Comment on Suzuki's rebuttal of Batra and Casas

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18 September 2015

Title: ‘Factor intensities and factor substitution in general equilibrium: A Comment’

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Abstract: Jones and Easton (1983) analyzed how commodity prices affect factor prices in the three-factor two-good general equilibrium trade model. These relationships determine whether ‘a strong Rybczynski result’ holds or not. In subsection 5.2.4., they found that the set of three equations holds for ‘the economy-wide substitution’ under the assumption of ‘perfect complementarity’. And they applied this to their analysis. In the following, I demonstrate that this is impossible, hence their proof is not plausible. In subsection 5.2.5., they analyzed similarly. But their proof is not plausible, either.

Keywords: three-factor two-good model; general equilibrium; Rybczynski result; economy-wide substitution; perfect complementarity.

1. Introduction

Jones and Easton (1983) (hereinafter JE) (p77) explored the question in the three-factor two-good neoclassical model which Batra and Casas (1976) (hereinafter BC) explored: ‘How do changes in relative commodity prices (at constant [factor] endowments) affect all factor prices?’ Here, the commodity prices are exogenous, and factor prices are endogenous (see BC (p23)). In a review of their article, I find that JE made an error in their proof which renders it implausible.

JE (p67) stated, ‘As is well understood in these models, commodity prices and factor prices are related in a manner that finds its dual counter-part in the factor endowment-commodity output relationships.‘ These relationships determine whether ‘a strong Rybczynski result’ (to use Thompson’s terminology, 1985, p617) holds or not. On the duality, see also BC (p36, eq. (31)-(33)).

For example, in subsection 5.2.4 (p90), JE found that the set of three equations holds for ‘the economy-wide substitution’ (hereinafter EWS) under the assumption of ‘perfect complementarity’ between extreme factors. On this, see eq. (1) shown below. In applying this to their analysis, they showed how commodity prices affected factor prices.

Thompson (1985) subsequently confirmed JE’s discussion. In a summary of the article, Thompson (1985, p617) suggests that, BC ‘claim a strong Rybczynski result [which holds for a two-factor, two-good model] stated in terms of extreme factors is also found in the three-factor model. Suzuki (1983) and Jones and Easton (1983) point out, as is done here, that a strong Rybczynski result is not necessary.'

In this paper, I return to JE's original article, finding that element of their proof is questionable. In section 4 in this article, we derive the important inequality among 'aggregate substitution' from the assumption of production functions. From this, in section 5, we get important relationship among EWS, which I find, is not consistent with eq. (1) shown below. In section 6, I also comment on subsection 5.2.5 (pp91-92) in JE which analyzed similarly. It is useful to use the Allen-partial elasticity of substitution (hereinafter AES) in my analysis.

According to Suzuki (1983, p141), BC contended in Theorem 6 (p34) that 'if commodity 1 is relatively capital intensive and commodity 2 is relatively labor intensive, an increase in the supply of labor increases the output of commodity 2 and reduces the output of commodity 1.' This is what 'a strong Rybczynski result' implies.

2. Definition of EWS

JE (p74) defined the parameter E_{ij}^k as follows,

' $E_{ij}^k \equiv \dfrac{\partial a_{ij}}{\partial w_k}\dfrac{w_k}{a_{ij}}$, [i, k=1, 2, 3, j=1, 2]. ' ,

where a_{ij} denotes the input of factor i required to produce a unit of output in the jth sector (or input output coefficient); w_k is the return earned by factor k. On the definition of these symbols, see JE (p68-69).

They stated, 'The sign of E_{ij}^k reflects the substitution or complementarity relationship between factors i and k. Of course 'own' substitution terms, E_{ij}^i , must all be negative.' And they stated that if factors i and k are substitutes (resp. complements), E_{ij}^k is positive (resp. negative).

Next, JE (p75) continued, 'Clearly, the substitution terms in the two industries are always averaged together. With this in mind we define the term σ_i^k to denote the economy-wide substitution [or EWS] towards or away from the use of factor i when the kth factor becomes more expensive, under the assumption that each industry's output is kept constant:

$\sigma_i^k \equiv \lambda_{i1} E_{i1}^k + \lambda_{i2} E_{i2}^k$, [i, k=1, 2, 3]',

where λ_{ij} refers to the fraction of the total supply of factor i used in the jth industry, (Thus, $\lambda_{ij} \equiv x_j a_{ij}/V_i$). On this, see JE (p70) and BC (1976, p23); x_j is the output of commodity j; V_i the factor endowments of factor i. On the definition of these symbols, see

JE (p68).

Replace $E_{ij}{}^{k}$, $\sigma_i{}^{k}$ respectively with $\varepsilon^{ij}{}_{h}$,g_{ih} for ease of notation:

$$g_{ih} = \Sigma_j \ \lambda_{ij} \varepsilon^{ij}{}_{h}, \quad i, h = T, K, L,$$

where T is the land; K capital; L labor. Of course, $E_{ij}{}^{i} < 0$ implies that $\sigma_i{}^{i} < 0$ and $g_{ii} < 0$.

3. 'Perfect complementarity' and its implication derived in subsection 5.2.4.

In subsection 5.2.4 (p90), JE stated,

'First we assume that the two extreme factors [factors 1 and 2] are perfect complements in the sense that any factor price change does not alter the ratio of the intensities of their use ($\sigma_1{}^{k} = \sigma_2{}^{k}$ [, k=1, 2, 3]).'

On the definition of extreme factors (and middle factor to be mentioned later), see JE (p68). Here, for them, 'perfect complementarity' implies $\sigma_1{}^{k} = \sigma_2{}^{k}$. This implies that

$$g_{Th} = g_{Kh},\ h = T, K, L \leftrightarrow g_{TT} = g_{KT},\ g_{TK} = g_{KK},\ g_{TL} = g_{KL}. \quad (1)$$

In other words, they found that the set of three equations holds for EWS under the assumption of 'perfect complementarity'.

Next, they used this set to prove how commodity prices affect factor prices. JE (p91) continued, 'Here, regardless of factor intensities, perfect complementarity between extreme factors drives their returns far apart, leaving the middle ground for $\hat{w}_3$:

$\hat{w}_1 > \hat{p}_1 > \ \hat{w}_3 > \hat{p}_2 \ > \hat{w}_2$.',

where p_j is the price of commodity j; a hat (^) over a variable denotes a relative change (Thus $\hat{x} \equiv dx/x$). On the definition of these symbols, see JE (p71). Of course, they assumed that $\hat{p}_1 > \hat{p}_2$.

Replace $\hat{w}_k$, k=1, 2, 3 with $\hat{w}_i$, i=T, K, L:

$\hat{w}_T > \hat{p}_1 > \ \hat{w}_L > \hat{p}_2 \ > \hat{w}_K$.',

4. Important inequality among 'aggregate substitution'

Thompson (1985, p618) stated, 'Aggregate substitution between factors h and k is expressed by the substitution term $s_{kh}=\Sigma_j\ x_j\ \partial a_{kj}/\partial w_h$ [, k, h=1, 2, 3]. The 3 X 3 matrix of substitution terms is symmetric and negative semidefinite. A result of cost minimizing behavior is $\Sigma_i\ s_{hi}w_i=0$, for every factor h [, h=1, 2, 3].' For definition of these symbols, it is similar to that in this paper.

But his explanation seems questionable. The 'cost minimizing behavior' implies $\Sigma_i\ \theta_{ij}\ \hat{a}_{ij}=0$, j=1, 2 (or $\Sigma_i\ w_i\ da_{ij}=0$). On this, see JE (p73, eq. (9)), BC (p24, n5). But without using this, we can derive Thompson's result, $\Sigma_i\ s_{hi}w_i=0$. Probably, we should prove it below.

From the definition of parameter ε^{ij}_h, we obtain

$$\partial a_{ij}/\partial w_h=\varepsilon^{ij}_h\ a_{ij}/w_h,\ i,\ h=T,\ K,\ L,\ j=1,\ 2,$$

where $\varepsilon^{ij}_h=\partial\log a_{ij}/\partial\log w_h=\theta_{hj}\sigma^{ij}_h$; σ^{ij}_h is the AES between the ith and the hth factors in the jth industry; θ_{hj} is the distributive share of factor h in the jth sector. For additional definition of these symbols, see Sato and Koizumi (1973, pp47-49), BC (1976, p24). Note that JE did not mention AES. Do not confuse σ^{ij}_h with σ_i^k.

Replace s_{kh} with s_{ih} : $s_{ih}=\Sigma_j\ x_j\ \partial a_{ij}/\partial w_h$, i, h=T, K, L. Rewrite this equation:

$$s_{ih}=\Sigma_j\ x_j\ \varepsilon^{ij}_h\ a_{ij}/w_h,\ i,\ h=T,\ K,\ L. \quad (2)$$

Because each a_{ij} function is homogeneous-of-degree-zero (see BC (p33)):

$$\Sigma_h\ \varepsilon^{ij}_h=\Sigma_h\ \theta_{hj}\sigma^{ij}_h=0,\ i=T,\ K,\ L,\ j=1,\ 2.$$

From the above:

$$\Sigma_h\ s_{ih}w_h=0,\ i=T,\ K,\ L. \quad (3)$$

This is equivalent to Thompson's result. AES's are symmetric in the sense that $\sigma^{ij}_h=\sigma^{hj}_i$. And according to BC (p33), 'Given the assumption that production functions are strictly quasi-concave and linearly homogeneous,' $\sigma^{ij}_i<0$. The former implies that $s_{ih}=s_{hi}$, namely, aggregate substitutions are symmetric. And the latter implies that $\varepsilon^{ij}_i<0$, hence $s_{ii}<0$.

Next, we analyze s_{LL} in a way similar to that which BC (p33) used in analyzing AES (σ^{Lj}_L). Delete s_{TL}, s_{KL} from eq. (3):

$$s_{LL}=\frac{1}{(w_L)^2}\{w_T\,(w_T\ s_{TT}+w_K s_{TK})+w_K\,(w_T\ s_{KT}+w_K s_{KK})\}.$$

Transform that

$$s_{LL}= x \cdot Ax \quad ,$$

where x is the vector, A the matrix, and $x \cdot Ax$ the inner product. That is, $x = \begin{bmatrix} w_K \\ w_T \end{bmatrix}, A = \begin{bmatrix} s_{KK} & s_{KT} \\ s_{TK} & s_{TT} \end{bmatrix}$.

Quote a passage from BC (p33): 'the quadratic form on the right-hand side of the expression above must be negative definite. This in turn implies that'

$$|A| = s_{KK}s_{TT} - s_{TK}s_{KT} > 0, \quad (4)$$

where $|A|$ is the determinant.

5. Important relationship among EWS

From eq. (2):

$$s_{ih}= \Sigma_j \lambda_{ij} \varepsilon^{ij}_h V_i/w_h = g_{ih}V_i/w_h, \quad i, h=T, K, L. \quad (5)$$

This equation shows how aggregate substitution and EWS are related. From this, g_{ih} is not symmetric. Namely, $g_{ih} \neq g_{hi}$, $i \neq h$ in general. Substitute eq. (5) into eq. (4):

$$g_{KK}g_{TT} - g_{TK}g_{KT} > 0. \quad (6)$$

If we compare inequality (6) with eq. (1), we find that the latter is not consistent with the former. That is, if eq. (1) holds, L.H.S. of eq. (6) equals zero. Hence, the JE's result is impossible. Specifically, they failed to explain what 'perfect complementarity' implies. In sum, their proof is not plausible.

6. Comment on subsection 5.2.5

In subsection 5.2.5 (p91), JE analyzed similarly to subsection 5.2.4. They stated, 'Now let an extreme factor, say factor 2, become a perfect complement with middle factor 3….In the basic definition for ξ_1 replace σ_3^1 by σ_2^1 since factors 2 and 3 are now assumed perfect complements.' That is, they derived $\sigma_3^1 = \sigma_2^1$. Apparently, in their context, they derived the equation $\sigma_3^k = \sigma_2^k$, k=1, 2, 3. This implies that

$$g_{Lh}=g_{Kh},\ h=T, K, L. \leftrightarrow g_{LT}=g_{KT}\,,\ g_{LK}=g_{KK}\,,\ g_{LL}=g_{KL}\,. \quad (7)$$

However, by analogy from eq. (6), we derive

$$g_{KK}g_{LL}-g_{LK}g_{KL}>0. \quad (8)$$

If we compare inequality (8) with eq. (7), we find that the latter is not consistent with the former. Hence, the JE's result is impossible. In sum, their proof is not plausible.

7. Conclusion

In subsection 5.2.4, JE examined how commodity prices affected factor prices in the three-factor two-good model using the fact that eq. (1) holds for EWS under the assumption of 'perfect complementarity', in their proof. This element, I find, is impossible, which renders the proof implausible. In subsection 5.2.5, they analyzed similarly. But their proof is not plausible, either.

They failed to notice the important relationship among EWS as shown in eq. (6) which I derived from the assumptions for the production functions. Their fellow researchers, including Thompson (1985), did not catch this oversight. We are now left to search for a new sufficient condition for 'a strong Rybczynski result' to hold (or not to hold).

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