\beta}$. The pure-strategy equilibrium of Proposition 2 must also fail, as $\alpha > \underline{\alpha}(\beta)$ implies that L could profitably deviate from $p_L^* = v_L$ to charging v_H . The equilibrium of Proposition 4 must fail, as $\alpha \leq \bar{\alpha}(\beta)$ implies $q_{H,\rho}^* \leq 0$.

Proposition 4 says that there is a unique mixed-strategy equilibrium with mass points on v_H and $\rho^* = v_H(1 - \beta)$ (by H) and v_L (by L) if $\beta < \bar{\beta}$ and $\alpha > \bar{\alpha}(\beta)$. Clearly, the equilibria

of Propositions 1 and 2 must fail due to the same reasons as given above (in particular, the equilibrium of Proposition 2 can be ruled out because $\alpha > \bar{\alpha}(\beta)$ implies $\alpha > \underline{\alpha}(\beta)$ for $\beta < \bar{\beta}$). Lastly, the equilibrium of Proposition 3 must fail because of the following. In this candidate equilibrium, L charges v_L with probability $q_L^* = \frac{1}{\beta} - \frac{v_H \alpha (1-\beta)}{v_L (1-\alpha + \alpha \beta)}$, while both L and H samples prices from an identical continuous interval $[\underline{p}, v_H]$, with $\underline{p} = \frac{v_L (1-\alpha + \alpha \beta)}{v_H \alpha \beta} > v_L$. Suppose that q_L^* is well-behaved, such that it falls in the range $(0, 1)$ (if this is not the case, the considered equilibrium fails anyway). Now there are three situations to consider. In parameter constellations where $q_L^*(\underline{p} - v_L) > s$, the flexible H -consumers would even find it optimal to search L if firm H charged its lowest equilibrium price \underline{p} . A potentially profitable deviation by H is then to charge a price $\rho' < \underline{p}$ that satisfies $q_L^*(\rho' - v_L) = s$ (i.e., $\rho' = \frac{s}{q_L^*} + v_L$), which is sufficient to discourage the flexible H -consumers from search. This gives H a deviation profit of $\rho' \alpha = \left(\frac{s}{q_L^*} + v_L \right) \alpha$, compared to the candidate equilibrium's profit of $v_H \alpha (1 - \beta)$. Hence, the equilibrium of Proposition 3 fails to exist if $\frac{s}{q_L^*} + v_L > v_H (1 - \beta)$. Since q_L^* is strictly decreasing in α and $q_L^* = \frac{s}{v_H (1-\beta) - v_L}$ for $\alpha = \bar{\alpha}(\beta)$, it is easy to check that this is indeed the case if $\alpha > \bar{\alpha}(\beta)$. For parameter constellations where $q_L^*(\underline{p} - v_L) < s$ (such that the flexible H -consumers would not want to search when facing a price of \underline{p} or slightly higher), the candidate equilibrium of Proposition 3 clearly breaks down. This is because firm H could sample (slightly) higher prices than \underline{p} without losing any demand, implying a profitable deviation. Finally, if it holds exactly that $q_L^*(\underline{p} - v_L) = s$, the flexible H -consumers would not search if H charges exactly \underline{p} , but they would do so for any higher price in the firm's pricing domain. It follows that H would have a profitable deviation by transferring probability mass from close above \underline{p} to \underline{p} .