Generalized Comparative Statics for Political Economy Models

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Generalized Comparative Statics for Political Economy Models.[☆☆]

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Abstract

I investigate the equilibrium properties of a deterministic voting model in which the policy space is multidimensional and politicians have limited ability to commit to platforms. This analysis is useful to answer Political Economy questions in which the multidimensional nature of the policy is crucial to model voters' trade-offs. The use of unidimensional models to study such problems is prevalent in the literature. This usually implies that one or more policy dimensions are assumed to be exogenous. This choice delivers sharp theoretical predictions, but implies an oversimplification of the problem, which often results in implausible or empirically inconsistent predictions.

I show that under suitable restrictions on the individuals' capability of committing to policy platforms and on individual preferences a Median Voter Theorem holds even if the policy space is multidimensional. Moreover, I show that the comparative statics of the equilibrium policy outcome induced by a change in the voters' distribution is monotone. I use this tool to extend the Meltzer-Richard model about size of the government. I show this model delivers empirically consistent predictions if a sufficiently rich policy space is assumed. Finally, I show that this framework can be used to study other Political Economy problems beyond simple voting problems.

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1. Introduction

The model of electoral competition proposed by Downs (1957) is a simple and useful tool that has proved to be extremely successful in the Political

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Economy literature. The model delivers very strong predictions: under suitable assumptions the Median Voter Theorem states that the unique equilibrium is the policy that is most preferred by the median voter, which implies that the levels and the comparative statics of the political equilibrium reduces to the ones of a single pivotal individual. The ease of calculation and interpretation of this prediction had to face a general lack of empirical support. A famous example is given by an influential paper by Meltzer and Richard (1981), who have shown that in a simple general equilibrium economy the size of the governmental expenditures is monotonic increasing in the mean to median income ratio. The large body of empirical studies[2] that has followed their paper has provided very little support to this hypotesis, with a majority of these analyses showing no statistically significant relationship between the two variables of interest and a number of papers that found a significant relationship but with opposite sign in comparison with the one implied by the Metlzer and Richard result.

A possible explanation for this poor performance of the model is a direct consequence of the restrictive assumptions that one has to impose in order to achieve the existence of a Majority Voting equilibrium in Downsian model, and one in particular is relevant for this and for several other examples: the unidimensionality of the policy space. In Melzer and Richard's paper a crucial consequence of this restriction is that the policy space is made of two variables: a linear tax rate and a lump-sum grant, that are connected by a balanced governmental budget constraint such that the effective choice of voters is reduced to a unidimensional policy space. The oversimplification of this setting is evident in several aspects. For instance in most countries direct redistribution is just one component of governmental spending and does not usually represent the largest share, given that usually expenditures in direct provision of Public Goods and services and other welfare policies accounts for a larger share of the public budget. Moreover the choice of a very simple tax system is likely to influence the result.

Unfortunately the attempt to model the political interaction in a Downsian model when the policy space is multidimensional has to face extremely restrictive assumptions in order to achieve existence of a pure strategy equilibrium (see section 2). Following a successful stream of literature (Levy 2004, 2005, Roemer 1999, etc.) my approach is to make the political interaction slightly more complex (and realistic) in comparison with the one implied in Downsian models by introducing coalitions (or factions) as intermediate bodies between the voters and the policies that have to be implemented.

The crucial idea of this approach is that individual citizens have limited ability to commit to specific policies, but they are allowed to form coalitions whose role is to increase the space of policies that a faction can credibily commit to implement after elections, namely a coalition can propose any policy in the Pareto set of its members. This assumption is common in the recent Political Economy literature (see Levy 2004, 2005, Roemer 1999).

[2]For a review of the literature about this topic see de Mello, Tiongson 2006.

In this paper I do not explicitly model the process of coalition formation, but I require coalitions to be stable in a peculiar and relatively weak way: a coalition $\mathcal{A}$ is stable if there is at least a vector of policies x in its Pareto set such that there is no other vector of policies x' with the following features: (i) x' makes each member of a subcoalition $\mathcal{A}' \subset \mathcal{A}$ strictly better off with respect to x; (ii) x' is in the Pareto set of the subcoalition $\mathcal{A}'$; (iii) there is no policy x'' in the Pareto set of the complementary subcoalition $\mathcal{A}\backslash\mathcal{A}'$ that is preferred to x' by the society as a whole according to some social preference relation (that is going to be Majority Voting). The details of this concept of stability will be given in section 2, but this description is sufficient to understand how flexible this concept of stability is: it is very unlikely that a coalition can be stable if for any policy that this coalition can put forward a subcoalition can deviate and propose a policy that makes all its member strictly better off (i), that is feasible for the deviators (ii), and such that the remaining members of the original coalition do not have access to any alternative that represents a "credible threat" and that can discourage this deviation (iii).

In this paper I show that under this notion of stability and some specific assumptions on individual preferences a Median Voter Theorem and a monotone comparative statics of the equilibrium outcomes can be derived in a multidimensional policy space. This result may be used to shed light on the effect of the restriction of unidimensionality of the policy space in some common applications in the literature.

The paper is structured as follows. Section 2 summarizes the existing literature about models of Political Economy in a multidimensional policy space and highlights why none of the existing models is suitable to analyze sufficiently complex problems of comparative statics. In section 3 I describe the model of political interaction and the notion of stability that I will use in the rest of the paper and the restriction on individual preferences that I need to impose. In section 4 I present the two main results of this paper: the Generalized Median Voter Theorem and the Monotone Comparative statics of the equilibrium outcome; moreover I show how my findings can be interpreted as a generalization of some results in the literature, I also describe some features of the coalition structures that can emerge in equilibrium and of the Social Choice that is implied by this model. Section 5 introduces a generalization of the model in which a more complex game is played such that agents do not only vote over different policy vectors but they also have access to a richer strategy space. I show that when the resulting game has some specific characteristics (similar to the ones of a game with strategic complementarities) a monotone comparative statics result similar to the one in section 4 can be derived. Section 6 describes two possible applications: the first one shows how the result in Meltzer and Richard's paper is not robust to a small change in the environment and that in a rather simple but more realistic setting their monotonicity result cannot be achieved or if it is it has the opposite direction in comparison with the one they derive in their simpler setting. This result may help to shed light on the reasons that underpin the lack of empirical support to the main prediction of their model. The second one is an application of the extension in section 5 and it shows that interesting prediction can be derived if a famous game, namely the *Arms Race*, is played in a Political Economy framework. Sections 6 concludes providing some comments about the importance and the limits of the results in the paper.

2. Literature

The seminal contributions of Hotelling (1929) , Black (1948) and Downs (1957) gave rise to the success in the Political Economy literature of the so-called Downsian models of political competition, which proved to be extremely successful and it is still popular in recent applications. The reason of this success relies in the simple and powerful result that this model delivers under suitable restrictions on individual preferences: the Median Voter Theorem. The crucial consequence of this result is that the equilibrium choice is going to be the policy that is most preferred by a single individual (Pivotal voter or Condorcet winner), which is the median individual. This implies in turn that all predictions about levels and comparative statics of the equilibrium outcomes are very easy to derive and to interpret in relations to the changes of the identity and preferences of the pivotal individual.

Unfortunately it is well known that the conditions for the existence of a Condorcet winner in a multidimensional policy space are extremely burdensome (see Plott 1967; Davis, DeGroot, Hinich 1972; and Grandmont, 1978). This implies that in order to study problems that are characterized by a sufficiently rich policy space one has to rely to an alternative model of political interaction.

In the Political Economy literature there are several examples of models that meet this requirement; in this section I will mention the most popular ones and explain why none of them is suitable to answer questions about the comparative statics of the equilibrium outcomes if the number of available policies is sufficiently large.

The first and popular example is given by Citizen-Candidate models first proposed by Osborne and Slivinski (1996) and Besley and Coate (1997); this class of models is based on the assumption each voter can run for elections but she cannot commit to any policy that is not in her set of ideal points. Under this (rather restrictive) assumption the existence of a political equilibrium is ensured, but multiplicity of equilibria[3] is a typical outcome. For instance with the same set of voters there may be equilibria with only one candidate running unopposed, equilibria with two candidates or more, and each of these cases is characterized by a different set of policies that are implemented in equilibrium. This implies that the model is not suitable to answer questions about policy outcomes and their comparative statics because the set of policy vectors that can be equilibria is usually too large to deliver any useful prediction. The

[3]There is a particular case, highlighted in Besley and Coate (1997) in which the Citizen Candidate model delivers an equilibium that exhibits features that are very similar to the ones of the equilibrium concept described in this paper. In section 4.3 I describe this case and show that it can be interpreted as a particular case of my more general setting.

problem of multiplicity is shared with the Party Unanimity Nash Equilibrium (PUNE) proposed by Roemer (1999).

The model developed by Levy (2004, 2005) is based on the idea that citizens can expand their ability to commit to policies different from their own ideal point by forming coalitions such that each coalition can propose any policy that is in the Pareto set of its members. A peculiar notion of coalition stability ensures existence of an equilibrium in a multimensional policy space even if the individual preferences are relatively complex (i.e. individuals differ in two paramenters that enter their utility function). On the other hand the predictions about levels and comparative statics of the equilibrium outcomes can be derived analytically only for problems in which the policy space or the individual preferences are restricted in such a way that a very small number of policy vectors can be chosen in an equilibrium. for instance in the application described in Levy (2005) there are only three policies that can be be chosen in any equilibrium). This make Levy's model unsuitable to analyze more complex problems.

Finally we have a relative large literature about Probabilistic Voting Models (Lindbeck and Weibull, 1987; Enelow and Hinich, 1989) that under not very restrictive assumptions ensure the existence and uniqueness of a voting equilibrium. These models do not deliver any result that makes the political equilibrium equivalent to the ideal point of a single "pivotal individual", therefore the kind of comparative statics that can emerge is generally more complex to derive and its interpretation is not always straightforward. Given that the equilibium outcomes of Probabilistic Models depend in principle on the preferences of all voters, then all comparative statics exercises have to be related to some charasteristic of the whole population, for instance a feature of its distribution that can be summarized by a unidimensional parameter. An example of this approach is in a paper by Dotti (2014) in which in a model of public provision of a private good the comparative statics of the equilibrium outcome induced by a marginal mean preserving spread in the income distribution of voters is derived.

In order to deal with more general comparative statics questions it is necessary to apply a tool that allows to summarizes the preferences of the society into the choice of a single individual, such that the comparative statics induced by changes in the environment can be derived and interpreted easily and condition for its monotonicity can be imposed in a simple and intuitive way. These features, that are ensured by the Median Voter Theorem in the unidimensional case, can be achieved in a multidimensional policy space thanks to the model presented in this paper.

3. The Model

3.1. Setting

Consider a voting game with n voters (n odd) such that each voter $i \in \mathcal{N}$ is denoted by a vector of parameters $\theta_i \in \Theta$. Assume $(\Theta, \preccurlyeq)$ is a totally ordered set for some transitive, reflexive, antisymmetric order relation $\preccurlyeq$. This allows me to establish a total order in the set of players $\mathcal{N}$, such that for all $i, j \in \mathcal{N}$ we have $i \leq j$ if and only if $\theta_i \preccurlyeq \theta_j$. For instance suppose θ is individual income, then $\theta \in [\underline{\theta}, \overline{\theta}]$ and Θ is a totally ordered set under the order relation $\leq$.

Each individual $i \in \mathcal{N}$ is endowed with a reflexive, complete and transitive preference ordering $\succeq^i$ that can be represented by a continuous and $\theta-concave$ utility function[4] $F : X \times \Theta \to R$.

The policy space X is a subset of the the d-dimensional real space R^d. In order to characterize X it is useful to recall some definitions.

Let $(L, \leqslant)$ be a partially ordered set, with the transitive, reflexive, antisymmetric order relation $\leqslant$. For x and y elements of X, let $x \vee y$ denote the least upper bound, or join, of x and y in X, if it exists, and let $x \wedge y$ denote the greatest lower bound, or meet of x and y in X, if it exists. The set L is a lattice if for every pair of elements x and y in L, the join $x \vee y$ and meet $x \wedge y$ do exist as elements of L. Similarly, a subset X of L is a sublattice of L if X is closed under the operations meet and join. A sublattice X of a lattice L is a convex sublattice of L, if $x \leqslant z \leqslant y$ and x, y in X implies that z belongs to X, for all elements x, y, z in L. Finally, a sublattice X of S is complete if for every nonempty subset X' of X, $\inf(X')$ and $\sup(X')$ both exist and are elements of X.

Recall the d-dimensional real space R^d is a partially ordered set under the transitive, reflexive, antisymmetric order relation $\leq$[5]. Moreover R^d is a lattice given the definition above. Now we have all the elements to characterize the policy space X. Let $X \subseteq R^d$ be a convex sublattice of R^d, then $(X, \leq)$ is a partially orderet set with order relation $\leq$. An example of a policy space that satisfies my assumption is given by the family of sets $Y = \{y | y \in [a, b]^d\}$ where $a, b \in R^d$.

Subset of voters can form coalitions $\mathcal{A} \subseteq \mathcal{N}$. The role of coalitions in this model is to increase the effective policy space available to the voters. Define $p_{X,\mathcal{A}}(a) \equiv \{b \in X : \ b \succeq^i a \, \forall i \in \mathcal{A}, b \not\preceq^i a \, \forall i \in \mathcal{A}\}$ to be the set of allocation in X that are Pareto superior to a for coalition $\mathcal{A}$. We assume that a coalition can propose any policy in the Pareto set of its members, i.e. $x^{\mathcal{A}} \in \mathcal{P}(\mathcal{A})$ where $\mathcal{P}(\mathcal{A}) \equiv \{a \in X : p_{X,\mathcal{A}}(a) = \oslash\}$. If a coalition is a singleton then the Pareto set reduces to the set of ideal points of its unique member (as in a citizen-candidate model).

3.2. Stability

In order to define a stability in this model we need to characterize a coalition structure and the preferences of each coalition. A coalition stucture is defined

[4]For any function f defined on the convex subset X of R^d , we say that f is concave in direction $v \neq 0$ if, for all x, the map from the scalar s to $f(x + sv)$ is concave. (The domain of this map is taken to be the largest interval such that $x + sv$ lies in X.) We say that f is $i-concave$ if it is concave in direction v for any $v > 0$ with $v_i = 0$. See Quah (2007).

[5]For $x, y \in R^d$ $x \leq y$ if and only if $x_i \leq y_i$ for all $i = 1, 2, ..., d$.

as a partition $\mathbb{P}$ of $\mathcal{N}$, i.e. a set of subsets of $\mathcal{N}$ such that $\oslash \notin \mathbb{P}$, $\cup_{\mathcal{A}\in\mathbb{P}}\mathcal{A} = \mathcal{N}$ and if $\mathcal{A}, \mathcal{B} \in \mathbb{P}$ with $\mathcal{A} \neq \mathcal{B}$, then $\mathcal{A} \cap \mathcal{B} = \oslash$.

We define a complete social preference relation $\succ$ and $\succeq$ such that $\succ$ is irreflexive i.e. $x \not\succ x$ and $\succeq$ is reflexive i.e. $x \succeq x$ and the weak and strong relations are dual, i.e. $a \succeq b \Leftrightarrow \neg b \succ a$ ($\succeq$ is not necessarily transitive). Given this preference relation we can define $P_A(a) \equiv \{b \in A : b \succ a\}$ where $A \subseteq X$ to be the strictly preferred set of a in A and $K(A) \equiv \{a \in A : P_A(a) = \oslash\}$ to be the set of $P - maximal$ alternatives in A, or the Core.

The crucial aspect of my concept of stability relies on the idea of "credible threat".

I define $S_{\mathcal{A}}(a) \equiv \{b \in \mathcal{P}(\mathcal{A}'), \mathcal{A}' \subseteq \mathcal{A} : b \succ^i a \,\forall i \in \mathcal{A}', b \succ c \,\forall c \in \mathcal{P}(\mathcal{A}\backslash\mathcal{A}')\}$ to be the set of "credible threats" to a. $S_{\mathcal{A}}(a)$ corresponds to the set of policies that are strictly preferred to a by each member of any subcoalition $\mathcal{A}' \subseteq \mathcal{A}$ and that are preferred by the society to any policy that can be proposed by the residual coalition $\mathcal{A}\backslash\mathcal{A}'$.

Using this concept we can define the S-Core (SK) to be the set of $S - maximal$ alternatives in A. i.e. $SK(\mathcal{A}) = \{a \in \mathcal{P}(\mathcal{A}) : S_{\mathcal{A}}(a) = \oslash\}$ is the set of policies that do not face any "credible threat" from any subcoalition of $\mathcal{A}$.

With this structure we can now define a concept of stability for a coalition structure in this game:

Definition 1. A coalition $\mathcal{A}$ is stable if and only if $SK(\mathcal{A})$ is nonempty.

It is useful to give an example of why a coalition that does not satisfy the definition above is unlikely to survive. Suppose $SK(\mathcal{A}) = \oslash$. Then for any $a \in \mathcal{P}(\mathcal{A})$, $\exists b \in \mathcal{P}(\mathcal{A}')$ and $\mathcal{A}' \subseteq \mathcal{A}$ such that $b \succ^i a \ \forall i \in \mathcal{A}'$ and $b \succeq c \,\forall c \in \mathcal{P}(\mathcal{A}\backslash\mathcal{A}')$, i.e. there exists a subset of the coalition $\mathcal{A}$ and a policy $b \in \mathcal{P}(\mathcal{A}')$ such that b is strictly preferred to a by all members of the subcoalition $\mathcal{A}'$ and b is also preferred by the society as a whole to any policy c that the remaining part of the original coalition $\mathcal{A}\backslash\mathcal{A}'$ can propose.

It is natural to consider this coalition structure unstable because for any policy chosen by this coalition in its Pareto Set (e.g. through some form of bargaining), the choice of this policy would not be self-enforcing because a subcoalition $\mathcal{A}'$ can deviate and propose a different policy that makes each member of the subcoalition strictly better off, that is preferred by the society as a whole, and such that the remaining part of the original coalition $\mathcal{A}\backslash\mathcal{A}'$ cannot prevent this deviation because there is no feasible "punishment" policy that can represent a credible threat for the deviators.

Definition 2. A stable coalition structure is a partition $\mathbb{P}$ of $\mathcal{N}$ such that all the coalitions $\mathcal{A}^j \in \mathbb{P}$ are stable.

3.3. Preferences

In order to establish our result I need to restrict individual and social preferences. The kind of restrictions I am going to use are very common in the many fields of Economic Theory.

About individual preferences the assumptions are *Supermodularity (SM)* and *Strict Single Crossing Property (SSCP).*

Recall that individual preferences can be represented by a function $F : X \times \Theta \rightarrow R$. A function F satisfies:

(i) *SM* if and only if $F(a \vee b, \theta) - F(a, \theta) \geq F(b, \theta) - F(a \wedge b, \theta)$ for all $\theta \in \Theta$ and for all $a, b \in X$.

(ii) *SSCP* in (a, θ) if and only if $F(a, \overline{\theta}) - F(b, \overline{\theta}) > F(a, \underline{\theta}) - F(b, \underline{\theta})$ for all $a \geq b$ and $a \neq b$, and for all $\overline{\theta}, \underline{\theta} \in \Theta$ such that $\overline{\theta} > \underline{\theta}$.

Given the individual preferences described above, I define $M(i)$ to be set of ideal points of an individual i, i.e. $M(i) \equiv \{y | y \in \arg\max_{x \in X} F(x, \theta_i)\}$[6].

About social preferences I am assuming *Majority Voting,* which is the most common and widely used criterion in order to establish a social preference relation. Formally $a \succeq b$ if and only if $\sum_{i=1}^{n} 1[F(a, \theta_i) \geq F(b, \theta_i)] > n/2$. Notice that this preference relation does not necessarily imply a *tournament*, i.e. it is possible that $a \neq b$ and $a \sim b$.

3.4. Equilibrium

Now that the setting is complete we need to define an equilibrium for this voting game.

Definition 3. A policy vector $a \in A$ is a winning policy if and only if it is in the strong Core of A, i.e. $a \in K(A)$.

Given that our Social preference relation is Majority Voting this is equivalent to say that a is a Condorcet Winner[7] of the set of alternatives A.

Suppose a coalition structure is made of h coalitions $\mathcal{A}^j$ for $j = 1, 2, ..., h$. then we can define an equilibrium for the voting game as follows.

Definition 4. A pure strategy equilibrium is a coalition structure $\mathbb{P}^* = \{\mathcal{A}^j\}_{j=1}^{h}$, a policy profile $A^* = \{a^j\}_{j=1}^{h}$ and a set of winning policies $W(A^*) \subseteq A^*$ such that:

(i) $\mathbb{P}^*$ is a stable coalition structure;

(ii)$a^j \in SK(\mathcal{A}^j)$ for all $j = 1, 2, .., h$;

(iii) $W(A^*)$ is nonempty.

In other words in an equilibrium each coalition is stable and is represented by one of the policy vectors that makes it stable, and the winning policy is a Condorcet Winner of the reduced games in which the policy space ir reduced to $A^* \subseteq X$.

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[6]Notice that the completeness of X implies compactness in the order-interval topology. On bounded sets in R^d, the order-interval topology coincides with the Eucilidean topology (Birkhoff 1967). Hence $M(i) \neq \oslash$ for all $i \in \mathcal{N}$.

[7]The relationship between the concepts of strong Core and Condorcet winner is described in Ordershook, 1986, pp. 347-349.

4. Results

The main result of this paper is stated in the following theorem:

Theorem 5. *(Median Voter Theorem). (i) An equilibrium of the voting game exists. (ii) In any equilibrium the set of winning policies W is a subset of the set of ideal points of the median voter m, i.e. $W \subseteq M(m)$. (iii) If the median voter has a unique ideal point, this policy is going to be the one chosen in any equilibrium.*

In order to prove this result we need to introduce some additional notation. Suppose the coalition $\mathcal{A}$ has k members. Consider a set of $k \times 1$ weighting vectors $\Lambda^{\mathcal{A}} \equiv \{\lambda : \sum_{i \in \mathcal{A}} \lambda_i = 1\}$ for each coalition $\mathcal{A}$ and a function $G : X \times \Lambda^{\mathcal{A}} \times \Theta \to R$ defined as follows: $G(x, \lambda, \Theta) = \sum_{i \in \mathcal{A}} \lambda_i F(x, \theta_i)$.

Lemma 6. *If F is a continuous function of x and X is a convex set then any point a in the Pareto set of $\mathcal{A}$ is a solution to $\max_{x \in X} G(x, \lambda^a, \Theta)$ for some vector $\lambda^a \in \Lambda^{\mathcal{A}}$.*

Proof. M.W.G., Proposition 16.E.2. □

We need to define four additional objects:

(i) a vector$\underline{\lambda}^{A,j}$ such that $\underline{\lambda}_i^{A,j} = \lambda_i \forall i \in \mathcal{A} : \theta_i < \theta_j$, $\underline{\lambda}_i^{A,j} = 0 \ \forall i \in \mathcal{A} : \theta_i > \theta_j$, $\underline{\lambda}_j^{A,j} = \sum_{\substack{i \in \mathcal{A} \\ i \geq j}} \lambda_i$;

(ii) a vector $\bar{\lambda}^{A,j}$ such that $\bar{\lambda}_i^{A,j} = \lambda_i \forall i \in \mathcal{A} : \theta_i > \theta_m$, $\bar{\lambda}_i^{A,j} = 0 \ \forall i \in \mathcal{A} : \theta_i < \theta_j$, $\bar{\lambda}_j^{A,j} = \sum_{\substack{i \in \mathcal{A} \\ i \geq j}} \lambda_i$;

(iii) the set $\Lambda^{A,j} = \{\underline{\lambda}^{A,j}, \lambda^A, \bar{\lambda}^{A,j}\}$;

(iv) an order relation $\leq^\lambda$given by: $\lambda^1 \leq^\lambda \lambda^2$ iff $\lambda_i^1 \geq \lambda_i^2 \ \forall i \leq m$ and $\lambda_i^1 \leq \lambda_i^2 \ \forall i \geq m$. It follows that $(\Lambda^{A,j}, \leq^\lambda)$ is a totally ordered set.

Lemma 7. *If F satisfies SM and SSCP then the Pareto Set $\mathcal{P}(\mathcal{A})$ of a coalition of players $\mathcal{A} \subseteq \mathcal{N}$ is such that $x \in \mathcal{P}(\mathcal{A})$ only if $x \geq \sup\{M(l)\}$ and $x \leq \inf\{M(h)\}$ where $l = \inf(\mathcal{A})$ and $h = \sup(\mathcal{A})$.*

Proof. See Appendix A. □

Lemma 8. *The function $G(x, \lambda, \Theta)$ satisfies (i) SM in x and (ii) SCP in (x, λ) $\forall \lambda \in \Lambda^{A,j}$.*

Proof. (i) *SM*. G is the sum of *SM* functions so it is supermodular (proof in Milgrom, Shannon, 1994). (ii) *SCP*. Using the definition of supermodularity, G is supermodular if and only if: $G(\bar{x}, \lambda^A, \Theta) - G(\underline{x}, \lambda^A, \Theta) \geq G(\bar{x}, , \underline{\lambda}^{a,j}, \Theta) - G(\underline{x}, \underline{\lambda}^{a,j}, \Theta) \ \forall \overline{x} \geq \underline{x}, \lambda \in \vec{\Lambda}^{a,j}$. Use the definitions of G and $\underline{\lambda}^{a,j}$:

$$[G(\bar{x}, \lambda^A, \Theta) - G(\underline{x}, \lambda^A, \Theta)] - [G(\bar{x}, \underline{\lambda}^{A,j}, \Theta) - G(\underline{x}, \underline{\lambda}^{A,j}, \Theta)] =$$

$$= \Big(\sum_{\substack{i \in \mathcal{A} \\ i \geq j}} \lambda_i [F(\bar{x}, \theta_i) - F(\underline{x}, \theta_i)] \Big) - \Big(\sum_{\substack{i \in \mathcal{A} \\ i \geq j}} \lambda_i \Big) [F(\bar{x}, \theta_j) - F(\underline{x}, \theta_j)] =$$

$$= \sum_{\substack{i \in \mathcal{A} \\ i \geq j}} \lambda_i \left([F(\bar{x}, \theta_i) - F(\underline{x}, \theta_i)] - [F(\bar{x}, \theta_j) - F(\underline{x}, \theta_j)] \right)$$

Notice that $[F(\bar{x}, \theta_i) - F(\underline{x}, \theta_i)] - [F(\bar{x}, \theta_j) - F(\underline{x}, \theta_j)] \geq 0 \ \forall i \geq j$ and $\lambda_i \geq 0$ $\forall i$ hence the sum above is also weakly positive, which implies $[G(\bar{x}, \lambda^A, \Theta) - G(\underline{x}, \lambda^A, \Theta)] - [G(\bar{x}, \underline{\lambda}^{A,j}, \Theta) - G(\underline{x}, \underline{\lambda}^{A,j}, \Theta)] \geq 0$. Similarly one can show that this is also true for $(\lambda^A, \bar{\lambda}^{A,j})$and $(\underline{\lambda}^{A,j}, \bar{\lambda}^{A,j})$. Finally notice that given that X is a convex sublattice of R^d and $\underline{x} \leq \tilde{x} \leq \bar{x}$, then $\tilde{x} \in X$. □

Lemma 9. *If $x \in \mathcal{P}(\mathcal{A})$ then $\exists x' \in \mathcal{P}(\mathcal{A}^{\leq j})$ with $\mathcal{A}^{\leq j} = \{i \in \mathcal{A} : i \leq j\}$ such that $x' \leq x$.*

Proof. Milgrom-Shannon's monotone comparative statics implies: $\widetilde{M}(\mathcal{A}, \lambda) = \arg\max_{x \in X, \lambda \in \Lambda} G(x, \lambda, \Theta)$ is monotone nondecreasing in λ. Hence $\exists x' \in \widetilde{M}(\mathcal{A}', \underline{\lambda}^{A,j})$ such that $x' \leq x$. Given that, together with the $\theta - concavity$ of F and the convexity of X (which together imply a convex utility possibility set), using the result in Mas Colell, Proposition 16.E.2, it follows that $x \in \mathcal{P}(\mathcal{A}^j)$, i.e. x is in the Pareto set of coalition $\mathcal{A}^j = \{i \in \mathcal{A} : i \leq j\}$. Q.E.D. □

Notice that we cannot exclude that $x' = x$.

Lemma 10. *If $x' \in \widetilde{M}(\mathcal{A}, \lambda^{A,j})$ and $x \in \widetilde{M}(\mathcal{A}', \underline{\lambda}^{A,j})$ and $x' \leq x$, $x' \neq x$, then $F(x', \theta_j) \geq F(x, \theta_j)$ and $F(x', \theta_i) > F(x, \theta_i)$ $\forall i < j$.*

Proof. We know $x' \leq x$ and $G(x', \underline{\lambda}^{A,j}, \Theta) \geq G(x, \underline{\lambda}^{A,j}, \Theta)$ from Monotone comparative statics. Suppose $F(x', \theta_j) < F(x, \theta_j)$. Then it must be true that $\sum_{i \in \mathcal{A}} \underline{\lambda}_i^{A,j} [F(x', \theta_i) - F(x', \theta_i)] > F(x', \theta_j) - F(x, \theta_j)$. Using $\sum_{i \in \mathcal{A}} \underline{\lambda}_i^{A,j} = 1$ the above can be rearranged as follows:

$$\sum_{i \in \mathcal{A}} \underline{\lambda}_i^{A,j} \left([F(x', \theta_i) - F(x', \theta_i)] - [F(x', \theta_j) - F(x, \theta_j)] \right) > 0$$

. Notice that $x' \leq x$ and $i \leq j \ \forall i \in \mathcal{A}$, hence *SSCP* implies $[F(x', \theta_i) - F(x', \theta_i)] - [F(x', \theta_j) - F(x, \theta_j)] \leq 0 \ \forall i \in \mathcal{A}$ and hence

$$\sum_{i \in \mathcal{A}} \underline{\lambda}_i^{A,j} \left([F(x', \theta_i) - F(x', \theta_i)] - [F(x', \theta_j) - F(x, \theta_j)] \right) \leq 0$$

which leads to a contradiction. Hence it must be true that $F(x', \theta_j) \geq F(x, \theta_j)$. Given that $x' \leq x$, $x' \neq x$, *SSCP* implies $F(x', \theta_i) > F(x, \theta_i)$ $\forall i < j$. □

Lemma 11. *The coalition $\mathcal{A}^m$ (could be a singleton) that includes the median voter m is stable only if $a^m \in M(m)$.*

Proof. Suppose $a^m \notin M(m)$ (A1).

(i) If $a^m \geq \underline{x}_m(\leq)$ for any $\underline{x}_m = \inf\{M(m)\}$ and $a^m \wedge x_m \in \mathcal{P}(\mathcal{A}^m)$. A1 implies $F(x_m, \theta_m) > F(a^m, \theta_m)$. *SSCP* implies $F(x_m, \theta_i) > F(a^m, \theta_i)$ and $x_m \succ^i a^m \; \forall i \in \mathcal{N} : \theta_i \leq \theta_m(\geq)$. Recall that any $c \in \mathcal{P}(\mathcal{A}^m \backslash \mathcal{A}^{\leq m})$ is $c \geq \underline{x}_m$(because of Lemma 7). Hence either $c \in M(m)$ or $\sum_{i=1}^n 1[F(\underline{x}_m, \theta_i) > F(c, \theta_i)] > n/2 \; \forall c \in \mathcal{P}(\mathcal{A} \backslash \mathcal{A}')$ which implies $x_m \succ c \rightarrow a^m \notin SK(\mathcal{A}^m)$.

(ii) If $a^m \ngeq x_m, a^m \nleq x_m$ and $a^m \wedge x_m \in \mathcal{P}(\mathcal{A})$., Consider $a^m \vee x_m(a^m \wedge x_m)$. Revealed preferences imply $F(x_m, \theta_m) \geq F(a^m \vee x_m, \theta_m)$. *QSM* implies $F(a^m \wedge x_m, \theta_m) \geq F(a^m, \theta_m)$. *SSCP* implies $F(a^m \wedge x_m, \theta_i) > F(a^m, \theta_i) \; \forall i \in \mathcal{N} : \theta_i \leq \theta_m$. Recall that any $c \in \mathcal{P}(\mathcal{A}^m \backslash \mathcal{A}^{\leq m})$ is $c \geq \underline{x}_m \geq a^m \wedge x_m$. Hence either $c \in M(m)$ or $\sum_{i=1}^n 1[F(a^m \wedge x_m, \theta_i) \geq F(c, \theta_i)] > n/2$ which implies $a^m \wedge x_m \succ c$.

(iii) If and $a^m \wedge x_m \notin \mathcal{P}(\mathcal{A}^m)$. $a^m \wedge x_m$. Recall that X is a convex set and $F(x, \theta)$ is $\theta - concave$, hence as $a^m \in A^m$ it has to be the solution to a problem in the form $a^m \in \arg\max_{x \in X} G(x, \lambda^m, \Theta)$. Now if $a^m \wedge \underline{x_m}$ is not part of the Pareto set of $\mathcal{A}^m$, consider the following alternative: $\tilde{x} \in \widetilde{M}(\mathcal{A}, \lambda^{a^m, m})$ (see Lemma 3). We know from Lemma 3 that $\tilde{x} \leq a^m$. First of all notice that $\widetilde{M}(\mathcal{A}, \lambda^{a^m, m}) = \widetilde{M}(\mathcal{A}', \lambda')$ for some λ', which imples that $\tilde{x} \in \mathcal{P}(\mathcal{A}')$, i.e. it is in the Pareto set of $\mathcal{A}'$. We need to show that $\tilde{x} \neq a^m$ and that $\tilde{x} \succeq^i a^m$ $\forall i \in \mathcal{A}'$. Suppose $\tilde{x} = a^m \rightarrow a^m \in \mathcal{P}(\mathcal{A}')$. But from point (b) we know that $F(a^m \wedge x_m, \theta_i) > F(a^m, \theta_i) \forall i \in \mathcal{N} : \theta_i \leq \theta_m \rightarrow a^m \notin \mathcal{P}(\mathcal{A}') \rightarrow$Contraddiction. Hence $\tilde{x} \neq a^m$ and $\tilde{x} \leq a^m$. Moreover, Lemma 4 implies $\tilde{x} \succeq^m a^m$. This means that $F(\tilde{x}, \theta_m) \geq F(a^m, \theta_m)$ and because $\tilde{x} \neq a^m$ SSCP implies $F(\tilde{x}, \theta_i) > F(a^m, \theta_i) \; \forall i \in \mathcal{N} : \theta_i \leq \theta_m$. Recall that any $c \in \mathcal{P}(\mathcal{A}^m \backslash \mathcal{A}^{\leq m})$ is $c \geq \underline{x}_m \geq \tilde{x}$. Hence either $c \in M(m)$ or $\sum_{i=1}^n 1[F(\tilde{x}, \theta_i) \geq F(c, \theta_i)] > n/2$ which implies $\tilde{x} \succ c \rightarrow a^m \notin SK(\mathcal{A}^m)$.

Now Suppose $a^m \in M(m)$, and in particular say $a^m = \bar{x}_m = \sup\{M(m)\}$ ($= \underline{x}_m \inf\{M(m)\}$). Consider any coalition $\mathcal{A}^{\leq m}$ such that $\theta_i < \theta_m(\geq) \; \forall i \in \mathcal{A}^{\leq m}$. From Lemma 7 we know that any $b \in \mathcal{P}(\mathcal{A}^{\leq m})$ it must be true that $b \leq x_m(\geq)$. Optimality implies $F(x_m, \theta_m) > F(b, \theta_m)$. *SSCP* implies $F(x_m, \theta_i) \geq F(b, \theta_i)$ and $x_m \succeq^i b \; \forall i \in \mathcal{N} : \theta_i \geq \theta_m(\leq)$. Hence $\sum_{i=1}^n 1[F(x_m, \theta_i) \geq F(b, \theta_i)] > n/2$ $\forall b \in \mathcal{P}(\mathcal{A}^{\leq m})$ which implies $x_m \succ b \; \forall b \in \mathcal{P}(\mathcal{A}^m) \rightarrow a^m \in SK(\mathcal{A}^m)$.

Finally Consider any coalition $\mathcal{A}^m$ such that $\theta_i \leq \theta_m(\geq) \; \forall i \in \mathcal{A}^m$ and $a^m = \overline{x}_m(\underline{x}_m)$. From Lemma 7 we know that any $b \in \mathcal{P}(\mathcal{A}^m)$ it must be true that $b \leq \overline{x}_m(\geq \underline{x}_m)$. This implies $F(\overline{x}_m, \theta_m) > F(b, \theta_m)$. *SSCP* implies $F(\overline{x}_m, \theta_i) > F(b, \theta_i)$ and $\overline{x}_m \succ^i b \; \forall i \in \mathcal{N} : \theta_i \geq \theta_m(\leq)$. Hence $\sum_{i=1}^n 1[F(\overline{x}_m, \theta_i) \geq F(b, \theta_i)] > n/2 \; \forall b \in \mathcal{P}(\mathcal{A}^m)$ which implies $\overline{x}_m \succ a \; \forall a \in \mathcal{P}(\mathcal{A}^m) \rightarrow P_{\mathcal{P}(\mathcal{A}^m)}(a^m) = \oslash \leftrightarrow a^m \in K(A^m)$. □

Lemma 12. *Any coalition $\mathcal{A}^j$ that does not contain the median voter m is stable only if $\exists a^j$ such that either of the following is true: (i) $a^j \in M(m)$; (ii) $a^j \geq x_m$ for all $x_m \in M(m)$; (iii) $a^j \leq x_m$ for all $x_m \in M(m)$.*

Proof. Suppose $a^j \notin M(m)$ and $a^j \ngeq x_m, a^j \nleq x_m$. There are three possible cases.

(i) say $x_k \in M(k)$ and $\forall k \in \mathcal{A}^j$ it is true either $x_k \geq a^j$ or $x_k \leq a^j$. Consider x_j such that $F(x_j, \theta_m) \geq F(x_k, \theta_m)$. In particular consider $\underline{x}_j = \inf\{\min_{\substack{k \in \mathcal{A}^j \\ k > m}} x_k\}$ or $\bar{x}_j = \sup\{\max_{\substack{k \in \mathcal{A}^j \\ k < m}} x_k\}$. Suppose $x_j = \underline{x}_j$ $(\bar{x}_j)$. Optimality implies $F(x_j, \theta_j) > F(a^j, \theta_j)$. Notice that because $x_j \neq a^j$ *SSCP* implies $F(x_j, \theta_i) > F(a^j, \theta_i)$ $\forall i \in \mathcal{N} : \theta_i > \theta_m(<)$. Notice that both $\underline{x}_j$ and $\bar{x}_j$ are in the Pareto set $\mathcal{P}(\mathcal{A}^{<j}) = \mathcal{P}(\{i \in \mathcal{A} : i < m\})$ $(\mathcal{P}(\mathcal{A}^{>j}) = \mathcal{P}(\{i \in \mathcal{A} : i > m\}))$ because they are the highest (lower) ideal points of some member of the subcoalition $\mathcal{A}^{<j}$ (see Lemma 7). Finally notice that any policy $b \in \mathcal{P}(\mathcal{A}^j \backslash \mathcal{A}^{<j})$ must be $b \in M(m)$ or $b \leq x_j$ $(\geq)$ (because of Lemma 7). Hence if $x_j \neq a^j$ then $\sum_{i=1}^n 1[F(x_j, \theta_i) \geq F(b, \theta_i)] > n/2$ $\forall b \in \mathcal{P}(\mathcal{A}^j \backslash \mathcal{A}^{<j})$ which implies $x_j \succ b \forall b \in \mathcal{P}(\mathcal{A}^j \backslash \mathcal{A}^{<j}) \rightarrow a^j \notin SK(\mathcal{A}^j)$.

(ii) $\exists x_k \in M(k)$, $k \in \mathcal{A}^j$, $\theta_k > \theta_m(<)$ such that $x_k \ngeq a^j$, $x_k \nleq a^j$ (A2). Consider $x_k \wedge a^j$. Notice that (A2) implies $x_k \wedge a^j \neq a^j$. Optimality implies $F(x_k, \theta_k) \geq F(x_k \vee a^j, \theta_k)$. *SM* implies $F(x_k \wedge a^j, \theta_k) \geq F(a^j, \theta_k)$. *SSCP* implies $F(x_k \wedge a^j, \theta_i) \geq F(a^j, \theta_i)$ $\forall i \in \mathcal{N} : \theta_i \leq \theta_m$. Hence $\sum_{i=1}^n 1[F(x_k \wedge a^j, \theta_i) \geq F(a^j, \theta_i)] > n/2$. which implies $x_k \wedge a^j \succ a$ $\forall a \in \mathcal{P}(\mathcal{A}^j)$.

(iii) Is $x_k \wedge a^j$ part of the Pareto set of $\mathcal{A}^{<j}$? Recall that X is a convex set and $F(x, \theta)$ is $\theta - concave$, hence as $a^j \in \mathcal{P}(\mathcal{A}^j)$ it has to be the solution to a problem in the form $a^j \in \arg\max_{x \in X} G(x, \lambda^j, \Theta)$. Now if $x_k \wedge a^j$ is not part of the Pareto set of $\mathcal{A}^j$, consider the following alternative: $\tilde{x} \in \widetilde{M}(\mathcal{A}, \lambda^{a^j,k})$ (see Lemma 6). We know from Lemma 9 that $\tilde{x} \leq a^j$. First of all notice that $\widetilde{M}(\mathcal{A}, \lambda^{a^j,k}) = \widetilde{M}(\mathcal{A}', \lambda')$ for some λ', which imples that $\tilde{x} \in \mathcal{P}(\mathcal{A}')$, i.e. it is in the Pareto set of $\mathcal{A}^{<j}$.We need to show that $\tilde{x} \neq a^j$ and that $\tilde{x} \succeq^i a^j$ $\forall i \in \mathcal{A}^{<j}$. Suppose $\tilde{x} = a^j \rightarrow a^j \in \mathcal{P}(\mathcal{A}^{<j})$. From point (ii) we know that $F(x_k \wedge a^j, \theta_i) > F(a^j, \theta_i)$ $\forall i \in \mathcal{N} : \theta_i \leq \theta_m \rightarrow a^j \notin \mathcal{P}(\mathcal{A}^{<j}) \rightarrow$ Contraddiction. Hence $\tilde{x} \neq a^j$ and $\tilde{x} \leq a^j$. Moreover, Lemma 10 implies $\tilde{x} \succeq^j a^j$. This means that $F(\tilde{x}, \theta_j) \geq F(a^j, \theta_j)$ and because $\tilde{x} \neq a^j$ *SSCP* implies $F(\tilde{x}, \theta_i) > F(a^j, \theta_i)$ $\forall i \in \mathcal{N} : \theta_i \leq \theta_j$. Recall that any $c \in \mathcal{P}(\mathcal{A} \backslash \mathcal{A}^{<j})$ is $c \geq \underline{x}_j \geq \tilde{x}$. Hence either $c \in M(m)$ or $\sum_{i=1}^n 1[F(\tilde{x}, \theta_i) \geq F(c, \theta_i)] > n/2$ which implies $\tilde{x} \succ c \rightarrow a^j \notin SK(\mathcal{A}^j)$. □

4.1. Proof of main result (Theorem 5)

The main result of this paper is stated in the following theorem:

> *(i) An equilibrium exists. (ii) In any equilibrium the set of winning policies W is a subset of the set of ideal points of the median voter m, i.e. $W \subseteq M(m)$. (iii) If the median voter has a unique ideal point, this policy is going to be the one chosen in any equilibrium.*

Proof. The results in Lemma 11 and Lemma 12 imply that the only policies that can be proposed by stable coalitions in equilibrium are either $a^m \in M(m)$ or $a^l \leq a^m$ or $a^h \geq a^m$. Recall optimality implies $F(a^m, \theta_m) > F(a^l, \theta_m)$ and *SSCP* implies $F(a^m, \theta_i) > F(a^l, \theta_i)$ $\forall i \in \mathcal{N} : \theta_i \geq \theta_m$. Similarly $F(a^m, \theta_m) >$

$F(a^h, \theta_m)$ and *SSCP* implies $F(a^m, \theta_i) > F(a^h, \theta_i)\ \forall i \in \mathcal{N} : \theta_i \leq \theta_m$. Also, the coalition structure in which every coalition is a singleton is always stable. Hence a Condorcet winner among the proposed policies exists, which is also the policy chosen in an equilibrium of the coalitional game (i). The total order in the policy available in all reduced games generated by a stable coalition structure implies the Condorcet winner must be always some $a^m \in M(m)$ (ii). The proof of (iii) is straighforward from (i) and (ii). □

Corollary 13. *(i) The equilibrium policy is in the Core of a winning coalition, i.e.* $x \in W \to x \in K(\mathcal{A}^m)$ *for some winning coalition* $\mathcal{A}^m$. *Moreover, (ii) the equilibrium policy is in the Core of the reduced game, i.e.* $x \in W \to x \in K(\mathcal{A}^s)$ *for any equilibrium policy profile* $\mathcal{A}^s$.

Proof. Straightforward from Theorem 5. □

4.2. Comparative statics

Define E to be the space of possible equilibrium policies, i.e. $x \in E$ if and only if x is a winning policy in the voting game.

Lemma 14. *The space of possible equilibrium policies* E *is a sublattice of* X.

Proof. Recall that a subset X of L is a sublattice of L if X is closed under the operations meet and join. It is easy to show that (i) if $\mathcal{A}^m = \{m\}$ and $M(m) \cap M(i) = \oslash$ for all $i \neq m$ then $E = M(m)$; (ii) if $\mathcal{A}^m = \{m\}$ and $M(m) \cap M(i) \neq \oslash$ for some $i \neq m$ then $E \in \{\{\overline{x}_m, \underline{x}_m\}, \{\overline{x}_m\}, \{\underline{x}_m\}\}$; (iii) if $\mathcal{A}^m \neq \{m\}$ then $E \in \{\{\overline{x}_m, \underline{x}_m\}, \{\overline{x}_m\}, \{\underline{x}_m\}\}$. Recall that $M(m)$ is a convex sublattice of X (see Milgrom, Shannon 1994). Moreover $E = \{\overline{x}_m, \underline{x}_m\}$ is a sublattice of X because $E \subseteq X$ and $\overline{x}_m \vee \underline{x}_m = \overline{x}_m$, $\overline{x}_m \wedge \underline{x}_m = \underline{x}_m$ hence $\{\overline{x}_m \vee \underline{x}_m, \overline{x}_m \wedge \underline{x}_m\} \in E$ therefore it satisfies the definition of sublattice. Finally $\{\overline{x}_m\}, \{\underline{x}_m\}$ are sublattices because they are singletons. □

The notion of monotonicity is the same as in Milgrom, Shannon (1994) and it is related to the Strong Set order, namely given two sets Y, Z we say that Y is grater than or equal to Z in the Strong Set order ($Y \geq_s Z$) if for any $y \in Y$ and $z \in Z$ we have $y \vee z \in Y$ and $y \wedge z \in Z$. This leads to the second important statement in this paper.

Theorem 15. *(Monotone Comparative Statics). The set of policies* E *that can be supported in an equilibrium of the voting game is monotonic nondecreasing in* θ_m.

Proof. From the proof of Lemma 11 consider case (i). Given that $E = M(m)$ and $M(m)$is monotonic nondecreasing in θ_m (Milgrom, Shannon 1994, Theorem 4), then E is as well. Consider cases (ii), (iii). Suppose $\theta_{m'} > \theta_m$, then given that $M(m') \geq_s M(m)$ (Milgrom, Shannon 1994), then it must be true that $\overline{x}_{m'} \geq \overline{x}_m$ and $\underline{x}_{m'} \geq \underline{x}_m$, which implies $\{\overline{x}_{m'}, \underline{x}_{m'}\} \geq_s \{\overline{x}_m, \underline{x}_m\}$, $\{\overline{x}_{m'}\} \geq \{\overline{x}_m\}$, $\{\underline{x}_{m'}\} \geq \{\underline{x}_m\}$. Q.E.D. □

This result is potentially very important in order to establish the direction of the change in policy induced by a change in the distribution of θ even if the individual objective function F is not C^2 and therefore the First Order Conditions of the maximization problem cannot be used in order to calculate the comparative statics of interest. The reason of this is that the result is based on Milgrom-Shannon's monotone comparative statics which is very general. One caveat is that in order to establish the existence of a Political Equilibrium in my model the additional assumption of continuity of F makes my comparative statics result slightly less general than the one in their paper.

4.3. Citizen Candidate Model

A class of models that allow for the existence of a political equilibrium even if the policy space is multidimensional is the one of Citizen-Candidate models (Besley and Coate 1997; Osborne and Slivinski, 1996). The crucial assumption of this class of models is that each voter can run for elections as a candidate, and that each candidate i can credibly commit only to a policy that is in the set of her ideal points $x_i \in M(i)$. The main shortcoming of this class of models if one aims to get predictions about the policy choice of a certain group of individuals is the multiplicity of equilibria. There is neverthless a case in which this model delivers a unique equilibrium, and this case is described in Corollary 2 (ii) of Besley and Coates 1997. In their model a citizen faces a cost δ to run for elections, and:

> (ii) if x_i is a strict Condorcet winner in the set of alternatives $\{x_j : j \in \mathcal{N}\}$ and if $x_i \neq x_0$, then a political equilibrium exists in which citizen i runs unopposed for sufficiently small δ.

Consider one particular stable coalition structure in the model presented in this paper, namely the one in which each coalition is a singleton. In this case the Pareto set of each coalition coincide with the set of ideal points of its single member. In this setting, my assumptions about individual preferences (*SM* and *SSCP*) are sufficient to ensure that there is at least one $x_m \in M(m)$ who is a Condorcet winner, as a consequence of Corollary 17 (ii).

Therefore we can conclude that:

(i) the result in Besley and Coate for $\delta \to 0$ can be interpreted as a particular case of equilibrium of the coalitional game model presented in this paper;

(ii) the restrictions on individual preferences I assumed in this paper (*SM* and *SSCP*) are also sufficient conditions for a unique equilibrium in which the median voter run unopposed and implements a policy that is in the set of her ideal points in the Citizen-Candidate model whenever $\delta \to 0$.

4.4. Coalition Structure

In this section I provide some example of stable and unstable coalition structures in this framework. A key aspect of a stable coalition structure in this model is given by the following statement.

Lemma 16. *(Lateral Coalitions). Any coalition $\mathcal{A}^j$ that include either (a) individuals with index ($i \leq m$) or (b) individuals with index ($j \geq m$) is always stable. Therefore a coalition structure $\mathbb{P}$ is stable if each coalition $\mathcal{A}^i \in \mathbb{P}$ satisfies either (a) or (b).*

Proof. Straightforward from Lemma 11 and Lemma 12 and the definition of a stable coalition structure. □

Lemma 17. *(Central Coalitions). (i) Any coalition $\mathcal{A}^j$ that include both (a) individuals with index ($i < m$) and (b) individuals with index ($j > m$) plus individual m is stable if at least one policy $x_m \in M(m)$ is in the Strong Core of a game $(\mathcal{N}, P(\mathcal{A}') \cup x_m, F_i)$ for all $\mathcal{A}' \subseteq \mathcal{A}$. (ii) If the (Strong) Core of the full game $(\mathcal{N}, X, F_i)$ is non-empty, then any "Central Coalition" is stable, including the Grand Coalition of all voters.*

Proof. Proof. (i) $x_m \in M(m)$ is in the Strong Core of a game $(\mathcal{N}, P(\mathcal{A}') \cup x_m, F_i)$ for all $\mathcal{A}' \subseteq \mathcal{A}^j$ implies that for any deviation of a subcoalition $\mathcal{A}' \subseteq \mathcal{A}$ any policy a in the Pareto set $P(\mathcal{A}')$ of this coalition is defeatead by x_m by majority voting. Moreover $x_m \in P(\mathcal{A} \backslash \mathcal{A}')$, hence a it is not a "credible threat" to x_m for coalition $\mathcal{A}$. As the statement implies that this is true for all possible $\mathcal{A}' \subseteq \mathcal{A}$, it implies that $x_m \in SK(\mathcal{A})$ and therefore $\mathcal{A}$ is stable. (ii) Notice that given that $P(\mathcal{A}') \cup x_m \subseteq X$ this implies that if the (Strong) Core of the full game $(\mathcal{N}, X, F_i)$ is non-empty, then any "Central Coalition" is stable, including the Grand Coalition of all voters. □

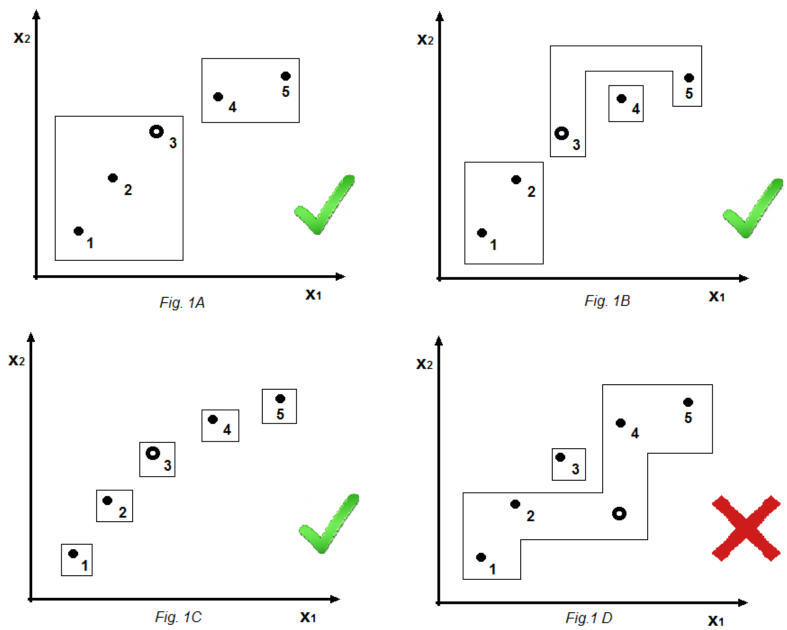
Definition 18. A Partisan Equilibrium is an equilibrium in which, given the policy profile $A_{\mathbb{P}}$ and the strategies of all other voters, it is a best response for each individual $i \in \mathcal{A}^j$ to vote for policy a^j.

The definition implies that in a Partisan Equilibrium no voter has a strict incentive to vote for a policy different from the one proposed by the coalition she is part of.

Lemma 19. *(Ends-Against-the-Middle Coalitions). (i) Any coalition $\mathcal{A}^j$ that include both (a) individuals with index ($i < m$) and (b) individuals with index ($j > m$) but it does not include individual m is stable if either of the following is true: (1) $a^j \in M(m)$; (2) $a^j \geq x_m$ for all $x_m \in M(m)$; (3) $a^j \leq x_m$ for all $x_m \in M(m)$. Therefore (ii) if $M(m) \cap P(\mathcal{A}^j) = \emptyset$, then there is no Partisan Equilibrium in which one or more coalitions are of the "Ends against the Middle" type.*

Proof. (i) is straightforward from Lemma 11 and Lemma 12 and the definition of a stable coalition structure. For (ii) notice that any coalition that satisfies (a) and (b) possess at least one member that strictly prefers any $x_m \in M(m)$ to a^j whenever $x_m \neq a^j$. Condition $M(m) \cap P(\mathcal{A}^j) = \emptyset$ implies $x_m \neq a^j$ for all $x_m \in M(m)$, therefore given that at the equilibrium there will be a coalition proposing a policy $x_m \in M(m)$ (because of Lemma 11), then there is at least one voter $i \in \mathcal{A}^j$ such that, whenever her vote determines the outcome, would prefer to vote for x_m rather than for a^j. □

For illustrative purposes it may be useful to analyse a case in which the policy space is bi-dimensional, i.e. $X \subseteq R^2_+$, 5 players i.e. $\mathcal{N} = 5$, $m = 3$ and in which individuals have unique ideal points (black dots). The black circle represents the winning policy for that coalition structure. Fig. 1A, 1B, 1C all represent stable coalition structures because condition (a) in Lemma 16 is satisfied. Notice that Fig. 1C corresponds to the case of the Citizen-Candidate model described in the previous section. Finally Fig. 1D represents a case in which condition (a) in Lemma 16 is violated, hence it may not represent a stable coalition structure.



These examples show that under my assumptions about individual preferences (*SM* and *SSCP*) the Median Voter Theorem result that emerge in the Citizen-Candidate model is robust to settings in which a much richer policy space is actually available to the voters, and hence it does not crucially depend on the strong restrictions that the model proposed by Besley and Coates implies on this aspect of the political interaction.

The political intuition that underpins the Median Voter result is that if preferences are ordered by the *SSCP* then in order to defeat the median voter it is necessary an Ends-Against-the-Middle Coalition, which is intuitively less stable than a "Lateral Coalition" or a "Central Coalition".

4.4.1. Conjecture: Ray-Vohra Stability

Under the assumptions of *SM* and *SSCP* a coalition structure is stable if it is stable in the sense of Ray and Vohra (1997).

This concept of stability is similar to the one I propose in this paper, except that the in our proposed solution the profitability from a deviation for a subcoalition is not evaluated keeping into account the possibility of future additional deviations (Recursive Consistency) or the possibility that a non profitable deviation can generate future deviations by other players and the final outcome is strictly better for all the deviators (Farsightedness). Moreover, potential deviators do not consider the potential threat represented by individuals that are outside the coalitions the deviators are part of.

The conjecture is correct if Recursive Consistency and Farsightedness can be incorporated in this model without changing the main result.

If this conjecture can be proved a strong link will arise between the political equilibrium concept presented in this paper and the one in Levy (2004, 2005), which is based on the stability concept proposed by Ray and Vohra, namely the equilibrium would be a *refinement* of the one in Levy's voting model.

4.5. Social Choice

It is useful to analyse the characteristics of the Social Preference Ordering generated by this Political Economy model. We know from Arrow's Impossibility Theorem (Arrow 1950) that there is no Social Preference Ordering that satisfies at the same time *unrestricted domain (UD)*, *non-dictatorship (ND), Pareto efficiency (PE), and independence of irrelevant alternatives* (*IIA*). Bergson (1976) has shown that Citizen-Candidate class of models imply a violation of *IIA*.

In my model *IIA* is not generally satisfied because the restrictions on the policy space X (which has to be a convex sublattice of R^d) are crucial in order to ensure a stable outcome that satisfies the Median Voter theorem.

On the other hand *Unrestricted Domain (UD)* is obviously violated given the restrictions on individual preferences, in the same way in which the Spence-Mirrlees conditions imply a violation of *UD* in the traditional Median Voter analysis of Downs (1957).

It is easy to verify that *ND* and *PE* are satisfied.

Finally the Gibbard-Satterthwaite Theorem (Gibbard 1973; Satterthwaite 1975) suggest that in this model voters may have an incentive to misrepresent their true preference orderings.

5. Games with Strategic Complementarities (incomplete)

Modify the setting in 2.1 in the following way. Consider a game with k players $j \in \mathcal{J}$ with a subset $\mathcal{N} \subset \mathcal{J}$ of n players who are also voters (n odd); Each player $j \in \mathcal{J}$ is endowed with a reflexive, complete and transitive preference ordering $\succeq^i$ that can be represented by a continuous and $\theta - concave$ utility function $F : X \times \Theta \times \Delta \to R$.

Define Δ as the set of vectors of strategies that each player can take outside of the voting game, with typical element $\delta = \{\delta_1, \delta_2, ..., \delta_k\}$, $\delta \in \Delta$.

The game can be either (i) simultaneous or (ii) sequential, i.e. at $t = 1$ voters play the voting game and the policy is chosen; at $t = 2$ each player j chooses $y \in Y_j$.

The voting game requires minimum changes in the definitions, in particular individual preference relations over policies have to be defined conditional on beliefs about other players' strategies, e.g. $\succeq^i (\tilde{\delta}_{-i})$ describes the preference relation of individual i given beliefs $\tilde{\delta}_{-i}$. Consequently the set of policies that are Pareto superior to a for coalition $\mathcal{A}$ becomes $p_{X,\mathcal{A}}(a, \{\tilde{\delta}_{-i}\}_{i\in\mathcal{A}}) \equiv \{b \in X : b \succeq^i (\tilde{\delta}_{-i})a \ \forall i \in \mathcal{A}, b \not\preceq^i (\tilde{\delta}_{-i})a \ \forall i \in \mathcal{A}\}$ and the Pareto set of coalition $\mathcal{A}$ becomes $\mathcal{P}(\mathcal{A}, \{\tilde{\delta}_{-i}\}_{i\in\mathcal{A}}) \equiv \{a \in X : \ p_{X,\mathcal{A}}(a, \{\tilde{\delta}_{-i}\}_{i\in\mathcal{A}}) = \oslash\}$. Similarly social preferences are now given by $\succeq (\{\tilde{\delta}_{-i}\}_{i\in\mathcal{N}})$ such that

$a \succeq (\{\tilde{\delta}_{-i}\}_{i\in\mathcal{N}})b$ if and only if $\sum_{i=1}^{n} 1[\max_{\delta_i} F(a, \theta_i, \delta_i, \tilde{\delta}_{-i}) \geq \max_{\delta_i} F(b, \theta_i, \delta_i, \tilde{\delta}_{-i})] > n/2$.

Similarly one can modify the definitions in section 3 as follows:

$P_A(a, (\{\tilde{\delta}_{-i}\}_{i\in\mathcal{N}})) \equiv \{b \in A : \ b \succ (\{\tilde{\delta}_{-i}\}_{i\in\mathcal{N}})a\}$ is the strictly preferred set;

$K(\mathcal{A}, (\{\tilde{\delta}_{-i}\}_{i\in\mathcal{N}})) \equiv \{a \in A : \ P_A(a, (\{\tilde{\delta}_{-i}\}_{i\in\mathcal{N}})) = \oslash\}$ is the Core;

$S_{\mathcal{A}}(a, \{\tilde{\delta}_{-i}\}_{i\in\mathcal{A}}) \equiv \{b \in \mathcal{P}(\mathcal{A}', \{\tilde{\delta}_{-i}\}_{i\in\mathcal{A}}), \mathcal{A}' \subseteq \mathcal{A} : b \succ^i (\tilde{\delta}_{-i})a \ \forall i \in \mathcal{A}', b \succ c \ \forall c \in \mathcal{P}(\mathcal{A}\backslash\mathcal{A}', \{\tilde{\delta}_{-i}\}_{i\in\mathcal{A}})\}$ is the set of "credible threats"

$SK(\mathcal{A}) = \{a \in \mathcal{P}(\mathcal{A}) : \ S_{\mathcal{A}}(a, \{\tilde{\delta}_{-i}\}_{i\in\mathcal{A}}) = \oslash\}$ is the S-Core.

Definition 20. A pure strategy equilibrium of this game is:

(a) a coalition structure $\mathbb{P}^* = \{\mathcal{A}^j\}_{j=1}^h$, a policy profile $A^* = \{a^j\}_{j=1}^h$ and a set of winning policies $W(A^*) \subseteq A^*$ such that such that *given beliefs* about the strategies $\tilde{y}$: (i) $\mathbb{P}^*$ is a stable coalition structure; (ii)$a^j \in SK(\mathcal{A}^j)$ for all $j = 1, 2, .., h$; (iii) $W(A^*)$ is nonempty;

(b) a strategy profile $\delta \in \Delta$ such that $\delta_i \in \arg\max_{\delta\in\Delta_i} F(\tilde{x}, \theta_i, \delta, \tilde{\delta}_{-i})$ for each $j \neq m$ and $(x, \delta_m) \in \arg\max_{x\in X, \delta\in\Delta_m} F(x, \theta_m, \delta, \tilde{\delta}_{-m})$ for $i \in m$;

(c) a set of beliefs about the strategies $\tilde{y} \in \tilde{Y}$ such that beliefs are correct at an equilibrium.

With this new definition of an equilibrium for the game we can state the following result:

Theorem 21. *(i) An equilibrium exists. (ii) In any equilibrium the set of winning policies W is a subset of the set of ideal points of the median voter m. (iii) The largest and smallest pure strategy equilibria (and serially undominated strategy profiles) (a_m^*, δ^*) and (a_{*m}, δ_*), are monotone nondecreasing functions of the parameter that identifies the median voter θ_m.*

Proof. See Appendix B (incomplete). □

6. Applications

6.1. Meltzer-Richard revisited

In one influential paper Meltzer and Richard (1981) analyse in a unidimensional political economy model the relationship between income distribution of a society and the extent of redistributive policies. One famous result in in their paper is that

> [..] An increase in mean income relative to the income of the decisive voter increases the size of the government.

This result imply a positive relationship between the size of the governamental sector and a measure of income skewness. It represents a strong prediction that has been tested empirically in several studies (for a survey of this literature see De Mello, Tiongson, 2006) with very little success. A majority of early papers based on cross-country data shows that the relationship between income skewness and some measure of the size of the government is not statistically significant (for instance Perotti 1992, 1994, 1996; Persson and Tabellini 1994). A subsequent stream of more recent studies based on panel data has shown a significant negative relationship between the two variables (Guveia and Masia 1998, Razin, Sadka and Swagel 2002). These results represent a strong rejection of the predictions of the theory and a puzzle for the Political Economy literature.

For instance Razin et al. (2002) use a Panel of 13 OECD countries for a total of 330 observations and use country fixed effects in their specification; their measure of income skewness is the is the ratio of the income share of the top quintile to the combined share of the middle three quartiles and their measure of the size of the government is the average tax rate. They find a negative and statistically significant relationship between income skewness and average tax rate (see Table 1). Interestingly they show that if they perform the same analysis using a different dependent variable, namely the total amount of social transfers, the relationship becomes positive (although not statistically significant). This represents a second challenge for the theory because the *MR* model implies that the size of the government and the amount of redistributive governmental intervention should move in the same direction as a result of a change in the degree of income skewness.

TABLE 1
DETERMINANTS OF LABOR TAX RATE AND SOCIAL TRANSFERS (330 Observations)

	LABOR TAX RATE		SOCIAL TRANSFERS	
	(1)	(2)	(3)	(4)
Dependency ratio	−.382	−.383	−7.493	−7.492
	(−4.02)	(−4.40)	(−8.81)	(−8.80)
Government jobs/ employment	.915	.729	4.467	4.611
	(12.17)	(10.01)	(6.64)	(6.47)
Trade openness	.198	.131	.740	.792
	(8.09)	(5.45)	(3.73)	(3.37)
Per capita GDP growth	−.187	−.127	−2.716	−2.762
	(−2.83)	(−2.09)	(−4.59)	(−4.63)
Rich/middle income share	−.055	−.049	.276	.271
	(−2.77)	(−2.66)	(1.55)	(1.52)
Unemployment rate		.480		−.370
		(7.82)		(−.62)
R^2	.753	.793	.617	.618

NOTE.—All specifications include country fixed effects (coefficients not shown).

Source: Razin et al., Journal of Political Economy, 2002.

In this section I will show how the result in Meltzer and Richard (*MR*) does not generally survive in a political equilibrium if a richer policy space is available to voters, and that the opposite prediction may emerge if one allows for a more realistic model. Moreover I will show that in such augmented model the progressivity of the tax sistem (and not the size of the government) is tweakly increasing in the degree of income skewness. Razin's findings suggest that a stronger degree of skewness in the income distribution should imply a lower average tax rate while the amount of social transfers should be unaffected or larger. Intuitively a tax system of this kind is a more progressive one, so both prediction of the augmented model seems to be more consistent with empirical evidence. The conclusion of this analysis is that the empirical puzzles described above can be partially explained by the excessively restrictive assumptions of the *MR* model.

6.1.1. Setting

The setup is similar to the one in *MR*, with the difference that the budget of the government is spent not only in in-cash redistribution, but also in Public Goods. For simplicity I am going to assume that the voters' population is a continuum with Lebesgue measure 1. This can be interpreted as a limit case in which he number of voters n is a very large number.

Policy Space. The policy space denoted by $X \subseteq R^2$ is bi-dimensional with tipical element $\chi = (x, Y)$. I am going to assume $\chi \in X$ if and only if $x \in [\underline{x}, \bar{x}]$ and $Y \in [0, \bar{Y}]$ with $0 < \underline{x} < x \leq 1$. Because a policy χ a vector (x, Y) any endogenous variable is going to be a function of (x, Y). Notice that the partially ordered set $(X, \leq)$ is a complete and convex sublattice of R^2.

Preferences. Each voter i has a C^2 concave utility function:

$$U_i = U(c_i, Y, l_i)$$

where c_i is i's consumption of private goods, Y is the the quantity of Public Goods that is provided by the government and l_i is leisure. I assume $c_i \geq 0$, $Y \geq 0$ and $T \geq l_i \geq 0$. Individuals allocate their time between consumption and leisure such that

$$l_i = T - h_i$$

where $h_i \in [0, T]$ is i's hours of work and T is the total endowment of time. Define $y_i = h_i \omega_i$ as the pre-tax income of an individual i with wage ω_i and that supplies an amount h_i of hours of labour. Individuals differ only in their wages that are distributed according to a continuous right-skewed distribution with cdf G.

Labour supply. The labour supply of each individual is endogenous in (ω_i, x, Y). In particular at each policy vector $\chi \in X$ an individual i solves a problem of choice over consumption and leisure (C-L problem). After substituting the time endowment constraint into the utility function the problem is in the form:

$$\max_{c\in[0,\bar{c}],h\in[0,T]} U(c,Y,T-h)$$

subject to an individual budget constraint $c - c(h\omega_i, x, Y) \leq 0$ for some function $c(h_i\omega_i x, Y)$. Given this that we can define the optimal labour supply function as a continuous function in the form:

$$h_i^* = h(\omega_i, x, Y) = \arg \max_{\substack{c \geq 0 \\ h \in [0,T]}} U(c,Y,T-h)$$

subject to $c - c(h_i\omega_i x, Y) \leq 0$. Moreover one can define a continuous function representing optimal earned income:

$$y_i^* = y(\omega_i, x, Y) = \omega_i h(\omega_i, x, Y)$$

Define the average income as:

$$\bar{y} = \bar{y}(x,Y,g,G(\omega)) = \int_{\underline{\omega}}^{\bar{\omega}} y(\omega,x,Y)dG(\omega)$$

Notice that $\bar{y}$ is itself endogenous in $(x, Y, G(\omega))$.

Tax system. The tax system is the same as in *MR*, namely individual post-tax income is determined by a linear tax rate t and by a lump-sum grant g. Define $x = 1 - t$. Therefore the after tax income that is equivalent to the amount of private good consumed by individual i will be given by:

$$c_i = xy_i + g$$

The government has to break even, such that the governmental budget constraint is given by:

$$(1-x)\bar{y}(x,Y,G(\omega)) - Y - g \geq 0$$

which simply states that the total governmental spending cannot exceed the total tax revenue. Assuming that the constraint above is binding in equilibrium (and later sufficient conditions for this will be provided) we can solve for g to get:

$$\hat{g} \equiv \hat{g}(x,Y) = (1-x)\bar{y}(x,Y) - Y$$

and then substitute $\hat{g}$ into the formula for c_i:

$$c_i = xy_i + (1-x)\bar{y} - Y$$

Therefore the pre-tax income of an individual i previously defined lies in the range $y_i^* \in [0, y_i^T]$, where $y_i^T = \omega_i T$ is the maximum income that individual i can achieve. Notice that $\hat{g}$ is itself a function of $\bar{y}$ and hence of h_i, but the effect of the individual choice of h_i on $\hat{g}$ disappears because the number of voters is infinitely large and therefore $\hat{g}$ is independent of h_i in this example.

Objective function . The objective function of an individual in the voting game is the indirect utility function of the *C-L* problem that represents the utility achieved by an individual at the optimal level of labour supply as a function of the policy vector (x, Y) and of the individual wage ω_i. Substituting the equations for $\hat{g}, c_i, y_i, \bar{y}$ from the previous paragraphs into U_i we can get the individual indirect utility function:

$$V(x, Y, ; \omega) = U(xy_i^* + (1-x)\bar{y} - Y, Y, T - h_i^*)$$

Notice that we also need to rule out the possibility that c_i is negative for some individuals at some $(x, Y) \in X$. It is sufficient to impose that $\bar{Y}$ is such that

$$\min_{x \in [\underline{x}, \bar{x}]} (1-x)\bar{y} - Y + xy_i^* \geq 0$$

for all i and all $Y \in [0, \bar{Y}]$ (this condition ensure that $\hat{g}$ cannot get too negative and induce negative consumption for some individual). Also notice that if $\bar{y} > 0$ for all $x \in [\underline{x}, \bar{x}]$ at $Y = 0$ the continuity of y_i^* and $\bar{y}$ ensures that there is always a $\bar{Y} > 0$ such that the condition above is satisfied.

Size of the Government and Progressivity. The other two elements to be defined are the size of the government and the progressivity of the tax system. In Meltzer and Richard's paper the size of the government is simply the marginal tax rate $t = 1 - x$. More correctly one should define it as either the total government spending or equivalently as the total tax revenue. A third possibility is to use the average tax rate. The formula for the total revenue is:

$$TR(x, Y) \equiv \int_{\underline{\omega}}^{\bar{\omega}} (1 - \mathbb{I}[(1-x)y(\omega_i, x, Y) - \hat{g} \leq 0]) ((1-x)y(\omega_i, x, Y) - \hat{g}) \, dG(\omega)$$

where

$$\mathbb{I}[a \leq 0] = \begin{cases} 1 & if \quad a \leq 0 \\ 0 & otherwise \end{cases}$$

$\mathbb{I}[a \leq 0]$is the indicator function that has value equal to 0 if individual i is a net tax payer and value equal to 1 if individual i is a net receiver of subsidies. So the integral defining TR represents the sum of net tax paid. Given that the net tax revenue is used to finance the provision of a public good Y and the subsidies to the individuals who pay no net taxes, one can define the total government spending as the sum of net subsidies plus the expenditure in public goods, namely:

$$S(x, Y) \equiv \left(\int_{\underline{\omega}}^{\bar{\omega}} \mathbb{I}[(1-x)y(\omega_i, x, Y) - \hat{g} \leq 0] [\hat{g} - (1-x)y(\omega_i, x, Y)] \, dG(\omega) \right) + Y$$

Another possibility is to use the average tax rate, which is defined as the total net tax revenues divided by total income:

$$AT(x,Y) = \left(\int_{\underline{\omega}}^{\bar{\omega}} [(1-x)y(\omega_i, x, Y) - \hat{g}]\, dG(\omega) \right) / \bar{y}$$

In the Meltzer and Richard's paper the simplification that the size of the government is simply equal to t has limited consequences: given that the policy space is unidimensional and there is no Public Good (equivalent to $Y = 0$ in the above formulas) it is easy to show that the size of the government is weakly increasing in t under relatively weak assumptions, hence the conclusions of their paper would survive even if we define the size of the government in any of the three ways suggested above. For instance if one chooses measure $S(x, 0, G(\omega))$ we have that in the Meltzer and Richard's case we get

$$\frac{\partial S}{\partial x} = \int_{\underline{\omega}}^{\bar{\omega}} \mathbb{I}\left[(1-x)y_i^* - \hat{g} \leq 0\right] \left[y - \bar{y} - (1-x) \left(\frac{d\bar{y}}{dx} - \frac{dy(\omega_i, x, Y)}{dx} \right) \right] dG(\omega)$$

Notice that the above is negative if the marginal effect of a change in x does not vary too much across different income levels (in particular it is easy to show that the above is negative if the tax elasticity of labout supply is negative and constant in ω_i). Therefore the size of the government tend to move in the same direction as $t = 1 - x$. In my setting instead I need to use one of the definitions above and I will fully keep into account the endogeneity of earned income. The progressivity of the tax system is defined using the index proposed by Slitor (1948). Consider the tax rate faced by individual i, defined as the ratio of individual net tax payment to gross income:

$$T_i(x, g, y_i) = \frac{(1-x)y_i - g}{y_i}$$

The progressivity $PR_i = PR(x, g, y_i)$ faced by individual i is defined as the change of the tax rate induced by a marginal increase in income:

$$PR(x, g, y_i) = \left. \frac{\partial T_i(x, g, y_i)}{\partial y_i} \right|_{x,g} = \frac{g}{y_i^2}$$

This measure of progressivity is income dependent, but given the structure of the tax system assumed in this example the sign of the index and the direction of the effect of the *MDIS* are the same for all individuals as they both depend uniquely on g (at constant y_i). Therefore in this framework we can define without loss of generality the average progressivity as:

$$\overline{PR}(x, g, G(\omega)) \equiv E_G\left[PR_i(x, g, y_i)\right] = E_G\left[\left. \frac{g}{y_i^2} \right| x, Y \right] = g\sigma(x, Y, G(\omega))$$

for $\sigma(x, Y, G(\omega)) = E_G\left[1/y^2 | x, Y\right]$.

6.1.2. Existence

In this section I state a set of sufficient conditions for the existence of the Coalitional voting equilibrium proposed in the previous sections of this paper. This represent only one possible set of restrictions and a more general description of the sufficient conditions for existence is described in Appendix C.

For the purposes of this section I will assume a simplified problem where voters have preferences represented by the utility function:

$$U(c_i, Y, l_i) = u(c_i) + a(Y) + \gamma l_i$$

where $u(\cdot)$ and $a(\cdot)$ are continuous, strictly increasing and strictly concave in c, Y. I will refer to this setting as the augmented *MR* model in the rest of this paper. The indirect utility becomes:

$$V(x, Y, \omega_i G(\omega)) = u(xy_i^* + (1-x)\bar{y} - Y/n) + a(Y) + \gamma(T - h_i^*)$$

The sufficient conditions for exsistence of a Coalitional Equilibrium are the following:

1. Individual income is such that $y^T > y_i^* > 0$ for all i and all $(x, Y) \in X$ (A1);
2. The derivative $\frac{dh_i^*}{dt} < 0$ for all $\omega_i > \bar{\omega}$ at all $(x, Y) \in X$ and it is finite for all i at all $(x, Y) \in X$ (A2).

Proof: See Appendix C.

The intuition for this result is that $\frac{dh_i^*}{dt} < 0$ corresponds to a relatively high elasticity of substitution between consumption and leisure. This condition ensure that and individual with higher wage has stronger preferences for a low marginal tax rate in comparison with an individual with low wage. Notice that condition 2 is equivalent in this setting to $+\infty > \frac{dy_i^*}{dx} > 0$. Also notice that

$$\frac{dy_i^*}{dx} = -\omega_i \frac{\partial h_i}{\partial t} + \omega_i \frac{\partial h_i}{\partial g} \frac{\partial \hat{g}}{\partial x}$$

It is easy to show that for general convex preferences $\frac{\partial h_i}{\partial g} < 0$ because an increase in g corresponds to a pure income effect for the individual. If $\frac{\partial \hat{g}}{\partial x} < 0$ at all $(x, Y) \in X$, then for condition 2 to be satisfied it is sufficient that $\frac{\partial h_i}{\partial t} < 0$ for all i at all $(x, Y) \in X$. Therefore if the elasticity of the labour supply with respect to the marginal tax rate is weakly negative for all i at all $(x, Y) \in X$. This assumption is reasonable and consistent with empirical findings (quote).

6.1.3. Comparative statics

In this section I will describe a Comparative Statics exercise and use the result in Theorem 15 in order to show that in this augmented model under certain conditions the predictions are very different from the ones in Metzer and Richard's paper.

Suppose that the sufficient conditions for a coalitional equilibrium described in the previous section are satisfied. Denote with (x^*, Y^*) an equilibrium policy vector. The exercise is the following.

1. Fix the policy vector at the equilibrium level (x^*, Y^*).
2. Change the wage distribution $G(\omega)$ such that the under the new wage distribution $\tilde{G}(\omega)$
   (a) the median income is strictly higher: $\tilde{y}_m^* > y_m^*$;
   (b) the change in median income $(\tilde{y}_m - y_m)$ is small;
   (c) the average income $\bar{y}$ is unchanged.
3. Find how the equilibrium level of the policy vector (x, Y) changes after this trasformation.

I will refer to this exercise as marginal decrease in income skewnwess (*MDIS*).

The exercise is complex because the labour supply of each individual $h(\omega_i, x, Y)$ is endogenous. This imply that a change in the wage distribution may affect the average income $\bar{y}$ and the median income y_m^* in non trivial ways even if the policy vector is kept constant. On the other hand we cannot directly manipulate the income distribution because it is endgenous in this model.

I am going to solve this problem by assuming the same restrictions that ensure existence of a coalitional equilibrium stated in secton 6.1.2.

One important consequence of those restrictions is that y_i is strictly increasing in ω_i for all i. This monotonicity result generally applies for less restrictive preferences because given that $1/x\omega_i$ can be interpreted as the relative price of consumption and leisure for an individual with wage ω_i, the the statement "y_i is strictly increasing in ω_i" is equivalent to assume that c_i is not a Giffen good, which is a very weak assumption (see Appendix C).

Define $R(y|x,Y)$ and $\tilde{R}(y|x,Y)$ as the endogenous cumulative income distributions function conditional on a certain policy vector (x, Y) corresponding respectively to the wage dstributions $G(\omega)$ and $\tilde{G}(\omega)$. The assumption that $\bar{y}$ is unchanged can be written as:

$$E_R\left[y|x,Y\right] = E_{\tilde{R}}\left[y|x,Y\right]$$

The relationship between G and R is easy to show under the assumption that y_i^* is strictly increasing in ω_i for all i such that the function $y_i^* = y(\omega_i, x, Y)$ that represents individual income conditional on the policy vector (x, Y) is a strictly increasing function of ω. Given the cumulative distribution $G(\omega)$ function denote with $g(\omega) = G'(\omega)$ the corresponding pdf. The result that under the restriction stated y_i is strictly increasing in ω_i implies $\omega < \omega_i \leftrightarrow y < y_i$ and therefore $R(y(\omega, x, Y)|x, Y) = G(\omega)$ for all $\omega \in [\underline{\omega}, \bar{\omega}]$. It also implies that $y_m^* = y(\omega_m, x, Y)$ is the median income under the distribution $R(y|x,Y)$ if and only if ω_m is the median wage under the distribution $G(\omega)$. These results allow one to restate the requirements of our Comparative Statics exercise as funcions of the wage distribution, namely:

1. Same mean income under g and $\tilde{g}$ (1) :

$$\int_{\underline{\omega}}^{\bar{\omega}} y(\omega, x, Y)\left[g(\omega) - \tilde{g}(\omega)\right] d\omega = 0$$

2. Median income that is strictly higher under distribution g in comparison with distribution $\tilde{g}$ (2):

$$\int_{\underline{\omega}}^{\omega_m} [g(\omega) - \tilde{g}(\omega)]\, d\omega > 0$$

where ω_m is the median wage under the distribution $G(\omega)$. Any distribution with pdf $\tilde{g}$ that satisfies these two requirements represent a Comparative Statics exercise that is equivalent to the *MDIS* described above. The advantage of this setting if that we can express the exercise in terms of the exogenous parameters ω_i and therefore we can use the results in Theorem 5 and Theorem 15 of this paper.

Under the restriction A1 and A2 stated in section 6.1.2 we can state the following results:

Lemma 22. *In the augmented MR model, if the progressivity of the tax system is sufficiently low, (i) the the size of the government and (ii) the expenditure in public goods are weakly increasing in the median income (at constant mean income).*

Proof. See Appendix C. □

This Lemma suggests that, differently from what is implied by the result in Meltzer and Richard's paper, a society that experiences a fall in income skewness may face an increase in the size of the government.

Lemma 23. *In the augmented MR model, the progressivity of the tax system is weakly decreasing in the in the median income (at constant mean income).*

Proof. See Appendix D □

Lemma 20 states an additional prediction that we can derive in the augmented *MR* model, which seems consistent with the findings in Razin and Sadka (2002). They find that the effect of an increase in income skewness on the average tax rate and on the total amount of social transfers have different signs, namely the average tax rate falls and the total amount of social transfers rises (this second result is not statisticaly significant) or is unchanged. The link with the predicitions of the model presented in this section is given by the fact that a tax system that has lower average tax rate and (weakly) higher social transfers is actually a more progressive tax system, which is in line with the prediction of Lemma 20.

6.2. Arms Race in a Democratic country

Consider a democratic country whose citizens vote about a linear tax rate, a lump-sum grant and the level of spending in national defence in presence of an external threating nation. Notice that all citizens have same preferences about national defence. The utility function of a citizen with income is:

$$U_i(x, y, I_i, Y) = f(x, y) + u(c_i, l_i)$$

where x is the national spending in defence, y is the level of spending of the rival country, c_i is the expenditure in consumption and l_i is the hours of leisure enjoyed by citizen i. Following the literature about arms race I assume $f_{xy} \geq 0$, i.e. an increase in expenditures in national defence in a rival country increases the marginal utility of governmental spending in national defence., and $u_{cl} \geq 0$. Individual i's consumption is given by her disposable income such that $c_i \leq (1-t)h_i\theta_i + g$ and the governmental budget constraint is balanced: $t\sum_{i=1}^{n} h_i\theta_i \geq x + ng$.

Lemma 24. *If the difference between median and mean pre-tax income is sufficiently small and a change in the tax system that benefit the riches and harm the poors is implemented by the government, then the same government will also increase the expenditure in national defence. Moreover in equilibrium all rival countries will increase their expenditures in national defence.*

Proof. See Appendix D. □

7. Conclusions

This paper proposes a model of political interaction in which voters can form coalitions in order to increase the space of policies that can be proposed and in which this coalitions are required to be stable in a peculiar sense. I show that the assumptions of Supermodularity and Strict Single Crossing Property of voters' objective functions are sufficient for the existence of a political equilibrium in a multidimensional policy space. Moreover I show that under the same assumptions a version of the Median Voter Theorem holds and as a consequence a monotone comparative statics result of the equilibrium outcomes is derived.

The paper describes a tool that can be useful to correct the predictions delivered by traditional Downsian models for some common Political Economy questions in the literature and potentially to explain the poor empirical performance of these predictions.

A feature that emerges is that the model is sufficiently flexible to deal with games that are more complex than simple voting games and can deliver interesting answer to questions in the field that cannot be easily analysed in the traditional framework.

I claim that this results can be applied to addess a number of different questions in the field and to light shed on some controversial results in the literature and that it represents an elegant an parsimonious way of dealing with one of the most commons problems that emerge if one aims to model Political Choices in an economic model. Moreover, the model is sufficiently general to be suitable to describe many different Political Economy problems and to incorporate some results that are well established in the literature as special cases of my framework.

Despite of these achivements the assumptions that must be satisfied in order to prove the two main results of the paper are quite restrictive so their applicability should be evaluated in relationship to the credibility of those assumptions in each specific application.

AppendixA. Lemma 5

If F satisfies SM and SSCP then the Pareto Set $\mathcal{P}(\mathcal{A})$ of a coalition of players $\mathcal{A} \subseteq \mathcal{N}$ is such that $x \in \mathcal{P}(\mathcal{A})$ only if $x \geq \sup\{M(l)\}$ and $x \leq \inf\{M(h)\}$ where $l = \min(\mathcal{A})$ and $h = \max(\mathcal{A})$.

Proof. Suppose $y \ngeq \overline{x}_l$ but $y \in \mathcal{P}(\mathcal{A})$. Because of the optimality of $\overline{x}_l$ and because X is a lattice, it must be true that $F(\overline{x}_l, \theta_l) \geq F(y \wedge \overline{x}_l, \theta_l)$. Supermodularity implies $F(y \vee \overline{x}_l, \theta_l) \geq F(y, \theta_l)$. Notice that $y \ngeq \overline{x}_l$ implies $y \vee \overline{x}_l \neq y$. Hence the Strict Single Crossing Property implies $F(y \vee \overline{x}_l, \theta_i) \geq F(y, \theta_i)$ $\forall \theta_i > \theta_l$. Given that $\theta_i > \theta_l$ is true for all $\theta \in \mathcal{A}, \theta \neq \theta_l$ we have that $\exists x \in X$ such that $F(x, \theta) \geq F(y, \theta)$ $\forall \theta \in \mathcal{A}$ and $F(x, \theta) > F(y, \theta)$ for at least one $\theta \in \mathcal{A}$, i.e $p_{X,\mathcal{A}}(y) \neq \oslash$. Hence $y \notin \mathcal{P}(\mathcal{A})$. Similarly one can show that $x \in A$ only if $x \leq \underline{x}_h$. Q.E.D. □

AppendixB. Theorem 18

Define the following game. A nonempty set N indexes the players, and each player's strategy set is S_i, partially ordered by $\geq$. The space of strategy profiles is then S, and player i has payoff function $\pi_i(z_i, z_{-i})$. Following Milgrom, Shannon 1994 such a game has (ordinal) strategic complementarities if for every i: (1) S_i is a compact lattice; (2) π_i is upper semi-continuous in z_i for z_{-i} fixed, and continuous in z_{-i} for fixed z_i; (3) π_n is quasisupermodular in z_i and satisfies the single crossing property in $(z_i; z_{-i})$. Say $z_i = \delta_i$, $z_{-i} = (x, \delta_{-i})$ for all $i \neq m$ and $z_m = (x, \delta_i)$, $z_{-m} = \delta_{-i}$.

INCOMPLETE.

AppendixC. Lemma 19

In the augmented MR model, if the progressivity of the tax system is sufficiently low, (i) the the size of the government and (ii) the expenditure in public goods are weakly increasing in the median income (at constant mean income).

AppendixC.1. Existence

Recall that the sufficient conditions for the existence of a Coalitional Equilibrium are:

(a) The Policy Space X is a convex and complete sublattice of R^d;

(b) The Objective Function V^i satisfies *SM* and *SSCP* for all i and all $\chi \in X$.

Restrictions for (a). (a) Condition (a) is always satisfied as the Policy Space assumed in this example is a convex and complete sublattice of R^2.

Restrictions for (b). The following section shows that assumptins A1, A2 are sufficient for *SM* and *SSCP*.

Proof. Denote $V^i_{ab} = \frac{\partial V^i}{\partial a \partial b}$. Recall that given that V^i is a C^2 function sufficient conditions for *SM* and *SSCP* are $V^i_{xY} \geq 0$, $V^i_{x\omega} > 0$ and $V^i_{Y\omega} > 0$ for all i and all $(x, Y) \in X$. First of all we need to calculate the marginal effects of x and Y on V_i, denoted with $\frac{\partial V_i}{\partial x}$ and $\frac{\partial V_i}{\partial Y}$ respectively. These derivatives correspond to the FOC of the maximization problem. Denote with $U^i_z = \frac{\partial}{\partial z}\left[U(xy^*_i + (1-x)\overline{y} - Y/n, Y, 1 - h^*_i)\right]$ for some variable z. Recall assumption 1 implies: $y^T > y^*_i > 0$ for all i and all $x, Y \in X$, which is equivalent to say that all individuals are in an internal maximum of their problem of utility maximization over consumption and leisure for any policy (x, Y). This assumption allows me to use an Envelope theorem when calculating $\frac{\partial V_i}{\partial x}$ and $\frac{\partial V_i}{\partial Y}$, for instance:

$$\frac{\partial V_i}{\partial x} = U^i_c\left(y^*_i - \overline{y} + (1-x)\frac{d\overline{y}}{dx}\right) + U^i_c x\frac{dy^*_i}{dx} + U^i_l\frac{dl^*_i}{dx}$$

Because we have assumed to be in an interior solution of of the consumption/leisure problem, then the FOC is: $U^i_c x\omega_i - U^i_l = 0$. Using this result into $\frac{\partial V_i}{\partial x}$ we get:

$$\frac{\partial V_i}{\partial x} = U^i_c\left(y_i - \overline{y} + (1-x)\frac{d\overline{y}}{dx}\right)$$

In the same way one can show that:

$$\frac{\partial V_i}{\partial Y} = U^i_c\left((1-x)\frac{d\overline{y}}{dx} - 1\right) + U^i_Y$$

SSCP. Calculate the derivative of V^i_x and V^i_Y w.r.t. ω_i. In this example we have:

$$V^i_{x\omega} = \left[U^i_{cc}\cdot\left(x + \frac{1-x}{n}\right)\frac{\partial y_i}{\partial \omega_i} - U_{cl}\cdot\frac{\partial h_i}{\partial \omega_i}\right]\left(y_i - \overline{y} + (1-x)\frac{d\hat{y}_i}{dx}\right) +$$
$$+ U^i_c\frac{\partial y_i}{\partial \omega_i}\left(1 - \frac{1}{n}\right) =$$

Notice that if the equilibrium exists and it is an internal one, then FOCs imply $y^*_m - \overline{y} + (1-x)\frac{d\overline{y}}{dx} = 0$ where y^*_m is the income of the median voter. This is true because $U_c(x\omega_m h_m + (1-x)\overline{y} - Y/n, 1 - h_m) > 0$. We are not going to use this reslt anyway because we do not want to exclude the possibility of a corner equilibrium.

$$V^i_{Y\omega} = \left[-U^i_{cc}\cdot x\frac{\partial y_i}{\partial \omega_i} + U_{cl}\cdot\frac{dh_i}{d\omega_i}\right]\left(\frac{1}{n} - (1-x)\frac{d\overline{y}}{dY}\right) + U^i_{cY}\cdot x\frac{\partial y_i}{\partial \omega_i} + U_{Yl}\cdot\frac{dh_i}{d\omega_i} > 0$$

SM. Calculate the cross derivative V^i_{xY}. In this example we have:

$$V^i_{xY} = \left[U^i_{cc}\cdot\left(x\frac{dy_i}{dY} + (1-x)\frac{d\overline{y}}{dY} - \frac{1}{n}\right) - U^i_{cl}\cdot\frac{dh_i}{dY} + U_{cY}\right]\left(y_i - \overline{y} + (1-x)\frac{d\overline{y}}{dx}\right) +$$

$$+U_c^i \cdot \left(\frac{dy_i}{dY} - \frac{d\bar{y}}{dY} + (1-x)\frac{d^2\bar{y}}{dxdY} \right)$$

Therefore a coalitional political equilibrium exists if the three ineqalites above are satisfied for all i at all $(x, Y) \in X$.

At this level of generality anyway it is had to understand what class of individual preferences and wage distributions will ensure existence, therefore it is useful to restrict preferences in a way that makes the condition easy to impose and to interpret. □

Additive Separability.. Say the utility function is additively separable, i.e.

$$U(c_i, Y, l_i) = u(c_i) + a(Y) + b(l_i)$$

Then the conditions for *SM* and *SSCP* reduce to:

$$V_{x\omega}^i = \left(u_{cc}^i \cdot x \left.\frac{dc_i}{dx}\right|_{h_i} + u_c^i \right) \frac{\partial y_i}{\partial \omega_i} > 0$$

$$V_{Y\omega}^i = u_{cc}^i \cdot x \frac{\partial y_i}{\partial \omega_i} \frac{dg}{dY} > 0$$

$$V_{xY}^i = u_{cc}^i \cdot \frac{dc_i}{dY} \left.\frac{dc_i}{dx}\right|_{h_i} + u_c^i \cdot \frac{d^2 c_i}{dxdY} \geq 0$$

Proof. Notice that at any point that is an internal maximum for some i it must be true that $\frac{dg}{dY} < 0$ (else there is a policy (x', Y') with $x' = x$, $Y' > Y$ implying $g' \geq g$, which is Pareto superior to (x, Y) and therefore it cannot be an optimum for any i). Neverthless $\frac{dg}{dY}$ may be positive in some other points of the policy space, hence if we do not add further assumptions we need to impose $\frac{dg}{dY} < 0$ for all $(x, Y) \in X$. Later I will show that $y_i^* > 0$ for all i and utility function linear in leisure this condition is unnecessary. If c is not a Giffen Good and n is large it must be that $\frac{\partial y_i}{\partial \omega_i} > 0$ for all individuals with $h > 0$. This result can be verified by noticing that for fixed policy vector (x, Y) and fixed wage distribution $G(\omega)$ the individual problem of choice over consumption and leisure can be interpreted as a problem of demand with endowments in the form:

$$\max_{c \geq 0, l \in [0,T]} u(c, Y, l)$$

s.t. $P_c c + P_l l \leq I_i$. Now we are going to use the governmental budget constraint to specify a functional form for P_c, P_l, I. Then the budget constraint can be written as:

$$\frac{1}{\omega_i x} c + l \leq T + \hat{g}$$

if n grows large $\hat{g}$ becomes independent of l_i conditional on (x, Y) and given that $G(\omega)$ is kept fixed, the above becomes:

$$\frac{1}{\omega_i x} c + l \leq T + \tilde{g}(x, Y)$$

for some function $\tilde{g}$. Therefore $P_c = \frac{1}{\omega_i x}$, $P_l = 1$, $I_i = T + \tilde{g}(x, Y)$. Finally notice that a (weak) Giffen Good is such that: $\frac{\partial c_i^*}{\partial P_c} \geq 0$. For large n we have $c_i^* = x y_i^* + \tilde{g}(x, Y)$ therefore

$$\frac{\partial c_i^*}{\partial P_c} = x \frac{\partial y_i^*}{\partial P_c}$$

So using the fact that $\frac{\partial P_c}{\partial \omega_i} = -\frac{1}{\omega_i^2 x}$ we can rewrite:

$$\frac{\partial y_i^*}{\partial \omega_i} = \frac{1}{x} \frac{\partial c_i^*}{\partial P_c} \frac{\partial P_c}{\partial \omega_i} = -\frac{\partial c_i^*}{\partial P_c} \frac{1}{\omega_i^2 x^2} > 0$$

and given that we have assumed $x > 0$, $\omega_i > 0$ this implies that the above is strictly positive whenever $\frac{\partial c_i^*}{\partial P_c} < 0$, i.e. c is not a Giffen Good, as stated. Therefore for internal solutions ($T > h_i^* > 0$) of the individual problem of choice over consumption and leisure the condition is satisfied.

Notice that if some individuals are in a corner solution with $h_i^* = T$ then $y_i^T = \omega_i T$ which is strictly increasing in ω_i so the condition $\frac{\partial y_i^*}{\partial \omega_i} > 0$ is also satisfied. Even if we establish that $\frac{\partial y_i^*}{\partial \omega_i} > 0$ for all i and we impose that $\frac{dg}{dY} < 0$ for all $(x, Y) \in X$ we are left with the following conditions: $\frac{dc_i}{dx}\Big|_{h_i} < \frac{u_c^i}{u_{cc}^i x}$ for all $y_i^* \geq \bar{y} - (1 - x)\frac{d\bar{y}}{dx}$ and $u_{cc}^i \cdot \frac{dc_i}{dY} \frac{dc_i}{dx}\Big|_{h_i} + u_c^i \cdot \frac{d^2 c_i}{dx dY} > 0$ for all i and all $(x, Y) \in X$. □

AppendixC.1.1. Sufficient Conditions.

These two conditions are hard to interpret and depend on endogenous objects (such as y_i^*) hence they are not easy to impose. The last step to simplify the conditions for existence is to impose the condition stated in section 6.1.2, namely:

Utility function additively separable and linear in leisure:

$$U(c_i, Y, l_i) = u(c_i) + a(Y) + \gamma l_i$$

(A1). Individual income is such that $y^T > y_i^* > 0$ for all i and all $(x, Y) \in X$;

(A2). The derivative $\frac{dh_i^*}{dt} < 0$ for all $\omega_i > \bar{\omega}$ at all $(x, Y) \in X$ and it is finite for all i at all $(x, Y) \in X$ (A2).

Now I am going to show that if one imposes these the three restrictions then the conditions for existence of a voting equilibrium are satisfied. Recall the conditions to verify are $V_{x\omega}^i > 0$, $V_{Y\omega}^i > 0$ and $V_{xY}^i \geq 0$ for all i at all $(x, Y) \in X$.

Preliminary steps. Using the FOC f the *C-L* problem to calculate $\frac{\partial y_i^*}{\partial \omega_i}$. Now in the case of interior solutions one can use the FOC of the *C-L* problem to calculate $\frac{\partial y_i}{\partial x}$:

$$u_c x \omega_i - \gamma = 0$$

Totally differentiate to get:

$$\frac{\partial y_i^*}{\partial \omega_i} = \begin{cases} \frac{-u_c^i}{u_{cc}^i \omega_i x} > 0 & for \quad interior \\ 0 & for \quad corner \end{cases}$$

for all i and all $(x, Y) \in X$. Notice that for all $y_i \leq \bar{y} - (1-x)\frac{\partial \bar{y}}{\partial x}$ it is immediate to verify that $u_{cc}^i \cdot x \left.\frac{dc_i}{dx}\right|_{h_i} + u_c^i > 0$ is always true because $u_c^i > 0$ and $u_{cc}^i < 0$ by assumption and $\left.\frac{dc_i^*}{dx}\right|_{h_i} = y_i^* - \bar{y} + (1-x)\frac{\partial \bar{y}}{\partial x}$. As before totally differentiate the FOC of the C-L problem to get:

$$\frac{\partial y_i}{\partial x} = \begin{cases} \frac{-u_c}{u_{cc} x^2} + \frac{1}{x}\left(y_i - \bar{y} + (1-x)\frac{\partial \bar{y}}{\partial x}\right) & for \quad interior \\ 0 & for \quad corner \end{cases}$$

A2 implies that $\frac{\partial y_i}{\partial x}$ is a finte number. A1 restrict the cases to internal solutions because it implies $T > h_i > 0$, hence we can solve:

$$\frac{\partial y_i}{\partial x} = \frac{-u_c^i}{u_{cc}^i x^2} - \frac{1}{x}\left(y_i - \bar{y} + (1-x)\frac{\partial \bar{y}}{\partial x}\right)$$

$$\frac{\partial \bar{y}}{\partial x} = \frac{1}{n}\sum_{i=1}^{n} \mathbb{I}(y > y_i^T > 0)\left[\frac{-u_c^i}{u_{cc}^i x^2} - \frac{1}{x}\left(y_i - \bar{y} + (1-x)\frac{\partial \bar{y}}{\partial x}\right)\right]$$

And using assumption A1 the above reduces to:

$$\frac{\partial \bar{y}}{\partial x} = \int_{\underline{\omega}}^{\bar{\omega}} \frac{-u_c^i}{u_{cc}^i x} dG(\omega)$$

Define RA_i to be the Arrow-Pratt Coefficient of Relative Risk Aversion, i.e. $RA_i = -\frac{u_{cc}^i}{u_c^i} c_i$. Then we can rewrite:

$$\frac{\partial \bar{y}}{\partial x} = \frac{1}{x}\int_{\underline{\omega}}^{\bar{\omega}} \frac{c_i^*}{RA_i} dG(\omega)$$

$$\frac{\partial y_i}{\partial x} = \frac{c_i^*}{RA_i x^2} - \frac{y_i - \bar{y}}{x} - \frac{(1-x)}{x^2}\int_{\underline{\omega}}^{\bar{\omega}} \frac{c_i^*}{RA_i} dG(\omega)$$

Notice that this formula imply that $\frac{\partial y_i}{\partial x}$ is a finite number for all i, as initially assumed.

Finally recall that in Section 6.1.1 we assumed that the governmental budget constraint it binding. A sufficient condition for that under restrictions A1, A2 is that $-\bar{y} + (1-x)\frac{\partial \bar{y}}{\partial x} < 0$ for all $(x, Y) \in X$.

SSCP: *$V^i_{x\omega} > 0$ for all i and all $(x, Y) \in X$.* Now recall that the condition we need to verify is:

$$V^i_{x\omega} = \left(u^i_{cc} \cdot x \left. \frac{dc_i}{dx} \right|_{h_i} + u^i_c \right) \frac{\partial y_i}{\partial \omega_i} > 0$$

Substituting the formulas for $\frac{\partial y_i}{\partial \omega_i}$ and $\left.\frac{dc_i}{dx}\right|_{h_i}$ into the above equation we get:

$$V^i_{x\omega} = \left[x \left(y_i - \bar{y} + (1-x) \frac{\partial \bar{y}}{\partial x} \right) u^i_{cc} + u^i_c \right] \frac{-u^i_c}{u_{cc} \omega_i x} =$$

$$= \left[x \left(y_i - \bar{y} + (1-x) \frac{\partial \bar{y}}{\partial x} \right) \frac{u_{cc}}{u_c} + 1 \right] \frac{-\left(u^i_c\right)^2}{u_{cc} \omega_i x}$$

Now substitute the formula for $\frac{\partial \bar{y}}{\partial x}$ into the formula for $V^i_{x\omega}$ to get:

$$= \left[-x \left(y_i - \bar{y} + (1-x) \left[\frac{1}{x} \int_{\underline{\omega}}^{\bar{\omega}} \frac{c^*_i}{RA_i} dG(\omega) \right] \right) - \frac{u^i_c c_i}{u^i_{cc} c_i} \right] \frac{u^i_c}{\omega_i x} =$$

$$= \left[- \left(y_i - \bar{y} + (1-x) \int_{\underline{\omega}}^{\bar{\omega}} \frac{c^*_i}{RA_i} dG(\omega) \right) + \frac{c_i}{xCRRA_i} \right] \frac{u^i_c}{\omega_i} =$$

Now recall the previous result:

$$\frac{\partial y^*_i}{\partial x} = \frac{c^*_i}{RA_i x^2} - \frac{y_i - \bar{y}}{x} - \frac{(1-x)}{x^2} \int_{\underline{\omega}}^{\bar{\omega}} \frac{c^*_i}{RA_i} dG(\omega)$$

$$V^i_{x\omega} = \left[\frac{\partial y^*_i}{\partial x} x + \frac{c^*_i}{xRA_i} \right] \frac{u^i_c}{\omega_i} > 0$$

The above is strictly greater than zero if $\frac{\partial y^*_i}{\partial x} > 0$, which is equivalent to $\frac{\partial y^*_i}{\partial t} < 0$. Therefore the condition for existence reduces to $\frac{\partial y^*_i}{\partial t} < 0$ for all i such that $y_i > \bar{y}$. Notice that if the utility function is Constant Relative Risk Aversion (CRRA) a sufficient condition for $\frac{\partial y^*_i}{\partial t} < 0$ is $RA \leq 1$. Q.E.D.

SSCP: *$V^i_{Y\omega} > 0$ for all i and all $(x, Y) \in X$.* Recall the condition previously derived is:

$$V_{Y\omega_i} = U^i_{cc} \cdot x \frac{\partial y_i}{\partial \omega_i} \frac{dg}{dY} > 0$$

Given the strict concavity of u we have $U^i_{cc} > 0$ and we have shown in the previous section that under Assumption 1 $\frac{\partial y_i}{\partial \omega_i} > 0$ for all i. So we need $\frac{dg}{dY} > 0$. Use the definition of $\hat{g}$ such that:

$$\frac{d\hat{g}}{dY} = (1-x)\frac{d\bar{y}}{dY} - 1$$

Because the utility function that is linear in leisure: $b(l_i) = \gamma l_i$. This implies:

$$\frac{\partial y_i^*}{\partial Y} = \frac{1-(1-x)\frac{d\bar{y}}{dY}}{x}$$

and from this we can get:

$$\frac{\partial \bar{y}}{\partial Y} = \frac{\partial y_i^*}{\partial Y} = 1$$

and therefore

$$V_{Y\omega_i} = -U^i_{cc} \cdot x \frac{-u^i_c x}{u^i_{cc}\omega_i x} = \frac{u^i_c x}{\omega_i} > 0$$

SM*: $V^i_{xY} \geq 0$ for all i and all $(x,Y) \in X$.*

$$V_{xY} = U_{cc} \cdot \frac{dc_i}{dY}\left.\frac{dc_i}{dx}\right|_{h_i} + U_c \cdot \frac{d}{dY}\left(\left.\frac{dc_i}{dx}\right|_{h_i}\right)$$

Proof. Recall that:

$$\frac{\partial y_i^*}{\partial Y} = \frac{1-(1-x)\frac{d\bar{y}}{dY}}{x}$$

and from this we can get:

$$\frac{\partial \bar{y}}{\partial Y} = \frac{\partial y_i^*}{\partial Y} = 1$$

For all i. Recall that $c_i = x\omega_i h_i + (1-x)\bar{y} - Y$ hence:

$$\frac{\partial c_i}{\partial Y} = x\frac{\partial y_i}{\partial Y} + (1-x)\frac{\partial \bar{y}}{\partial Y} - 1 = 0$$

and

$$\left.\frac{dc_i}{dx}\right|_{h_i} = y_i - \bar{y} + (1-x)\frac{d\bar{y}}{dx}$$

$$\frac{d}{dY}\left(\left.\frac{dc_i}{dx}\right|_{h_i}\right) = \frac{dy_i}{dx} - \frac{d\bar{y}}{dx} + (1-x)\frac{d^2\bar{y}}{dxdY} = 1 - 1 + 0 = 0$$

wsuch that

$$V^i_{xY} = 0$$

for all $(x,Y) \in X$ and for all i, as required.

Summarizing a set of sufficient conditions for existence of a voting equilibrium is: utility function additively separable in (c,Y,l) and linear in leisure, $y^T > y_i^* > 0$ for all i and all all $(x,Y) \in X$,$\frac{dy_i}{dt} \leq 0$ for all $y_i^* > \bar{y}$ for all $(x,Y) \in X$. Q.E.D. □

AppendixC.2. Comparative Statics.

If the sufficient conditions are satisfied the main theorem of this paper ensures that the equilibrium policy is going to be the unique ideal point of the median voter ω_m. The theorems 5 and 15 in this paper imply that if the conditions for existence are satisfied, then the optimal policy vector (x^*, Y^*) is:

1. An ideal point of the median voter m, i.e.

$$(x^*, Y^*) \in \arg \max_{(x,Y)\in X} V(x, Y, \omega_i G(\omega))$$

2. Monotone weakly increasing in the parameter tht identifies the median voter ω_m.

Notice that in this example $V(x, Y, \omega_i G(\omega))$ is strictly concave in (x, Y) therefore the ideal point of each individual is unique.

Define $\triangle G(\omega)$ to be any trasformation $\tilde{G}(\omega)$ of the income distribution $G(\omega)$ such that $\triangle\omega_m = \tilde{\omega}_m - \omega_m > 0$ and because under restrictions A1, A2 we have $\frac{dy_i^*}{d\omega_i} > 0$ for all i, this imply $\triangle y_m = \tilde{y}_m - y_m > 0$.

Denote the changes in $x^*$and Y^*induced by the Comparative Statics exercise *MDIS* with $\frac{\triangle x}{\triangle G}$ and $\frac{\triangle Y}{\triangle G}$. Theorem 15 implies: $\frac{\triangle x}{\triangle G} \geq 0$ and $\frac{\triangle Y}{\triangle G} \geq 0$. In order to use Theorem 15 anyway we need to ensure that the transformation $\triangle G$ does not cause a change in the individual objective function $V(x, Y, \omega_i G(\omega))$.

Recall that the objective function is

$$V(x, Y, \omega_i G(\omega)) = u(xy_i^* + (1-x)\bar{y} - Y) + a(Y) + \gamma(T - h_i^*)$$

We need to verify that the *CS* exercise *MDIS* as defined in Section 6 implies that $V(x, Y, \omega_i G(\omega))$ is independent of $G(\omega)$ conditional on (x, Y). Recall that y_i^* and $h_i^*$are not affected by changes in G that keep $\bar{y}$ unchanged (this is true because of the assumption that the voters are a continuum). On the other hand V_i is a function of $\bar{y}$. Hence if $\bar{y}$ is kept constant then V_i is not affected by a change in G. Hence the assumptions of theorems 5 and 15 are satisfied and we can claim that the equilibrium policy vector (x, Y) is weakly increasing in y_m at constant $\bar{y}$, i.e. $\frac{\triangle x}{\triangle G} \geq 0$ and $\frac{\triangle Y}{\triangle G} \geq 0$. This proves the part (ii) of Lemma 19.

In order to prove part (i) we need to use the definition of the size of the government as the total government spending:

$$S(x, Y, G(\omega)) \equiv \left(\int_{\underline{\omega}}^{\bar{\omega}} \mathbb{I}\left[(1-x)y(\omega_i, x, Y) - \hat{g} \leq 0\right]\left[\hat{g} - (1-x)y(\omega_i, x, Y)\right] dG(\omega) \right) + Y$$

For small changes in ω_m one can calculate for $\triangle\omega_m$ small enough:

$$\frac{\triangle S}{\triangle G} = \int_{\underline{\omega}}^{\bar{\omega}} \mathbb{I}\left[(1-x)y_i - g \leq 0\right] \frac{\triangle x}{\triangle G} \left(y_i - (1-x)\frac{\partial y_i}{\partial x} - \frac{\partial \hat{g}}{\partial x} \right) dG(\omega) + (1 - G(k)) \frac{\triangle Y}{\triangle G}$$

where $k = \int_{\underline{\omega}}^{\bar{\omega}} \mathbb{I}\left[(1-x)y_i - g \leq 0\right] dG(\omega)$. Notice that for not so small $\triangle G$ this formula would be an aproximation because we are not considering individuals that switch from being a net tax payer to a net grant receiver (or viceversa) as a consequence of the change in policy. Notice anyway that as the change in $\triangle G$ becomes arbitrarily small the set of this kind of voters reduces to individuals with $(1-x)y_i - g = 0$ (if any of them exists) and the formula becomes exactly correct. This is the change in the size of the government induced by an increase in the wage of the median voter. It is easy to realize that if $g \leq 0$ given that $x \leq 1$ then $(1-x)y_i - g > 0$ for all i, therefore

$$\frac{\triangle S}{\triangle G} = \frac{\triangle Y}{\triangle G} \geq 0$$

This is also linked with the progressivity of the tax system. Using a standard index of progressivity described in section 6:

$$\overline{PR}(x, g, G(\omega)) \equiv E_G\left[\frac{g}{y_i^2}\right] = g\sigma(x, Y, G(\omega))$$

$$\frac{\triangle S}{\triangle G} = \int_{\underline{\omega}}^{\bar{\omega}} \mathbb{I}\left[(1-x)y_i - \overline{PR}/\sigma(x, Y, G(\omega)) \leq 0\right] \frac{\triangle x}{\triangle G}\left(y_i - (1-x)\frac{\partial y_i}{\partial x} - \frac{\partial \hat{g}}{\partial x}\right) dG(\omega) +$$

$$+ (1 - G(k)) \frac{\triangle Y}{\triangle G}$$

Because $S(x, Y, G(\omega))$ is continuous in $\overline{PR}$ and $\frac{\triangle S}{\triangle G} \geq 0$ for $\overline{PR} \leq 0$, then there exists a threshold $J(x, Y.G(\omega))$ such that if at the equilibrium $\overline{PR} \leq J(x, Y.G(\omega))$, then $\frac{\triangle S}{\triangle G} \geq 0$, which proves part (ii).

The result states that if the tax system exhibits sufficiently low levels of progressivity (or it is regressive), then the size of the government is nondecreasing in y_m In the same way one can prove that the result holds also if the size of the government is defined as the total tax revenues $TR(x, Y, G(\omega))$ or the average tax rate $AT(x, Y, G(\omega))$ as defined in the previous section. Q.E.D.

AppendixC.3. Example

Proof. Consider the following example: there is a continuum of voters of Lebesgue measure 1, individuals have wages $\omega_i \in [1, \bar{\omega}]$ with $E(\omega) = \mu$ and cdf $G(\omega)$. Peferences are represented by the utility function:

$$u(c_i, Y, l_i) = \alpha \ln(c_i) + (1-\alpha)\ln(Y) + \gamma l_i$$

Say the policy space is (x, Y) with $x \in [\underline{x}, 1]$ and $Y \in [0, \overline{Y}]$ with $\underline{x}, \bar{Y}$ chosen such that $\underline{x} \geq \frac{\gamma \bar{Y}}{\alpha \mu} > 0$. Solve for the optimal labor supply conditional on the policy (x, Y) (notice that the assumptions about x, Y and ω_i imply an internal solution in T is large enough):

$$h_i^*(x, Y) = \frac{\alpha}{\gamma} - \frac{(1-x)\bar{y}}{x\omega_i} + \frac{Y}{x\omega_i}$$

This implies:

$$\bar{y}(x, Y) = \frac{\alpha x \mu}{\gamma} + Y$$

Substitute $h_i^*(x, Y)$ and $\bar{y}(x, Y)$ into V_i to get:

$$V_i = \alpha \ln(\alpha x \omega_i) - \alpha \ln(\gamma) + (1-\alpha)\ln(Y) - \alpha + (1-x)\frac{\alpha\mu}{\omega_i} - \frac{\gamma Y}{\omega_i}$$

$$V_x = \frac{\alpha}{x} - \frac{\gamma\mu}{\omega_i}$$

$$V_Y = \frac{1-\alpha}{Y} - \frac{\gamma}{n\omega_i}$$

$$V_{x\omega_i} = \frac{\gamma\mu}{\omega_i^2} > 0$$

$$V_{Y\omega_i} = \frac{\gamma}{n\omega} > 0$$

$$V_{xY} = 0$$

Hence SM and SSCP hold for (x, Y) and ω_i for all i and all $(x, Y) \in X$. Notice that the indirect utility function is strictly concave in x, Y if the is:

$$\begin{array}{rccc} x_i^* = \underline{x} & if & low & corner \\ x_i^* = \frac{\alpha}{\gamma}\frac{\omega_i}{\mu}, & if & interior & \\ x_i^* = 1 & if & high & corner \end{array}$$

$$Y_i^* = \max\left[\frac{(1-\alpha)\omega_i}{\gamma}, \overline{Y}\right]$$

confirming the expected result of the optimum being monotonic nondecreasing in ω_i. Now recall that

$$g = (1-x)\bar{y} - Y$$

in this example for $Y < \overline{Y}$ this is equivalent to:

$$\begin{array}{rcc} g = \frac{\alpha\omega_m}{\gamma^2}\left(\alpha - \frac{\alpha^2\omega_m}{\gamma\mu} - \frac{(1-\alpha)\omega_m}{\mu}\right) & if & \underline{x} < x < 1 \\ g = (\alpha\mu - (1-\alpha)\omega_m)x/\gamma & if & x = \underline{x} \\ g = -\frac{(1-\alpha)\omega_i}{\gamma} & if & x = 1 \end{array}$$

Notice that $g \leq 0$ if $x = 1$ or if $x < 1$and $\frac{\omega_m}{\mu} \geq \frac{\alpha\gamma}{\alpha^2+\gamma(1-\alpha)}$, but interior solutions occur only if $\frac{\omega_m}{\mu} < \frac{\gamma}{\alpha}$. This implies that for $\frac{\gamma}{\alpha} > \frac{\omega_m}{\mu} \geq \frac{\alpha\gamma}{\alpha^2+\gamma(1-\alpha)}$ there is an interior solution with $g \leq 0$. Moreover we have internal solutions with $g > 0$

for $\frac{\gamma}{\alpha} > \frac{\omega_m}{\mu} \geq \frac{\alpha\gamma}{2[\alpha^2+\gamma(1-\alpha)]}$ which is the thresholt that guarantees that the total tax revenue does not exceed the total income. Also one can prove that $\mu \geq \frac{\alpha}{\alpha^2-\gamma(1-\alpha)}$ and a suitable choice of $\underline{Y}$ is sufficient to ensure that the objective function is$\omega_i - concave$ for all i. Finally notice that for interior solutions:

$$\frac{\partial g}{\partial \omega_m} = \frac{\alpha}{\gamma^2}\left(\alpha - \frac{2\alpha^2}{\gamma\mu} - \frac{2(1-\alpha)}{\mu}\right) < 0$$

in all the range of parmeters for which the solution is interior. Q.E.D. □

AppendixD. Lemma 20

In the augmented MR model, the progressivity of the tax system is weakly decreasing in the in the median income (at constant mean income).

Proof. Recall the definitions of progressivity as the expectation of the Slitor index:

$$\overline{PR} = E_G\left(\left.\frac{\partial T_i}{\partial y_i}\right|_{x,g}\right) = E_G\left(\frac{\hat{g}}{y_i^2}\right)$$

The CS of interest is:

$$\left.\frac{\triangle \overline{PR}}{\triangle G}\right|_{y_i} = \left.\frac{\partial \overline{PR}}{\partial x}\right|_{y_i}\frac{\triangle x}{\triangle G} =$$

$$= E\left(\frac{1}{y_i^2}\right)\frac{\partial \hat{g}}{\partial x}\frac{\triangle x}{\triangle G} \leq 0 \forall i$$

The sign of the above is weakly negative because $\frac{\partial \hat{g}}{\partial x} < 0$ at any equilibrium point (x, Y) with $x < \bar{x}$ This is true because if not there exists another policy (x', Y) Such that $x' > x$ and $\hat{g}(x', Y, G(\omega)) \geq \hat{g}(x, Y, G(\omega))$. Such policy is preferred to (x, Y) by all i hence (x, Y) cannot be an equilibrium. on the other hand if if $x = \bar{x}$ it must be that $\frac{\triangle x}{\triangle G} = 0$; in both cases the above is weakly negative. Alternatively one may want to keep into account of the changes in earned income induced by the change in equilibrium policies:

$$\frac{\triangle \overline{PR}}{\triangle G} = \frac{\partial \overline{PR}}{\partial x}\frac{\triangle x}{\triangle G} + \frac{\partial \overline{PR}}{\partial Y}\frac{\triangle Y}{\triangle G} =$$

$$= E\left[-\frac{2g}{y_i^3}\left(\frac{dy_i^*}{dx}\frac{\triangle x}{\triangle G} + \frac{dy_i}{dY}\frac{\triangle Y}{\triangle G}\right) + \frac{1}{y_i^2}\left(\frac{\partial \hat{g}}{\partial x}\frac{\triangle x}{\triangle G} + \frac{\partial \hat{g}}{\partial Y}\frac{\triangle Y}{\triangle G}\right)\right] < 0 \forall i$$

Hence in this case a sufficient condition for the above to be negative for all i is $\frac{dy_i^*}{dx} \geq 0$ (or not strongly negative). Under assumptions A1,A2 this s alsways satisfied as $\frac{dy_i^*}{dx} = -\frac{dy_i^*}{dt} > 0$ by assumption A2. Q.E.D. □

AppendixE. Lemma 21

Proof. Substitute the individual and the governmental budget constraint into the utility function. The resulting objective function of citizen i is $V(x, y, T; \theta_i) = f(x, y) + u\left((1-t)h_i\theta_i + t\bar{y} - x/n, 1-h_i\right)$. First derivatives of V are: $V_x = f_x - u_c/n$; $V_t = -u_c\left(h_i\theta_i - \bar{y}\right)$. Second derivatives at constant $\bar{y}$ (see definition of V_{ij} in Appendix B) are: $V_{t\theta_i} = \left[-u_{cc}\left(h_i + \theta_i\frac{\partial h_i}{\partial \theta_i}\right)(1-t) + u_{cl}\frac{\partial h_i}{\partial \theta_i}\right](y_i - \bar{y}) - u_c\left(h_i + \theta_i\frac{\partial h_i}{\partial \theta_i}\right) < 0$; $V_{x\theta_i} = -u_{cc}\left(h_i + \theta_i\frac{\partial h_i}{\partial \theta_i}\right)(1-t) + u_{cl}\frac{\partial h_i}{\partial \theta_i} > 0$; $V_{xt} = u_{cc}\left(y_i - \bar{y}\right) > 0$. Notice that $\lim_{y_i \to \bar{y}} V_{t\theta_i} = 0$; $\lim_{y_i \to \bar{y}} V_{xt} = 0$; $\lim_{y_i \to \bar{y}} V_{tx\theta} > 0$. Hence there must be a threshold $\hat{y}(\bar{y})$ such that if $\hat{y}(\bar{y}) \leq y_m < \bar{y}$ then the comparative statics is: (i) x is increasing in y_m; (ii) g is decreasing in y_m; (iii) the comparative statics of t is ambiguous. This implies that a less progressive tax system will be implemented and the amount of transfers to poor individuals will fall and at the same time the expenditure in national defence will rise. Moreover, using Theorem 18 we know that the expenditure in defence of all other countries must be weakly higher in equilibrium. □

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