Myths about Beta-Convergence

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Abstract

A popular methodology of studying spatial income inequality is analysis of beta-convergence (i.e. an inverse relationship between current income per capita and its initial level). Its widespread use is based on a belief that the economic growth theory predicts income convergence among economies (countries or regions within a country), and that beta-convergence suggests decreasing income inequality. This article demonstrates that these are nothing but myths; hence, analyzing of beta-convergence cannot serve as an adequate methodology for studying and predicting the evolution of spatial income inequality.

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D63, O11, O40

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spatial income inequality, convergence, economic growth

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1. Introduction

The most important question which is to be answered while studying the evolution of income inequality among some geographical units is: Is there a trend to leveling of their incomes per capita, i.e. to income convergence? These geographical units can be countries, regions of different countries, or regions within a country (according to a particular level of political or economic geographical division); "economy" will be used hereafter as a general term. The term "income" is a general one as well. In specific researches, depending on their objectives and economies under consideration, its content differs, e.g. it can be GDP or gross regional product (GRP) per capita, personal income per capita, wage per worker, etc.

Income convergence means that dispersion of incomes in a set of economies under study diminishes over time (ideally, tends to zero). Thus, in fact, the case in point is the decrease in the "width" of income distribution density. All income inequality indicators, such as Gini, Theil, Atkinson indices, etc., are just statistics of income distribution, measuring its "width" in one way or another. One of inequality measures, standard deviation of income logarithms $\sigma_t = \sigma(\log(y_t))$, where y_t is incomes per capita at some point in time t, is directly a usual attribute of distributions (logarithms are used here to eliminate an even change in all incomes, e.g. due to inflation). It gave its name to a simple test for income convergence, σ-convergence. The latter occurs if $\sigma_{t+T} < \sigma_t$, where T is some period (e.g., a year, five years, etc.). Any other inequality index can be used in place of σ in this relationship. In this case, some authors keep the term "σ-convergence," while others include the name of inequality index in it (e.g., "Gini-convergence").

However, the most popular method for identification of income convergence is testing for β-convergence. An occurrence of β-convergence implies that the poorer is the economy (i.e. the lower the income per capita there in the initial point in time), the higher is its rate of growth of income per capita. Then, seemingly, the income gaps between economies will steadily decline.

The β-convergence concept owes its popularity to publications by Robert Barro and Xavier Sala-i-Martin, starting from Barro and Sala-i-Martin (1991, 1992). Their widely known textbook of economic growth, Barro and Sala-i-Martin (2004), also contributed significantly.[1] In the last two decades elapsed after appearance of the above articles, a few thousand applied researches exploiting analysis of β-convergence were published in the world; in Russia, they number at least a few tens.

[1] They are sometimes referred to as pioneers of β- and σ-convergence. However, analyzing β-convergence was applied well before (e.g., by Baumol (1986)), not to mention so simple and obvious method as testing for σ-convergence. Barro and Sala-i-Martin's credit is in that they have demonstrated the way the β-convergence equation follows from the neoclassical growth model as well as have given names "β-convergence" and "σ-convergence" to hitherto nameless concepts.

The authors of overwhelming majority of these papers misunderstand specificity of this criterion, resulting in misinterpretation of results obtained and thus in erroneous policy implications. It must be told that R. Barro and X. Sala-i-Martin are not to blame for this. Quite the contrary, they underline in almost each their publication that β-convergence (i.e. more rapid growth of poor economies) does not necessarily result in *decline of income inequality*, namely in σ-convergence (see, e.g., Barro and Sala-i-Martin (1991, 1992); Barro and Sala-i-Martin (2004, pp. 50–51 and 462–465)). Nonetheless (even citing sometimes Barro and Sala-i-Martin's point that the meaning of β- and σ-convergence is different), authors of empirical studies identify in fact β-convergence with decrease in income dispersion. The foundation of this mass fallacy is two widespread myths:

– the economic growth theory predicts income convergence among economies;

– β-convergence suggests decreasing income inequality (i.e. income convergence).

These myths (especially the second one) were debunked by Friedman (1992), Quah (1993), Durlauf and Quah (1999), Magrini (2004), Wodon and Yitzhaki (2006), and some others. However, the billow of empirical studies based of the β-convergence concept does not subside (and it seems even to grow in Russia). Therefore, it is appropriate to consider this topic once more, providing a concentrated demonstration of the nature of these myths and flimsiness of the β-convergence concept as a tool for empirical studying the evolution of income inequality.

2. The first myth: The economic growth theory predicts income convergence of economies

Most of empirical papers exploiting the β-convergence concept are of an applied character, aiming to reveal trends of income disparities in some specific groups of economies. The issues of economic growth as such are not a subject of such papers; the authors use an analysis of β-convergence simply as a methodological tool, referring to the growth theory as its theoretical basis. In doing so, sometimes either they explicitly claim that the growth theory predicts income convergence of economies, or sometimes faith in this myth is seen from the context. Apparently, the reason is superficial knowledge of the growth theory by the authors; because of this, they "generalize" a fairly particular result of the theory as a universal conclusion.

By now the growth theory is far from being something completed. It includes different growth models which progressively develop and supplement each other (as one way, by abandonment of old assumptions). These models not infrequently lead to distinct (sometimes directly opposite) conclusions. And the β-convergence concept is generated by only one of these models, namely, the neoclassical economic growth model (in some versions). Hence it follows that the correct use of the β-convergence concept in empirical analysis requires the analysis to take account of model prerequisites and peculiarities of derivation of the β-convergence equation (some aspects will be considered later). The faith in the "universality" of prediction regarding convergence does result in isolation of the analysis from the definite growth model, so denuding the analysis of a theoretical ground and making it to be scholastic.

For justice sake, it should be noted that more sophisticated authors are more accurate, asserting that it is the neoclassical growth model – or, more specifically, the Solow-Swan model – that predicts income convergence of economies (decreasing income inequality). However, as it will be demonstrated below, this assertion is overly strong as well. To do so, let us briefly discuss how the β-convergence equation is derived from the neoclassical growth model.

Consider a closed one-sector economy characterized by a neoclassical production function (with constant returns to scale and diminishing marginal returns to inputs as well as satisfying Inada conditions: infinite marginal product with zero quantity of an input, and zero marginal product with infinite quantity of an input) with exogenous labor-augmenting technical progress. We take the Cobb-Douglas function $Y(t) = K(t)^a (L(t)A(t))^{1-a} = K(t)^a \widetilde{L}(t)^{1-a}$ for definiteness sake. Here, t is time, $Y(t)$ is total output (hereafter, income), $K(t)$ is physical capital, $L(t) = L(0)e^{\nu t}$ is labor (the number of workers), ν is its growth rate, $A(t) = A(0)e^{\xi t}$ is the state of technology, ξ is the rate of technological progress, and $\widetilde{L}(t) \equiv L(t)A(t)$ is the number of "effective workers." The following equation describes the dynamics of capital:

$$dK(t)/dt = sY(t) - \delta K(t), \tag{1}$$

where δ is the depreciation rate of physical capital, and s is the saving rate. The latter is either exogenous like in the Solow-Swan model, or is a solution of the social-welfare maximization problem like in the Ramsey model in its Cass-Koopmans version. In the latter case, the saving rate depends on time, but it can be taken constant in the neighborhood of equilibrium growth – which solely is of interest to us (see, e.g., Durlauf and Quah (1999, pp. 247–248)).

Expressing in specific terms per effective worker, $\widetilde{y} = Y/\widetilde{L}$ and $\widetilde{k} = K/\widetilde{L}$, we get $\widetilde{y} = \widetilde{k}^a$. Since $dK/dt = d(\widetilde{k}\widetilde{L})/dt = \widetilde{L}d\widetilde{k}/dt + (\nu + \xi)\widetilde{k}\widetilde{L}$, Equation (1) takes the form

$$d\widetilde{k}/dt = s\widetilde{y} - (\delta + \nu + \xi)\widetilde{k}. \tag{2}$$

The steady state or the equilibrium (balanced) growth is defined as a situation in which capital per effective worker has an equilibrium, time-invariant value $\widetilde{k}(t) = \widetilde{k}_*$ (implying $\widetilde{y}(t) = \widetilde{y}_* = \widetilde{k}_*^a$).

Hence, $d\tilde{k}/dt = 0$ on the equilibrium growth path. Thus, the condition of equilibrium growth is

$$s\tilde{y}_* = (\delta + \nu + \xi)\tilde{k}_*. \tag{3}$$

Given this, as is seen, investment sY compensates the depreciation of capital as well as augments capital according to increase in the number of workers and their productivity (due to technical progress) so that capital per effective worker remains constant.

Transforming Equation (2), we get

$$(d\tilde{k}/dt)/\tilde{k} = d\log(\tilde{k})/dt = s\tilde{k}^{a-1} - (\delta + \nu + \xi) = se^{(a-1)\log(\tilde{k})} - (\delta + \nu + \xi) \equiv h(\log(\tilde{k})).$$

A Taylor series expansion about $\log(\tilde{k}_*)$ linearizes function $h(\log(\tilde{k}))$:

$$h(\log(\tilde{k})) \approx h(\log(\tilde{k}_*)) + dh/d\log(\tilde{k})\Big|_{\log(\tilde{k})=\log(\tilde{k}_*)} \cdot (\log(\tilde{k}) - \log(\tilde{k}_*)) = (a-1)s\tilde{k}_*^{a-1}(\log(\tilde{k}) - \log(\tilde{k}_*)).$$

Herefrom, benefiting from equality (3), a differential equation is arrived at:

$$d\log(\tilde{k})/dt = (a-1)(\delta + \nu + \xi)(\log(\tilde{k}) - \log(\tilde{k}_*)) \equiv \lambda(\log(\tilde{k}) - \log(\tilde{k}_*)). \tag{4}$$

Solving it gives $\log(\tilde{k}(t)) - \log(\tilde{k}_*) = (\log(\tilde{k}(0)) - \log(\tilde{k}_*))e^{\lambda t}$. To express this equation in terms of income, multiply its both sides by a. Then

$$\log(\tilde{y}(t)) - \log(\tilde{y}_*) = (\log(\tilde{y}(0)) - \log(\tilde{y}_*))e^{\lambda t}. \tag{5}$$

The value of λ quantifies the convergence rate of the growth path to the equilibrium one. Since $a < 1$, then $\lambda < 0$, hence, $\tilde{y}(t) \to \tilde{y}_*$ with $t \to \infty$ for any initial income $\tilde{y}(0)$.

Observable income per actual worker is $y(t) = Y(t)/L(t) = \tilde{y}(t)A(0)e^{\xi t}$. Then Equation (5) takes the form

$$\log(y(t)) = (\log(\tilde{y}_*) + \log(A(0)) + \xi t) + (\log(y(0)) - \log(\tilde{y}_*) - \log(A(0)))e^{\lambda t}. \tag{6}$$

The first bracketed summand in the right-hand side of Equation (6) is the equilibrium growth path; the second summand is a decaying (over time) deviation from this path (theoretically, it could be both negative and positive; in reality, it apparently may be only negative). Note that Equation (6) can be written in terms of the model parameters only, getting rid of $\tilde{y}_*$, as it follows from Equation (3) that $\tilde{y}_* = ((\delta + \nu + \xi)/s)^{a/(a-1)}$.

Let us take some point in time $t = T$ and denote $y(T) = y_T$ and $y(0) = y_0$. Regrouping terms in the right-hand side of Equation (6), the β-convergence equation is arrived at:

$$\log(y_T) = ((\log(\tilde{y}_*) + \log(A(0)))(1 - e^{\lambda T}) + \xi T) + e^{\lambda T}\log(y_0) \equiv \alpha + \beta_+\log(y_0). \tag{7$'$}$$

Since $\lambda < 0$, then $\beta_+ < 1$. Equivalent equations are used frequently, the left-hand side of which is the overall or annual growth rate of incomes per capita over period $[0, T]$:

$$\log(y_T / y_0) = \alpha + \beta \log(y_0), \quad (7'')$$

$$\log(y_T / y_0) / T = \alpha' + \beta' \log(y_0), \quad (7''')$$

where $\beta = \beta_+ - 1 < 0$, $\alpha' = \alpha/T$, $\beta' = \beta/T < 0$.

A number of conditions for adequate application of Equations (7′), (7′′), and (7′′′) in empirical studies are easily seen from the foregoing. First, the equations hold only in the neighborhood of the equilibrium growth. Thus, their application implies an implicit assumption that the economic growth paths of all economies covered are close to the equilibrium paths. This assumption seems to be overly strong. It is definitely false if economies under consideration are Russian regions. However, a reservation is needed that if the saving rate over period [0, T] is constant (i.e. the economies are described by the Solow-Swan model), then the β-convergence equation holds on any one segment of the growth path.

Second, by construction of the model, income per capita in it is value added – GDP or GRP – per worker (working efficiency in the economy). If value added per head of the total population is used as y, then adequate application of the β-convergence equation requires growth rates of population and the number of workers to coincide (though, more weak conditions are possible). In the case when y represents personal incomes per capita, the situation becomes more complex: additional conditions are needed that ensure "isomorphism" of the paths of value added per worker and personal incomes per capita (the same is true for the case of wage per worker). We will not consider relevant conditions, only notice that they are *never* mentioned and, all the more, tested in empirical studies.

It should be also remarked that when regions are taken as economies, there is no escape from taking into account the fact that they are not closed economies. This manifests itself in particular in redistribution of the national income among regions of a country and even different countries (as in the EU). As a result, the evolution of region's population incomes loses direct connection with the evolution of GRP. These evolutions can be even differently directed.[2] Besides, because of features of national statistics (specifically, the Russian one) or the use of simplified methodologies for evaluation of GRP, this indicator itself is far from always corresponding to value added produced in given region's territory (Zaitseva, 2012; Zubarevich, 2012).

Third, the β-convergence equation holds only if economic growth actually occurs. Its

[2] For example, GRP per capita in the Chukchi Autonomous Okrug (a region of Russia) in 2009 was equal to 4.1 relative to the national average, while personal income per capita equaled 2.1, the relative GDP per capita having increased and personal income per capita decreased as compared to the previous year (author's calculations based on data from Rosstat (2011, pp. 148–149 and 353–354)).

application to economies experiencing recession has no theoretical justification. For example, Mikheeva (1999) notes that the estimation of the Solow-Swan model on data for 76 Russian regions over 1990–1996 has not yielded statistically significant estimates. Hence, the model does not describe the given set of economies, which would be expected (nonetheless, further empirical analysis in Mikheeva (1999) exploits the β-convergence concept based on this model).

The above model describes dynamics of a single economy. What implications can be drawn, if there is a set of economies $\{i\}$ governed by the above model? If the economies are homogeneous, i.e. have the same values of structural parameters a, ξ, ν, δ, s, and $A(0)$, differing only in initial income per capita y_{0i} (determined by the initial level of capital per worker, $K_i(0)/L_i(0)$), then, as Equation (6) suggests, they will have the same equilibrium growth path. Their individual paths converge to the common equilibrium path, incomes per capita in poor economies (with lesser y_{0i}) growing faster than those in rich economies. Such an evolution is referred to as unconditional (or absolute) β-convergence; Figure 1a provides an example. Theoretically, it will eventually result in equalization of incomes per capita across economies: poor economies catch up with rich ones; in the limit $t \to \infty$ income inequality in the entire set of these economies becomes zero.

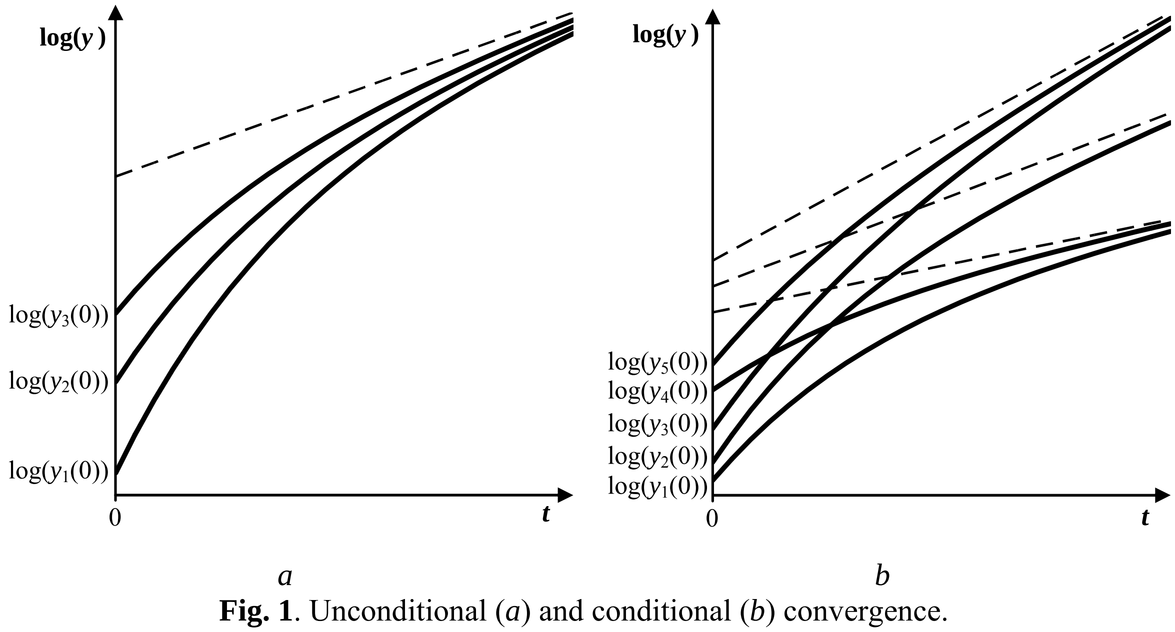

Fig. 1. Unconditional (*a*) and conditional (*b*) convergence.
Dashed lines denote equilibrium growth paths.

Given heterogeneous economies, it follows from Equation (6) that each economy has *its own* equilibrium growth path, to which its growth path converges. Such a situation is referred to as conditional β-convergence. It suggests only that income per capita in an economy grows faster the further it is from the equilibrium value *for the given economy* (Barro and Sala-i-Martin, 2004, p. 47). But it is uninformative as to how growth rates of different economies relate. Under conditional β-convergence, poor economies do not need to grow faster than rich ones. If the rich economy is further below its equilibrium growth path than the poor economy is below its one, then the rich economy can grow faster. Thus, conditional β-convergence gives no grounds for implications regarding the evolution of income dispersion in a sample of economies under study. Figure 1b provides an example of conditional β-convergence. There are two groups of homogeneous economies: the growth paths of economies 1 and 4 converge to the equilibrium path $\log(\tilde{y}_{*1,4}) + \log(A_{1,4}(0)) + \xi_{1,4}t$, those of economies 3 and 5 converge to $\log(\tilde{y}_{*3,5}) + \log(A_{3,5}(0)) + \xi_{3,5}t$, and economy 2 has its own equilibrium growth path $\log(\tilde{y}_{*2}) + \log(A_2(0)) + \xi_2 t$. By and large we observe income divergence.

Figure 1b shows only one possible pattern. Besides, the prerequisites of the neoclassical growth model hold only if technological progress is freely available to all firms (Barro and Sala-i-Martin, 2004, pp. 62–63). If this is true for any firm in the set of economies $\{i\}$, and not only for firms within a given economy, then the rate of technological progress, ξ, should be taken the same across economies. Then it follows from (6) that the equilibrium growth paths of heterogeneous economies have to be parallel to one another. Figure 2 demonstrates possible qualitatively different alternatives of the evolution of income distribution under conditional β-convergence, given uniform rate of technological progress. The upper panel of the figure depicts growth paths in some set of economies, and the bottom panel plots respective income distribution densities in the initial and final points in time (incomes are normalized to cross-sectional averages).

Depending on the values of structural parameters for specific economies (i.e. on the mutual arrangement of their equilibrium growth paths), and initial levels of incomes per capita, three qualitatively different patterns of income distribution dynamics can correspond to conditional β-convergence:

(*a*) Global income convergence of economies. It can occur when distance between extreme equilibrium growth paths is less that the spread of initial income levels. However, in contrast to unconditional β-convergence, income inequality does not vanish in the limit, eventually stabilizing at some constant level.

(*b*) Income divergence of economies (when distance between extreme equilibrium growth paths is more that the spread of initial income levels).

(*c*) Local or club income convergence (polarization). Convergence occurs within two or more groups of homogeneous economies, "convergence clubs." Under club convergence, inequality over all economies covered may either decrease or increase.

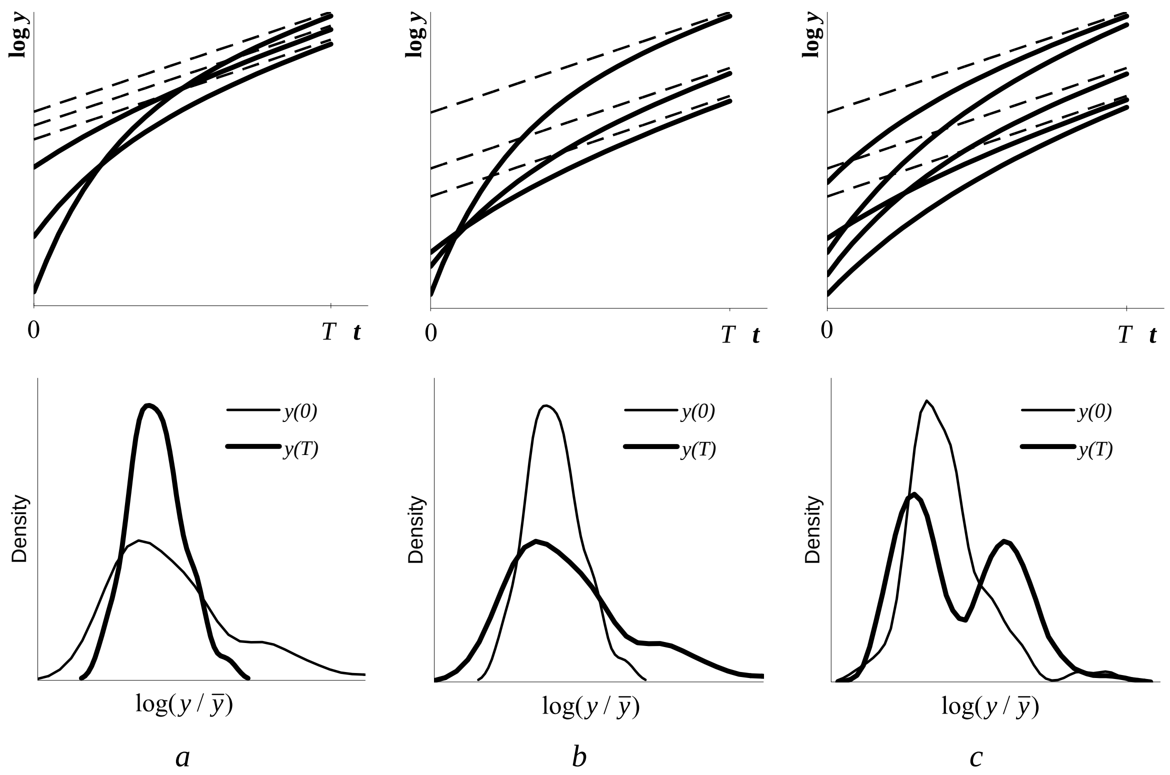

Fig. 2. Possible changes in income distribution under conditional β-convergence: (*a*) global income convergence; (*b*) income divergence; (*c*) local (club) convergence.
Dashed lines denote equilibrium growth paths.

The assumption of uniform rate of technological progress across *different* economies does not seem sufficiently justified, since, recall, closed economies are dealt with. It follows from free availability of the same technology for all economies that the state of technology in them, $A_i(t)$, including that at $t = 0$, should be the same, which evidently contradicts reality. Discarding this assumption (or more specifically, treating it as a particular case), the slopes of the equilibrium growth paths may differ. For instance, the pattern in the upper panel of Figure 2c changes to the pattern in Figure 1b; dynamics of growth depicted in Figures 2a and 2b change in a similar way. However, the evolutions of income distribution remain qualitatively similar to those in the bottom panels of Figure 2. The difference occurs only in the limit $t \to \infty$. Given uniform rate of technological progress, income dispersion tends to some finite value, whereas it infinitely increases if the rate differs across economies. But over a finite period [0, T], trends of inequality will be qualitatively similar.

Thus, regardless of uniform or different rates of technological progress across economies, conditional β-convergence can be accompanied by both decreasing and increasing income inequality. Hence, **conditional β-convergence has no (applied) analytical and predictive power** regarding trends of the evolution of spatial income inequality. It only makes it possible to conclude that the behavior of a set of economies under consideration is compatible with the neoclassical growth model, but does not suggest whether income gaps between the economies widens or narrow.[3]

What does it suggest then? In the case of conditional β-convergence, α or α' in Equations (7′), (7″), and (7‴) cannot be considered as a constant that is identical for all economies; it has its own value for each economy.[4] Then, e.g., Equation (7′) for economy i takes the form $\ln y_{Ti} = \alpha_0 + \alpha_i + \beta_+ \ln y_{0i}$, where α_0 is a common value for all economies (e.g., the cross-sectional average). To get a single uniform equation instead of economy-specific equation, subtract α_i from both sides. Equation $\ln y'_{Ti} = \alpha_0 + \beta_+ \ln y_{0i}$ is arrived at, where $\ln y'_{Ti} = \ln y_{Ti} - \alpha_i = \ln(y_{Ti} / \exp(\alpha_i))$. We thus obtain an equation of unconditional β-convergence, however with income per capita at T (or growth rate, if we would deal with Equation (7″) or (7‴)) adjusted for differences between economies (or more specifically, for differences between their equilibrium growth paths).[5] It is impossible to directly compute α_i, since – as (7′) suggests – they include a number of unobservable parameters. Because of this, some function $\alpha_i = \alpha(x_{i1}, \ldots, x_{in})$ is used instead; arguments x_{ij} (conditioning variables) are observable parameters (e.g., the average rate of growth of the number of workers ν_i) and proxies characterizing – as the researcher supposes – unobservable parameters $A_i(0)$, a_i, etc. Thus, an interpretation of conditional β-convergence decreases *subject to condition* that incomes are adjusted for differences between economies. Hence, we acquire information not on evolution of actual inequality, but on evolution of inequality in some speculative indicators that reflect nothing real.

Consider a simple numerical example. There are two countries, say, an “eastern” one (i = 1) and a “western” one (i = 2); $\ln y_0$ = (1.8, 2), $\ln y_T$ = (3.44, 4). A sole variable x equaling 0 for the

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[3] If the examined sample of economies contains groups of homogeneous economies, analyzing conditional β-convergence provides no means to identify them. Moreover, it cannot suggest at all whether there are such groups in the sample.

[4] Further still the value of β also cannot be assumed identical for all economies, as is seen from Equations (7′) and (4). Nevertheless, empirical analyses are *always* based on such an assumption (which would be true only under the equality of structural parameters a, s, ξ, ν, and δ across all economies under study).

[5] An actual example can be seen in Barro and Sala-i-Martin (2004, p. 523), where Figure 12.3 reports adjusted growth rates of real per capita GDP in 112 countries.

eastern country and 0.4 for the western one describes difference between their economies.[6] Conditional β-convergence holds in this set of economies: $\beta_+ = 0.8 < 1$ in equation $\ln y_{Ti} = \alpha_0 + x_i + \beta_+ \ln y_{0i}$. We obtain 3.44 = 2 + 0 + 0.8·1.8 for the eastern country, and 4 = 2 + 0.4 + 0.8·2 or 3.6 = 2 + 0.8·2 for the western country. Thus, $\ln y'_T$ = (3.44, 3.6). The income gap between countries has decreased: $\sigma(\ln y_0)$ = 0.1, a $\sigma(\ln y'_T)$ = 0.08. However, this conclusion is true for *adjusted* incomes. To eliminate differences between economies, we should decrease growth rate of the western country by a factor of circa 1.5 ($e^{0.4}$)! The gap between actual incomes, quite the contrary, increases: $\sigma(\ln y_T)$ = 0.28. Therefore, conditional β-convergence with the western country can hardly be an encouraging fact for the eastern country; this fact itself contains no helpful (from the practical viewpoint) information.

Turn to a more real situation. Let the question of interest is that of the evolution of income inequality between regions within a country. Based on the belief that differences between regional economies are adequately described by sectoral structure (taking conditioning variables $\mathbf{x}_i$, where x_{ij} is a proportion of sector j in GRP of region i), we find conditional β-convergence. This implies that we obtain the following answer: inter-regional income inequality would decrease, *if sectoral structure of all regional economies were the same* (with no information on whether inequality actually decreases or increases). Now what is a practical value of such an answer?

A failure of conditional β-convergence need not evidence increasing income gap. The reason may be that the neoclassical model cannot describe the economy sample under study, or that the conditioning variables taken by the researcher poorly characterize differences between economies. A violation of conditions for adequate application of β-convergence equations can also be the reason (note that it can cause a reverse effect, namely, falsely finding β-convergence).

The main implication of the growth model considered is that *always* the poorer the given economy (the lower the y_0), the higher its growth rate (y_T/y_0) – see Equation (7′′). However, even a small and fairly realistic modification leads to quite different result. The modification consists in discriminating the saving rates between two kinds of income: the saving rate out of wage income equals s_w, and the saving rate out of interest income equals s_r (Galor, 1996). Then the economy may have two stable equilibrium states $\tilde{y}_{*1}$ and $\tilde{y}_{*2}$, $\tilde{y}_{*1} < \tilde{y}_{*2}$. This implies that in a set of *homogeneous economies* the growth of economies with low y_0 converges to the equilibrium growth path with $\tilde{y}_{*1}$

[6] In econometric terms, x is a dummy equaling 0 for the eastern country and 1 for the western one, and the estimated coefficient on x equals 0.4.

(implying a "poverty trap"), and the growth of economies with high y_0 converges to the path with $\tilde{y}_{*2}$. Such model does not generate the β-convergence equation (as the equilibrium growth path itself proves to be dependent on initial condition y_0).

All the more models that abandon the neoclassical premises predict neither unconditional nor conditional β-convergence. For example, in the Romer (1986) model, knowledge is assumed to be an input in production that has increasing marginal productivity, which leads to increasing returns to scale. The implication of this model is that "The level of per capita output in different countries need not converge; growth may be persistently slower in less developed countries and may even fail to take place at all" (Romer, 1986, p. 1003). A similar "negative" conclusion follows from the Azariadis and Drazen (1990) model and a number of others.

So, the neoclassical model of economic growth (more exactly, a number of its versions) predicts income convergence (strictly speaking, when income is measured by value added per worker) for the only case: under the very strong and hardly realistic condition of homogeneity of economies under consideration. **Given heterogeneous economies, this model can suggest nothing definite regarding the evolution of income inequality.**

3. The second myth: Beta-convergence suggests diminishing inequality

Let us turn first to unconditional β-convergence. For its empirical analysis, econometric versions of Equations (7′), (7″), and (7‴) are used, namely,

$$\log(y_{Ti}) = \alpha + \beta_{+}\log(y_{0i}) + \varepsilon_i, \tag{8′}$$

$$\log(y_{Ti}/y_{0i}) = \alpha + \beta\log(y_{0i}) + \varepsilon_i, \tag{8″}$$

$$\log(y_{Ti}/y_{0i})/T = \alpha' + \beta'\log(y_{0i}) + \varepsilon_i, \tag{8‴}$$

where i indexes economies, $i = 1,\dots, N$; ε_i is a random shock (regression residual). Hypothesis to be tested for regression (8′) is H_0: $\beta_+ < 1$ (against H_a: $\beta_+ \geq 1$); that for regressions (8″) and (8‴) is H_0: β, $\beta' < 0$ (against H_a: β, $\beta' \geq 0$). If the zero hypothesis is not rejected, then β-convergence holds.

As Figure 1a shows, from β-convergence it seemingly follows steady decrease in inequality, i.e. σ-convergence: $\sigma_T < \sigma_0$. But, such a deduction would be true, if growth strictly followed the theoretical paths depicted in Figure 1a. As it is seen from Equation (6), income ranking of economies is time-invariant (to put it differently, there is no rank mobility of economies). In reality, however, the situation is different. Because of some contingencies unaccounted in the theoretical model (and described by random shocks ε_r in econometric version of the β-convergence equation), some of economies may "outrun" their theoretical path, hence leaving other regions behind (instead of catching up), and some of economies may "lag behind" the theoretical paths. All the more this can be in the neighborhood of the equilibrium growth path, which implies closeness of incomes per capita across these economies. Thus, actually rank mobility of economies can occur. In that instance, β-convergence do not necessarily implies σ-convergence.

A conclusion that the differences in values of some indicator across economic objects are narrowing (i.e. that inequality among them, judged by this indicator, is diminishing), based on the fact that the indicator of the objects with its higher values decreases, or increases slower than that of objects with lower values, and the indicator of the latter objects increases (faster that the indicator of the former objects), is a not infrequent delusion in empirical studies. It is known as Galton's Paradox. Regarding β-convergence, a number of authors pointed to this, among then Friedman (1992), Quah (1993), Wodon and Yitzhaki (2006), and others. Galton (1886) found that the adult children of tall parents tended to be shorter than their parents, and the children of short parents tended to be taller than parents (by the way, the term "regression" had its origin just in this). This fact seemingly implies that the heights of all adult people should even out over time.[7] Some details apart, the relationship found by Galton is formalized similarly to Equation (8′): H_{Ti} = const + $^2/_3 H_{0i}$ (H_{0i} is parent's height and H_{Ti} is child's height).

Let us strictly derive a relationship between β- and σ-convergence. According to the known formula, the estimate of β in bivariate regression (8″) equals

$$\beta = \frac{\operatorname{cov}(\log(y_0), \log(y_T))}{\sigma_0^2} - 1.$$

Benefiting from the fact that $\sigma^2(\log(y_T) - \log(y_0)) = \sigma_0^2 + \sigma_T^2 - 2\operatorname{cov}(\log(y_0), \log(y_T))$, we get

$$\beta = \frac{1}{2}\left(\left(\frac{\sigma_T^2}{\sigma_0^2} - 1\right) - \frac{\sigma^2(\log(y_T / y_0))}{\sigma_0^2}\right). \tag{9}$$

It follows herefrom that if income inequality decreases ($\sigma_T < \sigma_0$), β-convergence holds: $\beta < 0$. If inequality remains constant, $\sigma_T = \sigma_0$, we would expect to get $\beta = 0$. However, as it follows from Formula (9), the estimate of regression (8″) will *always* (except the only event when y_0 are y_T identical, yielding $\sigma^2(\log(y_T/y_0)) = 0$) suggest β-convergence. Moreover, β-convergence will take place even when income inequality rises, $\sigma_T > \sigma_0$, if $\sigma_T^2 - \sigma_0^2 < \sigma^2(\log(y_T / y_0))$ or, what is

[7] Contrary to certain assertions in the literature, Galton himself not in the least makes such an inference. Quite the reverse, he explains why this does not happen (Galton, 1886, p. 256). Thus the term "Galton's Paradox" or, all the more, "Galton's Fallacy" is unjust.

equivalent, $\mathrm{cov}(\log(y_0),\log(y_T)) < \sigma_0^2$. In this case (as well as in the case of $\sigma_T = \sigma_0$), β-convergence will be bi-directional, holding in both direct and reverse directions of time, which just contradicts the common sense.

It follows from the latter relationship that bi-directional β-convergence will be always observed (and without regard of the relation between σ_0 and σ_T) under statistical independence of y_0 and y_T (with $\mathrm{cov}(\log(y_0),\log(y_T)) = 0$, we get $\beta = -1$), as well as under a negative correlation between y_0 and y_T. The latter means that a considerable part of initially poorer economies keep ahead of initially richer economies. Such a situation can emerge, e.g., when a set of economies under consideration consists of economies with rather close incomes per capita.

Thus, if there is σ-convergence than it must be that there is β-convergence; however, β-convergence is not necessarily accompanied with σ-convergence[8] (i.e. diminishing income inequality). Hence, while analyzing trends of income inequality, it is more adequate to directly compare a measure of inequality at different dates. If income convergence occurs, then analyzing β-convergence yields no additional information. **If income convergence does not hold, then inference regarding the evolution of inequality, based on analyzing β-convergence, may prove to be false.**

Consider two real examples of such cases. The first example is (nominal) personal money incomes per capita across regions of Russia in 1995 and 2005 (the data source is Rosstat's Central Statistical Database, URL: www.gks.ru/dbscripts/Cbsd/DBInet.cgi?pl=2340019). Composite subjects of the Russian Federation are treated as single regions; then the number of regions (excluding the Chechen Republic) is 79. Income inequality in 2005 slightly decreased as compared to 1995: $\sigma_{1995} = 0.418$, while $\sigma_{2005} = 0.405$. The estimate of regression (8′) suggests β-convergence: $\log(y_{2005}) = 3.434 + 0.877\log(y_{1995})$. However, as $\mathrm{cov}(\log(y_{1995}),\log(y_{2005})) = 0.153 < \sigma_{2005}^2 = 0.175$, β-convergence is observed in the reverse direction of time as well: $\log(y_{1995}) = -2.128 + 0.935\log(y_{2005})$ (both estimates of β_+ are statistically significant at the 1-percent level).

The second, more prominent, example is real GDP per capita across 24 countries in Latin America[9] in 1960 and 1998 (the data source: Heston, Summers, and Aten (2002), variable *rgdpl*). Wodon and Yitzchaki (2006) have pointed to bi-directional β-convergence in this sample;

[8] This fact is proved in other ways in Barro and Sala-i-Martin (2004, p. 71), Quah (1993), Furceri (2005), and Wodon and Yitzchaki (2006).

[9] Argentina, Barbados, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Guyana, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Puerto Rico, Trinidad and Tobago, Uruguay, and Venezuela.

however, our estimates somewhat differ (apparently because of a further revision of data in its source). Inequality rose here over 1960–1998: σ_{1960} = 0.457 and σ_{1998} = 0.554. Nonetheless, β-convergence holds: $\log(y_{1998})$ = 2.003 + 0.823$\log(y_{1960})$. It is due to the above-derived relationship: $\mathrm{cov}(\log(y_{1960}), \log(y_{1998})) = 0.172 < \sigma_{1960}^2 = 0.307$. In the reverse direction, β-convergence also holds (which is to be expected, as income inequality decreases in this direction): $\log(y_{1960})$ = 3.179 + 0.561$\log(y_{1998})$ (estimates of β_+ are significant at the 1-percent level in both regressions). Figure 3 provides a graphical illustration. It contains scatter plots and regression lines. The raw data are normalized to the cross-sectional averages for more clearness (which change only the regression constant, keeping the estimate of β_+ intact); the dashed line indicates the diagonal corresponding to $\beta_+ = 1$.

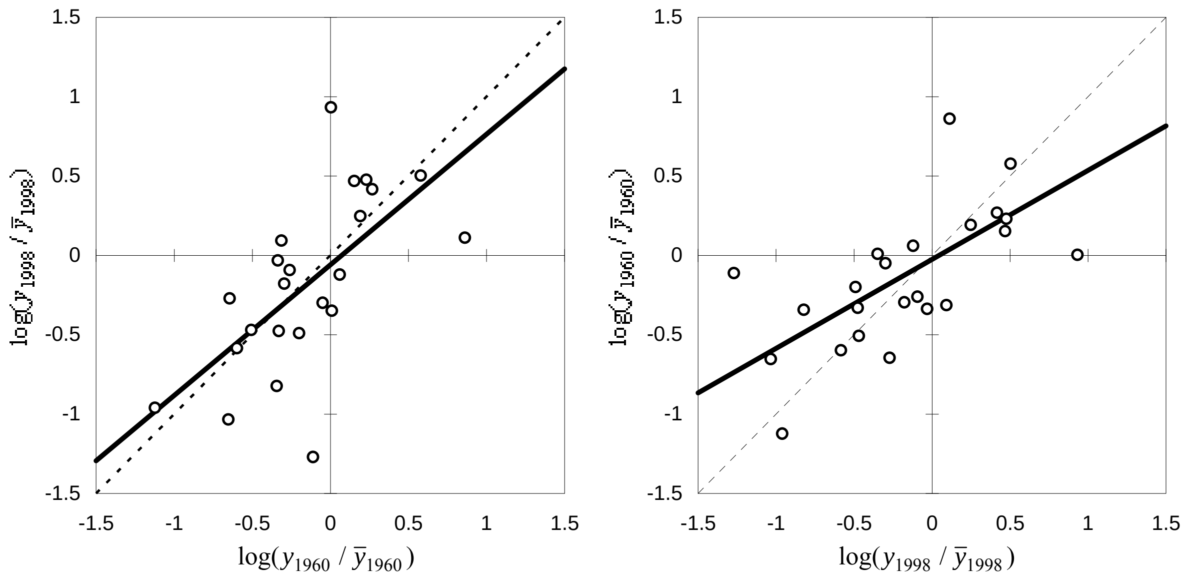

Fig. 3. Bi-directional β-convergence of real GDP per capita (Latin America, 1960 and 1998)

Intuitively, the reason for possible contradicting results of analyzing σ- and β-convergence may be explained by a conceptual discrepancy between them. One of requirements for inequality measures is the anonymity (symmetry) principle (Jenkins and van Kerm, 2009, p. 52). It asserts that the value of inequality index should depend only on income values in a set under consideration, irrespective of which element of the set possesses a specific income. In other words, the value of inequality is invariant to permutations of elements of the set (numbers of observations). Thus, a change in income inequality, σ_T/σ_0, is not dependent on where (in which specific economies) income has changed. For instance, if a part of economies simply "interchange" their incomes, inequality remains the same: as the economies are impersonal for the inequality index, no change will be observed in the final state as compared to the initial one.

A different situation arises with β. The anonymity principle does not hold for it, as an economy is identified by the number of observation, and β confronts observations (economies) with the same number at dates 0 and T. Thus, the value of β depends on income change in each specific economy, i.e. on the collection of $\log(y_{Ti}/y_{0i})$. To put it differently, β includes income mobility, term $\sigma^2(\log(y_T / y_0)/\sigma_0^2$ in Formula (9) measuring its total value. In the case when a part of economies simply "interchange" their incomes, this term quantifies rank mobility, i.e. a change in ranks of economies sorted in ascending order of income per capita.

Let us consider a simple example of two economies with $\log(y_0) = (0.99, 1.01)$ and $\log(y_T) = (1.01,0.99)$, and thus with time-invariable inequality ($\sigma_T = \sigma_0 = 0.01$). However, the logarithm of income per capita in the first economy has changed by 0,02, and that in the second economy has changed by –0.02. This yields $\sigma^2(\log(y_T / y_0)/\sigma_0^2$= 0.0004/0.0001 = 4 and β = –2, evidencing β-convergence.

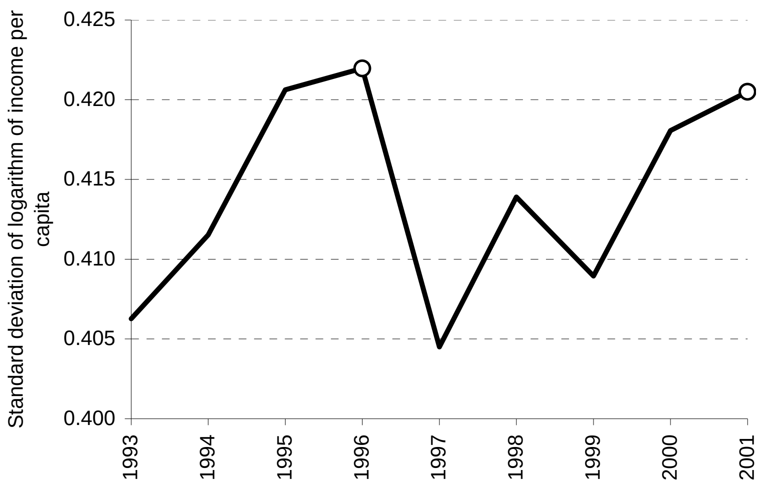

Fig. 4. The evolution of inequality in nominal incomes per capita (σ_t) among Russian regions in 1993–2001.
Source: author's calculations based on data from Rosstat's Central Statistical Database, URL: www.gks.ru/dbscripts/Cbsd/DBInet.cgi?pl=2340019.

It should be added that considering merely two dates, 0 and T, which is a characteristic feature of analyzing β-convergence, is fraught with erroneous conclusions regarding the trend of income inequality, even if it is smaller at date T than at the initial date. This can happen, e.g., if inequality has a U-shaped path. The loss of information on inequality changes within the time span between 0 and T in the analysis of β-convergence prevents from noticing this, while merely a graph of the evolution of σ_t (or other inequality index) over that time span yields a much richer pattern which makes possible to readily see the trend to rise in inequality. The evolution of (nominal) income inequality among Russian regions provides an example. It is depicted in Figure 4. Considering incomes in 1996 and 2001, β-convergence is found, while it is obvious from the graph that only a one-time fall of income inequality has occurred, then giving way to its rise (with a deviation from the latter trend in 1999).

As regards conditional β-convergence, it dispenses with the need for a detailed consideration, since, as has been demonstrated in the previous section, conditional β-convergence is not able to yield any information at all on the evolution of income inequality. Besides, all of the preceding regarding unconditional β-convergence is true for conditional β-convergence, as will readily be demonstrated. An analogue of Equation (8″) for conditional β-convergence looks like: $\log(y_{Ti}/y_{0i}) = \alpha_0 + \alpha(x_{i1},...,x_{in}) + \beta\log(y_{0i}) + \varepsilon_i$. Recall that $x_{i1},\dots, x_{in}$ are variables proxying (as the researcher believes) parameters involved in the neoclassical growth model. Function $\alpha(x_{i1},...,x_{in})$ is, as a rule, represented in the log-linear form, which gives the following typical equation of conditional β-convergence:

$$\log(y_{Ti}/y_{0i}) = \alpha_0 + \beta\log(y_{0i}) + \alpha_1\log(x_{i1}) + ... + \alpha_n\log(x_{in}) + \varepsilon_i . \tag{10}$$

As it is demonstrated in Section 2, to eliminate cross-economy differences described by variables $x_{i1},\dots, x_{in}$, one can adjust per capita incomes y_{iT} (and thus, the growth rates) for these differences: $\log(y'_{Ti}) = \log(y_{Ti}) - (\alpha_1\log(x_{i1}) + ... + \alpha_n\log(x_{in}))$. Then convergence becomes unconditional. Since parameters $\alpha_1, \dots, \alpha_n$ are unknown, the adjusted incomes are estimated as $\log(y'_{Ti}) = \hat{\eta}_i$, where $\hat{\eta}_i$ are estimates of the residuals in regression $\log(y_{Ti}) = \alpha_1\log(x_{i1}) + ... + \alpha_n\log(x_{in}) + \eta_i$. Then Equation (10) transforms to $\log(y'_{Ti}/y_{0i}) = \alpha_0 + \beta\log(y_{0i}) + \varepsilon_i$, i.e. a regression of the form (8″).

The use of panel data (being treated as a set of cross sections $\{y_{it}\}$ observed across a number of points in time $t_1,\dots, t_m$) also does not rescue the situation. Panel data analysis allows better taking into account heterogeneity of economies and characterizing time variation of parameters. However, using panel versions of Equations (8′), (8″), and (8‴), all fundamental problems related to β-convergence still stand (and new problems caused by features of the panel-data analysis arise in addition). We do not dwell upon this issue; see Durlauf and Quah (1999) and Magrini (2004).

So, **interpretation of β-convergence found with regression analysis as evidence of**

decreasing income dispersion is fallacious. The occurrence of β-convergence is compatible not only with a decrease of income inequality, but with its permanence and even increase as well.

4. Conclusion

As it has been demonstrated, **analysis of β-convergence** (both unconditional and conditional) **is useless** in applied researches aiming at identifying trends of the evolution of spatial income inequality. Its considerable use in such studies is based on a mass fallacy caused by the two considered widespread myths. Noted economists are responsible for a number of studies of this kind, which favors uncritical treatment of these studies and provides examples for imitation by novices. Eventually, this generates a "chain reaction" of applied researches based on the β-convergence concept.

The above consideration in no way implies that the β-convergence concept is fallacious. The matter is not the concept itself, but wrong interpretation, misuse of the concept. By means of the β-convergence analysis, researchers tackle a question which fundamentally cannot be answered by this method. Empirical analysis of β-convergence is able to shed light on the only issue: whether behavior of economies possesses properties emerging from the neoclassical growth model, and nothing more. Therefore, its area of application is fairly narrow: verification of theoretical models of economic growth.[10]

Worthy of mention is one more reason for popularity of analyzing β-convergence. Examining the evolution of spatial income inequality, the question asked is whether inequality has increased or decreased at $t = T$ as compared to $t = 0$. It can be easily answered, merely comparing two numbers, the values of an inequality index (e.g., σ_t or the Gini index) in those two points in time. However, the possibility to publish an article based on so unsophisticated method is very questionable. In contrast to this, accoutrements of regression analysis – that accompany a search for answer to the same question with the use of the β-convergence concept – impart an academese to the article, significantly boosting its chances of publication.

Analysis of β-convergence for applied purposes has to be qualified as "economic alchemy." It might be regarded as a formal academic exercise which is harmless but for informational noise caused by relevant publications. However, far-reaching conclusions are not infrequently made from such an analysis. For example, "If sufficiently fast unconditional β-convergence occurs…, then

[10] True enough, the possibilities of this method are limited in that respect as well, as pointed out by a number of authors, e.g., Durlauf, Quah (1999).

regional economic policy of the central government can be managed without at all" (Melnikov, 2005, p. 13). Thus, this line of economic analysis if far from innocuous; it can produce erroneous policy implications.

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