The Political Economy of Publicly Provided Private Goods

Dotti, Valerio

Department of Economics, University College London

27 February 2014

The Political Economy of Publicly Provided Private Goods☆,☆☆

V. Dotti[a]

[a]*Department of Economics, University College London*

Abstract

I study the relationship between income inequality and public intervention in education in a voting model. Traditional Political Economy models typically imply a strong relationship between income inequality and public intervention in redistributive policies. Empirical evidence suggests that this may hold true only for certain kinds of policies, e.g. cash transfers, but it may not hold true in the case of social security and education policies. I propose a method to study this relationship in the case in which forms of redistribution other than the public provision of education are available to voters. Moreover, I allow consumers to opt-out of the public education system and get private education. This assumption implies that individual preferences may exhibit non-convexities and therefore the equilibrium may not be unique.

I show that the relationship between income inequality and governmental intervention implied in the traditional literature is mainly a result of excessively restrictive assumptions. In particular, in a political equilibrium income inequality negatively affects public investment in education if the latter tends to reduce future inequality. This is consistent with the results in the empirical literature about public investment in education.

Keywords: Political Economy, Probabilistic models, public provision, income inequality

1. Introduction

The relationship between the degree of governmental intervention in redistributive policies and the features of the population in democratic political systems has been a major topic of research in Political Economy for over 50 years. Despite of the remarkable research effort and the large literature produced the existing theoretical models are not suitable to explain some of the patterns of

☆Preliminary and incomplete. Please do not circulate.

☆☆I would like to thank I. Preston and A. de Paula for the supervision of this work and V. Bhaskar, M. Bassetto and T. König for the useful comments and suggestions.

Email address: valerio.dotti@ucl.ac.uk (V. Dotti)

governmental intervention described in the empirical literature in the field and in particular the one that is the object of this paper, namely the relationship between income inequality and degree of public intervention in the provision of certain goods, such as public education.

Traditional models imply a strong relationship between these two variables, and this is an implication that is directly related to the redistributive effects[1] of the public provision; more specifically if the public provision is uniform and it is financed through a progressive income tax, then the typical implication is a positive effect of a rise in income inequality[2] on the degree of public intervention.

Empirical evidence suggests that this relationship may hold true only for certain kinds of policies, for instance cash transfers or education, but in the case of other policies with redistributive effects such as social security and some publicly provided private goods it may not hold true. In particular while there is some evidence about a positive relationship between governmental spending in schooling and income inequality (see Easterly and Rebelo, 1993 and Sylwester, 2000) this relationship does not seem to emerge about the provision of other goods such as Health Care.

A second, perhaps more foundamental puzzle that emerge from the traditional models is that they typically imply a positive level of public provision for pure private goods, which is something that it is inconsistent with facts about governmental intervention in democratic countries.

In this paper I aim to show the theoretical issues that make traditional models unsuitable for addressing this problem and I propose an alternative framework that may help to shed light on what really drives the realtionship of interest.

The main result is a method that allows me to derive direction of the effect of a marginal increase in income inequality on the equilibrium level of public provision of a private good with peculiar characteristics in a Probabilistic Voting Model, and the use of this method in order to show two important results.

The first result is that some of the predictions of the traditional models are mainly due to excessively burdensome assumptions and will not survive in less restrictive models. The second result is that the way in which the good is provided may play a crucial role in determining the direction of the relationship of interest, and this finding makes the model useful to describe the peculiar patterns of public intervention in education. More specifically I focus on the effects of the presence of private alternatives to public education and the possibility of opting out of the public sector. My findings suggest that public intervention in education may be affected by income inequality not because of the characteristics that this policy shares with other governmental policies (redistributive effects) but because of its peculiarities in the way in which the provision is

[1]Given that we are interested in in-kind policies the concept of redistributive effects has to be properly defined. See section 2.

[2]More precisely, of a specific feature of the income distribution that is typically the distance between mean and median income.

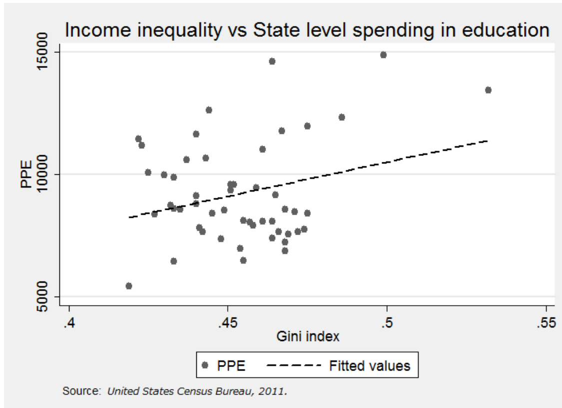
delivered.

Finally I provide an example of an an application of the tool developed in this paper that can be particulary promising: a model of parental investment in education. This is interesting because it potentially provides a testable implication for the mechanism that underpins the comparative statics in my model.

The paper is structured as follows. In section 2 I describe the findings of the empirical literature about public provision of Education and income inequality and in section 3 I explain why the traditional theoretical literature in the field is not suitable to explain some of these fact. In section 4 I describe a Probabilisting Model of Voting and I propose a tool that allows me to study the sign of the relationship between the equilibrium level of public provision of a good of interest and the degree of income inequality in the population of voters. In section 5 I show why this relationship is crucially driven by the way in which the public provision is delivered, and I argue that this finding has interesting consequences in the study of the relationship between income inequality and public intervention in education. In section 6 I provide some comments about the potential of these finding for future research and about the limits of this approach.

2. Facts

There is a large empirical literature about the relationship between income inequality and governmental spending in redistributive policies (see de Mello, Tiongson 2006 for a review of this literature). On the contrary the amount of research about the relationship between income inequality and the degree of public intervention in specific in-kind policies (and Education in particular) is very limited. There is evidence of positive correlation between income inequality and public intervention in schooling, but it is hard to establish a causal relationship between these two variables. The reason is that the relationship is potentially endogenous because of reverse cuasality; on one hand Political Economy models suggest that income inequality may affect the degree of public intervention in certain policies, but on the other hand it is likely that Public investment in Education induces a fall in the degree of future income inequality (see for instance Sylwester, 2002). Figure 1 shows the correlation between the pre-tax Gini index of income inequality in 2010 and the public expenditure per capita in education in the 50 U.S. States (2011).



Easterly and Rebelo (1993) unsing cross section data show that high level income inequality tend to be associated with future high level of public spending in education. Sylwester (2000) also finds a weak but significant positive correlation between income inequality and future public spending in education, even if the issue of reverse causality in the relationship is not completely addressed in his paper.

Despite of these issues is interesting to notice that if the existence of a non-positive impact of public education on income inequality is a theoretically and empirically robust channel that relates the two variables, then the fact that the two variables are positively correlated suggest that there may be a non negligible positive effect in the opposite direction of causality (even if other sources of endogeneity may affect this hypotesis).

In strong contrast with these facts, there is little evidence of any relationship between other forms of public provision and income inequality and the relationship between income inequality and transfers seems to be null or negative (see Bassett et al., 1999).

This implies that a model that aims to succesfully explain these facts must allow the researcher to separate the effects of governmental intervention in education from the ones of other source of public spending and must provide some theoretical reason for the different effects of income inequality on different public spending variables.

In the next section I will explain why traditional models are not suitable to address this question.

3. Downsian Models

The seminal works of Black (1948) and Downs (1957) provides a framework that allows to analyze problems of Political Economy that have public intervention in spending policies as object. One of the main advantages of their approach is that it delivers a characterization for the outcome of the political interaction that is very attractive both for its ease of interpretation (Majority Voting Equilibrium) and for its immediate and sharp implications in terms of social choice and comparative statics (the Median Voter Theorem). In the so-called Downsian models the socially preferred policy in equilibrium is the the one that maximizes the objective function of the Median Voter, therefore it is sufficient to analyze the choice of one specific individual in order to characterize the social choice (see AppendixA for an example).

The use of Downsian models to study the patterns of public intervention in specific policies such as the public provision of private goods had to face important theoretical issues due to the different ways in which the provision of this kind of goods can be carried out (see Stiglitz, 1974) but it proved successful in characterizing a Political equilibrium based on the concept of Pivotal Voter (see for instance Epple and Romano 1996a, 1996b and Gouveia, 1997). Therefore these models share the same interpretative advantages of the original model proposed by Downs but they do not always deliver similar sharp predictions about the comparative statics of interest.

Despite of the positive features listed above and the success in the traditional literature in Political Economy, I claim that this kind of models is not suitable the analysis of the relationship between income inequality and the degree of public intervention in imperfect public goods or private goods with externalities.

In order to understand why this is the case it may be useful to recall that the existence of a Majority Voting equilibrium relies on two main assumptions:

the first is that the policy space is restricted to a single dimension (i); the second is that voters' preferences satisfy the Spence-Mirrlees condition[3] in a unidimensional parameter (ii); it is also important to underline that the outcome of these models in equilibrium is characterized as the policy that is preferred by a single individual (Pivotal Voter) (iii).

Assumpion (i) implies that the public provision is the only channel through which redistribution can be achieved, hence these models imply that pure private goods should be publicly provided in equilibrium (see Usher, 1977); assumption (ii) implies that the equilibrium social choice depends uniquely on the distribution of this parameter and this generates a strong relationship between some feature of the distribution and the degree of public intervention in equilibrium. In the analysis of the effects of political interaction on public spending policies the natural choice for this parameter is some measure of income or earnings (see Usher 1977, Epple and Romano 1996a, 1996b). Finally result (iii) implies that the effects of a change in the policy on any voter but the Pivotal one do not affect the equilibrium outcome.

The analysis suggests that in Downsian models the strong relationship between the degree of income inequality among voters and the degree of governmental intervention in policies with redistributive effects is equivalent to the demand for public intervention of a single individual; in the analysis of public provision of private goods This means that the comparative statics of these models is equivalent to the change in the demand for the good of this specific

[3]Some papers the more general Single Crossing Property. The relationship between The S-M condition and SCP is described in Migrom, Shannon (1994).

individual induced by changes in income and in the tax-price of the good[4]. This implies in turn that goods that have similar feature in terms of demand on the private market should be characterized by similar patterns in the public provision, and that if the tax system is progressive enough the effect of income inequality on publicly provided private goods should be generally positive regardless of the features of the good (see AppendixB for an example).

Empirical evidence suggests that this may not be true, for instance the public provisions of Health Care and public goods seem to exhibit very different patterns in comparison with public education [5].

In order to solve this problem it is necessary to analyze the choice of voters in a multidimensional policy space in which at least another (possibly pure) redistributive variable is available to voters (for instance a uniform in-cash grant) such that the social demand for redistribution and for public intervention in a specific in-kind public provision can be disentangled. Moreover it would be desirable to develop a model in which the effects of a policy on all voters contribute in determining the political outcome, because the role of the Pivotal Voter in the traditional models is the result of an unrealistic description of the political process that may affect (and I claim in a decisive way in section 4) the comparative statics of interest. Unfortunately these two requirements cannot be achieved in the traditional framework because the unidimensionality assumption is essential for the existence of a Majority Voting equilibrium in Downsian Models[6], and because the deterministic nature of voting in these models implies that only ordinal factors matter (i.e. if for an individual i $x^A \succ x^B$, he will vote for candidate A with probability 1, independent of the *intensity* of this preference).

The literature in Political Economy accounts for different classes of model that provide one or both these desirable features. Citizen-candidate models (Besley and Coates, 1991, 1997) allow for a multidimensional policy space but their equilibrium outcomes depend on few individuals; more generally they are not suitable for studying the problem of interest because they do not usually deliver sharp prediction about the policy that should be chosen by a certain community. This is due to the multiplicity of equilibria that is a usual outcome in this models. More recent equilibium concepts such as the Party Unanimity Nash Equilibrium (Roemer, 1999) or the one in Levy (2005) meet both the requirements, but they are also not useful because of the same multiplicity issue and for the lack of a useful characterization of the policy chosen in equilibrium.

The choice of a Probabilistic Model relies on two appealing features of this kind of models: on one hand they meet the two requirments described above, on the other hand they imply a unique equilibrium under assumptions that are relatively mild and easy to impose and the equilibrium consists of a single chosen

---

[4]Defined as total taxes paid by the individual divided by the size of the public provision.

[5]For a review of the empirical literature about the relationship between income inequality and public spending see De Mello, Tiongson (2006)

[6]see Grandmond (1978) and Plott (1967) for an analysis of the effects of multidimensionality in Downsian models

policy. The main shortcoming in the use of this class of models in analysing problems similar to the one that is the object of this paper is the lack of analitical tools to study the effects of changes in the distribution of voter's characteristics and the policy chosen in equilibrium. The development of one of these tools is one of the main contributions of this paper. In the next section I propose a two-candidates Probabilistic Voting model in which individuals differ in one single observable dimension (income) and can vote over a multidimensional policy space and I derive the comparative statics induced by a marginal increase in the variance of the income distribution keeping the mean constant (i.e. the effect of a marginal mean preserving spread).

4. A Probabilistic Voting Model with Convex Utility

In this section I will present a relatively simple model of Probabilistic Voting that is substantially similar to the ones of this kind in the literature such as the one proposed by Lindbeck and Weibull (1987), Enelow and Heinrich (1989), Banks and Duggan (2004). The key feature of this models is that the vote of every individual (and therefore the outcome of the election) is not deterministic. This assumption eases dramatically the conditions for the existence of a Political Equilibrium when the policy space is multidimensional in comparison with Downsian models.

The price to pay is that we cannot obtain a simple characterization of the equilibrium outcomes and that there are very little results about the comparative statics in these models.

In the next section I will provide sufficient conditions for existence and uniqueness of a political equilibrium, while in section 4.2 I will propose a method to study the comparative statics induced by a marginal mean-preserving spread in the income distribution.

4.1. Existence and Uniqueness

In this section I will describe the model and I will state results of existence and uniqueness of the political equilibrium wich are minor modifications of the ones in Banks, Duggan (2005) and Enelow, Hinich (1989).

The consumer-voters differ from each other only in a unidimensional parameter $t \in T$ that is continuously distributed with c.d.f. $G(t)$.

A feasible policy is a vector $x \in X$ where $X := \left\{x | x \in R_+^n \cap C(x, G(t)) \leq 0\right\}$ is a convex set and $C(x, G(t)) \leq 0$ is the governmental budget constraint.

There are 2 parties: A and B. Before the election the two parties simultaneously choose a feasible policy x^A and x^B, respectively.

A voters supports party A if $v(x^A, t) - v(x^B, t) + \epsilon \geq 0$ where $v(x, t)$ is the indirect utility induced by policy x to an individual with parameter t and ϵ is a parameter unobserved by the parties that captures the idiosincratic preference of an individual for party A. Assume ϵ is i.i.d. across voters. then we can define the probability that an individual with parameter t votes for party A

given policies x^A, x^B as

$$Pr\left(\epsilon \le v(x^A,t) - v(x^B,t)\right) = \mathbb{P}(v(x^A,t) - v(x^B,t))$$

where $\mathbb{P}(\cdot)$ is an increasing C^2 function. The probability that an individual with parameter t votes for party B given policies x_A, x_B is simply $1 - \mathbb{P}\left(v(x_A,t) - v(x_B,t)\right)$. Therefore the expected plurality for party A from proposing policy x_A given the policy proposed by party B is given by:

$$V^A(x^A,x^B) \equiv E_t[\mathbb{P}(v(x^A,t) - v(x^B,t))] = \int_{\underline{t}}^{\overline{t}} [\mathbb{P}(v(x^A,t) - v(x^B,t))]g(t)dt$$

And streightforwardly the expected plurality for party B is $V^B(x^A,x^B) = 1 - V^A(x^A,x^B)$.

Each party maximizes the expected number of votes[7]. Therefore the best response for party A to policy $\overline{x}^B$proposed by party B is a policy x^A_{BR} such that:

$$x^A_{BR} \equiv \arg\max_{x\in X} V^A(x,\overline{x}_B)$$

This setting is relatively standard and because of that some results in Enelow, Hinich (1989) and Banks, Duggan (2004) hold with minor adjustments. I am going restate these results for this specific setting.

Theorem 1. *(Mixed strategies equilibria) Assume (i) X is compact and* (ii) for each t, $\mathbb{P}(v(x^A,t) - v(x^B,t))$ is jointly continuous in (x^A,x^B). *Then there exists a mixed strategy electoral equilibrium of the electoral game.*

Proof. See Banks, Duggan (2004).

I prove that condition (i) and (ii) are satisfied under the assumptions of the model.

(i) Compactness. Using the definition the definition of the policy space in our paper X is a closed and bounded subset of R^n_+, hence it is a compact set.

(ii) Joint continuity. This holds for this objective function.

Proof. Define $Y = X \times X$, $\tilde{v}^C : Y \to R$ such that $y = (x^A,x^B) \Rightarrow \tilde{v}^C(y,t) = v(x^C,t)$ for $i = A,B$.

Suppose $\tilde{v}^C : Y \to R$ are all continuous maps. Then, the map $h_i : Y \to R$ given by $h^C(y,t) = \mathbb{P}^C(\tilde{v}^A(y,t), \tilde{v}^B(y,t))$ is a jointly continuous map.

Similarly, the expectation is a continuous map of $h^C(y,t)$ for $\forall t \in T$ to R. Hence $V^C(x_A,x_B)$ is jointly continuous in (x_A,x_B). Q.E.D.

Theorem 2. *(Pure strategy equilibria) Assume (i) X is compact and convex, (ii) for each t, $\mathbb{P}(v(x^A,t) - v(x^B,t))$ is jointly continuous in (x^A,x^B), (iii) for each (x^A,x^B), $V^A(x^A,x^B)$ is quasi-concave in x^A and $V^B(x^A,x^B)$ is quasi-concave in x^B. Then there exist a pure strategy electoral equilibrium.*

---

[7] Aranson, Hinich and Ordeshook (1974) have shown that in Probabilistic Voting models this is equivalent to maximize the expected plurality and, as the number of voters approaches infinity, it is also equivalent to maximize the probability of winning the elections.

Proof. See Banks, Duggan (2004).

(i) Compactness. Same as in Theorem 1. *Convexity.* The space of mixed strategies for the candidates is convex if the governmental budget set is a convex set.

(ii) Joint continuity. Same as in Theorem 1.

(iii) Quasi-concavity. We will verify conditions for concavity, which implies the condition required. Theorem 3 establish sufficient conditions for concavity to hold.

Define $V(x, \overline{x}^B)$ as above, $d^A(x,t) \equiv v(x,t) - v(\overline{x}^B, t)$, H_v is the Hessian matrix of $v(x,t)$, i.e. $H_v^{ij} = \frac{\partial^2 v(x,t)}{\partial x_i \partial x_j}$.

Theorem 3. *(Existence). Assume $\mathbb{P}(\cdot)$ and $v(x,t)$ are C^2 functions in their domains. Then there xists a pure strategy electoral equilibrium if (i)$V^A(x, \overline{x}^B)$ is concave in x if for any $x \in X$ and (ii) for any vector $y \in R^n$ the following condition holds:*

$$\int_T \left(p'(d^A(x,t)) \left[y^T \nabla v(x,t) \right]^2 + p^A(d^A(x,t)) y^T H_v y \right) g(t) dt \leq 0$$

A similarly condition can be derived for concavity of $V^B(\overline{x}^A, x)$ in x.

Proof. See Enelow, Hinich (1989).

Notice that this condition implies that as long as $v(x,t)$ is strictly concave in x, a sufficient condition for this to hold is

$$\frac{p'(d^A(x,t))}{p(d^A(x,t))} \leq -y^T H_v y \left[y^T \nabla v(x,t) \right]^{-2}$$

i.e. the probability distribution $\mathbb{P}$ must be sufficiently “flat”[8].

Theorem 4. *(Uniqueness). Assume (i) X is compact and convex, (ii) for each t, $\mathbb{P}(v(x^A,t) - v(x^B,t))$ is jointly continuous in (x^A, x^B), (iii) for each (x^A, x^B), $V^A(x^A, x^B)$ is strictly concave in x^A and $V^B(x^A, x^B)$ is strictly concave in x^B. Then there is exactly one electoral equilibrium, and it is in pure strategies.*

(i) Compactness. Same as in Theorem 3. Convexity. In out problem is satisfied if and only if the Governmental Budget set $C(x, G(t)) \leq 0$ is a convex set. In problems with non-linear budget sets this conditions may fail to apply (see next sections).

(ii) Joint continuity. Same as in Theorem 1.

(iii) Strict concavity. Conditions are the same as in Theorem 3, except that we need to verify the negative definitiveness of the Hessian Matrix.

Theorem 5. *(Policy convergence): under assumptions (i), (ii), (iii) of Theorem 4, the unique electoral equilibrium is such that the two parties choose the same policy.*

[8] See Enelow, Hinich (1989) for an example of how this condition simplifies under specific parametric assumptions.

Proof. See AppendixC.3.

Theorem 6. *(Utilitarian outcome): if all individuals have the same Party preference distribution, i.e.* $\mathbb{P}(\cdot, t) = \mathbb{P}(\cdot)\forall t$ *then the policy chosen by both parties in equilibrium is the same as the policy that would be chosen by an omnipotent Benthamite government, i.e. the policy that maximizes the Utilitarian Social Welfare function.*

Proof. See AppendixC.4.

4.2. Main result

In this section I describe the way in which a change in income inequality is defined in this paper. I will use the concept of Mean Preserving Spread, i.e. a distribution $G(t)$ is a MPS of $F(t)$ if and only if $E_G(t) = E_F(t)$ and $VAR_G(t) > VAR_F(t)$. Notice how this concept is much more general and easy to interpret in comparison with the distance between mean and median that typically drives the comparative statics in Downsian models.

Mean preserving spread are imposed as follows: $F(t)$, $G(t)$ are two c.d.f.s such that $\int_{\underline{t}}^{t} [G(t) - F(t)]\, dt \geq 0$ for all $t \leq \bar{t}$ (i.e. $F(t)$ Second Order Stochastically Dominates $G(t)$), and $\int_{\underline{t}}^{\bar{t}} [G(t) - F(t)]\, dt = 0$ (i.e. $G(t)$ is a mean preserving spread of $F(t)$). Define:

$$r(t, \theta) \equiv f(t) + \theta\, [g(t) - f(t)]$$

where $f(t)$, $g(t)$ are the p.d.f.s of $F(t)$ and $G(t)$. Notice that $r(t, \theta)$ is a Mean Preserving Spread of $f(t)$ for all θ.

$$\int_{\underline{t}}^{\bar{t}} tr(t, \theta)dt = (1 - \theta) \int_{\underline{t}}^{\bar{t}} tf(t)dt + \theta \int_{\underline{t}}^{\bar{t}} tg(t)dt = E(t)$$

Moreover, the effect of moving θ in a neighborhood of $\theta = 0$ corresponds to the effect of increasing Income Inequality keeping the mean constant. Hence the derivative of the equilibrium value of a choice variable with respect to θ corresponds to the comparative statics of interest.

4.2.1. Comparative statics

Assume that the conditions of Theorem 4 are satisfied, and that $C(x, R(t, \theta))$ is an increasing and convex C^2 function. Then Theorem 6 can be used to derive the comparative statics induced by an increase in income inequality. In particular, given that the political equilibrium is the same as the Utilitarian outcome, the comparative statics will also be the same. Hence I will study the simpler problem:

$$\max_{C(x,R(t,\theta)) \leq 0} \int_{\underline{t}}^{\bar{t}} v(x, t) r(t, \theta) dt \tag{1}$$

The Lagrangean of this problem is:

$$L = \int_{\underline{t}}^{\bar{t}} v(x,t)r(t,\theta)dt - \lambda C(x, R(t,\theta))$$

Given that the objective function is strictly convex and the budget set is compact the solution is internal. The First Order Conditions are:

$$L_i = \int_{\underline{t}}^{\bar{t}} v_i(x,t)r(t,\theta)dt - \lambda C(x, R(t,\theta)) = 0\,\forall i$$

$$L_\lambda = C(x, R(t,\theta)) = 0$$

Differentiate F.O.C.s w.r.t. θ and evaluate them at $\theta = 0$ to get

$$\left(\sum_j L_{ij}\frac{\partial x_j(0)}{\partial\theta}\right) + L_{i\lambda}\frac{\partial\lambda}{\partial\theta} + L_{i\theta} = 0\,\forall i \tag{2}$$

This will give a sistem of $n+1$ equations and $n+1$unknown that can be solved using the Cramer's rule, giving:

$$\frac{\partial x_i(0)}{\partial\theta} = -\frac{\det\left[N^i(L,\theta)\right]}{\det\left[D^2(L)\right]}\,\forall i$$

where $N^i = (n+1 \times n+1)$ is a transformation of $D^2(L)$ in which the i_{th} column of $D^2(L)$ is replaced with ∇L_θ, which is the $(n+1\times 1)$ vector such that $\nabla\left(L_\theta\right)_i = L_{i\theta}$. We need to impose some more restrictions in order to derive the sign of the derivative of interest. Different assumption may deliver a useful characterisation of the comparative statics and may be more or less restrictive depending on the specific problem of interest. In the next section I present some sets of restrictions that may be interesting for the question that is the object of this paper.

The first restriction is to assume transferable utility, i.e. $v(x,U,t) = u(x,t) + sU$ where x is a $(n-1\times 1)$ vector.

Theorem 7. *(Monotonicity): Assume transferable utility. Suppose (x,U) is an internal political equilibrium and the Lagrangean L is supermodular in x and satisfies the single crossing property in (x,t), i.e. $L_{ij} \geq 0\,\forall i \neq j$ and $L_{i\theta} \geq 0\,\forall i$. Then x is weakly increasing in θ (notice the similarities with Milgrom, Shannon 1994).*

Proof. See AppendixD (missing).

Theorem 7 immediately suggest the main difficulty in deriving the sign of the comparative statics, namely how one can derive the sign of $L_{i\theta}$and L_{ij}, and this is going to be the object of the next paragraph.

Assume $C(x,U,R(t,\theta)) = \int_{\underline{t}}^{\bar{t}} c(x,t)r(t,\theta)dt + U$. Recall the first order condition of problem (1) with respect of x_i is:

$$L_i = \int_{\underline{t}}^{\bar{t}} v_i(x,t)r(t,\theta)dt - \lambda \int_{\underline{t}}^{\bar{t}} c_i(x,t)r(t,\theta)dt$$

Differentiate with respect to θ, which is equivalent to impose a marginal mean preserving spread in the distribution of t(see above), to get:

$$L_{i\theta} = \int_{\underline{t}}^{\bar{t}} \left[v_i(x,t) - \lambda c_i(x,t)\right]\left[g(t) - f(t)\right] dt$$

Finally, integrate it by parts twice and evaluate it at $\theta = 0$ to obtain:

$$L_{i\theta}(0) = \int_{\underline{t}}^{\bar{t}} \left[v_{itt}(x,t) - \lambda c_{itt}(x,t)\right] \left(\int_{\underline{t}}^{t} \left[G(s) - F(s)\right] ds \right) dt$$

Notice that $\int_{\underline{t}}^{t} [G(s) - F(s)]\, ds \geq 0 \,\forall t$ because $F(s)$ Second Order Stochastically Dominates $G(s)$ (see beginning of this section). Hence this implies:

$$sign(L_{i\theta}) = sign(v_{itt}(x,t,n) - \lambda c_{itt}(x,t))$$

If one is interested in the comparative statics of only one specific element of x, say x_i, there are other assumptions that are sufficient for monotone comparative statics. One interesting set of assumptions is for example additive separability and linear budget constraint.

Define x_{-i} as the $(n-2 \times 1)$ vector including all the elements of x but the element x_i.

Theorem 8. *(Monotonicity 2): Assume the objective function is in the form* $\tilde{v}(x_{-i}, x_i U, t) = a(x_{-i}, t) + u(x_i, t) + sU$ *and the governmental budget constraint is linear in* x_i*. Suppose* (x_{-i}, x_i, U) *is an internal political equilibrium. Then* x *is weakly increasing in* θ *if* $u_{itt}(x_i, t) \geq 0 \,\forall x_i, t$*.*

Proof. Use equation (2) and Theorem 7. Notice that if the indirect utility is additively separable in x_i and the budget constraint is linear in x_i then $L_{ij} = 0 \,\forall j \neq i$. Transferable utility implies λ is constant at equilibrium. Hence (2) simplifies to $\frac{\partial x_i(0)}{\partial \theta} = -\frac{L_{i\theta}}{L_{ii}}$. Now notice that L_{ii} must be negative because of the Second Order Condition of the maximization problem, hence $sign\left(\frac{\partial x_i(0)}{\partial \theta}\right) = sign(L_{i\theta})$. Finally notice that additive separability and linearity of the budget constraint in x_i imply $L_{i\theta} = \int_{\underline{t}}^{\bar{t}} u_i(x,t)\,[g(t) - f(t)]\, dt$. Integrate this by parts twice and evaluate it at $\theta = 0$ to get $L_{i\theta}(0) = \int_{\underline{t}}^{\bar{t}} u_{itt}(x_i,t) \left(\int_{\underline{t}}^{t} [G(s) - F(s)]\, ds\right) dt$. Because $F(s)$ SOSD $G(s)$ this imply $\frac{\partial x_i(0)}{\partial \theta} \geq 0$ if $u_{itt}(x_i,t) \geq 0 \,\forall x_i, t$. Q.E.D.

4.2.2. Example: Public Good

Assume linear Governmental Budget constraint: $C(x, Y, U, R(t, \theta))) = -\int_{\underline{t}}^{\overline{t}} -t+ I(x,t)r(t,\theta)dt + U + PY \leq 0$ where Y is the amount of Public Good provided by the government and x is a vector of tax and transfer variables such that the post-tax income of an individual with gross income t is $I(x, t)$. Assume additive separability of indirect utility between x and Y, and transferable utility, i.e. the indirect utility is $v(x, Y, U, t) = a(x, t) + u(Y, t) + sU$. Use Theorem 8 with $x_i = Y$ to get:

$$sign\left(\frac{\partial Y(0)}{\partial \theta}\right) = sign\left(\int_{\underline{t}}^{\overline{t}} u_{itt}(x_i, t)\left(\int_{\underline{t}}^{t} [G(s) - F(s)]\, ds\right) dt\right)$$

Notice that this depends only on the part of the utility function in which Y is an argument (all the redistributive elements disappeared).

Moreover, it is unaffected by the tax system chosen $T(x, t)$. The shortcoming is that economic theory does not give us hints about the sign of $u_{Ytt}(Y, t)$ and it does not seem a feature of voters' preferences that can be easily tested. Nevertheless, notice that the result depends on the third derivative $u_{itt}(x_i, t)$ which has small magnitude for many specifications of utility. In particular if $u_{itt}(x_i, t)$ is constant $\frac{\partial Y(0)}{\partial \theta} > 0$ because $\int_{\underline{t}}^{\overline{t}} \left(\int_{\underline{t}}^{t} [G(s) - F(s)]\, ds\right) dt = VAR_G(t) - VAR_F(t) > 0$. Hence, differently from Downsian Models, in this model it seems unlikely to get a strong effect of income inequality on the degree of governmental intervention in Public Goods, although the result does not imply an interpretation that is equally straightforward.

5. Non-convex Utility

This section is dedicated to model the effects of political interaction on those private goods that because of some special features (externalities, merit goods, etc.) are publicly provided at least to some extent in most countries. This group includse Health Care, Education, childcare, etc. Although in most of the examples I am not going to model explicitly the feature that induce a positive level of provision, it is worth to keep in mind that pure private goods are not going to be publicly provided in Probabilistic models if we allow sufficient flexibility in the tax system. This paragraph is going to play a crucial role in order to understand some peculiarities in the public provision of schooling that are crucial in shaping the comparative statics.

I assume that the good is uniformly provided across individuals. About the way in which the provision is delivered there are three main possibilities that have been described in the literature:

1. Exclusive provision (socialization of commodities). The publicly provided good is not available on the private market (an example is Usher, 1977). In terms of comparative statics this case is totally equivalent to the case of a Public Good described in section 4.

2. Top-up good. The dimension of choice is *quantitative*, hence individuals receiving a certain level of public provision can decide to supplement this quantity with private purchases (examples are in Epple and Romano 1996a and Gouveia, 1997). This case implies similar consequences for comparative statics as the one of case 1 (see AppendixE, missing).
3. Opting-out good. The dimension of choice is *qualitative*, this meaning that individuals can either enjoy the publicly provided good or purchase a different level of quality on the private market (no supplementation occurs).

This second case is particularly interesting both because it resembles the way in which some goods are provided (e.g. education), and because they imply additional complexity in modelling political interaction due to the non-linearity of the governmental budget constraint and the possibility of non-convexities in individual preferences. The effects of the opting-out assumption have been extensively analysed in Downsian models (Epple and Romano, 1996b). In the next section I will focus on this kind of goods in the framework of a Probabilistic Voting model in the form of the one presented in section 4.

5.1. Opting out

Suppose that a private good is uniformly publicly provided with quality level Q and is also available on the private market at different quality levels $\{q_1, q_2, \ldots, q_N\}$ at price Pq_n (the assumption of cost of private purchase linear in quality is a simplification that does not imply any loss of generality).

The first consequence of the opting out assumption is that the governamental budget set cannot be linear as in the case presented in section 4. In order to understand why this is true, consider the following way of modelling uniform provision of an opting out good. The quality of the good provided Q is equal to the total expenditure in the good E divided by the number of individuals that use the public service and the price per unit of quality P. keeping the assumptions that the consumers-voters are a continuum of size 1, this means $Q = \frac{E}{P\int 1(t)g(t)dt}$ where $1(t)$ is an indicator function such that $1(t) = 1$ if an individual with income t does not opt-out, $1(t) = 0$ otherwise. As the choice of opting out is endogenous it is evident that a governmental budget constraint in the form proposed in case 1, e.g. $\int_{\underline{t}}^{\bar{t}} (T(x,t) + PQ1(t)g(t))\, r(t,\theta)dt + U \leq 0$ it is not generally linear.

The second consequence is that the convexity assumption of the party's objective function may not always be met. First of all the opting out assumption makes the objective function more complex, for instance consider the indirect utility of an individual with income t:

$$v(x,t) = \max\left(v(x,t,n), v(x,t,m)\right)$$

Where$v(x,t,m)$ is the indirect utility if the individual decides to opt out and $v(x,t,n)$ is the indirect utility of an individual that does not opt-out. This implies that even if $v(x,t,n), v(x,t,m)$ are C^2 conacave functions, the function $v(x,t)$ may be neither differentiable in all the points of his domain nor concave.

5.2. Existence and uniqueness of a political equilibrium

In order to keep the problem tractable we will assume $v(x,t,n)+\eta(t)-v(x,t,m)$ is monotone in t (A1). This assumption implies that for each vector of policies x either $v(x,t)+\eta(t)\geq v(x,t,n)$ i.e. no opting out occurs, or $v(x,t)+\eta(t)\leq v(x,t,m)$ i.e. all individuals opt out, or $\exists\hat{t}(x)$ such that

$$v(x,t)=\begin{cases} v(x,t,n) & if \quad t\geq(\leq)\hat{t}(x) \\ v(x,t,m) & if \quad t<(>)\hat{t}(x) \end{cases}$$

This assumption simplifies the problem in such a way that we can restate Theorems 3-6 such that the same properties hold with the In AppendixC I provide sufficient conditions for this to hold. In the next subsection I will derive the comparative statics induces by a marginal mean preserving spread in the income distribution in this more complex case.

5.2.1. Comparative statics

Again, the complexity induced by the presence of an endogenous threshold implies the need of further restrictions in order to derive the sign of the comparative statics that is the object of this paper.

Assumption A2: the distribution of the variable T is *decomposable* into a uniform distribution plus another distribution such that $T=Z+\Sigma$ where $Z\sim R(z,\theta)$ and $\Sigma\sim Unif[\underline{\epsilon},\overline{\epsilon}]$ such that:

- $\underline{\epsilon}+\underline{z}\geq 0$ (this ensure no negative income)
- $\overline{\epsilon}-\underline{z}-pq_1\geq 0$ (technical assumption: ensures that even an individual with the lowest t opts-out with positive probability).
- Quasi-linear utility[9]: $v(x,Q,U,z,\epsilon,n)=I(x,z+\epsilon)+u(Q,z+\epsilon)+s(U)+\eta(z+\epsilon)$.
- Private market: discrete different quality levels are available $\{q_1,q_2,\ldots,q_N\}$ at price Pq_n.

$$v(x,U,z,\epsilon,m)=\max_{q_i} I(x,z+\epsilon-Pq_i)+u(q_i,z+\epsilon)+s(U)$$

$$v(x,Q,U,z,\epsilon)=\max\left(v(x,Q,z,\epsilon,n),v(x,Q,z,\epsilon,m)\right)$$

Define a new threshold level $\tilde{\epsilon}(x,Q,t)$ as:

$$v(x,Q,U,z,\tilde{\epsilon},n)=v(x,U,z,\tilde{\epsilon},m)$$

[9]The same sign for the derivative of interest can be achieved if the tax rate is linear with quadratic utility in consumption, and transferable in a third variable U i.e. $U(I,q,U,t)=I-\gamma I^2+u(q,t)+sU$ (work in progress).

Notice that given assumption A1 this threshold, if it exists, it must be unique given x, Q, U, z.

Result 1: the sign of the comparative statics is unaffected if we limit the set of alternatives available on the private market to only one specific option, namely: $\bar{q} = \arg\max_{q_i} \{I(x, z + \tilde{\epsilon}) - Pq_i + u(q_i, z + \tilde{\epsilon}) + sU\}$

Proof. See AppendixF.

From now on I will assume that only one option q is available on the private market. Denote with k the density of ϵ. The utilitarian outcome is given by:

$$V(x, Q, \theta) = \int_{\underline{z}}^{\overline{z}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}(z)} [I(x, z + \epsilon) + u(Q, z + \epsilon)]\, kd\epsilon r(z, \theta) dz +$$

$$\int_{\tilde{\epsilon}(z)}^{\overline{\epsilon}} [I(x, z + \epsilon) - Pq + u(q, z + \epsilon)]\, kd\epsilon r(z, \theta) dz + s(U)$$

Notice that the expected Social Welfare is now composed by two parts, which corresponds to individuals that opt-out and the ones that do not.

The governmental budget constraint is also more complex in comparison with the convex utility case because of the endogeneity of the threshold $\tilde{\epsilon}(z)$ as I described in section 3.1. Hence if one assumes additive budget constraint this will be as follows:

$$\int_{\underline{z}}^{\overline{z}} \left(\int_{\underline{\epsilon}}^{\overline{\epsilon}} -t - \epsilon + I(x, z + \epsilon) kd\epsilon + U + PQ \int_{\underline{\epsilon}}^{\tilde{\epsilon}(z)} kd\epsilon \right) r(z, \theta) dz + U \leq 0$$

Where $\int_{\underline{z}}^{\overline{z}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}(z)} kd\epsilon r(z, \theta) dz$ represents the share of individuals that do not opt out. Notice that the fact that an optimum is an internal solution allows us to substitute $I(x, z + \epsilon)$ from the budget constraint into the objective function (quasilinearity of the utility function does not harm the previous assumption of strict concavity of the objective function because $I(x, z+\epsilon)$ does not have to be linear). Hence the problem becomes equivalent to the unconstrained problem:

$$\max_{Q,U} \int_{\underline{z}}^{\overline{z}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}(z)} [-PQ + u(Q, z + \epsilon)]\, kd\epsilon r(z, \theta) dt + \int_{\tilde{\epsilon}(z)}^{\overline{\epsilon}} [-pq + u(q, z + \epsilon)]\, kd\epsilon r(z, \theta) dz + s(U) - U + E_{t,\epsilon}(z+\epsilon)$$

Using the same method I have shown in section 2, we need to derive the sign of $L_{Q\theta}, L_{QU}, L_{U\theta}$.

$$L_{Q\theta} = \int_{\underline{z}}^{\overline{z}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}(z)} [-P + u_q(Q, z + \epsilon)]\, kd\epsilon \,[g(z) - f(z)]\, dz =$$

$$= -k \int_{\underline{z}}^{\overline{z}} u_{qt}(Q, z + \underline{\epsilon}) \left(\int_{\underline{z}}^{z} [G(s) - F(s)]\, ds \right) dz$$

It is easy to show that $L_{QU} = 0$, $L_{U\theta} = 0$ because of the additive separability of U in the utility function. Hence the sign of the comparative statics is given by:

$$sign\left(\frac{\partial Q(0)}{\partial \theta}\right) = sign\left(-\int_{\underline{z}}^{\overline{z}} u_{qt}(Q, z + \underline{\epsilon}) \left(\int_{\underline{z}}^{z} [G(s) - F(s)]\, ds \right) dz \right)$$

This result has a much more immediate interpretation in comparison with the one in section 4. Namely, if $u(q,t)$ satisfies the Single Crossing Property in (q,t) then Q is decreasing in the degree of income inequality, while the opposite is true if $u(q,t)$ satisfies the Single Crossing Property in $(-q,t)$. This just means that in the former case individuals with relatively high income have stronger preferences for high quality, while in the latter case ndividuals with relatively high income have weaker preferences for high quality.

This implies that the comparative statics, as in case 2, is independent of the tax system and of redistributive motives. Hence all that matters for the sign of the comparative statics is how preferences towards the good of interest change with income.

Despite of being interetsing on a theoretical point of view, this result still relates the sign of the comparative statics to an unobservable feature of individual preferences. In the next subsection I will show how under specific restrictions this sign can be recovered from data in the case of publicly provided education.

5.2.2. Example: Education

Consider the following simplified version of the Becker-Tomes (1979) model of intergenerational mobility, in which parental investment in children's education in which the parents' (f) utility is a function of by parents' consumption $I(x, w^f)$ (in which x is a vector of tax and transfers variables and w^f is parental income), public goods U and expected income of their children w^s, i.e.:

$$U(x, q, Y, z) = I(x, w^f) + s(U) + \beta E\left[w^s | w^f\right]$$

Suppose income w is a function of human capital of an individual i in the form $w^i = \tilde{w}(H^i) + \eta^i$ and the Human capital production function of the child H^s is a function of the quality of the education q that the child receive and of the Human capital of the father H^f (i.e. there is a transmission mechanism of human capital from parents to children), such that $H^s = H(q, H^f)$. Using these two formulas we can derive the child's income as a function of q, w^f as follows:

$$w^s = \tilde{w}(H(q, H(q, \tilde{w}^{-1}\left(w^f - \eta^f\right)) + \eta^s) = w(q, w^f, \eta^f, \eta^s)$$

Assume η^s is independent of η^f, w^f. Substituting into the utility function we get:

$$U(x,q,Y,z) = I(x,w^f) + sY + \beta E_{\eta^s}\left[w(q,w^f,\eta^f,\eta^s)|w^f,\eta^f\right]$$

Define $u(q,w^f) \equiv \beta E_{\eta^s}\left[w(q,w^f,\eta^f,\eta^s)|w^f,\eta^f\right]$ and $w^f = z + \epsilon$ to get an utility function in the form proposed in section 5.2.1, which implies:

$$sign\left(\frac{\partial Q(0)}{\partial \theta}\right) = sign\left(-\int\limits_{\underline{z}}^{\overline{z}} u_{qt}(Q,z+\underline{\epsilon})\left(\int\limits_{\underline{z}}^{z}\left[G(s)-F(s)\right]ds\right)dz\right)$$

Again the sign of the comparative statics depends on the sign of $u_{qt}(\cdot)$. In this example we can calculate our derivative and derive a condition for $u_{qt}(q,t) \leq 0\,\forall t$, namely we have $u_{qt}(Q,z+\underline{\epsilon}) = H_q(Q,H^f)H_h(Q,H^f)\frac{\tilde{w}''(H^s)}{\tilde{w}'(H^f)} + H_{qh}(Q,H^f)\frac{\tilde{w}'(H^s)}{\tilde{w}'(H^f)}$ which implies:

$$u_{qt}(q,t) \leq 0 \Leftrightarrow -\frac{\tilde{w}''(H^s)}{\tilde{w}'(H^s)} \geq \frac{H_{qh}(Q,H^f)}{H_q(Q,H^f)H_h(Q,H^f)}$$

This means that the effect of an increase in income inequality on Q is going to be weakly positive if education and parents' human capital are not strong complements in the human capital production function. In this example it is perhaps more interesting to notice another result:

$$sign\left(\frac{\partial Q(0)}{\partial \theta}\right) = sign\left(-\frac{\partial^2 w^s}{\partial q \partial w^f}\right)$$

which has an intuitive interpretation: an increase in the political weight of relatively low income individuals imply a rise in the quality of public education if a better education reduces the intensity of the transmission mechanism of income from parents to children. This implication is potentially testable and can be the object of future empirical research.

Notice that if the derivative $\frac{\partial^2 w^s}{\partial q \partial w^f}$ is negative this implies that a stronger public intervention in education should imply a fall in future income inequality; on the other hand under the assumptions of section 5.2.2 the model I presented implies that an increase in income inequality should cause an increase in public intervention in education though the political channel. These two predictions seem to be coherent with the facts I described in section 2, but additional theoretical research and empirical evidence are required in order to support the predictions that are suggested by this example.

6. Conclusions

In this paper I provide a framework that is useful to analyse the effects of income inequality on public spending in education in democratic countries. The theoretical foundation of this relationship is provided through the channel of political interaction.

This paper shows that the strong relationship between income inequality and public intervention in public spending policies that typically emerges in equilibrium in Downsian models is mainly a result of the restrictive assumptions of these models and does not survive if some of these assumptions are relaxed. I also show that these traditional models are not suitable to separate the effect on income inequality on the extent of public intervention in a specific policy from the effect on total redistribution, because of their excessively restrictive assumptions. On the other hand I provide a potential theoretical explanation to the positive relationship between income inequality and public spending in education that is supported by some empirical literature. This explanation relies on the particular way in which this good is publicly provided.

I propose a model of Probabilistic Voting and I develop a tool that allows one to recover the direction of the effect of a marginal mean preserving spread in the income distribution on the degree of public intervention in publicly provided good with specific characteristics, and I show how the derived conditions for a weakly positive comparative statics have an interesting interpretation if the variable of interest is public spending in education.

The first insight of this analysis is that if voters have access to different ways of achieving redistribution they should prefer ways of achieving their redistibutive target that are more efficient than in-kind policies and in particular of publicly provided private goods. The second is that a credible way of modeling the political interaction in order to study the comparative statics of the equilibrium outcome should imply that the welfare of every single voter contributes in determining the choice of the society, and this is exactly what can be observed in the model I propose. A consequence of this result is that the direction of the effect of an increase in income inequality on the level of public provision of education should depend mainly on how preferences for this good relative to consumption (and other goods) change with different level of income for all income levels. I find that under a set of restriction on individual preference the outcome of the political process becomes completely independent of the tax system and the comparative statics is uniquely driven by the interaction between earning capacity and preferences for better quality of education.

Finally, I show that the sign of the comparative statics under additional assumptions can theoretically be recovered from data.

A limit of these results is that they rely on strong assumptions about voters' preferences and the sharp predictions I have shown would not survive if a more flexible set of assumptions would be imposed. This suggests that further research is required in order to achieve a clearer understanding of the determinants of the relationship of interest and to be able to provide some empirical support to the channel of causality proposed in this paper.

AppendixA. Simple Downsian Model

Consider the following simple Downsian model.

- n voters, each of them characterized by a unidimensional parameter $t \in T$ (income).
- Two choice variables x_1, x_2 related by a convex governmental budget set X such that $X \equiv \{(x_1, x_2) \mid (x_1, x_2) \in R^2_+ \cap C(x_1, x_2, E(t)) \leq 0\}$.
- Individual preferences: $u(x_1, x_2, t)$ continuous, increasing C^2 function.
- Spence-Mirrles condition: $M(x_1, x_2, t) = \frac{\partial}{\partial t}\left(\frac{u_1(x_1,x_2,t)}{u_2(x_1,x_2,t)}\right) > (<) 0 \forall x_1, x_2, t$.

In equilibrium the social choice is $(x_1, x_2) = \arg\max_{(x_1,x_2)\in X} u(x_1, x_2, t_m)$ where t_m is the median of t. Because of the Spence-Mirrles condition the preferred choice of an individual with parameter t is such that x_1 is increasing (decreasing) in t and x_2 is decreasing (increasing) in t. Hence the equilibrium social choice would also change in this way if the distribution of t is changed in such a way that the median voter has higher t and $E(t)$ is unchanged. This implies a monotone link between some features of the income distribution (in this case the difference between mean and median). Q.E.D.

AppendixB. Redistribution and Income Distribution in Downsian Models

In the model proposed in Appendix A, suppose $x_2$is a private good that in uniformly publicly provided (no private purchases are allowed in this simple example) and $t - x_1 T(t)$ is after tax income and the utility fuction is $u(x_1, x_2, t) = u(t - x_1 T(t), x_2))$. Governmental budget constraint is in the form: $-x_1 E(T(t)) + x_2 \leq 0$.

Notice that in an interior solution the budget contraint is binding hence the problem is equivalent to:

$$\max_{x_2} u(t - p(t)x_2, x_2)$$

where $p(t)$ is the tax-price of the good defined as total amount of tax paid divided by the size of the provision, i.e.:

$$p(t) = \frac{x_1 T(t)}{x_2(t)} = \frac{T(t)}{E(T(t))}$$

Equilibrium condition:

$$\frac{u_1(t - p(t)x_2, x_2)}{u_2(t - p(t)x_2, x_2)} = \frac{1}{p(t)}$$

Demand:

$$\frac{\partial x_2^g}{\partial t} = \frac{p(t)u_{11} - u_{21} + p'(t)\left(u_{21}x_2 + u_1 - p(t)u_{11}x_2\right)}{p(t)^2 u_{11} - 2p(t)u_{12} + u_{22}} =$$

$$\frac{\partial x_2^g}{\partial t} = \frac{\partial x_2^m}{\partial y} + \frac{\partial x_2^m}{\partial p} p'(t)$$

Using Slutsky equation:

$$\frac{\partial x_2^g}{\partial t} = \underbrace{\frac{\partial x_2^m}{\partial y}(1 - p'(t)x_2)}_{A} + \underbrace{\frac{\partial x_2^h}{\partial p} p'(t)}_{B}$$

This implies that:

1. If x_2 is an inferior good, then both the income effect A and the tax-price effect B are negative, hence $\frac{\partial x_2^g}{\partial t} < 0$, i.e. if the median voter becomes a relatively "richer" individual the level of public provision falls;
2. if the tax system is close to a lump-sum tax (i.e. $p'(t) = 0$) then the public demand for public provision of the good has the same comparative statics as the private demand.
3. even if the good is normal, the higher the marginal tax rate $p'(t)$ the smaller the income effect A and the larger the tax-price effect B.
4. if the tax system is progressive $\left(p'(t) > \frac{1}{E(t)}\right)$ and in equilibrium a high share of total income $\frac{x_2}{E(t)}$ is spent in the public provision, then $\frac{\partial x_2^g}{\partial t} < 0$ unless x_2 is very income elastic (luxury).

AppendixC. Existence and Uniqueness with opting out

Endogenous discontinuity. Define

$$v(x_i, t) = \max\{v(x_i, n, t), v(x_i, m, t)\}$$

where $v(x_i, n, t), v(x_i, m, t)$ are two different functions of x_i, t.
Suppose $v_t(x_i, n, t) - v_t(x_i, m, t) \geq 0 \forall x_i, t$ or $v_t(x_i, n, t) - v_t(x_i, m, t) \leq 0 \forall x_i, t$
Then there is at most one $\hat{t}$ such that $v(x_i, n, t) = v(x_i, m, t)$

This implies the existence of an endogenous threshold in t such that Party A's objective function becomes:

$$V(x_A, \overline{x}_B) = \int_{\underline{t}}^{\hat{t}} [\mathbb{P}(v(x_A, t, n) - v(\bar{x}_B, t))] g(t) dt + \int_{\hat{t}}^{\bar{t}} [\mathbb{P}(v(x_A, t, m) - v(\bar{x}_B, t))] g(t) dt$$

AppendixC.1. Existence

Theorem 3b: Assume $\mathbb{P}(\cdot)$ and $v(x, i, t)$ are C^2 functions in their domains. Suppose $v_t(x_A, t; n) - v_t(x_A, t; m) \geq 0 \ \forall x, t$. Then sufficient conditions for existence of an equilibrium are the same as in Enelow, Hinrich (1989). Namely, for any function $v(x, t)$ and any distribution $G(t)$, there is always some function $\mathbb{P}(\cdot)$ such that $V(x_A, \overline{x}_B)$ is concave in x_A for all $x_A \in X$ and $t \in T$.

Proof. Define $V_{jk}(x_A, \overline{x}_B)$ an element of the Hessian H_V, i.e.

$$H_V(j,k) \equiv V_{jk}(x_A, \overline{x}_B) = \frac{\partial^2 V(x_A, \overline{x}_B)}{\partial x^i \partial x^j}$$

Denote $d(x_A, t, i) \equiv v(x_A, t, i) - v(\bar{x}_B, t)$ with $i \in \{n, m\}$. Then (need proof! there are things disappearing):

$$V_{jk}(x_A) = \frac{\partial \hat{t}}{\partial x_k} p(d(x_A, \hat{t}))[d_j(x_A, \hat{t}; n) - d_j(x_A, \hat{t}; m)]g(\hat{t}) +$$

$$+ \int_{\underline{t}}^{\hat{t}} \frac{\partial^2}{\partial x_j \partial x_k} [\mathbb{P}(d(x_A, \hat{t}; n)]g(t)dt + \int_{\hat{t}}^{\overline{t}} \frac{\partial^2}{\partial x_j \partial x_k} [\mathbb{P}(d(x_A, \hat{t}; m)]g(t)dt$$

Define a matrix $M(x_A)$ such that each element is:

$$M_{jk}(x_A) = \frac{\partial \hat{t}}{\partial x_k} p(d(x_A, \hat{t}, n)[d_j(x_A, \hat{t}; n) - d_j(x_A, \hat{t}; m)]g(\hat{t})$$

And two matrixes $H(i)$ for $i = n, m$ such that each element is:

$$H_{jk}(i) = \frac{\partial^2}{\partial x_j \partial x_k} [\mathbb{P}(d(x_A, t; i)]$$

Recall that the sum of negative semidefinite matrixes is negative semidefinite. Hence we need:

$$x^T H_V x \leq 0$$

Using the matrixes defined above can be written as:

$$x^T H_V x = x^T M(x_A) x + \int_{\underline{t}}^{\hat{t}} \left[x^T H(n) x\right] g(t)dt + \int_{\hat{t}}^{\overline{t}} \left[x^T H(m) x\right] g(t)dt \leq 0$$

Define $H_v(i) \equiv D^2[d(x_A, t, i)]$ as the Hessian of individual indirect utility and $\nabla d(x_A, t; n)$ the gradient vector.

Following Enelow, Hinrich (1989) for the second and third component of $x^T H_V x$ we need:

$$x^T H(i) x = \int_{T(i)} \left(p'(d(x_A, t)) x^T \left[\nabla d(x_A, t; i)\right] \left[\nabla d(x_A, t; i)\right]^T x + p(d(x_A, t)) x^T H_v(i) x\right) g(t)dt \leq 0$$

Notice that as F becomes close to uniform this condition is equivalent to the matrix $H_v(i)$ to be negative semidefinite, which is equivalent to a concave utility function. But in comparison with Enelow, Hinrich (1989) we have an additional element: $M(x_A)$.

Consider the definition of $\hat{t}(x_A)$:

$$d(x_A, \hat{t}; m) = d(x_A, \hat{t}; n)$$

Differentiate this w.r.t. x_k and rearrange to get:

$$\frac{\partial \hat{t}}{\partial x_k} = -\frac{d_k(x_A, \hat{t}; m) - d_k(x_A, \hat{t}; n)}{d_t(x_A, \hat{t}; m) - d_t(x_A, \hat{t}; n)}$$

Substituting into $M_{jk}(x_A)$ we get:

$$M_{jk}(x_A) = -p(d(x_A, \hat{t}, n)) \frac{[d_j(x_A, \hat{t}; n) - d_j(x_A, \hat{t}; m)][d_k(x_A, \hat{t}; n) - d_k(x_A, \hat{t}; m)]}{d_t(x_A, \hat{t}; n) - d_t(x_A, \hat{t}; m)} g(\hat{t})$$

Hence

$$M(x_A) = -\frac{f(d(x_A, \hat{t}, n))}{d_t(x_A, \hat{t}; n) - d_t(x_A, \hat{t}; m)} \left[\nabla d(x_A, \hat{t}; n) - \nabla d(x_A, \hat{t}; m)\right] \left[\nabla d(x_A, \hat{t}; n) - \nabla d(x_A, \hat{t}; m)\right]^T g(\hat{t})$$

Given that $\left[\nabla d(x_A, \hat{t}; n) - \nabla d(x_A, \hat{t}; m)\right] \left[\nabla d(x_A, \hat{t}; n) - \nabla d(x_A, \hat{t}; m)\right]^T$ is the product of the same vector it is positive semidefinite, hence a sufficient condition for $x^T M(x_A) x \leq 0$ for all x is:

$$d_t(x_A, \hat{t}; n) - d_t(x_A, \hat{t}; m) > 0$$

Notice that this condition corrisponds roughly to SDI in Epple and Romano (1996) in the case of opting out (explain).

Hence the conditions for existence of a Political Equilibrium are the same as in Enelow, Hinrich (1989), plus the additional condition stated above.

Overall, the restrictiveness of this additional requirement depends on the specific application chosen (see section...)

On the other hand, it is possible that even if $d_t(x_A, \hat{t}; n) - d_t(x_A, \hat{t}; m) < 0$ the second and the third elements of $V_{jk}(x_A)$ are sufficient to guarantee concavity of the whole function.

AppendixC.2. Uniqueness

Theorem 4b: *Assume (i) X is compact and convex, (ii) for each t and C, $P_C(x_A, x_B, t)$ is jointly continuous in (x_A, x_B), (iii) for each (x_A, x_B), $\int P_A(x_A, x_B)g(t)dt$ is strictly concave in x_A and $\int P_B(x_A, x_B, t)g(t)dt$ is strictly concave in x_B. Then there is exactly one electoral equilibrium, and it is in pure strategies.*

(i) Compactness. Same as in Theorem 3. Convexity. In out problem is satisfied if and only if the Governmental Budget set $C(x, G(t)) \leq 0$ is a convex set. In problems with non-linear budget sets this conditions may fail to apply (see next sections).

(ii) Joint continuity. Same as in Theorem 1.

(iii) Strict concavity. Conditions are the same as in Theorem 3, except that we need to verify the negative dfinitiveness of the Hessian Matrix.

AppendixC.3. Policy convergence

Theorem 5b *(policy convergence): under assumptions* (i), (ii), (iii) *of Theorem 4, the unique electoral equilibrium is such that the two parties choose the same policy.*

Proof. Notice that the game described above can be modelled as a zero sum game because the expected plurality for Party B: is equal to $1 - V(x_A, x_B)$

Suppose $(\overline{x}_A, \overline{x}_B)$ is an equilibrium strategy and $\overline{x}_A \neq \overline{x}_B$ and $V(\overline{x}_A, \overline{x}_B) = k$. Each Party i can always achieve a certain value V^i by playing $x_i = x_{-i}$.

Hence if $k < V^A$ (a) then $\overline{x}_A$ cannot be a best response for Party A because it can profitably deviate to $x^A = \overline{x}^B$. If $k > V^A$ (b), then $V(\overline{x}^B, \overline{x}^A) = 1 - V(\overline{x}_A, \overline{x}_B) = 1 - k$. Then $\overline{x}^B$ cannot be a best response for Party B because it can deviate to $x^B = \overline{x}^A$ and get $V^B = 1 - V^A$ and (b) implies that this deviation is profitable. Hence in equilibrium it must be true that $V(x^A, x^B) = V^A$ and $V(x^B, x^A) = V^B$. Given that the function $V(x^A, x^B)$ is strictly concave in x^A the problem

$$\max_{x \in X} V(x, \overline{x}^B)$$

has got a unique solution, hence it must be the case that in equilibrium $x^A = x^B$.

AppendixC.4. Utilitarian outcome

Theorem 6b *(utilitarian outcome): if all individuals have the same Party preference distribution, i.e.* $\mathbb{P}(d,t) = \mathbb{P}(d) \forall t$ *then the policy chosen by both parties in equilibrium is the same as the policy that would be chosen by an omnipotent Benthamite governemt.*

Proof. Lagrangean for this problem is:

$$\int_{\underline{t}}^{\hat{t}} [\mathbb{P}(v(x_A, t, n) - v(\bar{x}_B, t))] g(t) dt + \int_{\hat{t}}^{\overline{t}} [\mathbb{P}(v(x_A, t, m) - v(\bar{x}_B, t))] g(t) dt - \lambda \left[C(x, G(t))\right]$$

First order conditions.

$$[x_i] : \int_{\underline{t}}^{\hat{t}} p\left[d(x_A, t, n)\right] v_i(x^A, t, n) g(t) dt + \int_{\hat{t}}^{\overline{t}} p\left[d(x_A, t, m)\right] v_i(x_A, t, m) g(t) dt = \lambda \left[C_i(x, G(t))\right]$$

Hence forany i, j:

$$\frac{\int_{\underline{t}}^{\hat{t}} p\left[d(x_A, t, n)\right] v_i(x^A, t, n) g(t) dt + \int_{\hat{t}}^{\overline{t}} p\left[d(x_A, t, m)\right] v_i(x_A, t, m) g(t) dt}{\int_{\underline{t}}^{\hat{t}} p\left[d(x_A, t, n)\right] v_j(x^A, t, n) g(t) dt + \int_{\hat{t}}^{\overline{t}} p\left[d(x_A, t, m)\right] v_j(x_A, t, m) g(t) dt} = \frac{C_i(x, G(t))}{C_j(x, G(t))}$$

Notice that at an equilibrium point $x^A = x^B$ (see above) hence $d(x_A, t, n) = d(x_A, t, m) = 0 \forall t$ and $\hat{t}(x^A) = \hat{t}(x^B)$.

Given that $p(d(x_A,t,i))$ is independent of t in this case, i.e. $p(d(x_A,t,i)) = p(0)$, then the previous equaltion becomes:

$$\frac{\int_{\underline{t}}^{\hat{t}} v_i(x^A,t,n)g(t)dt + \int_{\hat{t}}^{\overline{t}} v_i(x_A,t,m)g(t)dt}{\int_{\underline{t}}^{\hat{t}} v_j(x^A,t,n)g(t)dt + \int_{\hat{t}}^{\overline{t}} v_j(x_A,t,m)g(t)dt} = \frac{C_i(x,G(t))}{C_j(x,G(t))}$$

which is the same condition that we would find for the problem:

$$\max_{C(x,G(t))\leq 0} \int_{\underline{t}}^{\hat{t}} v(x,t,n)g(t)dt + \int_{\hat{t}}^{\overline{t}} v(x,t,m)g(t)dt$$

Which is the Utilitarian Social Optimum. Notice that this result does not hold if individuals with different t have different Party preference distributions.

AppendixD. TBC

AppendixE. TBC

AppendixF. Multiple Options in the Private Market

Suppose there are other options available on the private market that are chosen with positive probability at an equilibrium. Define new thresholds $\tilde{\epsilon}_i(x,Q,z)$ such that

$$I(x,z+\tilde{\epsilon}_i) - Pq_i + u(q_i,z+\tilde{\epsilon}_i) + sU = I(x,z+\tilde{\epsilon}_i) - Pq_{i+1} + u(q_{i+1},z+\tilde{\epsilon}) + sU$$

then the objective function becomes

$$\tilde{V}(x,Q,\theta) = \int_{\underline{z}}^{\overline{z}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}(z)} \left[-PQ + u(Q,z+\epsilon)\right] kd\epsilon r(z,\theta)dz + \sum_{i=1}^{n-1} \int_{\tilde{\epsilon}_i(z)}^{\tilde{\epsilon}_{i+1}(z)} \left[-Pq_i + u(q_i,z+\epsilon)\right] kd\epsilon r(z,\theta)dz +$$

$$+ \int_{\tilde{\epsilon}_i(z)}^{\overline{\epsilon}} \left[-Pq_i + u(q_i,z+\epsilon)\right] kd\epsilon r(z,\theta)dz + (s-1)\,U + E_{t,\epsilon}(z+\epsilon)$$

Then

$$\tilde{L}_{Q\theta} = \int_{\underline{t}}^{\overline{t}} \int_{\underline{\epsilon}}^{\tilde{\epsilon}_1(z)} \left[-P + u_q(Q,z+\epsilon)\right] kd\epsilon \left[g(z) - f(z)\right] dz = -\int_{\underline{z}}^{\overline{z}} u_{qt}(Q,z+\underline{\epsilon})s(z)dz = L_{Q\theta}$$

Hence the comparative statics is not affected by reducing the range of options on the private market to one single quality level as long as the level chosen is such that $\exists\tilde{\epsilon}(z)$ such that $-Pq + u(q_i,z+\tilde{\epsilon}) = u(Q,z+\tilde{\epsilon})$ for some $z \in (\underline{z},\overline{z})$ in the proximity of the political equilibrium. Q.E.D.

References

[1] Aranson, P., Hinich, M., and Ordeshook, P. (1974). “Election goals and strategies: Equivalent and nonequivalent candidate objectives”, American Political Science Review 68: 135-152.

[2] Bassett, W.F., Burkett, J.P., Putterman, L. (1999), “Income distribution, government transfers, and the problem of unequal influence”, European Journal of Political Economy Vol. 15 1999 207–228.

[3] Banks, J.S., and J. Duggan (2005). ”Probabilistic Voting in the Spatial Model of Elec-tions: The Theory of Office-Motivated Candidates”. In Social Choice and Strategic Decisions: Essays in Honor of Jeffrey S. Banks, ed. David Austen-Smith and John Duggan, 15-56. New York: Springer.

[4] Becker, G.S., Tomes, N. (1986). “Human Capital and the Rise and Fall of Families”. Journal of Labor Economics, University of Chicago Press, vol. 4(3), pages S1-39, July.

[5] Besley, T., Coate, S. (1991). “Public provision of private goods and the redistribution of income,” American Economic Review, 81, 979–984.

[6] Besley, T., Coate, S. (1997). “An Economic Model of Representative Democracy”, Quarterly Journal of Economics, 108(1), 85-114, 1997.

[7] Black, D. (1948). “On the Rationale of Group Decisionmaking”, Journal of Political Economy, 56: 23-34.

[8] De Mello, L. and Tiongson, E.R. (2006), “Income Inequality and Redistributive Government Spending”, Public Finance Review, 34, 282-305.

[9] Downs, A. (1957). An Economic Theory of Democracy. New York: Harper Collins.

[10] Enelow, J.M., Hinich, M.J. (1989). “A general probabilistic spatial theory of elections”, Publ& Choice 61: 101-113.

[11] Epple, D., Romano, R.E. (1996). “Public provision of private goods”, Journal of Political Economy, 104, 57–84.

[12] Epple, D. and Romano, R.E. (1996), “Ends against the middle: Determining public service provision when there are private alternatives”, Journal of Public Economics, Elsevier, vol. 62(3), pages 297-325, November.

[13] Easterly, W. and Rebelo, S. (1993). "Fiscal policy and economic growth: An empirical investigation," Journal of Monetary Economics, Elsevier, vol. 32(3), pages 417-458, December.

[14] Gouveia, M. (1997). "Majority rule and the public provision of health care", Public Choice, 93, 221–244.

[15] Grandmont, J.-M. (1978). "Intermediate preferences and the majority rule", Econometrica, 46, 317–330.

[16] Levy, G. (2005). "The Politics of Public Provision of Education", The Quarterly Journal of Economics, Vol. 120, No. 4, pp. 1507-1534

[17] Lindbeck A., Weibull, J.W. (1987). "Balanced-budget redistribution as the outcome of political competition", Public Choice, 52, Number 3, 273-297.

[18] Plott, C. R. (1967). A notion of equilibrium and its possibility under majority rule. American Economic Review 57 (Sept.): 787-806.

[19] Roemer, J.E. (1999). "The Democratic Political Economy of Progressive Taxation", Econometrica, 67, 1-19.

[20] Sylwester (2000). "Income inequality, education expenditures, and growth", Journal of Development Economics, Vol. 63, 379–398.

[21] Sylwester, K. (2002). "Can education expenditures reduce income inequality?", Economics of Education Review, 21, 43–52.

[22] Stiglitz, J.E. (1974). "The demand for education in public and private school systems", Journal of Public Economics, 3, 349–385. 61

[23] Usher, D. (1977). "The welfare economics of the socialization of commodities", Journal of Public Economics, Elsevier, vol. 8(2), pages 151-168, October.