A Complete Characterization of Equilibria in Two-type Common Agency Screening Games

Martimort, David and Semenov, Aggey and Stole, Lars

Paris School of Economics-CREST, University of Ottawa, University of Chicago

19 August 2015

Working paper

A COMPLETE CHARACTERIZATION OF EQUILIBRIA IN TWO-TYPE COMMON AGENCY SCREENING GAMES[1]

August 20, 2015

David Martimort, Aggey Semenov and Lars Stole

Abstract. We characterize the complete set of equilibrium allocations to a two-type intrinsic common agency screening game as the set of solutions to a *self-generating* optimization program. The program, in turn, can be thought of as a maximization problem facing a fictional "surrogate" principal with a simple set of incentive constraints that embed the non-cooperative behavior of principals in the underlying game. After providing a complete characterization of equilibrium outcomes, we refine the set by imposing a requirement of *biconjugacy* on equilibrium tariffs: In biconjugate equilibria, the surrogate principal's incentive constraints are described by marginal conditions. *Biconjugate equilibria* always exist, they are simple to compute, and they are robust in the sense that they remain equilibria when *"out-of-equilibrium"* output-price pairs are pruned. After characterizing the set of biconjugate equilibrium allocations, we ask what is the best equilibrium for the principals from an *ex ante* perspective. We show that the allocation that maximizes the principals' *ex ante* collective payoff among all possible equilibria is distinct from the best allocation in the refined set of biconjugate equilibria, although their qualitative properties are similar.

Keywords. Intrinsic common agency, aggregate games.

JEL codes. D82, D86.

1. INTRODUCTION

Motivation and Objectives. We consider a canonical class of common agency games in which the agent has private information, his action is publicly contractable by all principals, and he must either accept all contract offers from the principals or choose not to participate. Common agency is thus *public* and *intrinsic*. As a motivating example, suppose there are multiple government agencies (principals) who regulate a polluting, public utility (the common agent) who has private information about the cost of production. If the firm decides to produce, it is under the joint control of all regulators. Regulators, however, may have conflicting objectives. For example, an environmental agency wishes on the margin to reduce output; a public-utility commission instead prefers to increase output (and thereby increase consumer surplus). Each regulator simultaneously offers a menu of transfer-output pairs in order to influence the choice of the public utility.

One of the main theoretical difficulties when modeling such non-cooperative scenarios is the characterization of all of the equilibrium outcomes that might arise. Previous studies have typically focused on a particularly tractable equilibrium (e.g., differentiable equilibria in games with a continuum of types) rather than exploring the entire set of possibilities. Because of the arbitrariness of such selection, *a priori*, comparative statics and economic implications might be fragile. Moreover, such a narrow focus might fail

[1] The usual disclaimer applies.

[a]Paris School of Economics-CREST, david.martimort@parisschoolofeconomics.eu.

[b]University of Ottawa, aggey.semenov@uottawa.ca.

[c]University of Chicago, lars.stole@chicagobooth.edu.

to give a full account of the welfare cost of the principals' non-cooperative behavior. We view these two concerns as significant weaknesses of the common agency paradigm that presently hinder the development of common agency applications. A sounder, more complete approach – the tack of the present paper – would characterize the entire set of equilibria in order to understand the full import of a particular equilibrium refinement.

The goal of this paper is to make progress on three fronts: equilibrium characterization, equilibrium selection, and welfare comparisons. First, we characterize the complete set of equilibria of a canonical intrinsic common agency game where the agent's private information may only take two values. Although specific, this two-type screening model is widely adopted in applications. Second, we refine this set by means of a robustness criterion coined *biconjugacy*. Roughly speaking, this criterion selects equilibrium schedules which are robust in the sense that pruning *"out-of-equilibrium"* price-quantity pairs does not destroy the equilibrium. We provide a complete characterization of the set of *biconjugate equilibria*. Finally, for both the full set of equilibria and its biconjugacy refinement, we determine the best equilibrium from the principals' point of view. These characterizations and the corresponding welfare analysis provide a set of tools for researchers interested in strategic agency settings with competing principals.

Self-Generating problems. Our first step towards a full characterization of equilibria relies on the fundamental structure of intrinsic common agency games. As noted by Martimort and Stole (2012), intrinsic common agency games are a special case of *aggregate games:* Principal i's expected payoff depends only upon principal i's contract and the *aggregate contract* offered by the non-cooperating principals. Because each principal can always *"undo"* the aggregate contracts offered by others with his own contract and thereby induce his most preferred allocation within the set of incentive-feasible allocations, in equilibrium the principals must prefer to induce the same mapping from an agent's type to his choice. This property is the key ingredient in characterizing the set of *all* equilibrium outcomes as solutions to simple *self-generating* optimization problems (Proposition 1 below).[1] Such problems are *self-generating* in the sense that each solution maximizes an objective function that is a linear function of an aggregate contract which, in turn, implements the solution. Thus, a solution appears both in the maximand and as as a maximizer, generating a fixed-point.

A key difference between such *self-generating* problem and the cooperative problem that n principals would face if they were able to collude in the design of the agent's incentive contract, is that in the former the agent's rent is weighted n times more than in the latter. This n-fold excess weighting captures the fact that, in the non-cooperative scenario, each of the n principals attempts to extract the agent's information rent for himself. Beyond this difference with the cooperative design, the two problems are otherwise similar. Everything happens as if a *surrogate principal* maximizes the *self-generating* problem with its bias toward over extracting the agent's information rent.

[1] The intuition for this result was first explained in a moral hazard setting by Bernheim and Whinston (1986a) who observed that, under intrinsic common agency, each principal can always undo the aggregate offers made by his rivals without changing the set of implementable allocations. The same trick, referred to as an *Aggregate Concurrence Principle*, applies here also even though our model is one of adverse selection and it applies as well in a broader class of aggregate games as shown in Martimort and Stole (2012).

Characterization of Equilibrium Sets. Martimort and Stole (2012) have used the force of these self-generating problems to prove equilibrium existence in intrinsic common agency games under quite general conditions (general type spaces and action sets, general preferences). In the present paper, we restrict preferences so that the agent's payoff is bilinear in action and type and assume that the agent's type takes on one of two values in order to pursue our more ambitious goal of characterizing the set of equilibria and their corresponding welfare properties. Our first result (Theorem 1) characterizes the whole set of equilibrium allocations by means of a new set of incentive constraints that apply to a fictional "surrogate" principal's program.

The set of equilibrium allocations described by these constraints may be quite large, sometimes even including the cooperative outcome. More collusive outcomes can be facilitated by the principals' use of forcing contracts that sufficiently raise the costs to a deviating principal for inducing the agent to choose some *"out-of-equilibrium"* output. Our complete characterization stands in sharp contrast with earlier contributions (Laffont and Tirole (1993), Martimort (1992), Stole (1992), Martimort and Semenov (2008), Martimort and Stole (2009a, 2009b) among others) that studied intrinsic common agency screening games with a continuum of types, but with a narrow focus on smooth equilibria.

Biconjugate Equilibria. While we are interested in a complete characterization of all equilibria in our two-type setting, we are also interested in the subset of equilibria which are analogues to the continuously differentiable equilibria of games with a continuum of agent types. As we will argue below, the appropriate analogue to smooth equilibria are those in which principals offer proper concave schedules taking only finite values.[2] To this end, we define *biconjugate* tariffs as the least-concave schedules that implement a given allocation. This property is satisfied in models with a continuum of types by tariffs which are *"smooth"* at all equilibrium points: The aggregate contract and the information rent profile that such a schedule implements are always conjugates to each other in the sense of convex analysis. Even though this property is more demanding when the type space and the quantity space no longer have the same dimensionality, using *biconjugate* tariffs allows one to import the tractability of convex calculus to non-smooth environments. Unsurprisingly, the qualitative predictions obtained with *biconjugate equilibria* in our discrete-type setting replicate the smooth equilibria of continuous-type games (Theorem 2). *Biconjugate equilibria* feature two properties already found in continuous-type models: an output which is first-best for the low-cost agent, and distortions for the high-cost type that are more pronounced than in the cooperative solution. These distortions are reminiscent of the familiar n-fold marginalization found in common agency models with a continuum of types.

In addition to their desirable mathematical properties, biconjugate tariffs are also robust in a strategic sense. Pruning *'out-of-equilibrium"* price-output pairs from biconjugate menus has no effect on the equilibrium allocation. Insisting on such schedules thus offers an attractive refinement. Among other things, we show that collusive equilibrium outcomes which rely on extreme forcing contracts may fail to be outcomes of biconjugate equilibria because the players cannot impose sufficient costs on a deviating principal who attempts to induce an *"out-of-equilibrium"* output. Thus, biconjugate equilibria form a

[2]In another context, competition between buyers with exclusive contracts, Attar et al. (2014a and 2014b) have also analyzed the properties of schedules in discrete-type models.

strict subset of the equilibrium set (Proposition 4).

Welfare. The best biconjugate equilibrium from the principals' point of view is as close as possible to the cooperative outcome. Yet, this solution is always dominated by an equilibrium implemented with forcing contracts (Propositions 5 and 6). In other words, insisting on robust equilibrium allocations entails a welfare loss for the principals. Better equilibrium outcomes are possible if principals insist on forcing contracts.

Organization. Section 2 presents the model. Section 3 defines the *self-generating problems*. Section 4 characterizes all equilibrium allocations. Section 5 motivates and analyzes *biconjugate tariffs* and *biconjugate equilibria*. Section 6 presents our welfare analysis. Proofs are relegated to Appendix A. For completeness and as a benchmark that can be skipped in first reading, Appendix B analyzes the case of complete information.

2. AN INTRINSIC COMMON AGENCY GAME

The focus of this paper is on common agency games with n principals (indexed by $i \in \{1, .., n\}$), each of who contracts with a single common agent. We assume that common agency is *intrinsic* and the choice variable of the agent is *public* (i.e., commonly observable and contractible by all principals). The timing is typical of principal-agent screening games, but now with n principals contracting instead of one.

In period one, the agent privately learns his type (a cost parameter), $\theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$, where $\theta = \underline{\theta}$ with probability ν and $\theta = \overline{\theta}$ with probability $1-\nu$. We denote $\Delta\theta = \overline{\theta} - \underline{\theta} > 0$ as the difference between types, and we use the notation $\mathrm{E}_\theta[\cdot]$ as expectation operator for the type distribution.

In period two, each principal simultaneously offers the agent a tariff, $T_i : \mathcal{Q} \to \mathbb{R}$, which is a promise to pay $T_i(q)$ to the agent following the choice of $q \in \mathcal{Q} = [0, Q]$. Our assumption that common agency is *public* is captured by the fact that each principal contracts on the same observed choice by the agent.

In period three, the agent either accepts or rejects all of the principals' offers. Our assumption that common agency is *intrinsic* means that the agent must either accept all contracts or reject all contracts; partial participation is not an option.[3] We denote the agent's acceptance decision by the strategy $\delta = 1$ and rejection by $\delta = 0$. If all contracts are accepted, the agent then chooses $q \in \mathcal{Q}$ to maximize his utility and receives payments from each principal according to their contractual offers. We assume that if the agent rejects the contracts, then by default all transfers are nil. Thus, the agent's period-three strategy is a pair, $\{\delta, q\}$, depending upon the agent's type and the contracts offered by the principals.

Preferences. Each principal i and the agent have preferences over output, $q \in \mathcal{Q} \equiv [0, q_{\max}]$, and payments that are, respectively, defined as

$$S_i(q) - t_i \quad \forall i \in \{1, .., n\} \text{ and } \sum_{i=1}^{n} t_i - \theta q.$$

[3]Martimort and Stole (2015) analyze the case of delegated common agency where a proper subset of the principals' offers can be rejected (i.e., common agency is *delegated*) which amounts to impose that transfers are non-negative.

We assume that the payoff functions S_i (for $i \in \{1,..,n\}$) are concave and twice continuously differentiable. Without loss of generality, we normalize $S_i(0) = 0$, so $S_i(q)$ should be understood as the net utility of principal i relative to the non-contractual default, $q = 0$.[4] Observe that if the agent rejects the principals' contracts ($\delta = 0$), he will choose $q = 0$ obtain zero payoff.

Assumption 1

$$S'(0) > \overline{\theta} + \left(\frac{n\nu}{1-\nu} + n - 1\right)\Delta\theta \quad \text{and} \quad S'(q_{\max}) < \underline{\theta}.$$

Strategy spaces. From the *Delegation Principle*,[5] there is no loss of generality in studying pure-strategy common agency equilibria to require that principals' strategy spaces are restricted to tariffs from output to transfers. As such, we denote each principal's strategy space, $\mathcal{T}$, as the set of all upper semicontinuous mappings, T_i, from $\mathcal{Q}$ into $\mathbb{R}$ (for $i \in \{1,..,n\}$).[6] We denote $\mathbf{T} = (T_1, ..., T_n) \in \mathcal{T}^n$ as an arbitrary array of contracts. An *aggregate contract* (or, in short, an *aggregate*) is defined as $T(q) = \sum_{i=1}^n T_i(q)$. We also use the familiar notation $\mathbf{T}_{-i}$ and $T_{-i}(q) = \sum_{j\neq i} T_j(q)$ to denote, respectively, an array of contracts and the aggregate contract from all principals but i.

Equilibrium: Our focus in this paper is on equilibrium allocations that arise in a pure-strategy Perfect Bayesian equilibrium.

Definition 1 *An **equilibrium** is a $n+2$-tuple $\{\overline{T}_1, \ldots, \overline{T}_n, \overline{q}_0, \overline{\delta}_0\}$ (with aggregate $\overline{T}(q) = \sum_{i=1}^n \overline{T}_i(q)$) such that*

1. *$\overline{q}_0(\theta, \mathbf{T})$ and $\overline{\delta}_0(\theta, \mathbf{T})$ jointly maximize the agent's payoff:*

$$\{\overline{q}_0(\theta, \mathbf{T}), \overline{\delta}_0(\theta, \mathbf{T})\} \in \underset{q\in\mathcal{Q},\delta\in\{0,1\}}{\arg\max}\ \delta T(q) - \theta q.$$

2. *$\overline{T}_i$ maximizes principal i's expected payoff given the other principals' contracts $\overline{\mathbf{T}}_{-i}$:*

$$\overline{T}_i \in \underset{T_i\in\mathcal{T}}{\arg\max}\, E_\theta\left[S_i(\overline{q}_0(\theta, T_i, \overline{\mathbf{T}}_{-i})) - \overline{\delta}_0(\theta, \mathbf{T}) T_i(\overline{q}_0(\theta, T_i, \overline{\mathbf{T}}_{-i}))\right].$$

Remark. There always exist uninteresting, trivial equilibria induced by a coordination failures in which two or more principals require sufficiently negative payments for each $q \in \mathcal{Q}$ so that it is not profitable for any principal to induce agent participation. We will demonstrate that outcomes in which both types participate and produce positive outputs are also equilibria, given Assumption 1.

In what follows, it will be useful to refer to the set of type-allocation mappings that are implementable for some aggregate tariff, $\mathcal{I}$, and the set of type-allocation mappings that are implementable and arise in some equilibrium, $\mathcal{I}^{eq}$.

[4]More generally, suppose that the agent's payoff is $\sum_{i=1}^n t_i + S_0(q) - \theta q$, where S_0 is a concave function normalized at $S_0(0) = 0$. Redefine payments from each principal and their respective payoff functions so that $\tilde{t}_i = t_i + \frac{S_0(q)}{n}$ and $\widetilde{S}_i(q) = S_i(q) + \frac{S_0(q)}{n}$ (for $i \in \{1,..,n\}$). One can verify that $\widetilde{S}_i(0) = 0$ and the expressions for the principals' and the agent's utility functions can be written, respectively, as $\tilde{S}_i(q) - \tilde{t}_i$ and $\sum_{i=1}^n \tilde{t}_i - \theta q$, just as we assume in the main text.

[5]See Peters (2001) and Martimort and Stole (2002).

[6]Upper semi-continuity ensures existence of continuation equilibria following acceptance. There always exists an optimal output for any array of contracts that are accepted.

Definition 2 *We say that the type-allocation mapping, $\{U, q, \delta\}$, $U : \Theta \to \mathbb{R}$, $q : \Theta \to \mathcal{Q}$, $\delta : \Theta \to \{0, 1\}$, is* ***implementable*** *if there is an aggregate tariff, $T : \mathcal{Q} \to \mathbb{R}$ such that*

$$(q(\theta), \delta(\theta)) \in \arg\max_{q\in\mathcal{Q},\delta\in\{0,1\}} \delta T(q) - \theta q,$$

$$U(\theta) = \max_{q\in\mathcal{Q},\delta\in\{0,1\}} \delta T(q) - \theta q.$$

The set of all implementable allocations is denoted $\mathcal{I}$.

We say that the type-allocation mapping $\{\overline{U}, \overline{q}, \overline{\delta}\}$ is an ***equilibrium allocation*** *or* ***equilibrium implementable*** *if it is implementable by an aggregate tariff, $\overline{T}$, that arises as part of an equilibrium. The set of all equilibrium allocations is denoted $\mathcal{I}^{eq}$.*

For any equilibrium, $\{\overline{\mathbf{T}}, \overline{q}_0, \overline{\delta}_0\}$, we define the associated type-allocation functions

$$\overline{\delta}(\theta) = \overline{\delta}_0(\theta, \overline{\mathbf{T}}),$$

$$\overline{q}(\theta) = \overline{q}_0(\theta, \overline{\mathbf{T}}),$$

$$\overline{U}(\theta) = \delta(\theta)\overline{T}(\overline{q}(\theta)) - \theta\overline{q}(\theta),$$

and we refer to the triplet $\{\overline{U}, \overline{q}, \overline{\delta}\}$ as the equilibrium allocation. As we will see below in Proposition 1, for any equilibrium allocation $\{\overline{U}, \overline{q}, \overline{\delta}\}$, there exists another equilibrium allocation in which the agent always participates, $\delta(\theta) = 1$ for all θ, and so we will subsequently focus our attention on the pair $\{\overline{U}, \overline{q}\}$ and suppress the type-allocation mapping for δ.

First-Best Outcome. The first-best allocation $\{U^{fb}, q^{fb}\}$ is obtained when principals cooperate and know the agent's cost parameter. In this scenario, principals jointly request production at the first-best level, $q^{fb}(\theta)$, and set transfers which fully extract the agent's surplus:

$$S'(q^{fb}(\theta)) = \theta \text{ and } U^{fb}(\theta) = 0 \quad \forall\theta \in \theta.$$

This outcome is also one possible equilibrium of the intrinsic common agency game when it takes place under complete information. Under complete information, and in sharp contrast with the analysis under asymmetric information, the principals' non-cooperative behavior need not entail any welfare loss. However, many other inefficient equilibria exist.[7] For completeness, the analysis of this complete information setting is relegated to Appendix B.

Cooperative outcome. Another useful benchmark is that which arises when all principals cooperatively design an aggregate contract, but under asymmetric information. The solution to this monopolistic screening problem is well known in two-type models.

[7] Bernheim and Whinston (1986b) have analyzed delegated common agency games under complete information and proposed a specific refinement (*"truthfulness"*) to select among those equilibria.

The low-cost agent produces the first-best output, $q^{coop}(\underline{\theta}) = q^{fb}(\underline{\theta})$, and gets an information rent worth $U^{coop}(\underline{\theta}) = \Delta\theta q^{coop}(\overline{\theta})$. In contrast, the high-cost agent gets no rent, $U^{coop}(\overline{\theta}) = 0$ and produces a positive output (thanks to Assumption 1), $q^{coop}(\overline{\theta})$, which is less than the first-best level $q^{fb}(\overline{\theta})$:

$$S'(q^{coop}(\overline{\theta})) = \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta. \tag{2.1}$$

This cooperative outcome can be implemented with a variety of aggregate transfers. First, consider the following *forcing contract*:

$$\overline{T}(q) = \begin{cases} \overline{\theta}q^{coop}(\overline{\theta}) & \text{if } q = q^{coop}(\overline{\theta}) \\ \underline{\theta}q^{coop}(\underline{\theta}) + \Delta\theta q^{coop}(\overline{\theta}) & \text{if } q = q^{coop}(\underline{\theta}) \\ -\infty & \text{if } q \notin \{q^{coop}(\overline{\theta}), q^{coop}(\underline{\theta})\}. \end{cases}$$

With such forcing contract, the agent chooses his preferred output from the set $\{q^{coop}(\underline{\theta}), q^{coop}(\overline{\theta})\}$ and gets paid accordingly. Any other choice under contract acceptance would imply negative utility and the agent would be better off rejecting the offers.

There are other aggregate transfers which implement $\{U^{coop}, q^{coop}\}$ that are less extreme than the forcing contract above. Consider the aggregate contract T^{bc} (the superscript "bc" refers to the biconjugate property) which takes finite values over the whole domain $\mathcal{Q}$:

$$\overline{T}^{bc}(q) = \min_{\theta\in\theta} U^{coop}(\theta) + \theta q = \min\left\{U^{coop}(\underline{\theta}) + \underline{\theta}q; \overline{\theta}q\right\}.$$

Observe that $\overline{T}^{bc}$ is piecewise linear, increasing, concave and satisfies $\overline{T}^{bc}(0) = 0$. In particular, this schedule satisfies the following *biconjugacy* condition:

$$\overline{T}^{bc}(q) = \min_{\theta\in\theta}\left\{\max_{q'\in\mathcal{Q}}\left\{\overline{T}^{bc}(q') - \theta q'\right\} + \theta q\right\}.$$

It is straightforward to see that $\overline{T}^{bc}$ establishes a maximal implementing tariff in the sense that the cooperative allocation (U^{coop}, q^{coop}) can also be implemented by any other nonlinear tariff T that lies below $\overline{T}^{bc}$ and such that $T(q^{coop}(\theta)) = \overline{T}^{bc}(q^{coop}(\theta))$ for all $\theta \in \Theta$. In other words, $\overline{T}^{bc}$ is the least concave schedule that implements the cooperative outcome.

Although insisting on one implementing contract or the other has no impact in a monopoly screening environment, the addition of (meaningful) extra price-quantity options in a tariff may play a role in a non-cooperative setting because they may affect the cost to a principal of offering a deviating contract.

These preliminary remarks serve as a first motivation our analysis in Section 5 which demonstrates how the requirement of *biconjugacy* refines the equilibrium set.

3. EQUILIBRIA AS SOLUTIONS TO SELF-GENERATING PROBLEMS

Martimort and Stole (2012) demonstrate that intrinsic common agency games are aggregate games whose equilibria can be identified with the solution set to an optimization problem. Specializing the necessary and sufficient conditions in their Theorem 2' to our present setting, we obtain the following characterization of the *entire* equilibrium allocation set as solutions of *self-generating* optimization problems.

PROPOSITION 1 *$(\overline{U}, \overline{q})$ is an equilibrium allocation (i.e., $(\overline{U}, \overline{q}) \in \mathcal{I}^{eq}$) if and only if it solves the following self-generating maximization problem:*

$$(\overline{\mathcal{P}}): \quad \max_{(U,q)\in\mathcal{I}} E_\theta\left[S(q(\theta)) - \theta q(\theta) - nU(\theta) + (n-1)(\overline{T}(q(\theta)) - \theta q(\theta))\right],$$

where $\overline{T}$ is an aggregate tariff that implements $(\overline{U}, \overline{q})$ and satisfies $\overline{T}(0) = 0$.

The maximization problem $(\overline{\mathcal{P}})$ is *self-generating* in the sense that its solution is implemented by an aggregate $\overline{T}$ that also appears in the maximand, embedding the fixed-point nature of the equilibrium. This objective function also features n-times the extraction of the agent's surplus since each principal, independently of what others are doing, wants to harvest the agent's information rent. In the aggregate, everything thus happens as if a *surrogate principal* was now in charge of maximizing the principals' collective payoff with the only proviso that the agent's information rent is now counted negatively n times. The extra term in the maximand, which represents $n-1$ times the agent's payoff at the induced allocation, captures the fact that a given principal does not take into account the impact of this contract on other principals' payoffs.

The necessity part of Proposition 1 can be obtained by summing the individual optimization problems of all principals. An equilibrium allocation, since it maximizes each principal's problem, also maximizes their sum. This summation introduces the n-rent distortion. In any equilibrium with non-zero output, the agent's information rent will thus be overweighted by a factor of n (instead of a coefficient of 1 that would arise had principals cooperated). There is a *"tragedy of the commons"* as the n principals effectively over harvest the agent's information rent, leading to an n-fold marginalization. It is this noncooperative information-rent externality that the principals would like to mitigate in their equilibrium selection.

Establishing the sufficiency argument in Proposition 1 is more subtle. It is obtained by reconstructing each principal's individual maximization problem from $(\overline{\mathcal{P}})$ itself, so as to align all individual principal's objectives with those of the surrogate principal. Consider the following construction:

$$\overline{T}_j(q) = S_j(q) - \frac{1}{n}(S(q) - \overline{T}(q)) \quad \forall j \in \{1, ..., n\}. \tag{3.1}$$

Summing over j yields an aggregate worth $\overline{T}$. Summing instead over all principals but i gives:

$$\overline{T}_{-i}(q) = S_{-i}(q) - \frac{n-1}{n}(S(q) - \overline{T}(q)).$$

By *"undoing"* the aggregate offer $\overline{T}_{-i}$ of his competitors so constructed, principal i can always offer any aggregate T he likes, thereby inducing any implementable allocation (U, q). This construction gives principal an expected payoff of

$$\begin{aligned}&\mathrm{E}_\theta\left[S_i(q(\theta)) - T(q(\theta)) + \overline{T}_{-i}(q(\theta))\right]\\ &\qquad\equiv \mathrm{E}_\theta\left[S(q(\theta)) - T(q(\theta)) - \frac{n-1}{n}(S(q(\theta)) - \overline{T}(q(\theta)))\right],\end{aligned}$$

where the right-hand side equality follows from our previous equation for $\overline{T}_{-i}$. Expressing payments in terms of the agent's rent, we may simplify this payoff as

$$\frac{1}{n}\mathrm{E}_\theta\left[S(q(\theta))-\theta q(\theta)-nU(\theta)+(n-1)(\overline{T}(q(\theta))-\theta q(\theta))\right]. \tag{3.2}$$

Up to a positive scale, the objective function (3.2) exactly replicates that of the surrogate principal. Therefore, principal i's incentives to induce a particular implementable allocation (U,q) are identical to those of the surrogate principal. Moreover, his incentives to induce the agent not to participate are also similar since, following refusal, principal i also gets a reservation payoff of 0.

Remark. The individual offers (3.1) have an attractive property: All principals get the same incremental gains from agent participation and from the production of quantity q:

$$S_i(q)-\overline{T}_i(q)=\frac{1}{n}(S(q)-\overline{T}(q)) \quad \forall i\in\{1,...,n\}.$$

Proposition 2 below will show that a variety of other unequal payoff distributions can also be sustained for a given type-output allocation.

4. CHARACTERIZATION OF THE ENTIRE SET OF EQUILIBRIA

Our next Theorem describes the set of *equilibrium implementable* allocations.

THEOREM 1 *An allocation $(\overline{U},\overline{q})$ is equilibrium implementable if and only if it satisfies the following conditions:*

- *Surrogate-principal incentive compatibility:*

$$S(\overline{q}(\underline{\theta}))-\underline{\theta}\overline{q}(\underline{\theta})\geq S(\overline{q}(\overline{\theta}))-\underline{\theta}\overline{q}(\overline{\theta}); \tag{4.1}$$

$$S(\overline{q}(\overline{\theta}))-\left(\overline{\theta}+\frac{n\nu}{1-\nu}\Delta\theta\right)\overline{q}(\overline{\theta}) \tag{4.2}$$

$$\geq \max\left\{S(\overline{q}(\underline{\theta}))-\left(\overline{\theta}+\frac{n\nu}{1-\nu}\Delta\theta\right)\overline{q}(\underline{\theta})-(n-1)\Delta\theta(\overline{q}(\underline{\theta})-\overline{q}(\overline{\theta}));0\right\}.$$

- *Rent minimization:*

$$\overline{U}(\underline{\theta})=\Delta\theta\overline{q}(\overline{\theta})\geq\overline{U}(\overline{\theta})=0. \tag{4.3}$$

Surrogate principal's incentive compatibility. Condition (4.3) is familiar from screening problems. At the optimum of $(\overline{\mathcal{P}})$, the low-cost agent's incentive constraint and the high-cost agent's participation constraints are necessarily binding. When turning to the optimal conditions with respect to outputs, we recognize that (4.1) and (4.2) are actually incentive constraints that apply to the surrogate principal. These conditions are obtained by imposing the minimal *proviso* that the surrogate principal should prefer to induce the equilibrium output profile $(\overline{q}(\underline{\theta}),\overline{q}(\overline{\theta}))$ rather than any alternative choice $(q(\underline{\theta}),q(\overline{\theta}))$ that would still satisfy the monotonicity condition $q(\underline{\theta})\geq q(\overline{\theta})$, and hence be implementable. Among all possible alternatives, the surrogate principal could choose to induce pooling either at $\overline{q}(\overline{\theta})$ or $\overline{q}(\underline{\theta})$ or he could also choose to induce no participation from the high-cost type while keeping a low-cost agent active. The first of those incentive constraints gives condition (4.1) while the second and third give (4.2).

MONOTONICITY. Simple revealed preferences arguments show that any output profile that satisfy conditions (4.1) and (4.2) is necessarily non-increasing:

(4.4) $\quad \overline{q}(\underline{\theta}) \geq \overline{q}(\overline{\theta}).$[8]

This monotonicity condition is strict for separating allocations. As an example, the surrogate principal could choose to *"shut down"* production for the high-cost agent, i.e., $\overline{q}(\overline{\theta}) = 0$. Conditions (4.1) and (4.2) reduce to the following pair of inequalities for the positive production of the low-cost agent:

$$\frac{n}{1-\nu}\Delta\theta\overline{q}(\underline{\theta}) \geq S(\overline{q}(\underline{\theta})) - \underline{\theta}\overline{q}(\underline{\theta}) \geq 0.$$

Clearly these conditions are always satisfied for $\overline{q}(\underline{\theta}) = q^{fb}(\underline{\theta})$ provided that n is sufficiently large. The best contract with *"shut-down"* that could be offered had principals cooperated can still be implemented as an equilibrium outcome when n increases (keeping aggregate surplus constant).

Pooling allocations such that $\overline{q}(\theta) = \overline{q}$ for all θ might also arise in equilibrium. Under pooling, (4.1) and (4.2) reduce to the non-negativity of the surrogate surplus obtained from inducing production by a high-cost agent, namely:

$$S(\overline{q}) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right)\overline{q} \geq 0.$$

Fixing $\overline{q}$ to the optimal value of a pooling contract had principals cooperated, namely $q^{fb}(\overline{\theta})$, it can be readily seen that the latter condition is instead harder to satisfy as the number of principals increase (still keeping aggregate surplus constant).

IMPLEMENTATION WITH FORCING CONTRACTS. A forcing contract that would punish the agent fiercely for choosing outputs outside the equilibrium set suffices to implement any equilibrium allocation satisfying conditions in Theorem 1:

$$\overline{T}(q) = \begin{cases} \overline{\theta}\overline{q}(\overline{\theta}) & \text{if } q = \overline{q}(\overline{\theta}) \\ \underline{\theta}\overline{q}(\underline{\theta}) + \Delta\theta\overline{q}(\overline{\theta}) & \text{if } q = \overline{q}(\underline{\theta}) \\ 0 & \text{if } q = 0 \\ -\infty & \text{if } q \notin \{\overline{q}(\overline{\theta}), \overline{q}(\underline{\theta}), 0\}. \end{cases} \tag{4.5}$$

Given the aggregate contract in (4.5), individual equilibrium offers, $\overline{T}_i$, can be reconstructed using (3.1). In particular, all those individual offers are forcing contracts with infinite negative values for *"out-of-equilibrium"* outputs.

[8]Monotonicity arises for two reasons. First, monotonicity is a direct consequence of implementability, as is well known. Hence, it appears as an implicit requirement of the constrained set $\mathcal{I}$. Yet, even if the surrogate principal were to maximize a relaxed problem $(\overline{\mathcal{P}}^r)$ which would not be constrained by this monotonicity condition, the requirement of self-generation imposes the surrogate principal's incentive constraints (4.1) and (4.2). These conditions, in turn, imply monotonicity and thus the solution of the relaxed problem also solves the more constrained problem.

Forcing contracts act as a severe coordination device. Indeed, because they specify infinitely negative payments for *"out-of-equilibrium"* outputs, "undoing" such aggregate payments to implement an alternative allocation can never be attractive to any deviating principal. In the sequel, we will see how the set of equilibrium allocations are refined when contracts entail less severe *"out-of-equilibrium"* punishments.

Non-Unique Implementation. Even when restricting the aggregate to be a forcing contract, the array of equilibrium offers defined by (3.1) and (4.5) is not necessarily unique: Unequal distributions of payoffs might be achieved by slightly modifying these offers.

To see how, consider a balanced vector of payoffs $\boldsymbol{\mu} = (\mu_1, ..., \mu_n) \in \mathbb{R}^n$ such that $\sum_{i=1}^{n} \mu_i = 0$. We may add this vector to the individual tariffs defined in (3.1) and thereby reconstruct another family of contracts keeping constant the aggregate $\overline{T}$:

$$\overline{T}_i^{\boldsymbol{\mu}}(q) = \overline{T}_i(q) - \mu_i \quad \forall i \in \{1, ..., n\}. \tag{4.6}$$

With such contracts, principal i's payoff now differs from that of the surrogate principal by the addition of the slack μ_i:

$$E_\theta \left[\frac{1}{n}(S(q(\theta)) - \theta q(\theta)) - U(\theta) + \frac{n-1}{n}(\overline{T}(q(\theta)) - \theta q(\theta)) + \mu_i \right]. \tag{4.7}$$

On one hand, this formula shows that principal i's incentives to induce a given output profile following the agent's acceptance are still aligned with those of the surrogate principal. On the other hand, incentives to *"shut down"* production may differ, in particular when μ_i is sufficiently negative. Intuitively, principal i would be asked to pay too much for the agent's output in that case and would favor vetoing production by offering an infinitely negative payment for the agent's service. That all principals should agree on inducing production at a non-trivial equilibrium thus puts a limit on how much payoff inequality can be supported in an equilibrium.[9]

Proposition 2 *For any equilibrium allocation $(\overline{U}, \overline{q})$ with associated aggregate and individual transfers given by (4.5) and (3.1), respectively, there exists a non-empty set of balanced vectors, $\boldsymbol{\mu} = (\mu_1, ..., \mu_n) \in \mathbb{R}^n$, such that $(\overline{T}_1 - \mu_1, \ldots, \overline{T}_n - \mu_n)$ is an equilibrium profile that also implements $(\overline{U}, \overline{q})$.*

The set of balanced vectors has non-empty interior if

$$S(\overline{q}(\overline{\theta})) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right)\overline{q}(\overline{\theta}) > 0. \tag{4.8}$$

This property shows that, at least when (4.8) holds, it is possible to redistribute surplus among principals without changing the equilibrium allocation. This result stands in sharp contrast with what arises under delegated common agency, where the non-negativity of transfers may break this neutrality as demonstrated in Martimort and Stole (2015).

[9]This upper bound can be quite tight, in particular if the equilibrium allocation just leaves the surrogate principal indifferent between *"shutting down"* production for the high-cost agent or not. Then, there is no slack available to modify individual payments and the individual contracts defined in (3.1) are the only feasible ones.

5. BICONJUGATE EQUILIBRIA

MOTIVATION. The implementation of all equilibrium allocations described in Theorem 1 by means of forcing contracts shows the force of contracts that use infinitely-negative payments for *"out-of-equilibrium"* outputs. These severe payments, though never chosen by the agent on the equilibrium path, prevent deviations. *A contrario*, we may ask whether requiring less severe transfers may serve as a palatable refinement device. Imposing such a restriction on equilibria, and studying its properties, is the purpose of this section.

As a first step, we observe that any aggregate contract $T \in \mathcal{T}$ that implements an allocation (U, q) satisfies the following inequality

$$T(q) \leq U(\theta) + \theta q \quad \forall q \in \mathcal{Q}$$

with equality at $q = q(\theta)$. From this, we immediately obtain an upper bound on all implementing contracts:

$$T(q) \leq T^{bc}(q) \equiv \min_{\theta \in \theta} U(\theta) + \theta q \quad \forall q \in \mathcal{Q}. \tag{5.1}$$

The function T^{bc}, defined above, has several important properties.

- T^{bc} is increasing and concave because it is a minimum of linear functions. It is thus almost everywhere differentiable. When the low-cost agent's incentive constraint and the high-cost agent's participation constraints are both binding, T^{bc} has derivatives $T^{bc'}(q) = \underline{\theta}$ for $q > q(\underline{\theta})$ and $T^{bc'}(q) = \overline{\theta}$ for $q \in (0, q(\overline{\theta}))$. Thus T^{bc} is not differentiable at at $q(\overline{\theta})$ but it admits a super-differential which is a non-empty interval, namely $\partial T^{bc}(q(\overline{\theta})) = [\underline{\theta}, \overline{\theta}]$.[10]
- When the high-cost agent's participation constraint is binding at equilibrium (i.e., $U(\overline{\theta}) = 0$), we must also have $T^{bc}(0) = 0$ so that the agent is always weakly indifferent between accepting the offer T^{bc} and producing zero output, and refusing to participate.
- Importantly, T^{bc} also implements the allocation (U, q), i.e.,

  $$U(\theta) = \max_{q \in \mathcal{Q}} T^{bc}(q) - \theta q \quad \forall \theta \in \theta. \tag{5.2}$$

  Using the language of convex analysis, the dual conditions (5.1) and (5.2) show that U and T^{bc} are thus *conjugates* to each other. T^{bc} is the upper envelope, or the least-concave function among all possible upper semi-continuous functions implementing (U, q). Concavity is a particularly attractive requirement since it allows one to characterize the agent's optimization behavior and the optimality conditions for self-generating problems by means of convex calculus and super-differentials, even when tariffs might not be everywhere differentiable.

[10] The *super-differential* $\partial T(q)$ of a concave function T at q is defined as the following correspondence:

$$\partial T(q) = \{s \in \mathbb{R} | \quad T(x) \leq T(q) + s(x - q) \quad \forall x \in \mathcal{Q}\}.$$

From Theorem 4.3 in Aubin (1998), such *super-differential* exists on a dense subset of $\mathcal{Q}$ and is almost everywhere single-valued if T is concave and upper semi-continuous. The super-differential of a concave function is analogous to the subdifferential of a convex function.

Taken together, (5.1) and (5.2) suggest the following definition of *biconjugacy* which is independent of U.

DEFINITION 3 *An aggregate contract T is* ***biconjugate*** *if and only if*

$$T(q) = \min_{\theta \in \theta} \left\{ \max_{q' \in \mathcal{Q}} \{T(q') - \theta q'\} + \theta q \right\} \quad \forall q \in \mathcal{Q}.$$

Denote by $\mathcal{T}^{bc} \subset \mathcal{T}$ the set of such biconjugate contracts. We define the set of biconjugate equilibrium allocations, accordingly.

DEFINITION 4 *An equilibrium allocation $(\overline{U}, \overline{q}) \in \mathcal{I}^{eq}$ implemented by aggregate $\overline{T}$ is a* ***biconjugate equilibrium allocation*** *if and only if $\overline{T} \in \mathcal{T}^{bc}$.*

Biconjugate tariffs are attractive because, by construction, they can always implement any allocation (U, q) satisfying the rent minimization condition (4.3), but they do so in a very special way. By construction, T^{bc} has the property that all price-quantity pairs that are *strictly dominated* for both types have been eliminated. To illustrate, when the aggregate tariff is biconjugate, a low-cost agent is indifferent among all outputs $q \geq q(\overline{\theta})$, while he strictly prefers these high outputs to any $q < q(\overline{\theta})$. In contrast, a high-cost agent is indifferent among all outputs $q \in [0, q(\overline{\theta})]$, while he strictly prefers these low outputs to higher outputs, $q > q(\overline{\theta})$. Any other schedule T that would implement the allocation (U, q) would entail price-quantity pairs that are strictly dominated for at least one type. In other words, equilibria which are sustained with an aggregate that is not biconjugate rely on *"out-of-equilibrium"* price-quantity pairs that are used as threats to prevent the principals' deviations but which are strictly dominated on the equilibrium path from the agent's point of view.

To make this point more precisely, we return to our definition self-generating problem and offer the following definition.

DEFINITION 5 *An equilibrium allocation $(\overline{U}, \overline{q})$ which is implemented by an aggregate $\overline{T}$ (together with the individual offers associated given by (3.1)) is* ***undefeated*** *if and only if any alternative implementation of this allocation, $\tilde{T}$, is also an equilibrium aggregate.*

Suppose that the equilibrium allocation $(\overline{U}, \overline{q})$, which is implemented by an aggregate $\overline{T}$, is defeated. Although $(\overline{U}, \overline{q})$ is a solution to the self-generating problem, $\overline{\mathcal{P}}$, when $\overline{T}$ is expected to be played, it is no longer a solution when $\overline{T}$ is replaced by the alternative implementation $\tilde{T}$. In other words, this new aggregate $\tilde{T}$ contains attractive output-price pairs that make the surrogate principal (and thus at least one of the individual principals) willing to deviate. A defeated equilibrium allocation might thus not be robust to the addition of some output-price options. *A contrario*, an undefeated equilibrium is robust in the sense that *"out-of-equilibrium"* output-pairs may be pruned from the tariff and the equilibrium allocation is still a solution to the *self-generating* problem with the new tariff so constructed.

Next proposition shows the robustness of biconjugate equilibria in light of this robustness criterion.

PROPOSITION 3 *An equilibrium allocation $(\overline{U}, \overline{q})$ is undefeated if and only if it is implemented by a biconjugate aggregate $\overline{T}^{bc}$ with the individual offers given by (3.1).*

CHARACTERIZATION. The refinement of biconjugacy leads to a sharp characterization of equilibrium allocations.

THEOREM 2 *Suppose that Assumption 1 holds. An allocation $(\overline{U}, \overline{q})$ arises in a biconjugate equilibrium, $(\overline{U}, \overline{q}) \in \mathcal{I}^{bc}$, if and only if it satisfies the following properties:*

- *Surrogate Principal's marginal incentive constraints:*

$$S'(\overline{q}(\underline{\theta})) - \underline{\theta} = 0; \tag{5.3}$$

$$S'(\overline{q}(\overline{\theta})) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right) \geq 0 \geq S'(\overline{q}(\overline{\theta})) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right) - (n-1)\Delta\theta. \tag{5.4}$$

- *Rent minimization: (4.3) holds.*

Though biconjugate tariffs are not everywhere differentiable, they are still concave and this concavity allows us to use convex calculus to characterize equilibrium allocations. As a consequence, the *non-local* optimality conditions (4.1) and (4.2) that more broadly characterized the surrogate principal's incentive compatibility in Theorem 1 are now replaced by the analogous *marginal* optimality conditions (5.3) and (5.4). These marginal conditions are, of course, more restrictive.

PROPOSITION 4

$$\mathcal{I}^{bc} \subset \mathcal{I}^{eq}.$$

The marginal conditions (5.3) and (5.4) have a few immediate consequences. First, inefficiencies for the low-cost agent's output are now ruled out whereas such inefficient outputs are *a priori* possible in other equilibria as demonstrated by Theorem 1. This difference comes from the fact that biconjugate tariffs necessarily have slope $\underline{\theta}$ for q large enough. Roughly, applying the Envelope Theorem when solving $(\overline{\mathcal{P}})$ preserves the *"no-distortion-at-the-top"* result found with monopolistic screening.

Second, biconjugate equilibria also feature distortions below the cooperative outcome for the high-cost agent's output. All biconjugate equilibrium outputs $\overline{q}(\overline{\theta})$ must lie within the interval $[\tilde{q}_n(\overline{\theta}), q_n^{bc}(\overline{\theta})]$, where the upper bound $q_n^{bc}(\overline{\theta})$ and the lower bound $\tilde{q}_n(\overline{\theta})$ of the equilibrium set are, respectively, defined by

$$S'(q_n^{bc}(\overline{\theta})) = \overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta \text{ and } S'(\tilde{q}_n(\overline{\theta})) = \overline{\theta} + \left(\frac{n\nu}{1-\nu} + n - 1\right)\Delta\theta.$$ [11]

Note that the cooperative outcome never belongs to this interval. This illustrates the import of using biconjugacy as a refinement while still allowing for the contractual externalities among non-cooperating principals.

[11] For future reference, we may define the allocation (U_n^{bc}, q_n^{bc}) with $U_n^{bc}(\underline{\theta}) = \Delta\theta q_n^{bc}(\overline{\theta}) > U_n^{bc}(\overline{\theta}) = 0$ by requesting the low-cost agent to produce the first-best output level $q_n^{bc}(\underline{\theta}) = q^{fb}(\underline{\theta})$.

More generally, the interval $[\tilde{q}_n(\overline{\theta}), q_n^{bc}(\overline{\theta})]$ is a strict subset of the outputs described in Theorem 1. To see why, observe that, thanks to the strict concavity of S, Jensen's inequality implies the following:

$$S'(q_n^{bc}(\overline{\theta})) = \overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta < \frac{S(q_n^{bc}(\overline{\theta}))}{q_n^{bc}(\overline{\theta})}.$$

Hence, $q_n^{bc}(\overline{\theta})$ satisfies condition (4.2) as a strict inequality.

Remark. The above properties of biconjugate equilibria are reminiscent of outcomes which that arise in a variety of common agency games in the literature (Laffont and Tirole (1993), Martimort (1992), Stole (1992), Martimort and Stole (2009a, 2009b), among others): *"No-distortion-at-the-top"* and the n-fold distortion of the output for less efficient types. The existing literature has focused on continuous types models but has also arbitrarily restricted the analysis to *"smooth"* tariffs. The fact that the type space and the set of possible outputs have the same dimensionality tightens the conjugacy relationship between rent profiles and tariffs. All smooth equilibria satisfy the biconjugacy requirement. Although the outputs space and the types space have different dimensionality here, it is thus not surprising that the biconjugate equilibria developed in this discrete setting share the same qualitative properties. This is reassuring from a theoretical point of view. Modelers may rely on the more tractable discrete models and impose biconjugacy in order to capture the features of smooth equilibria.

6. *EX ANTE* OPTIMAL EQUILIBRIA FOR THE PRINCIPALS

If principals could meet *ex ante* and negotiate over the equilibrium to be played, a reasonable prediction would be that they would agree to play the equilibrium that maximizes their *ex ante* collective payoff. In this section, we consider solutions to this collective problem, both over the full equilibrium domain and over the restricted domain of biconjugate equilibria.

It is possible that the cooperative allocation can be supported as an equilibrium using forcing contracts. To illustrate, consider the case of a quadratic surplus function $S(q) = \alpha q - \frac{q^2}{2}$ (for α sufficiently large to ensure positive outputs under all circumstances). The cooperative allocation (U^{coop}, q^{coop}) belongs to $\mathcal{I}^{eq}$ if

$$q^{coop}(\overline{\theta}) = \alpha - \overline{\theta} - \frac{\nu}{1-\nu}\Delta\theta > 2\frac{(n-1)\nu}{1-\nu}\Delta\theta,$$

i.e., if n is small enough. Clearly, one can find parameter values such that this condition holds when $n = 2$; a case where one would already expect distortions to arise from the principals' non-cooperative behavior. In view of Theorem 2 and the discussion thereafter, this remark already suggests that the set $\mathcal{I}^{eq}$ may be too large to convey the basic intuition that the principals' non-cooperative behavior may entail some welfare cost for them. To focus our attention on the interesting case with welfare costs, we make an assumption that the cooperative outcome cannot be supported in any equilibrium.

Assumption 2

$$S(q^{coop}(\overline{\theta})) < \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right) q^{coop}(\overline{\theta}).$$

Assumption 2 holds when n is sufficiently large. The contractual externality among principals then becomes so significant that the cooperative solution can not arise in any equilibrium. The next Proposition shows that the best equilibrium is nevertheless chosen to be as close as possible to this cooperative solution.

PROPOSITION 5 *Suppose that Assumption 2 holds. The best allocation $(\hat{U}_n, \hat{q}_n)$ in $\mathcal{I}^{eq}$ from the principals' ex ante point of view has the low-cost agent producing at the first-best $q^{fb}(\underline{\theta})$ and the high-cost agent's output being less than the cooperative outcome; $\hat{q}_n(\overline{\theta}) < q^{coop}(\overline{\theta})$, where*

$$S(\hat{q}_n(\overline{\theta})) = \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right)\hat{q}_n(\overline{\theta}). \tag{6.1}$$

The corresponding information rents satisfy the rent minimization condition (4.3).

When n increases, $\hat{q}_n(\overline{\theta})$ decreases. In other words, the principals face greater difficulties to coordinate and contractual externalities are exacerbated as their number increases.

An immediate consequence of Proposition 2 is that the allocation $(\hat{U}_n, \hat{q}_n)$ is uniquely implemented. When choosing the best equilibrium, principals have no way of redistributing payoffs among themselves using the balanced payments suggested by Proposition 2. Doing so would mean that at least one of them would be ready to *"shut down"* production by the high-cost type.

BEST BICONJUGATE EQUILIBRIUM. We now look for the *ex ante* optimal equilibrium within the refined set of biconjugate equilibria, $\mathcal{I}^{bc}$. Again, the upper bound on this equilibrium set that now corresponds to the output distortion $q^{bc}(\overline{\theta})$ has attractive welfare properties.

PROPOSITION 6 *The best allocation (U_n^{bc}, q_n^{bc}) in $\mathcal{I}^{bc}$ from the principals' ex ante point of view has the low-cost agent producing at the first-best $q^{fb}(\underline{\theta})$ and the high-cost agent's output being less than the cooperative outcome; $\hat{q}_n^{bc}(\overline{\theta}) < q^{coop}(\overline{\theta})$, where*

$$S'(q_n^{bc}(\overline{\theta})) = \overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta, \tag{6.2}$$

and $q_n^{bc}(\overline{\theta}) < \hat{q}_n(\overline{\theta})$.

The definitions of the best equilibrium outputs in the full set $\mathcal{I}^{eq}$, namely (6.1), and in the biconjugate domain $\mathcal{I}^{bc}$, (6.2), bear some strong similarities. Moving from $\mathcal{I}^{eq}$ to $\mathcal{I}^{bc}$, the *non-local* definition (6.1) of the best equilibrium is again replaced by a marginal condition (6.2) for the best equilibrium. Qualitative properties of the two solutions are also closely related to the extent that the marginal aggregate surplus $S'(q)$ and the average aggregate surplus $S(q)/q$ vary in similar direction. Regardless of whether modelers choose to select the best equilibrium within $\mathcal{I}^{eq}$ or $\mathcal{I}^{bc}$, the comparative statics share a likeness.

Nevertheless, the restriction to biconjugate equilibria prevents the principals from reaching the highest possible equilibrium collective payoff that could be reached with a forcing aggregate of the kind (4.5). This fact is probably best seen when Assumption 2 holds.

Then, the *ex ante* optimal equilibrium allocation in $\mathcal{I}^{eq}$ is obtained at $(\hat{U}_n(\theta), \hat{q}_n(\theta))$. Yet, the strict concavity of $S(q)$, together with the fact that $\hat{q}_n(\overline{\theta})$ is positive, implies:

$$S'(\hat{q}_n(\overline{\theta})) \leq \frac{S(\hat{q}_n(\overline{\theta}))}{\hat{q}_n(\overline{\theta})} = \overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta = S_n'^{bc}(\overline{\theta})) \Leftrightarrow \hat{q}_n(\overline{\theta}) > q_n^{bc}(\overline{\theta}).$$

In other words, the optimal intrinsic equilibrium is never a biconjugate equilibrium. For the principals, there exists a trade-off between robustness and welfare.

APPENDIX A

PROOF OF PROPOSITION 1: The proof follows similar steps to those in Martimort and Stole (2012, Theorem 2'), though we explicitly treat the agent's participation decision, δ, here for completeness.

Necessity. Given $\overline{T}_{-i}$, the bilateral surplus between principal i and the agent of type θ when $(q, \delta) \in \mathcal{Q} \times \{0, 1\}$ is chosen is given by

$$\delta(S_i(q) - \theta q + \overline{T}_{-i}(q)) + (1-\delta)(S_i(0)) = \delta(S_i(q) - \theta q + \overline{T}_{-i}(q)).$$

Thus, principal i desires to implement the allocation $(U, q, \delta) : \Theta \to \mathbb{R}_+ \times \mathcal{Q} \times \{0, 1\}$ which maximizes

$$E_\theta[\delta(\theta)(S_i(q(\theta)) - \theta q(\theta) + \overline{T}_{-i}(q(\theta)) - U(\theta))],$$

subject to $(U, q, \delta) \in \mathcal{I}$. For $(\overline{U}, \overline{q}, \overline{\delta})$ to be an equilibrium allocation, it must be that

$$(\overline{U}, \overline{q}, \overline{\delta}) \in \arg\max_{(U,q,\delta)\in\mathcal{I}} E_\theta[\delta(\theta)(S_i(q(\theta)) - \theta q(\theta) + \overline{T}_{-i}(q(\theta)) - U(\theta))].$$

Note that every principal i faces the same domain of maximization, $\mathcal{I}$; the difference between the programs of any two principals, i and j, is entirely embedded in the differences in the aggregates $\overline{T}_{-i}$ and $\overline{T}_{-j}$. Following Martimort and Stole (2012), an equilibrium allocation must necessarily maximize the some of the principals' programs. Thus, $(\overline{U}, \overline{q}, \overline{\delta})$ must solve

$$\text{(A1)} \qquad (\overline{U}, \overline{q}, \overline{\delta}) \in \arg\max_{(U,q,\delta)\in\mathcal{I}} E_\theta\left[\delta(\theta)\left(S(q(\theta)) - \theta q(\theta) - nU(\theta) + (n-1)(\overline{T}(q(\theta)) - \theta q)\right)\right].$$

Note that if $(\overline{U}, \overline{q}, \overline{\delta})$ is implemented by the aggregate tariff $\overline{T}$, then $(\overline{U}, \overline{q}, \tilde{\delta} = 1)$ is implemented by the aggregate tariff

$$\tilde{T}(q) = \begin{cases} \overline{T}(q) & \text{if } q \neq 0 \\ 0 & \text{if } q = 0. \end{cases}$$

Under $\tilde{T}$, every agent type chooses to participate $\tilde{\delta}(\theta) = 1$. Because the objective function in (A1) has the same expected value at $(\overline{U}, \overline{q}, \overline{\delta})$ using $\overline{T}$ as it does at $(\overline{U}, \overline{q}, \tilde{\delta})$ using $\tilde{T}$, we conclude that

$$(\overline{U}, \overline{q}) \in \arg\max_{(U,q)\in\mathcal{I}} E_\theta\left[S(q(\theta)) - \theta q(\theta) - nU(\theta) + (n-1)(\tilde{T}(q(\theta)) - \theta q)\right],$$

which is $(\overline{\mathcal{P}})$.

Sufficiency. Consider a solution $(\overline{U}, \overline{q})$ to $(\overline{\mathcal{P}})$ which is implemented by the aggregate tariff $\overline{T}$. Note that because $(\overline{U}, \overline{q})$ is implemented by $\overline{T}$ with $\overline{T}(0) = 0$, we are considering the case where the agent always participates, $\delta = 1$.

Construct individual tariffs $\overline{T}_i$ satisfying

$$\overline{T}_i(q) = S_i(q) - \frac{1}{n}(S(q) - \overline{T}(q)) \quad \forall i \in \{1, ..., n\}.$$

By construction,

$$\sum_{i=1}^{n} \overline{T}_i(q) = \overline{T}(q).$$

We show that this contract profile $(\overline{T}_1, ..., \overline{T}_n)$ is an equilibrium. Suppose indeed that all principals j for $j \neq i$ offer $\overline{T}_j$. At a best response, principal i induces an allocation (U, q, δ) that solves:

$$(\mathcal{P}_i): \quad \max_{(U,q,\delta)\in\mathcal{I}} E_\theta \left[\delta(\theta)\left(S_i(q(\theta)) - \theta q(\theta) - U(\theta) + \overline{T}_{-i}(q(\theta))\right)\right].$$

Inserting the expressions of $\overline{T}_j$ (for $j \neq i$) using our construction above), the allocation that principal i would like to induce should solve

$$\max_{(U,q,\delta)\in\mathcal{I}} E_\theta \left[\delta(\theta)\left(S(q(\theta)) - \theta q(\theta) - nU(\theta) + (n-1)(\overline{T}(q(\theta)) - \theta q)\right)\right].$$

But this is the same maximization program in (A1), and hence principal i's choice $\overline{T}_i$ is a best response against $\overline{T}_{-i}$. *Q.E.D.*

PROOF OF THEOREM 1: *Necessity.* Standard arguments from discrete screening models first show that any equilibrium allocation must satisfy the monotonicity condition

$$\text{(A2)} \qquad \overline{q}(\underline{\theta}) \geq \overline{q}(\overline{\theta}).$$

Consider the relaxed program $(\overline{\mathcal{P}}^r)$ obtained from $(\overline{\mathcal{P}})$ by ignoring (A2) and focusing only on the incentive constraint of a low-cost type and the participation constraint of a high-cost type, namely:

$$\text{(A3)} \qquad U(\underline{\theta}) \geq U(\overline{\theta}) + \Delta\theta q(\overline{\theta});$$

$$\text{(A4)} \qquad U(\overline{\theta}) \geq 0.$$

At the optimum of $(\overline{\mathcal{P}}^r)$, (A3) and (A4) are both binding. Hence, the equilibrium profile of rents $\overline{U}$ must satisfy (4.3). Inserting the expressions of $U(\theta)$ obtained from (A3) and (A4) binding into the maximand of $(\overline{\mathcal{P}}^r)$, this maximand can be rewritten in terms of outputs only:

$$\nu\left(S(q(\underline{\theta})) - \underline{\theta}q(\underline{\theta}) - n\Delta\theta q(\overline{\theta}) + (n-1)\left(\overline{T}(q(\underline{\theta})) - \underline{\theta}q(\underline{\theta})\right)\right)$$

$$+(1-\nu)\left(S(q(\overline{\theta})) - \overline{\theta}q(\overline{\theta}) + (n-1)\left(\overline{T}(q(\overline{\theta})) - \overline{\theta}q(\overline{\theta})\right)\right).$$

By definition of an equilibrium, the non-increasing output schedule $(\overline{q}(\underline{\theta}), \overline{q}(\overline{\theta}))$ maximizes this expression. Thus, $(\overline{q}(\underline{\theta}), \overline{q}(\overline{\theta}))$ is weakly preferred to any other non-increasing pair $(q(\underline{\theta}, q(\overline{\theta}))$.

• Because $(\overline{q}(\underline{\theta}), \overline{q}(\overline{\theta}))$ is weakly preferred to $(\overline{q}(\overline{\theta}), \overline{q}(\overline{\theta}))$, we have

$$\text{(A5)} \quad S(\overline{q}(\underline{\theta})) - \underline{\theta}\overline{q}(\underline{\theta}) + (n-1)(\overline{T}(\overline{q}(\underline{\theta})) - \underline{\theta}\overline{q}(\underline{\theta}))$$

$$\geq S(\overline{q}(\overline{\theta})) - \underline{\theta}\overline{q}(\overline{\theta}) + (n-1)(\overline{T}(\overline{q}(\overline{\theta})) - \underline{\theta}\overline{q}(\overline{\theta})).$$

Since $\overline{T}$ implements $(\overline{U}, \overline{q})$ and (4.3) holds for this allocation, we obtain

$$\overline{T}(\overline{q}(\underline{\theta})) - \underline{\theta}\overline{q}(\underline{\theta}) = \overline{T}(\overline{q}(\overline{\theta})) - \underline{\theta}\overline{q}(\overline{\theta}).$$

Inserting into (A5) and simplifying immediately gives (4.1).

• Because $(\overline{q}(\underline{\theta}), \overline{q}(\overline{\theta}))$ is weakly preferred to $(\overline{q}(\underline{\theta}), \overline{q}(\underline{\theta}))$, we have

$$\text{(A6)} \quad S(\overline{q}(\overline{\theta})) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right)\overline{q}(\overline{\theta}) + (n-1)(\overline{T}(\overline{q}(\overline{\theta})) - \overline{\theta}\overline{q}(\overline{\theta})))$$

$$\geq S(\overline{q}(\overline{\theta})) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right)\overline{q}(\underline{\theta}) + (n-1)(\overline{T}(\overline{q}(\underline{\theta})) - \overline{\theta}\overline{q}(\overline{\theta})).$$

Since $\overline{T}$ implements the equilibrium allocation $(\overline{U}, \overline{q})$ and (4.3) holds for this allocation, we obtain

$$\text{(A7)} \quad \overline{T}(\overline{q}(\overline{\theta})) - \overline{\theta}\overline{q}(\overline{\theta}) = 0 \text{ and } \overline{T}(\overline{q}(\underline{\theta})) - \overline{\theta}\overline{q}(\underline{\theta}) = -\Delta\theta(\overline{q}(\underline{\theta}) - \overline{q}(\underline{\theta})).$$

Inserting into (A6) and simplifying immediately gives the first condition in (4.2).

• Because $(\overline{q}(\underline{\theta}), \overline{q}(\overline{\theta}))$ is weakly preferred to $(\overline{q}(\underline{\theta}), 0)$, we have

$$\text{(A8)} \quad S(\overline{q}(\overline{\theta})) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right)\overline{q}(\overline{\theta}) + (n-1)(\overline{T}(\overline{q}(\overline{\theta})) - \overline{\theta}\overline{q}(\overline{\theta}))) \geq 0,$$

where the right-hand side above uses the normalization $S(0) = 0$. Using now the first condition in (A7) and simplifying gives us the second condition in (4.2).

• Finally, summing together (4.1) and the first condition in (4.2) immediately gives us that any allocation that solves the relaxed problem $(\overline{\mathcal{P}}^r)$ also satisfies (A2) and thus solves $(\overline{\mathcal{P}})$.

Sufficiency. Take any allocation $(\overline{U}, \overline{q})$ satisfying constraints (4.1), (4.2) (4.3) and (A2). We check that such allocation solves $(\overline{\mathcal{P}})$ for an aggregate $\overline{T}$ that implements the allocation.

First, observe that for such allocation, (4.1) and (4.2) taken together implies:

$$S(\overline{q}(\underline{\theta})) - \underline{\theta}\overline{q}(\underline{\theta}) \geq S(\overline{q}(\underline{\theta})) - \overline{\theta}\overline{q}(\underline{\theta}) + \Delta\theta\overline{q}(\overline{\theta}) \geq \left(\frac{n\nu}{1-\nu} + 1\right)\Delta\theta\overline{q}(\overline{\theta}) \geq 0.$$

From this, together with (4.2) and (4.3), we deduce that the objective function in $(\overline{\mathcal{P}})$ is necessarily positive for an allocation $(\overline{U}, \overline{q})$ that is implemented by a contract $\overline{T}$.

Consider thus the aggregate $\overline{T}$ defined in (4.5). Two facts follow from this definition. First, $\overline{T}$ implements $(\overline{U}, \overline{q})$ and, in particular, (4.3) holds. Second,choosing any implementable output profile $q(\underline{\theta}) \geq q(\overline{\theta})$ other than $(\overline{q}(\underline{\theta}), \overline{q}(\overline{\theta}))$, $(\overline{q}(\overline{\theta}), \overline{q}(\overline{\theta}))$, $(\overline{q}(\underline{\theta}), \overline{q}(\underline{\theta}))$, $(\overline{q}(\underline{\theta}), 0)$ or $(0, 0)$ leads to an infinitively negative payoff for the surrogate principal. Thus a principal who maximizes $(\overline{\mathcal{P}})$ will choose to implement one of these allocations, all of which lead to a non-negative payoff. Conditions (4.1) and (4.2) ensure that the best such profile is indeed $(\overline{q}(\underline{\theta}), \overline{q}(\overline{\theta}))$.

From those two facts, it also follows that $(\overline{U}, \overline{q})$ solves $(\overline{\mathcal{P}})$. Using the construction (3.1) allows us to retrieve all individual equilibrium offers $\overline{T}_i$. *Q.E.D.*

PROOF OF PROPOSITION 2: From (4.7), principal i always induces output $\overline{q}$ from the agent when he chooses to induce participation. Principal i should also prefer to induce the equilibrium allocation $(\overline{U}, \overline{q})$ rather than deviating and implementing the allocation corresponding to the non-decreasing output profile $\{\overline{q}(\underline{\theta}), 0\}$ which leaves zero rent to the low-cost agent. Because $S_i(0) = 0$, this condition can be written as:

$$E_\theta \left[\frac{1}{n} \left(S(\overline{q}(\theta)) - \theta \overline{q}(\theta) \right) - \overline{U}(\theta) + \frac{n-1}{n} (\overline{T}(\overline{q}(\theta)) - \theta \overline{q}(\theta)) + \mu_i \right]$$

$$\geq \nu \left(\frac{1}{n} \left(S(\overline{q}(\underline{\theta})) - \underline{\theta} \overline{q}(\underline{\theta}) \right) + \frac{n-1}{n} \Delta\theta \overline{q}(\overline{\theta}) + \mu_i \right).$$

Using the fact that the high-cost agent's participation constraint is binding yields the following lower bound on μ_i

$$\text{(A9)} \qquad \mu_i \geq -\frac{1}{n} \left(S(\overline{q}(\overline{\theta})) - \left(\overline{\theta} + \frac{n\nu}{1-\nu} \Delta\theta \right) \overline{q}(\overline{\theta}) \right).$$

Because (4.2) holds, we have:

$$S(\overline{q}(\overline{\theta})) - \left(\overline{\theta} + \frac{n\nu}{1-\nu} \Delta\theta \right) \overline{q}(\overline{\theta}) \geq 0.$$

Therefore, there exists a non-empty set of vectors, $\boldsymbol{\mu} = (\mu_1, ..., \mu_n)$, which are balanced and satisfy (A9) for all i. This set has non-empty interior when (4.8) holds. *Q.E.D.*

PROOF OF PROPOSITION 3: *Necessity.* Consider an equilibrium allocation that is undefeated. By definition, it is also implemented by the equilibrium aggregate $\overline{T}^{bc}$:

$$\text{(A10)} \qquad \overline{T}^{bc}(q) = \min_{\theta \in \Theta} \overline{U}(\theta) + \theta q \quad \forall q \in \mathcal{Q}.$$

It is thus a biconjugate equilibrium.

Sufficiency. Suppose that $(\overline{U}, \overline{q})$ is implemented in equilibrium by the biconjugate aggregate contract

$$\overline{T}^{bc}(q) = \min_{\theta \in \Theta} \overline{U}(\theta) + \theta q \quad \forall q \in \mathcal{Q}.$$

By definition, $(\overline{U}, \overline{q})$ must solve the following self-generating problem:

$$(\overline{\mathcal{P}}): \qquad \max_{(U,q) \in \mathcal{I}} E_\theta \left[S(q(\theta)) - \theta q(\theta) - nU(\theta) + (n-1)(\overline{T}^{bc}(q(\theta)) - \theta q(\theta)) \right].$$

Because any contract $\overline{T}$ that implements $(\overline{U}, \overline{q})$ is such that $\overline{T}(q) \leq \overline{T}^{bc}(q)$ with equality at $q = \overline{q}(\theta)$ for all θ, the following series of conditions holds:

$$E_\theta \left[S(\overline{q}(\theta)) - \theta \overline{q}(\theta) - n\overline{U}(\theta) + (n-1)(\overline{T}(\overline{q}(\theta)) - \theta \overline{q}(\theta)) \right]$$

$$= E_\theta \left[S(\overline{q}(\theta)) - \theta \overline{q}(\theta) - n\overline{U}(\theta) + (n-1)(\overline{T}^{bc}(\overline{q}(\theta)) - \theta \overline{q}(\theta)) \right]$$

$$\geq E_\theta \left[S(q(\theta)) - \theta q(\theta) - nU(\theta) + (n-1)(\overline{T}^{bc}(q(\theta)) - \theta q(\theta)) \right)] \quad \forall (U, q) \in \mathcal{I}$$

$$\geq E_\theta \left[S(q(\theta)) - \theta q(\theta) - nU(\theta) + (n-1)(\overline{T}(q(\theta)) - \theta q(\theta)) \right] \quad \forall (U, q) \in \mathcal{I},$$

which proves that $\overline{T}$ also implements $(\overline{U}, \overline{q})$ in equilibrium. Hence, the equilibrium allocation $(\overline{U}, \overline{q})$ is undefeated. *Q.E.D.*

PROOF OF THEOREM 2: A biconjugate equilibrium allocations $(\overline{U}, \overline{q})$ is supported by a concave and proper aggregate schedule $\overline{T}^{bc}$. Therefore, $(\overline{\mathcal{P}})$ is a concave program whose solution is characterized by generalized first-order conditions using super-differentials instead of derivatives for $\overline{T}^{bc}$. Because $\overline{T}^{bc}$ is the least-concave function implementing $(\overline{U}, \overline{q})$, we have:

$$\partial\overline{T}^{bc}(q) = \begin{cases} \overline{\theta} & \text{if } q < \overline{q}(\overline{\theta}), \\ [\underline{\theta}, \overline{\theta}] & \text{if } q = \overline{q}(\overline{\theta}), \\ \underline{\theta} & \text{if } q > \overline{q}(\overline{\theta}). \end{cases} \tag{A11}$$

We can now return to program $(\overline{\mathcal{P}})$ and write the corresponding (necessary and sufficient) optimality conditions in both states of nature using super-differentials:

$$0 \in S'(\overline{q}(\underline{\theta})) - \underline{\theta} + (n-1)(\partial\overline{T}^{bc}(\overline{q}(\underline{\theta})) - \underline{\theta}), \tag{A12}$$

and

$$0 \in S'(\overline{q}(\overline{\theta})) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right) + (n-1)(\partial\overline{T}(\overline{q}(\overline{\theta})) - \overline{\theta}). \tag{A13}$$

Separating allocations. These allocations are such that (4.4) holds strictly. From (A11), $\partial\overline{T}^{bc}(\overline{q}(\underline{\theta})) = \underline{\theta}$. Inserting into (A12) yields (5.3) and (5.4).

Pooling allocations. From (A11), $\partial\overline{T}^{bc}(\overline{q}(\underline{\theta})) = [\underline{\theta}, \overline{\theta}]$. Inserting into (A12) yields that such a pooling allocation, $\overline{q}(\overline{\theta}) = \overline{q}(\underline{\theta}) = \overline{q}$, should satisfy

$$S'(\overline{q}) - \underline{\theta} + (n-1)\Delta\theta \geq 0 \geq S'(\overline{q}) - \underline{\theta}, \tag{A14}$$

and

$$S'(\overline{q}) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right) \geq 0 \geq S'(\overline{q}) - \left(\overline{\theta} + \frac{n\nu}{1-\nu}\Delta\theta\right) - (n-1)\Delta\theta. \tag{A15}$$

Clearly, the right-hand side inequality in (A14) and the left-hand side in (A15) are not compatible. This rules out pooling allocations as implemented by biconjugate equilibria. *Q.E.D.*

PROOF OF PROPOSITION 4: Immediate from the text. *Q.E.D.*

PROOF OF PROPOSITION 5: Since (4.3) holds for allocations in $\mathcal{I}^{eq}$, we may rewrite the maximand of $(\mathcal{P}^{eq})$ only in terms of outputs to get:

$$(\mathcal{P}^{eq}): \quad \max_{\overline{q}(\theta)} \nu\left(S(\overline{q}(\underline{\theta})) - \underline{\theta}\overline{q}(\underline{\theta}) - \Delta\theta\overline{q}(\overline{\theta})\right) + (1-\nu)\left(S(\overline{q}(\overline{\theta})) - \overline{\theta}\overline{q}(\overline{\theta})\right)$$

subject to (4.1) and (4.2).

The optimal output for the low-cost agent is thus first-best. When Assumption 2 holds, the cooperative outcome $q^{coop}(\overline{\theta})$ is not implementable and the best output for a high-cost agent is obtained when (4.2) is binding. *Q.E.D.*

PROOF OF PROPOSITION 6: From Theorem 2, we can now write the optimization problem as:

$$(\mathcal{P}^{eq}): \quad \max_{\overline{q}(\theta)} \nu \left(S(\overline{q}(\underline{\theta})) - \underline{\theta}\overline{q}(\underline{\theta}) - \Delta\theta\overline{q}(\overline{\theta}) \right) + (1-\nu) \left(S(\overline{q}(\overline{\theta})) - \overline{\theta}\overline{q}(\overline{\theta}) \right)$$

subject to $\overline{q}(\underline{\theta}) = q^{fb}(\underline{\theta})$ and $\overline{q}(\overline{\theta}) \in [\tilde{q}_n(\overline{\theta},), q_n^{bc}(\overline{\theta})]$.

The maximum of the principal's payoff is achieved at $q^{bc}(\overline{\theta})_n$ since $q_n^{bc}(\overline{\theta}) < q_n^{coop}(\overline{\theta})$. *Q.E.D.*

APPENDIX B: EQUILIBRIUM SET UNDER COMPLETE INFORMATION

To better understand the restrictions imposed by asymmetric information and for the sake of completeness, we now restate our characterization theorem when the agent's cost parameter is common knowledge *ex post* while contracting takes place under symmetric but incomplete information. The following definition follows.

DEFINITION B.1 *An allocation (U, q) is complete information-implementable if there exists a family of aggregate contributions $T(q|\theta) = \sum_{i=1}^n T_i(q|\theta))$ contingent on the realization of θ such that:*

$$\text{(B1)} \qquad U(\theta) = \max_{q \in \mathcal{Q}} T(q|\theta)) - \theta q \geq 0; \ \textit{and } q(\theta) \in \arg\max_{q \in \mathcal{Q}} T(q|\theta)) - \theta q \quad \forall \theta \in \Theta.$$

Let denote accordingly by $\mathcal{I}^{fb}$ the set of *individually rational* allocations $(\overline{U}(\theta), q(\overline{\theta}))$ that can be achieved with such state-contingent schedules $T(q|\theta)$. A simplified version of Proposition 1 still applies when θ is common knowledge *ex post* while contracting takes place under symmetric but incomplete information so that *self-generating* problems maximizes the surrogate principal's objective function in expectation over the possible realizations of θ.

PROPOSITION B.1 *An allocation $(\overline{U}, \overline{q})$ belongs to $\mathcal{I}^{fb}$ if and only if it solves the following self-generating maximization problem:*

$$(\mathcal{P}^{fb}): \quad \max_{(U,q) \in \mathcal{I}^{fb}} E_\theta \left(S(q(\theta)) - \theta q(\theta) - nU(\theta) + (n-1)(\overline{T}(q(\theta)|\theta)) - \theta q(\theta)) \right),$$

where $\overline{T}(q|\theta)$ is a family of state-contingent aggregate tariffs that implements $(\overline{U}, \overline{q})$.

It is straightforward to modify Theorem 1 to account for complete information.[12]

THEOREM B.1 *An allocation $(\overline{U}, \overline{q})$ belongs to $\mathcal{I}^{fb}$ if and only if it satisfies the following conditions.*

- *Maximization of the collective surplus:*

$$\text{(B2)} \qquad S(\overline{q}(\underline{\theta})) - \underline{\theta}\overline{q}(\underline{\theta}) \geq S(\overline{q}(\overline{\theta})) - \underline{\theta}\overline{q}(\overline{\theta});$$

$$\text{(B3)} \qquad S(\overline{q}(\overline{\theta})) - \overline{\theta}\overline{q}(\overline{\theta}) \geq \max\left\{ S(\overline{q}(\underline{\theta})) - \overline{\theta}\overline{q}(\underline{\theta}) - (n-1)\Delta\theta(\overline{q}(\underline{\theta}) - \overline{q}(\overline{\theta})); 0 \right\}.$$

[12] The proof is quite similar to the proof of Theorem 1 with the omission of the agent's incentive constraint and is thus omitted.

- *Rent minimization:*

$$\overline{U}(\underline{\theta}) = \overline{U}(\overline{\theta}) = 0. \tag{B4}$$

Under complete information, the solutions to the self-generating problem boils down to the maximization of the sum of the principals' profits. The only source of contractual externality in our model is thus asymmetric information and, when this source disappears, the objective of the surrogate principal does not differ from what principals can achieve by acting cooperatively. There are no transaction costs involved by having principals not cooperating under complete information. It is indeed straightforward to check that the efficient allocation $(U^{fb}(\theta) = 0, q^{bf}(\theta))$ is now equilibrium implementable. For instance, the following family of aggregate offers, which are forcing contracts conditioned on the realization of θ, implements this efficient outcome:

$$\overline{T}(q|\theta) = \begin{cases} \theta q^{fb}(\theta) & \text{if } q = q^{fb}(\theta) \\ -\infty & \text{if } q \notin \{q^{fb}(\overline{\theta}), q^{fb}(\underline{\theta})\}. \end{cases} \tag{B5}$$

Individual state-contingent offers can again be reconstructed by means of formula (3.1).

Proofs of Proposition B.1 and Theorem B.1: Those proofs follow the same steps as the proofs of Proposition 1 and Theorem 1 with the only proviso that tariffs are now type-contingent. Details are omitted. *Q.E.D.*

REFERENCES

[1] Aubin, J.P. (1998). *Optima and Equilibria.* Springer.
[2] Attar, A., Mariotti, T. and F. Salanié (2014a). Nonexclusive Competition under Adverse Selection. *Theoretical Economics*, **9:** 1-40.
[3] Attar, A., Mariotti, T. and F. Salanié (2014b). On Competitive Nonlinear Pricing. Working Paper Toulouse School of Economics.
[4] Bernheim, D. and M. Whinston, (1986a). Common Agency. *Econometrica*, **54**: 923-942.
[5] Bernheim, D. and M. Whinston (1986b). Menu Auctions, Resource Allocations and Economic Influence. *The Quarterly Journal of Economics*, **101**: 1-31.
[6] Laffont, J.J. and J. Tirole (1993). *A Theory of Incentives in Regulation and Procurement.* MIT Press.
[7] Martimort, D. (1992). Multi-Principaux avec Anti-Selection. *Annales d'Economie et de Statistiques*, **28**: 1-38.
[8] Martimort, D. and A. Semenov (2008). Ideological Uncertainty and Lobbying Competition. *Journal of Public Economics,* **92:** 456-481.
[9] Martimort, D. and L. Stole (2002). The Revelation and Delegation Principles in Common Agency Games. *Econometrica*, **70**: 1659-1674.
[10] Martimort, D. and L. Stole (2009a). Market Participation under Delegated and Intrinsic Common Agency Games. *The RAND Journal of Economics*, **40**: 78-102.
[11] Martimort, D. and L. Stole (2009b). Selecting Equilibria in Common Agency Games. *Journal of Economic Theory*, **144**: 604-634.
[12] Martimort, D. and L. Stole (2012). Representing Equilibrium Aggregates in Aggregate Games with Applications to Common Agency. *Games and Economic Behavior*, **76**: 753-772.
[13] Martimort, D. and L. Stole (2015). Menu auctions and influence games with private information. Working paper. February 2015.

[14] Pavan, A., and G. Calzolari (2010). Truthful Revelation Mechanisms for Simultaneous Common Agency Games. *American Economic Journal: Microeconomics,* **2**: 132-190.

[15] Peters, M. (2001). Common Agency and the Revelation Principle. *Econometrica*, **69**: 1349-1372.

[16] Stole, L. (1991). Mechanism Design under Common Agency. Mimeo University of Chicago.