Do vegetarian marketing campaigns promote a vegan diet?

James, Waters

Warwick Business School

17 September 2015

Do vegetarian marketing campaigns promote a vegan diet?

James Waters [a,1]

[a] *Warwick Business School, University of Warwick, Coventry, CV4 7AL, United Kingdom*

17 September 2015

WORKING PAPER – COMMENTS WELCOME

ABSTRACT

This paper examines whether vegetarian marketing campaigns promote a vegan diet. Our trivariate model of omnivorous, vegetarian, and vegan consumption is estimated using twenty years of UK data. For short-lived campaigns, we find no persistent effect, but observe a rise and fall in vegan numbers during adjustment. For long-running campaigns, we find that for every person who adopts a vegetarian diet in such a campaign, around 0.34 people adopt a vegan diet. In a campaign to market veganism, for every new vegan there are between 0.5 and 0.77 new vegetarians.

[1] E-mail: james.waters@warwick.ac.uk

1. Introduction

There have been many marketing campaigns in recent years promoting vegetarianism (Animal Aid, 2015; Peta, 2015; Vegetarian Society, 2015). Some of these are run by organisations such as People for the Ethical Treatment of Animals for whom vegetarianism is not their ultimate goal, which is the promotion of a vegan lifestyle with no use of animal products. Nevertheless, they see vegetarianism as a good intermediate step, and perhaps the best achievable (Fischer and McWilliams, 2015; Fastenberg, 2010).

Other people with similar vegan aims reject the use of intermediate steps. They argue that promoting vegetarianism reinforces use of animal products, and hinders the promotion of veganism (Dunayer, 2004, page 155; Francione, 2015). Taking as examples past successful campaigns for social reform, they argue on moral and practical grounds that campaigners should instead market veganism exclusively.

This paper examines the following questions. Do marketing campaigns that promote a vegetarian diet also promote a vegan diet? How does the effect differ if a vegetarian campaign only attracts people from an omnivorous diet, rather than from a vegan diet too? Would an alternative campaign that promotes a vegan diet also encourage people to adopt a vegetarian diet?

We formulate a modified Lanchester model of advertising in a competitive market. There are states measuring the numbers of omnivores, vegetarians, and vegans, with entry and exit between the states dependent on external advertising and internal word-

of-mouth effects. A trivariate system of differential equations is derived, and solved in its full and linearised form using UK data. The effect of transient vegetarian and vegan marketing campaigns are analysed using the differential field of the solved system, and persistent campaigns are analysed by examining the derivatives of equilibrium points with respect to system parameters.

We make three main contributions to the literature and to assist animal rights advocates. Firstly, we describe the interactive dynamics in the numbers of omnivorous, vegetarian, and vegan consumers in the UK. Although there have been previous studies of trends in vegetarian and veganism (for example, Beardsworth and Bryman (2004)), we are unaware of any previous marketing model studying their joint dynamics. We determine the extent of consumer interactions and transitions, finding equilibrium points and their dynamic stability.

Secondly, we show the effect on dietary preferences of transient marketing campaigns that promote vegetarianism. We show that vegetarian and vegan numbers tend towards a single stable equilibrium, no matter what the original distribution of dietary preference is. We give a complete graphical description of how the numbers react dynamically to transient vegetarian campaigns and show a tendency of vegan numbers to rise then fall after a transient vegetarian campaign, given current dietary preferences.

Our third contribution is to examine the effect of permanent campaigns. We estimate that a vegetarian marketing campaign that increases the equilibrium number of vegetarians by one also increases the equilibrium number of vegans by around 0.34.

There is little difference in the effect of campaigns that attract omnivores and vegans to the vegetarian diet, and those that attract omnivores alone. We also estimate that a vegan marketing campaign that increases the equilibrium number of vegans by one increases the equilibrium number of vegetarians by between 0.50 and 0.77.

The model in this paper has precursors in the marketing literature, many of which depart from Sorger's (1989) variant of the Lanchester model applied to competitive dynamic advertising. Chintagunta and Jain (1995) extend the model to include word of mouth effects, in common with us. Naik et al (2008) add multiple competitors to the model and apply an extended Kalman filtration estimation, as we do. We differ from these papers in that the Sorger (1989) model constrains the size of word-of-mouth effects to be determined by the extent of external advertising, whereas in our model they are derived to have an independent impact on adoption. Libai et al (2009) present a model in which churn between different consumption groups is modelled explicitly in a differential equations framework, as in our model. However, they assume that there is unexploited market potential whereas our market is saturated, and their word-of-mouth effects operate on the remaining market potential whereas in our model they operate as an additional churn influence.

There are some precursor papers in the economics literature that look at how animal rights campaigns and considerations affect the demand for animal goods. Both Bennett (1995) and Frank (2006) look at how disclosure of welfare information affects demand, while Waters (2015) examines how the number of animals killed varies in response to a number of different campaign types. The studies use static analysis unlike the dynamic approach given here, and do not distinguish between

vegetarian and vegan preferences. In the wider legal, philosophic, and sociological literature, there is debate and sharp disagreement on the subject of efficient marketing of veganism, and the consequences of it (DeCoux, 2009; Francione, 1997; Garner, 2006; Wrenn, 2012).

Section 2 presents our theoretical model, section 3 gives our estimation method, and section 4 describes the data. Section 5 presents the results and section 6 concludes.

2. Theoretical model

In this section we describe our model of adoption of vegetarian and vegan diets. It is similar to the dynamic part of the Chintagunta and Jain's (1995) model, but without an implicit constraint on the relation between the word-of-mouth effect (see Sorger (1989) and Chintagunta and Jain (1995) for a derivation and discussion of the constraint). Alternatively, it overlaps with the Libai et al (2009) model, but with a fully saturated market and word-of-mouth effects operating between different dietary states.

The model expresses adoption rates in terms of proportions of consumers rather than absolute numbers. We work with proportions because of the available empirical data and because they make it mathematically tidier to express our assumption on new entrants leaving proportions unchanged. The model's argument would be the same if we used absolute numbers instead.

There are three types of consumers, distinguished by their consumption of animal products. The first type is omnivorous consumers who eat all forms of animal

products. At time t, a proportion l_t of consumers are omnivorous. The second type is vegetarian consumers who do not eat meat but eat eggs and dairy products. They account for a proportion m_t of consumers at time t. The final type is vegan consumers who do not eat any products from animal sources, and they are a proportion h_t of consumers at time t. The proportions satisfy the identity $l_t + m_t + h_t = 1$ at time t.

Omnivorous consumers are subject to external advertising for the vegetarian diet, and are persuaded to adopt it at an instantaneous rate of $a_0 l_t$. Word-of-mouth additionally influences their adoption, at a instantaneous rate proportional to the share of current vegetarians, or $a_1 m_t l_t$. Omnivorous consumers are also subject to external advertising for a vegan diet, which they adopt at a rate of $b_0 l_t$, and word-of-mouth influence proportional to the share of vegans, giving an instantaneous adoption rate of $b_1 h_t l_t$. New consumers who enter the market at time t are omnivorous in the same share as existing consumers, so that their entrance leaves the proportion of omnivorous consumers unchanged. We may consider young consumers as having similar dietary preferences to their carers, or immigrants as having the same distribution of preferences as the host population. One way of modifying this assumption would be to create exogenous drifts in the rates of each dietary type, with the algebra adjusting accordingly, while another way would be to assume that the proportions of new entrants in each dietary type is fixed and then use data on the numbers of new entrants to estimate these proportions. With our dataset we cannot pursue the latter approach.

Vegetarian consumers experience external advertising for the omnivorous diet, which is adopted at a rate of $c_0 m_t$, and word-of-mouth influence leading to an adoption rate of $c_1 l_t m_t$. They experience external advertising for the vegan diet giving an adoption rate of $d_0 m_t$, and word-of-mouth influence for the vegan diet leading to an adoption rate of $d_1 h_t m_t$. New entrants to the market leave the proportions of people with the vegetarian diet unchanged.

Vegan consumers are acted on by external advertising for the omnivorous diet, so that it is adopted at a rate of $e_0 h_t$, and by word-of-mouth influence leading to an adoption rate of $e_1 l_t h_t$. They are subject to external advertising for the vegetarian diet leading to an adoption rate of $f_0 h_t$, and word-of-mouth influence for it resulting in an adoption rate of $f_1 m_t h_t$. Entry of new consumers leaves the proportion of vegan consumers unchanged.

Considering all entries and exits from each state of food consumption, it follows that the number of omnivorous consumers then satisfies the differential equation

$$\frac{dl_t}{dt} = -(a_0 + a_1 m_t)l_t - (b_0 + b_1 h_t)l_t + (c_0 + c_1 l_t)m_t + (e_0 + e_1 l_t)h_t$$

The number of vegetarian consumers satisfies

$$\frac{dm_t}{dt} = -(c_0 + c_1 l_t)m_t - (d_0 + d_1 h_t)m_t + (a_0 + a_1 m_t)l_t + (f_0 + f_1 m_t)h_t \qquad (1)$$

while the number of vegan consumers satisfies

$$\frac{dh_t}{dt} = -(e_0 + e_1 l_t)h_t - (f_0 + f_1 m_t)h_t + (b_0 + b_1 h_t)l_t + (d_0 + d_1 h_t)m_t.$$ (2)

Differentiating the population identity $l_t + m_t + h_t = 1$ gives

$$\frac{dl_t}{dt} + \frac{dm_t}{dt} + \frac{dh_t}{dt} = 0$$

It follows that there is linear dependence between the equation for $\frac{dl_t}{dt}$ and the equations for $\frac{dm_t}{dt}$ and $\frac{dh_t}{dt}$, so we can examine the last two equations alone without losing any information about the dynamics of the system.

In equation (1)

$$\frac{dm_t}{dt} = -(c_0 + c_1 l_t)m_t - (d_0 + d_1 h_t)m_t + (a_0 + a_1 m_t)l_t + (f_0 + f_1 m_t)h_t$$

we substitute for l_t using the population equation $l_t + m_t + h_t = 1$:

$$\frac{dm_t}{dt} = -(c_0 + c_1(1 - m_t - h_t))m_t - (d_0 + d_1 h_t)m_t$$
$$+ (a_0 + a_1 m_t)(1 - m_t - h_t) + (f_0 + f_1 m_t)h_t$$

or

$$\frac{dm_t}{dt} = -c_0 m_t - c_1(1 - m_t - h_t)m_t - (d_0 + d_1 h_t)m_t$$
$$+ a_0(1 - m_t - h_t) + a_1 m_t(1 - m_t - h_t) + (f_0 + f_1 m_t)h_t$$

or

$$\frac{dm_t}{dt} = -c_0 m_t - (c_1 m_t - c_1 m_t^2 - c_1 m_t h_t) - (d_0 m_t + d_1 m_t h_t)$$
$$+ (a_0 - a_0 m_t - a_0 h_t) + (a_1 m_t - a_1 m_t^2 - a_1 m_t h_t) + (f_0 h_t + f_1 m_t h_t)$$

or

$$\frac{dm_t}{dt} = -c_0 m_t - c_1 m_t + c_1 m_t^2 + c_1 m_t h_t - d_0 m_t - d_1 m_t h_t$$
$$+ a_0 - a_0 m_t - a_0 h_t + a_1 m_t - a_1 m_t^2 - a_1 m_t h_t + f_0 h_t + f_1 m_t h_t$$

or

$$\frac{dm_t}{dt} = +a_0 - a_0 m_t + a_1 m_t - c_0 m_t - c_1 m_t - d_0 m_t - a_0 h_t + f_0 h_t$$
$$- a_1 m_t h_t + c_1 m_t h_t - d_1 m_t h_t + f_1 m_t h_t - a_1 m_t^2 + c_1 m_t^2$$

or

$$\frac{dm_t}{dt} = a_0 + (-a_0 + a_1 - c_0 - c_1 - d_0)m_t + (-a_0 + f_0)h_t$$
$$+ (-a_1 + c_1)m_t^2 + (-a_1 + c_1 - d_1 + f_1)m_t h_t$$

Equation (2) describing the evolution in the number of people following a vegan diet

is

$$\frac{dh_t}{dt} = -(e_0 + e_1 l_t)h_t - (f_0 + f_1 m_t)h_t + (b_0 + b_1 h_t)l_t + (d_0 + d_1 h_t)m_t$$

or on using $l_t + m_t + h_t = 1$ we have

$$\frac{dh_t}{dt} = -(e_0 + e_1(1 - m_t - h_t))h_t - (f_0 + f_1 m_t)h_t$$
$$+ (b_0 + b_1 h_t)(1 - m_t - h_t) + (d_0 + d_1 h_t)m_t$$

or

$$\frac{dh_t}{dt} = -e_0 h_t - e_1(1 - m_t - h_t)h_t - (f_0 + f_1 m_t)h_t$$
$$+ b_0(1 - m_t - h_t) + b_1 h_t(1 - m_t - h_t) + (d_0 + d_1 h_t)m_t$$

or

$$\frac{dh_t}{dt} = -e_0 h_t - (e_1 h_t - e_1 m_t h_t - e_1 h_t^2) - (f_0 h_t + f_1 m_t h_t)$$
$$+ (b_0 - b_0 m_t - b_0 h_t) + (b_1 h_t - b_1 m_t h_t - b_1 h_t^2) + (d_0 m_t + d_1 m_t h_t)$$

or

$$\frac{dh_t}{dt} = -e_0 h_t - e_1 h_t + e_1 m_t h_t + e_1 h_t^2 - f_0 h_t - f_1 m_t h_t$$
$$+ b_0 - b_0 m_t - b_0 h_t + b_1 h_t - b_1 m_t h_t - b_1 h_t^2 + d_0 m_t + d_1 m_t h_t$$

or

$$\frac{dh_t}{dt} = b_0 - b_0 m_t + d_0 m_t - b_0 h_t + b_1 h_t - e_0 h_t - e_1 h_t - f_0 h_t$$
$$- b_1 m_t h_t + d_1 m_t h_t + e_1 m_t h_t - f_1 m_t h_t - b_1 h_t^2 + e_1 h_t^2$$

or

$$\frac{dh_t}{dt} = b_0 + (-b_0 + d_0)m_t + (-b_0 + b_1 - e_0 - e_1 - f_0)h_t$$
$$+ (-b_1 + d_1 + e_1 - f_1)m_t h_t + (-b_1 + e_1)h_t^2$$

We can write the equations as

$$\frac{dm_t}{dt} = \alpha_1 + \alpha_2 m_t + \alpha_3 h_t + \alpha_4 m_t h_t + \alpha_5 m_t^2 \qquad (3)$$

and

$$\frac{dh_t}{dt} = \beta_1 + \beta_2 m_t + \beta_3 h_t + \beta_4 m_t h_t + \beta_5 h_t^2. \qquad (4)$$

There are no cross-equation restrictions as the small letter parameters in the original

equations can be defined to solve for any set of Greek parameters (for example, set

$a_0 = \alpha_1$, $-a_0 + f_0 = \alpha_3$, and so on), with slight redundancy in the original set of 12 parameters in mapping to the new set of 10 parameters.

3. Estimation method

We adapt our model in equations (3) and (4) in stochastic form as

$$\frac{dm_t}{dt} = \alpha_1 + \alpha_2 m_t + \alpha_3 h_t + \alpha_4 m_t h_t + \alpha_5 m_t^2 + v_{1,t} \tag{5}$$

and

$$\frac{dh_t}{dt} = \beta_1 + \beta_2 m_t + \beta_3 h_t + \beta_4 m_t h_t + \beta_5 h_t^2 + v_{2,t} \tag{6}$$

where $\mathbf{v}_t = (v_{1,t}, v_{2,t}) \sim N(0, \mathbf{Q})$ is a normal error term with covariance matrix $\mathbf{Q}$.

We have discrete data but a continuous model. Estimation methods such as OLS that neglect the difference can give rise to biased estimates (Schmittlein and Mahajan, 1982). In the case of Bass (1969) type models of diffusion, the problem is handled by Schmittlein and Mahajan (1982) and Srinivasan and Mason (1986) who find exact expressions for the extent of diffusion at discrete intervals allowing for MLE or NLS solutions, under certain assumptions on the form and occurrence of errors.

Our model is more complicated than the Bass model in that there is two way movement between states, and three states rather than two. As a result, we do not have solutions for the exact expressions at discrete time periods. Instead of estimation

methods based on such expressions, we use two alternative techniques. The first

technique is seemingly unrelated regression, estimated on a discrete version of

equations (5) and (6) with monthly intervals:

$$\Delta m_t = \alpha_1 + \alpha_2 m_t + \alpha_3 h_t + \alpha_4 m_t h_t + \alpha_5 m_t^2 + v_{1,t}$$

and

$$\Delta h_t = \beta_1 + \beta_2 m_t + \beta_3 h_t + \beta_4 m_t h_t + \beta_5 h_t^2 + v_{2,t}$$

Although SUR neglects the continuous nature of the model, it offers the advantages of

producing stable estimates, being well established, and reducing to vector

autoregressive estimates when the model is linearised (which we describe shortly).

The second estimation method is a new way (for the marketing literature) of applying

the extended Kalman filter with continuous time and discrete observations, which

uses multi-step forecasting between discrete time periods to approximate the

continuous adjustment of the system. Xie et al (1997) have previously used the

extended Kalman filter in diffusion estimation for direct parameter estimation and

Naik (2008) have used it to track an endogenously determined variable in a study of

brand awareness in dynamic oligopolies. Our approach is to use the filter as a means

of state tracking in conjunction with classical parameter estimation. The method is

described in detail in Appendix A.

Our model has a quite high ratio of parameters to data points (in the case of the

extended Kalman Filter, 15 parameters and 168 data points), making estimates subject

to uncertainty. We also estimate more parsimonious models allowing us to find narrower standard errors, by linearising our main model:

$$\frac{dm_t}{dt} = \alpha_1 + \alpha_2 m_t + \alpha_3 h_t + v_{1,t}$$

and

$$\frac{dh_t}{dt} = \beta_1 + \beta_2 m_t + \beta_3 h_t + v_{2,t}$$

These equations represent a basic Lanchester model of bivariate competition, similar to the bivariate model of Case (1979). The equations remain informative about the larger system because at the small rates of non-omnivorous consumption in which we are interested, their behaviour is similar. In particular, the two systems have equilibria located near each other, and display comparable responses to animal advocacy campaigns, as described in the section 5.

Our estimation assumes that the parameters in the model are stable over the 1992-2015 period. In section 5, we assume that campaigns can induce changes in the parameters, and it is reasonable to think that earlier campaigns may also have changed them. To investigate whether the parameters were stable, we ran seemingly unrelated regressions on the linearised system over five year periods starting in 1992, with the last period from 2008-2012, using the data described in section 4. Appendix B shows the resulting parameter estimates. There are some fluctuations in the estimates, although these are smaller on the most consistently significant parameters: the lagged

vegetarian percentage in the equation describing the change in the number of vegetarians and the lagged vegan percentage in the equation describing the evolution in the number of vegans. The final parameter estimates over 2008-2012 are quite close to the estimates over the whole period reported in section 5, and it is the current parameters that we require in answering our research question. Thus, we treat the coefficients as constant over the 1992-2012 period.

R language code for the main estimates is given at the end of this paper.

4. Data

Our data is constructed from three sets of surveys of consumption by British households: the Family Expenditure Survey from January 1992 to March 2000, its successor the Expenditure and Food Survey from April 2001 to December 2007, and then its successor the Living Costs and Food module of the Integrated Household Survey from January 2008 to December 2012. The surveys were constructed to give representative samples on British households. They ran quarterly giving us 84 periods of data, and the number of households in our calculations varied across quarters from 1278 to 1915.

The surveys report consumption of different food and other goods by households and individuals within the households. We take households to be the consumers in our model, and consider a household to follow a vegetarian diet in a quarter if no individual within it consumed meat or fish in the survey period, and to follow a vegan diet if no individual in it consumed dairy or eggs either. In our model, influence may equally apply to households and individuals as consumers, and there are practical or

interpretational advantages of using households. Individual purchases are reported in our datasets, but they may be made for others in the household so we can't say that an individual is a vegetarian or vegan based on their purchases or absence of them. With household data, purchases are less likely to be made for a different unit and so are more likely to be an accurate reflection of behaviour. Additionally, consumption figures may be more accurate than self-reports of being a vegetarian or vegan, as the latter may be influenced by people's wish to identify with a particular lifestyle. Household consumption figures are less likely to be misreported to give the appearance of individual adherence to a diet, as consumption may plausibly be attributed to other people in the household so that there is less personal investment in an identity.

The data is shown in Appendix C, with figure 1 showing the rate of consumption of vegetarian and vegan diets over the surveyed period. Consumption of a vegetarian diet rose from 2.0 percent of households in early 1992 to 3.6 percent in late 2012, while consumption of the vegan diet rose 0.5 percent to 1.2 percent over the same period. Most of the growth had occurred by late 2004, with no clear trend in consumption rates subsequently.

Figure 1. Percentages of households following vegetarian and vegan diets
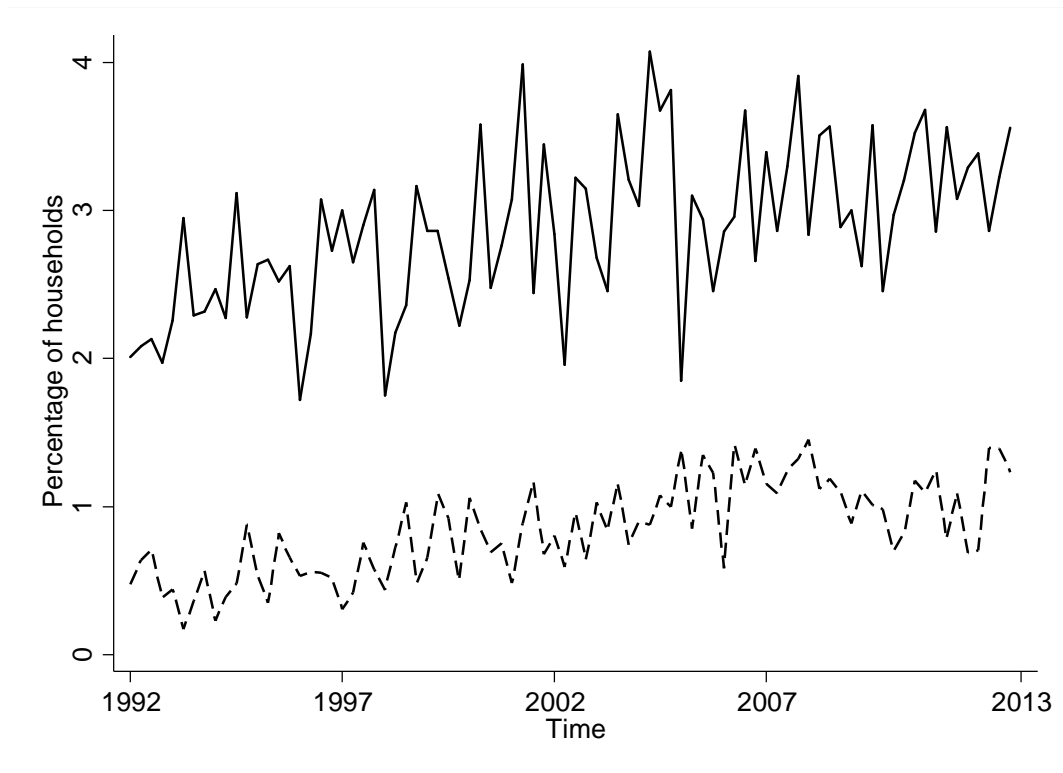
5. Results

5.1 Estimation results

Table 1. Estimation results

Dependent variable	Equation (a)	First differences in the percentage of vegetarians		Rate of change in the percentage of vegetarians		
	Equation (b)	First differences in the percentage of vegans		Rate of change in the percentage of vegans		
	Estimation method	SUR		EKF-CT/DO		
		(1)	(2)	(3)	(4)	
Equation (a):	Vegetarians$_{t-1}$	0.785	-0.886 ***	-1.098	-0.693	**
		0.855	0.103	1.849	0.328	
	Vegans$_{t-1}$	1.012	0.694 ***	0.695	0.396	
		1.076	0.176	4.820	0.655	
	Vegetarians$_{t-1}$ *Vegans$_{t-1}$	-11.708		-1.868		
		38.237		166.086		
	Vegetarians$_{t-1}^2$	-27.284 *		-1.945		
		16.516		20.994		
	Constant	-0.005	0.020 ***	0.028	0.017	**
		0.012	0.003	0.044	0.008	
Equation (b):	Vegetarians$_{t-1}$	-0.004	0.214 ***	0.401	0.340	
		0.157	0.052	1.199	0.382	
	Vegans$_{t-1}$	-1.234 **	-0.563 ***	-1.519	-1.138	
		0.610	0.088	3.182	0.723	
	Vegetarians$_{t-1}$ *Vegans$_{t-1}$	25.523		5.650		
		17.245		125.960		
	Vegans$_{t-1}^2$	-2.112		-0.964		
		25.264		52.381		
	Constant	0.004	-0.001	0.000	0.000	
		0.004	0.001	0.030	0.008	
	R^2 (eq 1)	0.498	0.474			
	R^2 (eq 2)	0.369	0.352			
	N	168	168	168	168	

Table 1 presents our estimation results. Column one reports the parameters for our full model estimated by seemingly unrelated regression. For the equation describing the dynamic in vegetarianism, the lagged squared vegetarian percentage has a ten percent significant negative effect on adoption of the diet, indicating its past adoption increasingly lowers the current rate of adoption. The equation for the change in veganism has a negative coefficient on the lagged vegan percentage with five percent

significance, so that past adoption slows current adoption. Column two shows the results for the linearised model estimated by SUR. In the equation for the dynamic in the vegetarian proportion, the lagged vegetarian proportion reduces current adoption. However, a higher rate of past veganism increases the current adoption of the vegetarian diet, and the constant term is significantly positive indicating advertising attracts omnivores to a vegetarian diet. In the equation for the evolution of the vegan share, a larger past vegetarian share increases adoption of veganism, while a bigger lagged proportion of vegans decreases adoption. All these coefficients are one percent significant. However, the constant term in the vegan dynamic equation is not significant, indicating the external advertising to omnivores does not attract them to adopt a vegan diet.

The third column shows the results for our model estimated using the extended Kalman filter with continuous time and discrete observations. None of the coefficients reach significance. Column four reports the estimated parameters from the linearised model, estimated using the same filter. In the equation describing the evolution of vegetarianism, the lagged proportion is negative and significant at five percent. As the proportion of vegetarians increases, the growth in the proportion falls. There is no significant effect of the vegan proportion on the vegetarian proportion, but there is a significant positive constant, indicating that external advertising to omnivores is successfully influencing them to adopt vegetarianism. In the equation describing the adoption of veganism, none of the coefficients are significant.

In summary, there is evidence that adoption of the vegetarian and vegan diets slows down as more people adopt them. Further, there is movement between vegetarian and

vegan dietary preferences. Omnivores are influenced to adopt the vegetarian diet, but there is no evidence for significant direct movement from an omnivorous diet to a vegan diet. The vegetarian diet seems to act as a stepping stone to the vegan diet.

5.2 Campaign effects

In this subsection we answer our research question by looking at the change in the number of vegans in response to campaigns that increase the number of vegetarians. We identify two types of campaigns. The first type of campaign alters the percentage of vegetarian diets without changing the underlying parameters. This type of campaign may be a temporary large push to increase the numbers of vegetarians. We can see the effect of such a campaign by calculating the differential field for our model, taking the estimated coefficients from specification one in table 1 and inserting them in our model from equations (3) and (4)

$$\frac{dm_t}{dt} = \alpha_1 + \alpha_2 m_t + \alpha_3 h_t + \alpha_4 m_t h_t + \alpha_5 m_t^2$$

and

$$\frac{dh_t}{dt} = \beta_1 + \beta_2 m_t + \beta_3 h_t + \beta_4 m_t h_t + \beta_5 h_t^2$$

to give the rates of change in the shares at each pair of vegetarian and vegan shares.

Figure 2 shows the differential field. The arrows represent the direction of change at any pair of (m_t, h_t). For example, when $(m_t, h_t) = (0.035, 0.025)$ the down-right

arrow indicates that vegetarianism is increasing and veganism is falling. There are two equilibrium points where $\dfrac{dm_t}{dt} = 0$ and $\dfrac{dh_t}{dt} = 0$, represented by circles on the figure. The lower of the two is at $(m_t, h_t) = (0.002, 0.003)$, which is an unstable equilibrium so that as a temporary campaign increases the rate of vegetarianism, the rates of vegetarianism and veganism move towards the higher equilibrium permanently. The higher equilibrium point is at $(m_t, h_t) = (0.029, 0.008)$, which is a stable equilibrium so that as a temporary campaign increases the rate of vegetarianism away from this equilibrium, the rates of consumption of the two diets subsequently restore to the equilibrium rates. An example path by which restoration occurs after a campaign raises m_t to 0.034 is shown by the thick black line in the lower right of the figure, with most of the adjustment occurring by a direct decline in the rate of vegetarian consumption and a slight rise and fall in vegan consumption. The upper equilibrium point is an attractor for all higher rates of vegetarianism and veganism as well, so that there is no large single campaign that will result in a permanent trend towards increased vegetarian and vegan consumption. The upper equilibrium point is also is close to the current UK rates which have been fluctuating around the same point since around 2008.

Figure 2. Differential field showing directions of movement in the vegetarian and vegan shares at different values of the shares
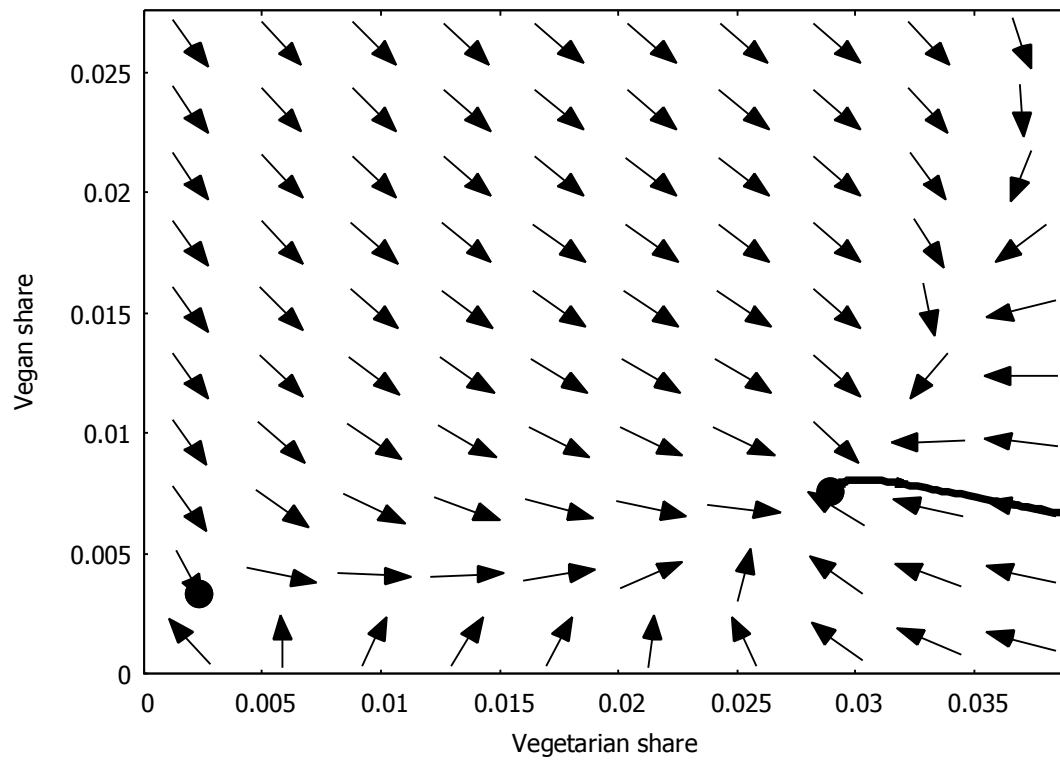
The second type of vegetarian campaign is one that permanently alters the model parameters, and so changes the equilibrium consumption of vegetarian and vegan diets. These campaigns achieve persistent gains for animal rights, in contrast to the transient effect of temporary campaigns. We calculate the effect of two permanent campaigns. The first campaign performs more advertising for vegetarian diets to attract people from both omnivorous and vegan diets. As given in equation (1), the vegetarian percentage follows the dynamic equation

$$\frac{dm_t}{dt} = -(c_0 + c_1 l_t)m_t - (d_0 + d_1 h_t)m_t + (a_0 + a_1 m_t)l_t + (f_0 + f_1 m_t)h_t$$

and the campaign raises a_0 (increasing adoption of vegetarian diets from omnivorous diets) and f_0 (increasing adoption of vegetarian diets from vegan diets). We represent the changes by adding a small scalar quantity q to a_0 and f_0. The campaign may also be considered to reduce c_0 (lowering exits from vegetarian to omnivorous diets) and d_0 (lowering exits from vegetarian to vegan diets), but we focus on the campaign as only attracting people to the vegetarian diet rather than additionally discouraging them from leaving it. In the next campaign we consider, the effects acting through a_0 and f_0 are isolated further.

After including q and transforming the dynamic equation as in section 2 we have

$$\frac{dm_t}{dt} = a_0 + q + (-a_0 - q + a_1 - c_0 - c_1 - d_0)m_t + (-a_0 - q + f_0 + q)h_t$$
$$+ (-a_1 + c_1)m_t^2 + (-a_1 + c_1 - d_1 + f_1)m_t h_t$$

or

$$\frac{dm_t}{dt} = a_0 + q + (-a_0 + a_1 - c_0 - c_1 - d_0 - q)m_t + (-a_0 + f_0)h_t$$
$$+ (-a_1 + c_1)m_t^2 + (-a_1 + c_1 - d_1 + f_1)m_t h_t$$

or in reduced coefficients

$$\frac{dm_t}{dt} = (\alpha_1 + q) + (\alpha_2 - q)m_t + \alpha_3 h_t + \alpha_4 m_t h_t + \alpha_5 m_t^2$$

The vegan percentage follows equation (2), or

$$\frac{dh_t}{dt} = -(e_0 + e_1 l_t)h_t - (f_0 + f_1 m_t)h_t + (b_0 + b_1 h_t)l_t + (d_0 + d_1 h_t)m_t.$$

which, after including q, transforms into

$$\frac{dh_t}{dt} = b_0 + (-b_0 + d_0)m_t + (-b_0 + b_1 - e_0 - e_1 - f_0 - q)h_t$$
$$+ (-b_1 + d_1 + e_1 - f_1)m_t h_t + (-b_1 + e_1)h_t^2$$

or in reduced coefficients

$$\frac{dh_t}{dt} = \beta_1 + \beta_2 m_t + (\beta_3 - q)h_t + \beta_4 m_t h_t + \beta_5 h_t^2.$$

We solve the original equations (3) and (4) at their equilibrium points

$$\frac{dm_t}{dt} = \alpha_1 + \alpha_2 m_t + \alpha_3 h_t + \alpha_4 m_t h_t + \alpha_5 m_t^2 = 0$$

and

$$\frac{dh_t}{dt} = \beta_1 + \beta_2 m_t + \beta_3 h_t + \beta_4 m_t h_t + \beta_5 h_t^2 = 0.$$

which yields solutions in (m, h). We are interested in the solution (m_S, h_S) close to their current rates, as the resulting analysis has most current policy relevance and it is more likely to be relevant at rates close to those used in estimation. We solve again for the perturbed equations under a value of q of 0.00001 to give corresponding solutions (m_{S+}, h_{S+}). The differential of the change in m_S and h_S with respect to q are approximated by $dm_S / dq = (m_{S+} - m_S) / q$ and $dh_S / dq = (h_{S+} - h_S) / q$, which describe the relative responses of the percentage of vegetarians and vegans to the campaign. These allow us to say how many people adopt a vegan diet following a campaign which persuades one extra person to adopt a vegetarian diet at equilibrium, using the quantity $(dh_S / dq) / (dm_S / dq)$.

The second campaign also performs more advertising for vegetarian diets but attracts people from omnivorous diets alone, leaving the direct movement from vegan diets unchanged. In the dynamic equation (1) for vegetarian numbers

$$\frac{dm_t}{dt} = -(c_0 + c_1 l_t)m_t - (d_0 + d_1 h_t)m_t + (a_0 + a_1 m_t)l_t + (f_0 + f_1 m_t)h_t$$

the campaign is represented by an increase in a_0 alone. Including q and transforming the dynamic equation as in section 2 gives

$$\frac{dm_t}{dt} = (a_0 + q) + (-a_0 - q + a_1 - c_0 - c_1 - d_0)m_t + (-a_0 - q + f_0)h_t$$
$$+ (-a_1 + c_1)m_t^2 + (-a_1 + c_1 - d_1 + f_1)m_t h_t$$

or in reduced coefficients

$$\frac{dm_t}{dt} = (\alpha_1 + q) + (\alpha_2 - q)m_t + (\alpha_3 - q)h_t + \alpha_4 m_t h_t + \alpha_5 m_t^2.$$

The vegan share follows equation (2)

$$\frac{dh_t}{dt} = -(e_0 + e_1 l_t)h_t - (f_0 + f_1 m_t)h_t + (b_0 + b_1 h_t)l_t + (d_0 + d_1 h_t)m_t$$

which is unchanged under the campaign. In reduced coefficients, the equation remains

$$\frac{dh_t}{dt} = \beta_1 + \beta_2 m_t + \beta_3 h_t + \beta_4 m_t h_t + \beta_5 h_t^2.$$

We then proceed in the same way as for the first campaign to calculate differentials of numbers of vegetarians and vegans, and the change in the number of vegans when the campaign increases the number of vegetarians by one. All campaign effects are calculated at the equilibrium close to the current rates of vegetarian and vegan consumption.

Only the central estimates of campaign response are reported. We could sample from the distribution of the parameter estimates to get alternative parameters for differential equations (3) and (4) and their linearised forms, which would be solved to find any equilibrium points. The equilibrium points at the campaign perturbed parameters could then be used to calculate responses at this sample point. The procedure could

be repeated to obtain a distribution for the policy responses. However, the size of the uncertainty in the parameter estimates produces some serious problems with the practical implementation of the procedure. The simultaneous quadratic equations which are solved to give the equilibrium points may have multiple solutions and there may be ambiguity about which one should be considered for calculating the campaign response, particularly if the solutions lie far from the current level of vegetarian and vegan consumption. Some parameterisations may not have any equilibrium points, with no campaign response available for calculation. We therefore only observe that there is wide uncertainty in the campaign response.

Table 2. Equilibrium shares and responses to vegetarian campaigns

| Model | Full | Linear | Full | Linear |
Estimation method	SUR	SUR	EKF-CT/DO	EKF-CT/DO
Equilibrium share				
Vegetarians	0.029	0.030	0.029	0.029
Vegans	0.008	0.010	0.009	0.009
Campaign one to boost the vegetarian diet at the expense of the omnivorous and vegan diet				
Change in number of vegans				
per extra vegetarian	0.35	0.37	0.32	0.29
Campaign two to boost the vegetarian diet at the expense of the omnivorous diet				
Change in number of vegans				
per extra vegetarian	0.36	0.38	0.33	0.30
Campaign three to boost the vegan diet at the expense of the omnivorous and vegetarian diet				
Change in number of				
vegetarians per extra vegan	0.76	0.77	0.50	0.53

Table 2 shows the results. The full and linear specifications, estimated under the SUR and EKF-CT/DO, have close agreement on the equilibrium level of consumption with current parameters. They put the equilibrium vegetarian consumption at around 2.9 percent of the total population, and equilibrium vegan consumption at around 0.9 percent of the population. There is slightly wider divergence in the effect of the campaigns, but they are still similar. In the case of the campaign one to increase vegetarianism at the expense of both an omnivorous diet and vegan diet, each extra

vegetarian is associated with between 0.29 and 0.37 extra vegans, depending on the model and estimation method. The campaign is associated with growth in consumption of the vegan diet despite it directly attracting people to vegetarianism from veganism. The reason is that the campaign also attracts people from the omnivorous diet to a vegetarian diet, and some of them then move to a vegan diet. As the equilibrium number of omnivores is much larger than the number of non-omnivores, many more people move from the omnivorous diet to a vegetarian one and then a vegan one than leave a vegan diet to adopt a vegetarian one. Thus, the net effect of the campaign is to increase the number of vegans.

In the case of the campaign two to increase vegetarian consumption by attracting from an omnivorous diet alone (and not a vegan one), each extra vegetarian is associated with an additional 0.30 to 0.38 vegans depending on the model and estimation method. This campaign has very little additional effect on vegan numbers compared with the campaign that also attracts from veganism. The reason is that the effect of the campaign is largely determined by the movement from the omnivorous diet to a vegetarian and then vegan diet. The numbers of people that the campaign encourages to abandon veganism for vegetarianism is small, so their exclusion does not alter net campaign effects.

Some campaigners have called for the resources spent on animal welfare campaigns to be redirected towards vegan campaigns, such as Gary-TV (2015). Our model allows us to see how the numbers of vegetarians would change in response to a campaign promoting the vegan diet. The campaign we examine attracts both

omnivores and vegetarians to veganism, and is represented algebraically in the dynamic equations (1) and (2)

$$\frac{dm_t}{dt} = -(c_0 + c_1 l_t)m_t - (d_0 + d_1 h_t)m_t + (a_0 + a_1 m_t)l_t + (f_0 + f_1 m_t)h_t$$

and

$$\frac{dh_t}{dt} = -(e_0 + e_1 l_t)h_t - (f_0 + f_1 m_t)h_t + (b_0 + b_1 h_t)l_t + (d_0 + d_1 h_t)m_t$$

by increases of q in b_0, the rate of externally induced adoption of the vegan diet from the omnivorous diet, and in d_0, the rate of externally induced adoption of the vegan diet from the vegetarian diet. After transformation and including the campaign parameter q, equation (3) describing the number of vegetarians becomes

$$\frac{dm_t}{dt} = a_0 + (-a_0 + a_1 - c_0 - c_1 - d_0 - q)m_t + (-a_0 + f_0)h_t$$
$$+ (-a_1 + c_1)m_t^2 + (-a_1 + c_1 - d_1 + f_1)m_t h_t$$

and equation (4) describing the number of vegans becomes

$$\frac{dh_t}{dt} = b_0 + q + (-b_0 + d_0)m_t + (-b_0 - q + b_1 - e_0 - e_1 - f_0)h_t$$
$$+ (-b_1 + d_1 + e_1 - f_1)m_t h_t + (-b_1 + e_1)h_t^2$$

The reduced equations are then

$$\frac{dm_t}{dt} = \alpha_1 + (\alpha_2 - q)m_t + \alpha_3 h_t + \alpha_4 m_t h_t + \alpha_5 m_t^2.$$

and

$$\frac{dh_t}{dt} = (\beta_1 + q) + \beta_2 m_t + (\beta_3 - q)h_t + \beta_4 m_t h_t + \beta_5 h_t^2.$$

The effect of the campaign is then calculated as for the other two campaigns, with $(dm_S/dq)/(dh_S/dq)$ estimating the change in the number of vegetarians when the number of vegans increases by one.

The results are shown in table 2. Estimates of the change in vegetarian numbers vary from 0.50 to 0.77, with the SUR estimates higher than the extended Kalman filter estimates. The vegan campaign is nearly as effective at generating new vegetarians as new vegans. Although the campaign results in people abandoning their vegetarian diet in favour of a vegan one, the movement from the omnivorous diet to the vegan diet is much larger in scale. Some of these new vegans subsequently adopt a vegetarian diet, and this movement is larger than the one in the other direction.

6. Conclusion

Our work indicates that vegetarian campaigns increase vegan numbers, and a policy-relevant question that follows this conclusion is whether an animal use abolitionist who only cared about vegan numbers could support a vegetarian campaign. To express the problem only in terms of cost, if it costs C_1 to persuade someone to adopt

a vegetarian diet through a vegetarian campaign, then the cost of one person adopting a vegan diet through the same campaign is about $C_1 / 0.34 \approx 2.94 C_1$. If it costs C_2 to persuade someone to adopt a vegan diet through a vegan campaign, then the abolitionist should prefer the vegetarian campaign route if and only if $2.94 C_1 < C_2$. Similar calculations can be made under other valuations, such as one that attributed a non-zero value to a vegetarian adoption.

This inequality ignores certain concerns of abolitionists. Among them is the entrenchment of use of animal products by non-vegan campaigns, which in our model would be represented by a decline in the transition rates into veganism in response to such campaigns. We didn't look fully at this dynamic link, so we can't exclude the possibility. Future work could examine whether it occurs and how critical it is, perhaps using the parameter tracking of the extended Kalman filter as in Naik et al (2008).

Our work points to a problem for the animal rights movement in the UK. Vegetarian and vegan numbers are now close to their equilibrium rates, and those equilibria have not changed much over the last twenty years. The recent flattening in the growth rates of the numbers of vegetarians and vegans is not the result of a failure or change of strategy, but an attainment of the potential of the advocacy approach followed over the same period. To move to a much higher rate requires a substantial adjustment to the approach.

This is not to say that it is a good idea to abandon the advertising, lobbying, and exposés of the meat industry that have been prominent features of UK animal rights

advocacy since the 1990s. Their loss would result in declines in the level of veganism, and constraints on them such as "ag-gag" laws (prohibiting disclosure of information about malpractices inside agricultural establishments by whistleblowers) should be challenged. The problem is in large part the overwhelming advertising and consumer access advantages of animal product industries. Even the largest animal rights organisations have tiny budgets by comparison (Counting Animals, 2015), and the net transition from omnivorous to non-omnivorous diets is commensurately small (Chintagunta and Vilcassim (1992) argue the transition coefficient in a duopoly is proportional to the square root of advertising expenditure). It may be helpful to analyse funding models in which a dominant good is supported by self-sustaining sales revenue and a substitute good is supported by charitable donations, to see if asymmetric positions can be used to the advantage of vegan promotion, rather than primarily hindering it.

Recent innovative campaigns have tried to adjust the word-of-mouth influence of people following vegetarian and vegan diets, rather than just the external advertising examined here. For example, a recent campaign Gary-TV (2015) relied on word-of-mouth diffusion over internet social media with periodic central intervention by the campaign coordinators. Another campaign, Direct Action Everywhere (2015), encourages its supporters to undertake high visibility actions and tell other people about cruelties in animal production without primarily focussing on the direct promotion of veganism. We could represent this in our model by increased word-of-mouth influence by vegetarians and vegans, but also by sympathetic omnivores who may persuade others (if not themselves) to adopt non-animal diets. An alternative

model may have stochastic connections between people and greater weak link formation due to the campaign (see Goldenberg et al, 2001).

There are some refinements in the empirical approach used in this paper that would be very welcome. The SUR method neglects the continuous nature of the underlying model, while the extended Kalman filter method has high levels of parameter uncertainty. A method that simultaneously solved these two problems would allow sharper results and interpretations. It is possible that the problem is primarily of colinearity in the specification, and that the full model given here should be reduced to the smaller Case (1979) model which does have high parameter significance, or another smaller alternative model used.

The theoretical model could be revised in various ways to make fuller use of the data or reflect contemporary developments in animal rights advocacy. Classifying people as omnivores, vegetarians, or vegans neglects the extent of use of animal products. Some animal advocates call for meat reduction to be a campaign target (Fischer and McWilliams, 2015; Ball, 2015). Meat and dairy use, and the effect of campaigns on them, could be examined in a bivariate or trivariate model. The wider arguments in the animal rights community on campaigns for animal welfare reforms and the consequences for animal rights outcomes could also be examined.

Appendix A

The Extended Kalman Filter in continuous time with discrete observations (EKF-CT/DO)

The continuous time and discrete observation extended Kalman filter has been previously used by Xie et al (1997), who use filter projection for simultaneous Bayesian updating of parameter estimates and sales, and Naik et al (2008) who use it to determine the behaviour of endogenous consumer awareness in a dynamic oligopoly. In contrast, our approach takes the parameters outside of the state variable, making them amenable to classical estimation and limiting the impact of prior beliefs on their assessment.

The extended Kalman filter with continuous time and discrete observations applies to state space models of the form

$$\frac{d\xi_t}{dt} = \mathbf{f}(\xi_t, \mathbf{u}_t) + \mathbf{v}_t$$

$$\mathbf{z}_t = \mathbf{h}(\xi_t) + \mathbf{w}_t$$

where ξ_t is a state vector of variables at time t some of which may be unobserved,

$\mathbf{u}_t$ is a vector of exogenous variables,

$\mathbf{z}_t$ is a vector of observed variables,

$\mathbf{f}$ and $\mathbf{h}$ are differentiable functions,

and $\mathbf{v}_t$ and $\mathbf{w}_t$ are mutually uncorrelated white noise with contemporaneous error variances of $E(\mathbf{v}_t \mathbf{v}_t^T) = \mathbf{Q}_t$ and $E(\mathbf{w}_t \mathbf{w}_t^T) = \mathbf{R}_t$ respectively.

After initialisation the filter has two repeated stages, consisting of successive forecasting and updating. In the first stage at time t using the information available at that time, the state variable is forecasted to give a value of $\xi_{t+1|t}$ and the mean squared error is forecasted to $\mathbf{P}_{t+1|t} = E((\xi_{t+1} - \xi_{t+1|t})(\xi_{t+1} - \xi_{t+1|t})^T)$. At the second stage, the forecasts are updated using information available at time $t+1$ to give $\xi_{t+1|t+1}$ and $\mathbf{P}_{t+1|t+1}$. We describe each stage more fully next.

Estimates are made of the starting state vector $\xi_{0|0}$ and its mean squared error $\mathbf{P}_{0|0}$ with information available at time 0. For forecasting **at** time t and starting from $\xi_t = \xi_{t|t}$ and $\mathbf{P}_t = \mathbf{P}_{t|t}$, we iteratively calculate at small intervals from time t to $t+1$ the equations

$$d\xi_t = \mathbf{f}(\xi_t, \mathbf{u}_t)dt$$

$$d\mathbf{P}_t = (\mathbf{F}_t \mathbf{P}_t^T + \mathbf{P}_t \mathbf{F}_t^T + \mathbf{Q}_t)dt$$

where $\mathbf{F}_t = d\mathbf{f}/d\xi_t^T$ is the Jacobian of $\mathbf{f}$ evaluated at $(\xi_t, \mathbf{u}_t)$. In our data, we calculate at ten steps between each data point. The final values give the forecasted state vector of $\xi_{t+1|t}$ and mean squared error of $\mathbf{P}_{t+1|t}$. They are then used to derive the forecasted observed vector $\mathbf{z}_{t+1|t}$ and its mean squared error $MSE(\mathbf{z}_{t+1|t})$ from the equations

$$\mathbf{z}_{t+1|t} = \mathbf{H}_{t+1}\xi_{t+1|t}$$

$$MSE(\mathbf{z}_{t+1|t}) = \mathbf{H}_{t+1}\mathbf{P}_{t+1|t}\mathbf{H}_{t+1}^T + \mathbf{R}_{t+1}$$

where $\mathbf{H}_{t+1} = d\mathbf{h}/d\xi^T$ is the Jacobian of $\mathbf{h}$ evaluated at $\xi_{t+1|t}$.

For updating the state forecasts at time $t+1$ we use the Kalman formulae

$$\xi_{t+1|t+1} = \xi_{t+1|t} + \mathbf{K}_{t+1}(\mathbf{z}_{t+1} - h(\xi_{t+1|t}))$$

$$\mathbf{P}_{t+1|t+1} = (\mathbf{I} - \mathbf{K}_{t+1}\mathbf{H}_{t+1})\mathbf{P}_{t+1|t}$$

where $\mathbf{I}$ is the identity matrix with column and row dimension equal to the number of variables in the state vector ξ_t , and

$$\mathbf{K}_{t+1} = \mathbf{P}_{t+1|t}\mathbf{H}_{t+1}{}^T (\mathbf{H}_{t+1}\mathbf{P}_{t+1|t}\mathbf{H}_{t+1}{}^T + \mathbf{R}_{t+1})^{-1}$$

Our model can be represented in a state space form suitable for use in the filter:

$$\xi_t = \begin{pmatrix} m_t \\ h_t \end{pmatrix}$$

$$\mathbf{f}(\xi_t, \mathbf{u}_t) = \begin{pmatrix} \alpha_1 + \alpha_2 m_t + \alpha_3 h_t + \alpha_4 m_t h_t + \alpha_5 m_t{}^2 \\ \beta_1 + \beta_2 m_t + \beta_3 h_t + \beta_4 m_t h_t + \beta_5 h_t{}^2 \end{pmatrix}$$

$$\mathbf{u}_t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{v}_t = \mathbf{v}_t$$

$$\mathbf{z}_t = \xi_t$$

$$\mathbf{h}(\xi_t) = \xi_t$$

$$\mathbf{w}_t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The Jacobian of f is calculated numerically.

In our empirical model, errors occur only in the state equation rather than the observation equation. Errors accumulate over time, as in empirical specifications of diffusion due to Bass (1969), Schmittlein and Mahajan (1982), Srinivasan and Mason (1986), Jain and Rao (1990), and Basu et al (1995). Our model is heavily parameterised, and the restriction on the observation errors reduces the number of parameters and improves convergence.

The updating phase of the Kalman filter allows us to track the likelihood function generated by our model and data, which we use in maximum likelihood estimation of the parameters.

Appendix B

SUR parameter estimates for the linearised model, over five year rolling periods of estimation

	Dependent variable	Change in vegetarian percentage			Change in vegan percentage		
	Variable	Vegetarian$_{t-1}$	Vegan$_{t-1}$	Constant	Vegetarian$_{t-1}$	Vegan$_{t-1}$	Constant
Data	1992-1996	-0.877	0.577	0.020	0.175	-0.610	0.000
period	1993-1997	-1.146	0.152	0.029	0.088	-0.839	0.002
	1994-1998	-0.927	0.393	0.022	-0.050	-0.956	0.007
	1995-1999	-0.826	0.129	0.021	-0.105	-0.896	0.009
	1996-2000	-0.930	0.504	0.022	-0.097	-0.793	0.008
	1997-2001	-1.012	0.428	0.025	0.036	-0.945	0.006
	1998-2002	-1.008	0.278	0.026	0.063	-1.293	0.009
	1999-2003	-1.153	0.005	0.033	0.031	-1.321	0.010
	2000-2004	-1.190	0.182	0.035	0.137	-1.092	0.005
	2001-2005	-1.109	-0.426	0.038	0.124	-1.037	0.006
	2002-2006	-1.019	0.171	0.029	0.116	-1.032	0.007
	2003-2007	-1.068	-0.077	0.034	0.156	-1.038	0.007
	2004-2008	-1.070	0.032	0.034	0.195	-1.056	0.006
	2005-2009	-1.142	0.582	0.028	0.279	-1.087	0.004
	2006-2010	-1.352	0.673	0.035	0.224	-0.914	0.003
	2007-2011	-1.267	0.380	0.037	0.291	-0.600	-0.003
	2008-2012	-1.158	0.458	0.032	0.280	-0.699	-0.002

Appendix C

Data, prior to conversion to percentages

year	qtr	omnivores	vegetarians	vegans
1992	1	1843	38	9
1992	2	1820	39	12
1992	3	1778	39	13
1992	4	1783	36	7
1993	1	1768	41	8
1993	2	1675	51	3
1993	3	1616	38	6
1993	4	1719	41	10
1994	1	1696	43	4
1994	2	1756	41	7
1994	3	1608	52	8
1994	4	1658	39	15
1995	1	1616	44	9
1995	2	1673	46	6
1995	3	1649	43	14
1995	4	1623	44	11
1996	1	1650	29	9
1996	2	1571	35	9
1996	3	1568	50	9
1996	4	1490	42	8
1997	1	1579	49	5
1997	2	1611	44	7
1997	3	1527	46	12
1997	4	1504	49	9
1998	1	1565	28	7
1998	2	1608	36	12
1998	3	1598	39	17
1998	4	1583	52	8
1999	1	1618	48	11
1999	2	1679	50	19
1999	3	1784	47	17
1999	4	1750	40	9
2000	1	1641	43	18
2000	2	1576	59	14
2000	3	1681	43	12
2000	4	1537	44	12
2001	1	1600	51	8
2001	2	1719	72	16
2001	3	1816	46	22
2001	4	1836	66	13
2002	1	1799	53	15

Data, prior to conversion to percentages (continued)

year	qtr	omnivores	vegetarians	vegans
2002	2	1643	33	10
2002	3	1697	57	17
2002	4	1652	54	11
2003	1	1688	47	18
2003	2	1732	44	15
2003	3	1644	63	20
2003	4	1678	56	13
2004	1	1713	54	16
2004	2	1518	65	14
2004	3	1685	65	19
2004	4	1622	65	17
2005	1	1672	32	24
2005	2	1580	51	14
2005	3	1564	48	22
2005	4	1726	44	22
2006	1	1655	49	10
2006	2	1618	50	24
2006	3	1580	61	19
2006	4	1515	42	22
2007	1	1407	50	17
2007	2	1409	42	16
2007	3	1535	53	20
2007	4	1504	62	21
2008	1	1384	41	21
2008	2	1361	50	16
2008	3	1443	54	18
2008	4	1397	42	16
2009	1	1409	44	13
2009	2	1394	38	16
2009	3	1415	53	15
2009	4	1376	35	14
2010	1	1233	38	9
2010	2	1288	43	11
2010	3	1299	48	16
2010	4	1217	47	14
2011	1	1309	39	17
2011	2	1343	50	11
2011	3	1402	45	16
2011	4	1401	48	10
2012	1	1361	48	10
2012	2	1371	41	20
2012	3	1301	44	19
2012	4	1312	49	17

References

Animal Aid, 2015. March is Veggie Month. Accessed at http://www.veggiemonth.com/ on 13th September 2015.

Ball, M., 2015. Understanding the Numbers for Better Advocacy. Accessed at http://www.mattball.org/2015/08/understanding-numbers-for-better.html on 13th September 2015.

Bass, F.M., 1969. A New Product Growth Model for Consumer Durables. Marketing Science 15, 215-227.

Beardsworth, A., Bryman, A., 2004. Meat consumption and meat avoidance among young people: An 11 year longitudinal study. British Food Journal 106, 313-327.

Bennett, R., 1995. The value of farm animal welfare. Journal of Agricultural Economics 46, 46-60.

Case, J.H., 1979. Economics and the Competitive Process. New York University Press.

Chintagunta,, P.K., Vilcassim, N.J., 1992. An Empirical Investigation of Advertising Strategies in a Dynamic Duopoly. Management Science 38, 1230-1244.

Chintagunta, P.K., Jain, D.C., 1995. Empirical Analysis of a Dynamic Duopoly Model of Competition. Journal of Economics & Management Strategy 4, 109-131.

Counting Animals, 2015. Meat industry advertising. Accessed at http://countinganimals.com/meat-industry-advertising/ on 13th September 2015.

DeCoux, E.L., 2009. Speaking for the modern Prometheus: the significance of animal suffering to the abolition movement. Animal Law 16, 9-64.

Direct Action Everywhere, 2015. The Open Model. Accessed at http://directactioneverywhere.com/theliberationist/2013/11/15/openness on 13th September 2015.

Dunayer, J., 2004. Speciesism. Ryce Publishing.

Fastenberg, D., 2010. Weekday Vegetarians. Time Magazine online. Accessed at http://content.time.com/time/magazine/article/0,9171,2010180,00.html on 13th September 2015.

Fischer, B., McWilliams, J., 2015. When Vegans Won't Compromise. New York Times Online. Accessed at http://opinionator.blogs.nytimes.com/2015/08/16/when-vegans-wont-compromise on 13th September 2015.

Francione, G.L., 1997. Animal rights theory and utilitarianism: relative normative guidance. Animal Law 3, 75-101.

Francione, G., 2015. Veganism, PETA, Farm Sanctuary, Peter Singer, "Personal Purity," and Principles of Justice. Accessed at http://www.abolitionistapproach.com/veganism-peta-farm-sanctuary-peter-singer-personal-purity-principles-justice on 13th September 2015.

Frank, J., 2006. Process attributes of goods, ethical considerations and implications for animal products. Ecological Economics 58, 538-547.

Garner, R., 2006. Animal welfare: a political defense. Journal of Animal Law and Ethics 1, 161-174.

Gary-TV, 2015. Gary-TV. Accessed at http://www.gary-tv.com/en/ on 13th September 2015.

Givon, M., Mahajan, V., Muller, E., 1995. Software Piracy: Estimation of Lost Sales and the Impact on Software Diffusion. Journal of Marketing 59, 29-37.

Goldenberg, J., Libai, B., Muller, E., 2001. Talk of the Network: A Complex Systems Look at the Underlying Process of Word-of-Mouth. Marketing Letters 12, 211-223.

Libai, B., Muller, E., Peres, R., 2009. The Diffusion of Services. Journal of Marketing Research 46, 163-175.

Naik, P.A., Prasad, A., Sethi, S.P., 2008. Building Brand Awareness in Dynamic Oligopoly Markets. Management Science 54, 129-138.

Peta, 2015. Ideas for Vegetarian Living. Accessed at http://www.peta.org/living/food/making-transition-vegetarian/ideas-vegetarian-living/ on 13th September 2015.

Schmittlein, D.C., Mahajan, V., 1982. Maximum Likelihood Estimation for an Innovation Diffusion Model of New Product Acceptance. Marketing Science 1, 57-78.

Sorger, G., 1989. Competitive Dynamic Advertising: A Modification of the Case Game. Journal of Economic Dynamics and Control 13, 55-80.

Srinivasan, V., Mason, C.H., 1986. Nonlinear Least Squares Estimation of New Diffusion Models. Marketing Science 5, 169-178.

Vegetarian Society, 2015. National Vegetarian Week 2015. Accessed at http://www.nationalvegetarianweek.org/ on 13th September 2015.

Waters, J., 2015. Ethics and the choice of animal advocacy campaigns. Ecological Economics 119, 107-117.

Wrenn, C., 2012. Abolitionist animal rights: critical comparisons and challenges within the animal rights movement. Interface 4, 438-458.

Xie, J., Song, X, Sirbu, M., Wang, Q., 1997. Kalman Filter Estimation of New Product Diffusion Models. Journal of Marketing Research 34, 378-93.

R language code implementing the estimation method

The code can be pasted directly into an R program, and requires the packages MASS, numDeriv, and systemfit. It runs using the data in appendix C which should be saved as UK_diet_rates.csv (for example as a csv file from a spreadsheet), for opening by the R code. Line 11 in the code is

```
data<-read.csv("C:\\Documents\\UK_diet_rates.csv", header=T)
```

and the directory will have to be changed to where UK_diet_rates.csv has been saved. Line four in the code is

```
.libPaths("C:/Documents/R/win-library/3.1")
```

and the directory will have to be changed to the local library directory for R. Type

```
.libPaths()
```

to see the current library directory.

The code structure is shown in the table. The preparation section should been run first, then any of the other sections can be run separately.

Line numbers	Purpose	Time taken on a medium pace laptop
1-23	Preparation	Seconds
28-52	Table 1, spec 1	Seconds
62-383	Table 1, spec 3	Tens of minutes
390-702	Table 1, spec 4	Tens of minutes

The code for the extended Kalman filter has *not* been extensively tested as at 17th September 2015. I'm uneasy about the convergence of the full model (table 1, specification 3), and more tests will be done on the code before formal use. The

consequences of the results are quite robust, however – the findings in table 2 are similar across the specifications, including from the SUR code. Given the insignificance of the parameters in table 1, specifications 1 and 3, there might be an issue with colinearity rather than non-convergence.

R CODE

```
######################################
#Lines 1-23 prepare for the later code sections
######################################
.libPaths("C:/Documents/R/win-library/3.1") #Replace this directory with the library directory.  Omitting this line might work if the default is used.
rm(list=ls())

library(MASS)
library(numDeriv)
library(systemfit)

data<-read.csv("C:\\Documents\\UK_diet_rates.csv", header=T)
attach(data)

vegetarian_perc<-vegetarians/(omnivores+vegetarians+vegans)
vegan_perc<-vegans/(omnivores+vegetarians+vegans)

summary(lm(vegan_perc~year))

X<-vegetarian_perc
Y<-vegan_perc

FD_vegetarian_perc<<-X[2:length(X)]-X[1:(length(X)-1)]
FD_vegan_perc<<-Y[2:length(Y)]-Y[1:(length(Y)-1)]




######################################
```

#Lines 28-52 generate the SUR results

#Spec 1 was originally written in a different language and has been ported to R. There are some small differences from the research paper due to different implementation combined with parameter uncertainty. Spec 2 is almost the same as the paper.

####################################

X_Y<-X*Y

X_X<-X*X

Y_Y<-Y*Y

m1<-FD_vegetarian_perc~X[1:(length(X)-1)]+Y[1:(length(X)-1)]+X_X[1:(length(X)-1)]+X_Y[1:(length(X)-1)]

m2<-FD_vegan_perc~X[1:(length(X)-1)]+Y[1:(length(X)-1)]+Y_Y[1:(length(X)-1)]+X_Y[1:(length(X)-1)]

table1_spec1 <- systemfit(list(m1,m2))

summary(table1_spec1)

#Table 1, spec 1

m1<-FD_vegetarian_perc~X[1:(length(X)-1)]+Y[1:(length(X)-1)]

m2<-FD_vegan_perc~X[1:(length(X)-1)]+Y[1:(length(X)-1)]

table1_spec2 <- systemfit(list(m1,m2))

summary(table1_spec2)

#Table 1, spec 2

####################################

#Lines 62-383 generate the EKF-CT-DO results

#for table 1, spec 3

####################################

```r
call_num<<-1


projectkalman<-function(xx) {

print(paste("Call number = ",call_num))

call_num<<-call_num+1

#Parameters

#Provided parameters

alpha1<-xx[1]

alpha2<-xx[2]

alpha3<-xx[3]^2

alpha4<-xx[4]

alpha5<-xx[5]

beta1<-xx[6]

beta2<-xx[7]

beta3<-xx[8]^2

beta4<-xx[9]

beta5<-xx[10]

q11<-(10^(-5))*xx[11]^2

q22<-(10^(-5))*xx[13]^2

q12<-(10^(-5))*sqrt(q11*q22)*xx[12]/(1+abs(xx[12]))

r11<-0

r22<-0

r12<-0


Q<-matrix(c(q11,q12,q12,q22),nrow=2)

R<-matrix(c(r11,r12,r12,r22),nrow=2)


LL<-0 #The value of the log likelihood


timeperiods<-length(Y)-1

steps<-10

dt<-1/steps


#Initial values

#The state vector

Xtt<-array(,c(timeperiods+1,2))

Xtt[1,1]<-X[1]

Xtt[1,2]<-Y[1]
```

```r
#The error's covariance matrix

Ptt<-array(,c(timeperiods+1,2,2))

Ptt[1,1,1]<-0

Ptt[1,1,2]<-0

Ptt[1,2,1]<-0

Ptt[1,2,2]<-0


#The log likelihood component vector for the output product

ll<-vector()

#The mean squared error components

mse_comp<-vector()


predict_observe<-matrix(nrow=timeperiods+1,ncol=2)

for (j in 1:timeperiods) {

#Prediction

X_pred<-Xtt[j,]

P_pred<-Ptt[j,,]


for (i in 1:steps) {


X1<-X_pred[1]

X2<-X_pred[2]



X_pred[1]<-X1+(alpha1*X1+alpha2*X2+alpha3+alpha4*X1*X2+alpha5*X1^2)*dt

X_pred[2]<-X2+(beta1*X1+beta2*X2+beta3+beta4*X1*X2+beta5*X2^2)*dt


f1<-function(x,y) alpha1*x+alpha2*y+alpha3+alpha4*x*y+alpha5*x^2

f2<-function(x,y) beta1*x+beta2*y+beta3+beta4*x*y+beta5*y^2


delta<-0.0001

F11<-(f1(X1+delta,X2)-f1(X1,X2))/delta

F12<-(f1(X1,X2+delta)-f1(X1,X2))/delta

F21<-(f2(X1+delta,X2)-f2(X1,X2))/delta

F22<-(f2(X1,X2+delta)-f2(X1,X2))/delta

F<-matrix(c(F11,F12,F21,F22),nrow=2,byrow=TRUE)
```

```
P_pred<-P_pred+(F%*%t(P_pred)+P_pred%*%t(F)+Q)*dt



}

Xttminus<-X_pred

Pttminus<-P_pred


Ht<-diag(2)


predict_observe[j+1,]<-t(t(Ht)%*%Xttminus)


Kt<-Pttminus%*%t(Ht)%*%ginv(Ht%*%Pttminus%*%t(Ht)+R)

Xtt[j+1,]<-Xttminus+Kt%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)

Ptt[j+1,,]<-(diag(2)-Kt%*%Ht)%*%Pttminus


#Log likelihood updating

LL<-LL+log((2*pi)^(-1/2))        +        log((det(t(Ht)%*%Pttminus%*%Ht+R))^(-1/2))        +        (-1/2)*t(c(X[j+1],Y[j+1])-
t(Ht)%*%Xttminus)%*%ginv(t(Ht)%*%Pttminus%*%Ht+R)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)


ll[j]<-log((2*pi)^(-1/2))        +        log((det(t(Ht)%*%Pttminus%*%Ht+R))^(-1/2))        +        (-1/2)*t(c(X[j+1],Y[j+1])-
t(Ht)%*%Xttminus)%*%ginv(t(Ht)%*%Pttminus%*%Ht+R)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)


mse_comp[j]<-t((c(X[j+1],Y[j+1])-t(Ht))%*%Xttminus)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)


}


predict_observe<<-predict_observe


actual_vegans_change<<-Y[2:(timeperiods)]-Y[1:(timeperiods-1)]

predicted_vegans_change<<-predict_observe[2:(timeperiods),2]-Y[1:(timeperiods-1)]

plot(c(0,1:(timeperiods-1)),c(-
0.01,predicted_vegans_change),col="Black",type="l",xlab="Time",ylab="Sales",main=paste("Vegans;        Black=actual,
red=predicted",";",alpha1,";"))

lines(1:(timeperiods-1),actual_vegans_change,col="Red",type="l")


actual_vegetarians_change<<-X[2:(timeperiods)]-X[1:(timeperiods-1)]

predicted_vegetarians_change<<-predict_observe[2:(timeperiods),1]-X[1:(timeperiods-1)]
```

```r
plot(c(0,1:(timeperiods-1)),c(-
0.01,predicted_vegetarians_change),col="Black",type="l",xlab="Time",ylab="Sales",main=paste("Vegetarians;      Black=actual,
red=predicted",";  a1=",alpha1,";  a2=",alpha2,";  a3=",alpha3,";  a4=",alpha4,";  a5=",alpha5))
lines(1:(timeperiods-1),actual_vegetarians_change,col="Red",type="l")


ll<<-ll


mse<<-sum(mse_comp)/timeperiods


LL<<-LL
print(paste("LL=",LL,sep=""))


as.numeric(LL)
}




projectkalman_original_params<-function(xx) {
print(paste("Call number = ",call_num))
call_num<<-call_num+1
#Parameters
#Provided parameters
alpha1<-xx[1]

alpha2<-xx[2]

alpha3<-xx[3]

alpha4<-xx[4]

alpha5<-xx[5]

beta1<-xx[6]

beta2<-xx[7]

beta3<-xx[8]

beta4<-xx[9]

beta5<-xx[10]

q11<-(10^(-5))*xx[11]

q12<-(10^(-5))*xx[12]

q22<-(10^(-5))*xx[13]

r11<-0

r12<-0

r22<-0
```

```
Q<-matrix(c(q11,q12,q12,q22),nrow=2)

R<-matrix(c(r11,r12,r12,r22),nrow=2)


LL<-0 #The value of the log likelihood


timeperiods<-length(Y)-1

steps<-10

dt<-1/steps


#Initial values

#The state vector

Xtt<-array(,c(timeperiods+1,2))

Xtt[1,1]<-X[1]

Xtt[1,2]<-Y[1]


#The error's covariance matrix

Ptt<-array(,c(timeperiods+1,2,2))

Ptt[1,1,1]<-0

Ptt[1,1,2]<-0

Ptt[1,2,1]<-0

Ptt[1,2,2]<-0


#The log likelihood component vector for the output product

ll<-vector()

#The mean squared error components

mse_comp<-vector()


predict_observe<-matrix(nrow=timeperiods+1,ncol=2)

for (j in 1:timeperiods) {

#Prediction

X_pred<-Xtt[j,]

P_pred<-Ptt[j,,]


for (i in 1:steps) {


X1<-X_pred[1]

X2<-X_pred[2]
```

```r
X_pred[1]<-X1+(alpha1*X1+alpha2*X2+alpha3+alpha4*X1*X2+alpha5*X1^2)*dt

X_pred[2]<-X2+(beta1*X1+beta2*X2+beta3+beta4*X1*X2+beta5*X2^2)*dt


f1<-function(x,y) alpha1*x+alpha2*y+alpha3+alpha4*x*y+alpha5*x^2

f2<-function(x,y) beta1*x+beta2*y+beta3+beta4*x*y+beta5*y^2


delta<-0.0001

F11<-(f1(X1+delta,X2)-f1(X1,X2))/delta

F12<-(f1(X1,X2+delta)-f1(X1,X2))/delta

F21<-(f2(X1+delta,X2)-f2(X1,X2))/delta

F22<-(f2(X1,X2+delta)-f2(X1,X2))/delta

F<-matrix(c(F11,F12,F21,F22),nrow=2,byrow=TRUE)


P_pred<-P_pred+(F%*%t(P_pred)+P_pred%*%t(F)+Q)*dt


}

Xttminus<-X_pred

Pttminus<-P_pred


Ht<-diag(2)


predict_observe[j+1,]<-t(t(Ht)%*%Xttminus)


Kt<-Pttminus%*%t(Ht)%*%ginv(Ht%*%Pttminus%*%t(Ht)+R)

Xtt[j+1,]<-Xttminus+Kt%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)

Ptt[j+1,,]<-(diag(2)-Kt%*%Ht)%*%Pttminus


#Log likelihood updating

LL<-LL+log((2*pi)^(-1/2))    +    log((det(t(Ht)%*%Pttminus%*%Ht+R))^(-1/2))    +    (-1/2)*t(c(X[j+1],Y[j+1])-
t(Ht)%*%Xttminus)%*%ginv(t(Ht)%*%Pttminus%*%Ht+R)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)


ll[j]<-log((2*pi)^(-1/2))    +    log((det(t(Ht)%*%Pttminus%*%Ht+R))^(-1/2))    +    (-1/2)*t(c(X[j+1],Y[j+1])-
t(Ht)%*%Xttminus)%*%ginv(t(Ht)%*%Pttminus%*%Ht+R)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)


mse_comp[j]<-t((c(X[j+1],Y[j+1])-t(Ht))%*%Xttminus)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)
```

```r
}

predict_observe<<-predict_observe

actual_vegans_change<<-Y[2:(timeperiods)]-Y[1:(timeperiods-1)]
predicted_vegans_change<<-predict_observe[2:(timeperiods),2]-Y[1:(timeperiods-1)]
plot(c(0,1:(timeperiods-1)),c(-
0.01,predicted_vegans_change),col="Black",type="l",xlab="Time",ylab="Sales",main=paste("Vegans;          Black=actual,
red=predicted",";",alpha1,";"))
lines(1:(timeperiods-1),actual_vegans_change,col="Red",type="l")

actual_vegetarians_change<<-X[2:(timeperiods)]-X[1:(timeperiods-1)]
predicted_vegetarians_change<<-predict_observe[2:(timeperiods),1]-X[1:(timeperiods-1)]
plot(c(0,1:(timeperiods-1)),c(-
0.01,predicted_vegetarians_change),col="Black",type="l",xlab="Time",ylab="Sales",main=paste("Vegetarians;      Black=actual,
red=predicted",";a1=",alpha1,"; a2=",alpha2,"; a3=",alpha3,"; a4=",alpha4,"; a5=",alpha5))
lines(1:(timeperiods-1),actual_vegetarians_change,col="Red",type="l")

ll<<-ll

mse<<-sum(mse_comp)/timeperiods

LL<<-LL
print(paste("LL=",LL,sep=""))

as.numeric(LL)
}




#Alternative varcov estimates
##############################################
#A wrapper for returning the vector of ll
##############################################
ll_projectkalman_original_params<-function(xx) {
projectkalman_original_params(xx)
ll
```

```
}


##############################################
#The I_{OP} matrix
##############################################
Iop_projectkalman_original_params<-function(xx) {
jac<-jacobian(ll_projectkalman_original_params,xx)
Iop<-matrix(0,nrow=ncol(jac),ncol=ncol(jac))
for (iopi in 1:nrow(jac)) {
Iop<-Iop+jac[iopi,]%*%t(jac[iopi,])
}
Iop<-Iop/nrow(jac)
Iop
}


##############################################
#Optimisation
##############################################


optimise_eqn<-function() {
alpha1<-0.1
alpha2<-0.1
alpha3<-0.1
alpha4<-0.1
alpha5<-0.1
beta1<-0.1
beta2<-0.1
beta3<-0.1
beta4<-0.1
beta5<-0.1
q11<-0.5
q12<-0.1
q22<-0.5



paramvalues<-c(alpha1,alpha2,alpha3,alpha4,alpha5,beta1,beta2,beta3,beta4,beta5,q11,q12,q22)


estvals<<-optim(paramvalues,projectkalman,hessian=FALSE,control=list(trace=3,maxit=4000,fnscale=-1))
```

```
#Stops after 3314 calls


pr<<-estvals$par


estpars<-

c(pr[1],pr[2],pr[3]^2,pr[4],pr[5],pr[6],pr[7],pr[8]^2,pr[9],pr[10],pr[11]^2,sqrt((pr[11]^2)*(pr[13]^2))*pr[12]/(1+abs(pr[12])),pr[1

3]^2)


call_num<<-1


Iop<-Iop_projectkalman_original_params(estpars)
varop<-(1/length(X))*solve(Iop)


#Stdevs from the second derivative method
stdevs<-sqrt(diag(varop))
pvalues<<-sapply(1:length(estpars),function(x) 2*(1-pnorm(abs(estpars[x]),0,stdevs[x])))
#The parameters and their standard deviations
print(rbind(estpars,stdevs,pvalues,mse,LL[1,1]))


output_vals<<-rbind(estpars,stdevs,pvalues,mse,LL[1,1])


}



##############################################
#Find the generating parameters
##############################################
optimise_eqn()
output_vals
#This is table 1, spec 3






######################################
#Lines 390-702 generate the EKF-CT-DO results
```

```r
#for table 1, spec 4

#####################################
call_num<<-1


projectkalman<-function(xx) {

print(paste("Call number = ",call_num))

call_num<<-call_num+1

#Parameters

#Provided parameters

alpha1<-xx[1]

alpha2<-xx[2]

alpha3<-xx[3]^2

beta1<-xx[4]

beta2<-xx[5]

beta3<-xx[6]^2

q11<-(10^(-5))*xx[7]^2

q22<-(10^(-5))*xx[9]^2

q12<-(10^(-5))*sqrt(q11*q22)*xx[8]/(1+abs(xx[8]))

r11<-0

r22<-0

r12<-0


Q<-matrix(c(q11,q12,q12,q22),nrow=2)

R<-matrix(c(r11,r12,r12,r22),nrow=2)


LL<-0 #The value of the log likelihood


timeperiods<-length(Y)-1

#timeperiods<-2

steps<-10

dt<-1/steps


#Initial values

#The state vector

Xtt<-array(,c(timeperiods+1,2))

Xtt[1,1]<-X[1]

Xtt[1,2]<-Y[1]
```

```r
#The error's covariance matrix

Ptt<-array(,c(timeperiods+1,2,2))

Ptt[1,1,1]<-0

Ptt[1,1,2]<-0

Ptt[1,2,1]<-0

Ptt[1,2,2]<-0


#The log likelihood component vector for the output product

ll<-vector()

#The mean squared error components

mse_comp<-vector()


predict_observe<-matrix(nrow=timeperiods+1,ncol=2)

for (j in 1:timeperiods) {

#Prediction

X_pred<-Xtt[j,]

P_pred<-Ptt[j,,]


for (i in 1:steps) {


X1<-X_pred[1]

X2<-X_pred[2]


X_pred[1]<-X1+(alpha1*X1+alpha2*X2+alpha3)*dt

X_pred[2]<-X2+(beta1*X1+beta2*X2+beta3)*dt


f1<-function(x,y) alpha1*x+alpha2*y+alpha3

f2<-function(x,y) beta1*x+beta2*y+beta3


delta<-0.0001

F11<-(f1(X1+delta,X2)-f1(X1,X2))/delta

F12<-(f1(X1,X2+delta)-f1(X1,X2))/delta

F21<-(f2(X1+delta,X2)-f2(X1,X2))/delta

F22<-(f2(X1,X2+delta)-f2(X1,X2))/delta

F<-matrix(c(F11,F12,F21,F22),nrow=2,byrow=TRUE)


P_pred<-P_pred+(F%*%t(P_pred)+P_pred%*%t(F)+Q)*dt
```

```
}

Xttminus<-X_pred

Pttminus<-P_pred

Ht<-diag(2)

predict_observe[j+1,]<-t(t(Ht)%*%Xttminus)

Kt<-Pttminus%*%t(Ht)%*%ginv(Ht%*%Pttminus%*%t(Ht)+R)

Xtt[j+1,]<-Xttminus+Kt%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)
Ptt[j+1,,]<-(diag(2)-Kt%*%Ht)%*%Pttminus

#Log likelihood updating
LL<-LL+log((2*pi)^(-1/2))      +      log((det(t(Ht)%*%Pttminus%*%Ht+R))^(-1/2))      +      (-1/2)*t(c(X[j+1],Y[j+1])-
t(Ht)%*%Xttminus)%*%ginv(t(Ht)%*%Pttminus%*%Ht+R)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)

ll[j]<-log((2*pi)^(-1/2))      +      log((det(t(Ht)%*%Pttminus%*%Ht+R))^(-1/2))      +      (-1/2)*t(c(X[j+1],Y[j+1])-
t(Ht)%*%Xttminus)%*%ginv(t(Ht)%*%Pttminus%*%Ht+R)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)

mse_comp[j]<-t((c(X[j+1],Y[j+1])-t(Ht))%*%Xttminus)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)

}

predict_observe<<-predict_observe

actual_vegans_change<<-Y[2:(timeperiods)]-Y[1:(timeperiods-1)]
predicted_vegans_change<<-predict_observe[2:(timeperiods),2]-Y[1:(timeperiods-1)]
plot(c(0,1:(timeperiods-1)),c(-
0.01,predicted_vegans_change),col="red",type="l",xlab="Time",ylab="Sales",main=paste("Vegans;          black=actual,
red=predicted",";",alpha1,";"))
lines(1:(timeperiods-1),actual_vegans_change,col="black",type="l")

actual_vegetarians_change<<-X[2:(timeperiods)]-X[1:(timeperiods-1)]
predicted_vegetarians_change<<-predict_observe[2:(timeperiods),1]-X[1:(timeperiods-1)]
```

```r
plot(c(0,1:(timeperiods-1)),c(-
0.01,predicted_vegetarians_change),col="red",type="l",xlab="Time",ylab="Sales",main=paste("Vegetarians;        black=actual,
red=predicted","; a1=",alpha1,"; a2=",alpha2,"; a3=",alpha3))
lines(1:(timeperiods-1),actual_vegetarians_change,col="black",type="l")


ll<<-ll


mse<<-sum(mse_comp)/timeperiods


LL<<-LL
print(paste("LL=",LL,sep=""))


as.numeric(LL)
}




projectkalman_original_params<-function(xx) {
print(paste("Call number = ",call_num))
call_num<<-call_num+1
#Parameters
#Provided parameters
alpha1<-xx[1]

alpha2<-xx[2]

alpha3<-xx[3]

beta1<-xx[4]

beta2<-xx[5]

beta3<-xx[6]

q11<-(10^(-5))*xx[7]

q12<-(10^(-5))*xx[8]

q22<-(10^(-5))*xx[9]

r11<-0

r12<-0

r22<-0


Q<-matrix(c(q11,q12,q12,q22),nrow=2)
R<-matrix(c(r11,r12,r12,r22),nrow=2)
```

```r
LL<-0 #The value of the log likelihood


timeperiods<-length(Y)-1

steps<-10

dt<-1/steps


#Initial values

#The state vector

Xtt<-array(,c(timeperiods+1,2))

Xtt[1,1]<-X[1]

Xtt[1,2]<-Y[1]


#The error's covariance matrix

Ptt<-array(,c(timeperiods+1,2,2))

Ptt[1,1,1]<-0

Ptt[1,1,2]<-0

Ptt[1,2,1]<-0

Ptt[1,2,2]<-0


#The log likelihood component vector for the output product

ll<-vector()

#The mean squared error components

mse_comp<-vector()


predict_observe<-matrix(nrow=timeperiods+1,ncol=2)

for (j in 1:timeperiods) {

#Prediction

X_pred<-Xtt[j,]

P_pred<-Ptt[j,,]


for (i in 1:steps) {


X1<-X_pred[1]

X2<-X_pred[2]


X_pred[1]<-X1+(alpha1*X1+alpha2*X2+alpha3)*dt

X_pred[2]<-X2+(beta1*X1+beta2*X2+beta3)*dt
```

```r
f1<-function(x,y) alpha1*x+alpha2*y+alpha3

f2<-function(x,y) beta1*x+beta2*y+beta3


delta<-0.0001

F11<-(f1(X1+delta,X2)-f1(X1,X2))/delta

F12<-(f1(X1,X2+delta)-f1(X1,X2))/delta

F21<-(f2(X1+delta,X2)-f2(X1,X2))/delta

F22<-(f2(X1,X2+delta)-f2(X1,X2))/delta

F<-matrix(c(F11,F12,F21,F22),nrow=2,byrow=TRUE)


P_pred<-P_pred+(F%*%t(P_pred)+P_pred%*%t(F)+Q)*dt


}


Xttminus<-X_pred

Pttminus<-P_pred


Ht<-diag(2)


predict_observe[j+1,]<-t(t(Ht)%*%Xttminus)


Kt<-Pttminus%*%t(Ht)%*%ginv(Ht%*%Pttminus%*%t(Ht)+R)

Xtt[j+1,]<-Xttminus+Kt%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)

Ptt[j+1,,]<-(diag(2)-Kt%*%Ht)%*%Pttminus


#Log likelihood updating

LL<-LL+log((2*pi)^(-1/2))      +      log((det(t(Ht)%*%Pttminus%*%Ht+R))^(-1/2))      +      (-1/2)*t(c(X[j+1],Y[j+1])-
t(Ht)%*%Xttminus)%*%ginv(t(Ht)%*%Pttminus%*%Ht+R)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)


ll[j]<-log((2*pi)^(-1/2))      +      log((det(t(Ht)%*%Pttminus%*%Ht+R))^(-1/2))      +      (-1/2)*t(c(X[j+1],Y[j+1])-
t(Ht)%*%Xttminus)%*%ginv(t(Ht)%*%Pttminus%*%Ht+R)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)


mse_comp[j]<-t((c(X[j+1],Y[j+1])-t(Ht))%*%Xttminus)%*%(c(X[j+1],Y[j+1])-t(Ht)%*%Xttminus)


}


predict_observe<<-predict_observe
```

```
actual_vegans_change<<-Y[2:(timeperiods)]-Y[1:(timeperiods-1)]

predicted_vegans_change<<-predict_observe[2:(timeperiods),2]-Y[1:(timeperiods-1)]

plot(c(0,1:(timeperiods-1)),c(-
0.01,predicted_vegans_change),col="red",type="l",xlab="Time",ylab="Sales",main=paste("Vegans;          black=actual,
red=predicted",";",alpha1,";"))

lines(1:(timeperiods-1),actual_vegans_change,col="black",type="l")


actual_vegetarians_change<<-X[2:(timeperiods)]-X[1:(timeperiods-1)]

predicted_vegetarians_change<<-predict_observe[2:(timeperiods),1]-X[1:(timeperiods-1)]

plot(c(0,1:(timeperiods-1)),c(-
0.01,predicted_vegetarians_change),col="red",type="l",xlab="Time",ylab="Sales",main=paste("Vegetarians;          black=actual,
red=predicted","; a1=",alpha1,"; a2=",alpha2,"; a3=",alpha3))

lines(1:(timeperiods-1),actual_vegetarians_change,col="black",type="l")


ll<<-ll


mse<<-sum(mse_comp)/timeperiods


LL<<-LL
print(paste("LL=",LL,sep=""))


as.numeric(LL)
}




#Alternative varcov estimates
############################################
#A wrapper for returning the vector of ll
############################################
ll_projectkalman_original_params<-function(xx) {
projectkalman_original_params(xx)
ll
}


############################################
#The I_{OP} matrix
```

```
#############################################

Iop_projectkalman_original_params<-function(xx) {

jac<-jacobian(ll_projectkalman_original_params,xx)

Iop<-matrix(0,nrow=ncol(jac),ncol=ncol(jac))

for (iopi in 1:nrow(jac)) {

Iop<-Iop+jac[iopi,]%*%t(jac[iopi,])

}

Iop<-Iop/nrow(jac)

Iop

}


#############################################

#Optimisation

#############################################


optimise_eqn<-function() {

alpha1<-0.1

alpha2<-0.1

alpha3<-0.1

beta1<-0.1

beta2<-0.1

beta3<-0.1

q11<-0.5

q12<-0.1

q22<-0.5



paramvalues<-c(alpha1,alpha2,alpha3,beta1,beta2,beta3,q11,q12,q22)


estvals<<-optim(paramvalues,projectkalman,hessian=FALSE,control=list(trace=3,maxit=4000,fnscale=-1))


pr<<-estvals$par


estpars<<-c(pr[1],pr[2],pr[3]^2,pr[4],pr[5],pr[6]^2,pr[7]^2,sqrt((pr[7]^2)*(pr[9]^2))*pr[8]/(1+abs(pr[8])),pr[9]^2)


call_num<<-1


Iop<-Iop_projectkalman_original_params(estpars)
```

```r
varop<-(1/length(X))*solve(Iop)


#Stdevs from the second derivative method

stdevs<-sqrt(diag(varop))

pvalues<<-sapply(1:length(estpars),function(x) 2*(1-pnorm(abs(estpars[x]),0,stdevs[x])))

#The parameters and their standard deviations

print(rbind(estpars,stdevs,pvalues,mse,LL[1,1]))



output_vals<<-rbind(estpars,stdevs,pvalues,mse,LL[1,1])

output_vals

}




#############################################

#Find the generating parameters

#############################################

optimise_eqn()

output_vals

#This is table 1, spec 4
```