# Government Expenditure and Economic Growth: A Demand-side Analysis

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**Abstract**

In this paper we analyze implication of different kinds of government expenditures on aggregate demand and economic growth in a neo-Kaleckian growth model with positive saving propensity out of wages. We distinguish government expenditure into consumption and investment expenditure. The basic idea is that certain kind of government investment expenditure does influence labour productivity. In the formal model we incorporate this idea by assuming labour productivity is an increasing function of government investment expenditure. We show under the assumption of balance budget, a shift from government consumption to investment expenditure unambiguously increases both aggregate demand and growth rate in case of an exhilarationist regime while it has ambiguous implications in a stagnationist regime. Once the balanced budget assumption is dropped, while in a stagnationist regime a rise in government investment expenditure may decrease aggregate demand and growth rate, it unambiguously raises both aggregate demand and growth rate in an exhilarationist regime. On the other hand, a rise in government consumption expenditure has positive effect in both the regimes in the absence of balanced budget assumption.

Keywords: fiscal policy, government debt, aggregate demand, economic growth, employment rate, Kaleckian

JEL codes: E11, E62, O41

## Introduction:

Recent financial crisis has forced economists and policymakers to rethink conventional wisdom regarding economic theories and policies. The old debate as to whether the government expenditure is able to stimulate the economic growth, has once again emerged before us in a new way. The view generally held by Keynesians is that government involvement in economic activity is vital for growth while others say that government operations are inherently bureaucratic and inefficient and therefore rather than promoting growth, stifle it. Recently there have been debates on the fact that whether government expenditure should increase or decrease to stimulate the economic growth.

There are several institutions in the literature discussing the relationship between government expenditure and the growth rate starting from Keynes. But we can find more formal analysis

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in the literature beginning with the work of Barro (1990). His work explains the role of public expenditure in economic growth from the supply side of the economy. The demand side analysis incorporating the effect of effective demand on economic growth is absent there.

In Keynesian analysis, although sufficient attention was paid regarding the implication of various kinds of government expenditures by its pioneers, the subject was largely overlooked later on. According to Commendatore and Pinto (2011), though Kaldor presented interesting insights regarding the relationship between the composition of government expenditure and long run growth, there was no formal analysis. The formal analysis of the impact of government expenditure on growth more or less starts with You and Dutt (1996).

While aiming to address the question of whether or not government debt worsens income distribution, You and Dutt show that fiscal expansion has a significant effect on the government debt-capital ratio, economic growth and income distribution. Their analysis implies a positive relationship between fiscal expansion and the rate of economic growth in the short-run. As fiscal expansion increases, aggregate demand and the degree of capacity utilization rises which in turn enhances the growth rate. But in the long run, the effect of fiscal expansion on the growth rate is ambiguous. This is because while fiscal expansion, through an increase in aggregate demand and the degree of capacity utilization, raises the growth rate, it can either increase or decrease the government debt-capital ratio. An increase in the government debt-capital ratio has a positive impact on the growth rate. When a rise in fiscal expansion raises the government debt-capital ratio, fiscal expansion unambiguously enhances the growth rate. However, when due to a rise in the fiscal expansion, the government debt-capital ratio falls then its effect on the growth rate is ambiguous and depends on the strength of change in debt-capital ratio due to change in the ratio of government expenditure to capital.

In a later contribution in the neo-Kaleckian tradition, Commendatore and Pinto (2011) analyse the impact of different kinds of government expenditure on capacity utilization and growth. In a single-good closed economy framework with numeraire good price, they introduce two different types of public expenditure: government consumption expenditure and public provision of capital, to analyse the impact of those different kinds of government expenditure on capacity utilization and growth. According to them, the government consumption expenditure through increase in effective demand, increases equilibrium degree of capacity utilization. Increase in equilibrium level of capacity utilization on the other hand increases equilibrium growth rate. Public investment expenditure influences the level of capacity utilization in three ways. First, it increases the effective demand. Second, it crowds-in private investment. Third, they assume public provision of capital positively affects the capital productivity by enhancing potential output-capital ratio. On the other hand capital productivity itself has a negative impact on the equilibrium degree of capacity utilization. It influences the capital productivity which in turn has a negative effect on level of capacity utilization. So the final effect of a rise in public investment expenditure on equilibrium degree of capacity utilization and capital accumulation is ambiguous and depends on the strength of negative effect on aggregate demand which comes through the enhancement of capital productivity and the strength of positive effect on aggregate demand which comes from the increase in investment demand due to crowding-in effect.

But the ratio of physical capital to output is nearly constant. It is one of the stylized facts given by Kaldor. The long-term data also shows the same result[3]. On the other hand, government investment expenditure like expenditure on streets and highways, electricity, gas and water supply, hospital, education can enhance labour productivity as well. But the analysis of impact of government expenditure on labour productivity is absent here (in Commendatore and Pinto (2011)).

Dutt (2013) analyses the impact of different kinds of government expenditure on aggregate demand and growth in the short run as well as in the long run in a single-good closed economy framework. Unlike Commendatore and Pinto (2011), in his analysis potential output to capital ratio is fixed and is not influenced by the government investment expenditure. Thus actual output to capital ratio can be used as a proxy for the degree of capacity utilization. In his analysis he assumes that the government budget is balanced and the government does not carry any debt.

In the short run, both kinds of government expenditure: government consumption and investment expenditure enhance aggregate demand and hence degree of capacity utilization. Increase in capacity utilization increases the growth rate. So, both kinds of government expenditure have positive effects on aggregate demand, degree of capacity utilization and accumulation rate. But government investment expenditure due to its ‘crowding-in’ effect on private investment increases investment and hence aggregate demand further. Thus the degree of capacity utilization and the growth rate both are higher in this case compared to the case of an increase in government consumption expenditure. In a balanced budget situation, when total revenue is given in the short run, a switch from government consumption to investment expenditure, does not increase the level of aggregate demand directly, but its

[3] Barro and Sala-i-Martin (2004).

indirect effect through 'crowding-in' of private investment increases aggregate demand, the degree of capacity utilization and the growth rate.

Then he introduces the endogenous technological change where the long-run rate of growth of the economy is determined by both demand and supply forces. In the long run, both kinds of government expenditure have positive effect on growth rate. But a switch from government consumption to government investment expenditure increases the growth rate. In other word, government investment expenditure is more effective in the long run too. The reason is two folded. First, it 'crowds-in' private investment. Second, it influences the speed of adjustment for technological change positively.

Then he relaxes the balanced budget assumption by allowing the government to run a deficit and incur debt. Here again both kinds of government expenditure have positive effects on aggregate demand, degree of capacity utilization and accumulation rate. But again the degree of capacity utilization and the growth rate both are higher in government investment expenditure compared to that of an increase in government consumption expenditure. But this analysis does not take into account the issue of income distribution. Taking account of the issue related to income distribution may affect the results in several ways.

In the next section following a 'neo-Kaleckian' framework of growth theory we want to verify whether the government expenditures at all influence the growth rate and if so how is it different from the previous literature. In our model we incorporate the fact that certain kinds of investment expenditure can influence labour productivity. Labour productivity on the other hand through its impact on share of profit influences the current profitability of the private capital formation. The novelty of the model of this paper lies in taking into account this fact. Unlike Commendatore and Pinto (2011) and Dutt (2013), in our model, the investment function also depends on the rate of profit. The main objective of this dissertation is to know whether both kinds of government expenditure have a positive impact on degree of capacity utilization and economic growth. The next sub-section is about the impact of changes in fiscal policy on the equilibrium employment rate from the short run perspective. We will show that an increase in saving rate causes a fall in the equilibrium level of employment rate while an increase in either of autonomous investment and tax rate leads to an increase in the equilibrium level of employment rate. But the effect of a rise in government investment expenditure at the cost of government consumption expenditure on the equilibrium level of employment rate is ambiguous. In the last sub-section we consider the effect of deficit and government debt on the economy in the short as well as in the long-run. We also want to know whether allowing the government to run in deficit and incur debt necessarily leads to the public debt to rise without bound.

## The Model:

We assume a simple one-sector neo-Kaleckian growth model in which the economy consists of two classes: capitalists and workers. Workers save a fraction $s_W$ of their wage income while capitalist's saving propensity is $s_P$ . We also assume capitalists saving propensity ($s_P$) is higher than that of workers. We introduce $s_W$ for two purposes. Firstly, it is a more general case than that of where only capitalists save. Secondly, introducing saving out of wages we are able to provide the possibility of exhilarationist regime along with the stagnationist regime in the economy.

Income is distributed between wages and profits in the following way:

$$pY = WL + rpK$$

Where $p$ is price level, $Y$ is real income, $W$ is nominal wage rate, $L$ is total amount of labour employment, $K$ is the existing capital stock, $r$ is the real rate of profit.

There is excess supply of labour and no depreciation of capital in the economy. The production function is of Leontief type i.e.

$$Y = \min\{aL, bK\} = aL, b = \frac{Y^P}{K} > \frac{Y}{K}$$

Where, $Y^P$ is the potential output level. So the actual output is below the potential output level.

The market is oligopolistic in nature where price is determined by mark-up on prime cost. For simplicity we assume away cost of raw materials and overhead cost. We assume here that the only cost is the labour cost. So price is given by

$$p = (1 + \lambda)\frac{WL}{Y}$$

$$\rightarrow p = (1 + \lambda)\frac{W}{a}$$

Where, $\lambda$ is the rate of mark-up and $a = \frac{Y}{L}$ is labour productivity.

Total wage share $= \frac{WL}{pY} = \frac{w}{a}$ , where $w$ is real wage rate.

So, share of profit $\pi = (1 - \frac{w}{a})$.

From this equation we can conclude that share of profit depends on labour productivity and real wage rate.

Real wage rate itself depends on labour productivity i.e. $w = w(a)$. But the rate of change in the real wage rate with respect to labour productivity depends on the bargaining power of the workers which in turn depends on the prevailing employment rate and the extent of unionization. We assume $\varepsilon_{w,a} < 1$ i.e.elasticity of real wage rate with respect to labour productivity is less than one[4]. As a consequence, if labour productivity increases, wage share $\frac{w}{a}$ decreases which in turn increases the share of profit. Thus, $\pi'(a) > 0$ i.e. change in share of profit due to change in labour productivity is positive.

We assume that there are two types of government expenditure: government consumption expenditure, denoted by $C_G$ and government investment expenditure denoted by $I_G$. We also assume that government investment expenditure is proportional to the aggregate real income i.e. $I_G = \theta\, Y$ , where $\theta$ represents government investment-output ratio. Government raises revenue through an income tax. Total tax revenue is $T = tY$ , where $t$ is the tax rate. For simplicity, we assume that the government budget is balanced. So,

$$t.Y = C_G + I_G$$

If $t$ and $\theta$ are fixed, this equation can be satisfied through adjustment in $C_G$. Given the tax rate, if $\theta$ increases then for a given income level, government consumption expenditure must fall. Thus for a given aggregate government expenditure, a change in the parameter $\theta$ represents a change in fiscal policy i.e. here changes in $\theta$ represents changes in fiscal policy related to the government's decision as how much to spend on consumption and how much to spend on investment purposes.

Total savings as a proportion of capital stock is expressed as

$$\frac{S}{K} = (s_P - s_W)(1-t)r + s_W(1-t)u$$

Where,

$$r = \frac{P}{K} = \frac{P}{Y}.\frac{Y}{K} = \pi(a).u$$

[4]In developing countries, a large number of workers are employed in unorganized sectors (eg. India, Pakistan, Bangladesh) where either they don't have any organized labour union or the union is too weak to have strong bargaining power. On the other hand, in developed countries as well, the workers may not be able to fully internalize the increase in productivity through proportionate increases in the real wage rate. (Carter (2007), Sharpe et al. (2008a), (2008b))

$P$ represents total profit, $\frac{P}{Y} = share\ of\ profit = \pi(a)$ and $u$ is the output-capital ratio which is used as a proxy for degree of capacity utilization[5] (Dutt 1984, 1987, 1990).

We assume that there is excess capacity in the economy (*i.e.* $u < 1$)

The investment function in the economy is given by

$$I = \left[\gamma + \gamma_1 u + \gamma_2 (1-t).r + \gamma_3 \left(\frac{I_G}{K}\right)\right] K$$

Or,

$$\frac{I}{K} = [\gamma + \gamma_1 u + \gamma_2 (1-t).\pi(a).u + \gamma_3.\theta.u]$$

Where $\gamma, \gamma_1, \gamma_2, \gamma_3$ all are positive parameters.

$\gamma$ represents the autonomous part of the investment function. We assume that investment depends positively on the degree of capacity utilization $(u)$, the rate of profit $(r)$ and the ratio of government investment to capital stock $(\frac{I_G}{K})$. $\gamma_1$ indicates the responsiveness of investment to a change in $u$ . Similarly $\gamma_2$ and $\gamma_3$ indicate the responsiveness of investment due to a change in the rate of profit and the ratio of government investment to capital respectively. The positive effect of $u$ is the static equivalent of the accelerator effect. The argument for rate of capacity utilization entering in the investment function comes from Steindl (1952). According to Steindl, because of indivisibilities in capital equipment, it is profitable for profit maximizing firms to have a certain desired level of excess capacity due to fluctuations in demand or expected growth in demand. Thus when capacity utilization rises above the desired level, firms would like to invest more; while the capacity utilization falls below the desired level, firm would like to increase utilization by disinvesting and hence by reducing the stock of capital. Rate of profit enters in the investment function as a proxy for the expected rate of return. It also provides internal funding for accumulation plans. For firms depending on external finance, it is also easier to raise that external finance while rate of profit is higher. For simplicity we assume that actual rate of profit is equal to the expected profit rate.

Now let us focus on the last variable in the investment function- the ratio of government investment expenditure to capital stock. Following Dutt (2013) and Taylor (1991) we can say

[5]As long as the potential output-capital ratio is fixed, actual output-capital ratio can be used as a proxy for the degree of capacity utilization.

that government investment expenditure has a positive impact on private investment because of a “crowding in[6]” effect.

Certain kinds of government investment expenditure (like expenditure on part of infrastructure, education and health facilities, water and electricity supply) have a positive impact on labour productivity as well. So we can say $a = a(\theta)$ and $a'(\theta) > 0$ i.e. labour productivity depends positively on the ratio of government investment to output[7].

In the short run, the good market is cleared through changes in the level of output and capacity utilization.

In equilibrium, saving must be equal to investment. i.e.

$$\frac{I}{K} = \frac{S}{K}$$

$$or, u^* = \frac{\gamma}{(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta}$$

$u^*$ is the equilibrium level of capacity utilization.

The equilibrium is stable if and only if the induced increase in saving as $u$ rises is greater than the induced increase in investment i.e.

$$(s_P - s_W)(1-t)\pi(a) + s_W(1-t) > \gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta$$

Or, $$(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta > 0$$

In other word, for the equilibrium to be stable the denominator of $u^*$ must be positive.

So the stability condition can be satisfied if $s_W > \frac{[\gamma_1+\gamma_3\theta-(s_P-\gamma_2)(1-t)\pi]}{[(1-t)(1-\pi)]}$. Thus the stability condition imposes a lower boundary to the savings propensity of the workers.

[6]Government investment on infrastructure, education, water supply, health facilities etc. boosts private investment through its complementary and other external effects. Again it raises the future profitability of private capital formation.

[7]Government investment expenditures like expenditure on health, roads, electricity and water supply have an impact on labour productivity within a fairly short period. On the other hand, a few kind of government investment expenditures like expenditure on education affects labour productivity with a long time lag. For simplicity, we assume that government investment expenditure is mainly of the first type. This type of government expenditure has the potential to influence the current profitability of private investment and thus this is different from the crowding in effect where the future profitability of private capital formation is influenced.

We get the equilibrium growth rate in terms of equilibrium degree of capacity utilization as,

$$g^* = \gamma + \gamma_1 u^* + \gamma_2(1-t).\pi(a).u^* + \gamma_3.\theta.u^*$$

Now let's discuss when the economy is in stagnationist regime and when it is in the exhilarationist regime. The following proposition due to Blecker (2002) provides the sufficient condition for the economy to be on either of those regime.

***Proposition 1: Whether the economy is in a stagnationist regime or in an exhilarationist regime depends on the value of $s_W$ as follows: (i) if $s_W < (s_P - \gamma_2)$ then the economy is in a stagnationist regime and (ii) if $s_W > (s_P - \gamma_2)$ then the economy is in an exhilarationist regime.***

Proof : Differentiating the equilibrium level of $u^*$ with respect to $\pi$ we get,

$$\frac{du^*}{d\pi} = \frac{-\gamma(s_P - s_W - \gamma_2)(1-t)}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}$$

Thus if $s_W < (s_P - \gamma_2)$ then $\frac{du^*}{d\pi} < 0$

And if $s_W > (s_P - \gamma_2)$ then $\frac{du^*}{d\pi} > 0$

If the saving propensity out of wages is high enough then due to redistribution of income from workers to capitalists, consumption demand falls by $(s_P - s_W)$ per unit income transferred from wages to profits. On the other hand, due to increase in profitability, investment demand rises by $\gamma_2$ per unit increase in profits. If the latter effect dominates the former, the equilibrium degree of capacity utilization rises due to a rise in the share of profit. In this case, the economy is said to be in exhilarationist regime. Following a similar argument, we can say that when $(s_P - s \quad - \gamma_2) > 0$ then the economy is in a stagnationist regime (i.e. $\frac{du^*}{d\pi} < 0$) . So depending on the sign of $(s_P - s_W - \gamma_2)$ the economy may be either in a stagnationist or in an exhilarationist regime.

Note that an increase in either of $\gamma, \gamma_1, \gamma_2, \gamma_3$ leads to an increase in the equilibrium level of $u$. But the effect of a rise in $t$ on the equilibrium level of $u$ is ambiguous in the exhilarationist regime, while in the stagnationist regime a rise in $t$ rises it.

As $\gamma_1$ increases, the accelerator effect of $u$ on investment demand rises, which in turn raises aggregate demand and hence equilibrium level of $u$.

As $\gamma_2$ increases, equilibrium level of $u$ also increases. This is because, an increase in $\gamma_2$ , for a given profit rate, leads to an increase in investment demand which in turn increases the aggregate demand and hence the degree of capacity utilization. Similarly, as $\gamma_3$ increases, for a given $\theta$ , investment demand increases which in turn raises the aggregate demand and hence the degree of capacity utilization.

The tax rate has a positive impact on the equilibrium degree of capacity utilization. This is mainly because of the balanced budget assumption. Per unit increase in tax rate reduces consumption for capitalists by $(1 - s_P)\pi$ unit, while the consumption for workers decreases by $(1 - s_W)(1 - \pi)$ unit. By reducing the after tax profit rate, it also reduces investment demand by $\gamma_2\pi$ unit. But the entire tax revenue is spent by the government and so the aggregate demand increases by one unit. As the increase in the government spending is higher than the reduction of consumption and investment demand[8], an increase in the tax rate increases the equilibrium level of degree of capacity utilization.

Now let us check, at the cost of government consumption expenditure what is the effect of an increase in government investment expenditure on the aggregate demand and the capacity utilization.

***Proposition 2 : An increase in $\boldsymbol{\theta}$ leads to an unambiguous increase in the equilibrium degree of capacity utilisation in the exhilarationist regime, while in the stagnationist regime, the effect of a rise in $\boldsymbol{\theta}$ on the equilibrium degree of capacity utilisation depends on the product of $\boldsymbol{\varepsilon_{\pi,a}}$ and $\boldsymbol{\varepsilon_{a,\theta}}$ as follows: $\boldsymbol{\frac{du^*}{d\theta} \leq> 0}$ according to whether $\boldsymbol{\varepsilon_{\pi,a}.\varepsilon_{a,\theta} \geq< \psi}$, where $\boldsymbol{\psi = \frac{\gamma_3\theta}{(s_P - s_W - \gamma_2)(1-t)\pi(a)}}$ , $\boldsymbol{\varepsilon_{\pi,a} = \left(\frac{d\pi}{da}\right)\left(\frac{a}{\theta}\right)}$ and $\boldsymbol{\varepsilon_{a,\theta} = \left(\frac{da}{d\theta}\right)\left(\frac{\theta}{a}\right)}$.***

Proof: Let's discuss the exhilarationist regime first.

Differentiating the equilibrium degree of capacity utilisation with respect to $\theta$ we get,

$$\frac{du^*}{d\theta} = \frac{\partial u^*}{\partial \pi}\frac{d\pi}{da}\frac{da}{d\theta} + \frac{\partial u^*}{\partial \theta}$$

If the economy is in exhilarationist regime then $\frac{\partial u^*}{\partial \pi} > 0$. $\frac{d\pi}{da}, \frac{da}{d\theta}$ $and$ $\frac{\partial u^*}{\partial \theta}$ all are also positive.

So, $\frac{du^*}{d\theta}$ is unambiguously positive.

Now suppose the economy is in stagnationist regime. Then,

$$\frac{du^*}{d\theta} = \frac{\partial u^*}{\partial \pi}\frac{d\pi}{da}\frac{da}{d\theta} + \frac{\partial u^*}{\partial \theta}$$

$$= \frac{-\gamma[(s_P - s_W - \gamma_2)(1-t)\frac{d\pi}{d\theta} - \gamma_3]}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}$$

[8] As $(1 - s_W)(1 - \pi) + (1 - s_P)\pi\,\gamma_2\pi \; < 1$

$$= \frac{-\gamma \frac{\pi(a)}{\theta}[(s_P - s_W - \gamma_2)(1-t)\boldsymbol{\varepsilon_{\pi,a}}.\boldsymbol{\varepsilon_{a,\theta}} - \gamma_3 \frac{\theta}{\pi(a)}]}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}$$

So, $\frac{du^*}{d\theta} \leq> 0$ according to whether $\varepsilon_{\pi,a}.\varepsilon_{a,\theta} \geq< \frac{\gamma_3\theta}{(s_P - s_W - \gamma_2)(1-t)\pi(a)} = \psi$

Above proposition implies that while in the exhilarationist regime an increase in $\theta$ always has a positive impact on the equilibrium degree of capacity utilization, in the stagnationist regime the effect of a rise in $\theta$ on $u^*$ depends on some critical value of the product of $\boldsymbol{\varepsilon_{\pi,a}}$ and $\boldsymbol{\varepsilon_{a,\theta}}$. If the product of $\boldsymbol{\varepsilon_{\pi,a}}$ and $\boldsymbol{\varepsilon_{a,\theta}}$ exceeds (less than) the critical value (let's say $\psi$) then the effect of a rise in $\theta$ on $u^*$ is negative (positive). Note that if we assume the case where workers don't save, the economy then is only in a stagnationist regime. There also the effect of a rise in $\theta$ on $u^*$ depends on the product of $\boldsymbol{\varepsilon_{\pi,a}}$ and $\boldsymbol{\varepsilon_{a,\theta}}$. In that sense, in the case of a stagnationist regime, there is no qualitatively difference between the situations where workers save and where they don't. Only the critical value changes.

In proposition 3 we discuss the impact of a change in $\theta$ on the equilibrium rate of capital accumulation.

***Proposition 3: An increase in $\boldsymbol{\theta}$ leads to an unambiguous increase in the equilibrium rate of accumulation in the exhilarationist regime while in the stagnationist regime, the effect of a rise in $\boldsymbol{\theta}$ on the equilibrium rate of capital accumulation depends on the product of $\boldsymbol{\varepsilon_{\pi,a}}$ and $\boldsymbol{\varepsilon_{a,\theta}}$ as follows: $\frac{dg^*}{d\theta} \geq< 0$ according to whether $\boldsymbol{\varepsilon_{\pi,a}}.\boldsymbol{\varepsilon_{a,\theta}} \leq> \boldsymbol{\rho}$ where $\boldsymbol{\rho} = \frac{\{(s_P - s_W)\pi + s_W\}\gamma_3\theta}{\{(\gamma_1+\gamma_3\theta)(s_P - s_W) - s_W\gamma_2(1-t)\}\pi}$, $\boldsymbol{\varepsilon_{\pi,a}} = \frac{d\pi}{da}\frac{a}{\theta}$ and $\boldsymbol{\varepsilon_{a,\theta}} = \frac{da}{d\theta}\frac{\theta}{a}$.***

Proof: Let's discuss the exhilarationist regime first. Differentiating the equilibrium rate of accumulation with respect to $\theta$ we get,

$$\frac{dg^*}{d\theta} = \frac{\partial g^*}{\partial u^*}\frac{du^*}{d\theta} + \frac{\partial g^*}{\partial \pi}\frac{d\pi}{d\theta} + \frac{\partial g^*}{\partial \theta}$$

From Proposition 2 we know that when the economy is in exhilarationist regime, $\frac{du^*}{d\theta}$ is positive.

$\frac{\partial g^*}{\partial u^*}, \frac{\partial g^*}{\partial \pi}, \frac{d\pi}{d\theta}, \frac{\partial g^*}{\partial \theta}$ all are also positive.

So, $\frac{dg^*}{d\theta}$>0.

Now suppose the economy is in stagnationist regime. Then $(s_P - s_W - \gamma_2) > 0$.

Differentiating the equilibrium rate of capital accumulation with respect to $\theta$ we get,

$$\frac{dg^*}{d\theta} = \frac{\partial g^*}{\partial u^*}\frac{du^*}{d\theta} + \frac{\partial g^*}{\partial \pi}\frac{d\pi}{d\theta} + \frac{\partial g^*}{\partial \theta}$$

$$= \{\gamma_1 + \gamma_2(1-t).\pi(a) + \gamma_3.\theta\}\frac{du^*}{d\theta} + \gamma_2(1-t).u^*\frac{d\pi}{d\theta} + \gamma_3.u^*$$

$$= \{\gamma_1 + \gamma_2(1-t).\pi(a) + \gamma_3.\theta\}\left[\frac{-\gamma[(s_P - s_W - \gamma_2)(1-t)\frac{d\pi}{d\theta} - \gamma_3]}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}\right]$$
$$+ \left[\gamma_2(1-t).\frac{d\pi}{d\theta} + \gamma_3\right]\left[\frac{\gamma}{(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta}\right]$$

$$= \frac{\gamma(1-t)\pi[\{(s_P - s_W)\pi + s_W\}\gamma_3\frac{\theta}{\pi} - \{(\gamma_1 + \gamma_3\theta)(s_P - s_W) - s_W\gamma_2(1-t)\}\varepsilon_{\pi,a}.\varepsilon_{a,\theta}]}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2}$$

Thus $\frac{dg^*}{d\theta} \gtrless 0$ according to whether $\boldsymbol{\varepsilon_{\pi,a}.\varepsilon_{a,\theta}} \lessgtr \frac{\{(s_P - s_W)\pi + s_W\}\gamma_3\theta}{\{(\gamma_1+\gamma_3\theta)(s_P - s_W) - s_W\gamma_2(1-t)\}\pi}$

So when the economy is in an exhilarationist regime, due to a rise in $\theta$, the equilibrium rate of capital accumulation unambiguously rises. But when the economy is in a stagnationist regime, the effect of a rise in $\theta$ on $g^*$ depends on the value of the product of $\varepsilon_{\pi,a}$ and $\varepsilon_{a,\theta}$. If the product of $\varepsilon_{\pi,a}$ and $\varepsilon_{a,\theta}$ exceeds (less than) the critical value (let's say $\rho$ ) then the effect of a rise in $\theta$ on $g^*$ is negative (positive). Again in that sense there is no qualitatively difference from the situation where workers do not save. Only the critical value changes.

**Issues regarding the employment rate:**

Now let us we focus on the employment rate in the economy. Here we will discuss about the impact of different kinds of government expenditures on labour employment rate from the short run perspective.

Equilibrium level of employment rate $e^*$ can be written as:

$$e^* = \frac{L}{N} = \frac{Y}{K}\frac{L}{Y}\frac{K}{N} = u^*\frac{1}{a}k_0 = u^*k$$

where,$u$ is the degree of capacity utilization, $N$ is the total supply of labour which is fixed in the short-run, $k_0$ is the ratio of capital stock to total supply of labour and $k(=\frac{K}{aN})$is the ratio of capitalstock to the productive labour supply.

An increase in either of $\gamma, \gamma_1, \gamma_2, \gamma_3$ and t leads to an increase in the equilibrium level of employment rate e*. As in the short-run $K$ and $N$ both are fixed, $k_0$, which is the ratio of the capital stock to the labour supply, is also fixed in the short-run. Then as long as the labour productivity is not influenced by any change in parameters, $k$ is also fixed. Thus a change in any parameter which does not have an impact on $a$, can change the equilibrium rate of employment only through change in $u^*$.

**P*roposition 4: An increase in $\theta$ leads to an ambiguous effect on the equilibrium level of employment rate both in the exhilarationist regime and in the stagnationist regime. The effect of a rise in $\theta$ on the equilibrium level of employment rate depends on $\varepsilon_{\pi,a}$ and $\varepsilon_{a,\theta}$ as follows:* $\frac{de^*}{d\theta} \geq< 0$ *according to whether* $\varepsilon_{\pi,a} \leq> \frac{\gamma_3\theta+(\gamma_1+\gamma_3\theta-s_W(1-t))\varepsilon_{a,\theta}}{(s_P-s_W-\gamma_2)(1-t).\pi(a)\varepsilon_{a,\theta}} - 1$, *where* $\varepsilon_{\pi,a} = \frac{d\pi}{da}\frac{a}{\theta}$ *and* $\varepsilon_{a,\theta} = \frac{da}{d\theta}\frac{\theta}{a}$.**

Proof:

$$e^* = u^*k = \frac{u^*k_0}{a}$$

$$\frac{de^*}{d\theta} = \frac{\partial e^*}{\partial u^*}\frac{du^*}{d\theta} + \frac{\partial e^*}{\partial a}\frac{da}{d\theta}$$

$$= \frac{k_0}{a}.\frac{-\gamma[((s_P - s_W - \gamma_2)(1-t)\frac{d\pi}{da}\frac{da}{d\theta} - \gamma_3]}{[(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta]^2} - \frac{k_0}{a^2}\left\{\frac{\gamma}{(s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) - \gamma_1 - \gamma_3\theta}\right\}\frac{da}{d\theta}$$

$$= \nabla[\frac{\pi a}{\theta}\{(s_P - s_W - \gamma_2)(1-t)(\varepsilon_{\pi,a}.\varepsilon_{a,\theta} + \varepsilon_{a,\theta})\} - \frac{a}{\theta}\{(\gamma_1 + \gamma_3\theta)\varepsilon_{a,\theta} - s_W(1-t)\varepsilon_{a,\theta} + \gamma_3\theta\}]$$

Where $\nabla = -\frac{k_0\gamma}{[a\{(s_P-s_W-\gamma_2)(1-t)\pi(a)+s_W(1-t)-\gamma_1-\gamma_3\theta\}]^2} < 0$

Thus $\frac{de^*}{d\theta} \geq< 0$ according to whether $\varepsilon_{\pi,a} \leq> \frac{\gamma_3\theta+(\gamma_1+\gamma_3\theta-s_W(1-t))\varepsilon_{a,\theta}}{(s_P-s_W-\gamma_2)(1-t).\pi(a)\varepsilon_{a,\theta}} - 1$

In the exhilarationist regime, $\frac{du^*}{d\theta} > 0$. So the effect of a rise in $\theta$ on the equilibrium employment rate is ambiguous. In the stagnationist regime, if $\frac{du^*}{d\theta} > 0$ the effect of a rise in $\theta$ on the equilibrium employment rate is ambiguous. But if $\frac{du^*}{d\theta} < 0$ , a rise in $\theta$ always decreases the equilibrium employment rate.

**Effect of government deficits and debt:**

So far we have assumed that the government budget is balanced and there is no government debt. Now we will relax the assumption. Let's assume there is a budget deficit and the government incurs debt.

Let's assume that the aggregate government tax revenue is given by $T = t(Y + iD)$. Where, $t$ is the tax rate, $Y$ is the real aggregate productive income, $i$ is the interested rate that is paid by the government, and $D$ is the real stock of government debt.

In this section we assume that government consumption expenditure and government investment expenditure depend on the income level of the economy. Let us assume that current government consumption expenditure is now given by $C_G = \eta Y$ and investment expenditure is given by $I_G = \theta\, Y$. Following Dutt (2013) for the sake of simplicity we ignore monetary and other assets. The entire government deficit is financed by issuing government debt. So, the change in debt with respect to time is given by,

$$\frac{dD}{d\tau} = (C_G + I_G) - T + iD$$

Aggregate private saving in the economy is given by,

$$S = (1-t)[s_P P + s_W W + s_P iD]$$

$$\rightarrow \frac{S}{K} = (s_P - s_W)(1-t)r + s_W(1-t)u + (1-t)iD$$

The investment function in the economy is given by

$$I = \left[\gamma + \gamma_1 u + \gamma_2(1-t)r + \gamma_3\left(\frac{I_G}{K}\right) - \gamma_4\delta\right]K$$

$$\text{So, } \frac{I}{K} = [\gamma + \gamma_1 u + \gamma_2(1-t).\pi(a).u + \gamma_3.\theta.u - \gamma_4\delta]$$

Where $\gamma, \gamma_1, \gamma_2, \gamma_3, \gamma_4$ all are positive parameters.

$\gamma_4$ is the coefficient measuring responsiveness of investment due to change in $\delta$. Here the fifth term entering in the investment function, represents the financial crowding out effect[9].

In the short run equilibrium, the following equation must be satisfied,

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[9] We introduce it to show that, even allowing the neo-classical argument of financial crowding-out of private investment due to rise in public debt, when we introduce government deficits and the dynamics of the government debt into our analysis, the model does not necessarily become unstable and $\delta$ does not rises without bound. We also will show that our long-run result differs from Dutt (2013) as here are two equilibrium values of $\delta$ and the smaller one represents the stable equilibrium value while in Dutt (2013) this is not necessarily the case.

$$\frac{S}{K}+\frac{T}{K}=\frac{I}{K}+\frac{G}{K}$$

Putting the values we get the equilibrium degree of capacity utilization as

$$u^{*}=\frac{\gamma-[\{s_P(1-t)+t\}i+\gamma_4]\delta}{(s_P-s_W-\gamma_2)(1-t)\pi(a)+s_W(1-t)+t-\gamma_1-\eta-\theta(1+\gamma_3)}$$

The equilibrium is stable when the induced increase in private savings and revenue income as $u$ rises must be greater than the induced increase in private investment and government expenditure. That is when the following equation is satisfied:

$$(s_P-s_W)(1-t)\pi(a)+s_W(1-t)+t>\gamma_1+\gamma_2(1-t)\pi(a)+\gamma_3\theta+(\eta+\theta)$$

That is,

$$(s_P-s_W-\gamma_2)(1-t)\pi(a)+s_W(1-t)+t-\gamma_1-\eta-(1+\gamma_3)\theta>0$$

In other word, for the equilibrium to be stable the denominator of $u^{*}$ must be positive.
But for a meaningful positive equilibrium degree of capacity utilization the numerator also should be positive. So we need,

$$\gamma-[\{s_P(1-t)+t\}i+\gamma_4]\delta>0$$

That is, $\gamma>[\{s_P(1-t)+t\}i+\gamma_4]\delta$

Putting the equilibrium value of degree of capacity utilization in the investment function equation we get the equilibrium value of growth rate as,

$$g^{*}=\gamma+\gamma_1u^{*}+\gamma_2(1-t).\pi(a).u^{*}+\gamma_3.\theta.u^{*}-\gamma_4\delta$$

Now we will focus on the effect of a change in model parameters on the equilibrium degree of capacity utilization. An increase in either of $\delta, i, \gamma_4, t$ ; cause a fall in the equilibrium level of $u$ while an increase in either of $\gamma, \gamma_1, \gamma_2, \gamma_3$ and $\eta$ leads to an increase in the equilibrium level of $u$.

Here again whether the economy is in a stagnationist regime or in an exhilarationist regime depends on the value of $s_W$. If $s_W<(s_P-\gamma_2)$ then the economy is in a stagnationist regime and if $s_W>(s_P-\gamma_2)$ then the economy is in an exhilarationist regime.

An increase in either of $\delta, i, \gamma_4$ causes a fall in the equilibrium level of $u$ while an increase in either of $\gamma, \gamma_1, \gamma_2, \gamma_3$ leads to an increase in the equilibrium level of $u$ irrespective of which regime the economy is in. An increase in government consumption expenditure $(\eta)$ leads to an increase in the aggregate demand and the equilibrium level of capacity utilization. However the effect of an increase in government investment expenditure $(\theta)$ on the equilibrium capacity utilization depends on which regime the economy is in. It is discussed in proposition 5. Note that here a rise in $\theta$ means solely an increase in government investment expenditure, not the increase in government investment expenditure at the cost of government consumption expenditure.

***Proposition 5: An increase in $\theta$ leads to an unambiguous increase in the equilibrium degree of capacity utilisation in the exhilarationist regime, while in the stagnationist regime, the effect of a rise in $\theta$ on the equilibrium degree of capacity utilisation depends on the product of $\varepsilon_{\pi,a}$ and $\varepsilon_{a,\theta}$ as follows: $\frac{du^*}{d\theta} \geq< 0$ according to whether $\varepsilon_{\pi,a}.\varepsilon_{a,\theta} \leq> \varphi$ where $\varphi = \frac{(1+\gamma_3)\theta}{(s-\gamma_2)(1-t).\pi}$, $\varepsilon_{\pi,a} = \frac{d\pi}{da}\frac{a}{\theta}$ and $\varepsilon_{a,\theta} = \frac{da}{d\theta}\frac{\theta}{a}$***

Proof: Let's discuss the exhilarationist regime first.

Differentiating the equilibrium degree of capacity utilisation with respect to $\theta$ we get,

$$\frac{du^*}{d\theta} = \frac{\partial u^*}{\partial \pi}\frac{d\pi}{da}\frac{da}{d\theta} + \frac{\partial u^*}{\partial \theta}$$

If the economy is in exhilarationist regime then $\frac{\partial u^*}{\partial \pi} > 0. \frac{d\pi}{da}, \frac{da}{d\theta}$ and $\frac{\partial u^*}{\partial \theta}$ [10] all are also positive. So, $\frac{du^*}{d\theta}$ is unambiguously positive.

Now suppose the economy is in stagnationist regime. Then $(s_P - s_W - \gamma_2) > 0$.

Differentiating the equilibrium degree capacity utilization equation with respect to $\theta$ we get,

$$\frac{du^*}{d\theta} = \frac{\partial u^*}{\partial \pi}.\frac{d\pi}{da}.\frac{da}{d\theta} + \frac{\partial u^*}{\partial \theta}$$

Putting the values of $\frac{\partial u^*}{\partial \pi}$ and $\frac{\partial u^*}{\partial \theta}$ on the above equation we get,

$$= \frac{-[\gamma - \Gamma\delta][(s_P - s_W - \gamma_2)(1-t)\frac{d\pi}{da}.\frac{da}{d\theta} - (1+\gamma_3)]}{\Lambda^2}$$

$$= \frac{-[\gamma - \Gamma\delta]\frac{\pi}{\theta}[(s_P - s_W - \gamma_2)(1-t).\varepsilon_{\pi,a}.\varepsilon_{a,\theta} - \frac{(1+\gamma_3)\theta}{\pi}]}{\Lambda^2}$$

Where, $\Gamma = \{s_P(1-t) + t\}i + \gamma_4$ and $\Lambda = (s_P - s_W - \gamma_2)(1-t)\pi(a) + s_W(1-t) + t - \gamma_1 - \eta - \theta(1+\gamma_3)$ and both $\Gamma$ and $\Lambda$ are positive.

So, $\frac{du^*}{d\theta} \geq< 0$ according to whether $\varepsilon_{\pi,a}.\varepsilon_{a,\theta} \leq> \frac{(1+\gamma_3)\theta}{(s_P - s_W - \gamma_2)(1-t).\pi} = \varphi$.

It follows that in the stagnationist regime, if the product of the elasticity of profit share with respect to labour productivity ($\varepsilon_{\pi,a}$) and the elasticity of labour productivity with respect to the ratio of government investment to output ($\varepsilon_{a,\theta}$) is greater than (less than) a critical value (let's say $\varphi$ ) then the effect of $\theta$ on $u^*$ is negative ( positive).

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[10] $\frac{\partial u^*}{\partial \theta} = \frac{[\gamma - \Gamma\delta](1+\gamma_3)}{\Lambda^2} > 0$

A rise in $\theta$ raises labour productivity which in turn raises profit share. In the stagnationist regime a change in profit share has a negative impact on the equilibrium degree of capacity utilization. On the other hand, a rise in $\theta$ directly raises the investment demand through the crowding-in effect leading to a rise in aggregate demand and the degree of capacity utilization. So the final impact of a change in $\theta$ on the degree of capacity utilization depends on the relative strength of the direct effect of $\theta$ on $u$ and its indirect effect on $u$ through the change in the share of profit. When the elasticities have lower values the indirect effect of $\theta$ on $u$ through the change in the profit share is comparatively lower and as a result the direct effect of $\theta$ on $u$ dominates the indirect effect. On the other hand when the above elasticities have sufficiently high values then the indirect effect of $\theta$ on $u$ dominates the direct effect and thus the impact of $\theta$ on $u$ is negative.

Let's focus on the effect of $\theta$ on $g^*$ now.

***Proposition 6: An increase in $\theta$ leads to an unambiguous increase in the equilibrium rate of accumulation in the exhilarationist regime, while in the stagnationist regime, the effect of a rise in $\theta$ on the equilibrium rate of capital accumulation depends on the product of $\varepsilon_{\pi,a}$ and $\varepsilon_{a,\theta}$ as follows: $\frac{dg^*}{d\theta} \geq< 0$ according to whether $\varepsilon_{\pi,a}.\varepsilon_{a,\theta} \leq> \rho'$ where $\rho' = \frac{\theta\{\gamma_1+\gamma_3(t-\eta)+(1-t)\pi(a)\{\gamma_2+(s_P-s_W)\gamma_3\}+s_W(1-t)\}}{\pi(1-t)\{(s_P-s_W-\gamma_2)(\gamma_1+\theta\gamma_3)-(t-\gamma_1-(1+\gamma_3)\theta-\eta+s_W(1-t))\gamma_2\}}$, $\varepsilon_{\pi,a} = \frac{d\pi}{da}\frac{a}{\theta}$ and $\varepsilon_{a,\theta} = \frac{da}{d\theta}\frac{\theta}{a}$.***

Proof: Let's discuss the exhilarationist regime first.

Differentiating the equilibrium rate of capital accumulation with respect to $\theta$ we get,

$$\frac{dg^*}{d\theta} = \frac{\partial g^*}{\partial u^*}\frac{du^*}{d\theta} + \frac{\partial g^*}{\partial \pi}\frac{d\pi}{d\theta} + \frac{\partial g^*}{\partial \theta}$$

Now,
$$\frac{\partial g^*}{\partial u^*} = [\gamma_1 + \gamma_2(1-t)\pi(a) + \gamma_3\theta] > 0$$

$$\frac{\partial g^*}{\partial \pi} = \gamma_2(1-t)u^* > 0$$

$$\frac{\partial g^*}{\partial \theta} = \gamma_3 u^* > 0$$

$$\frac{d\pi}{d\theta} = \frac{d\pi}{da}\frac{da}{d\theta} > 0$$

And from the proposition 6 we know in the exhilarationist regime $\frac{du^*}{d\theta} > 0$.

So, in the exhilarationist regime $\frac{dg^*}{d\theta} > 0$.

Now suppose the economy is in stagnationist regime. Then $(s_P - s_W - \gamma_2) > 0$.

Differentiating the equilibrium rate of capital accumulation with respect to $\theta$ we get,

$$\frac{dg^*}{d\theta} = \frac{\partial g^*}{\partial u^*}\frac{du^*}{d\theta} + \frac{\partial g^*}{\partial \pi}\frac{d\pi}{d\theta} + \frac{\partial g^*}{\partial \theta}$$

$$= \{\gamma_1 + \gamma_2(1-t).\pi(a) + \gamma_3.\theta\}\frac{du^*}{d\theta} + \gamma_2(1-t).u^*\frac{d\pi}{d\theta} + \gamma_3.u^*$$

$$= \{\gamma_1 + \gamma_2(1-t).\pi(a) + \gamma_3.\theta\}\left[\frac{-(\gamma - \Gamma\delta)\{(s_P - s_W - \gamma_2)(1-t)\frac{d\pi}{d\theta} - (1+\gamma_3)\}}{\Lambda^2}\right]$$
$$+\left\{\gamma_2(1-t).\frac{d\pi}{d\theta} + \gamma_3\right\}\{\frac{(\gamma - \Gamma\delta)}{\Lambda}\}$$

$$= \frac{(\gamma - \Gamma\delta)}{\Lambda^2}\left[-\{\gamma_1 + \gamma_2(1-t).\pi(a) + \gamma_3.\theta\}\left\{(s_P - s_W - \gamma_2)(1-t)\frac{d\pi}{d\theta} - (1+\gamma_3)\right\}\right]$$
$$+\frac{(\gamma - \Gamma\delta)}{\Lambda^2}[\left\{\gamma_2(1-t).\frac{d\pi}{d\theta} + \gamma_3\right\}\{(s_P - s_W - \gamma_2)(1-t)\pi(a) + t - \gamma_1$$
$$- (1+\gamma_3)\theta - \eta + s_W(1-t)\}]$$

$$= \frac{(\gamma - \Gamma\delta)}{\Lambda^2}\frac{\pi}{\theta}\left[[\gamma_1 + \gamma_3(t-\eta) + (1-t)\pi(a)\{\gamma_2 + (s_P - s_W)\gamma_3\} + s_W(1-t)]\frac{\theta}{\pi}\right]$$
$$- \frac{(\gamma - \Gamma\delta)}{\Lambda^2}\frac{\pi}{\theta}[(1-t)\varepsilon_{\pi,a}.\varepsilon_{a,\theta}\{(s_P - s_W - \gamma_2)(\gamma_1 + \theta\gamma_3)$$
$$- \big(t - \gamma_1 - (1+\gamma_3)\theta - \eta + s_W(1-t)\big)\gamma_2\}]$$

So, $\frac{dg^*}{d\theta} \geq< 0$ according to whether

$$\varepsilon_{\pi,a}.\varepsilon_{a,\theta} \leq> \frac{\theta\{\gamma_1 + \gamma_3(t-\eta) + (1-t)\pi(a)\{\gamma_2 + (s_P - s_W)\gamma_3\} + s_W(1-t)\}}{\pi(1-t)\{(s_P - s_W - \gamma_2)(\gamma_1 + \theta\gamma_3) - (t - \gamma_1 - (1+\gamma_3)\theta - \eta + s_W(1-t))\gamma_2\}} = \rho'$$

Now, we will focus on the effect of $\delta$ on $g^*$.

***Proposition 7: An increase in $\boldsymbol{\delta}$ decreases the equilibrium value of the rate of capital accumulation.***

Proof: Differentiating the equilibrium rate of capital accumulation with respect to $\boldsymbol{\delta}$ we get,

$$\frac{dg^*}{d\delta} = \frac{\partial g^*}{\partial u^*}\frac{du^*}{d\delta} + \frac{\partial g^*}{\partial \delta}$$

$$= -\left[\{\gamma_1 + \gamma_2(1-t).\pi(a) + \gamma_3.\theta\}\frac{\Gamma}{\Lambda} + \gamma_4\right] < 0$$

An increase in $\boldsymbol{\delta}$ decreases the equilibrium level of $g$ in two ways: (1) directly through financial crowding-out effect and (2) indirectly through decrease in the equilibrium degree of capacity utilization. Let's discuss its ($\boldsymbol{\delta}$) effect on $u^*$first. In the short run an increase in $\delta$ decreases the equilibrium degree of capacityutilization. Due to one unit increase in $\delta$, the ratio of private saving to capitalstock increases by $s_P(1-t)i$ unit while the ratio of government revenue income to capital stock increases by $it$ unit. Thus, due to one unit increase in $\delta$ consumption demand decreases by $\{ s_P(1-t) + t\}iK$ unit. On the other hand due to one unit rise in $\delta$ , investment demand decreases by $\gamma_4 K$ unit.[11] Thus aggregate demand and hence the equilibrium degree of capacity utilization decreases.

A rise in debt-capital ratio due to the financial crowding out effect, directly leads to a fall in the equilibrium growth rate. So, the effect of a change in $\boldsymbol{\delta}$ on the equilibrium growth rate is negative.

## The long-run analysis:

Now we will analyze the long-run dynamics of the government debt and the capital stock. We will say that long-run equilibrium is attained when the government debt-capital ratio ($\boldsymbol{\delta}$) remains constant over time.

We know, $\delta = \frac{D}{K}$

So, $\hat{\delta} = \widehat{D} - \widehat{K}$

Again,

$$\frac{dD}{d\tau} = (C_G + I_G) - T + iD$$

$$\rightarrow \frac{\frac{dD}{d\tau}}{D} = \frac{1}{D}[(C_G + I_G) - T + iD]$$

$$\widehat{D} = (\eta + \theta - t)\frac{u^*}{\delta} + i(1-t)$$

Now, $\hat{\delta} = \widehat{D} - \widehat{K}$

[11]This is because of the financial crowding out effect.

$$\rightarrow \hat{\delta} = (\eta + \theta - t)\frac{u^*}{\delta} + i(1-t) - g^*$$

$$\rightarrow \frac{d\delta}{d\tau} = (\eta + \theta - t) + i(1-t)\delta - g^*\delta$$

$$\rightarrow \frac{d\delta}{d\tau} = \frac{(\eta + \theta - t)\gamma}{\Lambda} + \left[-\frac{(\eta + \theta - t)\Gamma}{\Lambda} + i(1-t) - \gamma - \frac{\{\gamma_1 + \gamma_2(1-t).\pi + \gamma_3.\theta\}\gamma}{\Lambda}\right]\delta + [\frac{\{\gamma_1 + \gamma_2(1-t).\pi + \gamma_3.\theta\}\Gamma}{\Lambda} + \gamma_4]\delta^2$$

$$\rightarrow \frac{d\delta}{d\tau} = D_0 + D_1\delta + D_2\delta^2$$

Here, $D_2 > 0$. Let's assume $D_0 > 0$ . This assumption ensures that even when $\delta = 0$, the government runs a deficit and so $\delta$ increases over time.

In Dutt (2013 both of $D_1$ and $D_2$ don't have any definite sign. So, various possibilities regarding $\frac{d\delta}{d\tau}$ may occur depending on the sign of $D_1$ and $D_2$. But in our analysis $D_2$ is unambiguously positive. Then depending on the sign of $D_1$, $\delta$ can either have a stable equilibrium value or it can rise without bound.
Now let us discuss the conditions for existence and stability of equilibrium.

If the interest rate $(i)$ is not too high $\left(if\ i < \frac{\frac{(\eta + \theta - t)\Gamma}{\Lambda} + \gamma + \frac{\{\gamma_1 + \gamma_2(1-t).\pi + \gamma_3.\theta\}\gamma}{\Lambda}}{(1-t)}\right)$ then $D_1$ can have a negative value. Then the change in debt-capital ratio with respect to $\delta$ would be "U" shaped and there is a possibility of existence of equilibrium.

Then the necessary and sufficient condition for existence of equilibrium is: minimum value of $\frac{d\delta}{d\tau}$ must be $\leq 0$ . 0. Minimum value of $\frac{d\delta}{d\tau}$ can be attained at $\delta = -\frac{D_1}{2D_2}$.

Then the minimum value of $\frac{d\delta}{d\tau} = D_0 - \frac{{D_1}^2}{4D_2}$. Thus the necessary and sufficient condition for existence of equilibrium is $\left(D_0 - \frac{{D_1}^2}{4D_2}\right) \leq 0$.

Then the necessary and sufficient condition for existence of a stable equilibrium is: minimum value of $\frac{d\delta}{d\tau}$ must be $< 0$ . Thus $\left(D_0 - \frac{{D_1}^2}{4D_2}\right) < 0$ ensures the necessary and sufficient condition for existence of a stable equilibrium.

Three different diagrams are given below. In figure 1 there is no equilibrium. Figure 2 represents existence of unique but unstable equilibrium. Figure 3 represents existence of multiple equilibria where one of them[12] is stable.

But if $i \geq \frac{\frac{(\eta+\theta-t)\Gamma}{\Lambda}+\gamma+\frac{\{\gamma_1+\gamma_2(1-t).\pi+\gamma_3.\theta\}\gamma}{\Lambda}}{(1-t)}$) then $\delta$ increases without bound. In diagram2 there is only one equilibrium value of $\delta$ which is unstable.

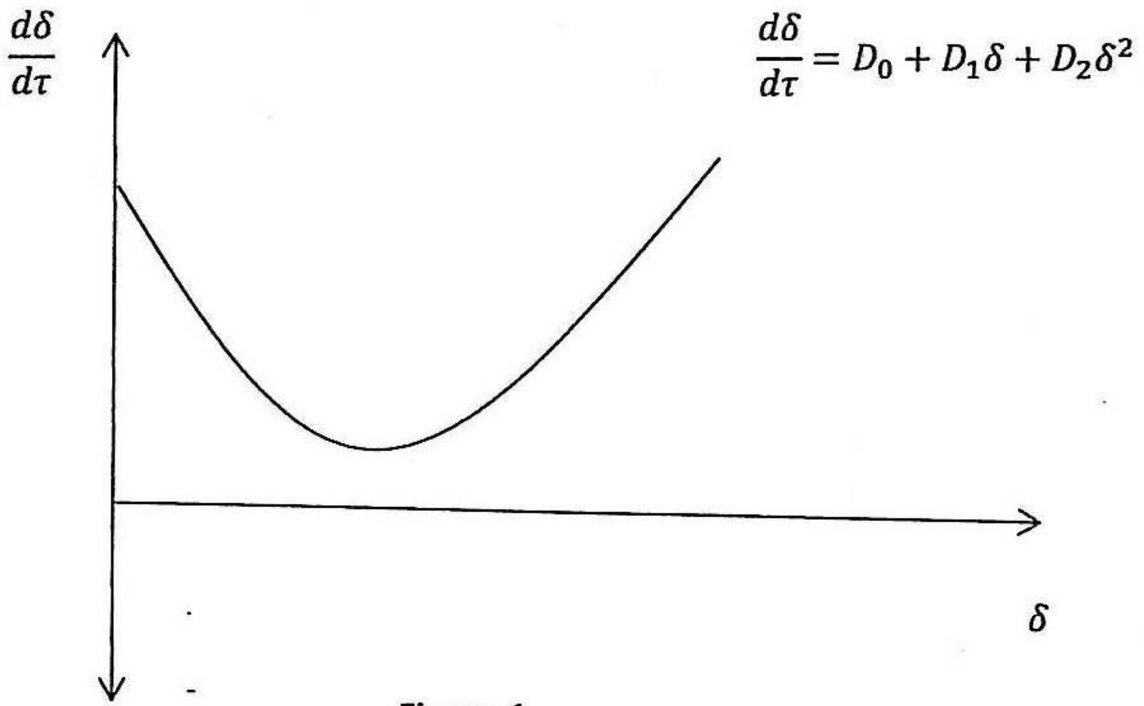

Figure-1

[12] Note that between those two equilibrium values of $\delta$ , the low equilibrium value of $\delta$ (i.e.$\delta^*$ )gives the stable equilibrium.

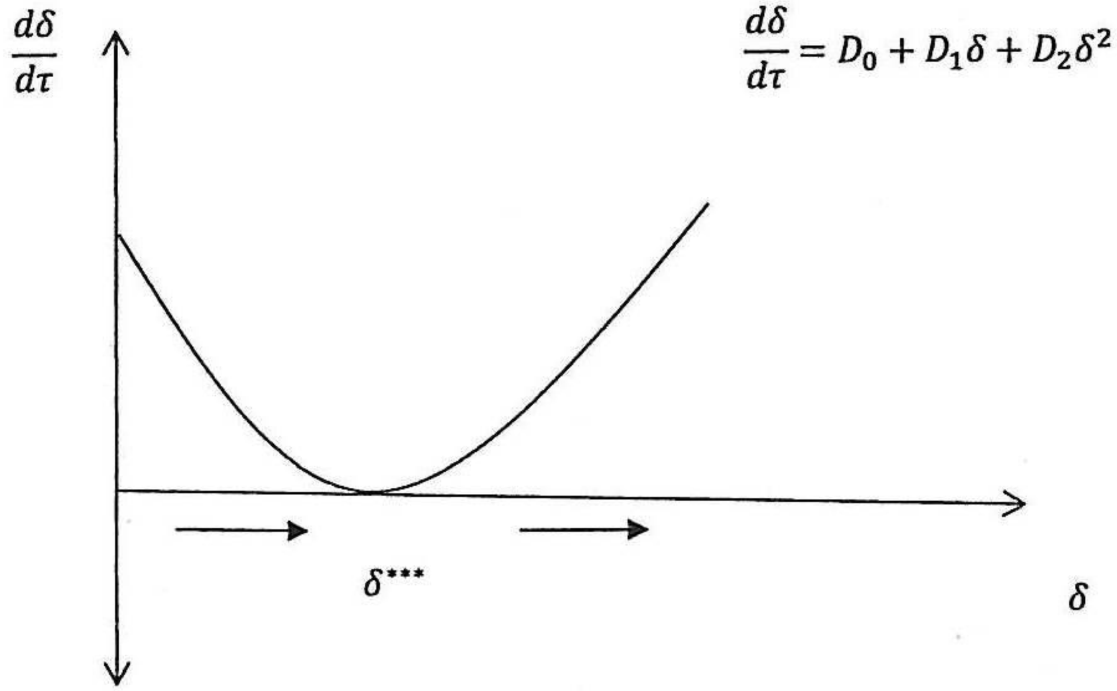


Figure-2

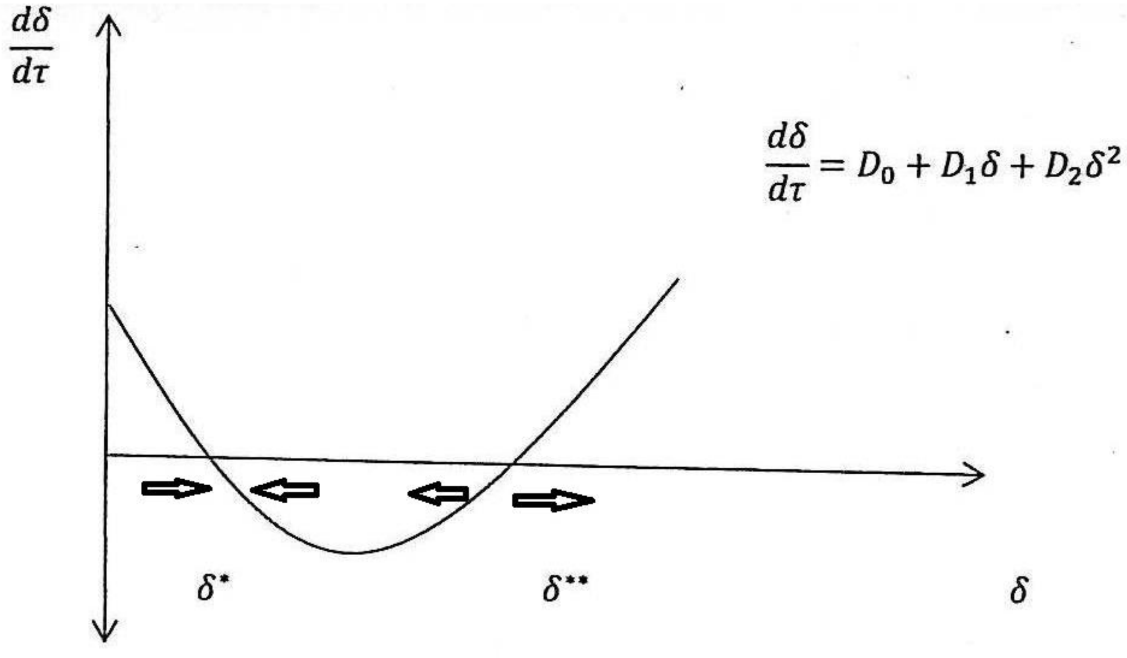


Figure-3

**Conclusion:**

Following a neo-Kaleckian framework we have tried to analyze the impact of expansionary fiscal policy on the growth rate. In the short-run, we have found that an increase in the government consumption expenditure increases the aggregate demand, equilibrium level of capacity utilization and the equilibrium growth rate.

Following Blecker (2002) we have shown that when workers also save, the possibility of an exhilarationist regime arises. When the economy is in exhilarationist regime, an increase in $\theta$ unambiguously raises both the equilibrium level of capacity utilization and the equilibrium growth rate. But when the economy is in the stagnationist regime, the results are not same. Unlike Dutt (2013), a switch in government expenditure from consumption to investment purposes does not always lead to a rise in the equilibrium degree of capacity utilization and the equilibrium growth rate in the stagnationist regime. As a rise in $\theta$ represents simply a switch in government expenditure from consumption to investment purposes, it does not increase aggregate demand and capacity utilization directly. It may raise the aggregate demand through its indirect 'crowding in' effect. On the other hand public investment expenditure through its effect on labour productivity can lead to a rise in share of profit in the economy which in turn decreases[13] aggregate demand and the degree of capacity utilization. Thus the final outcome of a rise in $\theta$ depends on the relative magnitudes of these opposing effects. In this case, although our findings are similar to Commendatore and Pinto (2011), the reason behind it is that unlike a change in capital productivity here it is a change in labour productivity that influences the equilibrium level of capacity utilization which in turn has an impact on the equilibrium growth rate.

When the balanced budget assumption is dropped, an increase in government debt-capital ratio leads to a decrease in the equilibrium level of capacity utilization and the equilibrium growth rate. This is in contrast to the analysis by Dutt (2013), where a rise in the government debt-capital ratio has an ambiguous effect on the equilibrium levels of capacity utilization and the accumulation rate. We also find that a rise in the current government consumption expenditure to output ratio raises the aggregate demand, capacity utilization and the accumulation rate. But a rise in $\theta$[14] has an ambiguous effect on both the equilibrium level of capacity utilization and the accumulation rate. This result differs from Dutt (2013) as there is a positive relation between $\theta$ and $u^*$ and $\theta$ and $g^*$ in his analysis. This is because a rise in public investment expenditure through its effect on labour productivity leads to a rise in share of profit which in turn mitigates the positive effect of a rise in $\theta$ on $u^*$ and $g^*$.

[13] When the economy is in stagnationist regime.

[14] When balanced budget assumption is dropped, a rise in $\theta$ does not imply a rise in government investment expenditure at the cost of consumption expenditure. It simply implies a rise in the ratio of government investment expenditure to income.

Regarding the employment issue, we also have seen that a change in any model parameter, which increases the equilibrium degree of capacity utilization without affecting labour productivity, necessarily increases the employment rate. But the effect of a rise in $\theta$ on the employment rate is ambiguous and it depends on the elasticity of the share of profit and the elasticity of labour productivity with respect to the ratio of government investment to output.

We have also seen that in the long run, a stable government debt-capital ratio is possible, provided that the interest rate is smaller than a critical value.

It should be noted that the results of our analysis are based on a very simple model. We have taken a homogenous tax rate for different classes in the economy. Introduction of different tax rates may change the results. Further, our model is based on the closed economy assumption. Introduction of an open economy framework may significantly change our findings.

In the long-run, we only have considered the dynamics of the government debt-capital ratio and the capital stock. If instead of assuming constant level of labour supply, the profit share and the technological growth, we allow these to vary in the long-run, then the analysis will be more interesting and the results may vary. Hope later on we might incorporate those issues and try to make the analysis more robust.

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