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2015

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Abstract

In many countries, the government pays almost identical nominal wages to workers living in regions with notable economic disparities. By developing a two-region general equilibrium model with endogenous migration and search frictions in the labour market, I study the differences in terms of unemployment, real wages, and welfare between a regional wage bargaining process and a national one in the public sector. Adopting the latter makes residents in the poorer region better off and residents of the richer region worse off. Private sector employment decreases in the poorer region and it increases in the richer one. Under some conditions, the unemployment rate in the poorer region soars. Simulation results also show that a regional bargaining scheme may increase inequality.

Keywords: public sector wages; centralized pay regulation; unemployment.
JEL codes: J45, J50, R13.

*I thank Giorgio Brunello, Maurizio Conti, Bruno Decreuse, Thomas Gall, Andrea Ichino, Christian Holzner, Andrey Launov, Claudio Lucifora, Fabien Postel-Vinay, Bruno Van der Linden, Klaus Wälde, and the participants to the EEA conference in Toulouse, the LAGV conference in Marseille, the Mainz Workshop in Labour Economics, the SAM conference in Bristol, and the "Brucchi Luchino" conference in Rome for useful comments and suggestions. Any errors are my own.

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1 Introduction

In many countries, public sector wages are very similar in nominal terms for employees of regions with different private sector productivities and costs of living. The spatial distribution of public sector earnings is very compressed in the five largest EU economies (Germany, France, UK, Italy, and Spain) (see Elliot *et al.*, 2007)[1]. Albeit in a weaker form, even the US federal government regional pays are substantially unaffected by local market conditions, while different is the case for state and local public employees (Katz and Krueger, 1991). Among the several explanations for the greater compression of public sector wage structures compared to private sector ones, the highly centralized structure of the public pay systems is one of the most relevant (Elliot *et al.*, 2007)[2].

In times in which many governments face the twofold challenge of improving the efficiency of the public sector and restraining spending, the poor responsiveness of civil servants to local labor market conditions has become an important issue to deal with for economists and policy makers[3]. This paper enters the debate by contrasting a centralized public sector pay system with a regional one and looking at the differences in terms of welfare, employment, and real earnings.

In both scenarios, public sector employment and wages are determined through

[1]In Italy, Spain, and Germany this is accompanied by a pronounced income disparity between regions (see, respectively, Dell'Arringa *et al.*, 2007, Garcia-Perez and Jimeno, 2007, and Heitmueller and Mavromaras, 2007). For France, see Meurs and Edon (2007). Some of these papers look at real wage spatial distributions but, since they use a national price index, their results also apply to nominal pay variations.

[2]The differences between private and public wage structures may be the result of worker self-selection. However, recent research shows that the wage compression associated with public sector pay scales has a large causal component (Melly and Puhani, 2013). The higher degree of unionization in public sector also plays a role. On the role of unions in compressing wage inequality see Kahn (2000) and Lemieux (1998).

[3]See the discussion in the next section.

efficient bargaining between a public authority that aims to maximize revenues net of production costs and a union. The two frameworks differ in that under the national bargaining both public sector vacancies in each region and the common wage are bargained over by the central government and a union whose members are all the civil servants in the country, whereas in the regional bargaining process the actors are the local government and a union that only cares about the utility of public sector employees of the region.

These two different public sector wage settings are nested into a two-region general equilibrium model in which private tradable and public nontradable goods are produced and the labor market exhibits search and matching frictions. Migration between regions is endogenous and unemployed workers are free to apply either for a private sector job or for a public sector one. To account for the differences between the US and the European labor markets, I consider both the case of individual bargaining and the case of decentralized collective negotiation in the private sector. The model is analytically tractable in steady-state. The conclusions on wage, employment, and welfare are obtained via comparative statics and they do not depend on the type of private sector wage system considered.

The first result of the paper is maybe not surprising. A centralized public sector pay system is a redistributive tool that shifts resources from the richer regions to the poorer ones. What is perhaps less expected is the extent of such a redistribution, as it does not involve only civil servants but all workers. Under centralized bargaining, real wages in both sectors in the richer region decrease, whereas real wages in both sectors in the poorer region go up. The welfare results are along the same line, with residents in the richer region that, regardless of their employment status (i.e if they are employed or unemployed, civil servants or private sectors worker), are worse off under national bargaining, whereas residents in the poorer region are better off.

The rationale behind these results is the following. Under a national negotiation, the public sector union must account for the marginal productivities and fall-back positions of workers of all regions. So, it ends up accepting a nominal wage that is lower (resp. higher) than the one that would accrue to employees in the richer (resp. poorer) region under a decentralized bargaining scheme. This has an impact on labor supply. In comparison with a regional bargaining system, public sector jobs become more enticing for workers in the poorer region while more residents of richer region will apply for private jobs. In turn, the difficulties in filling public sector vacancies in the richer region raises the production cost of the public nontradable good and, in turn, the cost of living. Therefore, in the richer region real wages and welfare are lower under a centralized pay system than with a regional bargaining scheme. On the contrary, in the poorer regions public authorities find it easier to fill a vacancy, so the same chain of events occurs with an opposite sign.

Comparative statics offers other insights. First, the redistributive effects of centralized bargaining decrease migration towards the richer region. Second, a national pay system lowers private sector employment rate in the poorer region while raising it in the richer one. Third, under a centralized wage-setting the unemployment rate can rise in the poorer region. This is because the larger number of applications for a civil service job in the poorer region stemming from centralized bargaining tends to "overload" the public sector labor market. In some circumstances (for instance in periods when many jobs are destroyed, lengthening the unemployment lines) such congestion effects are so strong that the unemployment rate soars in the poorer region.

Further results are obtained by calibrating and simulating on the basis of Italian data. Italy is an interesting case study, for it exhibits huge economic disparities between the North-Center regions and the South ones, the former being 50 % richer in terms of average disposable income per capita (ISTAT, 2013). Moreover, no statistically significant difference emerges in nominal public wages paid across regions (Alesina et al., 2001; Dell'Arringa et al., 2007). Numerical exercises show that a centralized negotiation in the public sector leads to a higher unemployment rate and a lower output in the country compared to a regional bargaining scenario. On the other hand, a national bargaining, by decreasing real pays in the richer parts of the country and increasing them in the poorer ones, tends to reduce inequality: both the Gini index and the wage variance go down.

The paper is organized as follows. Section 2 discusses the literature on the topic. Section 3 and 4 present the basic model. Section 5 contrasts the regional public sector model with the national one. Sections 6 extends the results of the baseline model to the case of collective bargaining in the private sector. Section 7 explains the calibration procedure. Section 8 shows the numerical exercises. Section 9 concludes.

2 Related Literature

Recent papers have warned about the negative consequences of regulated pays across heterogeneous labor markets. In their analysis of the UK economy, Corry *et al.* (2011) argue that the existing centralized wage bargaining leads to excessively high public sector wages in some areas of the country, making it difficult for the private sector to attract skilled labor and impairing business activity. On the other hand, Propper and van Reenen (2010) and Propper and Britton (2012) provide empirical evidence about the negative impact on the quality of the public good in regions where the regulated pay is lower than the market wage[4]. In this case it is the public authority that struggles to attract skilled labor, and the quality of the service will be worse. Another sort of criticism is advanced by Alesina *et al.* (2001). They note that paying civil servants in poorer areas the same nominal salary of their peers in other parts of the country can be viewed as a subsidy designed to discourage internal migration (Caponi, 2008) or to redistribute income. More importantly, the hidden form of such

[4]In detail, Propper and van Reenen find out that the number of hospital deaths for emergency heart attacks is larger in English regions where the market wage is higher than the pay received by nurses, that is uniform across the country. Propper and Britton find that the centralized wage regulation of teachers in England decreases educational output in regions where market wages are high.

a transfer makes it politically attractive (Coate and Morris, 1995) and, in the case of EU countries, points to circumvent the competition rules that forbid direct subsidies to disadvantaged regions[5].

As concerns the theoretical literature, the emphasis on the interplay between public sector labor institutions and the private sector relates the present model to the works of Courant *et al.* (1979), Holmlund (1993; 1997), Strøm (1997; 1999), and Forni and Giordano (2003). The main difference between these papers and the present one is the link through which the public sector affects the economy. While in my model the change in the relative cost of the public nontradable good compared to the private tradable one is key, these papers mainly focus on the fiscal system.

Courant *et al.* (1979) notice that stronger public employees market power leads to larger government spending, but that public sector growth can be limited by the simple optimizing behavior of civil servants, if private sector workers retain their right to migrate in case of rising tax rates. If taxpayers are not mobile across communities, Strøm (1999) shows that wage moderation is more likely to be attained under a decentralized financing system than via a centralized one. Under the latter, public sector unions in one local community do not take into account the consequences of a wage increase in terms of a higher per capita tax rate and a lower availability of public services.

In a related paper, Strøm (1997) looks at the differences between a decentralized bargaining scheme in the private sector - in which unions ignore the fact that the public good is financed by taxes on wages - and a centralized setting. He shows the latter puts downward pressure on wages and raises employment. Even in Holmlund (1993), the preeminent link are the taxes levied on private sector workers to finance civil servants' pays. Neglecting this mechanism, public sector unions ask for inefficiently higher wages and public employment crowd out private employment[6]. Holmlund (1997) argues that

[5] For Alesina *et al.* (2001), about half of the public wage bill in the South of Italy can be read as a transfer of resources from the North.

[6] Empirical evidence on the crowding out impact of public employment is found by Boeri *et al.* (2000), Demekas and Kontolemis (2000), Algan *et al.* (2002), and Afonso and Gomes (2014).

the reservation utility of workers plays a crucial role: through that channel, an increase in the civil servants' pay leads to higher private sector wages, stifling labor demand and employment. Forni and Giordano (2003) consider another link through which public employment and wages may affect the rest of economy, namely the degree of cooperation between public sector and private sector unions. With a high degree of cooperation, private sectors' unions are more willing to ask for pay rises. The reason is that the employment losses these demands would entail in the private sector would be partially offset by government's creation of new public sector jobs to keep the unemployment rate low.

Finally, this paper bears some similarities with two recent research areas. One analyses cross-regional unbalances in the labour market and how they influence the impact of a local policy or shock (see Moretti, 2011). The other strand of research studies the interaction between imperfect labour markets and land/housing markets to address issues like urban unemployment, ghettos, and the spatial mismatch between jobs and ethnic minorities's residence (see Zenou, 2009 for a thorough presentation of the main models). Similarly to the present paper, part of this literature incorporates search and matching frictions in a framework with geographical disparities in productivity to study migration and unemployment[7]. The difference is that these models focus on the labor market effects of changes in commuting costs and the land prices, whereas my framework considers the variation in the cost of the public nontradable good.

3 The Basic Framework

3.1 Production and Matching Technology

Time is continuous and the model is developed in steady-state. I consider a country composed by two regions, say a and b. Regions differ only in terms of private sector

[7]See Zenou (2009, chapter 3).

productivity, while all the other product and labor market parameters are assumed to be the same. Besides the gain in simplicity, this also allows to isolate more starkly the effects of different public wage policies on the regional disparities that will result from the model.

As regards the structure of the product market, I follow the standard approach of Obstfeld and Rogoff (1996, chapter 4, pages 204 - 228). In each region, two intermediate goods and one final consumption good are produced. One intermediate good is produced in the private sector and can be traded across the regions at a competitive price, the other one is public and not tradable[8]. The consumption good is also sold in a competitive market, but it is not tradable. Its production function takes a CES form:

$$Y_i = \left[Q_{p,i}^{\frac{s-1}{s}} + Q_{g,i}^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}} \quad \text{with } i \in \{a, b\} \text{ and } \quad s > 0 \,, \tag{1}$$

in which $Q_{p,\,i}$ and $Q_{g,\,i}$ respectively denote the intermediate good produced in the private sector and the intermediate good produced in the public sector in region i. Imposing the elasticity of substitution s greater than 0 both gross substitutability and gross complementarity between private and public goods are allowed.

Let P_i and $P(Q_{p,\,i})$ be the prices of the consumption good and the private intermediate good in region i. I consider $Q_{p,i}$ as the *numeraire* for the economy of region i. So its price is normalized to 1 and it is equal across the regions. The final good firm in region i minimizes its cost by taking these prices and the amount of the public good provided by the government $Q_{g,i}$ as given. This leads to the following F.O.C.:

$$p_i \cdot \left(\frac{Q_{p,i}}{Y_i} \right)^{-\frac{1}{s}} = 1 \qquad \text{with } p_i \equiv \frac{P_i}{P(Q_{p,\,i})} \tag{2}$$

In the entire country there is a measure normalized to L of workers that are infinitely-lived and risk-neutral. Workers endogenously choose the region to live in, according to a maximization rule it will be presented in the next section. Any private

[8]Police service, environmental protection, the administration of justice are all examples of goods that cannot be traded.

(resp. public) employed worker in region i produces y_i (resp. 1) units of the private (resp. public) intermediate good, with $i \in \{a, b\}$ and $y_a > y_b > 1$:[9]

$$\begin{aligned} Q_{p,i} &= y_i \cdot E_{p,i} \\ Q_{g,i} &= E_{g,i} \qquad\qquad i \in \{a, b\}. \end{aligned} \tag{3}$$

$E_{p,i}$ and $E_{g,i}$ respectively define the level of employment in the private sector and in the public sector of region i.

There are frictions in the labor markets. The flow of hires in sector $n \in \{g, p\}$ of region $i \in \{a, b\}$, $M_{n,i}$, is a function of the number of vacancies, $V_{n,i}$ and the number of unemployed people living in region i and searching for a job in sector n, $U_{n,i}$. There is no on-the-job search. The matching function is written $M_{n,i} = m\,(U_{n,i}, V_{n,i})$. Following most of the literature (see Petrongolo and Pissarides, 2001), I impose it is homogeneous of degree 1 and increasing and concave in both arguments. Labor market tightness in sector $n \in \{g, p\}$ of region $i \in \{a, b\}$ is denoted by $\theta_{n,i} \equiv V_{n,i}/U_{n,i}$. The rate at which vacant jobs become filled is $q(\theta_{n,i}) \equiv m\,(U_{n,i}, V_{n,i})\,/V_{n,i}$, with $q'(\theta_{n,i}) < 0$. A job-seeker moves into employment at a rate $f(\theta_{n,i}) \equiv m\,(U_{n,i}, V_{n,i})\,/U_{n,i} = \theta_{n,i}\,q(\theta_{n,i})$ with $f'(\theta_{n,i}) > 0$.[10] I also define $\eta \equiv -q'(\theta_{n,i})\theta_{n,i}/q(\theta_{n,i})$, the opposite of the elasticity of the job-filling rate, and I assume to be constant[11]. At an exogenous rate δ_g (resp. δ_p) a public (private) job is destroyed. Working in the public sector has a longer duration: $\delta_p > \delta_g$.

Let L_i designate the labor force in region $i \in \{a, b\}$, with $L_a + L_b = L$. Then one can write $E_{p,i} + U_{p,i} = (1 - \phi_i)L_i$ and $E_{g,i} + U_{g,i} = \phi_i\, L_i$, the term $\phi_i \in (0, 1)$ being the endogenous fraction of the labor force in region $i \in \{a, b\}$ belonging to the public sector.

[9] Measuring output for public sector services is problematic. In some numerical exercises I drop the assumption that the public sector's productivity is lower than the private one. Results are robust to this change. See section 8.1.

[10] Moreover, it is assumed that $\lim_{\theta_{n,i} \to 0} q(\theta_{n,i}) = +\infty$, $\lim_{\theta_{n,i} \to +\infty} q(\theta_{n,i}) = 0$, $\lim_{\theta_n \to 0} f(\theta_{n,i}) = 0$ and $\lim_{\theta_n \to +\infty} f(\theta_{n,i}) = +\infty$.

[11] This is the case under the standard assumption of a Cobb-Douglas matching function.

The equality between flows in and out each workers' status leads to the following equations:

$$\begin{aligned} E_{p,i}\,\delta_p &= [\,(1-\phi_i)\,L_i \;-\; E_{p,i}\,]\; f(\theta_{p,i}) \\ E_{g,i}\,\delta_g &= (\,\phi_i L_i \;-\; E_{g,i}\,)\; f(\theta_{g,i}) \qquad \text{with } i \in \{a,\, b\}. \end{aligned} \tag{4}$$

Rearranging, one gets the following expression for the levels of employment for each sector in each region:

$$\begin{aligned} E_{p,i} &= \frac{f(\theta_{p,i})}{\delta_p \,+\, f(\theta_{p,i})}(1-\phi_i)L_i \\ E_{g,i} &= \frac{f(\theta_{g,i})}{\delta_g \,+\, f(\theta_{g,i})}\phi_i L_i \end{aligned} \tag{5}$$

with $i \in \{a,\, b\}$. The employment rate for each sector in each region is defined as $e_{n,i} \equiv E_{n,i}/L_i$, with $i \in \{a, b\}$ and $n \in \{p,\, g\}$. The unemployment rate in region i is equal to:

$$u_i \;=\; \frac{\delta_p}{\delta_p \,+\, f(\theta_{p,i})}(1-\phi_i) \,+\, \frac{\delta_g}{\delta_g \,+\, f(\theta_{g,i})}\phi_i\,, \qquad \text{with } i \in \{a,\, b\}. \tag{6}$$

Substituting equations (3) and (5) in the demand function (2), p_i can be written as:

$$p_i = \left[1 \,+\, \left(\frac{\phi_i}{y_i(1-\phi_i)} \cdot \frac{f(\theta_{g,i})}{\delta_g \,+\, f(\theta_{g,i})} \cdot \frac{f(\theta_{p,i})+\delta_p}{f(\theta_{p,i})}\right)^{\frac{s-1}{s}}\right]^{\frac{1}{1-s}} \qquad \text{with } i \in \{a,\, b\}. \tag{7}$$

The price of the consumption good in region i positively depends on y_i and $\theta_{p,i}$, while it is decreasing in ϕ_i and $\theta_{g,i}$. As I proceed I will investigate the general implications of this equation.

3.2 Workers' Preferences

Let r be the discount rate common to all agents. As usual in the standard search and matching literature (Pissarides, 2000, chapter 1), I impose the one firm - one job assumption in the private sector.

The expected discounted utility of the unemployed worker j searching for a job of type $n \in \{g, p\}$ in region $i \in \{a, b\}$, $W^U_{j,n,i}$ verifies the following Bellman equation:

$$rW^U_{j,n,i} = z_{j,i} + f(\theta_{n,i}) \left[W^E_{j,n,i} - W^U_{j,n,i} \right] \tag{8}$$

where the random term $z_{j,i}$ stands for the idiosyncratic preference for region i and $W^E_{j,n,i}$ is the discounted present value of being employed in the n sector in region i.[12] This and the following Bellman equations have a standard interpretation. Being unemployed is like holding an asset that gives you a dividend $z_{j,i}$ and a capital gain, occurring at the rate $f(\theta_{n,i})$, equal to the term inside the square brackets. A higher $z_{j,i}$ means a stronger attachment to region i for worker j.

To determine the measure of workers searching for a job in either sector and the measure of workers choosing to live in either region, I introduce two conditions. The first is the no arbitrage condition $W^U_{j,p,i} = W^U_{j,g,i} = W^U_{j,i}$, that ensures in equilibrium there is no gain in choosing to apply for either sector. From equation (8), this implies:

$$f(\theta_{g,i}) \left[W^E_{j,g,i} - W^U_{j,i} \right] = f(\theta_{p,i}) \left[W^E_{j,p,i} - W^U_{j,i} \right] \tag{9}$$

The second condition, borrowed from Moretti (2011), imposes that a generic worker j's relative preference for region a over region b is:

$$z_{j,a} - z_{j,b} \sim h\left[-\lambda, \lambda\right], \tag{10}$$

[12] An intuitive way of deriving the Bellman equation is the following. The expected lifetime utility of an unemployed worker $W^U_{j,n,i}(t)$, with $n \in \{g, p\}$ and $i \in \{a, b\}$ takes this form:

$$W^U_{j,n,i}(t) = \frac{1}{1+rdt} \left[z_{j,i}dt + f(\theta_{n,i})dt \cdot W^E_{j,n,i}(t+dt) + (1 - f(\theta_{n,i})dt) \cdot W^U_{j,n,i}(t+dt) \right].$$

In words, $W^U_{j,n,i}(t)$ is equal to the discounted sum of the flow $z_{j,i}dt$ in the interval dt and of the discounted expected future utilities. With probability $f(\theta_{n,i})dt$ this utility coincides with the expected value of being employed. With probability $1 - f(\theta_{n,i})dt$, it coincides with the expected value of being unemployed. Doing some algebra and taking the limit $dt \to 0$, this equation coincides in steady-state with (8). All the other Bellman equations in the model can be derived in the same way. See Cahuc and Zylberberg (2004, Appendix D) and Zenou (2009, Appendix B) for a detailed exposition.

with $h(.)$ being a probability density function. Parameter λ captures the importance of the preference for location and therefore the degree of labor mobility. If λ is large, people's willingness to move in order to reap the benefits of higher real wages or shorter unemployment spells is limited. Conversely, if λ is small, workers are more willing to migrate in search of better economic conditions. With $\lambda = 0$, nobody is attached to a region compared to the other, and there is perfect worker mobility. One can define the value λ^* that belongs to the marginal worker j^*, the one indifferent between searching for a job in region a or in b:

$$rW^U_{j^*,b} - rW^U_{j^*,a} = 0.$$

If $\lambda^* \equiv z_{j^*,a} - z_{j^*,b}$, from equation (8) I get:

$$\lambda^* = f(\theta_{n,b}) \left[W^E_{j^*,n,b} - W^U_{j^*,n,b} \right] - f(\theta_{n,a}) \left[W^E_{j^*,n,a} - W^U_{j^*,n,a} \right] \tag{11}$$

with $n \in \{g, p\}$. The labor forces in both regions can be written as:

$$\begin{aligned} L_b &= H(\lambda^*)\, L \\ L_a &= (1 - H(\lambda^*))\, L\,, \end{aligned} \tag{12}$$

with $H(.)$ being the cumulative density function. Finally, the Bellman equation for a worker of region $i \in \{a, b\}$ employed in sector $n \in \{g, p\}$ is:

$$rW^E_{j,n,i} = z_{j,i} + \frac{w_{n,i}}{p_i} + \delta_n \left(W^U_{j,i} - W^E_{j,n,i} \right), \tag{13}$$

where $w_{n,i}/p_i$ is the real wage in sector n of region i.

3.3 Firms in the Private Sector

On the other side of the market, the Bellman equation for an active private firm is:

$$rJ^E_{p,i} = \frac{y_i - w_{p,i} - p_k \cdot k}{p_i} + \delta_p \left(J^V_{p,i} - J^E_{p,i} \right), \quad \text{with } i \in \{a, b\} \tag{14}$$

The first term in the RHS of (14) is the firm's revenues, namely the amount of the units of the intermediate good produced y_i net of the wage bill and the rental cost of capital equipment. I assume that each firm needs an amount k of capital, whose price p_k is determined in the international markets. The second term in the RHS is the capital loss occurring at rate δ_p, with $J^V_{p,\,i}$ being the expected value of a vacancy and defined as follows:

$$rJ^V_{p,\,i} \;=\; -\frac{p_k \cdot k}{p_i} \;+\; q(\theta_{p,\,i})\left(J^E_{p,\,i} \;-\; J^V_{p,\,i}\right), \quad \text{with } i \in \{a,\, b\}. \tag{15}$$

The expected value of vacancy is given by the sum of the rental cost of equipment and the capital gain that accrues from the match, multiplied by the job filling rate.

As common in search and matching models, a free-entry zero profit condition determines the equilibrium values of tightness $\theta_{p,\,i}$, conditional on the nominal wage and the price level in sector i. Free-entry of vacancies and zero profits imply that $J^V_{p,i} \;=\; 0$. Substituting this into (14) and (15), one gets:

$$\frac{y_i \;-\; w_{p,\,i} \;-\; p_k \cdot k}{r \;+\; \delta_p} \;=\; \frac{p_k \cdot k}{q(\theta_{p,\,i})} \quad \text{with } i \in \{a,\, b\}. \tag{16}$$

Firms' expected discounted revenues (the LHS of (16)) are equal to the expected cost of posting a vacancy (the RHS of (16)).

3.4 Private Sector Wage: Individual Bargaining

I assume that the wage in the private sector is negotiated between each firm and worker at individual level. This assumption better fits the U.S. wage setting. In section 6, I consider the case of collective negotiation between unions of firms and workers. As we will see, this setting differs in that unions take the negative effect of the wage on employment into account, but the main results of the paper hold true.

Assuming an axiomatic Nash solution to split the surplus $W^E_{j,\,p,\,i} - W^U_{j,\,i} + J^E_{p,\,i} - J^V_{p,\,i}$ originated from a match, the nominal wage $w_{p,\,i}$ solves the following problem:

$$w_{p,\,i} \;=\; \text{argmax}\left[W^E_{j,\,p,\,i} \;-\; W^U_{j,\,i}\right]^{\beta}\left[J^E_{p,\,i} \;-\; J^V_{p,\,i}\right]^{1-\beta}, \tag{17}$$

with $i \in \{a, b\}$. Parameter β denotes the exogenous bargaining power of a worker $(0 < \beta < 1)$. Knowing that $J^V_{p,i} = 0$, the F.O.C. of this problem is:

$$W^E_{j,p,i} - W^U_{j,i} = \beta \left[J^E_{p,i} + W^E_{j,p,i} - W^U_{j,i} \right] \tag{18}$$

Each worker takes a share β of the total surplus. Using equations(8), (13), and (14) one gets:

$$w_{p,i} = \beta \cdot [\, y_i + p_k \cdot k\,(\theta_{p,i} - 1)] \quad \text{with } i \in \{a, b\}. \tag{19}$$

This wage equation is similar to the one usually obtained in search and matching models. The nominal pay positively depends on the amount of the intermediate good produced, y_i, and labor market tightness $\theta_{p,i}$. Substituting this expression for the nominal wage in equation (16) yields:

$$\mathbb{ZP}(\theta_{p,i}) \equiv (1 - \beta)\, y_i - p_k \cdot k \cdot \frac{r + \delta_p + (1-\beta) q(\theta_i) + \beta f(\theta_i)}{q(\theta_i)} = 0\,, \tag{20}$$

with $i \in \{a, b\}$. This implicit function is denoted $\mathbb{ZP}(\theta_{p,i}) = 0$ because of the zero-profit condition that determines the vacancy/unemployment ratio in the private sector. Notice that $\frac{d\,\mathbb{ZP}(\theta_{p,i})}{d\,\theta_{p,i}} < 0$. For the limit conditions imposed on functions $f(\theta_{p,i})$ and $q(\theta_{p,i})$, it is easy to see that there exists a unique $\theta_{p,i}$ that solves $\mathbb{ZP}(\theta_{p,i}) = 0$. Moreover, one also get that $\theta_{p,a} > \theta_{p,b}$ because $y_a > y_b$.

3.5 No arbitrage condition

Rearranging the Bellman equations (8) and (13), the no arbitrage condition (9) can be written as:

$$f(\theta_{g,i})\,\frac{w_{g,i}}{r + \delta_g + f(\theta_{g,i})} = f(\theta_{p,i})\,\frac{w_{p,i}}{r + \delta_p + f(\theta_{p,i})} = \frac{\beta}{1-\beta}\, p_k \cdot k\ \theta_{p,i} \tag{21}$$

with $i \in \{a, b\}$. The second equality is obtained via the free-entry condition $J^V_{p,i} = 0$ and equations (8), (14) and (18). So, the equation (11) defining the value of λ^* can be expressed as:

$$\lambda^* = \frac{\beta}{1-\beta}\, p_k \cdot k \left(\frac{\theta_{p,b}}{p_b} - \frac{\theta_{p,a}}{p_a} \right) \tag{22}$$

This equation says that the tighter the private sector labor market and the lower the cost of living in region a (resp. b), the larger the share of workers $(1 - H(\lambda^{*}))\ L$ (resp. $H(\lambda^{*})\, L$) living there[13].

4 Public Sector Bargaining

In the public sector, the nominal wage level and the amount job vacancies are determined according two mutually exclusive scenarios:

1. *Regional Bargaining.* The negotiation takes place at regional level by local public authorities and unions.

2. *Centralized Bargaining.* The negotiation takes place at the central level by national unions and the central government.

In both cases we are in an "efficient bargaining" situation, as the parties jointly decide the optimal level of job vacancies and wages. This arrangement better fits the empirical evidence than a monopoly union model, that predicts a (counterfactual) relation between wage increases and employment losses[14].

[13] Moreover, if function h (.) were symmetric along the vertical axis, a negative (resp. positive) value for λ^{*} would imply more (resp. less) workers living in region a compared to region b

[14] See the seminal paper of McDonald and Solow (1981) and Gregory and Borland (1999) for an extensive discussion. Inman (1981), Oswald *et al.* (1984) and Fernandez-de-Cordoba *et al.* (2012) consider a similar bargaining scheme.

4.1 Public Sector Regional Bargaining

As concerns the first scenario, the optimal values of $w_{g,i}$ and $\theta_{g,i}$ are obtained by solving the following problem:

$$\begin{aligned} &\max_{w_{g,i},\,\theta_{g,i}} \left[E_{g,\,i}\left(W^{E}_{j,\,g,\,i} - W^{U}_{j,\,i}\right)\right]^{\beta} \, [\Pi_i]^{1-\beta} \\ &\text{with } \Pi_i = \frac{T_i}{p_i} - E_{g,\,i}\frac{w_{g,\,i}}{p_i} - (V_{g,\,i} + E_{g,\,i})\frac{p_k \cdot k}{p_i} \\ &\text{and } \frac{T_i}{p_i} = Y_i - \frac{1}{p_i}\cdot Q_{p,\,i} \end{aligned} \tag{23}$$

with $i \in \{a,\, b\}$. Unions care about the sum of the utilities of its members, that for simplicity are all the workers in the public sector. On the other side of the bargaining, the public authority of region i wants to maximize its revenues, namely the total amount of taxes raised in region i, denoted by T_i, net of the wage bill and the equipment costs. Government's revenues are expressed in real terms, so every term in Π_i is divided by the price of the final good p_i. The third equation in (23) states that the local government's taxes T_i are levied on the final good firms and they are equal to their profits.[15]. A corollary of the third equation in (23) is the absence of fiscal redistribution among regions: taxes raised in one region does not finance expenses in the other one.

The utility function Π_i means that the local government wants to maximize taxes revenues over public spending. This assumption deserves some comments. There are two main routes taken in the literature to model public employer's decisions. One approach considers a public authority guided only by either efficiency or equity considerations (i.e. the maximization of the welfare of citizens, or the cost minimization of the public good, or the reduction of the unemployment rate).[16] The other approach

[15]Final good firms' profits in real terms are equal to the amount of the consumption good produced Y_i net of the purchase of the private intermediate good, that is $Q_{p,\,i}$ multiplied by its value in terms of the consumption good $1/p_i$.

[16]Papers along this line are Ashenfelter and Ehrenberg (1975), Ehrenberg and Goldstein (1975) and, more recently, Forni and Giordano (2003).

takes another perspective by looking at the personal objectives of politicians and bureaucrats: vote-maximization or some budget targets are the most studied examples in the literature.[17] In assuming that the public authority maximizes the surplus of tax revenues over government spending, this paper follows the second approach, specifically borrowing from the so-called "Leviathan model" introduced by Brennan and Buchanan (1980). However, in a separate Appendix, available on request, I show that such a behavior delivers the same equilibrium results that would be obtained by a social planner aiming to maximize the utility of all workers in the region, given a budget constraint. So the choice of either approach in modeling public employer's decisions does not affect the main predictions of the paper.

For simplicity I assume that workers' bargaining power β is identical across sectors and regions. In finding the optimal values of $w_{g,i}$ and $\theta_{g,i}$, both parts take the price of the consumption good, p_i as given. I discuss the implications of this assumption in section 6.1.

The F.O.C. for $w_{g,i}$ that satisfies (23) is:

$$E_{g,i}\left(W^E_{j,g,i} - W^U_{j,i}\right) \;=\; \beta\left[\Pi_i + E_{g,i}\left(W^E_{j,g,i} - W^U_{j,i}\right)\right] \qquad \text{with } i \in \{a, b\} \tag{24}$$

As in the private sector, workers get a fraction β of the total surplus. Before computing the F.O.C. for $\theta_{g,i}$, note first that the constant returns to scale property of the final good production functions and equation (2) allow to write taxes T_i as follows:

$$\frac{T_i}{p_i} = Y_i - \frac{1}{p_i} Q_{p,i} = E_{g,i} \cdot (dY_i / dQ_{g,i}) \qquad \text{with } i \in \{a, b\}. \tag{25}$$

Note also that in steady-state $V_{g,i} = \delta_g \cdot E_{g,i}/q(\theta_{g,i})$. Inserting this expression into Π_i, the F.O.C. for $\theta_{g,i}$ is:

$$\beta \frac{\frac{dE_{g,i}}{d\theta_{g,i}}\left(W^E_{j,g,i} - W^U_{j,i}\right)}{E_{g,i}\left(W^E_{j,g,i} - W^U_{j,i}\right)} = -(1-\beta) \frac{\frac{dE_{g,i}}{d\theta_{g,i}}\left[\frac{dY_i}{dQ_{g,i}} - \frac{w_{g,i}}{p_i} - \frac{p_k \cdot k}{p_i}\left(1 + \frac{\delta_g}{q(\theta_{g,i})}\right)\right] + E_{g,i}\frac{p_k \cdot k}{p_i}\frac{q'(\theta_{g,i})}{q^2(\theta_{g,i})}\delta_g}{\Pi_i}$$

[17] For an overview of this research area, see Mueller (2003).

with $i \in \{a, b\}$. Using (24), the equation above can be written in the following way:

$$\frac{d\,E_{g,\,i}}{d\,\theta_{g,\,i}}\left[\frac{d\,Y_i}{d\,Q_{g,\,i}} - \frac{w_{g,\,i}}{p_i} - \frac{p_k\cdot k}{p_i} + W^E_{j,\,g,\,i} - W^U_{j,\,i}\right] = \frac{\delta_g\,p_k\cdot k}{p_i\,q(\theta_{g,\,i})}\left(\frac{d\,E_{g,\,i}}{d\,\theta_{g,\,i}} - \frac{E_{g,\,i}\,q'(\theta_{g,\,i})}{q(\theta_{g,\,i})}\right)$$

with $i \in \{a, b\}$. At the LHS the surplus obtained by one more public job (the term inside the square brackets) is multiplied by the employment gain due to higher labor market tightness, $dE_{g,\,i}/d\theta_{g,\,i}$. At the RHS we have the marginal cost of an increase in tightness, namely the larger expenditures in capital equipment implied by more vacancy creation.[18] At the equilibrium, marginal cost and marginal revenues must be equal.

Using the no arbitrage condition (21) and computing $d\,E_{g,\,i}/d\,\theta_{g,\,i}$ via equation (5), the F.O.C.s for $w_{g,\,i}$ and $\theta_{g,\,i}$ respectively become:

$$w_{g,\,i} = p_i\,\frac{d\,Y_i}{d\,Q_{g,\,i}} - p_k\cdot k\left(1 + \frac{\delta_g}{q(\theta_{g,\,i})} + \frac{\theta_{p,\,i}}{f(\theta_{g,\,i})}\right), \tag{26}$$

$$p_i\,\frac{d\,Y_i}{d\,Q_{g,\,i}} = w_{g,\,i} + p_k\cdot k\left[1 + \frac{\delta_g}{q(\theta_{g,\,i})} - \frac{\beta}{1-\beta}\frac{\theta_{p,\,i}}{f(\theta_{g,\,i})} + \frac{\eta}{1-\eta}\frac{\delta_g + f(\theta_{g,\,i})}{q(\theta_{g,\,i})}\right] \tag{27}$$

with $i \in \{a, b\}$.[19] Combining (26) and (27) to get rid of $w_{g,i}$ yields:

$$\theta_{g,\,i}\,(\delta_g + f(\theta_{g,\,i})) = \frac{1-\eta}{\eta\,(1-\beta)}\,\theta_{p,\,i} \qquad \text{with } i \in \{a, b\}. \tag{28}$$

Since $f(.)$ is an increasing function, this equation implies a positive relationship between $\theta_{g,\,i}$ and $\theta_{p,\,i}$. This stems from the no arbitrage condition. The stronger the vacancy creation in the private sector, the shorter the unemployment spell for those searching for a job there. So the government needs to post more vacancies (raising $\theta_{g,\,i}$) to make the value of searching for a public job equally appealing.

[18] Notice that the term at the RHS is equal to $\frac{p_k\cdot k}{p_i}\cdot\frac{d\,V_{g,\,i}}{d\,\theta_{g,\,i}}$ with $V_{g,\,i} = \frac{\delta_g\,E_{g,\,i}}{q(\theta_{g,\,i})}$ for the steady-state equality of labor market flows.

[19] It is easy to see that equation (26) implies that $\Pi_i > 0$, meaning that government is running a surplus. To keep things as simple as possible, I then assume that it is distributed evenly to all residents via a lump-sum subsidy that does not affect the agents' behavior.

Using equations (2), (3), and (5) to re-express the derivative $dY_i/dQ_{g,\,i}$ and the expression for the nominal wage in the first equality of (21), equation (27) can be written as:

$$\begin{aligned} & y_i^{1/s} \cdot \left(\frac{\phi_i}{1-\phi_i} \cdot \frac{\delta_p + f(\theta_{p,\,i})}{f(\theta_{p,\,i})} \cdot \frac{f(\theta_{g,\,i})}{\delta_g + f(\theta_{g,\,i})} \right)^{-1/s} = \\ & = p_k \cdot k \left[1 + \frac{\delta_g}{q(\theta_{g,\,i})} + \frac{\eta}{1-\eta} \frac{\delta_g + f(\theta_{g,\,i})}{q(\theta_{g,\,i})} + \frac{\beta}{1-\beta}\, \theta_{p,\,i} \frac{r + \delta_g + f(\theta_{g,\,i}) - 1}{f(\theta_{g,\,i})} \right] \end{aligned} \tag{29}$$

Definition 1 *A general equilibrium of the public sector regional bargaining model is a vector* $[\,\theta_{p,\,i},\ \theta_{g,\,i},\ \phi_i,\ w_{g,i},\ p_i,\ Y_i\,]$ *for* $i \in \{a,\,b\}$*, and value of* λ^* *satisfying: (i) the zero profit condition* $\mathbb{ZP}(\theta_i) = 0$*; (ii) the F.O.C.s for* $w_{g,i}$ *and* $\theta_{g,\,i}$ *(26) and (28); (iii) the equation for* ϕ_i*, (29); (iv) the equation for* λ^**, (22); (v) the equation for* Y_i *and the F.O.C in the intermediate sector, (1) and (7).*

The following Lemma presents the main results of the public sector regional bargaining framework.

Lemma 1 *If* $r+\delta_g < 1$*, a general equilibrium of the public sector regional bargaining model exists and it is unique. Moreover, we have:*

1. $\theta_{n,a} > \theta_{n,b}$ *for* $n \in \{p,\,g\}$.
2. $p_a > p_b$.
3. $w_{n,a} > w_{n,b}$, *for* $n \in \{p,\,g\}$.
4. *If function* $h(.)$ *is symmetric along the vertical axis,* $L_a > L_b$.

The formal proof of the existence is in Appendix A. Here I focus on the properties of the equilibrium. As regards point 1, recall that the private intermediate good sector is more productive in region a than in region b ($y_a > y_b$). So, for the zero-profit equation $\mathbb{ZP}_i(\theta_{p,\,i}) = 0$ to hold, firms need to post more vacancies in the former than in the latter. This implies $\theta_{p,\,a} > \theta_{p,\,b}$. In turn, we also have $\theta_{g,\,a} > \theta_{g,\,b}$, as explained when interpreting equation (28).

To see the inter-regional differences in the cost of living, some algebraic computations (presented in Appendix A) allow to express the price of the consumption good p_i as an increasing function of $\theta_{g,\,i}$ only. Since $\theta_{g,\,a} > \theta_{g,\,b}$, then $p_a > p_b$. This means that the region with a higher productivity in the tradable good compared to nontradable exhibits a higher cost of living. It is the well-known *Harrod-Balassa-Samuelson effect*[20]. Because of labor mobility across the private tradable and the public nontradable sectors, a higher marginal productivity in the former, y_i, drives up the marginal value of employment in the public sector (that is equal to 1 multiplied by the tax per unit of public good levied on the final firm, $T_i/E_{g,\,i}$). A higher cost - paid via taxes - of the public nontradable good translates into a higher price of the composite consumption good, p_i.

A greater productivity and labor market tightness in the private sector of region a also entails a higher nominal wage: $w_{p,\,a} > w_{p,\,b}$. For the no arbitrage condition, such a gap is present in the public sector too: $w_{g,\,a} > w_{g,\,b}$. As concerns the real wages, results are ambiguous, for it is not possible to verify to which extent higher nominal pays in region a are gobbled up by the more expensive cost of living. Finally, inspecting equation (22), we have that the threshold parameter $\lambda^* < 0$ if $\theta_{p,a}/p_a > \theta_{p,\,b}/p_b$. After some computations (see Appendix A), one gets that this last inequality holds true. So, if the distribution of preferences for location is symmetric, better labor market conditions pushes the majority of workers to stay in region a.

What cannot be checked at the analytical level are the employment differences among regions. Although region a exhibits a tighter labor market in both sectors, this does not imply a lower unemployment rate, whose value also depends on the fraction of workers ϕ_i searching for a public job. From equation (6), it is not possible to check if $u_a < u_b$.

[20]See Obstfeld and Rogoff, 1996, chapter 4, for a detailed exposition.

4.2 Public Sector National Bargaining

In this second scenario, I assume that the nominal public sector wage (identical across regions) is bargained over at the national level. A national union representing all the civil servants in the economy negotiates with the central government. Labor market tightness in the public sector of each region is also decided at national level by unions and the central government. The Nash bargaining problem is:

$$\max_{w_g,\, \theta_{g,a},\, \theta_{g,\, b}} \left[E_{g,\, a}\left(W^E_{j,\, g,\, a} - W^U_{j,\, a}\right) + E_{g,\, b}\left(W^E_{j,\, g,\, b} - W^U_{j,\, b}\right)\right]^{\beta} \left[\Pi_a + \Pi_b\right]^{1-\beta} \tag{30}$$

Workers' utilities and fall-back positions, as well as the government's value functions, are the same as in the section 4.1. The only difference is that in the regional case there are two different bargaining problems, one for each region, whereas in this scenario there is only one. The F.O.C.s for w_g and $\theta_{g,i}$ ($i \in \{a,\, b\}$) are, respectively:

$$\begin{aligned}
\beta \sum_{i=a,\, b} \Pi_i &= (1-\beta) \sum_{i=a,\, b} E_{g,\, i}\left(W^E_{j,\, g,\, i} - W^U_{j,\, i}\right), \\
\beta \frac{-\frac{d\, E_{g,\, i}}{d\, \theta_{g,\, i}}\left(W^E_{j,\, g,\, i} - W^U_{j,\, i}\right)}{\sum_{i=a,\, b} E_{g,\, i}\left(W^E_{j,\, g,\, i} - W^U_{j,\, i}\right)} &= (1-\beta) \frac{\frac{d\, E_{g,\, i}}{d\, \theta_{g,\, i}}\left[\frac{d\, Y_i}{d\, Q_{g,\, i}} - \frac{w_{g,\, i}}{p_i} - \frac{p_k \cdot k}{p_i}\left(1 + \frac{\delta_g}{q(\theta_{g,\, i})}\right)\right] + E_{g,\, i} \frac{p_k \cdot k}{p_i} \frac{q'(\theta_{g,\, i})}{q^2(\theta_{g,\, i})} \delta_g}{\sum_{i=a,\, b} \Pi_i}
\end{aligned}$$

with $i \in \{a,\, b\}$. Using the first equation to get rid of the denominators in the second equation, we get that the F.O.C.s for $\theta_{g,a}$ and $\theta_{g,\, b}$ are identical to the same conditions (27) in the regional bargaining scenario. So, the same steps made in the previous scenario lead to equation (29), one of the equilibrium conditions in the centralized bargaining model[21]. Proceeding as in the previous section, the F.O.C on w_g leads to the following wage equation:

$$w_g = \frac{\sum_{i=a,\, b} \frac{E_{g,\, i}}{p_i}\left[p_i \frac{d\, Y_i}{d\, Q_{g,\, i}} - p_k \cdot k\left(1 + \frac{\delta_g}{q(\theta_{g,\, i})} + \frac{\theta_{p,\, i}}{f(\theta_{g,\, i})}\right)\right]}{\sum_{i=a,\, b} E_{g,\, i}/p_i}. \tag{31}$$

[21] It is also clear that the F.O.C.s would have been the same if I had left to local governments and unions the choice of the amount of job vacancies, conditional on the nominal wage bargained at national level.

Let compare this expression with the corresponding equation (26). Under national bargaining, the nominal public sector wage is a weighted average of the pays obtained via regional negotiation. The endogenous weight are $\frac{E_{g,i}/p_i}{\sum_{i=a,\,b} E_{g,i}/p_i}$ for $i \in \{a,\, b\}$. A high value for $E_{g,i}$ means that a big share of the union's members belongs to region i. This strengthens their implicit bargaining power and makes the nominal wage w_g closer to productivity and tightness of that region. Conversely, the price p_i has a negative impact on the implicit bargaining power of civil servants of region i. The more expensive the cost of the consumption good in one region, the lower is the real value of a public worker there compared to his peers in the other region.

Notice also that, unlike the regional bargaining case, this centralized scenario may involve fiscal redistribution across regions. From equation (30), government maximizes the total amount of taxes raised in the country, net of the sum of labor and capital costs in both regions. So there is no constraint that prevents the public authority from using taxes collected in one region to pay workers and capital rents in the other one.

For a better comparison between this scenario and the regional bargaining one, I also rearrange equations (29) and (31) to get an expression without the nominal wage w_g:

$$\frac{\phi_a\, L_a}{p_a}\left[\theta_{g,\,a} - \frac{1-\eta}{\eta\,(1-\beta)}\frac{\theta_{p,\,a}}{\delta_g + f(\theta_{g,\,a})}\right] + \frac{\phi_b\, L_b}{p_b}\left[\theta_{g,\,b} - \frac{1-\eta}{\eta\,(1-\beta)}\frac{\theta_{p,\,b}}{\delta_g + f(\theta_{g,\,b})}\right] = 0. \quad (32)$$

While in the regional bargaining scenario both expressions in the square brackets of (32) are equal to zero, in the national case this is not necessarily the case. In the next section, I discuss this equation more in detail.

Since the nominal public sector wage w_g is identical across regions, the no arbitrage conditions (21) in region a and b can be rearranged to get:

$$\theta_{p,\,a}\,\frac{r\,+\,\delta_g\,+\,f(\theta_{g,\,a})}{f(\theta_{g,\,a})} \;=\; \theta_{p,\,b}\,\frac{r\,+\,\delta_g\,+\,f(\theta_{g,\,b})}{f(\theta_{g,\,b})} \qquad (33)$$

This condition creates a link between the labor markets of each region. In particular, notice that $\theta_{p,\,a} > \theta_{p,\,b}$ leads to $\theta_{g,\,a} > \theta_{g,\,b}$, for the fractions in both sides of the equation are increasing in $\theta_{g,\,i}$ ($i \in \{a,\, b\}$). This means that the central government needs to have a tighter labor market for public jobs in the more productive region, where stronger vacancy creation makes competition to attract workers fiercer.

The public intermediate good is financed via the same tax on the final good firm as in the previous section. Definition 2 describes the equilibrium in this scenario.

Definition 2 *A general equilibrium of the public sector national bargaining model is a vector* $[\,\theta_{p,\,i},\ \theta_{g,\,i},\ \phi_i,\ p_i,\ Y_i\,]$ *for* $i \in \{a,\, b\}$*, and values of* w_g *and* λ^* *satisfying: (i) the zero profit condition* $\mathbb{ZP}(\theta_i) = 0$*; (ii) the F.O.Cs for* θ_g *and* w_g *(29) and (32); (iii) the no arbitrage condition (33); (iv) the equation for* λ^**, (11); (v) the equation for* Y_i *and the F.O.C in the intermediate sector, (1) and (7).*

Lemma 2 presents the main features in this second scenario:

Lemma 2 *If* $r + \delta_g < 1$*, a general equilibrium of the public sector national bargaining model exists and it is unique. Moreover, we have:*

1. $\theta_{n,a} > \theta_{n,b}$ *for* $n \in \{p,\, g\}$.
2. $p_a > p_b$.
3. $w_{p,a} > w_{p,b}$.

The formal proof is in Appendix B. Notice that the main properties of the equilibrium (such as the Harrod-Balassa-Samuelson effect) hold even under in the case of centralized bargaining. The only notable difference is that under national negotiation one cannot claim that $L_a > L_b$ in case of symmetric distribution of preferences $h(.)$. As it will be clearer in the next section, this depends on the fact that in the central bargaining scenario public employees in a (resp. b) receive a lower (resp. higher) pay than under the regional bargaining case. This has a negative effect on migration towards the more productive region a.

5 Regional vs. National Public Sector Bargaining

In this section, I wonder what are the consequences on employment, prices, and real wages of applying either regime. Let denote with superscript ^{N} the equilibrium values in the national bargaining scenario and the superscript ^{R} the ones in the regional case. Lemma 3 presents some intermediate outcomes, while Proposition 1 and 2 respectively describe the welfare and the employment effects. Table 1 summarizes all the results.

Lemma 3 *Compared to a regional negotiation in the public sector, the adoption of a national bargaining delivers the following results:*

1. *Labor market tightness decreases in region b and increases in region a:* $\theta^N_{g,a} > \theta^R_{g,a}$ *and* $\theta^N_{g,b} < \theta^R_{g,b}$.
2. *The fraction of the labor force belonging to the public sector increases in region b and decreases in region a:* $\phi^N_a < \phi^R_a$ *and* $\phi^N_b > \phi^R_b$.

Notice first that labor market tightness in the private sectors $\theta_{p,i}$ with $i \in \{a, b\}$ is uniquely determined by the zero profit condition $\mathbb{ZP}(\theta_{p,i}) = 0$ and it is therefore unaffected by the change in the public sector bargaining process. Point (1) of Lemma 3 is formally proved in Appendix C, but the rationale is easy to understand. Under national bargaining, w_g is a weighted average of the pays civil servant get in the regional case, $w_{g,a}$ and $w_{g,b}$. This means that public sector employees in the more productive region a are paid less under central negotiation, while public employees in region b are paid more. For the no arbitrage condition, that pushes more (resp. less) unemployed workers of region b (resp. region a) to search for a public job, reducing (resp. increasing) the vacancy-unemployment ratio in the public sector: $\theta^N_{g,a} > \theta^R_{g,a}$ and $\theta^N_{g,b} < \theta^R_{g,b}$. In turn, the fraction of the labor force belonging to the public sector also increases in region b and decreases in region a (point (2) of Lemma 3).

Proposition 1 *Compared to a regional negotiation in the public sector, the adoption of a national bargaining delivers the following results:*

1. *The inter-regional differences in the cost of living increase:* $p_a^N > p_a^R$ *and* $p_b^N < p_b^R$.

2. *Real wages go up in region* b *and down in region* a. *The ratio* $w_{g,i}/w_{p,i}$ *decreases in region* a *and increases in region* b.

3. *All residents in region* a *are worse off, while residents in region* b *are better off.*

Proposition 1 tells that a centralized public sector bargaining widens the differences in the cost of living between regions, as the unique consumption good becomes even more expensive in a and even cheaper in b. Computations are in Appendix C but the intuition is the following. A centralized negotiation enhances the unitary cost of the public good (paid via taxes $T_i/Q_{g,i}$) in the richer region, whereas it lowers it in the poorer one. In turn, a more (resp. less) expensive public good translates into a dearer (cheaper) composite consumption good in region a (resp. b). This may appear counter-intuitive, as it is region a (region b) that pays lower (higher) public sector wages compared to the regional bargaining scenario. However, as an inspection of the F.O.C. (29) makes clear, the cost of producing one unit of the public good (the RHS of 29, equal to the tax per unit $T_i/Q_{g,i}$) not only depends on the wage bill, but also on labor market tightness $\theta_{g,i}$. A tighter labor market means it takes more time to fill a vacancy, raising government' expected costs. So, the reduction in the number of public sector job applications in region a raises the unitary cost of the public good more than a lower wage bill reduces it[22]. The opposite occurs in region b. This chimes well with the empirical evidence presented by Propper and van Reenen (2010) and Propper and

[22]Such a result does not depend on the assumption of efficient bargaining, but it is reinforced by it. It can be shown that a higher $\theta_{g,i}$ reduces workers's quasi-rents from the match $W_{j,g,i}^E - W_{j,i}^U$, as it becomes easier to find another job in case of disagreement in the negotiation. Under efficient bargaining this effect is taken into account, raising even more the marginal cost of $\theta_{g,i}$.

Britton (2012). They emphasize the recruitment problems that centralized pay systems may have in regions where public wages are lower than private ones and the negative consequences on public output that may entail.
The effects on real wages are obvious. From equation (19), the nominal wage $w_{p,\,i}$ depends only on labor market tightness $\theta_{p,\,i}$ and it is therefore unaffected by the change in the public sector negotiation. Given that $p_a^N > p_a^R$ and $p_b^N < p_b^R$, private sector real wages decrease in region a and increase in region b. Since $w_{g,\,a} > w_g > w_{g,\,b}$, public sector real wages move in the same direction not just for the change in the cost of living but also for the variations in the nominal pays. The impact on the ratio $w_{g,\,i}/w_{p,\,i}$ is straightforward: it goes down in region a and it increases in region b. Unfortunately, it not possible to see at the analytical level if there exists a regional wage premium (i.e. if $w_{g,\,i} > w_{p,\,i}$ with $i \in \{a, b\}$).[23]

The variations in the cost of living influence the welfare of all workers in the economy. Using equations (8), (9), (13), and (21), the expected discounted utility of the unemployed worker j can be written as:

$$rW_{j,\,i}^U = z_{j,\,i} + p_k \cdot k\,\frac{\beta}{1-\beta} \cdot \frac{\theta_{p,\,i}}{p_i} \qquad \text{with } i \in \{a,\, b\}.$$

The price p_i is the only variable at the LHS affected by the introduction of a national public sector bargaining. Thus unemployed workers are better off in region b and worse off in region a. Using (8), (9), and (21) the expected discounted utility for employed workers can be written as:

$$\begin{aligned} rW_{j,p,i}^E &= z_{j,\,i} + \frac{r + f(\theta_{p,\,i})}{r + \delta_p + f(\theta_{p,\,i})}\,\frac{w_{p,\,i}}{p_i}\,, \\ rW_{j,g,i}^E &= z_{j,\,i} + \frac{r}{r + \delta_g + f(\theta_{g,\,i})}\,\frac{w_{g,\,i}}{p_i} + \frac{f(\theta_{p,\,i})}{r + \delta_p + f(\theta_{p,\,i})}\,\frac{w_{p,\,i}}{p_i}\,. \end{aligned}$$

with $i \in \{a,\, b\}$. It is easy to see that private sector employees' utility $rW_{j,p,a}^E$ (resp. $rW_{j,p,b}^E$) decreases (increases), because national bargaining raises p_a (lowers p_b). By the

[23]In the calibration, I impose a small wage premium in both regions in line with recent empirical research. See section 7.

same token, the rise in labor market tightness $\theta_{g,\,a}$ and the real wages reduction make civil servants in region a worse off. The opposite occurs to civil servants in region b.

Proposition 2 *Compared to a regional negotiation in the public sector, the adoption of a national bargaining delivers the following results:*

1. *Migration towards the more productive region decreases: $L_a^N < L_a^R$ and $L_b^N > L_b^R$.*
2. *The private sector employment rate increases in region a and decreases in b: $e_{p,\,a}^N > e_{p,\,a}^R$ and $e_{p,\,b}^N < e_{p,\,b}^R$.*
3. *If $\delta_g^{\frac{2-\eta}{1-\eta}}(1-\beta)\frac{\eta}{1-\eta} > \delta_p^{\frac{1}{1-\eta}}$, the unemployment rate in region b increases.*

A direct result of the change in the price levels is the decrease in migration towards the richer region a. This can be seen analytically through equations (12) and (22). The threshold value λ^* that makes the marginal worker indifferent between searching for a job in region a and b depends positively on p_a and negatively on p_b. Since national bargaining makes region a relatively less affordable, more workers decide to stay in region b.

As concerns private sector employment, the model predicts a decrease in the share of private sector workers out the labor force $e_{p,\,i} \equiv E_{p,\,i}/L_i$ in the poorer region, where a centralized negotiation make civil service jobs more attractive (from Lemma 3, $\phi_b^N > \phi_b^R$). The opposite occurs in region a, in which $\phi_a^N < \phi_a^R$.[24]

Finally, Proposition 2 presents a sufficient condition under which a centralized pay system in the public sector raises the unemployment rate in region b (computations are in Appendix C). To grasp the rationale of this result, let consider the equation (6). The unemployment rate in region b, u_b, is influenced by the change in $\theta_{g,\,b}$, that under

[24] Notice that I consider the rate and not the levels of private sector employment. Since a centralized bargaining also lowers (raises) the labor force in region a (b), the sign of the change in the number of private sector workers in both regions cannot be ascertained.

a centralized pay system goes down, and ϕ_b, that goes up (Lemma 3). A lower labor market tightness $\theta_{g,b}$ tends to raise u_b, as it takes more time for a job-seeker to find a public work in region b. On the other hand, the increase in the share of the labor force belonging to the public sector ϕ_b has an ambiguous impact on u_b. Intuitively, a larger ϕ_b raises the unemployment rate if such a change in the composition of the labor force adds more congestions on the public sector labor market than it reduces them in the private sector. This is more likely to occur when, in comparison with the private sector, there are large flows out of public sector employment (δ_g is relatively high, δ_p is relatively low) and small flows into ($f(\theta_{g,b})$ is relatively low, $f(\theta_{p,b})$ relatively high). Indeed, the sufficient condition in Proposition 2 is more likely to be fulfilled if δ_g is high and δ_p low. Moreover, a weaker workers' bargaining power β, by boosting private sector vacancy creation, raises $f(\theta_{p,b})$ and makes the public sector comparatively more congested than the private sector.

6 Collective Bargaining in the Private Sector

Individual bargaining in the private sector is common in the U.S. labor market but quite rare in Europe. In this section, I consider the case of collective negotiation between unions of workers and firms and I show the results of the previous sections hold true even under this scenario.

More precisely, I assume a "moderate" degree of geographic centralization in the private sector pay system, namely that unions negotiate over the wage at regional level. This implies workers' compensation is not decided at a firm level but it is still affected by some macro features of the area in which the firm is located. For simplicity, I assume that workers' (resp. employers') union represents all the workforce (the active firms) in the private sector of a region $i \in \{a, b\}$. Moreover each union behaves in an utilitarian way, caring about the sum of the utilities of its members. The Nash bargaining maximization problem becomes:

$$w_{p,i}^{EU} \;=\; \text{argmax}\; \left[\, E_{p,i} \cdot \left(W_{j,p,i}^{E} \;-\; \cdot W_{j,i}^{U}\right)\,\right]^{\beta} \left[\, E_{p,i} \cdot \left(J_{p,i}^{E} \;-\; J_{p,i}^{V}\right)\,\right]^{1-\beta}\;, \tag{34}$$

with $i \in \{a,\, b\}$. If the negotiation fails, all workers become unemployed while firms need to post job vacancies again.[25]. The notable difference compared to the previous setting is that unions take the negative effect of the wage on the level of employment into account. So, the F.O.C. of equation (34) is:

$$\beta \cdot \frac{\frac{E_{p,\,i}}{p_i} \;+\; \frac{d\,E_{p,\,i}}{d\,w_{p,\,i}} \cdot \left(W_{j,\,p,\,i}^{E} \;-\; W_{j,\,i}^{U}\right)}{E_{p,\,i} \cdot \left(W_{j,\,p,\,i}^{E} \;-\; W_{j,\,i}^{U}\right)} \;=\; (1-\beta) \cdot \frac{\frac{E_{p,\,i}}{p_i} \;-\; \frac{d\,E_{p,\,i}}{d\,w_{p,\,i}} \cdot J_{p,\,i}^{E}}{E_{p,\,i} \cdot J_{p,\,i}^{E}},$$

with $i \in \{a,\, b\}$. Rearranging, one gets:

$$W_{j,\,p,\,i}^{E} - W_{j,\,i}^{U} \;=\; \beta \left[\, J_{p,\,i}^{E} \;+\; W_{j,\,p,\,i}^{E} \;-\; W_{j,\,i}^{U} \,\right] \;+\; J_{p,\,i}^{E}\; \left[\, W_{j,\,p,\,i}^{E} \;-\; W_{j,\,i}^{U} \,\right] \cdot \frac{d\,E_{p,\,i}}{d\,w_{p,\,i}}\, \frac{p_i}{E_{p,\,i}}, \tag{35}$$

with $i \in \{a,\, b\}$. It is easy to see that, if $\frac{d\,E_{p,\,i}}{d\,w_{p,\,i}} = 0$, the F.O.C in (35) coincides with the individual bargaining F.O.C. in (18). Using the expression for $E_{p,\,i}$ in (5) and free-entry zero profit condition (16), I obtain:

$$\frac{d\,E_{p,\,i}}{d\,w_{p,\,i}} \;=\; -\frac{1-\eta}{\eta} \cdot \frac{\delta_p}{\delta_p + f(\theta_{p,\,i})} \cdot \frac{E_{p,\,i}}{y_i \;-\; w_{p,\,i} \;-\; p_k \cdot k} \;<\; 0, \tag{36}$$

with $i \in \{a,\, b\}$. Since unions consider the negative effect of the wage on vacancy creation and employment, the share of the surplus going to workers is decreased by the second term in the RHS of (35) compared to the individual bargaining setting. Of course, firms' part of the surplus is increased by the same extent. Using equations (8),

[25] Notice that the utility of the unemployed people $U_{p,\,i}$ disappears, as their situation does not change whatever the result of the negotiation. Rosen (1997) and, more recently, Hall and Milgrom (2008) question the hypothesis that in a collective negotiation the threat points are identical to the individual bargaining case, pointing that a wage disagreement usually implies a delay in the production, strikes, not massive lay-offs. In another version of the paper, I examine this approach and find no significant difference in terms of the main results of the model.

(13), and (14) and after some computations, one gets the expression for the wage in the collective bargaining case:

$$w_{p,i}^{EU} = \beta\,\eta\,(y_i - p_k \cdot k) \cdot \frac{[\delta_p + f(\theta_{p,i})] \cdot [r + \delta_p + f(\theta_{p,i})]}{\eta\,[r + \delta_p + \beta f(\theta_{p,i})] \cdot [\delta_p + f(\theta_{p,i})] \,+\, (1-\eta)\delta_p}, \tag{37}$$

with $i \in \{a,\, b\}$. Despite being more cumbersome, this expression exhibits the same important features of equation(19): the nominal private sector wage is increasing with labor market tightness $\theta_{p,i}$ and with y_i. Therefore, following the same steps of section 3.4, one can obtain an implicit function determining the equilibrium value of $\theta_{p,i}$ under the collective regional bargaining setting:

$$\mathbb{ZP}_{EU}(\theta_{p,i}) \equiv y_i \,-\, w_{p,i}^{EU}(\theta_{p,i}) \,-\, p_k \cdot k \,-\, \frac{p_k \cdot k}{q(\theta_{p,i})}\,(r \,+\, \delta_p) \,=\, 0\,, \tag{38}$$

with $i \in \{a,\, b\}$. All the other equations of the model (i.e. the F.O.C.s for $\theta_{g,i}$ and the nominal public wage, the no arbitrage condition) remain the same. So it is straightforward to see that all the results of the previous section remain unchanged[26].

6.1 A Remark on the Price-Taking Behavior of the Agents

In this paper unions (both in the private and in the public sector) and the government take the price of the consumption good as given. It is natural to wonder whether the results of the model hold true if such "big agents" internalized the effects of their decisions on the cost of living. The question might appear particularly appropriate in the light of a well-known literature (e.g. Calmfors and Driffil, 1988 and Alesina and Perotti, 1997) that stresses the advantages of a national negotiation compared to a more decentralized one in that, under the former, unions are more conscious of the impact of their wage demands on inflation. However, this line of reasoning does not apply to the framework considered in the present paper. Since the price of the consumption

[26] The only equality that does not hold under regional bargaining is the second equality in (21), as workers' share is no longer equal to $\frac{\beta}{1-\beta} p_k \cdot k \frac{\theta_{p,i}}{p_i}$. This does not affect the results of the paper. Details are available on request.

good is determined at regional level, even under a regional bargaining scheme unions' and governments' choices could have an impact on its value. So, abandoning the price-taking assumption does not add a further distinction between the centralized and the regional public sector pay system, that remains the main objective of the present work.

7 Calibration

The structure of the model may offer further useful insights that unfortunately cannot be derived at the analytical level. So I perform a numerical exercise to study the effects on employment, real wages and inequality resulting from the adoption of either scenario. I take the quarter as unit of time. Data refer to the period 2012 in Italy. This country is an interesting case study. It exhibits huge economic disparities between the North-Center regions and the South ones, the former being 50% richer in terms of average disposable income per capita (ISTAT, 2013). Moreover, no statistically significant difference emerges in nominal public wages paid across regions (Alesina *et al.*, 2001; Dell'Arringa *et al.*, 2007). So the second scenario is the baseline model that I will consider for the parametrization.

Results are summarized in Table 2. The discount rate is fixed at 5% on an annual basis. Elasticity in the final good production functions s is set equal to 2.3, implying a low level of substitution between public and private goods[27]. A sensitivity analysis is performed for this and other parameters. I consider a Cobb-Douglas matching function $M_{n,i} = m \cdot U_{n,i}^{\eta} V_{n,i}^{1-\eta}$, with $i \in \{a, b\}$ and $n \in \{p, g\}$. Matching parameter η is fixed equal to 0.5. The Italian Institute of Statistics (ISTAT, 2013a; 2013b) provides data on the labor force L_i, the employment levels E_i, and the unemployment rates u_i in each region[28]. The number of civil servants in each region, $E_{g,i}$, are obtained via the

[27] If it is not unrealistic to suppose a low degree of substitutability between private and public goods, the case is even stronger for that subset of public goods that are nontradable: police services, justice administration, etc...

[28] I split the Italian regions as follows. In region a I include the North (Piedmont, Valle d'Aosta,

annual report of the Finance Ministry (MEF, 2013).[29]. As respects to the (quarterly) separation rates, I set $\delta_p = 0.018$ and $\delta_g = 0.015$. The latter is taken from MEF (2013). The former is taken from ISTAT (2011), the latest available source on workers' mobility, and it is in the middle ground between the estimates of Hobijn and Sahin (2007) (about 0.0207) and Jolivet *et al.* (2006) (about 0.0133). Once the numerical values for $E_{n,i}$, L_i, and δ_n (with $i \in \{a, b\}$ and $n \in \{p, g\}$) are inserted in the steady state equations (4), it is easy to see that ϕ_i and $\theta_{g,i}$ can be written as explicit functions of the unknown variables $\theta_{p,i}$ and m, the matching coefficient.

The rest of the calibration procedure is presented in detail in Appendix D. In short, rearranging eqs. (33), $\mathbb{ZP}_a(\theta_{p,a}) = 0$, and (29) (evaluated both at $i = a$ and $i = b$), I obtain explicit expressions of $\theta_{p,b}$, $p_k \cdot k$, y_a, and y_b in terms of $\theta_{p,a}$, m, and β. As I make clear in Appendix D, these equations need a low value for β to avoid that y_a and y_b are negative. So I set $\beta = 0.12$. This number is in line with the results of Cahuc *et al.* (2006), that estimate a bargaining power for workers with no managerial tasks between 0 and 0.2.

The equilibrium values of $\theta_{p,a}$ and m are then obtained by solving a system composed by equation (32) and a combination of $\mathbb{ZP}_a(\theta_{p,a}) = 0$ and $\mathbb{ZP}_b(\theta_{p,b}) = 0$. With the numerical values of $\theta_{p,a}$ and m pinned down, all the other variables of the model are then easily derived.

All the figures are presented in Table 2. Notice the tiny difference between y_a and y_b, whose calibrated values are 3.35 and 3.33 respectively. As equation (46) in Appendix D makes clear, this stems from the fact that the value for y_i depends on the levels of employment, and the entry and exit rates in region i. So Italian data on

Lombardy, Trentino, Alto Adige - South Tyrol, Veneto, Fiuli-Venezia Giulia, and Liguria) and the Center (Tuscany, Umbria, and Marche). In region b there are Abruzzo, Molise, Campania, Puglia, Basilicata, Calabria, Sicily, and Sardinia. I decided to exclude Lazio from the computations because its high share of public employment is due to the presence of Rome, the capital, in its territory.

[29]In 2012 in Italy 11.7% of the employees in the North-Center regions were working in the publc sector. In the South the share was 18.5%

the labor market stocks and flows do not allow a great divergence in private sector productivities across regions. However, this does not prevent from getting plausible results in terms of real wage gaps, with civil servants' real earnings in region b being slightly lower (resp. higher) than those obtained by private sector (resp. public sector) workers in region a and workers in the private sector in region b getting instead a much smaller pay.

As respects to the public sector wage premium, from the calibration results I get $w_g/w_{p,a}$ equal to 0.92 and $w_g/w_{p,b}$ equal to 1.7. It is difficult to compare these figures with the empirical estimates in the literature, for the only work that looks at wage differentials across regions is Dell'Arringa *et al.* (2007). They get a public sector wage premium of 1.26 in the South and 1.12 in the North-Center, but their results are questioned for endogeneity issues by more recent research[30]. Despite the calibrated values are different form those obtained by Dell'Arringa *et al.*, they confirm the empirical evidence that wage differentials are more favorable for civil servants in the South than for their peers in the North.

8 Numerical Results

I compare the national bargaining scenario calibrated in the previous section with a regional bargaining setting. Numerical results are summarized in Table 3. The first five rows of this Table confirm the conclusions of Lemma 3 and Propositions 1 and 2. More interestingly, results on output per capita and unemployment tell that the advantages of a regional negotiation seem greater than its costs. If nominal public sector wages are determined at local level, public sector jobs become more attractive in the richer region a. This lowers the recruitment costs for public employers and in

[30]For instance, Ghinetti (2014) finds a value of 9% at national level, not statistically significant once selection effects in sector and education are taken into account. Other papers show a rather small wage premium (see the overview of the literature in Ghinetti, 2014).

turn it reduces the price of the consumption good. The opposite occurs in region b: the cost of living in region a decreases by 2.2%, whereas in region b p_b goes up by 1.1%. The change in the price of the consumption good has an obvious impact on real earnings: pays are up to 8% higher in the richer region. In region b, the fall in real wages is almost negligible in the private sector, but it reaches a -44% decrease in the public sector. In accordance with Proposition 1, the adoption of a regional negotiation pushes the public sector wage premium up in region a and down in region b. Moreover, such changes in real terms affect migration: the labor force in region a increases by 1.6% (or, equivalently, 3.4% of the people in region b migrate in search for better job opportunities).

As concerns the regional unemployment rates, region a experiences a modest increase (from 7.6% to 7.9%) while in region b it plunges from 17.1% to 10.6%. As explained in the comment on Proposition 2, a regional bargaining process reduces unemployment in the poor region if the public sector labor market is highly congested, i.e. the number of job applicants is large compared to the available job vacancies. According to the calibration results, this is the case for region b, as the vacancy/unemployment ratio is 32 times lower in the public sector than in the private one (see Table 2). Therefore, by reducing the attractiveness of a public sector occupation in b and pushing more people to search for a job in the private sector, a regional negotiation leads to a large decrease in the unemployment rate. In turn, such a fall in the number of jobless people drives output per capita up, with an increase by 8% in region b compared to the national bargaining scenario.

As respects to the aggregate variables, the second part of Table 3 shows that choosing a regional bargaining scheme implies a decrease in the national unemployment rate (from 10.6% to 8.6%). Similarly, output per capita results 2.6% higher once a regional public sector negotiation takes place. The average real wage results 1% lower, as the increase in compensations in the region a is outweighed by their fall in region b.

Regional bargaining also seems to have a modest but positive impact on inequality in the entire country: the Gini index remains substantially unchanged, but the wage variance increase by 36%. There are two effects at work that help to understand such results. First, a national negotiation in the public sector pushes real pays of civil servants in the poor region to the level enjoyed by private and public sector employees in the rich region (see Table 2). Workers in the private sector in the poor region are the only ones with a much lower pay. Conversely, the introduction of a regional bargaining negotiation lowers the pay of civil servants in region b by such an extent that the cross-regional gap in real earnings gets wider. Wage variance increases.

We do not observe a similar change in the Gini index because a regional negotiation in the public sector also decreases unemployment, so squeezing the number of the poorest people in the economy (recall that in the model unemployed workers get no monetary subsidy).

8.1 Robustness

I check the robustness of the results presented in section 8 by changing the value of some crucial variables and parameters of the model. In detail, I wonder how sensible the effects of the introduction regional bargaining negotiation are to the share of public sector workers in each region, the degree of substitutability between private and public goods (parameter s), and to the assumption that $y_a > y_b > 1$. Results are presented in Table 4. There are no qualitative differences with respect to the baseline model considered in the previous section. However, the size of the effects differs depending on the sensitivity exercises considered.

More in detail, the first rows in Table 4 show that, as long as the private and public goods are gross substitutes ($s = 1.5$ or $s = 6$), there are no substantial departures from the results obtained in the baseline model with $s = 2.3$. Conversely, the size of the percentage changes in the variables of interest gets much larger if the goods are gross complements ($s = 0.2$). In that case, passing to a regional bargaining scheme in the public sector makes the consumption good much cheaper in region a (-43%)

and very expensive in region b (+22%). In turn, these huge variations in the cost of living produce sizable effects in terms of output per capita, migration, real wage, and inequality. Given the multiplicity of the effects at work in this model, it is difficult to nail down a single reason for such quantitative differences. A possible explanation is the following. Regional negotiation makes the public good in region b more expensive. Taxes raised to finance it, as well as the price of the consumption good, go up. This tax hike is partially offset by a decrease in the demand for the public good, that is greater the more substitutable the goods are. So, it makes sense that under gross complementarity region b experiences a larger increase in the cost of living[31].

In another exercise, I drop the assumption of the theoretical model that public sector is less productive than the private sector and I look at the case $1 > y_a > y_b$. Recall that in the calibration procedure the values for y_a and y_b are not arbitrarily chosen, but they depend on the labor market flow rates and stocks of the corresponding region[32]. So to look at the properties of the model when $1 > y_a > y_b$, I have to consider an economy with other labor market conditions, namely lower levels of employment in both regions[33]. In an economy in which E_a and E_b are 1.5 lower than the calibrated values, we have $y_a = 0.1$ and $y_b = 0.09$. As Table 4 illustrates, the sign of the change in the variables of interest is the same as in the baseline model, but with a larger magnitude. This is probably due to the fact that the elasticity of the price of the consumption good p_i with respect to $\theta_{g,i}$ is decreasing in y_i. So for small values of y_a there is a greater percentage reduction in the cost of living once the increase in the number of workers applying for public sector jobs lowers $\theta_{g,a}$. The greater percentage variation in p_a in turn translates into percentage changes in the other variables of larger

[31]Of course, the same reasoning with opposite sign applies to region a.

[32]See equation (46) in Appendix D.

[33]More in detail, from equation (46) in Appendix D, we have that y_i positively depends on $E_{p,i}$ for $i \in \{a, b\}$. So by decreasing the level of employment E_i while keeping the same fraction of public sector jobs out of the total, I get values of y_i lower than 1.

magnitude.[34]

In the last two exercises, I look at the predictions of the model when the share of civil servants in the labor force is considerably different from the calibrated version. First, I consider an economy in which the share of public sector employees out of total employment in the poor region is about 35% larger than the value for Southern Italy in 2012 (i.e. 25% compared to 18.5%). As illustrated in Table 4, the qualitative predictions of the model do not change but the size of the percentage changes is larger. Because of decreasing returns to labor, a large number of civil servants in the poor region lowers public sector productivity there, widening the gap with the public sector productivity in the rich region. So the price and income effects that the introduction of regional bargaining ensues gets larger.

In a specular exercise, I consider an economy where the shares of civil servants do not differ much across regions. In detail, I impose the fraction of public sector employees out of total employment in region a is 16%, 36% larger than in the baseline model and almost equal to the same ratio in region b. Results are roughly the same of the baseline model.

9 Conclusions

In the last two decades many European countries have experienced a transition towards more decentralized pay systems in the private sector. For some economists this change in the system of industrial relations is key to understanding the economic success of some countries compared to others (e.g. see Dustmann *et al.*, 2014 on Germany's "miracle" of the last fifteen years). Decentralization seems to combine a greater flexibility in response to demand shocks with wage restraints to keep labor costs low.

Since such a trend has not involved the public sector, this paper tries to assess the differences between a centralized and a decentralized wage bargaining in the civil

[34]The same chain of events with opposite sign occurs in region b.

service in a country with relevant regional disparities in productivity.

The first conclusion is that neither wage-setting mechanism is a Pareto improvement with respect to the other. Indeed, a national pay system in the public sector has beneficial effects on residents in the poorer areas of a country and a negative impact on the welfare of residents in the richer ones. This may help to explain the resistance that a change towards more decentralized pay systems often stirs. It also leaves room for interesting political economy reflections this model is not designed to deal with. They are left for future research.

The model also predicts that national pay system impairs the creation of private sector jobs in the poorer regions while boosting it in the richer ones. So, on the one hand centralized wage setting can be viewed as a subsidy towards poor regions. On the other hand, it pushes towards a greater economic diversification within the same country, with rich regions specialized in private sector tradable activities and poor regions more involved in the production of nontradable public goods. The long run effects in terms of growth and human capital are difficult to ascertain, but they may well deserve a further scrutiny.

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Appendix A: Proof of Lemma 1

For the existence and uniqueness of the equilibrium, it is sufficient to find the values of ϕ_i, $\theta_{p,\,i}$, and $\theta_{g,\,i}$ (with $i \in \{a,\, b\}$) and all the remaining endogenous variables of the model are uniquely determined. We have already seen that there is a unique $\theta_{p,\,i}$ that solves $\mathbb{ZP}_i(\theta_{p,\,i}) = 0$. In turn, $\theta_{g,\,i}$ is found via equation (28).

To determine ϕ_i, I rewrite (29) as an implicit function of $\theta_{g,\,i}$ and ϕ_i ($i \in \{a,\, b\}$):

$$\mathbb{G}(\theta_{g,\,i};\phi_i) \;\equiv\; y_i^{1/s} \cdot \left(\frac{\phi_i}{1-\phi_i} \cdot \frac{\delta_p + f(\theta_{p,\,i})}{f(\theta_{p,\,i})} \cdot \frac{f(\theta_{g,\,i})}{\delta_g + f(\theta_{g,\,i})} \right)^{-1/s} \;+$$

$$-\; p_k \cdot k \left[1 + \frac{\delta_g}{q(\theta_{g,\,i})} \;+\; \frac{\eta}{1-\eta} \frac{\delta_g + f(\theta_{g,\,i})}{q(\theta_{g,\,i})} \;+\; \frac{\beta}{1-\beta}\,\theta_{p,\,i} \frac{r + \delta_g + f(\theta_{g,\,i}) - 1}{f(\theta_{g,\,i})} \right] \;=\; 0$$

Notice that $\frac{\partial\,\mathbb{G}_i}{\partial\,\phi_i} \;<\; 0$ and $\frac{\partial\,\mathbb{G}_i}{\partial\,\theta_{g,\,i}} \;<\; 0$ if $r + \delta_g \;<\; 1$, for $i \in \{a,\, b\}$. For the implicit function theorem, we have $\frac{d\,\phi_i}{d\,\theta_{g,\,i}} \;<\; 0$, for $i \in \{a,\, b\}$. For the Inada conditions imposed on the matching function, we also get that $\phi_i \to 0$ as $\theta_{g,\,i} \to +\infty$, for $i \in \{a,\, b\}$. In addition, it can be shown (details are available on request) that as $\phi_i \to 1$ the value of $\theta_{g,\,i}$ tends to a positive finite number, denoted by $\bar{\theta}^G_{g,\,i}$ ($i \in \{a,\, b\}$). It can also be proved that such a value is always lower than the equilibrium value of $\theta_{g,\,i}$ obtained via equation (28). So the model does not exhibit corner solutions. I conclude that there exists a unique $\phi_i \,\in (0,\, 1)$ that solves $\mathbb{G}(\theta_{g,\,i};\phi_i) = 0$ for any value of $\theta_{g,\,i} \;>\; \bar{\theta}^G_{g,\,i}$ and $i \in \{a,\, b\}$. Figure 1 illustrates that.

Point 2 of Lemma 1 is proved as follows. I use equation (28) to express the RHS of (29) as a function of $\theta_{g,\,i}$. Then equation (7) becomes:

$$p_i \;=\; \left\{ 1 \;+\; \left[p_k \cdot k \left(1 + \frac{\delta_g}{q(\theta_{g,\,i})} + \frac{\eta}{1-\eta}\,(\delta_g + f(\theta_{g,\,i}))\, \frac{1 - \beta + \beta\,(r + \delta_g + f(\theta_{g,\,i}))}{q(\theta_{g,\,i})} \right) \right]^{1-s} \right\}^{\frac{1}{1-s}}$$

The expression at the RHS is increasing in $\theta_{g,\,i}$. Since $\theta_{g,\,a} \;>\; \theta_{g,\,b}$, then $p_a \;>\; p_b$.

As regards point 4 of Lemma 1, I show that $\theta_{p,a}/p_a \;>\; \theta_{p,\,b}/p_b$, I express p_i as a function of the RHS of (29). Then substitute $\theta_{p,i}$ via equation (28) to get:

$$\frac{\theta_{p,\,i}}{p_i} \;=\; \left\{ \theta^{s-1}_{p,i} \;+\; \left[p_k \cdot k \left(\frac{1-\eta}{\eta(1-\beta)} \frac{\delta_g + q(\theta_{g,i})}{f(\theta_{g,\,i})(\delta_g + f(\theta_{g,\,i}))} \;+\; \frac{1 - \beta + \beta(r + \delta_g + f(\theta_{g,\,i}))}{(1-\beta) f(\theta_{g,\,i})} \right) \right]^{1-s} \right\}^{\frac{1}{s-1}}$$

This expression is increasing in $\theta_{p,i}$ and $\theta_{g,i}$, so $\theta_{p,a}/p_a > \theta_{p,b}/p_b$.

Appendix B: Proof of Lemma 2

As for the regional bargaining case, to prove that a national bargaining equilibrium exists and it is unique, it is sufficient to show that there exists a 3-tuple $(\theta_{p,i}; \theta_{g,i}; \phi_i)$ for $i \in \{a, b\}$ that satisfy the equations (29), (32), and (33). All the other remaining variables of the model are easily determined once $(\theta_{p,i}; \theta_{g,i}; \phi_i)$ are found.

As concerns $\theta_{p,i}$, I have already shown that it is uniquely determined by the implicit equation $\mathbb{ZP}(\theta_{p,i}) = 0$ for $i \in \{a, b\}$. Moreover, from the previous Appendix we have that there is a unique $\phi_i \in (0, 1)$ that solves $\mathbb{G}(\theta_{g,i}; \phi_i) = 0$ for any value of $\theta_{g,i} > \bar{\theta}^G_{g,i}$ ($i \in \{a, b\}$). Therefore, the existence and uniqueness of the equilibrium is proved if there exists only one couple $(\theta_{g,a} > \bar{\theta}^G_{g,a}; \theta_{g,b} > \bar{\theta}^G_{g,b})$ solving the system composed by equations (32) and and (33). To simplify the notation, I rewrite these equations as implicit functions of the endogenous variables $(\theta_{g,i}; \phi_i)$, for $i \in \{a, b\}$:

$$\Phi(\theta_{g,a}; \phi_a; \theta_{g,b}; \phi_b) \equiv$$
$$\frac{\phi_a L_a}{p_a}\left[\theta_{g,a}(\delta_g + f(\theta_{g,a})) - \frac{(1-\eta)\theta_{p,a}}{\eta(1-\beta)}\right] + \frac{\phi_b L_b}{p_b}\left[\theta_{g,b}(\delta_g + f(\theta_{g,b})) - \frac{(1-\eta)\theta_{p,b}}{\eta(1-\beta)}\right] = 0.$$
$$\Omega(\theta_{g,a}; \theta_{g,b}) \equiv \theta_{p,a}\frac{r + \delta_g + f(\theta_{g,a})}{f(\theta_{g,a})} - \theta_{p,b}\frac{r + \delta_g + f(\theta_{g,b})}{f(\theta_{g,b})} = 0.$$

I proceed in two steps. First, I prove that $\Omega(\theta_{g,a}; \theta_{g,b}) = 0$ describes an increasing relationship in the positive hortant of the $(\theta_{g,a}; \theta_{g,b})$ space. Second, I show that, after substituting ϕ_a and ϕ_b with the corresponding expressions via equations $\mathbb{G}(\theta_{g,a}; \phi_a) = 0$ and $\mathbb{G}(\theta_{g,b}; \phi_b) = 0$, the implicit function $\Phi(\theta_{g,a}; \phi_a; \theta_{g,b}; \phi_b) = 0$ describes a decreasing relationship in the positive hortant of the $(\theta_{g,a}; \theta_{g,b})$ space. Thus, the system admits a unique solution in $(\theta_{g,a}; \theta_{g,b})$.

Let consider the first step. Differentiating $\Omega(\theta_{g,a}; \theta_{g,b}) = 0$ yields to $\frac{\partial\Omega}{\partial\theta_{g,a}} < 0$ and $\frac{\partial\Omega}{\partial\theta_{g,b}} > 0$. For the implicit function theorem we have $\frac{d\theta_{g,b}}{d\theta_{g,a}} > 0$. Moreover, it is easy to see that $(0;0)$ is a solution of $\Omega(\theta_{g,a};\theta_{g,b}) = 0$. I conclude that $\Omega(\theta_{g,a};\theta_{g,b}) = 0$ starts from the origin and it describes a positive relationship in the space $(\theta_{g,a};\theta_{g,b})$.

Let denote with $\phi_i = g(\theta_{g,i})$ the explicit function of $\mathbb{G}(\theta_{g,i};\phi_i) = 0$ for $i \in \{a, b\}$. In the second step, I study the implicit equation $\Phi(\,\theta_{g,a};\, g(\theta_{g,a});\, \theta_{g,b};\, g(\theta_{g,b})\,) = 0$. The implicit function theorem does not help in this case, for it is difficult to check the sign of the derivatives. So I consider another approach. Let define $\mathbb{H}_i(\theta_{g,i}) \equiv \theta_{g,i}\,(\delta_g + f(\theta_{g,i})) - \frac{(1-\eta)\theta_{p,i}}{\eta(1-\beta)}$ and denote by $\bar{\theta}^{\Phi}_{g,i}$ the solution to the equation $\mathbb{H}_i(\theta_{g,i}) = 0$ for $i \in \{a, b\}$. It easy to see that $\frac{\partial\,\mathbb{H}_i}{\partial\,\theta_{g,i}} > 0$, that $\mathbb{H}_i \to +\infty$ as $\theta_{g,i} \to +\infty$, and that $\mathbb{H}_i < 0$ as $\theta_{g,i} \to 0$ for $i \in \{a, b\}$. So $\bar{\theta}^{\Phi}_{g,i}$ exists and it is unique ($i \in \{a, b\}$). It can also be proved (details are available on request) that $\bar{\theta}^{\Phi}_{g,i} > \bar{\theta}^{G}_{g,i}$ for $i \in \{a, b\}$. Notice that the couple $\left(\bar{\theta}^{\Phi}_{g,a};\, \bar{\theta}^{\Phi}_{g,b}\right)$ is a solution of the implicit equation $\Phi(\,\theta_{g,a};\, g(\theta_{g,a});\, \theta_{g,b};\, g(\theta_{g,b})\,) = 0$. In addition, for each $\theta_{g,a} \in \left(\bar{\theta}^{G}_{g,a},\, \bar{\theta}^{\Phi}_{g,a}\right)$, we have $\mathbb{H}_a < 0$, for the function $\mathbb{H}_a$ is increasing and it crosses the horizontal axis at point $\bar{\theta}^{\Phi}_{g,a}$. This implies that the first term at the RHS of $\Phi(\theta_{g,a}; g(\theta_{g,a}); \theta_{g,b}; g(\theta_{g,b})) = 0$ is negative. For $\Phi(\theta_{g,a}; g(\theta_{g,a}); \theta_{g,b}; g(\theta_{g,b}))$ to be equal to 0, the second term at the RHS must be positive. This means that $\mathbb{H}_b > 0$, that is the case for $\theta_{g,b} > \bar{\theta}^{\Phi}_{g,b}$. This is equivalent to say that $\Phi(\theta_{g,a}; g(\theta_{g,a}); \theta_{g,b}; g(\theta_{g,b})) = 0$ describes a negative relationship in the space $(\theta_{g,a};\theta_{g,b})$.

For the results in Step 1 and 2, there exists a unique positive couple $(\theta_{g,a};\theta_{g,b})$ that verify the equations $\Omega(\theta_{g,a};\theta_{g,b}) = 0$ and $\Phi(\theta_{g,a}; g(\theta_{g,a}); \theta_{g,b}; g(\theta_{g,b})) = 0$. Figure 2 illustrates that. For the properties of $\mathbb{G}(\,\theta_{g,i};\, \phi_i\,) = 0$, $\theta_{g,i}$ determines the equilibrium value of ϕ_i ($i \in \{a, b\}$).

The proof of point (1) of Lemma 2 follows closely the one presented in Lemma 1. Because of higher productivity, labor market tightness in the private sector is higher in region a. In turn, for equation (33), one also gets $\theta_{g,a} > \theta_{g,b}$.

To check that $p_a > p_b$ (point 2 of Lemma 2) differentiating equation (7) yields:

$$\frac{d\,p_i}{d\,\theta_{g,i}} = \frac{\partial\,p_i}{\partial\,\phi_i} \cdot \frac{\partial \phi_i}{\partial \theta_{g,i}} + \frac{\partial p_i}{\partial \theta_{g,i}},$$

for $i \in \{a, b\}$ and in which $\partial\phi_i/\partial\theta_{g,i}$, obtained by totally differentiating equation $\mathbb{G}(\theta_{g,i};\, \phi_i) = 0$, is negative (see Appendix A). After some algebra one gets:

$$\frac{d\,p_i}{d\,\theta_{g,i}} = \frac{p_i^s}{\theta_{g,i}} \cdot \left(\frac{\phi_i}{y_i(1-\phi_i)} \cdot \frac{f(\theta_{g,i})}{\delta_g + f(\theta_{g,i})} \cdot \frac{f(\theta_{p,i}) + \delta_p}{f(\theta_{p,i})} \right) \cdot A_i > 0 \tag{39}$$

with

$$A_i \equiv p_k \cdot k \left[\eta\,\theta_{g,i} + \frac{\eta^2}{1-\eta} \frac{\delta_g + f(\theta_{g,i})}{q(\theta_{g,i})} + \frac{\eta\,\delta_g}{q(\theta_{g,i})} + \frac{\beta \cdot \theta_{p,i}}{1-\beta} \cdot \frac{(1-\eta)(1-r-\delta_g)}{f(\theta_{g,i})} \right] > 0$$

The positive sign of such a derivative implies that $p_a > p_b$ as $\theta_{g,a} > \theta_{g,a}$.

Finally, an inspection of equation (19) makes clear that $w_{p,a} > w_{p,b}$ because $\theta_{p,a} > \theta_{p,b}$ and $y_a > y_b$.

Appendix C: Computations for Lemma 3 and Propositions 1 and 2

As concerns point (1) of Lemma 3, I re-write the no arbitrage condition (33) as follows:

$$\frac{\theta_{p,a}}{\theta_{g,a}} \frac{r + \delta_g + f(\theta_{g,a})}{q(\theta_{g,a})} = \frac{\theta_{p,b}}{\theta_{g,b}} \frac{r + \delta_g + f(\theta_{g,b})}{q(\theta_{g,b})} \tag{40}$$

The second factor at the LHS is greater than the second factor at the RHS, because they are both increasing functions of $\theta_{g,i}$ ($i \in \{a, b\}$) and, for Lemma 2, $\theta_{g,a} > \theta_{g,b}$. For the equation (40) to hold, it must be that $\frac{\theta_{p,a}}{\theta_{g,a}} < \frac{\theta_{p,b}}{\theta_{g,b}}$. This inequality and the fact that $\theta_{g,a} > \theta_{g,b}$ allow to get the sign of the terms inside the square brackets in equation (32):

$$\delta_g + f(\theta_{g,a}) - \frac{(1-\eta)}{\eta\,(1-\beta)} \frac{\theta_{p,a}}{\theta_{g,a}} > 0 > \delta_g + f(\theta_{g,b}) - \frac{(1-\eta)}{\eta\,(1-\beta)} \frac{\theta_{p,b}}{\theta_{g,b}} \tag{41}$$

Under the regional bargaining scenario, both expressions in (41) are equal to zero (see equation 28). Since $\theta_{p,i}$ for $i \in \{a, b\}$ takes the same value under both scenarios, I conclude that the $\theta_{g,a}$ (resp. $\theta_{g,b}$) is greater (resp. lower) under national bargaining than under the regional case.

Point (2) of Lemma 3 comes directly from the fact that for equation (29) ϕ_i is a decreasing function of $\theta_{g,i}$, for $i \in \{a,\, b\}$ (see Appendix A). Thus, under national bargaining, we jointly have a higher $\theta_{g,a}$ (resp. lower $\theta_{g,b}$) and a lower ϕ_a (resp. higher ϕ_b).

As concerns point (1) of Proposition 1, recall from equation (39) that p_i is an increasing function of $\theta_{g,i}$ (after taking into account that $\phi_i \;=\; g(\theta_{\theta_{g,\,i}})$. This implies that under national bargaining p_a is greater and p_b is lower compared to the regional bargaining scenario.

The proof of point (3) of Proposition 3 is the following. Differentiating equation (6), one gets:

$$\frac{d\,u_b}{d\,\theta_{g,\,b}} \;=\; \frac{d\,\phi_b}{d\,\theta_{g,\,b}} \left[\frac{\delta_g}{\delta_g \,+\, f(\theta_{g,\,b})} \;-\; \frac{\delta_p}{\delta_p \,+\, f(\theta_{p,\,b})}\right] \;-\; \frac{(1-\eta)\,\delta_g\,\phi_b\,q(\theta_{g,\,b})}{(\delta_g \,+\, f(\theta_{g,\,b}))^2},$$

in which $d\,\phi_b/d\,\theta_{g,\,b}$ is obtained by totally differentiating $\mathbb{G}(\,\theta_{g,\,b};\,\phi_b\,) \;=\; 0$ and it is negative (see Appendix A). So $d\,u_b/d\,\theta_{g,\,b} \;<\; 0$ if the term inside the square brackets is positive. In turn, this is the case if $\delta_p\,f(\theta_{g,\,b}) \;<\; \delta_g\,f(\theta_{p,\,b})$. With a Cobb-Douglas matching function, we can write that

$$\frac{d\,u_b}{d\,\theta_{g,\,b}} \;<\; 0 \;\;\longleftrightarrow\;\; \frac{\theta_{p,\,b}}{\theta_{g,\,b}} \;>\; \left(\frac{\delta_p}{\delta_g}\right)^{\frac{1}{1-\eta}} \tag{42}$$

From (41) we know that $\frac{\theta_{p,\,b}}{\theta_{g,\,b}} \;>\; \frac{\eta\,(1-\beta)}{1-\eta}\;(\delta_g + f(\theta_{g,b}))$. Using equation (42), a sufficient condition for $d\,u_b/d\,\theta_{g,\,b} \;<\; 0$ is

$$\delta_g\,\frac{\eta\,(1-\beta)}{1-\eta} \;>\; \left(\frac{\delta_p}{\delta_g}\right)^{\frac{1}{1-\eta}}.$$

Rearranging, we get the same expression in point (3) of Proposition 2. Since $\theta_{g,\,b}$ is lower under national bargaining than under regional bargaining, the respect of this condition implies that the unemployment rate is higher under under national bargaining than under regional bargaining.

Appendix D: Calibration procedure

Rearranging the first steady-state equation in (4), I get the following expressions for ϕ_i:

$$\phi_i \;=\; 1 \;-\; \frac{E_{p,\,i}\;(\delta_p \;+\; f(\theta_{p,\,i}))}{L_i \cdot f(\theta_{p,\,i})}, \tag{43}$$

for $i \in \{a,\, b\}$. Inserting this expression into the second steady state equality in 4 and considering a Cobb-Douglas matching function, I get:

$$\theta_{g,i} \;=\; \left(\frac{1}{m} \cdot \frac{E_{g,\,i}\,\delta_g\,f(\theta_{p,\,i})}{U_i \cdot f(\theta_{p,\,i}) \;-\; \delta_p \cdot E_{p,\,i}}\right)^{\frac{1}{1-\eta}}, \tag{44}$$

for $i \;\in \{a,\, b\}$. So ϕ and $\theta_{g,i}$ only depend on the unknown variables m and $\theta_{p,i}$ for $i \in \{a,\, b\}$.

Rearranging $\mathbb{ZP}_i(\theta_{p,i}) \;=\; 0$ for $i \;\in \{a,\, b\}$, I get:

$$p_k \cdot k \;=\; \frac{(1-\beta)y_a q(\theta_{p,a})}{r+\delta_p+(1-\beta)q(\theta_{p,a})+\beta f(\theta_{p,a})} \;=\; \frac{(1-\beta)y_b q(\theta_{p,b})}{r+\delta_p+(1-\beta)q(\theta_{p,b})+\beta f(\theta_{p,b})} \tag{45}$$

Inserting these expressions for $p_k \cdot k$ into equation (29), I derive an explicit expression for y_i:

$$y_i \;=\; \left(\frac{\left(\frac{E_{g,i}}{E_{p,i}}\right)^{-\frac{1}{s}}\;[r+\delta_p+q(\theta_{p,i})(1-\beta)+\beta f(\theta_{p,i})]}{(1-\beta)q(\theta_{p,i})C_i \;+\; \beta\frac{f(\theta_{p,i})}{f(\theta_{g,i})}\,[r+\delta_g+f(\theta_{g,i})-1]}\right)^{\frac{s}{s-1}} \tag{46}$$

with

$$C_i \;\equiv\; 1 + \frac{\delta_g}{q(\theta_{g,\,i})} \;+\; \frac{\eta}{1-\eta} \cdot \frac{\delta_g + f(\theta_{g,i})}{q(\theta_{g,i})},$$

for $i \;\in \{a,\, b\}$. So y_i depends on the unknown variables m, $\theta_{p,i}$, and β. Notice that because of the -1 term at the denominator, y_i might be negative[35]. To avoid that, I need to impose a low value for β, specifically 0.12.

[35] Recall that the calibrated values for δ_g and r are much smaller than 1.

Inserting equations (43) and (44) into the no arbitrage condition (33) and doing some algebra, I get:

$$
\begin{aligned}
&\frac{1}{\delta_g E_{g,a}} \cdot \left[\theta_{p,\,a}\,(rU_a + \delta_g\,(U_a + E_{g,\,a})) \,-\, m^{-1}(r+\delta_g)\delta_p E_{p,\,a}\,\theta^{\eta}_{p,a}\right] \,= \\
&\frac{1}{\delta_g E_{g,b}} \cdot \left[\theta_{p,\,b}\,(rU_b + \delta_g\,(U_b + E_{g,\,b})) \,-\, m^{-1}(r+\delta_g)\delta_p E_{p,\,b}\,\theta^{\eta}_{p,b}\right]
\end{aligned}
\tag{47}
$$

It is easy to see that with $\eta = 0.5$, this equation can be easily rearranged as an explicit function of $\theta_{p,b}$ in terms of m and $\theta_{p,a}$.

So far, I used equations (4), (29), and (33) to respectively express ϕ_i, $\theta_{g,i}$, y_a, y_b, and $\theta_{p,b}$ as explicit functions of m and $\theta_{p,a}$. Inserting these functions into equation (32) and the second equality in (45), I get a system with two unknowns, m and $\theta_{p,a}$. The solutions of this system are $m = 3.68$ and $\theta_{p,a} = 0.048$. Once these two variables are pinned down, all the remaining variables are easily obtained. The threshold value λ^* is derived via equation (22). Finally, I get the value of λ by solving equation (12) under the hypothesis of a uniform distribution $h(.)$:

$$
L_a \,=\, (\,\lambda \,-\, \lambda^*\,)\,\frac{L}{2\lambda}
$$

Analytical Results						
Differences across regions						
VARIABLES	$\theta_{p,a} > \theta_{p,b}$	$\theta_{g,a} > \theta_{g,b}$	$p_a > p_b$	$w_{p,a} > w_{p,b}$	$w_{g,a} > w_{g,b}$	$L_a > L_b$
REGIONAL	yes	yes	yes	yes	yes	yes*
NATIONAL	yes	yes	yes	yes	-	?

From regional to national bargaining in the public sector										
VARIABLES	$\theta_{p,a}$	$\theta_{p,b}$	$\theta_{g,a}$	$\theta_{g,b}$	ϕ_a	ϕ_b	p_a	p_b	$\frac{w_{n,a}}{p_a}$	$\frac{w_{n,b}}{p_b}$
RESULTS	$=$	$=$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
VARIABLES	$rW^U_{j,a}$	$rW^E_{j,n,a}$	$rW^U_{j,b}$	$rW^E_{j,nb}$	L_a	L_b	u_b	$e_{p,a}$	$e_{p,b}$	
RESULTS	$-$	$-$	$+$	$+$	$-$	$+$	$+^{**}$	$+$	$-$	
From national to regional bargaining in the public sector										
VARIABLES	$\theta_{p,a}$	$\theta_{p,b}$	$\theta_{g,a}$	$\theta_{g,b}$	ϕ_a	ϕ_b	p_a	p_b	$\frac{w_{n,a}}{p_a}$	$\frac{w_{n,b}}{p_b}$
RESULTS	$=$	$=$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$
VARIABLES	$rW^U_{j,a}$	$rW^E_{j,n,a}$	$rW^U_{j,b}$	$rW^E_{j,nb}$	L_a	L_b	u_b	$e_{p,a}$	$e_{p,b}$	
RESULTS	$+$	$+$	$-$	$-$	$+$	$-$	$-^{**}$	$-$	$+$	

Table 1: Analytical results. Subscript $n \in \{p, g\}$. * If function $h(.)$ is symmetrical along the vertical axis. ** If the condition in Proposition 2, point 3, is fulfilled.

Calibration			
Variables	Values	Interpretation	Source
r	0.0122	discount rate	5% on annual basis.
L_a, E_a	15.66, 14.4	labor force and employment in a	ISTAT (2013b)
L_b, E_b	7.46, 6.18	labor force and employment in b	ISTAT (2013b)
$E_{g,a}$, $E_{g,b}$	1.68, 1.13	public employment in a and b	MEF (2013)
δ_p,	1.8%	separation rate in sector p	ISTAT (2010)
δ_g	1.5%	separation rate in sector g	MEF (2013)
ϕ_a, ϕ_b	0.11, 0.24	share of the labor force in g	eqs. (43)
y_a, y_b	3.35, 3.33	private sector productivities	eqs. (46)
β	0.12	workers' bargaining power	Cahuc *et al.* (2006)
m	3.69	matching coefficient	eq. (32)
$p_k \cdot k$	3.30	cost of capital	$\mathbb{ZP}(\theta_{p,a}) = 0$
$\theta_{p,a}$	0.048	tightness in sector p, region a	eq. $\mathbb{ZP}(\theta_{p,b}) = 0$
$\theta_{p,b}$	0.025	tightness in sector p, region b	eq. (47)
$\theta_{g,a}$, $\theta_{g,b}$	0.215, 0.00078	tightness in sectors g	eq. (44)
$w_{p,a}/p_a$, $w_{p,b}/p_b$	0.028, 0.015	real wages in p	eq. (19)
w_g/p_a, w_g/p_b	0.026, 0.027	real wages in g	eq. (31)
p_a, p_b	0.89, 0.85	prices of the consumption good	eq. (7)
λ^*	-0.011	marginal worker's utility parameter	eq. (22)
λ	0.032	preference for location	Data on L_a and L_b

Table 2: Calibration procedure. Labor force data in millions.

Comparison between the National and Regional Bargaining		
Variables	**Region** a	**Region** b
Cost of living	-2.2	1.1
Share of public employment	33.8	-11.8
Real wage in the private sector	2.2	-1.1
Real wage in the public sector	7.9	-44.6
Labor force	1.6	-3.4
Unemployment rate*	0.2	-6.5
Output per capita	0.2	8.1
Public sector wage premium	5.6	-43.9

Aggregate Variables	
Unemployment rate*	-2
Output per capita	2.6
Average real wage	-1.0
Average public sector real wage	-9.3
Gini index	0.1
Highest to lowest wage in the economy	8.2
Wage variance	36

Table 3: Simulation Results. Percentage changes when regional bargaining is introduced in the public sector. * For the unemployment rate the change is in % points.

Robustness analysis								
Regional variables								
MODEL	${u_a}^*$	${u_b}^*$	p_a	p_b	Y_a/L_a	Y_b/L_b	L_a	L_b
Baseline	0.2	−6.5	−2.2	1.1	0.2	8.1	1.6	−3.4
$s = 1.5$	0.25	−6.7	−3.5	1.8	0.5	8.1	2.6	−5.5
$s = 6$	0.25	−6.6	−0.8	0.4	−0.1	8.0	0.6	−1.2
$s = 0.2$	0.4	−6.4	−43.0	22.0	7.2	8.6	17.0	−35.7
$y_b < y_a < 1$	0.25	−6.5	−25.3	13.2	4.4	8.1	23.4	−49.2
share of civil servants in b 25%	0.2	−7.1	−4.9	2.4	1.9	8.7	4.4	−9.3
share of civil servants in a 16%	0.4	−6.5	−0.7	0.4	−0.3	8.0	0.5	−1.1

Aggregate Variables				
MODEL	u^*	real wage	output	Gini
Baseline	−2.0	−1.0	2.6	0.1
$s \in (1.5, 6)$	−2.0	[−2.3, 0.3]	[2.4, 2.8]	[−0.5, 0.5]
$s = 0.2$	−2.1	44.8	9.0	−15.6
$y_b < y_a < 1$	−2.3	26.4	6.8	28.7
share of civil servants in b 25%	−2.2	1.2	3.8	1.2
share of civil servants in a 16%	−1.9	−2.2	2.2	−0.3

Table 4: % change in the variables of interest once regional bargaining in the public sector is introduced. Results under different scenarios. * For the unemployment rate the change is in % points.

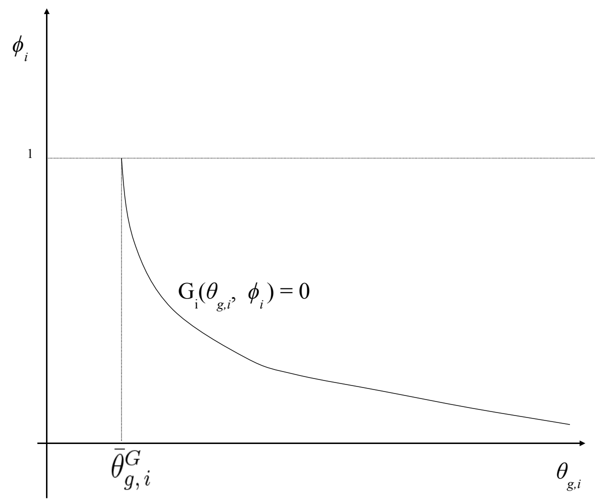

Figure 1: The implicit function $\mathbb{G}(\theta_{g,i};\, \phi_i) = 0$ for $i \in \{a,\, b\}$

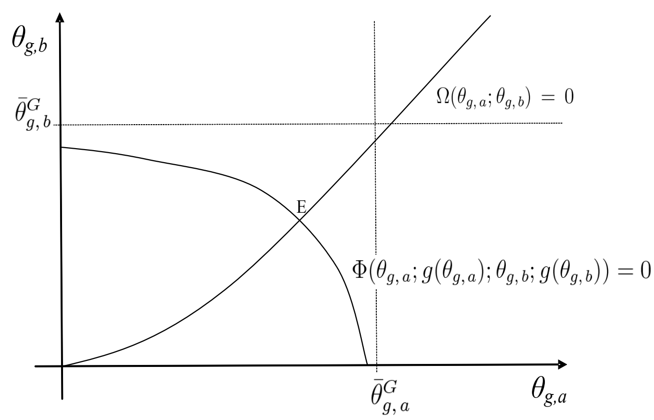

Figure 2: Equilibrium values of $\theta_{g,a}$ and $\theta_{g,b}$