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CONSUMER MODELS AND THE COMMON INFLUENCE OF INCREASING VAT AND DECREASING WEDGES

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Abstract: In this paper we study the common impact of increasing VAT and decreasing incomes in consumer models. The considered models are linear ones (see [3], [4] and [8]). It is in fact the extension of the study [2], where there was performed the study of the impact of only one of the mentioned government decision (increasing VAT). We have already noticed that applying the simple three rule is not appropriate. But the problems that arise come from the common impact. It is possible that if it is applied only the decreasing of the wedges (25%), the incomes from selling products decreases by the ratio β, if we apply only increasing VAT the income decreases by the ratio α (we have obtained in [2] $\alpha = 4.01653$ ignoring the dependence of quantity on wedges), but if there are applied both the income decreases by the ratio $\gamma \neq 1-(1-\alpha)(1-\beta)$.

This is the general case, and the explanation of such phenomenon comes from analogous reasons as in [2]: the total income is the sold quantity multiplied by the price, hence we have not linearity. Another explanation comes from the least squares method: in the obtained linear system for estimating the three parameters (intercept, coefficient of prices and coefficient of wedges) both variables influence the result.

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1. Introduction

Consider n points in R^{k+1}, $X^{(1)},\ldots,X^{(n)}$, where $X^{(i)} = \left(X_1^{(i)}, X_2^{(i)}, \ldots, X_k^{(i)}, Y_i\right)$. The regression hyper-plane has the equation (see [7,3])

$$H : Y = A_0 + \sum_{i=1}^{k} A_i X_i \text{ such that} \tag{1}$$

$$\sum_{i=1}^{n} u_i^2 \text{ is minimum,} \tag{1'}$$

where the residues u_i have the formula

$$u_i = Y_i - A_0 - \sum_{j=1}^{k} A_j X_j^{(i)}. \tag{1''}$$

For the computation of A_i from (1) we have to solve the system (see [7,4])

$$\sum_{j=0}^{k} \overline{X_i \cdot X_j} \cdot A_j = \overline{X_i \cdot Y},\ i = \overline{0,k}, \tag{2}$$

where $\overline{X_0 \cdot X_i} = \overline{X_i}$ and $\overline{X_0^2} = 1$.

If we apply the linear regression to a consumer model, the dependent variable is the sold quantity. One of the explanatory variables is the unit price, and there are some other explanatory variables for the multi-linear model, as income and advertising. As we expect, the coefficient of price is negative.

Next we will present some notion about the theory of interest in order to point out the errors that appear when we replace the multiplicative model by the additive one.

The interest for a financial operation represents the sum added periodically to the initial sum S_0 during the operation time T, until its end. The interest can be simple or compound (see [6]).

Denoting by S_t the amount at the moment t from the initial moment to the maturity, T, and by R the interest rate (the ratio of the initial sum added periodically), we obtain

$$S_t = S_0 (1 + R \cdot t). \tag{3}$$

The above interest rate can be expressed even as a number between 0 and 1 (the ratio between the periodically added sum and the initial one), even as percentage: $100 \times R = p\%$. If we want to compute the initial capital in terms of the final capital, the duration of the operation and the interest rate, we obtain

$$S_0 = \frac{S_T}{1 + R \cdot T}. \tag{3'}$$

In the case of compound interest the maturity is divided into n periods, T_1, T_2,..., T_n. The final capital is in this case (see [6])

$$S_T = S_0 \cdot \prod_{i=1}^{n} (1 + R_i T_i), \tag{4}$$

where S_0 is the initial capital, and R_i is the interest rate on the period T_i.

Usually T is divided into equal time periods, the common length of these time periods becoming time unit $(T_i = 1)$. If the interest rate is constant, R, over all the duration T, the formula (4) becomes

$$S_T = S_0 (1 + R)^T. \tag{4'}$$

Remark 1. *Sometimes the maturity,* T, *is not supposed to be integer, considering* $T = n + T_{n+1}$ *with* $T_{n+1} \in (0,1)$. *In this case the above value* S_t *is the trading solution, and the rational solution is* $S_T = S_0 (1 + R)^n \cdot (1 + R \cdot T_{n+1})$.

In the case of deposits at given term, it is used also the solution with lost interest: $S_T = S_0 (1 + R)^n$.

The initial capital is computed using (4) and (4'), as in the case of simple interest. We obtain the general formula

$$S_0 = \frac{S_T}{\prod_{i=1}^{n} (1 + R_i T_i)}, \tag{5}$$

and in the particular case $T_i = 1$ and $R_i = R$

$$S_0 = \frac{S_T}{(1 + R)^T}. \tag{5'}$$

In [2] we have pointed out some errors that can appear in consumer models. First one is that if VAT increases from 19% to 24% even the prices do not increase by 5%: the increasement of prices is only 4.20168%. Another error is to believe that the income of selling company increase by this percentage. In the application we have found that in fact the income decreases by 4.01653%.

2. The common impact

The above decrease is explained because the sold quantity Y is expressed as linear regression on prices X [3,4]

$$Y = b_0 + b_1 X, \tag{6}$$

where b_1 is negative. The explanation of the decrease of income is that the total income of the selling company is the sum of the product between X_i and Y_i, hence modifications of X_i yields to modification of Y_i.

But in [3,4] in consumer models the sold quantity Y can be expressed as multi-linear regression on wedge X_1 and the price X_2:

$$Y = a_0 + a_1 X_1 + a_2 X_2, \tag{7}$$

where a_1 is positive a_2 is negative.

The wedges, by the coefficient a_1 from (7) takes a part of the influence of the prices, but their presence modify the intercept, and by this modification they change the linear part of the model (the total income is the sum of prices multiplied by quantities, hence the linear part is the intercept multiplied by the sum of prices).

The above differences can be noticed from expressing the income Z in terms of prices in (6), namely

$$Z = b_0 X + b_1 X^2, \tag{8}$$

which is a parabolic cylinder, respectively expressing the income in terms of both variables (wedges and prices)

$$Z = a_0 \cdot X_2 + a_1 \cdot X_1 \cdot X_2 + a_2 \cdot X_2^2, \tag{9}$$

which is a hyperbolic paraboloide, because the matrix of the conic given by the plane section $Z = z_0$ is

$$A = \begin{pmatrix} 0 & \frac{a_1}{2} & 0 \\ \frac{a_1}{2} & a_2 & \frac{a_0}{2} \\ 0 & \frac{a_0}{2} & -z_0 \end{pmatrix}. \tag{10}$$

3. Application

Consider the following consumer model, where X_1 is the wedge, X_2 is the unit price and Y the sold quantity (see [3,4]).

Table 1: The consumer model of sold quantity in terms of wedge and price

X_1	3	2	0.8	2.5	2	1.4	2.5	2.5	3	1.4	1	1.2	1.6
X_2	1.3	2.8	1.5	0.2	1.8	4	1.8	2	0.5	2.8	3.2	2.5	1.3
Y	2	0.5	1.5	3	1	0.2	2.1	1.8	3	0.7	0.5	1	1.4
X_1	0.5	2.8	3.2	2.5	1.3	2.2	3.5	1.1	0.001	0.2	2	1.2	
X_2	2.2	3.5	1.1	0.1	0.2	2	1.2	3	3	0.6	3.2	0.3	
Y	1.2	0.8	2.3	3.5	3.8	1.8	2.6	0.8	1.2	4.2	0.8	2.5	

The total income maintaining the above data is 52.6975, and the linear regression is

$$Y = 1.98335 + 0.44061 X_1 - 0.6386 X_2.$$

We consider four transformations and we study the impact of both variables separately (maintaining the values of the other explanatory variable constant), respectively simultaneously (modifying both variables simultaneously). First transformation starts from initial data, the wedges decrease by 25% and the prices increase by 4.20168%.

The second transformation starts from new data obtained by the above simultaneous transformation, the wedges increase by 5% and the prices decrease by 12.09677% (in fact the last value is from decreasing VAT from 24% to 9%: ($\left(1-\frac{109}{124}\right)\cdot 100 = 12.09677$).

The last two transformations start from the data obtained after the second transformation. For the third transformation the wedges increase by 5% and the prices increase by 2.75%, and for the fourth transformation the wedges decrease by 2.5% and the prices decrease by 5%. The results are presented in the following table.

Table 2: The modifications on incomes after the four transformations

Transformation	Wedges only		Prices only		Wedges and prices		$100\left(\left(1+\frac{\alpha}{100}\right)\left(1+\frac{\beta}{100}\right)-1\right)$
	Value	Percentage	Value	Percentage	Value	Percentage	
First transformation	43.6469	-17.17462	51.63935	-2.00798	42.20847	-19.90422	-18.83774
Second transformation	43.6231	3.35153	45.73207	8.34808	46.97558	11.29419	11.9794
Third transformation	48.28126	2.77949	46.49553	-1.02191	47.83711	1.83402	1.72918
Fourth transformation	46.32273	-1.38975	47.6054	1.34075	46.9852	0.02049	-0.06763

4. Conclusions

In our example if we decrease/ increase wedge maintaining constant price we have an increasement (of course, if the income decrease we consider an increasement by a negative value) of $\alpha\%$, and analogously we have an increasement of $\beta\%$ if we modify only the prices. But if we modify both variables we have an increasement of $\gamma\%$, where $\gamma \neq 100\left(\left(1+\frac{\alpha}{100}\right)\left(1+\frac{\beta}{100}\right)-1\right)$. Some explanations of this phenomenon are the non-linearity and the fact that in the obtained linear system for estimating the three parameters (intercept, coefficient of prices and coefficient of wedges) both variables influence the result.

Another phenomenon that appears in our example, analogous to the resonance when people cross a bridge, is that in the above relation γ is greater if both measures decrease the sold quantity, namely decreasing wedges and increasing VAT. For instance in the case of decreasing wedges by 25% and increasing VAT (and prices) by 4.20168%, we have $\alpha = 17.17462$ and $\beta = 2.00798$, and $100\left(1-\left(1+\frac{\alpha}{100}\right)\left(1+\frac{\beta}{100}\right)\right) = 18.83774$. But if we apply the modifications for both variable, the decrease of income is 19.90422%.

We notice that the income is not linear in wedges and prices, but according (9) it is linear on price, price times wedges and the square of price. An open problem is to estimate the linear regression of incomes Z from the above formula in terms of the above mentioned explanatory variables. We can also test the significance of the intercept, wich does not appear in (9).

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