A Note on Endogenous Growth with Public Capital

Bhattacharyya, Chandril

Economic Research Unit, Indian Statistical Institute

5 May 2014

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Chandril Bhattacharyya[*]

Economic Research Unit,

Indian Statistical Institute,

203, B.T. Road,

Kolkata 700 108,

India.

20/08/2015

Abstract:

This paper develops a two sector model of endogenous economic growth with public capital where private goods and public investment goods are produced with different production technologies. The government buys public investment goods produced by private producers; and the government is a monopsonist in this market to determine the price. However, growth rate maximising buying price of public investment good is not identical with the competitive price of the final good and the growth rate maximising income tax rate in the steady state equilibrium is independent of the technology in public good production. It is also shown that the welfare maximising solution is not necessarily identical to the growth rate maximising solution even in the steady state equilibrium.

JEL classification: H41; H21; O41

Keywords: Income taxation; Price of public good; Endogenous growth; Steady-state equilibrium; Public capital

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[□] This is related to a part of the research work in progress for the Ph.D. degree of Indian Statistical Institute. Helpful and interesting comments received from Professor Manash Ranjan Gupta, Rajit Biswas and Dr. Sattwik Santra are acknowledged with gratitude. Remaining errors are of mine alone.

[*] E-mail address: chandrilbhattacharyya@gmail.com .

1. Introduction

The literature on growth theory emphasizes the pivotal role played by public capital in the process of fostering growth. World Bank (1994), identifies public capital as the 'wheels' of economic growth. In a seminal contribution, Barro (1990) makes the first attempt to incorporate the productive role of public infrastructure in an endogenous growth model; and also determines and analyses the properties of optimal income tax used to finance this productive public expenditure. Futagami et al. (1993), extends Barro (1990) model by considering public capital as a stock variable rather than a flow variable (as assumed by the latter). Other important contributions that followed these two models are, Eicher and Turnovsky (2000), Tsoukis and Miller (2003), Turnovsky (2000) and Irmen and Kuehnel (2009). Interestingly, in all these models, it is assumed, that producers of both the public good and the final private good use identical technology. The state buys public goods using tax revenue and then, freely provides the whole stock of public good to producers as public input.[1] Moreover, the government chooses optimal tax rate such that the rate of growth and / or the welfare level is maximized.

This type of modelling has two major problems. First of all, these models assume that the aggregate production functions of both goods are identical. In Barro's own words, "*As long as the government and the private sector have the same production functions, the results would be the same if the government buys private inputs and does its own production, instead of purchasing only final output from the private sector, as I assume.*" However, this simplifying assumption is too simple to model the real world. Productive public capitals, such as, ports, roads, bridges, dams, rail etc. may have different input elasticities than from the input elasticities of other private goods, such as agricultural products, clothing, computers, bicycles etc. In fact, a fairly large number of contributions (see for example, Pereira and Roca-Sagales (2001), Pereira and Andraz (2007), Cantos et al. (2005), Ammad and Ahmed (2013), Annala et al. (2004) and Feng and Serletis (2013)) have empirically shown that output elasticities of public capital are very different for different sectors. For example, Pereira and Roca-Sagales (2001) has shown that the long term accumulated elasticities with respect to public capital are 0.81, 1.23 and 0.37 in manufacturing sector, construction sector and in service sector respectively.[2] This implies that production functions are different for different sectors. Since aggregate production functions of public capital and of final private goods are weighted averages of production functions of these sectors with different sets of weights; so identical aggregate production functions for final private good and public capital is hardly possible. This in turn implies that it is important to derive the properties of optimal income tax rate where private goods and public goods are produced with different production technologies. Few papers, such as Dasgupta (1999, 2001), Dasgupta and Shimomura (2006),

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[1] In Barro's own words, "*But conceptually, it is satisfactory to think of the government as doing no production and owning no capital. Then the government just buys a flow of output (including services of highways, sewers, battleships, etc.) from the private sector. These purchased services, which the government makes available to households, correspond to the input that matters for private production ……..*".

[2] Though these are long term accumulated elasticities with respect to public capital, but their very different values clearly indicate that the direct output elasticities with respect to public capital are very different. Otherwise, the long term accumulated elasticities would have been the same.

Pintea and Turnovsky (2006), Turnsovsky and Pintea (2006) consider different production functions for producing private goods and public goods. However, Pintea and Turnovsky (2006) and Turnsovsky and Pintea (2006) do not derive the optimal tax rate analytically. On the other hand, Dasgupta (2001) and Dasgupta and Shimomura (2006) do not consider income taxation[3]. Only Dasgupta (1999), derives the optimal tax rate. However, Dasgupta (1999) shows that the optimal income tax rate is zero and the government should earn the entire revenue only by charging the private sector firms for usage of public services on a per unit basis. This may be impossible to implement when public services are non-rival and non-excludable in nature, since, firms will try to take a free ride. So Barro (1990) model's idea of freely distributing services of public capital and of charging income taxes to finance its cost is better from the viewpoint of implementation.

The second problem with Barro (1990) type of modelling is more severe. In this entire genre of literature, it is assumed that the government buys public goods from private producers at a given price and this price is equal to the competitive price of the final good. However, why the government should act as a price-taker is not clear. The government is the only buyer; and so it should act as a monopsonist and thus use the relative price as a tool to maximize its objective.

These two issues motivate us to develop the present model. Otherwise building closely on Futagami et al. (1993), we assume a two sector economy with different production functions for producing the final good and a public investment good. Here, we attempt not only to analyse the properties of optimal income tax rate used to finance investment in public capital but also analyse the properties of the optimal buying price of the public investment good. In this model, the private sector produces public investment good and sells it to the government who has a monopsony power to set the buying price. Thus, this price is also used to control allocation of resources between these two sectors.

We derive many interesting results from this model. First of all, unlike Barro (1990) type of modelling with one sector, the government cannot control the quantity of productive public capital formation using income tax rate. Rather, the government can affect intersectoral allocation of resources by altering the buying price of public investment good. This is so because, in a two sector economy, resource allocation depends upon the marginal productivity of the resources across sectors. As the income tax rate reduces the value of marginal productivity in the same ratio across all sectors and the value of marginal productivity depends upon the price of the good produced, so the government in a two sector economy can affect the intersectoral allocation of resources by altering the buying price of the public investment good via the value of marginal productivity. Secondly, growth rate maximising buying price of public investment good is not necessarily equal to the competitive price of final good. In fact, if we assume identical production functions for both goods like Barro (1990) type of modelling, then also this growth rate maximising buying price of public investment good is not necessarily equal to the competitive price of the final good. This result stems from the fact, that in case the productivity of private capital is lesser

[3] Dasgupta and Shimomura (2006) considers lump sum taxes but not per unit income tax.

(greater) than the productivity of public capital, a benevolent state would choose an optimal price of the public investment good greater (lesser) than the price of the final good to attract (drive away) resources and to enhance its (final good's) production. Thus, the optimal buying price of the public investment good can become a mechanism for promoting economic growth. Thirdly, the growth rate maximising income tax rate is equal to the elasticity of output with respect to public capital in the production of final private goods only but is independent of the production technology to produce public investment good. This is so because, public investment good sector uses public capital as input only to produce additional public capital. There exists only one final good sector to receive the service of public capital free of cost. If there is exchange, it is optimal for the final good sector to buy public investment good at the competitive price. So in the absence of exchange, it is optimal to charge a tax rate which is equal to the competitive output share of public capital in the final good sector. As a result, optimal tax rate is independent of the production technology of public capital. Lastly, welfare maximising buying price of public capital, welfare maximising income tax rate and welfare maximising allocation of private capital are different from their corresponding growth rate maximising values even in the steady state growth equilibrium. However, if we consider identical production functions for both goods, then welfare maximising solutions and growth rate maximising solutions become identical. Since Barro (1990) and Futagami et al. (1993) consider identical production functions for both goods, so in those models, growth rate maximising fiscal policies are identical with welfare maximising fiscal policies in the steady state equilibrium. Thus the present model generalises these previous result. These results are new in the literature of endogenous growth with public capital.

Rest of the paper is organized as follows. Section 2 describes the structure of the model. Section 3 deals with steady state growth equilibrium and growth rate maximizing policies. Section 4 compares between growth rate maximizing fiscal policies and optimal (welfare maximizing) fiscal policies in the steady state equilibrium; and section 5 concludes the paper.

2. The Model

The representative household-producer produces both final good and public investment good using private capital and public capital. Public investment good is defined as the additional stock of non-rival public capital. Production functions of two sectors with different technologies are given by

$$Y = A(\theta K)^{\alpha} G^{1-\alpha} \quad \text{where} \quad \alpha \in (0,1) \quad \text{and} \quad A > 0 \quad ; \tag{1}$$

and

$$\dot{G} = B[(1-\theta)K]^{\beta} G^{1-\beta} \quad \text{where} \quad \beta \in (0,1) \quad \text{and} \quad B > 0 \quad . \tag{2}$$

Here, Y, K, G and θ denote level of output of final good, stock of private capital, stock of public capital and the share of private capital allocated to production of final goods respectively. $\dot{G}$ represents the level of output of public investment good. The government sets the relative price of $\dot{G}$; and the household–producer determines the allocation of resources between production of these two goods. Public capital does not depreciate over time.

The government buys all $\dot{G}$ at the relative price, μ; and freely provides the whole stock of G to the household-producers, thereby incurring an expenditure equal to $\mu\dot{G}$. On the other hand, the government charges an income tax at the rate, τ, on the representative household producer's total income, $(Y + \mu\dot{G})$. So the government's balanced budget equation becomes

$$\tau Y + \tau\mu\dot{G} = \mu\dot{G} \ . \tag{3}$$

The representative household is infinitely lived; and she derives instantaneous utility from consumption of final goods only; and maximizes her discounted present value of instantaneous utility subject to her intertemporal budget constraint. The optimization problem is given by the following.

$$Max \int_0^\infty \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t}\, dt \tag{4}$$

subject to, $\dot{K} = (1-\tau)Y + (1-\tau)\mu\dot{G} - c$; (5)

$K(0) = K_0$;

and $\theta \in [0\,,1]$.

Here c is the level of consumption of the final good and K_0 is historically given initial private capital stock. σ represents the elasticity of marginal utility with respect to consumption and ρ denotes the constant rate of discount. Savings is always invested; and there is no depreciation of private capital.

Here c and θ are two control variables and K is the only state variable. Solving this dynamic optimisation problem, we obtain[4]

$$(1-\tau)A\alpha\theta^{\alpha-1}K^{\alpha}G^{1-\alpha} = \mu(1-\tau)B\beta(1-\theta)^{\beta-1}K^{\beta}G^{1-\beta} \quad ; \tag{6}$$

and

$$\frac{\dot{c}}{c} = \frac{(1-\tau)A\alpha\theta^{\alpha}K^{\alpha-1}G^{1-\alpha} + \mu(1-\tau)B\beta(1-\theta)^{\beta}K^{\beta-1}G^{1-\beta} - \rho}{\sigma} \quad . \tag{7}$$

Equation (6) shows the efficient allocation of private capital between two sectors. It implies that the after tax value of the marginal product of private capital is same in both these two sectors. Equation (7) describes the demand rate of growth of consumption which is defined as

[4] Derivation of equations (6) and (7) are shown in the appendix.

the excess of after tax marginal return of private capital accumulation over the rate of discount normalized with respect to the elasticity of marginal utility.

3. The Steady State Equilibrium

The equations of motion of the system are given by equations (2), (5) and (7). In the steady-state growth equilibrium,

$$g = \frac{\dot{G}}{G} = \frac{\dot{K}}{K} = \frac{\dot{c}}{c} \qquad , \tag{8}$$

where g is the balanced growth rate of the economy.

Now, from equation (6), we obtain

$$\frac{(1-\theta)^{1-\beta}}{\theta^{1-\alpha}} = \frac{\mu B\beta}{A\alpha}\left(\frac{K}{G}\right)^{\beta-\alpha} \qquad ; \tag{6a}$$

Equation (6a) shows that the allocation share of private capital to the final goods sector, θ, is independent of the income tax rate, τ. This implies that the government cannot control the allocation of resources between two sectors and thereby production of public investment good using τ. In Barro (1990), Futagami et al. (1993) and in their extended one sector models, the income tax rate, τ, is a tool to determine the level of production of public investment good. However, in this model, τ does not play any such role but the relative price, μ, can be used as a tool for this purpose. As equation (6a) shows, that a change in μ has two different effects on θ in the steady – state equilibrium. First, it has a direct negative effect obtained for a given value of (*K/G*), i.e., the ratio of two types of capital. However, (*K/G*) also changes with a change in μ[5]. So in the case of identical production technologies, i.e., $A = B$ and $\alpha = \beta$, the effect through change in (*K/G*) vanishes and θ varies inversely with the buying price of public investment good, μ. This is shown below.

$$\left(\frac{1-\theta}{\theta}\right)^{1-\alpha} = \mu \quad . \tag{6b}$$

The intuition behind this result is as follows. The income tax rate reduces the marginal productivity of capital in both sectors in the same proportion. So θ becomes independent of τ. However, given other things constant, an increase in μ raises the after tax value of marginal product of private capital in the public good producing sector; and so the share of private capital is increased in that sector. Equation (6b) also shows that if we assume $\mu = 1$ along with identical production functions like Barro (1990) type of models, then θ = ½. This in turn implies that, if the relative price is equal to unity, private capital will be allocated equally between two sectors.

[5] We cannot determine analytically the effect of μ on the steady state (*G/K*). However, this effect is not important for the main results of this paper and so we overlooked it.

Using equations (1), (2) and (3), we obtain

$$\left(\frac{\tau}{1-\tau}\right)=\frac{\mu B(1-\theta)^{\beta}}{A\theta^{\alpha}}\left(\frac{K}{G}\right)^{\beta-\alpha} \qquad . \tag{9}$$

Using equations (6a) and (9), we obtain

$$\tau=\frac{\alpha(1-\theta)}{\alpha(1-\theta)+\beta\theta}<1 \qquad . \tag{9a}$$

Equation (9a) shows that τ varies inversely with θ. Equations (6a) and (9a) simultaneously indicate that a change in μ has a direct positive effect obtained for a given (*K/G*), and an ambiguous indirect effect working through change in (*K/G*). However in the case of identical production functions, τ becomes independent of (*K/G*) and varies positively with μ. This is so because an increase in the relative buying price, μ, raises the government expenditure in a higher proportion than the government revenue; and to balance the budget, the revenue must rise. A rise in μ is also associated with a fall in θ and hence a fall in Y and a rise in $\dot{G}$. So the tax rate, τ, must rise to balance the budget. Since a fall in θ is associated with a fall in Y and a rise in $\dot{G}$, so τ is inversely related with θ.

The above discussion can be summarized as the following proposition.

Proposition 1: Government cannot (can) affect intersectoral allocation of private capital by changing the income tax rate (relative price of public investment good). In case of identical production functions, the budget balancing income tax rate as well as the allocative share of private capital to the public investment good producing sector vary positively with the government's buying price of public investment good.

Now, Using equations (2) and (8), we have

$$\left(\frac{G}{K}\right)=\frac{(1-\theta)B^{\frac{1}{\beta}}}{g^{\frac{1}{\beta}}} \qquad . \tag{2a}$$

Now using equations (2a), (6), (7), (8) and (9), we have[6]

$$\rho+\sigma g=\frac{\beta B^{\frac{1}{\beta}}\mu g^{\frac{\beta-1}{\beta}}}{1+\alpha\left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}\beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} \qquad . \tag{10}$$

Equation (10) solves for the balanced growth rate, *g*; and this equation also shows the nature of the relationship between the buying price of the public investment good, μ, and the balanced growth rate, *g*. Here μ is not a parameter in this model. μ is an instrument to solve

[6] Derivation of equation (10) is shown in appendix.

the optimisation problem of the government. Ideally, the government's objective should be to maximise the welfare level of the representative household, ω, given by

$$\omega = \underset{\{c\}}{Max} \int_0^\infty \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t}\, dt \qquad . \tag{11}$$

Unfortunately, we cannot solve for the welfare maximising buying price of the public investment good due to technical complications. Rather, we solve for its steady-state equilibrium growth rate maximising solution in this section; and examine, in the next section, whether it deviates from its welfare maximising solution. Now we maximize g given by equation (10) with respect to μ; and, using the first order condition, we obtain the following.[7]

$$\mu = \frac{A(1-\alpha)^{1-\alpha}}{\alpha^{1-2\alpha} B^{\frac{\alpha}{\beta}} g^{\frac{\beta-\alpha}{\beta}} \beta^{\alpha}} \qquad . \tag{12}$$

Using equations (10) and (12), we have

$$(\rho + \sigma g) g^{\frac{1-\alpha}{\beta}} = A(1-\alpha)^{1-\alpha} \beta^{1-\alpha} \alpha^{2\alpha} B^{\frac{1-\alpha}{\beta}} \qquad . \tag{13}$$

Equation (13) solves for the maximum value of g, which is the endogenous rate of growth of the economy in the steady-state equilibrium.

Denoting this maximum value of g by g^* and putting it in equation (12), we obtain[8]

$$\mu^* = \frac{A(1-\alpha)^{1-\alpha}}{\alpha^{1-2\alpha} B^{\frac{\alpha}{\beta}} (g^*)^{\frac{\beta-\alpha}{\beta}} \beta^{\alpha}} \qquad . \tag{14}$$

This equation (14) shows that the growth rate maximising μ is not necessarily equal to unity, i.e., the competitive price of the final good. Even if we consider identical production technology in both the sectors, i.e., $A = B$ and $\alpha = \beta$, then also

$$\mu^* = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \qquad ; \tag{14a}$$

and hence $\mu^* = 1$ if and only if $\alpha = 1/2$, i.e., if and only if production function is symmetric in terms of its arguments. This equation also shows that μ^* varies inversely with α. This is so because, suppose α decreases, thereby implying that private capital, K, (public capital, G,) becomes less (more) productive. The optimal value of the price of the public investment good, μ^* rises in order to allocate more resources towards public capital formation. So the government may set higher buying prices of public investment good to enhance economic

[7] Derivation of equation (12) is shown in the appendix.

[8] The second order condition of maximisation of growth rate with respect to μ is satisfied. From equation (10), it can be shown very easily that $\frac{d^2 g}{d\mu^2} < 0$ when equation (12) holds.

growth even the cost of its provision is higher. This result is stated in the following proposition.

Proposition 2: The steady-state equilibrium growth rate maximising buying price of public investment good is not necessarily equal to the competitive price of the final good. The equality is obtained if production technology in both the sectors are identical and symmetric.

In Barro (1990) and Futagami et al. (1993), where production functions are identical, this symmetry assumption is not made but the government's buying price of public good is set to be equal to the competitive price of the final good.

Equations (2), (6), (9) and (14) can be used to obtain

$$\theta^* = \frac{\alpha^2}{\alpha^2 + \beta(1-\alpha)} \quad ; \tag{15}$$

and

$$\tau^* = 1 - \alpha \quad . \tag{16}$$

θ^* represents the growth rate maximising allocation of private capital to the final goods producing sector in the steady state growth equilibrium. Equation (15) shows that θ^* varies inversely with β and positively with α. This is so because, as β (α) rises, productivity of private capital rises in the public investment good (final good) sector relative to the other sector; and, as a result, allocative share of private capital to public investment good (final good) sector goes up. In the case of identical production technology, $\theta^* = \alpha$. This is stated in the following proposition.

Proposition 3: The growth rate maximising allocative share of private capital to final good (public investment good) producing sector varies positively (inversely) with the private capital elasticity of output of final good and varies inversely (positively) with the private capital elasticity of output of public investment good.

Equation (16) does not involve β. So this leads to the following proposition.

Proposition 4: The steady state equilibrium growth rate maximising income tax rate is equal to the elasticity of output of final good with respect to public capital but is independent of the production technology in the public investment good producing sector.

Public investment good sector uses public capital as input only to produce additional public capital. There exists only one final good sector to receive the service of public capital free of cost. If there is exchange, it is optimal for the final good sector to buy public investment good at the competitive price. So in the absence of exchange, it is optimal to charge a tax rate which is equal to the competitive output share of public capital in the final good sector.

In Barro (1990) and Futagami et al. (1993), input elasticities of output are same in both the sectors. So this problem does not arise.

4. Welfare Maximization

In this section, we examine whether the growth rate maximising buying price of public investment good is identical with the welfare maximising buying price of public investment good. We use equations (1), (2), (5), (6), (7), (9) and (11) to obtain the welfare level of the representative household, denoted by ω. This is identical to her discounted present value instantaneous utilities over the infinite horizon. It is derived as[9]

$$\omega = \frac{\left[\frac{\rho}{\alpha} + g\left(\frac{\sigma}{\alpha} - 1\right) + \frac{(\alpha-\beta) g^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}} A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha-2}{1-\alpha}} \mu^{\frac{2-\alpha}{1-\alpha}} B^{\frac{1}{\beta(1-\alpha)}} \beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta + \frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right]^{1-\sigma}}{{K_0}^{\sigma-1}(1-\sigma)[\rho - g(1-\sigma)]} + constant \quad . \tag{17}$$

If $\sigma > \alpha$ and if $\rho - g(1-\sigma) > 0$, then equation (17) shows that ω varies positively with g when $\alpha = \beta$. So the growth rate maximising solution is identical to the welfare maximising solution in the steady state equilibrium when $\alpha = \beta$, i.e., production technologies are identical in these two sectors. However, when $\alpha \neq \beta$, i.e., when production technologies are not identical, then the welfare maximising solution is not identical to the growth rate maximising solution even in the steady state equilibrium. From equation (17), we differentiate ω with respect to μ and then evaluate it at $\mu = \mu^*$. Hence we obtain[10]

$$\left.\frac{d\omega}{d\mu}\right|_{\mu=\mu^*} = \left\{\frac{\left[\frac{\rho}{\alpha} + g^*\left(\frac{\sigma}{\alpha} - 1\right) + \frac{A(\alpha-\beta) {g^*}^{\frac{\alpha-1}{\beta}} B^{\frac{1-\alpha}{\beta}} \beta^{1-\alpha}}{[\alpha^2 + \beta(1-\alpha)](1-\alpha)^{\alpha-2}\alpha^{1-2\alpha}}\right]^{-\sigma}}{{K_0}^{\sigma-1}[\rho - g^*(1-\sigma)]}\right\} \cdot \left\{\frac{(\alpha-\beta) {g^*}^{\frac{\beta-1}{\beta}} B^{\frac{1}{\beta}} \alpha^2 \beta}{[\alpha^2 + \beta(1-\alpha)]^2}\right\} \quad . \tag{18}$$

We assume $\sigma > \alpha$ and $\rho > g^*(1-\sigma)$. This ensures that the right hand side of equation (18) is positive (zero) (negative) when α > (=) (<) β[11]. This implies that the welfare maximising value of μ is higher (lower) than the growth rate maximising value of μ even in the steady

[9] See appendix for derivation of equation (17).

[10] See appendix for derivation of equation (18).

[11] When $\beta > \alpha$, then also the first term in the R.H.S. of equation (18) is positive as c_0 cannot be negative.

state equilibrium when the final private good sector is more (less) private capital intensive than the public investment good sector. We refer welfare maximising μ as $\bar{\mu}$.

Now, we compare growth rate maximising solutions τ^* and θ^* to welfare maximising solutions $\bar{\tau}$ and $\bar{\theta}$. When $\alpha > \beta$, then $\left.\frac{d\omega}{d\mu}\right|_{\mu=\mu^*}$ is positive and as a result, $\bar{\mu} > \mu^*$. So the growth rate corresponding to $\bar{\mu}$, i.e., $\bar{g}$, is less than g^* as g^* is the maximum value of balanced growth rate. As a result, $\bar{\mu}\bar{g}^{\frac{\beta-\alpha}{\beta}} > \mu^* g^{*\frac{\beta-\alpha}{\beta}}$.

Using equations (2a), (6a) and (9), we obtain[12]

$$\theta = \frac{1}{1 + \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}} \quad ; \tag{19}$$

and

$$\tau = \frac{\alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}}{\alpha \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}} + \beta} < 1 \quad . \tag{20}$$

Equations (19) and (20) show that θ and τ vary inversely and positively with $\mu g^{\frac{\beta-\alpha}{\beta}}$ respectively. So welfare maximising θ, i.e., $\bar{\theta}$, is less than θ^* but welfare maximising τ, i.e., $\bar{\tau}$, is higher than τ^*. Similarly, when $\beta > \alpha$, then $\mu^* > \bar{\mu}$ and $g^* > \bar{g}$. This implies that, $\bar{\mu}\bar{g}^{\frac{\beta-\alpha}{\beta}} < \mu^* g^{*\frac{\beta-\alpha}{\beta}}$; and as a result, $\bar{\theta}$ is greater than θ^* but $\bar{\tau}$ is less than τ^*.

Barro (1990) and Futagami et al. (1993) show that growth rate maximising income tax rate is identical to the welfare maximising income tax rate in the steady state equilibrium. However, we find that the welfare maximising solution is different from the growth rate maximising solution even in the steady state equilibrium when we consider different production functions for different goods. However these two solutions are always identical with identical production technology. So our result generalises the result of Barro (1990) and Futagami et al. (1993). This result is stated in the following proposition.

Proposition 5: When the final good sector is more (less) private capital intensive than the public investment good sector, welfare maximising buying price of public investment good, income tax rate and the allocation share of private capital to the public investment good sector exceeds (falls short of) their corresponding growth rate maximising values even in the steady state equilibrium.

[12] See appendix for derivation of equations (19) and (20).

5. Conclusions

This paper constructs a simple two sector endogenous growth model with public capital; and derives the properties of optimal fiscal policies in the steady state equilibrium. Both the final good and the public investment good are produced by the private sector using different production technologies. However, in this model, the government buys public good from private producers at a monopsony price and this buying price is a tool to control allocation of resources between these two sectors. This is how the present model differs from models like Barro (1990), Futagami et al. (1993) etc.

Various interesting findings are obtained here. First, the government can affect inter-sectoral allocation of private capital not by changing the income tax rate but by altering the buying price of public investment good. Secondly, growth rate maximising buying price of public investment good is not necessarily equal to the competitive price of the final good even in the case with identical production technologies. Thirdly, the growth rate maximising income tax rate is equal to the elasticity of output of final good with respect to public capital but is independent of the production technology of public good. At last, welfare maximising solutions are different from growth rate maximising solutions even in the steady state equilibrium.

This model can be extended in various possible directions. One very pertinent direction will be to incorporate the congestion effect of capital on productivity. Moreover, non-productive public services can directly affect households' utility. Political incentives remain a powerful alternative that can replace our assumption of a benevolent government in this set up. All these remain as possible projects for future research.

References

Ammad, S., & Ahmed, Q. (2013). *Dynamic Effects of Energy Sector Public Investment on Sectoral Economic Growth: Experience from Pakistan Economy.* presented at the 29th AGM of Pakistan Society of Development Economist.

Annala, C. N., Batina, R. G., & Feehan, J. P. (2004). *ESRI International Collaboration Project - The Impact of Public Inputs on the Private Economy: Further Evidence for the Japanese Economy; Part 2: Public Inputs and the Private Economy: A study of Japan.* Retrieved from http://www.esri.go.jp/jp/prj-rc/macro/macro15/09-2-R.pdf

Barro, R. J. (1990). Government Spending in a Simple Model of Endogeneous Growth. *Journal of Political Economy, 98(5)*, S103-S125.

Cantos, P., Gumbau-Albert, M., & Maudos, J. (2005). Transport infrastructures and regional growth: evidence of the Spanish case. *MPRA Paper No. 15261.* University Library of Munich, Germany.

Dasgupta, D. (1999). Growth Versus Welfare in a Model of Nonrival Infrastructure. *Journal of Development Economics, 58(2)*, 359–385.

Dasgupta, D. (2001). Lindahl Pricing, Nonrival Infrastructure, and Endogenous Growth. *Journal of Public Economic Theory, 3(4)*, 413–430.

Dasgupta, D., & Shimomura, K. (2006). Public infrastructure, employment and sustainable growth in a small open economy with and without foreign direct investment. *The Journal of International Trade & Economic Development: An International and Comparative Review 15(3)*, 257-291.

Eicher, T., & Turnovsky, S. J. (2000). Scale, Congestion and Growth. *Economica 67(267)*, 325-346.

Feng, G., & Serletis, A. (2013). Public Infrastructure And Externalities In US Manufacturing: Evidence From The Price-Augmenting Aim Cost Function. *Macroeconomic Dynamics, 17(7)*, 1367-1410.

Futagami, K., Morita, Y., & Shibata, A. (1993). Dynamic Analysis of an Endogenous Growth Model with Public Capital. *The Scandinavian Journal of Economics, 95(4)*, 607-625.

Irmen, A., & Kuehnel, J. (2009). Productive Government Expenditure and Economic Growth. *Journal of Economic Surveys, 23(4)*, 692–733.

Pereira, A. M., & Andraz, J. M. (2007). Public investment in transportation infrastructures and industry performance in Portugal. *Journal of Economic Development, 32(1)*, 1-20.

Pereira, A. M., & Roca-Sagales, O. (2001). Infrastructures and private sector performance in Spain. *Journal of Policy Modeling, 23(4)*, 371–384.

Pintea, M., & Turnovsky, S. J. (2006). Congestion and Fiscal Policy in a Two-Sector Economy with Public Capital: A Quantitative Assessment. *Computational Economics 28(2)*, 177-209.

Tsoukis, C., & Miller, N. J. (2003). Public services and endogenous growth. *Journal of Policy Modeling, 25(3)*, 297–307.

Turnovsky, S. J. (2000). Fiscal policy, elastic labor supply, and endogenous growth. *Journal of Monetary Economics, 45(1)*, 185–210.

Turnovsky, S. J., & Pintea, M. (2006). Public and private production in a two-sector economy. *Journal of Macroeconomics 28(2)*, 273–302.

World Bank. (1994). *World Development Report 1994 : Infrastructure for development - executive summary.*

Appendix:

Derivation of equations (6) and (7):

Using equations (4) and (5), we construct the Current Value Hamiltonian as given by

$$H_c = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda\big[(1-\tau)Y + (1-\tau)\mu\dot{G} - c\big] \quad . \qquad (A.1)$$

Here λ is the co-state variable. Incorporating equations (1) and (2) in equation (A.1); and then maximising it with respect to c and θ, we obtain following first order conditions.

$$c^{-\sigma} - \lambda = 0 \quad ; \qquad (A.2)$$

and

$$\lambda(1-\tau)A(K)^{\alpha}G^{1-\alpha}\alpha\theta^{\alpha-1} = \lambda\mu(1-\tau)B[K]^{\beta}G^{1-\beta}\beta(1-\theta)^{\beta-1} \quad . \qquad (A.3)$$

From equation (A.3), we obtain equation (6) in the body of the paper.

Again from equation (A.1), we have

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1-\tau)AK^{\alpha-1}G^{1-\alpha}\alpha\theta^{\alpha} - \mu(1-\tau)BK^{\beta-1}G^{1-\beta}\beta(1-\theta)^{\beta} \quad ; \qquad (A.4)$$

and from equation (A.2), we have

$$\frac{\dot{\lambda}}{\lambda} = -\sigma\frac{\dot{c}}{c} \quad . \qquad (A.5)$$

Using equations (A.4) and (A.5), we have equation (7) in the body of the paper.

Derivation of equation (10):

From equation (7), we have

$$\rho + \sigma g = (1-\tau)A\alpha\theta^{\alpha}\left(\frac{G}{K}\right)^{1-\alpha} + \mu(1-\tau)B\beta(1-\theta)^{\beta}\left(\frac{G}{K}\right)^{1-\beta} \quad . \qquad (A.6)$$

From equation (2), we have

$$\left(\frac{G}{K}\right) = \frac{B^{\frac{1}{\beta}}(1-\theta)}{g^{\frac{1}{\beta}}} \quad . \qquad (A.7)$$

From equations (2), (6), (9), (A.6) and (A.7), we obtain equation (10) in the body of the paper.

Derivation of equation (12):

Taking log on both sides of equation (10) and then differentiating it with respect to μ and assuming $\frac{dg}{d\mu} = 0$, we obtain

$$\frac{1}{\mu}=\frac{\frac{\alpha}{1-\alpha}\mu^{\frac{\alpha}{1-\alpha}}\left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}\beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{1}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}{1+\alpha\left[\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}\beta^{\frac{\alpha}{1-\alpha}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]} \quad . \qquad (A.8)$$

From equation (A.8), we obtain equation (12) in the body of the paper.

<u>Derivation of equation (17):</u>

From equation (11), we obtain

$$\omega=\frac{{c_0}^{1-\sigma}}{[\rho-g(1-\sigma)](1-\sigma)}+constant \quad . \qquad (A.9)$$

Here, $c(0)=c_0$.

From equation (5), we obtain

$$c_0=K_0\left\{(1-\tau)A(\theta)^{\alpha}\left(\frac{G_0}{K_0}\right)^{1-\alpha}+(1-\tau)\mu B(1-\theta)^{\beta}\left(\frac{G_0}{K_0}\right)^{1-\beta}-g\right\} \quad . \qquad (A.10)$$

Using equations (7) and (A.10), we obtain

$$c_0=K_0\left\{\frac{\rho+\sigma g}{\alpha}+(1-\tau)\mu B(1-\theta)^{\beta}\left(\frac{G_0}{K_0}\right)^{1-\beta}\left(\frac{\alpha-\beta}{\alpha}\right)-g\right\} \quad . \qquad (A.11)$$

Using equations (2) and (A.11), we obtain

$$c_0=K_0\left\{\frac{\rho+\sigma g}{\alpha}+(1-\tau)\mu B(1-\theta)\left(\frac{B^{\frac{1}{\beta}}}{g^{\frac{1}{\beta}}}\right)^{1-\beta}\left(\frac{\alpha-\beta}{\alpha}\right)-g\right\} \quad . \qquad (A.12)$$

Using equations (2), (6), (9) and (A.12), we obtain

$$c_0=K_0\left\{\frac{\rho}{\alpha}+g\left(\frac{\sigma}{\alpha}-1\right)+\frac{(\alpha-\beta)g^{\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}}A^{\frac{1}{\alpha-1}}\alpha^{\frac{\alpha-2}{1-\alpha}}\mu^{\frac{2-\alpha}{1-\alpha}}B^{\frac{1}{\beta(1-\alpha)}}\beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1+\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta+\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right\} \quad . \qquad (A.13)$$

Using equations (A.9) and (A.13), we obtain equation (17) in the body of the paper.

<u>Derivation of equation (18):</u>

Differentiating equation (17) with respect to μ and evaluating it at $\mu = \mu^*$, we obtain

$$
\left.\frac{d\omega}{d\mu}\right|_{\mu=\mu^*}
= \left\{\frac{\left[\frac{\rho}{\alpha}+g^*\left(\frac{\sigma}{\alpha}-1\right)+\frac{(\alpha-\beta)g^{*\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}}A^{\frac{1}{\alpha-1}}\alpha^{\frac{\alpha-2}{1-\alpha}}\mu^{*\frac{2-\alpha}{1-\alpha}}B^{\frac{1}{\beta(1-\alpha)}}\beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1+\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta+\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right]^{-\sigma}}{{K_0}^{\sigma-1}[\rho-g^*(1-\sigma)]}\right\}
$$

$$
\cdot\left\{\frac{(\alpha-\beta)g^{*\frac{2\beta-1-\alpha\beta}{\beta(1-\alpha)}}A^{\frac{1}{\alpha-1}}\alpha^{\frac{\alpha-2}{1-\alpha}}\mu^{*\frac{2-\alpha}{1-\alpha}}B^{\frac{1}{\beta(1-\alpha)}}\beta^{\frac{2-\alpha}{1-\alpha}}}{\left[1+\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]\left[\beta+\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right\}
$$

$$
\cdot\left\{\left(\frac{2-\alpha}{1-\alpha}\right)\left(\frac{1}{\mu^*}\right)-\frac{\frac{\mu^{*\frac{\alpha}{1-\alpha}}}{1-\alpha}\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}}{\left[1+\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}-\frac{\frac{\mu^{*\frac{\alpha}{1-\alpha}}}{1-\alpha}\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}}{\left[\beta+\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]}\right\}. \quad (A.14)
$$

Now, from equations (2), (6) and (9), we find that the last bracket term is equal to $\left(\frac{1}{\mu^*}\right)\left\{\frac{2-\alpha}{1-\alpha}-\frac{1}{1-\alpha}[(1-\theta^*)+\tau^*]\right\}$. Again, from equations (2) and (6), it appears that $\left[1+\frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]$ is equal to $\left(\frac{1}{\theta^*}\right)$; and from equations (2), (6) and (9), we find that $\left[\beta+\frac{\alpha B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g^*)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}}\left(\frac{\mu^*\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}}\right]$ is equal to $\frac{\alpha(1-\theta^*)}{\theta^*\tau^*}$. Incorporating all these equalities and putting values of μ^*, τ^* and θ^* from equations (14), (15) and (16), we obtain equation (18).

Derivations of equations (19) and (20):

From equation (2), we obtain

$$\frac{G}{K} = B^{\frac{1}{\beta}}(1-\theta)g^{-\frac{1}{\beta}} \quad . \tag{A.15}$$

Using equations (6) and (A.15), we obtain

$$\frac{(1-\theta)^{1-\alpha}}{\theta^{1-\alpha}} = \mu g^{\frac{\beta-\alpha}{\beta}} \frac{B^{\frac{\alpha}{\beta}}}{A} \frac{\beta}{\alpha} \quad . \tag{A.16}$$

From equation (A.16), we obtain equation (19) in the body of the article.

Now, from equation (9) and (A.15), we obtain

$$\left(\frac{\tau}{1-\tau}\right) = \frac{\mu(1-\theta)^{\alpha}}{A\theta^{\alpha}} g^{\frac{\beta-\alpha}{\beta}} B^{\frac{\alpha}{\beta}} \quad . \tag{A.17}$$

Using equations (A.16) and (A.17), we obtain

$$\left(\frac{\tau}{1-\tau}\right) = \frac{\alpha}{\beta} \frac{B^{\frac{\alpha}{\beta(1-\alpha)}}}{(g)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}} \left(\frac{\mu\beta}{A\alpha}\right)^{\frac{1}{1-\alpha}} \quad . \tag{A.18}$$

From equation (A.18), we obtain equation (20) in the body of the paper.