Vehicle Fuel-Efficiency Choices, Emission Externalities, and Urban Sprawl

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Abstract

This paper shows that a city where both a congestion externality and an externality from greenhouse gas emissions are corrected by efficient policies is more compact than the laissez-faire equilibrium city. Motivated by recent empirical studies showing a positive relationship between population density and vehicle fuel-efficiency, the consumer is assumed to choose vehicle fuel-efficiency jointly with housing consumption and residential location. By incorporating the consumer’s vehicle choice into the urban spatial model, we can represent the total amount of vehicle emissions released by the city residents. We first establish the well-known result that the congestion externality as a source of market failure is associated with excessive urban sprawl. We then show that vehicle emissions are an additional source of market failure, which also leads to excessive urban sprawl. The source of excessive sprawl arising from the emission externality is the use of larger and less-fuel efficient vehicles in more sprawled cities, which is different from that of the congestion externality. We also analyze the effect of the Corporate Average Fuel Economy (CAFE) standards on urban spatial structure and its efficacy as a second-best tool for correcting the emission externality.

Keywords: urban sprawl; vehicle fuel-efficiency; emission externality; congestion

JEL Classification Numbers: R14, R41, Q53

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1 Introduction

The phenomenon of urban sprawl, which characterizes the land development pattern in the US since 1950, has become a major concern for policymakers in many countries, especially in the US. While the standard urban model suggests that cities’ spatial expansion is a natural consequence of changes in several fundamental economic forces,\(^1\) the expansion may be excessive compared to the socially desirable level. Specifically, cities’ spatial expansion is excessive when the operation of sprawl-inducing forces involves market failures or equivalently when the urban developer fails to fully account for the social cost of suburban development (Brueckner (2000; 2001), Brueckner and Helsley (2011)).

Among other sources, traffic congestion is the most studied source of market failures associated with urban sprawl.\(^2\) With unpriced traffic congestion, the social cost of commuting exceeds the private cost because every driver on the road slows down other drivers while this external congestion cost is ignored by himself. Commute trips are thus excessively long, and the market equilibrium would generate the city that is too spread out compared to the socially desirable level. Consistent with this intuition, the congested-city models suggest that a city where congestion externalities are internalized by congestion tolls is denser and spatially smaller than the other city where congestion externalities are left uncorrected (e.g., Arnott (1979), Wheaton (1998), Brueckner (2007)).

While traffic congestion is a widely-recognized vehicle-related externality leading to excessive urban sprawl, there are also other kinds of vehicle-related externalities in the urban economy. These externalities include global air pollution (especially greenhouse gas emissions), local air pollution, the country’s oil dependence, and traffic accidents (Parry et al. (2007)). Like congestion externalities, these kinds of externalities, especially air pollution,

\(^1\)The fundamental forces leading to urban spatial growth include rising incomes, lowered transport cost, and falling agricultural rents (Wheaton (1974), Brueckner (1987)).

\(^2\)For example, Wheaton (1998) and Brueckner (2007) investigate the effect of traffic congestion in a closed-city model. Anas and Pines (2008) extend the analysis to a system of congested cities. Beside traffic congestion, other sources of market failure leading to excessive sprawl of urban areas include the failure to account for the amenity value of open space and the failure to fully account for the social cost of infrastructure development (Brueckner (2000; 2001), Brueckner and Helsley (2011)).
are also potentially related to the phenomenon of urban sprawl because longer commutes and increased passenger vehicle travels induced by low-density suburban development would mean greater air pollution released by the city residents. This hypothesis is supported by a number of empirical papers, which find that lower population density of the resident’s neighborhood increases vehicle mileage traveled and energy consumption (e.g., Boarnet and Crane (2001), Bento et al. (2005), Brownstone and Golob (2009), Kim and Brownstone (2013)). Moreover, the increased vehicle travels are responsible for a significant portion of the increased greenhouse gas emissions (Glaeser and Kahn (2010), Glaeser (2011)). Therefore, according to these empirical papers, real-world cities are too spread out in the sense that such sprawled cities emit too much air pollution.\(^3\) While vehicle emissions have been recognized as an important source of urban externalities in the empirical studies, urban economic models analyzing this kind of externality are relatively rare compared to the large literature on urban traffic congestion.\(^4\) This paper fills this gap by analyzing both the congestion and the emission externalities in an urban economic model framework to ask whether the optimal city is more compact than the laissez-faire city.

We treat vehicle fuel-efficiency as a key variable in analyzing the emission externality because vehicle fuel-efficiency (or vehicle size and weight) chosen by the city residents is a key determinant of the city’s emission level. Specifically, the city’s emission amount will be greater as the consumers choose bigger and less-fuel efficient vehicles, holding their vehicle utilization levels fixed.\(^5\) Indeed, vehicle fuel-efficiency has been recognized as an important policy target by the government in its goal of reducing greenhouse gas emissions. For ex-

\(^3\)While a majority of urban economists are in favor of high density, based on the empirical effect of density on the environment (e.g., Glaeser (2011)), Gaigné, Riou, and Thisse (2012) argue that this conclusion is not always true if the general equilibrium effect of firms’ and consumers’ location choices on the emission level is taken into account.

\(^4\)An exception is Riley (1974), who considers local pollution and its interaction with traffic congestion. A recent paper by Hirte and Tscharaktschiew (2010) also investigates the effect of congestion tolls and emission charges on urban spatial structure in a polycentric city model framework, relying on numerical simulations.

\(^5\)Fatal traffic accidents are another example of vehicle externalities that are greater as the consumer’s vehicle size is greater. There is recent empirical evidence that the probability of committing a fatal traffic accident is significantly greater for heavier and larger vehicles (Anderson and Auffhammer (2014), Van Ommeren et al. (2013)).
ample, the US government has been implementing various energy policies for vehicles, such as fuel taxes, vehicle fuel-efficiency standards, and financial subsidies and penalties for the purchase of high- and low- efficiency vehicles.\(^6\) Along the same lines, we also treat vehicle fuel-efficiency as a key variable in analyzing vehicle emission externalities.

There is another group of relevant empirical studies that motivate us to treat vehicle fuel-efficiency as a key variable. These empirical papers suggest that consumers’ vehicle fuel-efficiency choices (or emissions per vehicle) interact with their location and housing consumption choices. Specifically, consumers residing in less-dense suburban areas tend to choose less fuel-efficient vehicles than those located in denser areas, controlling for the consumers’ other aspects such as incomes (Brownstone and Golob (2009), Kim and Brownstone (2013)).\(^7\) According to these papers, consumers living in lower-density neighborhoods tend to emit disproportionate air pollution because they not only drive more but also choose less fuel-efficient vehicles. This suggests that urban expansion will be inefficient unless vehicle emission externalities are corrected by efficient policies.

Our model provides a theoretical framework to see whether this intuition is correct. In the model, the consumer is assumed to choose vehicle fuel-efficiency jointly with housing consumption, conditional on her residential location, and as a result, the empirical relationship between population density and vehicle fuel-efficiency emerges in equilibrium.\(^8\) By endogenizing consumers’ vehicle fuel-efficiency choices in this way, we are able to represent the total amount of vehicle emissions released by the city residents. The city’s total vehicle emission is proportional to the residents’ aggregate fuel consumption, more accurately to the weighted summation of residents’ commute distances with weights set at vehicle sizes

\(^6\)See Small (2012) for discussion of the costs and effectiveness of these energy policies.

\(^7\)Similar vehicle-choice patterns are found in West (2004) and Fang (2008). Note that these studies indicate a negative relationship between fuel efficiency and vehicle usage when these variables are interacted with residential density. But, holding residential density fixed, improved fuel efficiency may cause additional travel by reducing the monetary cost of travel. Researchers have long estimated this “rebound-effect” (e.g., Small and Van Dender (2007)).

\(^8\)Kim (2012) also considers a similar framework, where the consumer chooses vehicle size jointly with housing consumption. But, there is no efficiency analysis in Kim (2012) while it is the main focus of this paper.
(capturing fuel-inefficiency). After analyzing the consumer’s problem, we next turn to the social planner’s problem to investigate how the social planner’s choices are different from the consumers’ choices.

We first establish that a city where congestion externalities are internalized via appropriate congestion tolls is more compact than the laissez-faire city. We then incorporate the vehicle emission externality into the model to see how the result would be modified. We show that under the presence of both congestion and emission externalities, the optimal city, where both kinds of externalities are corrected by efficient policies, is more compact than the city where the externalities are left uncorrected. The sprawl effect of the emission externality does not rely on the effect of the congestion externality, which implies that the emission externality is an independent source of market failures leading to excessive urban sprawl. While both the congestion and the emission externalities are associated with excessive urban sprawl, the source of this outcome is different between the two cases. Under traffic congestion, urban sprawl is undesirable because longer commute distances induced by urban sprawl generate more external congestion costs. Meanwhile, under emission externalities, urban sprawl is undesirable because residents in more sprawled cities tend to use excessively larger and less-fuel efficient vehicles, emitting disproportionate air pollution.

In our model, the first-best optimal policy for correcting the emission externality is vehicle fuel taxes. But, we also analyze the effect of an alternative vehicle fuel-efficiency regulation, in particular Corporate Average Fuel Economy (CAFE) standards in the US, on the variables of interest including land-use patterns, emission levels, the consumer welfare. The analysis shows that the CAFE regulation reduces the city’s spatial size and increases consumer welfare from the laissez-faire equilibrium, but not by as much as the first-best optimal fuel taxes. There have been many papers to evaluate the efficiency implications of various second-best anti-sprawl policies. However, the emission externality is additionally considered in our paper, and therefore this is to best of our knowledge the first attempt to

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9For example, Bento et al. (2006) compare the effects of various policies such as urban growth boundaries, development taxes, property taxes, and fuel taxes on efficiency and on land-use patterns.
evaluate the effect of the CAFE regulation as a second-best anti-sprawl policy in a spatial framework.

The rest of the paper is organized as follows. Section 2 proposes the model and analyzes the laissez-faire equilibrium. Section 3 analyzes the central planner’s problem to show whether traffic congestion and vehicle emissions are sprawl-inducing externalities. Section 4 characterizes the equilibrium under the policy of the CAFE standards. Section 5 provides numerical examples of various policy regimes to numerically investigate the effects of various policies. Finally, Section 6 concludes.

2 The model

2.1 The setup

We adopt a variant of the simple two-zone city framework that is also used in Brueckner and Helsley (2011). The city is monocentric and has two zones: central zone, denoted by $c$, and suburban zone, denoted by $s$. Land area of the central zone is normalized to unity. The suburban zone is comprised of developed land and potentially developable open space. There is the central business district (CBD) at the left end of the city. The central residents must cross the central bridge while the suburban residents must cross both the central bridge and the suburban bridge to reach the CBD. The two bridges have the same length and are congestible. Figure 1 shows the regional map of the city. The city is closed, which means that the city’s total population is exogenously fixed.

[Figure 1 about here]

Only passenger vehicle travel is available for the city residents. For simplicity, commuting cost within a zone is assumed to be zero while the cost for crossing a bridge is positive. So, residents within a zone are homogeneous while central- and suburban- residents differ by the number of bridges they cross.

Commuting costs (costs for crossing the bridges) are comprised of congestion-unrelated
costs, denoted by $f$, and congestion costs, denoted by $I$. The congestion-unrelated cost ($f$) is monetary fuel cost that is proportional to distance traveled. The consumer’s choice of $f$ is equivalent to her choice of vehicle size because larger vehicles are typically less fuel-efficient and therefore are more costly. So, $f$ indicates fuel-inefficiency, and we often call $f$ vehicle size below.$^{10}$

The congestion cost ($I$) captures driving inconvenience and time cost associated with traffic congestion. $I$ also captures part of fuel costs wasted due to traffic congestion. Following an approach similar to that of Kim (2012), congestion cost ($I$) is assumed to depend on $f$ and $Q$, where $f$ captures vehicle size and $Q$ is traffic flows on a bridge.$^{11}$ Congestion costs are assumed to be higher as traffic flows are higher, so that $\partial I/\partial Q > 0$. Also, $\partial I/\partial f < 0$ holds, because a larger car offers a greater driving convenience for a given level of traffic flows, but at a diminishing rate, so that $\partial^2 I/\partial f^2 > 0$. But as roads become congested, this advantage of large cars lessens because of the difficulty of maneuvering in heavy traffic, so that $\partial^2 I/\partial f \partial Q > 0$ (making $\partial I/\partial f$ less negative). This cross-partial derivative also indicates that $\partial I/\partial Q$ increases with vehicle size $f$, which means that additional fuel costs wasted due to congestion are higher as the vehicle gets larger and less fuel-efficient.

To summarize, it is assumed that $I = I(f, Q)$, with $\partial I/\partial Q > 0$, $\partial I/\partial f < 0$, $\partial^2 I/\partial f^2 > 0$, and $\partial^2 I/\partial f \partial Q > 0$. Under the maintained assumptions, the central commuter’s total annual

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$^{10}$In effect, we are assuming that the cars in the two zones have the same fixed cost (cost for purchasing a vehicle, which is normalized to zero) but only differ in fuel efficiency and thus in operating cost. If capital cost were considered, the congestion-unrelated cost ($f$) would be less than proportional to distance traveled, so the coefficient on $f_s$ in (2) would be smaller than 2. But, it can be shown that the results of the paper, notably Lemma 1, would be unchanged with this modification. It is more straightforward in the subsequent analysis to focus on the operating cost of travel, which is proportional to distance traveled and to vehicle fuel-efficiency.

$^{11}$Kim (2012) introduces a similar form of commuting cost, but his version of congestion cost is driving inconvenience that depends on population density instead of traffic flows. Anas and Hiramatsu (2012)’s commuting-cost formulation is also similar to ours. However, instead of being comprised of congestion and congestion-unrelated costs, their commuting cost is composed of monetary and time-cost parts. Like the congestion-cost component in our formulation, they assume non-separability of travel speed and vehicle size in the monetary cost component, so the interaction effect of travel speed and vehicle size on commuting cost exists.
commuting cost, denoted by $t_c$, is given by

$$t_c = f_c + I(f_c, N), \quad (1)$$

where $f_c$ denotes the central resident’s choice of $f$ and $N$ denotes the total population in the city. Note that the traffic flow on the central bridge is $N$ because all residents (including both the central residents and the suburban residents) cross that bridge.

For the suburban resident, who travels twice as far as the central resident, the congestion-unrelated cost (i.e., cost that is proportional to distance traveled) is $2f_s$, where $f_s$ represents vehicle size chosen by the suburban resident. Recognizing the different levels of traffic on the two bridges, the total congestion cost for the suburban resident is given by $I(f_s, N) + I(f_s, n_s)$, where $n_s$ denotes the number of suburban residents, i.e., the traffic flow on the suburban bridge. Summing up, the suburban commuter’s total annual commuting cost, denoted by $t_s$, is given by

$$t_s = 2f_s + I(f_s, N) + I(f_s, n_s). \quad (2)$$

Consumer preferences depend on housing consumption, denoted $q$, and consumption of a non-housing composite good, denoted $e$. For analytical tractability, preferences are assumed to be quasi-linear, with the utility function given by $u = e + v(q)$, where $v’ > 0$ and $v’’ < 0$. All consumers earn incomes of $y$ by commuting to the CBD. Letting $p_i$ denote the price per unit of housing in zone $i$ ($\in \{c, s\}$), the budget constraint of a consumer in zone $i$ is given by $e_i + p_i q_i = y - t_i$, where lower subscripts imply the choice made by the consumer in zone $i$ and where the price of the non-housing good ($e$) is normalized to unity. By substituting $e_c$ from the budget constraint into the utility function, we write the central resident’s utility, denoted by $u_c$, as follows:

$$u_c = y - f_c - I(f_c, N) - p_c q_c + v(q_c). \quad (3)$$
In the same way, the suburban resident’s utility is written

\[ u_s = y - 2f_s - I(f_s, N) - I(f_s, n_s) - psqs + v(q_s). \] (4)

Note that housing production is suppressed in our model, so that housing consumption is equivalent to land consumption.

### 2.2 The laissez-faire equilibrium

The consumer in the central zone maximizes (3) by choices of \( q_c \) and \( f_c \). The first-order condition for choice of \( q_c \) is

\[ v'(q_c) = p_c, \] (5)

and the first-order condition for \( f_c \) is

\[ 1 + \frac{\partial I(f_c, N)}{\partial f_c} = 0. \] (6)

In the same manner, the suburban resident maximizes (4) by choices of \( q_s \) and \( f_s \), and the first-order condition for \( q_s \) is given by \( v'(q_s) = p_s \). But, the usual condition is that land rent at the edge of the city must equal an exogenous agricultural rent, \( \bar{p} \), so that \( p_s = \bar{p} \). The first-order condition for \( q_s \) is then written

\[ v'(q_s) = \bar{p}. \] (7)

The first-order condition for \( f_s \) is

\[ 2 + \frac{\partial I(f_s, N)}{\partial f_s} + \frac{\partial I(f_s, n_s)}{\partial f_s} = 0. \] (8)
Next, for spatial equilibrium, the consumers in the city must achieve a common utility level regardless of their location (zones). By setting (3) equal (4), this condition is written

\[ v(q_c) - p_c q_c - [f_c + I(f_c, N)] = v(q_s) - \bar{p} q_s - [2f_s + I(f_s, N) + I(f_s, n_s)]. \]  

(9)

The final equilibrium condition requires that the population fit inside the city. Letting \( n_c \) denote the number of central residents, this condition is written

\[ N = n_c + n_s, \]

where \( N \) is the exogenous city population. The total land consumption in the central zone is \( n_c q_c \), and using the assumption that the central zone’s land area is unity, it follows that \( n_c = 1/q_c \). Thus, the population condition is rewritten

\[ n_s = N - 1/q_c. \]  

(10)

Equations (5)-(10) determine the equilibrium values for \( q_c, q_s, f_c, f_s, p_c \), and \( n_s \).

The key properties of the laissez-faire equilibrium are summarized in Lemmas 1 and 2.

**Lemma 1** Vehicle size is smaller (vehicle fuel-efficiency is higher) in the central zone than in the suburban zone, so that \( f_c < f_s \).

**Proof.** Suppose \( f_c \geq f_s \). Then, \( 1 + \frac{\partial I(f_s, N)}{\partial f_s} \leq 1 + \frac{\partial I(f_c, N)}{\partial f_c} \) holds, given \( \partial^2 I/\partial f^2 > 0 \). As a result, \( 1 + \frac{\partial I(f_s, N)}{\partial f_s} \leq 0 \) holds, given \( 1 + \frac{\partial I(f_c, N)}{\partial f_c} = 0 \) from (6). Also, manipulation of (8) yields \( -1 - \frac{\partial I(f_s, n_s)}{\partial f_s} = 1 + \frac{\partial I(f_c, N)}{\partial f_c} \), and from this equality and using \( 1 + \frac{\partial I(f_c, N)}{\partial f_c} \leq 0 \), \( -1 - \frac{\partial I(f_s, n_s)}{\partial f_s} \leq 0 \) follows. The inequality, \( -1 - \frac{\partial I(f_s, n_s)}{\partial f_s} \leq 0 \), is rewritten using (6) as \( -1 - \frac{\partial I(f_s, n_s)}{\partial f_s} \leq 0 = -1 - \frac{\partial I(f_c, N)}{\partial f_c} \). As a result, \( \frac{\partial I(f_c, n_s)}{\partial f_s} \geq \frac{\partial I(f_c, N)}{\partial f_c} \) holds, which is however impossible given \( f_c \geq f_s \), \( N > n_s \), \( \partial^2 I/\partial f^2 > 0 \), and \( \partial^2 I/\partial f \partial Q > 0 \). So, \( f_c \geq f_s \) is impossible, implying \( f_c < f_s \). ■

**Lemma 2** The housing price is higher and housing consumption is lower in the central zone than in the suburban zone, so that \( p_c > \bar{p} \) and \( q_c < q_s \).
Proof. First, note that $t_c < t_s$ holds. To see this point, the fact that $f_c$ minimizes $f_c + I(f_c, N)$ means $f_c + I(f_c, N) \leq f_s + I(f_s, N)$. It then follows that $t_c = f_c + I(f_c, N) \leq f_s + I(f_s, N) < 2f_s + I(f_s, N) + I(f_s, n_s) = t_s$. With $t_c < t_s$, $v(q_c) - p_c q_c < v(q_s) - \bar{p}q_s$ holds from (9). Letting $g(p) \equiv v(q) - pq, g'(p) = (v'(q) - p)(\partial q/\partial p) - q = -q < 0$ holds both at $q_c$ and $q_s$ using (5) and (7). So, for $v(q_c) - p_c q_c < v(q_s) - \bar{p}q_s$ to hold, $p_c > \bar{p}$ must hold. Given $v''(q) < 0$ and using (5) and (7), $q_c < q_s$ follows.

Since the suburban resident’s commuting cost is higher ($t_c < t_s$), the consumer should be compensated by a lower housing price to be equally well-off, the same principle as in the standard model. But, the current model has the additional vehicle choice pattern shown in Lemma 1. Note that $q_c < q_s$ means a higher population density in the central zone than in the suburban zone. So, $f_c < f_s$ along with $q_c < q_s$ capture the empirical vehicle choice pattern, i.e., larger vehicles for households residing in the less-dense suburban area.

2.2.1 Discussion on relationship between residential location and vehicle choice

There can be several sources of the relationship between vehicle size and residential location. Among others, our model suggests the congestion (speed) effect on vehicle choices as its main source. We exploit an engineering relationship between fuel efficiency and congestion, particularly an observation that a car consumes more fuel per distance as the congestion level gets higher (as the speed falls).\footnote{Anas and Hiramatsu (2012)’s commuting cost incorporates a more general U-shaped relationship between fuel-inefficiency (gasoline consumption per mile) and the speed of travel, motivated by Barth and Boriboonsomsin (2008) and Davis and Diegel (2004). While an engineering relationship indicates that fuel costs per mile increase with speed when speed is beyond a certain level, our commuting cost does not have this property. The underlying assumption is that drivers do not waste fuel, and more specifically, that drivers, even when the road is not congested, do not increase speed to the level where fuel efficiency begins to fall.} This observation would imply that fuel waste due to congestion is bigger for a larger and less fuel-efficient car than for a smaller car, motivating us to assume $\partial^2 I/\partial f \partial Q > 0$. Since a larger car gets more costly as congestion gets severe (as speed falls), consumers suffering from severe congestion will choose a more fuel-efficient vehicle, while a larger vehicle is more suitable for travelers experiencing lower congestion,
with a higher speed.

In our spatial framework, only the suburban resident crosses both bridges for commuting, and therefore she takes the traffic condition of the less-congested suburban bridge as well as that of the central bridge into account in choosing a car. As a result, given the suitability of a larger car for the less-congested suburban bridge, the suburban resident chooses a larger vehicle than the central resident. In other words, the suburban resident chooses a larger car because she experiences a lower level of congestion, or equivalently *a higher average speed on the commute route*. Thus, in our model, the heterogeneity in the average commute speed explains the relationship between vehicle choice and residential location.

We can verify this argument in a different perspective. In particular, we carry out a comparative static analysis of vehicle size \((f)\) with respect to population \((N)\) to see how vehicles sizes differ in cities with different overall levels of congestion (speed). The comparative static derivation shown in Appendix A indicates that increased population (corresponding to a lower speed both for the central and the suburban residents) induces a fall in both \(f_c\) and \(f_s\). This result is due to the congestion (speed) effect, which leads consumers in a city with a higher overall level of congestion (lower speed) to choose a more fuel-efficient car.

While the suburban resident experiences a higher average speed, the suburban resident also travels a longer distance and therefore meets more traffic and consumes more fuel in total than the central resident. So, in reality, holding the speed of travel fixed, consumers with a longer commute distance might tend to economize on fuel consumption by choosing a smaller car. However, the heterogeneity in commute distance, setting aside the speed effect, plays no role in explaining the vehicle choice pattern in our model.

To explain why, note first that while the consumer takes the additional fuel cost of a larger car into account in choosing a car, the consumer also takes the benefit of a larger car into account. In our commuting cost, while \(f\) (fuel cost) rises with the car size, \(I(f, Q)\) falls with \(f\), which implies that there are both a cost and a benefit involved with a larger \(f\). At the optimum, the marginal cost of \(f\) equals its marginal benefit (see (6) and (8)). Moreover,
we can see from (6) and (8) that, if the travel speed were held fixed on the two bridges, the first-order conditions for $f$ would effectively be the same for the central- and the suburban residents, which implies that the $f$ choices would be invariant to the distance traveled if the speed were held fixed.\footnote{Assume for example that the traffic is not up to the level where congestion arises, so it continues to move at the speed limit on both bridges. Since the congestion cost $I(f, Q)$ is invariant to $Q$ for this level of traffic, we can effectively set $N = n_s$ in (6) and (8). Since the first-order conditions are then the same, we have $f_c = f_s$.} Since both the marginal cost and the marginal benefit of a higher $f$ are proportional to the distance traveled, the distance effects on them cancel and thus have no net effect.\footnote{The marginal cost of a higher $f$ for the suburban resident is 2 and that for the central resident is 1. The marginal benefit of a higher $f$ for the suburban resident is $2\partial I/\partial f$, which, holding the speed fixed, is also twice of that for the central resident (i.e., $\partial I/\partial f$).}

To incorporate a distance effect on vehicle choices, more general preferences would have to be used. For example, we could use a general quasi-concave utility function, $u(y - f - pq, q, I(f, Q))$, where $I(f, Q)$ is entered as a separate component of utility while $f$ is directly subtracted from consumption. Given $u_{11} < 0$, the marginal decrease in utility from a higher $f$ will depend on distance (i.e., whether $f$ is multiplied by 1 or 2), which would generate the force that we expect regarding the distance effect. While this alternative formulation will have an advantage of generating various vehicle-choice patterns with various sets of parameters used, the preferences that we use allow various analytic outcomes in the paper. We leave the use of more general preferences for future work.

In addition to the distance effect, we could also incorporate the income effect and allow income heterogeneity. We could then generate various equilibrium configurations regarding the location and vehicle choices patterns with various parameters used. However, we again leave this extension for future work in part because the empirical studies motivating us also controls for the household income. Our goal in this paper is to investigate the implications of the vehicle choice pattern for the urban economy and the environment rather than to provide more comprehensive explanations for the pattern.
3 Sprawl-inducing externalities and effects of corrective policies

In this section, we investigate how the laissez-faire equilibrium diverges from the social optimum. To find the social optimum, we consider a problem that would be solved by the social planner and compare the resulting socially optimal values to the laissez-faire equilibrium values. We first investigate the case where only the congestion externality exists. We then add the vehicle emission externality to the model.

3.1 Congestion externalities and the effect of congestion toll

The planner’s problem we consider is a utility maximization problem and is written

$$\max_{\{u,q_c,q_s,f_c,f_s,n_s\}} Nu \tag{11}$$

s.t. (i) $$\ (N-n_s)q_c = 1,$$

(ii) $$\ (N-n_s)(u-v(q_c)) + n_s(u-v(q_s)) + ((N-n_s)q_c + n_sq_s)\bar{p}$$

$$\ + (N-n_s)(f_c + I(f_c, N)) + n_s(2f_s + I(f_s, N) + I(f_s, n_s)) = Ny.$$ 

The constraint (i) is the condition requiring that land area of the central zone is fixed at unity, where the population constraint, $$n_c = N - n_s$$, has been substituted. The constraint (ii) is the aggregate income constraint, where $$e_i = u - v(q_i) \ (i \ni \{c, s\})$$ are substituted. Note also that this formulation assumes that toll revenues are returned in lump-sum fashion and that aggregate land rent also comes back as income. As a result, only the land’s opportunity cost must be paid, and tolls do not show up in the income constraint.

\footnote{The previous version of this paper considered a resource-cost-minimization problem. The resource-cost-minimization framework generates the same optimal choices as for the utility maximization problem. See also Brueckner and Helsley (2011), who also adopted the resource-cost-minimization approach.}
The Lagrangian expression for the maximization problems is

\[ L = N u + \rho [1 - (N - n_s)q_c] 
+ \lambda [Ny - (N - n_s)(u - v(q_c)) - n_s(u - v(q_s)) - ((N - n_s)q_c + n_s q_s)\bar{p} 
- (N - n_s)(f_c + I(f_c, N)) - n_s(2f_s + I(f_s, N) + I(f_s, n_s))]. \]  

(12)

The choice variables are \( u, q_c, q_s, f_c, f_s, \) and \( n_s \). The first-order condition for \( u \) yields \( \lambda = 1 \).

Using \( \lambda = 1 \), the first-order conditions for the \( q \) variables are given by

\[ v'(q_c) = \bar{p} + \rho, \quad v'(q_s) = \bar{p} \]  

(13)

The first-order conditions for \( f_c \) and \( f_s \) are the same as those of the laissez-faire equilibrium (see (6) and (8)).

While the laissez-faire equilibrium value of \( n_s \) is determined by the equal-utility condition (see (9)), the choice for \( n_s \) is explicitly considered by the social planner. The first-order condition for \( n_s \) is written

\[ v(q_c) - (\bar{p} + \rho) q_c - [f_c + I(f_c, N)] = v(q_s) - \bar{p} q_s - [2f_s + I(f_s, N) + I(f_s, n_s)] - n_s \frac{\partial I(f_s, n_s)}{\partial n_s}. \]  

(14)

We can see that (14) is almost identical to the laissez-faire equal-utility condition, (9), except for the presence of \( \bar{p} + \rho \) in place of \( p_c \) and the subtraction of \( n_s \frac{\partial I(f_s, n_s)}{\partial n_s} \) on the RHS in (14). To find the socially optimal values, \( \bar{p} + \rho \), interpreted as the shadow price of a housing in the central zone, is set at its socially optimal value, denoted by \( p^*_c \). The socially optimal value for \( q_c \), denoted by \( q^*_c \), is then recovered, since \( p^*_c \) is associated with \( q^*_c \) via \( p^*_c = v'(q^*_c) \). Under this interpretation and rewriting (14), the socially optimal values satisfy

\[ v(q^*_c) - v'(q^*_c) q^*_c - v(q^*_s) + \bar{p} q^*_s = -t^*_s + t^*_c - n^*_s \frac{\partial I(f^*_s, n^*_s)}{\partial n_s}, \]  

(15)
where the superscript * denotes the optimal values and \( t_s^* = 2f_s^* + I(f_s^*, N) + I(f_s^*, n_s^*) \) and \( t_c^* = f_c^* + I(f_c^*, N) \).

Letting \( \hat{t} \) denote the laissez-faire equilibrium values, the equal-utility condition (9) is rewritten as

\[
v(\hat{q}_c) - v'(\hat{q}_c)\hat{q}_c - v(\hat{q}_s) + p\hat{q}_s = -\hat{t}_s + \hat{t}_c, \tag{16}
\]

where \( \hat{t}_s = 2\hat{f}_s + I(\hat{f}_s, N) + I(\hat{f}_s, \hat{n}_s) \) and \( \hat{t}_c = \hat{f}_c + I(\hat{f}_c, N) \). Note that suburban housing consumption \( q_s \) is the same in both cases from \( v'(q_s^*) = v'(\hat{q}_s) = \bar{p} \). Also, since the first-order conditions for \( f_c \) are identical in both cases, given the common \( N, \hat{f}_c = f_c^* \) and thus \( t_c^* = \hat{t}_c \) holds. For the remaining variables, we compare (15) and (16) to see the differences in the optimal and the laissez-faire equilibrium values. The results are summarized as follows:

**Proposition 1** \( n_s^* < \hat{n}_s \) holds, meaning that there are too many suburban residents at the laissez-faire equilibrium, compared to the socially optimal level. Also, \( q_c^* < \hat{q}_c \) and \( p_c^* > \bar{p}_c \) hold. Finally, while \( f_c^* = \hat{f}_c \) holds, the optimal vehicle size is larger than the laissez-faire equilibrium level in the suburban zone, so that \( f_s^* > \hat{f}_s \).

**Proof.** Suppose \( n_s^* \geq \hat{n}_s \). Then, \( q_c^* \geq \hat{q}_c \) holds given \( n_s = N - 1/q_c \). Since \( v(q) - v'(q)q \) is increasing in \( q \) (the derivative of the expression is \( -v''(q)q > 0 \)), the LHS of (15) is greater than (or equal to) the LHS of (16), given \( q_s^* = \hat{q}_s \). Therefore, the RHS of (15) must also be greater than (or equal to) the RHS of (16). Given \( -n_s^*\frac{\partial I(f_s^*, n_s^*)}{\partial n_s} < 0 \) and \( t_c^* = \hat{t}_c, t_s^* < \hat{t}_s \) must then hold. But, note that \( \frac{\partial v^*}{\partial n_s} = n_s^*\frac{\partial I(f_s^*, n_s^*)}{\partial n_s} > 0 \) holds by the envelope theorem. Then, given \( n_s^* \geq \hat{n}_s \), \( t_s^* \geq \hat{t}_s \) follows, which is contradictory. Thus, we can rule out \( n_s^* \geq \hat{n}_s \), so that \( n_s^* < \hat{n}_s \) holds. Then, \( q_c^* < \hat{q}_c \) follows given \( n_s = N - 1/q_c \), and \( p_c^* > \bar{p}_c \) holds given \( v'(q_c) = p_c \) and \( v'' < 0 \). Finally, to compare \( f_s \) values, total differentiation of (8) and the maintained assumptions on \( I \) imply \( \partial f_s / \partial n_s < 0 \). Since \( f_s \) is solely determined by \( n_s \) given the exogeneity of \( N, f_s^* > \hat{f}_s \) follows from \( n_s^* < \hat{n}_s \). ■

Since commuters do not account for the external costs imposed on other drivers, there
is too much traffic on the suburban bridge, and equivalently there is too large a suburban population at the laissez-faire equilibrium. So, congestion tolls should be imposed to capture the commuters’ external congestion costs. The above analysis shows that imposition of the appropriate congestion toll, whose amount is \( n_s^* \frac{\partial I(f_s^*, n_s^*)}{\partial n_s} \) for each resident, shifts the suburban population to the central zone and population density in the central zone increases in response.\(^{16}\) With this toll subtracted from the RHS of (16), the conditions (16) and (14) become equivalent so that the equilibrium in the presence of the toll is social optimum.

Note also that \( 1 + q_s^* n_s^* < 1 + \hat{q}_s \hat{n}_s \) holds, given \( q_s^* = \hat{q}_s \), \( n_s^* < \hat{n}_s \), and \( q_c n_c = 1 \). This means that the optimal city is spatially smaller than the laissez-faire city. Observe also that imposition of the congestion toll influences the suburban resident’s vehicle size, \( f_s \), via the effect on \( n_s \). In particular, with the smaller suburban population and the reduced traffic flow on the suburban bridge, the suburban residents choose larger vehicles at the optimum than at the laissez-faire equilibrium. The central resident’s vehicle size, \( f_c \), is unchanged since the traffic flow on the central bridge is fixed at \( N \) regardless of congestion tolling.

### 3.2 Adding vehicle emission externalities

We now assume that city residents care about the level of air pollution in the city. Specifically, the utility function for the consumer in zone \( i (\ni \{c, s\}) \) is modified to \( u_i = e_i + v(q_i) - z(E) \), where \( E \) represents the total vehicle emissions released by all city residents, with \( z'(E) > 0 \), so that greater vehicle emissions reduce the consumer’s utility. Note that \( E \) does not involve a subscript, which means that city residents are equally affected by air pollution regardless of their locations.\(^{17}\) An emission externality exists because individual consumers ignore the effect of their location and vehicle fuel-efficiency choices on the city’s emission level.

\(^{16}\)Imposition of the congestion toll is actually equivalent to regulating lot size or population density (Wheaton (1998)).

\(^{17}\)We could instead introduce local air pollution that varies by zone, but it would not change the main results of the paper. We instead refer to Riley (1974), who considers local air pollution in a similar framework but without explicit vehicle-size choices and finds some ambiguous outcomes regarding the optimal and laissez-faire population densities.
The emission amount will be proportional to the total fuel consumption in the city. Total fuel consumption will depend on the number of residents in each zone, fuel-efficiency of each vehicle, and distance traveled by each vehicle. In particular, we model the total vehicle emissions in the city as

\[ E \equiv n_c f_c + \beta n_s f_s, \]  

(17)

where \( \beta \) represents distance-related extra emissions of suburban vehicles, with \( \beta \geq 1 \). If there were no traffic congestion, emissions released by a car would simply be proportional to distance traveled, controlling for the car’s fuel-efficiency, in which case \( \beta = 2 \) would hold. But, when congestion is considered, the additional emissions generated by the suburban vehicle would be less than proportional to the distance traveled, because only suburban vehicles cross the less-congested suburban bridge and the lower congestion would correspond to a less pollution per vehicle on the bridge, in which case \( \beta < 2 \) would hold. An implicit assumption is that lower driving speed (more congestion) raises pollution per mile. We assume that \( \beta \) can take any value between 1 and 2.\footnote{The main result below does not rely on the value of \( \beta \) as long as \( \beta \) is greater than or equal to 1. A \( \beta \) value that is less than 1 does not make sense, since it would mean a lower emission for longer distance holding fuel-efficiency fixed.}

To compare the laissez-faire equilibrium and the social optimum, note first that all the first-order conditions in the laissez-faire equilibrium are unchanged with the modification because \( E \) is taken as fixed in the individual consumer’s problem. The equal-utility condition, (9), is also unchanged because \( z(E) \) enters into both sides of the equation. But, the planner’s problem should be modified because the social planner takes the effect of her choices on the \( E \) level into account. In particular, the social planner is assumed to solve the following
\[
\max_{\{u, q_c, q_s, f_c, f_s, n_s\}} L = Nu + \rho [1 - (N - n_s)q_c] \\
+ \lambda [N y - (N - n_s)(u - v(q_c) + z((N - n_s)f_c + \beta n_s f_s))] \\
- n_s(u - v(q_s) + z((N - n_s)f_c + \beta n_s f_s)) - ((N - n_s)q_c + n_s q_s)\bar{p} \\
- (N - n_s)(f_c + I(f_c, N)) - n_s(2f_s + I(f_s, N) + I(f_s, n_s))],
\]

(18)

where (17) is used to substitute for \(E\).

The first-order conditions for \(u\) and the \(q\) variables are the same as in the previous case where only the congestion externality is considered. But, the first-order condition for \(f_c\) is different, being written

\[1 + \frac{\partial I(f_c, N)}{\partial f_c} + Nz'(E) = 0.\]

(19)

The first-order condition for \(f_s\) is given by

\[2 + \frac{\partial I(f_s, N)}{\partial f_s} + \frac{\partial I(f_s, n_s)}{\partial f_s} + \beta Nz'(E) = 0.\]

(20)

These conditions imply that the consumers should be charged vehicle fuel taxes (or emission taxes), whose amounts are \(Nz'(E)f_c\) for the central resident and \(\beta Nz'(E)f_s\) for the suburban resident. Note that \(Nz'(E)f_c < \beta Nz'(E)f_s\) holds, meaning that the suburban resident should be charged higher fuel taxes because she consumes more fuel and thus emit more pollution due to her longer commute trips (\(\beta \geq 1\)) and lower fuel-efficiency \((f_c < f_s)\).\footnote{Again, emission tax revenues discussed below are returned in lump-sum fashion, so they do not appear in the budget constraint.}

\footnote{The proof of \(f_c < f_s\) in the first-best case is as follows. Suppose \(f_c \geq f_s\). Then, 0 = 1 + \frac{\partial I(f_c, N)}{\partial f_c} + Nz'(E) \geq 1 + \frac{\partial I(f_c, N)}{\partial f_c} + Nz'(E) holds since \(\frac{\partial I(f_c, N)}{\partial f_c} \geq \frac{\partial I(f_s, N)}{\partial f_s}\) holds given \(\partial^2 I/\partial f^2 > 0\) and \(\partial^2 I/\partial f\partial Q > 0\). Using this inequality and by manipulation of (20), we can see that 1 + \frac{\partial I(f_c, N)}{\partial f_c} + Nz'(E) = 0 \leq 1 + \frac{\partial I(f_s, n_s)}{\partial f_s} + (-1 + \beta)Nz'(E) must hold, which is impossible because \(Nz'(E) \geq (-1 + \beta)Nz'(E)\) (since 1 \(\leq \beta \leq 2\)) and \(\frac{\partial I(f_c, N)}{\partial f_c} > \frac{\partial I(f_s, n_s)}{\partial f_s}\) given \(f_c \geq f_s\) and \(N > n_s\) and \(\partial^2 I/\partial f^2 > 0\) and \(\partial^2 I/\partial f\partial Q > 0\). Since \(f_c \geq f_s\) is ruled out, the only possibility is \(f_c < f_s\).}
The first-order condition for \( n_s \) is given by

\[
v(q_c) - (\bar{p} + \rho) q_c - [f_c + I(f_c, N)] - Nz'(E)f_c = v(q_s) - \bar{p}q_s - [2f_s + I(f_s, N) + I(f_s, n_s)] - n_s \frac{\partial I(f_s, n_s)}{\partial n_s} - \beta Nz'(E)f_s. \tag{21}
\]

According to this condition, the optimal \( n_s \) is achieved when consumers are charged both congestion tolls and vehicle fuel taxes. As explained above, we can recover the optimal solution chosen by the planner by setting the shadow price of a central housing at its optimal value. Setting \( p^*_c = \bar{p} + \rho = v'(q^*_c) \) and rearranging \((21)\), the optimal values, denoted by \( * \), satisfy the following:

\[
v(q^*_c) - v'(q^*_c)q^*_c - v(q^*_s) + \bar{p}q^*_s = -t^*_s + t^*_c - n_s \frac{\partial I(f^*_s, n^*_s)}{\partial n_s} - Nz'(E^*)(-f^*_c + \beta f^*_s), \tag{22}
\]

where \( t^*_s = 2f^*_s + I(f^*_s, N) + I(f^*_s, n^*_s) \) and \( t^*_c = f^*_c + I(f^*_c, N) \).

Since the first-order conditions for \( q_s \) are unchanged, \( \hat{q}_s = q^*_s \) again holds, where \( \hat{\cdot} \) denotes the laissez-faire solution. But, note that the first-order conditions for the \( f \) variables at the optimum (see \((19)\) and \((20)\)) are different from those at the laissez-faire equilibrium (see \((6)\) and \((8)\)), which should be incorporated for comparison. With this incorporation, we compare \((22)\) and \((16)\) to see how the optimal solution diverges from the laissez-faire equilibrium solution. The comparison results are summarized as follows:

**Proposition 2** \( n^*_s < \hat{n}_s \) holds, meaning that the suburban population is too large when both the congestion and the emission externalities are uncorrected, compared to when these externalities are corrected by the appropriate congestion tolls and vehicle fuel taxes. Also, \( q^*_c < \hat{q}_c \) and \( p^*_c > \hat{p}_c \) hold. Finally, the optimal vehicle size in the central zone is smaller than the laissez-faire equilibrium size, so that \( f^*_c < \hat{f}_c \). But, the optimal suburban vehicle size may be smaller or larger than the laissez-faire size.

**Proof.** See Appendix B. \(\blacksquare\)
Thus, when both the congestion and the emission externalities exist, the laissez-faire city is too spread-out compared to the optimal city, where both kinds of externalities are corrected by the appropriate policies. Importantly, this result does not rely only on congestion externalities but is due to the operation of emission externalities, implying that the emission externality is an independent source of market failures leading to excessive sprawl. To see this point, we isolate the effect of emission taxes on city size by comparing two cities, one where emission externalities are corrected and the other where emission externalities are left uncorrected, while assuming that congestion externalities are ignored and uncorrected in both cities. To compare these cities, we simply drop the congestion toll term in (22) and compare the equation to (16). This modification does not affect the above result, which can be seen from the proof of Proposition 2 (see Appendix B).\footnote{In particular, in Appendix B, the congestion toll term is dropped from (37), and the condition for a contradiction then becomes (38). This means that the congestion toll term has no role in the proof of Proposition 2. To check whether emission externalities are an independent source of excessive urban sprawl differently, we could alternatively eliminate the effect of traffic congestion by assuming that $\partial I/\partial Q = 0$. We can see that the proof of Proposition 2 apparently does not depend on the assumption $\partial I/\partial n_s > 0$.} \footnote{Since emission taxes alone lead to a more compact city, the model implies that emission taxes reduce congestion externalities by reducing the suburban population, so that emission taxes can be seen as a second-best policy for congestion externalities. Indeed, many real-world cities impose fuel taxes, while ignoring congestion externalities, viewing fuel taxes as a substitute for congestion tolling.}

The effects of charging for emissions have a simple explanation. With emission taxes, the vehicle sizes ($f$) tend to fall, an effect that raises $t$ values. Note that $t$ values are minimized at the laissez-faire equilibrium with no emission taxes (see (6) and (8)). In addition, since the emission taxes themselves are added, the consumers’ commuting costs increase, and the resulting higher commuting costs cause people to move toward the center, shrinking the city.

While this story is intuitively simple, the mechanism behind this effect is fundamentally different from that of the congestion externality. Specifically, under congestion externalities, urban sprawl is undesirable because commuters drive excessively long distances and thus residents in more sprawled cities generate more congestion externalities (distance-related source of excessive sprawl). The emission externality not only has this distance-related source of sprawl, but it also has “vehicle-choice-related source of excessive sprawl,” meaning
that residents in more sprawled cities use excessively less fuel-efficient vehicles than those in more compact cities. Moreover, this vehicle-choice-related source of sprawl (separately from the distance-related source) makes the optimal city more compact compared to the laissez-faire city. To argue this point, we set the distance-related extra emission of suburban vehicles (β in (17)) at unity to eliminate its effect in the proof of Proposition 2.

Another question is whether the first-best optimal city, where both kinds of externalities are remedied by the appropriate policies, is spatially smaller than the city where only the congestion externality is corrected by imposing the congestion toll. In effect, we are asking whether the optimal city in the current model is more compact than the other optimal city that the standard congested-city model yields (Arnott (1979), Wheaton (1998), Brueckner (2007)). Since our model has an additional source of externalities (vehicle emissions), intuition would suggest that the optimal city in our model should be more compact than that of the standard congested-city model. To check whether this intuition is correct, we compare (22) to the following expression:

\[ v(q_{c}^{**}) - v'(q_{c}^{**})q_{c}^{**} - v(q_{s}^{**}) + \bar{p}q_{s}^{**} = -t_{s}^{**} + t_{c}^{**} - n_{s}^{**} \frac{\partial I(f_{s}^{**}, n_{s}^{**})}{\partial n_{s}} , \quad (23) \]

where ** denotes the case where only the congestion externality is corrected while the emission externality is left uncorrected. Note that there are no terms for fuel taxes but only the congestion toll term in (23). The fuel-tax terms in (19) and (20) are also be dropped in this case. The comparison result is given as follows:

**Proposition 3** The optimal city, where both the congestion and the emission externalities are remedied by the appropriate policies, is not necessarily smaller than the city where only the congestion externality is corrected by optimal tolling while the emission externality is left uncorrected.

**Proof.** See Appendix C. ■

This result suggests that when we consider a city where congestion tolling is already
implemented, emission taxation may decrease or increase the city’s spatial size. This ambiguity arises due to the tension between congestion tolling and emission taxation whose effects differ between the regimes. First note that the congestion toll term in (23) is greater than that in (22) under $\partial^2 I/\partial f \partial Q > 0$ for a given $n_s$, since $f_s$ is larger with no emission taxation (i.e., $f_s^{**} > f_s^*$). The relative sizes of the congestion tolls are ultimately ambiguous between the two regimes because the $n_s$ values are different, but if $f$ choices are highly responsive to emission taxation and the second partial derivative, $\partial^2 I/\partial f \partial Q > 0$, is sufficiently large, then the congestion toll is larger at the regime ** than at *. The higher toll under the regime of congestion tolling alone (**) would contribute to the outcome $n_s^{**} < n_s^*$, setting aside the effect of emission taxation.

But, given $\beta N z'(E)f_s > N z'(E)f_c$, emission taxes imposed under the first-best regime (*) pull the suburban population toward the center, contributing to a smaller $n_s$ at * than at **, an opposite force to the effect of congestion tolling. The overall relative sizes between $n_s^*$ and $n_s^{**}$ depend on which force is stronger. If the effect of the higher congestion toll in (23) outweighs the effect of emission taxes, then the higher toll pulls the suburban residents to the central zone more strongly, yielding a more compact city at ** than at *, and vice versa. The numerical examples below verify that both results are possible. This tension exists in our model because the congestion toll depends not only on traffic flows but also on vehicle fuel efficiency, an element that is absent in the standard congested-city models.

4 The effect of Corporate Average Fuel Economy standards

Since the choice of vehicle fuel-efficiency interacts with urban spatial structure in our model, it implies that policies that influence vehicle choices will also influence urban spatial structure. In this section, we analyze the effects of Corporate Average Fuel Economy (CAFE) standards on urban spatial structure and its efficacy as a second-best tool for remedying the
emission externality.

The CAFE standards regulate the average fuel economy of new vehicles sold at the company level. So, we consider the following CAFE constraint:

$$n_c f_c + n_s f_s \leq K,$$

where $K$ is the limit of the available $f$’s, a variable set by the regulator. Recognizing that $f$ captures vehicle fuel-inefficiency, (24) indicates that the weighted average of $f_c$ and $f_s$, with the weights set at sales volumes, should be smaller than a certain level. More precisely, dividing (24) by $N$ would give a condition that average fuel economy of vehicles is less than some number, but that condition would be the same as (24) with the appropriate modification of $K$. We can see that the CAFE regulation, (24), is a modified version of the total emission expression, (17), with 1 in place of $\beta$. This implies that the CAFE regulation does not fully account for how much the vehicle is utilized and thus their impact on emissions.

4.1 The equilibrium conditions under CAFE standards

Under the CAFE regime, the planner (regulator) would set the limit of available vehicle sizes ($f$’s) by choosing $K$ while the individual consumers would choose all other variables conditional on $K$. So, the framework to analyze the CAFE effect should be a mix of decentralized and centralized problems.

To analyze the effect of the CAFE regulation, we exploit an intuition that the introduction of the CAFE constraint, (24), will raise the unit price of $f$ (capturing vehicle fuel-inefficiency) by limiting the supply. The underlying assumptions are first, that the CAFE constraint is

---

\(^{23}(24)\) is approximately equivalent to the CAFE regulation in practice, which also requires that a sales-weighted average of fuel-efficiency (fuel-inefficiency) for the company’s vehicles be greater (smaller) than a certain level. See EPA website (www.epa.gov/fueleconomy/regulations.htm) for how the company’s average fuel economy is calculated. For details of CAFE regulation, its history, and policy in practice, see Anderson et al. (2011).
binding for vehicle producers, and second, that the incidence of the CAFE regulation falls on consumers.\\(^{24}\)

Letting \( w \) denote the increase in the unit price of \( f \), the central- and the suburban-residents’ commuting costs under the CAFE regime, denoted by \( \tilde{t}_c \) and \( \tilde{t}_s \), are given by

\[
\tilde{t}_c = f_c + I(f_c, N) + wf_c, \tag{25}
\]

and

\[
\tilde{t}_s = 2f_s + I(f_s, N) + I(f_s, n_s) + wf_s, \tag{26}
\]

Note that \( w \) is the *increase* in unit price of \( f \), since the capital costs of purchasing a car in both zones were normalized to zero (see footnote 10). We impose a common \( w \) value in (25) and (26), implying that the car producer does not impose different unit prices of \( f \) in the two zones. Allowing \( w \) values to be different may be unrealistic in that it implies price discrimination by the car producers depending on a consumer’s place of residence.\\(^{25}\)

Note however that the price increases in levels differ by the location after all since \( w \)'s are interacted with consumers’ vehicle choices (\( f \)) that depend on the location.

Substitution of (25) into the utility, (3), and differentiation gives the first-order condition for \( f_c \) under the CAFE regime as

\[
1 + \frac{\partial I(f_c, N)}{\partial f_c} + w = 0. \tag{27}
\]

---

\(^{24}\)Regarding the market structure, it would be easy for interpretation to think of a monopolistic supplier who considers the CAFE constraint in the production of cars. But, we could also imagine multiple producers who jointly supply vehicles while their total production meets the CAFE constraint.

\(^{25}\)Allowing different \( w \) values is also technically difficult in the model, because we have only one additional equilibrium condition, (30), so we can add only one additional endogenous variable to solve the equation system.
In the same manner, the first-order condition for \( f_s \) is given by

\[
2 + \frac{\partial I(f_s, N)}{\partial f_s} + \frac{\partial I(f_s, n_s)}{\partial f_s} + w = 0. \tag{28}
\]

The first-order conditions for the \( q \) variables are the same as in the laissez-faire case. As usual, spatial equilibrium requires equal utility across zones. Using (25) and (26), the equal-utility condition is written

\[
v(q_c) - p_c q_c - \tilde{t}_c = v(q_s) - \bar{p} q_s - \tilde{t}_s. \tag{29}
\]

Finally, individual choices under the CAFE regime should meet the CAFE constraint. The CAFE constraint is assumed to be binding, since otherwise \( w = 0 \) will hold and the CAFE regime will be equivalent to the laissez-faire equilibrium, which we rule out for relevancy of the CAFE regulation. So, the individual choices satisfy the following equation:

\[
(N - n_s)f_c + n_s f_s = K. \tag{30}
\]

The conditions consisting of (27)-(30), (5), (7), and (10) determine the equilibrium values under the CAFE regime of the variables \( q_c, q_s, p_c, n_s, f_c, f_s, \) and \( w \) for a given \( K \). So, while the CAFE regime has an additional condition, (30), it also has an additional variable, \( w \), compared to the laissez-faire case.

While individuals’ choices are made conditional on \( K \), the CAFE regulator chooses \( K \) (the limit of available vehicle fuel-inefficiencies). The regulator’s objective is to maximize the consumers’ aggregate utility, and the problem is written as

\[
\max_{\{K\}} \quad Nu = (N - n_s)u_c + n_s u_s \tag{31}
\]

\[
= (N - n_s) \left[ y - \tilde{t}_c - p_c q_c + v(q_c) - z(E) \right] + n_s \left[ y - \tilde{t}_s - \bar{p} q_s + v(q_s) - z(E) \right].
\]
As described above, the variables in (31), including $E$, are determined in the decentralized setting for a given $K$ and thus are implicit functions of $K$. In the numerical section below, we compute the equilibrium values under the CAFE regulation.

Note that (31) shows a key feature of the planner’s decision in choosing $K$. Specifically, there is a trade-off between the individuals’ commuting costs ($\tilde{t}$) and the city’s emission level ($E$). When $K$ is low, the price increase of cars will be high because of the reduction in the available $f$’s. The benefit of a low $K$, however, is a lower level of vehicle emissions. On the contrary, if $K$ is high, the price increase of cars will be limited, but the emission level will be high. The planner balances this trade-off to maximize the total utility of the consumers. This trade-off is illustrated in the numerical examples below. In the next section, equilibria under CAFE as well as the other policy regimes are found using a numerical method.

5 Numerical examples

In this section, we find equilibria under various policy regimes and compare equilibrium values across the regimes.

5.1 Finding equilibria under alternative regimes

We use $I(f, Q) = b - df^\nu Q^{-\theta}$, with $b > 0$, $d > 0$, $\theta > 0$, and $0 < \nu < 1$, as the functional form for the congestion cost. The functional form for utility is $u(e, q, E) = e + q^\alpha - \sigma E$, with $\sigma > 0$ and $0 < \alpha < 1$. Under the maintained functional forms, the first-order conditions for the $q$ variables are given by $p_c = \alpha q_c^{\alpha-1}$, where $q_c = \ell/(N - n_s)$ ($\ell$ is land area of the central zone), and $q_s = (\bar{p} \alpha^{-1})^{-\frac{1}{\alpha-1}}$. While these conditions hold regardless of the policy regimes, the key condition that distinguishes one regime from the others is the condition for $n_s$. The first-order conditions for $f$ choices also differ by the policy regime.\footnote{Under the maintained functional form, the first-order conditions for $f_c$ and $f_s$ with emission taxation are $f_c = [N^\theta(1 + N\sigma)/dv]^{-\frac{1}{\nu+1}}$ and $f_s = [(2 + \beta N\sigma)/dv (N^{-\theta} + n_s^{-\theta})]^{-\frac{1}{\nu+1}}$, respectively. The laissez-faire conditions for $f$’s are $f_c = (N^\theta/dv)^{\frac{1}{\nu+1}}$ and $f_s = [2/dv (N^{-\theta} + n_s^{-\theta})]^{-\frac{1}{\nu+1}}$, respectively.}
Let’s begin by describing the procedure finding the equilibrium under CAFE. Under the maintained functional forms, the first-order condition for $f_c$ under CAFE, (27), is written

$$f_c = \left[ \frac{N^\theta(1+w)}{d\nu} \right]^{1/\nu}.$$

and the first-order condition for $f_s$, (28), is written

$$f_s = \left[ \frac{2+w}{d\nu(N^{-\theta} + n_s^{-\theta})} \right]^{1/\nu}.$$

As well as these conditions, the equilibrium values under CAFE also satisfy the equal-utility condition, (29), and the CAFE constraint, (30), for a given $K$. To find the optimal $K$, we find sets of decentralized choices with varying $K$ values and substitute the choices into (31) to compute the corresponding total utilities. The planner then picks the $K$ value that gives the maximum of the total utility, and the corresponding values are determined as the equilibrium values.

In addition to CAFE, we can consider 5 other alternative policy regimes. The equilibrium values in each regime are searched with adjustments to corresponding conditions. For example, we find the equilibrium under the policy of both CAFE and congestion tolling in the same way described above but by adding the congestion toll term to (29). We can also find the first-best choices by using (19), (20), and (22). Note that the first-best policy imposes both congestion tolls and emission taxes. To find the equilibrium values under the regime of emission taxation alone, we get rid of the congestion toll term from (22) while using the same conditions as in the first-best regime. Note that the emission taxes in this regime equal the marginal externality costs as in the first-best regime (i.e., $Nz'(E)f_c$ and $\beta Nz'(E)f_s$). The equilibrium under tolling alone satisfies condition (23) as well as (6) and (8). The congestion toll in this regime is also the marginal external congestion cost, i.e., $n_s (\partial I/\partial n_s)$.

---

27So, the toll in this regime can be thought of a first-best policy whose goal is however limited to remedying only the congestion externality. It cannot be thought of as a second-best policy that would be helpful to remedy the emission externality since the emission level ($E$) under tolling alone is actually higher than under
and its equilibrium values are searched with adjustment for the fact that emission taxes are not imposed by finding the values satisfying (6), (8), and (23) simultaneously. Finally, the laissez-faire choices satisfy (6), (8), and (9).

The parameter values are given as follows. The population size \( N \) is set at 100. The agricultural land rent is \( \bar{p} = 0.04 \), and the land area of the central zone is \( \ell = 500 \) (\( \ell = 1 \) was used in the analytical section). The parameters in the utility function are set at \( \alpha = 0.32 \) and \( \sigma = 0.01 \). The distance-related extra emission is set at \( \beta = 1.5 \). The parameters in the congestion cost function are set at \( b = 0.4, d = 5, \nu = 0.6, \) and \( \theta = 0.358 \), and we also use \( \nu = 0.35 \) and \( \theta = 0.7 \) as an alternative parameter set.

5.2 Numerical results

Let’s begin by illustrating how the equilibrium under CAFE is identified. Figure 2 shows a graph representing the magnitudes of the total utility with varying \( K \) under the policy of CAFE. As the figure shows, the curve is globally convex and smooth over the relevant \( K \) values. So, we can identify the optimal \( K \) that maximizes total utility. Figure 3 shows the total commuting cost of the city residents, \((N - n_s)\tilde{t}_c + n_s\tilde{t}_s\), and the city’s emission level, \( E \), with varying values of \( K \). These lines illustrate the trade-off faced by the CAFE regulator in choosing the optimal \( K \).

[Figures 2 and 3 about here]

We now focus on the effects of alternative policies on the suburban population \( n_s \). Table 1 shows the equilibrium values under all possible combinations of the corrective policies. We first investigate the effects of emission taxation and CAFE, i.e., the policies aimed at correcting the emission externality, by comparing the \( n_s \) values shown in columns (1), (3), and (4). Note that we are looking at the cases where congestion tolls are not imposed while the regimes differ only by the policy aimed at correcting the emission externality. The

laissez-faire. The emission taxes under emission taxation alone are also a first-best policy specifically targeting the emission externality. But, emission taxation has a second-best effect on the congestion externality in that it reduces congestion by lowering \( n_s \).
simulation results indicate that $n_s$ under CAFE is 63.23, which is smaller than that under laissez-faire (68.25) but larger than that under the policy of emission taxation (60.63). We conclude that imposition of CAFE reduces the city’s spatial size from the laissez-faire, but not by as much as the emission taxes.

To further investigate the effects of emission taxation and CAFE on $n_s$, we can alternatively compare the regimes where congestion tolling is imposed at all cases but differ by the policy aimed at correcting the emission externality. Columns (2), (5), and (6) illustrate such cases. We find that $n_s$ in a city where tolls and emission taxes are imposed (first-best city) is 52.92, which is smaller than $n_s$ in a city where only tolls are imposed (54.73). The $n_s$ value in a city where tolls and CAFE are imposed is 54.72, which is between these two regimes.

This result, however, can be reversed depending on the parameter values. Table 2 shows the results from the simulation using an alternative parameter set of $\nu = 0.35$ and $\theta = 0.7$ (while maintaining the other parameters). We find that $n_s$ in columns (6) is 48.93, which is larger than $n_s$ in column (2) (48.77), implying that when we consider a city where congestion tolling is already implemented, emission taxation increases the city’s spatial size. Since this result is opposite to that in Table 1, it confirms the ambiguous result shown in Proposition 3.

As explained above, this ambiguity arises due to the tension between congestion tolling and emission taxation whose effects differ between the regimes. Specifically, in both tables, the toll value under the policy of tolling alone (in column (2)) is greater than that under the first-best policy (in column (6)). In Table 1, the difference in the toll values between the regimes, i.e., the gap in $n_s(\partial I/\partial n_s)$ between the two columns, is 0.12, while the difference in the emission taxes between the suburban and the central residents, i.e., $\beta Nz' (E) f_s^* - Nz' (E) f_c^*$, is 0.08. The ratio of these numbers is 1.5 ($= 0.12/0.08$). The outcome $n_s^* < n_s^{**}$ in Table 1 suggests that the force generated by the gap in the congestion tolls does not outweigh the force generated by emission taxation. The same kind of ratio calculated from Table 2 is 1.6, which is larger than that in Table 1 and thus regarded as a source of the outcome $n_s^* > n_s^{**}$. 
in Table 2. Simulations with other parameter sets show that as the ratio computed in this way gets higher, \( n_s^* > n_s^{**} \) is more likely to emerge.

[Tables 1 and 2 about here]

The effects of congestion tolling on \( n_s \) are also consistent with the theoretical predictions. Table 1 shows that the \( n_s \) value in column (2) is 54.73, which is smaller than that in column (1) (68.25), meaning that congestion tolling reduces the city’s spatial size from the laissez-faire. To further investigate the effect of tolling, we can alternatively compare the \( n_s \) values in columns (4) and (6), where emission taxes are imposed in both cases. We can also compare columns (3) and (5), where the CAFE standards are imposed in both cases. Regardless of the comparison frame, the city shrinks spatially in response to imposition of tolling.

We now compare welfare across the alternative policy regimes. The welfare measure we use is the consumers’ total utility plus rebates of congestion toll revenue, emission tax revenue, CAFE revenue, and total land rents paid to absentee landlords.\(^{28}\) The consumer utility is computed by substituting the equilibrium values to the utility function \textit{ex post} charge of the corrective policies. This implies that the revenues from the corrective policies must be included in the welfare measure as they can potentially be paid back to the consumers. Note in particular that the CAFE revenue, which equals \((N - n_s)f_cw + n_sf_sw\), is included in the welfare since it captures the increased vehicle revenue due to the price increase under CAFE and therefore is a benefit to somebody.\(^{29}\) The last row in Table 1 shows the welfare calculated in this way. As expected, the first-best regime imposing both tolls and emission taxes yields the highest welfare while the laissez-faire yields the lowest welfare. The CAFE standards increase welfare from the laissez-faire, but not by as much as the emission taxes. As expected, imposition of congestion tolling increases the city’s welfare. The simulation

\(^{28}\)Note that rent per unit of land in the suburban zone is fixed at \( \bar{p} \). Since the land area of the suburban zone (used either for agriculture or for rental to urban residents) is also fixed, its land value is not affected by the policies. So, the welfare measure includes only the land value of the central zone, which equals \( p_c q_c (N - n_s) \).

\(^{29}\)In this remark, we are implicitly ignoring costs incurred by the supply side, which is suppressed in our model. Without inclusion of the CAFE revenue, the welfare under CAFE alone is 138.26 and that under both CAFE and congestion tolling is 139.25. So, conclusions we draw from the welfare comparison are unchanged with or without inclusion of the CAFE revenue.
result shown in Table 2 verifies that these results are robust to variations in parameter values.

We can also compare values for other endogenous variables across the alternative regimes. For example, vehicle sizes are smaller under the policy of emission taxation than under laissez-faire (see the \( f \) values in columns (1) and (4)). The \( f_s \) value under the first-best policy is also smaller than that under laissez-faire (see columns (1) and (6)), which clears up the ambiguity shown in Proposition 2.\(^{30}\) But, imposition of tolling increases the \( f_s \) size from the laissez-faire, confirming the result of Proposition 1 (see columns (1) and (2)). The effects of CAFE on \( f \) sizes depend on the magnitude of \( w \) (increase in unit price of \( f \)). According to Table 1, \( w \) is 1.14, which is between the emission tax per \( f_c \) (= 1) and that per \( f_s \) (= 1.5). Consequently, the \( f_c \) value is smaller under CAFE than under emission taxation while \( f_s \) is larger under CAFE than under emission taxation (see columns (3) and (4)). The tables also present other endogenous variables, such as the city’s emission levels whose relative sizes are consistent with the theory.

6 Conclusion

There have been growing concerns about the potential influences of urban sprawl on the environment, especially on greenhouse gas emissions. Few urban economic models have analyzed this issue, presumably because of the lack of a reliable way of representing the city’s total emission amount. We overcome this challenge by treating consumers’ vehicle fuel-efficiency choices endogenously. The analysis shows that under the presence of both congestion and emission externalities, a city without appropriate corrective policies is too spread out compared to the socially optimal city. While this result is the same as in the standard congested-city models that abstract from the emission externality, it is shown that the source of this outcome is different and includes the use of larger and less fuel-efficient vehicles in more sprawled cities.

\(^{30}\)This result implies that the effect of emission taxation on \( f_s \) outweighs the effect of congestion tolling that operates in the opposite direction. This result is natural because while emission taxes are imposed directly on vehicle sizes, congestion tolling influences \( f_s \) via \( n_s \) and therefore its effect is indirect.
Congestion pricing is uncommon in the US, and the rate of US fuel taxes is relatively low not only compared to the socially desirable level but also by international standards (Parry et al. (2007), Small (2010), Glaeser (2011)). Based on our model, the failure to implement the first-best policies in the US means that the US cities are too spread out, justifying criticisms of urban sprawl. However, land-use restrictions such as urban growth boundaries, while potentially appealing as anti-sprawl policies, are inefficient alternatives to congestion pricing and higher fuel taxes (Brueckner (2007), Anas and Pines (2008)). This paper additionally analyzes the CAFE standards as an alternative second-best tool for correcting emission externalities and restricting excessive growth of urban areas. While the CAFE regulation potentially improves the welfare of the city residents by making the choice of less fuel-efficient cars more costly, the analysis shows that it is nevertheless limited in its ability to charge properly for the external costs involved in the choices of vehicles and residential locations.
References


Barth, M., Boriboonsomsin, K., 2008. Real-world carbon dioxide impacts of traffic congestion. Transportation Research Record: Journal of the Transportation Research Board 2058, 163-171.


A  The effect of an increase in \( N \) on \( f \)

For comparative static derivation, differentiation of (8) with respect to \( N \) gives

\[
\frac{\partial^2 I(f_s, N) \partial f_s}{\partial f_s^2 \partial N} + \frac{\partial^2 I(f_s, N)}{\partial f_s \partial N} + \frac{\partial^2 I(f_s, n_s) \partial f_s}{\partial f_s^2 \partial n_s} + \frac{\partial^2 I(f_s, n_s) \partial n_s}{\partial n_s \partial N} = 0, \tag{34}
\]

and rearrangement yields

\[
\frac{\partial f_s}{\partial N} = - \left( \frac{\partial^2 I(f_s, N)}{\partial f_s \partial n_s} + \frac{\partial^2 I(f_s, n_s) \partial n_s}{\partial f_s^2 \partial n_s} \right) \left( \frac{\partial^2 I(f_s, N)}{\partial f_s^2} + \frac{\partial^2 I(f_s, n_s)}{\partial f_s^2} \right)^{-1} < 0, \tag{35}
\]

given \( \partial^2 I/\partial f \partial Q > 0 \) and \( \partial^2 I/\partial f^2 > 0 \) and \( \partial n_s / \partial N \geq 0 \). We rule out an implausible case of \( \partial n_s / \partial N < 0 \) (or \( \partial n_c / \partial N > 1 \)), which implies some \textit{ex-ante} suburban population migrate into the central zone in response to the increase in \( N \). Differentiation of (6) with respect to \( N \) and rearrangement yields

\[
\frac{\partial f_c}{\partial N} = - \left( \frac{\partial^2 I(f_c, N)}{\partial f_c \partial n_c} \right) \left( \frac{\partial^2 I(f_c, N)}{\partial f_c^2} \right)^{-1} < 0, \tag{36}
\]

where the sign is determined by \( \partial^2 I/\partial f \partial Q > 0 \) and \( \partial^2 I/\partial f^2 > 0 \).

B  Proof of Proposition 2

We begin by showing the relative sizes of \( f_c \) values. Note first that \( \frac{\partial I(f_c^*, N)}{\partial f_c} < \frac{\partial I(\hat{f}_c, N)}{\partial f_c} \) holds from (6) and (19). This inequality implies \( f_c^* < \hat{f}_c \) given \( \partial^2 I/\partial f^2 > 0 \).

Next, to show \( n_s^* < \tilde{n}_s \), suppose \( n_s^* \geq \tilde{n}_s \). Then, \( q_c^* \geq \hat{q}_c \) holds given \( n_s = N - 1/q_c \). Since \( v(q) - v'(q)q \) is increasing in \( q \), the LHS of (22) is greater than (or equal to) the LHS of (16), given \( q_c^* \geq \hat{q}_c \) and \( q_s^* = \hat{q}_s \). So, the RHS of (22) must also be greater than (or equal to) the RHS of (16), or equivalently the following must hold:

\[
-Nz'(E^*)(-f_c^* + \beta f_s^*) \geq (\tilde{r}_c - t_c^*) - (\tilde{r}_s - t_s^*) + n_s^* \frac{\partial I(f_s^*, n_s^*)}{\partial n_s}. \tag{37}
\]

37
To show a contradiction of \( n_s^* \geq \hat{n}_s \), we can show (37) does not hold. But, it is sufficient for establishment of a contradiction of \( n_s^* \geq \hat{n}_s \) to show the following:

\[
-Nz'(E^*)(-f_c^* + f_s^*) \leq (\hat{t}_c - t_c^*) - (\hat{t}_s - t_s^*),
\] (38)

since the addition of \( n_s^* \frac{\partial H(f^*_s, n^*_s)}{\partial n_s} > 0 \) to the RHS of (38) wouldn’t change the inequality when (38) holds. Also, \( \beta \) in (37) is set at unity since allowing \( \beta > 1 \) would not change the inequality in (38) when (38) holds. In other words, (38) is a sufficient condition for a contradiction of \( n_s^* \geq \hat{n}_s \). So, we prove \( n_s^* < \hat{n}_s \) by showing (38) under \( n_s^* \geq \hat{n}_s \).

The second term on the RHS of (38) is written

\[
\hat{t}_s - t_s^* = \delta(\hat{f}_s - f_s^*),
\] (39)

where \( \delta \) satisfies

\[
-Nz'(E^*) \leq \delta \equiv \frac{\hat{t}_s - t_s^*}{\hat{f}_s - f_s^*} \leq 0.
\] (40)

To see (39) holds, note first that \( \partial t_s/\partial f \) evaluated at \( f_s^* \) equals \( -Nz'(E^*) \) from (20) while the derivative evaluated at \( \hat{f}_s \) equals zero from (8). Since the relationship between \( t_s \) and \( f \) is convex, so that the derivative is monotonic, the change in \( t_s \) (i.e., \( \hat{t}_s - t_s^* \)) equals the change in \( f \) times a value intermediate between the derivatives at the starting and ending \( f \) values, a number that therefore lies between \( -Nz'(E^*) \) and 0. In the same manner, the first term on the RHS of (38) is written

\[
\hat{t}_c - t_c^* = \gamma(\hat{f}_c - f_c^*),
\] (41)

where \( -Nz'(E^*) \leq \gamma \leq 0 \) holds for the same reason as (40).
Using (39) and (41), the condition for a contradiction, (38), is rewritten

\[-f^*_c + f^*_s \geq -\frac{1}{Nz'(E_\ast)} \left[ (\hat{t}_c - t^*_c) - (\hat{t}_s - t^*_s) \right] \]

\[= -\frac{1}{Nz'(E_\ast)} \left[ \gamma (\hat{f}_c - f^*_c) - \delta (\hat{f}_s - f^*_s) \right] \]

\[= -\frac{\delta}{Nz'(E_\ast)} \left[ \frac{\gamma}{\delta} (\hat{f}_c - f^*_c) - (\hat{f}_s - f^*_s) \right]. \tag{42} \]

Since \(0 \leq -\frac{\delta}{Nz'(E_\ast)} \leq 1\) from (40), the sufficient condition for a contradiction, i.e., (42) equivalent to (38), reduces to

\[-f^*_c + f^*_s \geq \frac{\gamma}{\delta} (\hat{f}_c - f^*_c) - (\hat{f}_s - f^*_s) \tag{43} \]

\[\iff 0 \geq \hat{f}_c - \hat{f}_s + \left( 1 - \frac{\gamma}{\delta} \right) (f^*_c - \hat{f}_c). \tag{44} \]

The reason is that if (43) holds, then the fact that the RHS of (43) is larger than the RHS (42) (given \(0 \leq -\frac{\delta}{Nz'(E_\ast)} \leq 1\)) will establish that (42) is satisfied. If the RHS of (43) is negative, (43) is always satisfied since \(-f^*_c + f^*_s > 0\) (see footnote 20). (43) is manipulated to yield (44), where we have \(\hat{f}_c - \hat{f}_s < 0\) and \(f^*_c - \hat{f}_c < 0\). It is also shown below that \(1 - \frac{\gamma}{\delta} \geq 0\) (\(\iff \delta \leq \gamma \leq 0\)) holds, so that (44) or equivalently (43) is satisfied.

Here, we check \(\delta \leq \gamma\), which means that the change in \(t_s\) is greater (in absolute value) than that in \(t_c\) when \(f\) changes from the optimal value to the laissez-faire value. Setting \(\hat{f}_c = \hat{f}_s \equiv \hat{f}\) (which is possible since we are comparing the changes in \(t\)), we write

\[\frac{\partial t_s}{\partial f} - \frac{\partial t_c}{\partial f} = \left( 2 + \frac{\partial I(f, N)}{\partial f} + \frac{\partial I(f, n_s)}{\partial f} \right) - \left( 1 + \frac{\partial I(f, N)}{\partial f} \right) \]

\[= 1 + \frac{\partial I(f, n_s)}{\partial f} < 0, \quad \forall f < \hat{f}. \tag{45} \]

The inequality in (45) is shown by noting that \(t_c\) is decreasing with \(f_c\) over the \([f^*_c, \hat{f}_c]\) interval, so that \(1 + \frac{\partial I(f, N)}{\partial f} < 0\), and this inequality implies \(1 + \frac{\partial I(f, n_s)}{\partial f} < 0\) since \(\frac{\partial I(f, N)}{\partial f} > \frac{\partial I(f, n_s)}{\partial f}\) holds given \(\partial^2 I/\partial f \partial Q > 0\). From \(\frac{\partial t_s}{\partial f} < \frac{\partial t_c}{\partial f}\) (see (45)), \(\delta \leq \gamma\) follows, establishing
The sufficient condition for a contradiction, (42) (equivalently (38)), is then established. Hence, \( n^*_s \geq \hat{n}_s \) is contradictory, so that \( n^*_s < \hat{n}_s \). Under \( n^*_s < \hat{n}_s \), \( q^*_c < \hat{q}_c \) holds given \( n_s = N - 1/q_c \), and \( p^*_c > \hat{p}_c \) follows given \( v'(q_c) = p_c \) and \( v'' < 0 \).

We finally show both \( f^*_s \leq \hat{f}_s \) and \( f^*_s > \hat{f}_s \) are possible under \( n^*_s < \hat{n}_s \). Note first that (8) and (20) imply, given \( N z'(E^*) > 0 \),

\[
\frac{\partial I(f^*_s, N)}{\partial f_s} + \frac{\partial I(f^*_s, n^*_s)}{\partial f_s} < \frac{\partial I(\hat{f}_s, N)}{\partial f_s} + \frac{\partial I(\hat{f}_s, \hat{n}_s)}{\partial f_s}. \tag{46}
\]

(46) is satisfied when \( f^*_s \leq \hat{f}_s \), since both \( \frac{\partial I(f^*_s, N)}{\partial f_s} < \frac{\partial I(f^*_s, N)}{\partial f_s} \) and \( \frac{\partial I(f^*_s, n^*_s)}{\partial f_s} < \frac{\partial I(f^*_s, \hat{n}_s)}{\partial f_s} \) then hold under \( n^*_s < \hat{n}_s \). However, (46) can still be satisfied even when \( f^*_s > \hat{f}_s \), because \( \frac{\partial I(f^*_s, n^*_s)}{\partial f_s} < \frac{\partial I(\hat{f}_s, \hat{n}_s)}{\partial f_s} \) is still possible under \( n^*_s < \hat{n}_s \), so that a contradiction of \( f^*_s > \hat{f}_s \) is not established. So, we cannot rule out the \( f^*_s > \hat{f}_s \) outcome, and therefore the relative sizes of \( f_s \) values are ambiguous.

## C Proof of Proposition 3

Following the same logic as in the proof of Proposition 2, to show \( n^*_s < n^*_{s*} \), we have to establish a condition for a contradiction of \( n^*_s \geq n^*_{s*} \), which is written

\[
-N z'(E^*)(-f^*_c + \beta f^*_s) \leq (t^*_{c*} - t^*_c) - (t^*_{s*} - t^*_s) - n^*_s \frac{\partial I(f^*_s, n^*_s)}{\partial n_s} + n^*_s \frac{\partial I(f^*_s, n^*_s)}{\partial m_s}. \tag{47}
\]

However, (47) may or may not hold under \( n^*_s \geq n^*_{s*} \). Note first that \( f^*_s < f^*_{s*} \) holds under \( n^*_s \geq n^*_{s*} \) (the proof of this remark is the same as in Proposition 2). Then, given \( n^*_s \geq n^*_{s*} \) and \( \partial^2 I/\partial f \partial Q > 0 \), the relative sizes of \( n_s \frac{\partial I(f^*_s, n_s)}{\partial m_s} \) values are ambiguous between the * and the ** cases, and thus (47) may not hold. Hence, we cannot establish a contradiction of \( n^*_s \geq n^*_{s*} \). Since this result does not rule out \( n^*_s < n^*_{s*} \) either, the relative sizes of \( n_s \) values are ambiguous. The ambiguities in the relative sizes in \( q_c \) and \( p_c \) values then follow.
Table 1: Equilibrium values under alternative regimes (with $\nu = 0.6$ and $\theta = 0.358$)

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<td>NA</td>
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<td>NA</td>
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<tr>
<td>$E$ (Total emission)</td>
<td>38.91</td>
<td>39.02</td>
<td>11.00</td>
<td>8.93</td>
<td>8.88</td>
<td>8.80</td>
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<td>127.89</td>
<td>147.15</td>
<td>147.74</td>
<td>147.91</td>
<td>148.07</td>
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Table 2: Equilibrium values under alternative regimes (with $\nu = 0.35$ and $\theta = 0.7$)

<table>
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<tr>
<th></th>
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<tr>
<td></td>
<td>Laissez-faire</td>
<td>Toll alone</td>
<td>CAFE alone</td>
<td>Emission tax</td>
<td>Toll &amp; CAFE</td>
<td>Emission tax</td>
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<tr>
<td>Congestion toll</td>
<td>No</td>
<td>Yes</td>
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<td>No</td>
<td>No</td>
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<td>No</td>
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<td>48.77</td>
<td>54.63</td>
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<td>9.76</td>
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<td>NA</td>
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<td>NA</td>
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<tr>
<td>$\beta Nz'(E)f_s$ (Emission tax in s)</td>
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<td>NA</td>
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<td>0.0049</td>
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<td>$E$ (Total emission)</td>
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<td>2.73</td>
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<td>1.08</td>
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<td>1.09</td>
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<td>138.43</td>
<td>138.62</td>
<td>139.36</td>
<td>139.28</td>
<td>139.46</td>
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</table>
Figure 1: Regional Map
Figure 2: Total utility as a function of $K$

Figure 3: Total commuting cost and total emission as a function of $K$