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Selection of an estimation window in the presence of data revisions and recent structural breaks*

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Abstract

In this paper, we analyze the forecasting performance of a set of widely used window selection methods in the presence of data revisions and recent structural breaks. Our Monte Carlo and empirical results for U.S. real GDP and inflation show that the expanding window estimator often yields the most accurate forecasts after a recent break. It performs well regardless of whether the revisions are news or noise, or whether we forecast first-release or final values. We find that the differences in the forecasting accuracy are large in practice, especially when we forecast inflation after the break of the early 1980s.

Keywords: Recent structural break, choice of estimation window, forecasting, real-time data

JEL codes: C22, C53, C82

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1. Introduction

Macroeconomic time series are often serially correlated. This implies that their own past values are themselves useful predictors. Therefore, it is not surprising that autoregressive (AR) models are used extensively in economic forecasting. The previous literature has found that it is difficult to outperform AR models in practice. For example, Rossi (2013) and Stock and Watson (2003) find that only a few macroeconomic predictors systematically improve upon the AR benchmark when forecasting inflation and output growth.

However, the parameters of AR models fitted to many macroeconomic time series are unstable over time (see, e.g., Stock and Watson, 1996). This observed parameter instability can arise as a result of several reasons. For instance, changes in tastes, technology, legislation, institutional arrangements, or government policy can cause changes in the dynamics of the economy. Structural breaks are crucial because they often have a major impact on forecasting performance: a forecasting model that performed well before the break might perform extremely poorly after the break (see, e.g., Clements and Hendry, 1998; Rossi, 2013). Because tastes, technology, legislation, institutional arrangements, and government policy are likely to change in the future, structural breaks are also likely to happen in the future. Therefore, information about the forecasting performance of AR models when these models undergo structural breaks is needed. Given the empirical success of AR models and their widespread use in practice, we believe that this is an important area to investigate.

A key question in the presence of structural instability is how many observations to use to estimate the parameters of a model so that, when used to generate a forecast, a loss function such as the root mean squared forecast error (RMSFE) will be minimized. This issue has been analyzed by Eklund *et al.* (2013), Giraitis *et al.* (2013), Pesaran and Pick (2011), Pesaran and Timmermann (2005, 2007), and Pesaran *et al.* (2013). This literature typically assumes that the break has occurred in the distant past. In such a case, the standard solution to the window selection problem is to test for breaks and use only observations after the most recent break. The estimates of the timing of the break(s) can be obtained, for example, using methods developed by Altissimo and Corradi (2003), Andrews (1993), Andrews *et al.* (1996), and Bai and Perron (1998, 2003). In the presence of recent breaks, this so called post-break window strategy is not feasible. As noted by Eklund *et al.* (2013), structural break tests are not designed for detecting recent breaks. Instead, the breaks are observed with a long lag. Even if real-time detection were possible, the post-break window strategy would not be useful. The parameters of the forecasting model are estimated with adequate accuracy only if the number of observations is at least two to three times the number of parameters (see, e.g., the discussion in Pesaran and Timmermann, 2005). Hence, the post-break window strategy is applicable only when the (last) break has occurred sufficiently long ago.

Forecasting after a recent break has received very little attention in the literature. However, in practice, forecast errors are often very large after structural breaks (Clements and Hendry, 2006). This suggests that improving forecast accuracy after a recent break is a central issue in economic forecasting.

Another issue that has often been overlooked in the literature is the real-time nature of the data used in many applications. For example, GDP and inflation series are published with a lag and are subject to revisions. These revisions are usually quite large and hence forecasts based on final revised data may differ considerably from those based on real-time data. Practical forecasting is inherently a real-time exercise, and therefore ingnoring the real-time nature of the data leads to a wide discrepancy between theory and practice.

We introduce two innovations on the existing literature. First, we focus on forecasting in the presence of recent breaks. To this end, several break processes are considered, including changes in the intercept, autoregressive parameter, and error variance. Second, we take into account that most macroeconomic time series are subject to data revisions. We follow the standard practice in the literature and allow revisions to be characterized either as news or noise, in the sense of Mankiw and Shapiro (1986). To the best of our knowledge, there are no other papers analyzing the window selection problem when the data are subject to revision.

The end of the Great Moderation and the Financial Crisis of 2008 provide an excellent motivation for our exercise. It is well-known that the volatility of many U.S. macroeconomic series has declined since the mid-1980s (see, e.g., McConnell and Perez-Quiros, 2000). Recent data suggest that this phenomenon, called the Great Moderation, came to an end with the Financial Crisis. Furthermore, monetary policy has changed fundamentally since the beginning of the crisis. The nominal short-term interest rate has been stuck at the zero lower bound and the Federal Reserve has used unconventional monetary policy, both of which should change the dynamics of key macro variables. So, forecasting these days, one would certainly run into the aforementioned too-few-data-after-the-break problem and the results of this paper will be relevant.

We consider a set of widely used methods for forecasting in the presence of structural instability. These methods include rolling windows, exponentially weighted moving average models, and the average window method advocated by Pesaran and Pick (2011) and Pesaran and Timmermann (2007). The potential gains in forecasting performance from using these methods compared to the expanding window method are demonstrated through Monte Carlo simulations and empirical examples.

The main finding from this study is that, at least for macroeconomic time series such as U.S. real GDP and inflation (defined as the growth rate of the GDP deflator), the expanding window estimator tends to produce more accurate forecasts than the alternative window selection methods considered here. Our simulation results indicate that the expanding window method performs particularly well when the parameters remain constant over time or when the innovation variance changes. Our empirical results suggest that the expanding window estimator is overwhelmingly the best estimation strategy when we forecast inflation after the break in the early 1980s. In this case, the alternative methods produce 7.5–52.9 percent larger forecast errors than the expanding window estimator. The expanding window method also performs well when we make real-time GDP growth forecasts for the period 2008:Q4–2011:Q1. However, we find that, in this case, the differences in relative performances are more modest.

The remainder of the paper is organized as follows. Section 2 introduces the notation and the statistical framework. Section 3 provides a brief overview of the window selection methods. Section 4 presents the Monte Carlo simulation results and Section 5 presents the empirical results. Section 6 concludes. The appendices at the end of the paper provide the technical details.

2. Statistical framework

An important feature of real-time data is that the data for a period are not released until some time has passed after the end of that period. Therefore, for instance, a forecaster at period $T+1$ has access to the vintage $T+1$ values of real GDP and inflation up to time period T. Furthermore, the data are revised over time, so the first-released values and the final values may differ considerably. Although the real-time nature of macroeconomic time series clearly matters for forecasting, data revisions are rarely incorporated into the theoretical models. One exception is the statistical framework suggested by Jacobs and van Norden (2011) and further developed by Clements and Galvão (2013). This framework for modeling data revisions, which we will closely follow, relates a data vintage estimate to the true value plus an error or errors. In particular, the period $t+s$ vintage estimate of the value of y in period t, denoted by y_t^{t+s} [1], where $s = 1,...,l$ [2], can be expressed as a sum of the true value $\tilde{y}_t$, a news component v_t^{t+s}, and a noise component ε_t^{t+s}, so that $y_t^{t+s} = \tilde{y}_t + v_t^{t+s} + \varepsilon_t^{t+s}$.

This framework follows the standard practice in the literature and assumes that revisions either add news or reduce noise. Data revisions are said to be news if revisions are uncorrelated with the previously published vintages, $cov(y_t^{t+k}, v_t^{t+s}) = 0 \ \forall k \leq s$. This implies that the initially released data are optimal forecasts of the later data. On the other hand, data revisions reduce noise if each vintage release is equal to the true value plus a noise, so that noise revisions are uncorrelated with the truth, $cov(\tilde{y}_t, \varepsilon_t^{t+s}) = 0$. For further discussion of the properties of news and noise revisions, see Croushore (2011) and Jacobs and van Norden (2011). The distinction between news and noise revisions is important in practice because revisions to different macroeconomic time series have different characteristics. For example, Clements and Galvão (2013) find that, at least since the mid-1980s, data revisions to output growth appear to be mainly news whereas those to inflation are mainly noise.

Following Clements and Galvão (2013) and Jacobs and van Norden (2011), we stack

[1] Throughout this paper, superscripts refer to vintages and subscripts to time periods. This notation has become standard in the literature.

[2] For simplicity, we assume that we observe l different estimates of y_t before the true value, $\tilde{y}_t$, is observed. In practice, however, the true value may never be observed.

the l different vintage estimates of y_t, v_t and ε_t into vectors $\boldsymbol{y}_t = (y_t^{t+1}, ..., y_t^{t+l})'$, $\boldsymbol{v}_t = (v_t^{t+1}, ..., v_t^{t+l})'$ and $\boldsymbol{\varepsilon}_t = (\varepsilon_t^{t+1}, ..., \varepsilon_t^{t+l})'$, respectively. Now we can express each vintage of y_t as follows

$$\boldsymbol{y}_t = \boldsymbol{i}\tilde{y}_t + \boldsymbol{v}_t + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\boldsymbol{i}$ is an $l \times 1$ vector of ones. For the true values we consider the following AR(1) process subject to a single structural break at time T_1

$$\tilde{y}_t = \begin{cases} \rho_1 + \sum_{i=1}^{l} \mu_{v1_i} + \beta_1 \tilde{y}_{t-1} + \sigma_1 \eta_{1t} + \sum_{i=1}^{l} \sigma_{v1_i} \eta_{2t,i}, & \text{for } t < T_1, \\ \rho_2 + \sum_{i=1}^{l} \mu_{v2_i} + \beta_2 \tilde{y}_{t-1} + \sigma_2 \eta_{1t} + \sum_{i=1}^{l} \sigma_{v2_i} \eta_{2t,i}, & \text{for } t \geq T_1, \end{cases} \tag{2}$$

where η_{1t} and $\eta_{2t,i}$ ($i = 1, \ldots, l$) are $NIID$ (0,1) disturbances.[3]

The news and noise processes of each vintage are specified by

$$\boldsymbol{v}_{1t} = \begin{bmatrix} v_{1t}^{t+1} \\ v_{1t}^{t+2} \\ \vdots \\ v_{1t}^{t+l} \end{bmatrix} = - \begin{bmatrix} \sum_{i=1}^{l} \mu_{v1_i} \\ \sum_{i=2}^{l} \mu_{v1_i} \\ \vdots \\ \mu_{v1_l} \end{bmatrix} - \begin{bmatrix} \sum_{i=1}^{l} \sigma_{v1_i} \eta_{2t,i} \\ \sum_{i=2}^{l} \sigma_{v1_i} \eta_{2t,i} \\ \vdots \\ \sigma_{v1_l} \eta_{2t,l} \end{bmatrix}, \boldsymbol{\varepsilon}_{1t} = \begin{bmatrix} \varepsilon_{1t}^{t+1} \\ \varepsilon_{1t}^{t+2} \\ \vdots \\ \varepsilon_{1t}^{t+l} \end{bmatrix} = - \begin{bmatrix} \mu_{\varepsilon 1_1} \\ \mu_{\varepsilon 1_2} \\ \vdots \\ \mu_{\varepsilon 1_l} \end{bmatrix} + \begin{bmatrix} \sigma_{\varepsilon 1_1} \eta_{3t,1} \\ \sigma_{\varepsilon 1_2} \eta_{3t,2} \\ \vdots \\ \sigma_{\varepsilon 1_l} \eta_{3t,l} \end{bmatrix} \tag{3}$$

for $t < T_1$ and

[3] We focus on the shortest possible lag length, because we want to minimize the number of possible breaks in the autoregressive structure. Furthermore, it is easier to calibrate the parameters (see the discussion below) when the lag order is one. Eklund *et al.* (2013) and Pesaran and Timmermann (2005) also consider an AR(1) specification in the presence of breaks.

$$\boldsymbol{v}_{2t} = \begin{bmatrix} v_{2t}^{t+1} \\ v_{2t}^{t+2} \\ \vdots \\ v_{2t}^{t+l} \end{bmatrix} = - \begin{bmatrix} \sum_{i=1}^{l} \mu_{v2_i} \\ \sum_{i=2}^{l} \mu_{v2_i} \\ \vdots \\ \mu_{v2_l} \end{bmatrix} - \begin{bmatrix} \sum_{i=1}^{l} \sigma_{v2_i}\eta_{2t,i} \\ \sum_{i=2}^{l} \sigma_{v2_i}\eta_{2t,i} \\ \vdots \\ \sigma_{v2_l}\eta_{2t,l} \end{bmatrix}, \boldsymbol{\varepsilon}_{2t} = \begin{bmatrix} \varepsilon_{2t}^{t+1} \\ \varepsilon_{2t}^{t+2} \\ \vdots \\ \varepsilon_{2t}^{t+l} \end{bmatrix} = - \begin{bmatrix} \mu_{\varepsilon 2_1} \\ \mu_{\varepsilon 2_2} \\ \vdots \\ \mu_{\varepsilon 2_l} \end{bmatrix} + \begin{bmatrix} \sigma_{\varepsilon 2_1}\eta_{3t,1} \\ \sigma_{\varepsilon 2_2}\eta_{3t,2} \\ \vdots \\ \sigma_{\varepsilon 2_l}\eta_{3t,l} \end{bmatrix} \quad (4)$$

for $t \geq T_1$.

The shocks are assumed to be mutually independent, i.e., if $\boldsymbol{\eta}_t = \left[\eta_{1t}, \boldsymbol{\eta}_{2t}', \boldsymbol{\eta}_{3t}'\right]'$, then $E(\boldsymbol{\eta}_t) = \mathbf{0}$ and $E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') = I$. We assume that $\tilde{y}_t$ is a stationary process, so that $|\beta_j| < 1$ (for j = 1,2). Because $\tilde{y}_t$ is a stationary process and both the news and noise terms are stationary, (1) implies that $\boldsymbol{y}_t$ is also a stationary process. Note that the means of the news and noise terms, denoted by μ_{vj_i} and $\mu_{\varepsilon j_i}$ (for j = 1,2 and i = 1,...,l), are allowed to be non-zero. This is an important feature because in practice revisions to macroeconomic data have non-zero means (see, e.g., Aruoba, 2008; Clements and Galvão, 2013; Croushore, 2011).

As discussed earlier, this framework is similar to that adopted in Clements and Galvão (2013) and Jacobs and van Norden (2011). The main point of departure from their framework is that we allow the process of the true values to be subject to a recent structural break. Our setup is quite general and allows for changes in intercept, slope, and error variance immediately after the break. Another novelty of our framework is that the means and variances of the news and noise revisions are also allowed to change.

3. Forecasting methods

In the presence of data revisions and structural breaks, a forecaster faces two key questions. First, a forecaster has to decide how to take into account the real-time nature of the data when estimating the parameters of the forecasting model. The most commonly used approach, called the end-of-sample vintage approach (EOS), uses observations from the latest available

$(T+1)$ vintage

$$y_t^{T+1} = \alpha_0 + \alpha_1 y_{t-1}^{T+1} + e_{t,EOS}, \qquad \text{for } t = 2, ..., T. \tag{5}$$

The forecast of y_{T+1} is conditioned on the latest available vintage value of the forecast origin data, so that $\hat{y}_{T+1,EOS} = \hat{\alpha_0} + \hat{\alpha_1} y_T^{T+1}$. Although popular in practice, the EOS approach has a fundamental shortcoming: a large part of the data used in model estimation has been revised many times (early in the sample), while the forecast is conditioned on first-release data (the latest observation).

An alternative estimation strategy is the real-time vintage approach (RTV) suggested by Koenig *et al.* (2003). The central idea in the RTV approach is that the data used in estimation and the data on which the forecast is conditioned should be of a similar maturity. Therefore, the forecasting model is estimated on first-release data

$$y_t^{t+1} = \beta_0 + \beta_1 y_{t-1}^{t} + e_{t,RTV}, \qquad \text{for } t = 2, ..., T, \tag{6}$$

and the corresponding forecast is $\hat{y}_{T+1,RTV} = \hat{\beta}_0 + \hat{\beta}_1 y_T^{T+1}$. Note that the two forecasts are conditioned on exactly the same data. The only difference between the two approaches is the data used in the estimation.

The results in Clements and Galvão (2013) and Koenig *et al.* (2003) indicate that the RTV approach produces more accurate forecasts than the EOS approach. However, it is not known whether this result holds in the presence of structural instability. Thus, our plan is to shed light on the relative accuracy of these two methods in the presence of recent breaks.

The second question a forecaster faces is how much data to use to estimate the parameters of the forecasting model. One solution to this window selection problem is to test for breaks and use only observations over a post-break window. If the structural break has occurred recently, this post-break window strategy is infeasible for two reasons. First, it is difficult or even impossible to estimate accurately the timing of a recent break. Second, even if an accurate detection of a recent break were possible, the post-break window strategy is infeasible because a sufficient number of post-break observations, say at least two to three times the number of parameters, is required for accurate estimation. Once the real-time nature of the data is taken into account, the problems associated with the post-break window strategy get compounded since the break may not be as apparent in real-time. Moreover, post-break observations are less 'mature', which will cause problems with accuracy.

An alternative solution is to use robust estimation strategies. An estimation strategy is said to be robust if no information about the structural break is needed for its implementation. Therefore, robust methods are also valid in the presence of recent breaks. In this paper, we focus exclusively on robust methods. We compare the forecasting performance of a set of widely used estimation strategies when the underlying time series process has undergone a recent structural break. Common to all of these strategies is that the estimation window should exceed a minimum length, denoted by $\underline{\omega}$.

The first strategy is the expanding window estimator

$$\hat{\boldsymbol{\beta}}_{T,EXP} = \left(\sum_{t=1}^{T} \boldsymbol{x}_{t-1} \boldsymbol{x}_{t-1}^{'} \right)^{-1} \sum_{t=1}^{T} \boldsymbol{x}_{t-1}^{'} y_t,$$

where $\boldsymbol{x}_t = (1, y_t)^{'}$. The expanding window estimator uses the whole data sample available at the forecast origin. The expanding window forecast for period $T+1$ is computed by $\hat{y}_{T+1,EXP} = \hat{\boldsymbol{\beta}}_{T,EXP}^{'} \boldsymbol{x}_T$.

The second strategy is the rolling window estimator

$$\hat{\boldsymbol{\beta}}_{T,ROLL}(m) = \left(\sum_{t=T-m+1}^{T} \boldsymbol{x}_{t-1} \boldsymbol{x}_{t-1}^{'} \right)^{-1} \sum_{t=T-m+1}^{T} \boldsymbol{x}_{t-1}^{'} y_t,$$

where $m \in \underline{\omega}, ..., T$ is the length of the rolling window. The parameters are estimated using the m most recent observations. The resulting forecast for period $T+1$ is computed by $\hat{y}_{T+1,ROLL}(m) = \hat{\boldsymbol{\beta}}_{T,ROLL}(m)^{'} \boldsymbol{x}_T$. Giacomini and White (2006) argue that when the forecasting model is misspecified (due to inadequately modeled dynamics, inadequately modeled heterogeneity, incorrect functional form, or any combination of these), the rolling window estimator often provides more reliable forecasts than the expanding window estimator.

The third alternative is the exponentially weighted moving average (EWMA) method.

This method, unlike rolling regressions, gives a positive weight to each observation. The central idea is that if the relation of interest has changed over time, the most recent observations are more informative than the earlier ones. Thus, the most recent observations receive the highest weight in the estimation:

$$\hat{\boldsymbol{\beta}}_{T,EWMA} = \left(\lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} \boldsymbol{x}_{t-1} \boldsymbol{x}_{t-1}^{'} \right)^{-1} \lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} \boldsymbol{x}_{t-1}^{'} y_t,$$

where $0 < \lambda < 1$ is the down-weighting parameter. The forecast for period $T+1$ is computed by $\hat{y}_{T+1,EWMA} = \hat{\boldsymbol{\beta}}_{T,EWMA}^{'} \boldsymbol{x}_T$. Pesaran and Pick (2011) find that the choice of the down-weighting parameter greatly affects the forecasting performance of the EWMA method.

A final alternative is the average window (AveW) method suggested by Pesaran and Timmermann (2007). This method builds on the common finding that forecast combinations often reduce forecast errors (see, e.g., Timmermann, 2006). Therefore, rather than selecting a single estimation window, the AveW method combines forecasts from models estimated on different observation windows. The AveW method gives an equal weight to each forecast,

$$\hat{y}_{T+1,AveW} = (T - \underline{\omega} + 1)^{-1} \sum_{m=\underline{\omega}}^{T} \hat{y}_{T+1,ROLL}(m),$$

where $\hat{y}_{T+1,ROLL}(m)$ denotes the forecast generated by a rolling window of size m.

4. Monte Carlo simulations

In this section, we evaluate the forecasting performance of alternative window selection methods in a set of Monte Carlo experiments. These experiments are based on the statistical framework introduced in Section 2. Our interest in this paper lies in the point forecasts shortly after a structural break. Therefore, we assume that a single break has occurred at time $T_1 = T$. One-step ahead forecasts are made recursively for the next ten periods, i.e., for the periods $T+1,...,T+10$. We assume that no breaks occur during the forecasting period.

To ensure that our simulation results are empirically relevant, we calibrate the parameters on actual U.S. output and inflation data. We start by considering the case where the parameters remain stable over time (experiment 1 in Table 1). In this case the mean of the true process lies between 2.0 and 2.5, which corresponds roughly to the average annual inflation and real GDP growth since the mid-1980s. The parameters of this model are used as benchmarks in the rest of the experiments. We consider both moderate (0.25) and large (0.5) changes in the autoregressive parameter in either direction (experiments 2–5). We also consider changes in the error variance. We allow σ to increase from 1.5 to 4.5 (experiment 6) and decrease from 1.5 to 0.5 (experiment 7). Finally, we study the effects of breaks in the constant term (experiments 8–9).

We assume that the revisions are either pure news ($\sigma_{v_i} \neq 0, \sigma_{\varepsilon_i} = 0$ for $i = 1, ..., l$) or pure noise ($\sigma_{v_i} = 0, \sigma_{\varepsilon_i} \neq 0$ for $i = 1, ..., l$). This allows us to analyze whether the properties of the revision process matters for the window selection problem. We set $l = 14$, so that we observe 14 different estimates of y_t before the true value, $\tilde{y}_t$, is observed. Following Clements and Galvão (2013), we assume that the first and the fifth revisions are non-zero mean. The means of these revisions are set to four and two percent of the mean of the first-release data, y_t^{t+1}, both before and after the break. Similarly, the standard deviation of the first revision is set to 40 percent of the standard deviation of the first-release data. The standard deviations of revisions 2–13 and 14 are set to 20 and 10 percent of the standard deviation of the first-release data, respectively. For convenience, the parameter values used in the Monte Carlo experiments are reported in Table 1.[4]

We examine the ability of various window selection methods to forecast both the first-release values (y_{t+1}^{t+2}) and the final values (y_{t+1}^{t+16}). Because we assume that revisions have non-zero mean, the final values differ systematically from the first-release values. As a consequence, the forecasting models in (5) and (6) produce unbiased forecasts for the first-release values, but biased forecasts for the final values. In order to produce unbiased forecasts for the final values, we use the bias correction method suggested by Clements and Galvão (2013). The

[4]Formulas for the means and standard deviations of the first-release and final data are presented in Appendix A. This appendix also presents the formulas for the means and standard deviations of data revisions when the revisions are either pure news or pure noise. Appendix B gives the means and standard deviations of the first-release and final data for each experiment.

Table 1: Simulation setup

True process						
Experiments	ρ_1	ρ_2	β_1	β_2	σ_1	σ_2
1: No break	1	1	0.5	0.5	1.5	1.5
2: Moderate break in β (increase)	1	1	0.5	0.75	1.5	1.5
3: Moderate break in β (decrease)	1	1	0.5	0.25	1.5	1.5
4: Large break in β (increase)	1	1	0.25	0.75	1.5	1.5
5: Large break in β (decrease)	1	1	0.75	0.25	1.5	1.5
6: Increase in post-break variance	1	1	0.5	0.5	1.5	4.5
7: Decrease in post-break variance	1	1	0.5	0.5	1.5	0.5
8: Break in mean (increase)	1	1.5	0.5	0.5	1.5	1.5
9: Break in mean (decrease)	1	0.5	0.5	0.5	1.5	1.5

News										
Experiments	μ_{v1_1}	μ_{v2_1}	μ_{v1_5}	μ_{v2_5}	σ_{v1_1}	σ_{v2_1}	$\sigma_{v1_{2,\ldots,13}}$	$\sigma_{v2_{2,\ldots,13}}$	$\sigma_{v1_{14}}$	$\sigma_{v2_{14}}$
1: No break	0.085	0.085	0.043	0.043	0.783	0.783	0.391	0.391	0.196	0.196
2: Moderate break in β (increase)	0.085	0.195	0.043	0.098	0.783	2.238	0.391	1.119	0.196	0.560
3: Moderate break in β (decrease)	0.085	0.054	0.043	0.027	0.783	0.634	0.391	0.317	0.196	0.158
4: Large break in β (increase)	0.054	0.195	0.027	0.098	0.634	2.238	0.317	1.119	0.158	0.560
5: Large break in β (decrease)	0.195	0.054	0.098	0.027	2.238	0.634	1.119	0.317	0.560	0.158
6: Increase in post-break variance	0.085	0.085	0.043	0.043	0.783	2.348	0.391	1.174	0.196	0.587
7: Decrease in post-break variance	0.085	0.085	0.043	0.043	0.783	0.261	0.391	0.130	0.196	0.065
8: Break in mean (increase)	0.085	0.128	0.043	0.064	0.783	0.783	0.391	0.391	0.196	0.196
9: Break in mean (decrease)	0.085	0.043	0.043	0.021	0.783	0.783	0.391	0.391	0.196	0.196

Noise										
Experiments	$\mu_{\varepsilon1_1}$	$\mu_{\varepsilon2_1}$	$\mu_{\varepsilon1_{2,\ldots,5}}$	$\mu_{\varepsilon2_{2,\ldots,5}}$	$\sigma_{\varepsilon1_1}$	$\sigma_{\varepsilon2_1}$	$\sigma_{\varepsilon1_{2,4,\ldots,14}}$	$\sigma_{\varepsilon2_{2,4,\ldots,14}}$	$\sigma_{\varepsilon1_{3,5,\ldots,13}}$	$\sigma_{\varepsilon2_{3,5,\ldots,13}}$
1: No break	0.113	0.113	0.038	0.038	0.728	0.728	0.188	0.188	0.325	0.325
2: Moderate break in β (increase)	0.113	0.226	0.038	0.075	0.728	0.953	0.188	0.246	0.325	0.426
3: Moderate break in β (decrease)	0.113	0.075	0.038	0.025	0.728	0.651	0.188	0.168	0.325	0.291
4: Large break in β (increase)	0.075	0.226	0.025	0.075	0.651	0.953	0.168	0.246	0.291	0.426
5: Large break in β (decrease)	0.226	0.075	0.075	0.025	0.953	0.651	0.246	0.168	0.426	0.291
6: Increase in post-break variance	0.113	0.113	0.038	0.038	0.728	2.183	0.188	0.564	0.325	0.976
7: Decrease in post-break variance	0.113	0.113	0.038	0.038	0.728	0.243	0.188	0.063	0.325	0.108
8: Break in mean (increase)	0.113	0.170	0.038	0.057	0.728	0.728	0.188	0.188	0.325	0.325
9: Break in mean (decrease)	0.113	0.057	0.038	0.019	0.728	0.728	0.188	0.188	0.325	0.325

bias correction is the sample estimate of the difference between the final value and the first-release value calculated using data up to the forecast origin. To be more specific, the forecast for the final value is computed using the formula $\hat{y}_{t+1}^{t+16} = \hat{y}_{t+1}^{t+2} + (t-14)^{-1} \sum_{i=1}^{t-14} (y_i^{i+15} - y_i^{i+1})$. An alternative approach, of course, would be to use the fully revised data as the left hand side variable in (5) and (6). As discussed in Clements and Galvão (2013), these two approaches are asymptotically equivalent. However, the bias correction method yields more accurate forecasts in small samples.

We focus on a set of widely used robust estimation strategies, including the rolling window, the exponentially weighted moving average (EWMA), and the average window (AveW) method. We analyze the forecasting performance of a short rolling window using the most recent 20 observations and a long rolling window using the most recent 40 observations. These rolling windows correspond to five and 10 years of quarterly data, respectively. The down-weighting parameter, λ, in the EWMA method is set to 0.05 (henceforth EWMAS). In addition, we follow Eklund *et al.* (2013) and consider a method that combines different down-weighting parameters. More specifically, we calculate an equally weighted forecast using down-weighting parameters of 0.1, 0.2 and 0.3 (henceforth EWMAA). We assume that the minimum estimation window length, $\underline{\omega}$, in the AveW method is 10 observations.

The expanding window estimator is the most efficient estimation method when the underlying time series process is stable over time. Therefore, it is used as a benchmark in our Monte Carlo simulations. For each robust estimation strategy we compute RMSFE values relative to those produced by the expanding window benchmark. Values below (above) unity indicate that the candidate method produces more (less) accurate forecasts than the benchmark. Relative RMSFE values are computed with sample sizes $T = 50$, 100, and 150. The results are based on 10,000 replications and are shown in Tables 2–5.

First, we compare the forecasting performance of alternative window selection methods when the revisions are pure news. The results, presented in Tables 2 and 3, reveal that the forecasting methods that generate the lowest RMSFE values in most of the experiments are the expanding window benchmark and the EWMAS method. Indeed, the expanding window estimator produces the most accurate forecasts in 52 of the 108 dependent variable/experiment/vintage approach/sample size combinations considered here. It performs particularly well when the parameters remain stable over time (experiment 1) or when the variance of the time series changes (experiments 6 and 7). There is a simple explanation for these findings. When the parameters remain fixed over time, it is optimal to use as many observations as possible in the estimation. Similarly, when a break only affects the volatility of the series, the variance of the parameter estimation error can be reduced by using a longer estimation window. The expanding window estimator performs poorly only when the autoregressive parameter is subject to large changes (experiments 4 and 5). Such breaks imply huge changes in the mean of the process, and are thus unlikely to occur in practice. Therefore, the weak performance of the expanding window estimator in the presence of large slope shifts should not be overemphasized. Interestingly, we find that it is more difficult to outperform the benchmark when the RTV approach is used. Similarly, the expanding window method performs better when the first-release values are the ones to be forecast.

Another prominent window selection method is the EWMAS approach. This approach performs well when the autoregressive parameter increases substantially after the break (experiment 4). In this case, the improvements over the expanding window benchmark are quite large, ranging from 0.6 to 8 percent. The EWMAS method also does particularly well in experiment 5 when we forecast first-release values, and in experiment 9 when we forecast the final values. Note that the EWMAS method improves upon the benchmark more often when the EOS approach is used.

In the few cases where the expanding window or EWMAS approach do not dominate, the EWMAA and AveW methods generate the best forecasts. The AveW method produces forecasts that are very close to those produced by the expanding window estimator. Therefore both the gains and losses in relative accuracy are more modest than with the other methods. The EWMAA method, on the other hand, performs well when the slope parameter decreases substantially after the break (experiment 5) and we forecast the final values, but extremely poorly in the vast majority of the experiments. The rolling windows fare no better: they rarely improve upon the benchmark and never produce the most accurate forecasts.

Our results indicate that the choice between the EOS and RTV approaches is not clear-

Table 2: Relative RMSFE values when revisions are news

							First-release						
			EOS							RTV			
Exp.	RMSFE	m = 20	m = 40	EWMAS	EWMAA	AveW	T = 50	RMSFE	m = 20	m = 40	EWMAS	EWMAA	AveW
1	1.743	1.042	1.007	1.007	1.078	1.011		1.732	1.036	1.006	1.010	1.096	1.011
2	3.906	1.016	1.001	0.999	1.059	1.002		3.925	1.030	1.004	1.014	1.126	1.010
3	1.673	1.029	1.005	0.999	1.054	1.001		1.646	1.023	1.003	1.003	1.071	1.001
4	4.159	0.984	0.990	0.971	1.010	0.980		4.160	1.004	0.995	0.992	1.087	0.992
5	2.208	1.061	1.014	0.954	0.954	0.998		2.114	1.043	1.008	0.968	0.982	0.997
6	5.213	1.046	1.009	1.034	1.174	1.022		5.248	1.053	1.010	1.044	1.213	1.027
7	0.654	1.171	1.038	0.997	1.030	1.035		0.621	1.137	1.028	1.007	1.063	1.033
8	1.789	1.033	1.006	0.998	1.056	1.005		1.794	1.025	1.004	1.000	1.070	1.004
9	1.822	1.027	1.005	0.993	1.044	1.002		1.794	1.025	1.004	0.999	1.068	1.004
							$T = 100$						
1	1.714	1.056	1.020	1.015	1.093	1.008		1.707	1.047	1.016	1.017	1.110	1.007
2	3.917	1.011	0.996	0.991	1.051	0.992		3.920	1.028	1.003	1.008	1.121	0.997
3	1.665	1.040	1.016	1.006	1.061	1.001		1.642	1.030	1.011	1.007	1.075	1.000
4	4.302	0.955	0.962	0.941	0.980	0.962		4.275	0.980	0.972	0.966	1.062	0.970
5	2.116	1.111	1.057	0.982	0.993	1.009		2.053	1.079	1.037	0.987	1.011	1.005
6	5.149	1.057	1.021	1.041	1.187	1.015		5.166	1.072	1.027	1.057	1.243	1.020
7	0.612	1.240	1.109	1.043	1.098	1.034		0.592	1.185	1.080	1.040	1.112	1.028
8	1.782	1.041	1.015	1.004	1.066	1.002		1.793	1.032	1.010	1.003	1.077	1.001
9	1.809	1.037	1.013	0.999	1.055	1.001		1.784	1.034	1.011	1.004	1.079	1.002
							$T = 150$						
1	1.711	1.061	1.025	1.020	1.099	1.006		1.707	1.052	1.021	1.020	1.113	1.005
2	3.944	1.008	0.993	0.988	1.050	0.988		3.939	1.026	1.000	1.007	1.120	0.992
3	1.661	1.042	1.018	1.008	1.066	1.000		1.641	1.031	1.012	1.009	1.078	0.999
4	4.414	0.933	0.940	0.920	0.956	0.952		4.383	0.959	0.951	0.945	1.042	0.957
5	2.085	1.122	1.071	0.994	1.004	1.009		2.030	1.085	1.048	0.995	1.019	1.005
6	5.128	1.061	1.024	1.044	1.188	1.011		5.138	1.076	1.032	1.061	1.242	1.015
7	0.598	1.268	1.136	1.067	1.122	1.030		0.583	1.197	1.095	1.053	1.127	1.021
8	1.766	1.046	1.019	1.008	1.072	1.002		1.780	1.036	1.013	1.006	1.082	1.001
9	1.811	1.040	1.017	1.001	1.056	1.001		1.788	1.035	1.014	1.005	1.079	1.001

Notes: The experiments are as defined in Table 1. Method m = 20 denotes rolling window of size 20, whereas m = 40 denotes rolling window of size 40. EWMAS denotes EWMA method with $\lambda = 0.05$ and EWMAA denotes EWMA method with $\lambda = 0.1$, 0.2 and 0.3. AveW denotes the average window method. The sample size is T. The break occurs at period $T_1 = T$. One-step ahead forecasts are generated recursively for periods $T+1,...,T+10$. The first column in each panel shows the RMSFE for the expanding window estimator. In subsequent columns, RMSFE values are computed relative to those produced by the expanding window estimator.

Table 3: Relative RMSFE values when revisions are news

							Final value						
			EOS							RTV			
Exp.	RMSFE	m = 20	m = 40	EWMAS	EWMAA	AveW	T = 50	RMSFE	m = 20	m = 40	EWMAS	EWMAA	AveW
1	2.383	1.018	1.003	1.000	1.035	1.003		2.366	1.017	1.003	1.004	1.049	1.005
2	5.991	1.006	1.000	0.998	1.024	1.000		6.001	1.012	1.001	1.005	1.054	1.004
3	2.146	1.012	1.002	0.995	1.023	0.997		2.114	1.010	1.001	1.000	1.038	0.999
4	6.155	0.990	0.995	0.984	0.999	0.989		6.151	0.999	0.997	0.994	1.035	0.995
5	2.895	1.004	1.002	0.945	0.909	0.979		2.777	1.000	1.000	0.958	0.937	0.983
6	7.044	1.025	1.005	1.018	1.099	1.012		7.066	1.029	1.006	1.025	1.122	1.015
7	0.932	1.059	1.013	0.978	0.967	0.999		0.881	1.057	1.012	0.993	1.007	1.008
8	2.428	1.012	1.002	0.994	1.021	0.999		2.424	1.011	1.002	0.997	1.032	1.000
9	2.460	1.010	1.001	0.992	1.015	0.998		2.428	1.010	1.001	0.997	1.031	1.000
							$T = 100$						
1	2.345	1.027	1.010	1.006	1.048	1.003		2.335	1.024	1.009	1.008	1.059	1.003
2	6.008	1.002	0.997	0.994	1.018	0.996		6.008	1.010	1.000	1.001	1.049	0.998
3	2.129	1.021	1.009	1.001	1.032	0.999		2.103	1.017	1.006	1.003	1.043	0.999
4	6.277	0.977	0.981	0.971	0.986	0.981		6.256	0.989	0.986	0.983	1.026	0.985
5	2.688	1.051	1.030	0.970	0.952	0.999		2.613	1.035	1.019	0.976	0.970	0.998
6	7.012	1.031	1.011	1.022	1.105	1.008		7.023	1.040	1.015	1.031	1.138	1.011
7	0.852	1.111	1.050	1.008	1.023	1.010		0.823	1.090	1.039	1.014	1.044	1.011
8	2.398	1.020	1.007	0.999	1.031	1.000		2.403	1.017	1.005	1.000	1.039	1.000
9	2.431	1.017	1.006	0.996	1.024	0.999		2.405	1.017	1.006	1.000	1.040	1.000
							$T = 150$						
1	2.340	1.031	1.013	1.009	1.052	1.003		2.333	1.027	1.010	1.010	1.062	1.003
2	6.026	1.000	0.996	0.993	1.018	0.994		6.022	1.009	0.999	1.001	1.050	0.996
3	2.113	1.023	1.010	1.003	1.036	0.999		2.092	1.018	1.007	1.004	1.045	0.999
4	6.371	0.963	0.968	0.957	0.971	0.975		6.348	0.975	0.973	0.968	1.011	0.978
5	2.623	1.064	1.041	0.980	0.966	1.002		2.559	1.042	1.026	0.983	0.980	1.000
6	6.957	1.033	1.013	1.024	1.105	1.006		6.964	1.042	1.017	1.033	1.138	1.008
7	0.827	1.135	1.069	1.026	1.045	1.012		0.806	1.102	1.049	1.023	1.058	1.010
8	2.393	1.022	1.009	1.001	1.033	1.000		2.401	1.017	1.006	1.001	1.041	1.000
9	2.430	1.019	1.008	0.997	1.025	0.999		2.408	1.017	1.007	1.000	1.038	1.000

See the notes to Table 2.

cut: the RTV approach yields more accurate forecasts in experiments 1, 3, 5, 7, and 9, whereas the EOS approach yields more accurate forecasts in experiment 6. The evidence for experiments 2, 4, and 8 is mixed. Hence, the EOS approach can be recommended only when the volatility of the series increases after a break. Another point worth noticing is that the sample size also matters for forecasting accuracy. We find that the selection of the estimation window becomes more important when the sample size increases.

The results for noise revisions are reported in Tables 4 and 5. These results are qualitatively similar to those presented in Tables 2 and 3, suggesting that the news versus noise issue does not matter much for the relative ranking of the alternative window selection methods. If anything, the view that emerges from Tables 2–5 is that the expanding window estimator performs slightly better when the revisions reduce noise. In such cases, it produces the best forecasts in 55 of the 108 cases. The EWMAS method also performs quite well when the revisions reduce noise. However, the evidence for its predictive ability is not as convincing as it is when the revisions are news. The results for noise revisions imply that it is more difficult to improve upon the benchmark when the EOS approach is used. This is a surprising result because the opposite was the case when the revisions were news. Again, the expanding window estimator performs better when the first-release values are the ones to be forecast.

When the revisions reduce noise, the ranking between the EOS and RTV approaches is very different. The EOS approach produces more reliable forecasts in experiments 2, 4, 5, and 8, whereas the RTV approach produces more accurate forecasts in experiments 3, 6, and 9. The evidence for experiments 1 and 7 is mixed. Once again, the choice of the estimation window matters more when the sample size is large. The results reported in Tables 2–5 reveal that the differences in the forecasting accuracy are larger when the revisions reduce noise. This result suggests that the choice of correct estimation window is more important when the revisions reduce noise.

To sum up, our results are consistent with the view that the news versus noise issue does not matter much for the relative ranking of alternative window selection methods. We find that the expanding window estimator often produces the best forecasts after a recent break—regardless of whether the revisions add news or reduce noise. However, the news versus noise

Table 4: Relative RMSFE values when revisions are noise

							First-release						
			EOS							RTV			
Exp.	RMSFE	m = 20	m = 40	EWMAS	EWMAA	AveW	T = 50	RMSFE	m = 20	m = 40	EWMAS	EWMAA	AveW
1	1.733	1.031	1.006	1.012	1.102	1.009		1.734	1.036	1.007	1.009	1.091	1.010
2	2.023	1.006	0.998	0.988	1.049	0.993		2.101	0.998	0.996	0.972	1.013	0.986
3	1.769	1.017	1.003	1.005	1.080	1.000		1.746	1.024	1.004	1.005	1.080	1.003
4	2.260	0.954	0.982	0.936	0.953	0.957		2.368	0.944	0.980	0.925	0.922	0.952
5	2.038	1.010	1.001	0.999	1.047	0.995		2.063	1.004	0.999	0.980	1.007	0.988
6	5.297	1.056	1.011	1.050	1.246	1.029		5.263	1.054	1.010	1.045	1.224	1.027
7	0.615	1.108	1.025	1.007	1.056	1.026		0.624	1.130	1.031	1.004	1.048	1.030
8	1.780	1.025	1.004	1.007	1.085	1.005		1.810	1.025	1.003	0.999	1.067	1.003
9	1.797	1.023	1.004	1.004	1.081	1.004		1.797	1.025	1.005	0.999	1.068	1.004
							$T = 100$						
1	1.717	1.039	1.013	1.017	1.110	1.005		1.714	1.046	1.016	1.015	1.102	1.006
2	2.027	1.004	0.995	0.983	1.045	0.988		2.112	0.993	0.990	0.964	1.007	0.982
3	1.755	1.022	1.007	1.007	1.085	0.999		1.727	1.033	1.011	1.009	1.088	1.001
4	2.352	0.919	0.945	0.902	0.919	0.945		2.464	0.911	0.943	0.892	0.888	0.944
5	2.040	1.017	1.007	1.003	1.050	0.997		2.058	1.014	1.008	0.987	1.017	0.993
6	5.191	1.072	1.027	1.060	1.264	1.019		5.164	1.070	1.026	1.055	1.243	1.019
7	0.594	1.149	1.059	1.031	1.091	1.020		0.597	1.181	1.073	1.033	1.091	1.023
8	1.774	1.024	1.005	1.006	1.084	1.005		1.802	1.023	1.005	0.998	1.065	1.003
9	1.798	1.028	1.009	1.006	1.084	1.001		1.797	1.031	1.011	1.002	1.073	1.001
							$T = 150$						
1	1.723	1.042	1.016	1.020	1.117	1.004		1.720	1.050	1.019	1.019	1.110	1.005
2	2.031	0.999	0.991	0.978	1.042	0.986		2.121	0.986	0.983	0.957	0.999	0.981
3	1.750	1.022	1.007	1.008	1.084	0.998		1.721	1.032	1.012	1.010	1.086	0.999
4	2.411	0.894	0.921	0.878	0.891	0.941		2.526	0.887	0.919	0.869	0.862	0.941
5	2.030	1.018	1.010	1.006	1.053	0.998		2.046	1.016	1.012	0.990	1.019	0.996
6	5.172	1.078	1.033	1.066	1.271	1.015		5.145	1.075	1.032	1.060	1.250	1.014
7	0.584	1.157	1.071	1.041	1.103	1.014		0.586	1.194	1.092	1.049	1.109	1.019
8	1.767	1.031	1.011	1.010	1.092	1.000		1.795	1.032	1.012	1.003	1.073	0.999
9	1.788	1.029	1.011	1.008	1.087	1.000		1.786	1.034	1.012	1.004	1.075	0.999

See the notes to Table 2.

Table 5: Relative RMSFE values when revisions are noise

							Final value						
			EOS							RTV			
Exp.	RMSFE	m = 20	m = 40	EWMAS	EWMAA	AveW	T = 50	RMSFE	m = 20	m = 40	EWMAS	EWMAA	AveW
1	1.576	1.037	1.007	1.014	1.121	1.010		1.574	1.044	1.009	1.011	1.110	1.013
2	1.829	1.003	0.997	0.980	1.047	0.988		1.922	0.994	0.994	0.961	1.003	0.981
3	1.656	1.019	1.003	1.005	1.089	1.000		1.628	1.028	1.005	1.006	1.091	1.004
4	2.130	0.937	0.977	0.917	0.923	0.945		2.251	0.929	0.976	0.907	0.891	0.942
5	1.998	1.007	1.000	0.993	1.033	0.993		2.037	1.000	0.998	0.973	0.989	0.985
6	4.823	1.067	1.013	1.060	1.291	1.035		4.785	1.065	1.012	1.054	1.266	1.033
7	0.576	1.122	1.028	1.008	1.064	1.029		0.576	1.160	1.038	1.011	1.071	1.041
8	1.637	1.028	1.004	1.006	1.095	1.005		1.670	1.029	1.004	0.997	1.075	1.003
9	1.665	1.025	1.004	1.003	1.088	1.004		1.663	1.028	1.005	0.997	1.074	1.004
							$T = 100$						
1	1.560	1.048	1.017	1.021	1.133	1.007		1.555	1.057	1.021	1.019	1.125	1.008
2	1.834	0.997	0.990	0.971	1.039	0.982		1.935	0.984	0.984	0.950	0.994	0.976
3	1.640	1.024	1.008	1.007	1.094	0.998		1.608	1.037	1.013	1.010	1.099	1.001
4	2.238	0.895	0.931	0.876	0.882	0.933		2.363	0.890	0.932	0.870	0.852	0.934
5	1.995	1.014	1.007	0.997	1.037	0.996		2.028	1.009	1.007	0.979	0.999	0.992
6	4.704	1.087	1.032	1.073	1.313	1.024		4.675	1.085	1.032	1.067	1.290	1.023
7	0.547	1.174	1.070	1.036	1.107	1.023		0.545	1.220	1.090	1.044	1.117	1.030
8	1.635	1.027	1.005	1.006	1.095	1.005		1.667	1.027	1.005	0.997	1.073	1.003
9	1.665	1.030	1.010	1.005	1.092	1.000		1.663	1.035	1.013	1.001	1.080	1.000
							$T = 150$						
1	1.563	1.050	1.018	1.023	1.139	1.004		1.557	1.060	1.023	1.022	1.133	1.005
2	1.843	0.991	0.985	0.965	1.035	0.980		1.950	0.976	0.977	0.942	0.983	0.975
3	1.636	1.024	1.008	1.008	1.093	0.997		1.603	1.037	1.014	1.011	1.096	0.999
4	2.308	0.866	0.903	0.849	0.848	0.930		2.437	0.862	0.905	0.843	0.822	0.932
5	1.986	1.015	1.010	0.999	1.039	0.998		2.018	1.011	1.011	0.981	0.999	0.995
6	4.685	1.094	1.040	1.080	1.323	1.018		4.656	1.091	1.038	1.073	1.299	1.017
7	0.536	1.185	1.084	1.049	1.121	1.017		0.534	1.233	1.111	1.061	1.135	1.023
8	1.626	1.034	1.012	1.010	1.102	1.000		1.659	1.036	1.013	1.001	1.080	0.998
9	1.651	1.032	1.012	1.006	1.094	0.999		1.649	1.037	1.014	1.002	1.082	0.999

See the notes to Table 2.

issue matters for the relative accuracy of the EOS and RTV approaches. In general, our results suggest that the RTV approach yields more accurate forecasts when the revisions add news, whereas the EOS approach generates more reliable forecasts when the revisions reduce noise. This result is consistent with the findings in Clements and Galvão (2013).

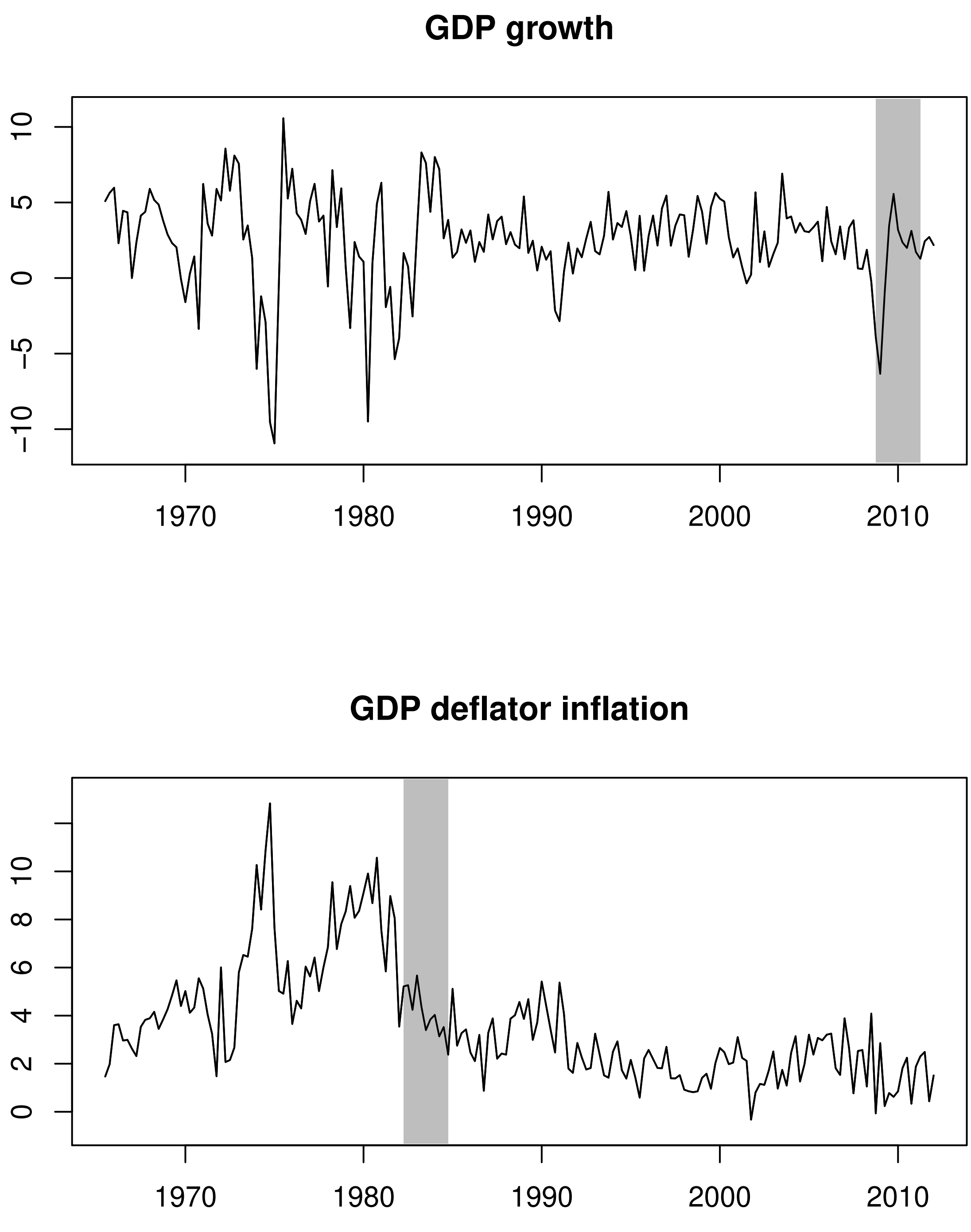
5. Empirical application

In this section, we compare the forecasting performance of the alternative window selection methods discussed above using actual U.S. data. We consider one-step ahead forecasts of real GDP and GDP deflator inflation (at an annualized rate). All forecasts are out-of-sample. In other words, at each forecast origin $t+1$, the $t+1$ vintage estimates of data up to period t are used to estimate the parameters of a forecasting model that is then used to generate a forecast for period $t+1$. All real-time data is quarterly and the sample period runs from 1965:Q4 to 2012:Q2. Different vintages of real GDP and GDP deflator series are obtained from the Federal Reserve Bank of Philadelphia's real-time database.

The goal of our application is to compare the different forecasting performances in the presence of a recent break. As discussed in the Introduction, structural break tests provide inaccurate estimates of the timing of the break(s). Therefore, a problem that arises in this analysis is how to select the relevant forecasting periods. To this end we consider the following strategy. In Figure 1, we plot the first-release quarterly growth rates of real GDP and GDP deflator over the 1965:Q4–2012:Q2 period. A time period is considered as a starting point of a forecasting period if the latest available observation differs considerably from the earlier ones. Our approach suggests that 2008:Q3 is a potential break point in the dynamics of the GDP growth. As a result, the GDP forecasts are made for the period 2008:Q4–2011:Q1. On the other hand, we find that the behavior of the inflation series changed after 1982:Q1 and hence the GDP deflator inflation forecasts are made for the period 1982:Q2–1984:Q3.

The performance of the various window selection methods compared to the expanding window benchmark is summarized in Table 6. Panel A shows the results for the real GDP forecasts, whereas Panel B has the inflation forecasts. The first row in both Panels provides

Figure 1: First-release growth rates



Notes: The figure depicts the quarterly growth rates of real GDP and GDP deflator (annualized) over the 1965:Q4–2012:Q2 period. The shaded areas denote out-of-sample forecasting periods.

Table 6: Out-of-sample relative RMSFE values

A. GDP growth

	First-release		Final value	
	EOS	RTV	EOS	RTV
Expanding window	3.219	2.776	4.419	3.967
m = 20	1.081	1.210	1.031	1.128
m = 40	1.047	1.120	1.003	1.077
EWMAS	1.033	1.166	1.002	1.103
EWMAA	1.052	1.273	1.019	1.157
AveW	0.998	1.084	0.991	1.056

B. GDP deflator inflation

	First-release		Final value	
	EOS	RTV	EOS	RTV
Expanding window	1.003	0.961	1.559	1.484
m = 20	1.529	1.502	1.460	1.488
m = 40	1.347	1.328	1.293	1.315
EWMAS	1.104	1.115	1.123	1.149
EWMAA	1.426	1.075	1.280	1.143
AveW	1.217	1.202	1.212	1.227

Notes: Forecasting periods for real GDP growth and GDP deflator inflation are 2008:Q4–2011:Q1 and 1982:Q2–1984:Q3, respectively. The first row in each panel shows the root mean squared forecast error for the expanding window estimator. Subsequent rows show the ratio of the RMSFE of a candidate window selection method to the RMSFE of the benchmark expanding window estimator. Forecasts of final values are bias corrected first-release forecasts. The correction is based on the sample mean of the difference between the final values, y_t^{t+15}, and the first-release values, y_t^{t+1}, calculated with data up to the forecast origin. We use y_{t+1}^{t+16} as true values for GDP deflator inflation and the vintage 2012:Q2 values as true values for real GDP growth.

the RMSFE value of the benchmark expanding window estimator. The subsequent rows show the RMSFE of a candidate window selection method relative to the RMSFE of the benchmark. Forecasts of final values are bias corrected first-release forecasts. The correction is based on the sample mean of the difference between the final values, y_t^{t+15}, and the first-release values, y_t^{t+1}, calculated with data up to the forecast origin. We use y_{t+1}^{t+16} as true values for inflation and the vintage 2012:Q2 values as true values for real GDP growth. To ensure that our empirical results are comparable to our Monte Carlo results, we consider an AR(1) specification.[5]

The AveW and the expanding window estimator produce the most accurate real GDP

[5]We also considered AR(2) and AR(4) models. The results for these specifications are qualitatively similar to those presented in Table 6.

forecasts. When we use the EOS approach, the AveW method does marginally better than the expanding window estimator. By contrast, the expanding window estimator turns out to be the best method when the RTV approach is used. For the GDP deflator inflation, the expanding window estimator is overwhelmingly the best estimation window method. It produces the most accurate forecasts in each of the four dependent variable/vintage approach combinations considered here. The differences in the forecasting abilities are very large. The relative RMSFE values range between 1.075 and 1.529, indicating that the alternative window selection methods produce 7.5-52.9 percent larger forecast errors than the expanding window benchmark.

Our simulation results are useful in explaining why it is difficult to outperform the expanding window estimator after a recent break. For example, if the break only affects the innovation variance, σ^2, our simulation results indicate that the expanding window estimator produces the most accurate forecasts. The two breaks considered here most likely caused changes in the innovation variance. In particular, the results in the literature indicate that the variance of the inflation series has reduced substantially since the early 1980s. This would explain why none of the alternative methods systematically improve upon the expanding window benchmark. Another reason for the good performance of the expanding window estimator lies in the fact that the means of the series have declined after the breaks (at least temporarily). The simulation results show that when the mean declines after the break, the expanding window estimator performs well relative to the alternatives (see experiments 3 and 9). Note also that the differences in the relative predictive abilities are larger for the GDP deflator inflation. As discussed in Section 2, revisions to the GDP deflator inflation are mainly noise, whereas those to the GDP are mainly news (see, e.g., Clements and Galvão, 2013). Thus, our results suggest that the differences in the relative predictive abilities are larger when the revisions reduce noise. In addition, our results indicate that, in general, the expanding window method performs better when the first-release values are the ones to be forecast. These two findings are consistent with our simulation results in Tables 2–5.

The rolling window methods and the EWMAA method perform poorly in our empirical applications. In particular, a short rolling window typically produces forecasts that are substantially worse than those produced by the expanding window benchmark. These empirical findings are in line with our Monte Carlo simulations. Indeed, our simulation results suggest that these methods rarely outperform the expanding window estimator.

The results in Table 6 also indicate that one key determinant of the forecasting performance is the choice of how to use the real-time data to estimate the parameters of the forecasting model. A substantial amount of the literature on real-time forecasting uses the EOS approach. In our empirical examples, the RTV approach produces more accurate forecasts after a recent break regardless of whether we consider forecasting the real GDP or the GDP deflator inflation. We find that the RTV approach yields improvements of 4.8%–13.8% over the EOS approach.

6. Conclusions

This paper analyzes the forecasting performance of various window selection methods after a recent break when the data are subject to revision. Several practical recommendations for choosing the estimation window emerge from our analysis. First, our Monte Carlo and empirical results suggest that the expanding window method usually provides the most accurate forecasts after a recent break. It performs well regardless of whether the revisions add news or reduce noise, or whether we forecast the first-release or the final values. Thus, the evidence in favor of the expanding window estimator seems well established. Second, we find that rolling windows perform the worst of all the methods. They never produce the most accurate forecasts in any of the cases considered here. Furthermore, they rarely improve upon the expanding window estimator. This is an important result because rolling windows are used extensively in the literature. In short, our results suggest that the use of rolling windows should be rethought, at least when making forecasts after a recent break. Third, our results imply that whether the revisions add news or reduce noise does not matter much for the relative ranking of the alternative window selection methods. Finally, no clear ranking between the EOS and RTV vintage approaches emerges. In general, our Monte Carlo results suggest that the RTV approach produces more accurate forecasts when the revisions add news, whereas the EOS approach yields more reliable forecasts when the revisions reduce noise. The RTV approach performs particularly well in our empirical examples.

Our results could be extended in several ways. We have considered only cases where the autoregressive process has been subject to a single, recent break. In practice, however, autoregressive processes are likely to be subject to multiple breaks. Therefore, analyzing the forecasting performance in the presence of multiple breaks might be a fruitful area for future research. In addition, our statistical framework neglects some important features of the actual data revision process, including time variations in the revision mean and variance. Incorporating these features into the statistical framework may lead to a better understanding of the relative forecasting accuracy of alternative window selection methods in the presence of data revisions.

Appendix A

In this Appendix, we derive formulas for the means and variances of the first-release data, y_t^{t+1}, and final data, $\tilde{y}_t$. Recall that $\tilde{y}_t = \rho + \sum_{i=1}^{l} \mu_{v_i} + \beta\tilde{y}_{t-1} + \sigma\eta_{1t} + \sum_{i=1}^{l} \sigma_{v_i}\eta_{2t,i}$ and $y_t^{t+1} = \tilde{y}_t - \sum_{i=1}^{l} \mu_{v_i} - \sum_{i=1}^{l} \sigma_{v_i}\eta_{2t,i} - \mu_{\varepsilon_1} + \sigma_{\varepsilon_1}\eta_{3t,1}$. Both y_t^{t+1} and $\tilde{y}_t$ are (covariance) stationary processes. We set $l = 14$, so that we observe 14 different estimates of y_t before the true value, $\tilde{y}_t$, is observed. The expected value of $\tilde{y}_t$ is

$$E(\tilde{y}_t) = \mu_{\tilde{y}} = \frac{\rho + \sum_{i=1}^{l} \mu_{v_i}}{1-\beta}.$$

Therefore, the expected value of y_t^{t+1} is

$$\begin{aligned} E(y_t^{t+1}) &= E(\tilde{y}_t) - \sum_{i=1}^{l} \mu_{v_i} - \mu_{\varepsilon_1} \\ &= \frac{\rho + \beta \sum_{i=1}^{l} \mu_{v_i}}{1-\beta} - \mu_{\varepsilon_1}. \end{aligned}$$

If the revisions are pure news, the expected values of the first-release and final data are

$$E(\tilde{y}_t) = \frac{\rho + \sum_{i=1}^{l} \mu_{v_i}}{1-\beta} \quad \text{and} \quad E(y_t^{t+1}) = \frac{\rho + \beta \sum_{i=1}^{l} \mu_{v_i}}{1-\beta}.$$

If the revisions are pure noise, the expected values of the first-release and final data are

$$E(\tilde{y}_t) = \frac{\rho}{1-\beta} \quad \text{and} \quad E(y_t^{t+1}) = \frac{\rho}{1-\beta} - \mu_{\varepsilon_1}.$$

The revisions are defined by $r_t^i = y_t^{t+1+i} - y_t^{t+i}$, for $i = 1,...,l$. For example, the first revision at time t is equal to $r_t^1 = y_t^{t+2} - y_t^{t+1}$, i.e., the difference between the second-release value and the first-release value. Equations (1), (2), and (3) imply that $y_t^{t+1} = \rho + \beta\tilde{y}_{t-1} + \sigma\eta_{1t} - \mu_{\varepsilon_1} + \sigma_{\varepsilon_1}\eta_{3t,1}$ and $y_t^{t+2} = \rho + \mu_{v_1} + \beta\tilde{y}_{t-1} + \sigma\eta_{1t} + \sigma_{v_1}\eta_{2t,1} - \mu_{\varepsilon_2} + \sigma_{\varepsilon_2}\eta_{3t,2}$.

Hence,

$$r_t^1 = y_t^{t+2} - y_t^{t+1} = \mu_{v_1} + \sigma_{v_1}\eta_{2t,1} - \mu_{\varepsilon_2} + \sigma_{\varepsilon_2}\eta_{3t,2} + \mu_{\varepsilon_1} - \sigma_{\varepsilon_1}\eta_{3t,1}.$$

Following Clements and Galvão (2013), we assume that the first and the fifth revisions have non-zero mean. To be more specific, we assume that the means of the first and fifth revisions are, respectively, δ and $\delta/2$ times the mean of the first-release data. In what follows, we set $\delta = 0.04$. Our assumptions imply that for news revisions,

$$\begin{aligned}
E(r_t^1) &= \mu_{v_1} \\
E(r_t^2) &= \mu_{v_2} \\
&\vdots \\
E(r_t^{14}) &= \mu_{v_{14}},
\end{aligned}$$

so that $\mu_{v_2} = \mu_{v_3} = \mu_{v_4} = \mu_{v_6} = ... = \mu_{v_{14}} = 0$, $E(r_t^1) = \mu_{v_1}$ and $E(r_t^5) = \mu_{v_5}$. Setting $E(r_t^1) = \delta E(y_t^{t+1})$ yields

$$\mu_{v_1} = \delta \frac{\rho + \beta \sum_{i=1}^{l} \mu_{v_i}}{1 - \beta}.$$

Using the fact that $E(r_t^1) = 2E(r_t^5)$, i.e., $\mu_{v_1} = 2\mu_{v_5}$, we can express μ_{v_1} and μ_{v_5} as

$$\mu_{v_1} = \frac{\delta\rho}{1 - (1 + 1.5\delta)\beta} \quad \text{and} \quad \mu_{v_5} = \frac{\mu_{v_1}}{2}.$$

The situation is more complicated if the revisions are pure noise. The structure of the DGP and our assumptions imply that

$$
\begin{aligned}
E(r_t^1) &= -\mu_{\varepsilon_2} + \mu_{\varepsilon_1} = \delta E(y_t^{t+1}) \\
E(r_t^2) &= -\mu_{\varepsilon_3} + \mu_{\varepsilon_2} = 0 \\
E(r_t^3) &= -\mu_{\varepsilon_4} + \mu_{\varepsilon_3} = 0 \\
E(r_t^4) &= -\mu_{\varepsilon_5} + \mu_{\varepsilon_4} = 0 \\
E(r_t^5) &= -\mu_{\varepsilon_6} + \mu_{\varepsilon_5} = \frac{\delta}{2} E(y_t^{t+1}) \\
E(r_t^6) &= -\mu_{\varepsilon_7} + \mu_{\varepsilon_6} = 0 \\
&\vdots \\
E(r_t^{13}) &= -\mu_{\varepsilon_{14}} + \mu_{\varepsilon_{13}} = 0 \\
E(r_t^{14}) &= \mu_{\varepsilon_{14}} = 0.
\end{aligned}
$$

Revisions 6–14 have zero mean, which implies that $\mu_{\varepsilon_6} = \mu_{\varepsilon_7} = ... = \mu_{\varepsilon_{13}} = \mu_{\varepsilon_{14}} = 0$. Because $\mu_{\varepsilon_6} = 0$, μ_{ε_5} equals $\frac{\delta}{2} E(y_t^{t+1})$. This finding implies that $\mu_{\varepsilon_2} = \mu_{\varepsilon_3} = \mu_{\varepsilon_4} = \mu_{\varepsilon_5} = \frac{\delta}{2} E(y_t^{t+1})$. Finally, we find that $\mu_{\varepsilon_1} = \frac{3\delta}{2} E(y_t^{t+1})$. So, if the revisions are pure noise,

$$
\mu_{\varepsilon_1} = \frac{1.5}{(1+1.5\delta)} \frac{\delta\rho}{(1-\beta)}, \quad \mu_{\varepsilon_2} = ... = \mu_{\varepsilon_5} = \frac{\delta}{2(1+1.5\delta)} \frac{\rho}{(1-\beta)}, \quad \mu_{\varepsilon_6} = ... = \mu_{\varepsilon_{14}} = 0.
$$

Next, we derive the variance of $\tilde{y}_t$. The true values can be expressed as follows

$$
(\tilde{y}_t - \mu_{\tilde{y}}) = \beta(\tilde{y}_{t-1} - \mu_{\tilde{y}}) + \sigma\eta_{1,t} + \sum_{i=1}^{l} \sigma_{v_i} \eta_{2t,i}, \tag{7}
$$

where $\mu_{\tilde{y}}$ denotes the expected value of $\tilde{y}_t$. The variance of $\tilde{y}$ can be found by multiplying (7) by $(\tilde{y}_t - \mu_{\tilde{y}})$ and taking expectations:

$$
E(\tilde{y}_t - \mu_{\tilde{y}})^2 = \beta E\left[(\tilde{y}_t - \mu_{\tilde{y}})(\tilde{y}_{t-1} - \mu_{\tilde{y}})\right] + E\left[(\tilde{y}_t - \mu_{\tilde{y}})\sigma\eta_{1t}\right] + E\left[(\tilde{y}_t - \mu_{\tilde{y}}) \sum_{i=1}^{l} \sigma_{v_i} \eta_{2t,i}\right]. \tag{8}
$$

Note that

$$E\left[(\tilde{y}_t - \mu_{\tilde{y}})\sigma\eta_{1t}\right] = \sigma^2 E(\eta_{1t}^2) = \sigma^2 \qquad \text{and}$$

$$E\left[(\tilde{y}_t - \mu_{\tilde{y}})\sum_{i=1}^{l}\sigma_{v_i}\eta_{2t,i}\right] = \sum_{i=1}^{l}\sigma_{v_i}^2 E(\eta_{2t,i}^2) = \sum_{i=1}^{l}\sigma_{v_i}^2.$$

Thus, (8) can be rewritten as

$$\gamma_0 = \beta\phi_1\gamma_0 + \sigma^2 + \sum_{i=1}^{l}\sigma_{v_i}^2, \tag{9}$$

where γ_0 denotes the variance and ϕ_1 the first autocorrelation coefficient. Using the fact that for an AR(1) process, $\phi_1 = \beta$, we have

$$\gamma_0 = \frac{\sigma^2 + \sum_{i=1}^{l}\sigma_{v_i}^2}{1 - \beta^2}.$$

The variance of y_t^{t+1} can be derived as follows

$$var(y_t^{t+1}) = var(\tilde{y}_t - \sum_{i=1}^{l}\mu_{v_i} - \sum_{i=1}^{l}\sigma_{v_i}\eta_{2t,i} - \mu_{\varepsilon_1} + \sigma_{\varepsilon_1}\eta_{3t,1})$$

$$var(y_t^{t+1}) = var(\tilde{y}_t) + \sum_{i=1}^{l}\sigma_{v_i}^2 var(\eta_{2t,i}) + \sigma_{\varepsilon_1}^2 var(\eta_{3t,1}) - 2\sum_{i=1}^{l}\sigma_{v_i} cov(\tilde{y}_t, \eta_{2t,i})$$

$$+2\sigma_{\varepsilon_1} cov(\tilde{y}_t, \eta_{3t,1}) - 2\sum_{i=1}^{l}\sigma_{v_i}\sigma_{\varepsilon_1} cov(\eta_{2t,i}, \eta_{3t,1}).$$

Because $cov(\tilde{y}_t, \eta_{2t,i}) = \sum_{i=1}^{l}\sigma_{v_i}$, $cov(\tilde{y}_t, \eta_{3t,1}) = 0$, and $cov(\eta_{2t,i}, \eta_{3t,1}) = 0$, we have

$$var(y_t^{t+1}) = var(\tilde{y}_t) + \sum_{i=1}^{l}\sigma_{v_i}^2 + \sigma_{\varepsilon_1}^2 - 2\sum_{i=1}^{l}\sigma_{v_i}^2$$

$$= \frac{\sigma^2 + \sum_{i=1}^{l} \sigma_{v_i}^2}{1-\beta^2} - \sum_{i=1}^{l} \sigma_{v_i}^2 + \sigma_{\varepsilon_1}^2$$

$$= \frac{\sigma^2 + \beta^2 \sum_{i=1}^{l} \sigma_{v_i}^2}{1-\beta^2} + \sigma_{\varepsilon_1}^2.$$

Therefore, when the revisions are pure news, we have

$$\sigma_{y_t^{t+1}}^2 = \frac{\sigma^2 + \beta^2 \sum_{i=1}^{l} \sigma_{v_i}^2}{1-\beta^2}.$$

When the revisions are pure noise, the variance is

$$\sigma_{y_t^{t+1}}^2 = \frac{\sigma^2}{1-\beta^2} + \sigma_{\varepsilon_1}^2.$$

Next, we derive the variances of the data revisions. Let $\sigma_{r_i}^2$ (for i = 1,...,l) denote the variance of the ith revision. The variance of the first revision is

$$var(r_t^1) = var(y_t^{t+2} - y_t^{t+1}) = var(\mu_{v_1} + \sigma_{v_1}\eta_{2t,1} - \mu_{\varepsilon_2} + \sigma_{\varepsilon_2}\eta_{3t,2} + \mu_{\varepsilon_1} - \sigma_{\varepsilon_1}\eta_{3t,1}).$$

If the revisions are pure news, $var(r_t^1) = \sigma_{r_1}^2 = var(\mu_{v_1} + \sigma_{v_1}\eta_{2t,1}) = \sigma_{v_1}^2 var(\eta_{2t,1}) = \sigma_{v_1}^2$. If the revisions are pure noise, $var(r_t^1) = \sigma_{r_1}^2 = var(-\mu_{\varepsilon_2} + \sigma_{\varepsilon_2}\eta_{3t,2} + \mu_{\varepsilon_1} - \sigma_{\varepsilon_1}\eta_{3t,1}) = \sigma_{\varepsilon_2}^2 var(\eta_{3t,2}) + \sigma_{\varepsilon_1}^2 var(\eta_{3t,1}) = \sigma_{\varepsilon_2}^2 + \sigma_{\varepsilon_1}^2$.

We set $\sigma_{r_1} = \alpha\sigma_{y_t^{t+1}}$, where α denotes the ratio of the standard deviation of the first revision to the standard deviation of the first-release data. Furthermore, we assume that $\sigma_{r_2,\ldots,r_{13}} = \frac{\alpha}{2}\sigma_{y_t^{t+1}}$ and that $\sigma_{r_{14}} = \frac{\alpha}{4}\sigma_{y_t^{t+1}}$. In what follows, we set $\alpha = 0.4$. Thus, the variance of the first revision, when the revisions are pure news, can be found by solving the equation

$$\sigma_{v_1}^2 = \alpha^2 \frac{\sigma^2 + \beta^2 \sum_{i=1}^{l} \sigma_{v_i}^2}{1-\beta^2}.$$

Note that $1/4\sigma^2_{v_1} = \sigma^2_{v_2} = ... = \sigma^2_{v_{13}}$ and $1/16\sigma^2_{v_1} = \sigma^2_{v_{14}}$. This implies that $\sum_{i=1}^{l}\sigma^2_{v_i} = 4.0625\sigma^2_{v_1}$. Using this fact we can express the variance of the first revision as

$$\sigma^2_{v_1} = \alpha^2 \frac{\sigma^2 + 4.0625\beta^2\sigma^2_{v_1}}{1-\beta^2}.$$

After some algebra, we find that

$$\sigma^2_{v_1} = \frac{\alpha^2\sigma^2}{1-(1+4.0625\alpha^2)\beta^2}.$$

So, the formulas for the standard deviations are

$$\sigma_{v_1} = \sqrt{\frac{\alpha^2\sigma^2}{1-(1+4.0625\alpha^2)\beta^2}}, \quad \sigma_{v_2} = ... = \sigma_{v_{13}} = \sigma_{v_1}/2, \quad \sigma_{v_{14}} = \sigma_{v_1}/4.$$

Next, we consider noise revisions. We have

$$\begin{aligned}
\sigma^2_{r_1} &= \sigma^2_{\varepsilon_2} + \sigma^2_{\varepsilon_1} \\
\sigma^2_{r_2} &= \sigma^2_{\varepsilon_3} + \sigma^2_{\varepsilon_2} \\
&\vdots \\
\sigma^2_{r_{13}} &= \sigma^2_{\varepsilon_{14}} + \sigma^2_{\varepsilon_{13}} \\
\sigma^2_{r_{14}} &= \sigma^2_{\varepsilon_{14}}.
\end{aligned}$$

Using the fact that revisions 2–13 have equal variance, we find that

$$\sigma^2_{\varepsilon_2} = \sigma^2_{\varepsilon_4} = ... = \sigma^2_{\varepsilon_{12}} = \sigma^2_{\varepsilon_{14}} \quad \text{and} \quad \sigma^2_{\varepsilon_3} = \sigma^2_{\varepsilon_5} = ... = \sigma^2_{\varepsilon_{13}}.$$

Note that $\sigma_{r_{13}} = \alpha/2\sigma_{y_t^{t+1}}$ and $\sigma_{r_{14}} = \alpha/4\sigma_{y_t^{t+1}}$, implying that $4\sigma^2_{r_{14}} = \sigma^2_{r_{13}}$. Therefore,

$$4\sigma^2_{\varepsilon_{14}} = \sigma^2_{\varepsilon_{14}} + \sigma^2_{\varepsilon_{13}},$$

which in turn implies that $\sigma^2_{\varepsilon_{13}} = 3\sigma^2_{\varepsilon_{14}}$. Plugging $\sigma^2_{\varepsilon_2} = ... = \sigma^2_{\varepsilon_{14}} = (\frac{\alpha}{4})^2\left[\frac{\sigma^2}{1-\beta^2} + \sigma^2_{\varepsilon_1}\right]$ into

$\sigma^2_{\varepsilon_2} + \sigma^2_{\varepsilon_1} = \alpha^2 \sigma^2_{y_t^{t+1}}$ yields

$$\frac{\alpha^2}{16}\left[\frac{\sigma^2}{1-\beta^2} + \sigma^2_{\varepsilon_1}\right] + \sigma^2_{\varepsilon_1} = \alpha^2\left[\frac{\sigma^2}{1-\beta^2} + \sigma^2_{\varepsilon_1}\right].$$

After some algebra, we find that

$$\sigma^2_{\varepsilon_1} = \frac{15\alpha^2}{16-15\alpha^2}\frac{\sigma^2}{1-\beta^2}.$$

So, the formulas for the standard deviations are

$$\sigma_{\varepsilon_1} = \sqrt{\frac{15\alpha^2}{16-15\alpha^2}\frac{\sigma^2}{1-\beta^2}},$$

$$\sigma_{\varepsilon_2} = \sigma_{\varepsilon_4} = ... = \sigma_{\varepsilon_{14}} = \sqrt{\left(\frac{\alpha}{4}\right)^2 \frac{16}{16-15\alpha^2}\frac{\sigma^2}{1-\beta^2}},$$

$$\sigma_{\varepsilon_3} = \sigma_{\varepsilon_5} = ... = \sigma_{\varepsilon_{13}} = \sqrt{3\left(\frac{\alpha}{4}\right)^2 \frac{16}{16-15\alpha^2}\frac{\sigma^2}{1-\beta^2}}.$$

Appendix B

Means and standard deviations

News Experiment	$E(\tilde{y}_{1t})$	$E(\tilde{y}_{2t})$	$E(y_{1t}^{t+1})$	$E(y_{2t}^{t+1})$	$\sigma_{\tilde{y}_{1t}}$	$\sigma_{\tilde{y}_{2t}}$	$\sigma_{y_{1t}^{t+1}}$	$\sigma_{y_{2t}^{t+1}}$
1	2.255	2.255	2.128	2.128	2.514	2.514	1.957	1.957
2	2.255	5.171	2.128	4.878	2.514	7.187	1.957	5.595
3	2.255	1.442	2.128	1.361	2.514	2.035	1.957	1.584
4	1.442	5.171	1.361	4.878	2.035	7.187	1.584	5.595
5	5.171	1.442	4.878	1.361	7.187	2.035	5.595	1.584
6	2.255	2.255	2.128	2.128	2.514	7.541	1.957	5.871
7	2.255	2.255	2.128	2.128	2.514	0.838	1.957	0.652
8	2.255	3.383	2.128	3.191	2.514	2.514	1.957	1.957
9	2.255	1.128	2.128	1.064	2.514	2.514	1.957	1.957
Noise Experiment	$E(\tilde{y}_{1t})$	$E(\tilde{y}_{2t})$	$E(y_{1t}^{t+1})$	$E(y_{2t}^{t+1})$	$\sigma_{\tilde{y}_{1t}}$	$\sigma_{\tilde{y}_{2t}}$	$\sigma_{y_{1t}^{t+1}}$	$\sigma_{y_{2t}^{t+1}}$
1	2.000	2.000	1.887	1.887	1.732	1.732	1.879	1.879
2	2.000	4.000	1.887	3.774	1.732	2.268	1.879	2.460
3	2.000	1.333	1.887	1.258	1.732	1.549	1.879	1.680
4	1.333	4.000	1.258	3.774	1.549	2.268	1.680	2.460
5	4.000	1.333	3.774	1.258	2.268	1.549	2.460	1.680
6	2.000	2.000	1.887	1.887	1.732	5.196	1.879	5.636
7	2.000	2.000	1.887	1.887	1.732	0.577	1.879	0.626
8	2.000	3.000	1.887	2.830	1.732	1.732	1.879	1.879
9	2.000	1.000	1.887	0.943	1.732	1.732	1.879	1.879

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