Frictions in Internet Auctions with Many Traders: a Counterexample

Donna, Javier and Schenone, Pablo and Veramendi, Gregory

The Ohio State University, Arizona State University, Arizona State University

1 September 2015

Frictions in Internet Auctions with Many Traders: a Counterexample

Javier Donna
The Ohio State University

Pablo Schenone
Arizona State University

Gregory Veramendi
Arizona State University

September 1, 2015

Abstract

Peters and Severinov (2006) (PS henceforth) characterize a perfect Bayesian equilibrium (PBE) in a competing auctions environment, where all buyers are linked to all the sellers. PS characterize a PBE using a simple bidding rule, whereby buyers select in which auction to bid. In this note we show that when buyers are linked with a subset of the sellers (*i.e.* when there are search frictions), the PS bidding rule is no longer guaranteed to be efficient nor a PBE of the competing auctions game of PS. Our results indicate that researchers should be cautious when using the PS bidding rule to make inference about the behavior of buyers and sellers in a market where frictions are present such as eBay.

JEL Codes: C73; C78; D44.

Keywords: Auctions; Internet; Frictions; Networks.

Javier Donna
Department of Economics
The Ohio State University
1945 N High St
417 Arps Hall
Columbus, OH 43210
Phone: 614-688-0364
Email: donna.1@osu.edu

Pablo Schenone
Department of Economics
W.P. Carey School of Business
Arizona State University
501 E. Orange Street
Tempe, AZ 85287
Phone: 480-965-5596
Email: pablo.schenone@asu.edu

Gregory Veramendi
Department of Economics
W.P. Carey School of Business
Arizona State University
501 E. Orange Street
Tempe, AZ 85287
Phone: 480-965-0894
Email: gregory.veramendi@asu.edu

1 Introduction

In a seminal paper, Peters and Severinov (2006) (PS henceforth) characterize a perfect Bayesian equilibrium (PBE) in a competing auctions game similar to buyer-seller trading platforms such as eBay, Amazon, or Taobao. In their setting, sellers offer a homogeneous good, differ in their valuation, and hold second-price auctions. Buyers differ in their valuation (which is private information) and have single unit demand. Bids increase in discrete amounts. Their environment is frictionless in the sense that any buyer may participate in any auction.

In their paper, PS characterize a PBE in the competing auctions game. The PBE bidding strategies specify that buyers bid in the auction with the lowest standing price, using a simple tie breaking rule when relevant. Having identified the auction they will bid on, bidders bid the standing price plus the minimum bid increment. (See subsection 2.2 for a full description of the game and the bidding rule.) This rule is appealing because it is based on observable market data. The only information that a bidder needs are the standing bid and whether the standing bid has changed since the last change of the winning bidder. This information is typically observable in buyer-seller trading platforms. In addition, their proposed bidding rule "constitutes a PBE in the bidding process independently of buyers' beliefs about other buyers' valuations, and even the number of other buyers. The outcome of this equilibrium is efficient provided that sellers set their reserve prices equal to their true costs. [...] The remarkable part [...] is that the outcome of the bidding process is efficient and sequentially rational (*i.e.* optimal at every information set given the traders' beliefs and their strategies), yet looks very much like a simple algorithmic price adjustment procedure." (PS p. 223).

Because the PS strategies are simple and based on easily observed data, a branch of the empirical auctions literature has used some of the theoretical predictions of the bidding rule from PS to investigate bidder behavior in online competing auctions environments such as eBay. For example, Anwar, McMillan, and Zheng (2006), empirically investigate whether bidders' behavior in eBay corresponds to the equilibrium bidding rule in PS. Bapna, Chang, Goes, and Gupta (2009), empirically investigate the prediction of the bidding rule in PS that bidders bid in multiple auctions and the resulting law of one price (*i.e.* no price price dispersion). Hasker and Sickles (2010) use the incremental bidding prediction of the bidding rule in PS as a simple explanation for sniping in eBay. Zeithammer and Adams (2010) use data on eBay auctions to reject the hypothesis that these auctions resemble second-price sealed-bid auctions and use PS bidding rule as potential model consistent with some of their findings. Backus, Podwol, and Schneider (2014) consider the case of two auctions with frictions using the framework of PS to investigate price dispersion using eBay data. A detailed discussion of the empirical evidence supporting the equilibrium bidding rule in PS can be found in the survey of online auctions by Ockenfels, Reiley, and Sadrieh (2006).

However, in internet platforms such as eBay it is costly for buyers to interact with all the sellers. Due to search costs, frictions in eBay are important.[1] Thus, it is unclear that the PS assumption of a frictionless market is appropriate when working with such data. In eBay, search frictions arise for two main reasons. First, two bidders that perform the exact same search query at a given time observe the same, say, 25 listings in the first page of results.[2] So certain sellers will rarely show up in the first page of results for most buyers. Second, buyers seldom perform the exact same search query. So the 25 products displayed in the first page will typically differ among buyers, depending

[1] This is documented for the case of internet auctions by Bajari and Hortacsu 2004, p. 483 and for eBay by Backus, Podwol, and Schneider 2014, p. 181.

[2] Twenty five is the default number of listings displayed by eBay in the first page.

on their search query and on the sellers' title for the product listing.[3] In this paper we show that when search frictions are present—so that buyers do not participate in all the auctions—the PS bidding rule is no longer guaranteed to be efficient nor a PBE of the competing auctions game of PS. Our results indicate that researchers should be cautious when using the PS bidding rule to make inference about the behavior of buyers and sellers in a market where frictions are present such as eBay.

We proceed as follows. In section 2, we describe the game and the bidding rule of PS. In section 3, we present a simple example where the bidding rule of PS produces an outcome that is no longer guaranteed to be efficient nor a PBE of the competing auctions game. Finally, in section 4 we conclude.

2 The Model

2.1 Setup

Consider a set of buyers and a set of sellers. Sellers differ in their valuation and offer one unit of an homogeneous good. Assume that sellers have no idiosyncratic preferences over the buyer they sell to. Buyers differ in their valuation and have single unit demand. A buyer with valuation ν that buys from a seller at price p has utility $\nu - p$ and 0 otherwise. The seller's utility is the price, p, if they sell the good, and their valuation, b, if they do not.

To model search frictions in buyer-seller markets we find it useful to use the formalism of bipartite networks. We think of buyer-seller markets as a bipartite network that consists of a set of sellers, a set of buyers, and a set of links connecting buyers with sellers. A buyer can obtain a good from the seller only if the two are linked. The interpretation is that when a buyer is linked with a seller, the buyer may participate in the seller's auction. When the network is fully connected (*i.e.* when all buyers are linked with all sellers) we say that the market is frictionless. In that case, the bidding strategies in PS are a PBE of the sequential auctions game. In this paper we assume that the network is common knowledge, and focus on the case where the network is not (necessarily) fully connected (*i.e.* when search frictions are present in the market).

2.2 Peters and Severinov (2006)

PS Sequential Game

The PS sequential game is as follows. Consider the setup from subsection 2.1. Sellers hold second-price auctions. Buyers (*i.e.* bidders) are ordered randomly in a queue and they arrive sequentially. When a new bidder arrives, the bidder can submit bids to one or more of the sellers to whom it is linked. After all bidders submit their bids (or decide not to bid), the bidding queue restarts. That is, bidders may sequentially update their bids (either in the same auction they bid before, or by bidding in different auctions), or decide not to. The process ends when all bidders in the queue decide not to place further bids. The buyers' valuations and sellers' valuations are distributed on the grid $\mathcal{D} \equiv \{\underline{p}, \underline{p}+\Delta, \underline{p}+2\Delta, ..., \overline{p}\}$ that has a step size $\Delta > 0$. The minimum bid increment is Δ.

For each seller, the standing bid is the second highest bid received (or the valuation if the seller received less than two bids). The standing bid is publicly observed for each seller at each moment. The highest bid is not publicly observed nor is the identity of the winner. If more than one bidder

[3] Bidders use these results to decide in which of the listed auctions to participate. Most users are reluctant to use other than the default settings in a search (Chau, Fang, and Liu Sheng 2005; Cone, Franklin, Ryan, and Stalker 2005).

submits the same bid, the winner is the bidder who submitted the bid first. In this case, the standing bid coincides with the highest bid. Because sellers hold second-price auctions, the winner in each auction is the holder of the highest bid, but this bidder only pays the standing bid.

PS Bidding Rule

Now we focus on the bidding rule proposed by PS. Because these are second-price auctions, if bidder j bids the standing bid plus Δ (the minimum bid increment), two things could happen. First, it could be that the new bid exceeds the current highest bid. In this case, bidder j becomes the highest bidder (or winning bidder) displacing the previous highest bidder. But the standing bid does not change: the second highest bid is still the original standing bid. Second, the new bid may tie with the current highest bid. In this case, bidder j does not displace the previous highest bidder. But now the highest and second highest bid coincide. That is, the winning bid and highest bidder do not change, but the standing bid increases and matches the winning bid.

To identify which sellers have the lowest winning bid (recall that due to the second-price structure, winning bids are unobservable), it is convenient to identify auctions such as the ones in the second case: auctions where the standing bid has changed since the last change in the highest bidder. These are preferable auctions.[4]

Definition (preferable auctions). *Preferable auctions are auctions that satisfy one of the two following conditions: (i) the standing bids have changed after the last change of the highest bidder or (ii) auctions that have not yet received any bids.*

Definition (PS bidding rule). *This is the PS bidding rule for bidder j. We have five cases:*

When it is the bidder's turn to bid, the bidder places no bid:

(i) If the bidder is the highest bidder at any auction, or

(ii) If the lowest standing bid (in linked auctions) is above the bidder's valuation.

Otherwise, j selects an auction to bid in as follows:

(iii) If there is a unique lowest standing bid (amongst those sellers j is linked to), the bidder submits a bid in that auction, or

(iv) If there is more than one lowest standing bid (amongst those sellers j is linked to), the bidder randomly chooses one auction among the preferable auctions, or

(v) If there is more than one lowest standing bid and no preferable auctions (amongst those sellers j is linked to), the bidder randomly chooses one auction among the ones with the lowest standing bid.

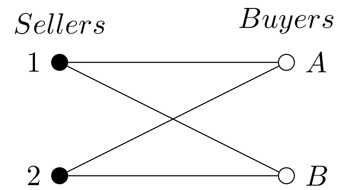
In this case, j bids the current standing bid plus Δ.

3 Two Examples

In this section we present two simple examples with two sellers and two buyers: (1) a network without frictions (fully connected) and (2) a network with frictions (not fully connected). We implement the bidding rule of PS in these two examples. In both examples the bidding order in the queue is (A, B).

[4] In this setting it is not a dominant strategy for bidders to bid their valuations in the first stage (see pp. 225-6 in PS for an example).

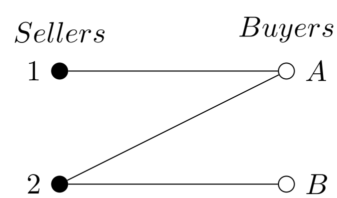
Example 1. Network without Frictions. Assume that buyers A and B are ordered in their valuations ($\nu_A > \nu_B = \Delta > 0$) and sellers 1 and 2 have the same valuation (normalized to 0). The minimum bid increment is $\Delta = \nu_B$. Consider the following network without frictions (*i.e.* fully connected network):



Round	Standing Bid		Winning Bid		Winner	
	Seller 1	Seller 2	Seller 1	Seller 2	Seller 1	Seller 2
1	-	0	-	Δ	-	A
2	0	0	Δ	Δ	B	A

Assume the bidding order for the buyers is (A, B). In round 1 buyer A must select which auction, if any, to bid in. At the beginning of round 1 none of the sellers have received any bid, so buyer A selects randomly. For concreteness, assume that A bids for 2. In this case buyer A bids Δ and becomes the winning bidder. At the beginning of round 2 buyer B observes that the auction of seller 1 has not yet received a bid. Buyer B also observes that the winning bidder in the auction of seller 2 has changed, but the standing bid has not. Thus, the auction of seller 1 is the unique most preferred auction for buyer B, so buyer B bids in this auction. Since after round 2 no further bids are placed, then the bidding ends. Each buyer wins an auction, and both pay zero. Not only do these strategies constitute a PBE, but they also yield an efficient outcome.

Example 2. Network with Frictions. As in the previous example, assume that buyers A and B are ordered in their valuations ($\nu_A > \nu_B = \Delta > 0$) and sellers 1 and 2 have the same valuation (normalized to 0). The minimum bid increment is $\Delta = \nu_B$. Now consider the following network that is not fully connected:



Round	Standing Bid		Winning Bid		Winner (Last Bidder)	
	Seller 1	Seller 2	Seller 1	Seller 2	Seller 1	Seller 2
1	-	0	-	Δ	-	A
2	-	Δ	-	Δ	-	A(B)

The above table shows an outcome that occurs with positive probability under the PS bidding strategies. Indeed, in round 1 both auctions are most preferred auctions for buyer A. If buyer A bids for seller 2 in round 1, the final outcome is that buyer A wins the auction and pays Δ. Had buyer A bid for seller 1, the outcome would be the same as in example 1: both buyers win an auction, and both pay 0. This implies that buyer A has a profitable deviation from the PS strategy: always bid for seller 1.[5] Moreover, this example shows that with positive probability the PS strategies yields an inefficient outcome. The key element in this example is that a losing bidder (in this example, bidder B) might place a bid whose only role is to raise the price that the winning bidder (in this example, bidder A) has to pay. For this reason the winning bidder (bidder A) no longer wants to win the auction at the current standing bid. But since bids cannot be reneged, the winning bidder is "stuck" with that auction.

4 Concluding Remarks

In this paper we showed that the bidding rule proposed by Peters and Severinov (2006) for their frictionless competing auctions environment is not a PBE when frictions are present.[6] Moreover, the outcome from the PS strategies need not be efficient. While the counterexample might seem stylized, it shows a general feature why the PS bidding rule is no longer guaranteed to be efficient nor a PBE of the competing auctions game when frictions are present. When the standing bid is below the winning bid in a given auction, a new bid increases the (second) price in that auction. A bidder that is winning before the increase in the price may prefer to release that auction and bid in an auction that has a lower standing bid. However, this is not possible since bids cannot be reneged on. Because the PS strategies are simple and based on easily observed data, a branch of the empirical auctions literature has used some of the theoretical predictions of the bidding rule from PS to investigate bidder behavior in online competing auctions environments such as eBay. However, it is well known that eBay is not a frictionless environment. Therefore in environments such as eBay, the PS strategies need not be equilibrium strategies, nor have any sort of efficiency property. Our results indicate that researchers should be cautious when using the PS bidding rule to make inference about the behavior of buyers and sellers in a market where frictions are present such as eBay, Amazon, or Taobao.

References

Anwar, S., R. McMillan, and M. Zheng (2006): "Bidding behavior in competing auctions: Evidence from eBay," *European Economic Review*, 50(2), 307–322. (Cited on page 1)

Backus, M. R., J. U. Podwol, and H. S. Schneider (2014): "Search Costs and Equilibrium Price Dispersion in Auction Markets," *European Economic Review,* forthcoming. (Cited on page 1)

Bajari, P., and A. Hortacsu (2004): "Economic Insights from Internet Auctions," *Journal of Economic Literature*, pp. 457–486. (Cited on page 1)

Bapna, R., S. A. Chang, P. Goes, and A. Gupta (2009): "Overlapping online auctions: Empirical characterization of bidder strategies and auction prices," *MIS quarterly*, pp. 763–783. (Cited on page 1)

[5] Indeed, this strategy yields payoff ν_A with probability 1. The PS strategy yields expected utility $\nu_A - \frac{1}{2}\mathbb{E}(\nu_B)$, where $\mathbb{E}$ denotes expectation with respect to A's prior over ν_B. If buyer A did not know the network, but instead had a prior over the possible links that bidder B had available, this would still be a profitable deviation provided bidder A places a sufficiently high prior on the network being as described.

[6] Although, to the best of our knowledge, there is no paper showing a PBE solution in the PS game when frictions are present, the reader can look at Donna, Schenone, and Veramendi (2015) for a different approach to solving the problem.

CHAU, M., X. FANG, AND O. R. LIU SHENG (2005): "Analysis of the query logs of a web site search engine," *Journal of the American Society for Information Science and Technology*, 56(13), 1363–1376. (Cited on page 2)

CONE, J., G. FRANKLIN, G. RYAN, AND W. STALKER (2005): "Adaptive search engine," US Patent App. 11/195,225. (Cited on page 2)

DONNA, J., P. SCHENONE, AND G. VERAMENDI (2015): "Networks, Frictions, and Price Dispersion," *Working Paper*. (Cited on page 5)

HASKER, K., AND R. SICKLES (2010): "eBay in the economic literature: Analysis of an auction marketplace," *Review of Industrial Organization*, 37(1), 3–42. (Cited on page 1)

OCKENFELS, A., D. REILEY, AND A. SADRIEH (2006): "Online auctions," Discussion paper, National Bureau of Economic Research. (Cited on page 1)

PETERS, M., AND S. SEVERINOV (2006): "Internet auctions with many traders," *Journal of Economic Theory*, 130(1), 220–245. (Cited on pages , 1, and 5)

ZEITHAMMER, R., AND C. ADAMS (2010): "The sealed-bid abstraction in online auctions," *Marketing Science*, 29(6), 964–987. (Cited on page 1)