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Abstract

Promoting competition in domestic markets is very often an important policy concern of governments in context of developmental objectives. Direct government intervention of different forms to promote competition becomes all the more necessary especially in the markets that have higher tendencies to concentrate. For example, in the market for telecom spectrum licenses, many countries impose ceilings on the number of licenses that a single individual company can possess. It is commonly believed that in the markets where permission from government is required for fresh operation or expansion of operation, e.g. through licenses, larger number of licenses lead to higher competition. But some earlier literature show that increasing the number of licenses might actually be detrimental to competition contrary to popular belief. This paper considers a situation where there is an incumbent monopolist in a market; the government is auctioning two new licenses, one for this same market and another one for a completely new market where no firm had been operating so far. A number of potential entrants are willing to bid for both the licenses. The incumbent firm is allowed to purchase only one of these licenses. If it purchases the license for its own market it can retain its monopoly position. The selling procedure dictates that only the potential entrants will be bidding and in order to purchase the license in its existing market, the incumbent monopolist has to match the highest bid in that auction. Alternatively, it can bid for the entry license for the new market. This paper tries to identify under what conditions the incumbent firm will bid for the outside market. It also tries to find under what conditions providing some other options to the incumbent firm leads to increased competition in the existing market, thus contributing to developmental prospects by enhancing social welfare.

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1 Introduction

It is broadly believed that development of an economy depends to a large extent on its market organisation (Stiglitz(1989))[5]. Simple economic intuitions suggest that the higher the degree of competition in markets, the lower would be the amount of dead weight loss. This view is supported by Cowling and Mueller (1978)[1]. Thus higher competition leads to higher social welfare. Generally in less developed countries, markets are not well organised. In such cases, direct government intervention becomes necessary to ensure competition. The role of government becomes more important especially in context of those markets where entry or expansion of existing operation require permission from the government, e.g. the market for telecommunications services, mining activities etc. Such permissions are usually obtained through licenses. As noted by Hoppe, Jehiel and Moldovanu (2006)[2], market structure in an industry is shaped by license auctions. It is believed commonly that by offering a higher number of licenses, the government can ensure more competition in an industry. But Hoppe, Jehiel and Moldovanu (2006)[2] establish that this is not the case always, a higher number of licenses not necessarily enhances competition, and in fact, as their analysis suggests, a higher number of licenses may keep the market more concentrated.

We must note here, that some markets have higher tendencies to concentrate than others due to advantages of incumbents from the concerned activities[1]. The market for telecommunications services is one of them. This is why in various countries the governments adopt different policy measures to prevent concentration of the telecom markets. For example the Government of India sets ceilings on the number of licenses that a single bidding firm can win in the telecom license auctions[2]. In UK, the Government had reserved a particular license only for the new entrants, and thus no incumbent was permitted to bid for it[3].

We consider a case where there is a single firm operating as a monopolist in a market, where fresh operation or any expansion of the existing operation are both subject to licensing. There is a fresh market, where no firm has been operational so far. The government is going to offer two licenses, one for the existing market and another one for the fresh market, through auction. There are potential bidders for both these licenses other than the incumbent. Now, the government has imposed ceilings on the number of licenses that a single firm can hold. Due to this ceiling, the incumbent firm can get hold of any one of these licenses. The selling procedure dictates, that only the potential entrants will be bidding for the license in the market where the incumbent monopolist

[1]As observed by Klemperer (2004)[4], promoting competition in British telecom spectrum auctions was an important objective and new entrants needed to be encouraged in order to ensure their participation in the telecom spectrum auctions.

[2]Internship Report, Competition Commission of India (May, 2012)[3].

[3]Klemperer (2004)[4].

operates. In order to win the license in its existing market, the incumbent has to match the highest bid among those submitted by the potential entrants. For the license for the fresh market, however, all the firms will be bidding.If the incumbent firm wins the license for its existing market, then it can retain its monopoly there. If it instead wins the license for the fresh market, its existing market becomes duopolisitic, while it becomes a monopolist in the fresh market. One important fact to note here is that, under both the situations mentioned above, for the industry as a whole, only two firms operate, if the incumbent wins any one of the licenses; however, when the incumbent wins the license for its existing market, the industry consists of two monopoly markets, while when the incumbent firm wins the license for the fresh market, its existing market turns more competitive by becoming a duopoly. Thus in the latter case,the industry as a whole is more competitive.

This paper, in terms of a very simple model, makes an attempt to identify under what conditions the incumbent will be interested to bid for the license for the fresh market. It also tries to find under what conditions providing some other options to the incumbent firm leads to increased competition in the existing market, thus contributing to developmental prospects by enhancing social welfare. The following Section 2 discusses the model and the policy implications and Section 3 concludes the paper.

2 The Model

We assume that there are two markets, viz. market A and market B. The inverse demand function of market A is $P_A = a_A - Q_A$ and that of market B is $P_B = a_B - Q_B$ respectively, where Q_A and Q_B are the market outputs and $a_A > 0$ and $a_B > 0$. In market A there is currently only one firm enjoying monopoly; we denote this firm as firm I. In market B, no firm is operating currently. The marginal cost of firm I in market A is $c_A > 0$. If firm I gets the right to operate in market B then her marginal cost will be $c_B > 0$. Government is auctioning two licenses, one for market A and another one for market B. There are $N_A > 1$ potential bidders for market A and $N_B > 1$ potential bidders for market B. The marginal cost of each of these bidders is private information to the concerned bidder. Although it is common knowledge that these marginal costs of the new entrants in market A are distributed according to the distribution function $F_{[A]}(\cdot)$ and the density function $f_{[A]}(\cdot)$ over the interval $[\underline{c}^{[A]}, \overline{c}^{[A]}]$ and has full support. It is also common knowledge that the marginal costs of the new entrants in market B are distributed according to the distribution function $F_{[B]}(\cdot)$ and the density function $f_{[B]}(\cdot)$ over the interval $[\underline{c}^{[B]}, \overline{c}^{[B]}]$ and has full support. Firm I can either bid for market B or she can outbid the highest bidder in market A to retain her monopoly. In the second case Government will disclose the highest bid to firm I and ask whether she wants to pay the amount and obtain the license. If firm I agrees to pay the amount to the Government, then she retains her monopoly. Otherwise one new firm will enter the market and thus market A becomes a duopoly market. However, firm I cannot bid in market B and outbid the new entrant in market A simultaneously. So she has to decide whether to bid for market B or outbid the new entrant before the auctions start. In all the cases the Government will declare all the bids after the auction.

Let us assume that a new entrant will participate in the auction if the marginal cost of that new entrant is less than or equal to $\hat{c}_A$[4]. Similarly, we assume that the incumbent firm will bid in market B if her marginal cost for market B is less than or equal to $\hat{c}_B$[5]. We know that $\underline{c}_A \leq \hat{c}_A \leq \overline{c}_A$, accordingly we define

- $G_{[A]}(z \,|\, \hat{c}_A) := \left(1 - \frac{F_{[A]}(z)}{F_{[A]}(\hat{c}_A)}\right)^{N_A - 1}$, as the conditional probability of winning of a potential entrant bidding firm in market A, given that the threshold value of marginal cost is $\hat{c}_A$, when the firm is signaling her type to be z,

  and

- $H_{[A]}(z \,|\, \hat{c}_A) := \left(1 - \frac{F_{[A]}(z)}{F_{[A]}(\hat{c}_A)}\right)^{N_A}$ as the conditional probability distribution of the lowest order statistic among the types of new entrants bidding for the license for market A given that the threshold value of marginal cost for market A is $\hat{c}_A$[6].

The current monopoly profit of firm I in market A is given by $\Pi_{IM}^{[A]}(c_A) = \left(\frac{a_A - c_A}{2}\right)^2$. The monopoly profit in market B, if firm I bids for market B and wins the auction, is given by $\Pi_{IM}^{[B]}(c_B) = \left(\frac{a_B - c_B}{2}\right)^2$[7]. Let us denote the new entrant in market A by firm N. Suppose the marginal cost of the new entrant in market A is $c_N > 0$. The profit of firm I in market A then is $\Pi_{ID}^{[A]}(c_A, c_N) = \left(\frac{a_A - 2c_A + c_N}{3}\right)^2$. The profit of firm N in market A then is $\Pi_{ND}^{[A]}(c_A, c_N) = \left(\frac{a_A - 2c_N + c_A}{3}\right)^2$[8].

2.1 Incumbent chooses to outbid in market A

If the incumbent firm does not bid for entry in market B and decides to outbid the new entrant in market A, then two factors need to be considered. First if the new entrant bids more than $\left[\Pi_{IM}^{[A]}(c_A) - \Pi_{ID}^{[A]}(c_A, c_N)\right]$ the incumbent firm cannot outbid the bidder. And second the probability that the incumbent firm is not bidding for market B is $\left(1 - F_{[B]}(\hat{c}_B)\right)$. Let us denote by $O_{ND}^{[A]}(c_A, c_N, \hat{c}_B) := \left(1 - F_{[B]}(\hat{c}_B)\right) \left[\Pi_{IM}^{[A]}(c_A) - \Pi_{ID}^{[A]}(c_A, c_N)\right]$,

[4] We refer to this value of marginal cost $\hat{c}_A$ as threshold value of marginal cost for market A subsequently.

[5] We refer to this value of marginal cost $\hat{c}_B$ as threshold value of marginal cost for market B subsequently.

[6] The conditional probabilities $G_{[A]}(z \,|\, \hat{c}_A)$ and $H_{[A]}(z \,|\, \hat{c}_A)$ have been written for simplicity as $G_{[A]}(z)$ and $H_{[A]}(z)$ respectively in the subsequent analysis.

[7] These monopoly profits can be calculated routinely.

[8] These duopoly profits can be calculated by the routine method for Cournot competition involving two firms with constant but different marginal costs.

which is the minimum bid that the new entrant must quote. It leads to the following lemma.

Lemma 2.1. *A new entrant will participate in the auction only if her type is less than or equal to* $\hat{c}_A$*.*

$$\Pi_{ND}^{[A]}(c_A, \hat{c}_A) = O_{ND}^{[A]}(c_A, \hat{c}_A, \hat{c_B})$$

The above lemma determines the value of c_A as a function of c_B. We will determine the value of c_B as a function of c_A in the next section. Together, both c_A and c_B will be determined uniquely. However note that c_A can be calculated by any agent including the Government. So it becomes common knowledge.

Since all the bidders in market A knows that no bidder has marginal cost greater than c_A, if a bidder bids as if her marginal cost is z, her winning probability is given by $G_{[A]}(z)$. Therefore, the expected profit function of the new entrant is given by

$$\left[\Pi_{ND}^{[A]}(c_A, c_N) - b_{[A]}(z)\right] G_{[A]}(z)$$

where $b_{[A]}(\cdot)$ is the symmetric and decreasing equilibrium bid function of the new entrant. The objective of the new entrant is to maximize this expected profit function, subject to the constraint $b_{[A]}(z) \geq O_{ND}^{[A]}(c_A, c_N, \hat{c}_B)$. Note that this constraint will act as a self imposed reserve price which is a function of the new entrant's own type (i.e. each bidder will face different reserve prices) so this auction in effect becomes an auction with dynamic reserve prices.

The following proposition derives the symmetric equilibrium bidding strategy of a new entrant.

Proposition 2.2. *The symmetric and decreasing equilibrium bidding strategy of a new entrant is given by*

$$b_{[A]}(c_A, c_N, \hat{c}_A, \hat{c}_B) = O_{ND}^{[A]}(c_A, c_N, \hat{c}_B) - \frac{1}{G_{[A]}(c_N)} \int_{c_N}^{\hat{c}_A} \left[\Pi_{ND}^{[A]}(c_A, z) - O_{ND}^{[A]}(c_A, z, \hat{c}_B)\right] dG_{[A]}(z)$$

Proof. The proof of this proposition is provided in the Appendix A. □

2.2 Incumbent bidding for license to enter market B

Here we are assuming that the incumbent firm is bidding in market B. Our objective is to find $\hat{c}_B$. It is important to assume that in market B all the other firms do not know that the incumbent firm is already operating in market A. So, all the other firms perceive the incumbent firm to be just another regular firm participating in the auction. The expected profit of the incumbent firm if she decides to bid in market B and bids as if her real marginal cost is z is given by

$$E\Pi_{IM}^{[B]}(z, c_B) = \left(\Pi_{IM}^{[B]}(c_B) - b(z)\right)\left(1 - F_{[B]}(z)\right)^{N_B}$$

We are interested in symmetric and decreasing equilibrium bidding strategy. The following proposition not only shows that such an equilibrium strategy exists, but also explicitly derives the form of this bidding strategy.

Proposition 2.3. *The corresponding symmetric decreasing equilibrium bid function is*

$$b_{IM}^{[B]}(c_B) = \frac{-1}{\left(1 - F_{[B]}(c_B)\right)^{N_B}} \int_{c_B}^{\bar{c}_B} \Pi_{IM}^{[B]}(z) d\left(1 - F_{[B]}(z)\right)^{N_B}$$

Proof. The proof of this proposition is provided in the Appendix B. □

Next we proceed to calculate the expected profit of the incumbent firm when she chooses to bid for the license permitting entry into market B. The following lemma does this.

Lemma 2.4. *The expected profit of the incumbent firm if she decides to bid for the license for market B in equilibrium is*

$$E\Pi_{IM}^{[B]}(c_B) = -\int_{c_B}^{\underline{c}_B} \Pi_{IM}^{[B]\prime}(z)\left(1 - F_{[B]}(z)\right)^{N_B} dz$$

Proof. We already know that the expected profit of the incumbent firm in market B is $\left(\Pi_{IM}^{[B]}(c_B) - b(c_B)\right)\left(1 - F_{[B]}(c_B)\right)^{N_B}$. Also the symmetric and decreasing equilibrium bid function is $\frac{-1}{\left(1-F_{[B]}(c_B)\right)^{N_B}} \int_{c_B}^{\underline{c}_B} \Pi_{IM}^{[B]}(z) d\left(1 - F_{[B]}(z)\right)^{N_B}$. After substituting for the bid function in the expected profit function we get

$$\left(\Pi_{IM}^{[B]}(c_B) - \frac{-1}{\left(1 - F_{[B]}(c_B)\right)^{N_B}} \int_{c_B}^{\underline{c}_B} \Pi_{IM}^{[B]}(z) d\left(1 - F_{[B]}(z)\right)^{N_B}\right)\left(1 - F_{[B]}(c_B)\right)^{N_B}$$

Simplifying the above equation we get

$$\Pi_{IM}^{[B]}(c_B)\left(1 - F_{[B]}(c_B)\right)^{N_B} + \int_{c_B}^{\underline{c}_B} \Pi_{IM}^{[B]}(z) d\left(1 - F_{[B]}(z)\right)^{N_B}$$

Finally after integrating by parts we get the result. □

The above expected profit is one part of the opportunity cost of not bidding in market B. Note that $\Pi_{IM}^{[B]\prime}(\cdot) < 0$, therefore, $E\Pi_{IM}^{[B]}(c_B)$ is positive. If the incumbent firm does not bid in market B and outbid in market A then her expected payment is the expected highest bid of the new entrant, which is given by

$$Eb_{ND}^{[A]}(c_A, \hat{c}_A, \hat{c}_B) = \int_{\underline{c}_A}^{\hat{c}_A} b_{[A]}(c_A, c_N, \hat{c}_A, \hat{c}_B) dH_{[A]}(c_N)$$

This is another part of the opportunity cost of not bidding for the license for market B. Finally, the expected profit of the duopoly is given by

$$E\Pi_{ID}^{[A]}(c_A) = \int_{\underline{c}_A}^{\hat{c}_A} \Pi_{ID}^{[A]}(c_A, c_N) dH_{[A]}(c_N)$$

If firm I bids for the license for market B then her expected profit is the sum of the expected duopoly profit and the expected profit in market B. However, otherwise her expected profit is the monopoly profit minus the expected highest bid. So, firm I will bid in market B if the first one is greater than the second one. This is formally stated in the next proposition.

Proposition 2.5. *The incumbent firm will not bid in market B if*

$$\Pi_{IM}^{[A]}(c_A) - Eb_{ND}^{[A]}(c_A, \hat{c}_A, \hat{c}_B) \geq E\Pi_{ID}^{[A]}(c_A) + E\Pi_{IM}^{[B]}(c_B)$$

We know firm I is indifferent between bidding and not bidding in market B if her marginal cost in market B is $\hat{c}_B$. The final lemma below establishes $\hat{c}_B$ as a function of $\hat{c}_A$.

Lemma 2.6.

$$\Pi_{IM}^{[A]}(c_A) - Eb_{ND}^{[A]}(c_A, \hat{c}_A, \hat{c}_B) = E\Pi_{ID}^{[A]}(c_A) + E\Pi_{IM}^{[B]}(\hat{c}_B)$$

From Lemma 2.1 and Lemma 2.6 we have two equations and two unknowns (viz. $\hat{c}_A$ and $\hat{c}_B$). So we can solve for the values uniquely.

2.3 Policy implications

Government knows all the distribution functions and c_A, though c_B and the marginal cost of any new entrant are completely unknown to the Government. However, Government can still calculate $\hat{c}_A$ and $\hat{c}_B$. Some important policy implications emerge from the above analysis.

First, we know that the Government's objective is to enhance competition at the existing monopoly market, i.e. market A, but not with firms who have marginal costs higher than $\hat{c}_A$ (let us refer to the firms with marginal cost below or equal to $\hat{c}_A$ efficient and the firms with marginal costs above $\hat{c}_A$ inefficient). In this mechanism, neither any direct reserve price is set by the Government, nor is any license reserved exclusively for the new entrants. Therefore, if any new entrant actually wins the license for market A, that firm is definitely an efficient one, whose marginal cost is not too high as per Government's intended standard. Thus, competition is enhanced in this case involving efficient firms. Instead, had there been a direct reserve price such that the incumbent does not bid for the license for market A at all, that might have failed to satisfy the individual rationality constraints (i.e. participation constraints) of the new entrants as well. In that case the license would have remained unallocated, thus resulting in a socially inefficient outcome. Alternatively, in case of reservation of license for new entrants only, it might very well happen that all the potential entrants are actually quite inefficient in terms of very high marginal costs. Then also the resulting allocation would have been socially inefficient in the sense that the best among the lot is not good enough. As noted earlier in this mechanism we effectively have a dynamic reserve price which is a function of the bidder's own type. Now, fixed reserve prices and dynamic reserve prices have different effects on bidders. For fixed reserve prices, the profit margin that the firms enjoy is lower than what they enjoy under dynamic reserve price. So the firms prefer dynamic reserve prices over fixed ones. Thus the Government is no worse off and the firms are better off under such a reserve price, and we can say that there is a Pareto improvement in a way due to this dynamic reserve price.

The second implication relates to the knowledge of $\hat{c}_B$. Here we have considered only one additional market (B) along with the existing monopoly market (A). The results derived suggest that $\hat{c}_A$ is functionally related to $\hat{c}_B$. Now, let us consider the case where there are more additional markets. For simplicity, let us consider just one more additional market C. and let $\hat{c}_C$ be the marginal cost, such that the incumbent will participate in the auction of license for entry into market C if her marginal cost is below this value. The Government can calculate both $\hat{c}_B$ and $\hat{c}_C$. Now, if the Government intends that the incumbent bids for license in any other market, then it has the choice to offer the license for the market that has a higher threshold value (i.e. $\hat{c}$ value) above which the incumbent will not bid for the license for the concerned market. A higher $\hat{c}$ value for a fresh market provides more incentive to the incumbent since it involves a higher probability of winning for the incumbent[9]. Thus the knowledge of $\hat{c}_i$ values can help the Government design an incentive compatible mechanism, for which the incumbent chooses to bid for an outside market in its own interest allowing for more competition in its existing market.

The third implication relates again to the knowledge of the threshold value of marginal cost in outside markets. If the Government wants the new entrant in market A to be very efficient, then the Government will try to keep the threshold value of marginal cost above which no new entrant participates in the auction in market A (i.e. $\hat{c}_A$) low. The Government can do so quite easily by offering the license for that alternative outside

[9]This means to say that as $F_i(\hat{c}_i)$ (where $i = B, C$) increases with higher values of $\hat{c}_i$, the probability that the incumbent will not bid for the entry license for market i given by $(1 - F_{[i]}(\hat{c}_i)$ decreases.

market which has a lower threshold value of marginal cost[10]. In that case if any new entrant wins the license for market A, that firm will definitely have quite low marginal cost.

The analysis above suggests that the Government is able to ensure competition in a monopoly market and that too with more efficient firms (efficient as defined above) with a higher probability than the commonly used alternative policies like setting reserve price or reserving licenses for new entrants. Reserve price, even if it succeeds in making an efficient firm enter the existing monopoly market, can do so with less probability. In case of reserving some license for new entrants, the entering firm might very well be one with very high marginal cost, i.e.inefficient in our defined sense.

Thus we observe that the Government can fulfill more than one policy objectives with a single policy instrument in our specified mechanism. We must note a few more aspects of this mechanism here. First, as mentioned above, this mechanism can successfully work if we consider many additional markets. Second, the prevailing structure of the additional market (i.e. whether there is no firm, or there is monopoly or oligopoly) is not decisive about the outcomes. Thus the assumptions we have made here about a single outside market where no firm is operating prior to the license auction are merely simplifying assumptions without any loss of generality and so this mechanism can be successfully extended to a much more general framework. However, market A being a monopoly is a crucial assumption.

An implicit implication of this analysis is that the government can, with some commonly used policy tools like tax concession or provision of subsidy, provide incentives to the incumbent monopolist so that it does not feel interested to outbid the potential entrants and attain the same outcome as suggested by the mechanism discussed in this paper. Even in the cases of tax concession or subsidy the inefficiency problems, which are inherent in case of setting reserve prices or reserving licenses for new entrants can be avoided. Thus the Government can enjoy a lot of flexibilities in the policy making, in terms of both choosing the policy instruments as well as directions and get more than objectives fulfilled at the same time with much less direct intervention in the market operation.

3 Conclusion

Promoting competition in domestic markets has remained an important policy objective for many governments across the world. Direct government intervention becomes all the more necessary especially for the markets those are more prone to concentrate. In order to ensure competition in markets where there are just single firms or a few large firms, the governments often try to provide incentives to new entrants so that they feel interested

[10] This can be done since $\hat{c}_A$ is function of the threshold value of the outside market.

to enter those markets. Governments can control these entries directly for markets where any fresh operation or expansion of existing operation requires government's permission in the form of licenses. Auction is regarded largely an effective way of allocating such licenses. As discussed earlier in the paper, the governments can reserve certain licenses for the new entrants, forbidding the incumbents to bid for these licenses. If the governments can calculate the price of the licenses which will definitely discourage the incumbents from bidding, then the government can also set that level of price as the reserve price. But as discussed above, both these policy instruments involve some amount of inefficiency. This paper has therefore proposed an alternative mechanism for allocating licenses that involves much less direct intervention by the government in the market operations with higher probabilities of efficiency in the system and much more flexible policy choices. The proposed policy here deals with a situation where an incumbent monopolist can choose to outbid any new entrant in its existing market or bid for a license allowing entry to an outside market. Further research question in this direction lies in analysing a situation where the incumbent is first asked to quote a bid for a new license in its own market and the potential entrants are given the option to match this bid. The fundamental difference between the current and this proposed analysis lies in the fact that in the current analysis, the potential entrants do not get to know the type of the incumbent prior to submitting their own bids, but in the proposed mechanism, they obtain this information before bidding. It will be interesting to compare the outcomes under these two different types of specifications and figure out under what conditions the government will choose one policy over the other.

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Appendix

A Proof of Proposition 2.2

Note that the equilibrium bid function must consist of two components, one for the incumbent firm and one for the other competitors. The firm must bid $O^{[A]}_{ND}(c_A, c_N, \hat{c}_B)$, otherwise the incumbent firm may outbid her. Therefore, the remaining profit for entering the market is $\Pi^{[A]}_{ND}(c_A, z) - O^{[A]}_{ND}(c_A, z, \hat{c}_B)$. The rest of the proof is similar to the proof of proportion 2.3 noting that $\partial O^{[A]}_{ND}(c_A, c_N, \hat{c}_B)/\partial c_N < 0$.

B Proof of Proposition 2.3

The objective of the incumbent firm is to maximize $E\Pi^{[B]}_{IM}(z, c_B)$ by choosing z. The first order condition of this maximization is given by

$$\Pi^{[B]}_{IM}(c_B)d\left(1 - F_{[B]}(z)\right)^{N_B} = d\left[b(z)\left(1 - F_{[B]}(z)\right)^{N_B}\right]$$

At a symmetric equilibrium bidding according to her true type is optimal. So, at equilibrium we have

$$\Pi^{[B]}_{IM}(c_B)d\left(1 - F_{[B]}(c_B)\right)^{N_B} = d\left[b(c_B)\left(1 - F_{[B]}(c_B)\right)^{N_B}\right] \tag{B.1}$$

First Fundamental Theorem of Calculus gives

$$b^{[B]}_{IM}(c_B) = \frac{-1}{\left(1 - F_{[B]}(c_B)\right)^{N_B}} \int_{c_B}^{\bar{c}_B} \Pi^{[B]}_{IM}(z)d\left(1 - F_{[B]}(z)\right)^{N_B}$$

To check the second order condition for this maximization, note that

$$E\Pi^{[B]\prime}_{IM}(z, c_B) = \Pi^{[B]}_{IM}(c_B)d\left(1 - F_{[B]}(z)\right)^{N_B} - d\left[b(z)\left(1 - F_{[B]}(z)\right)^{N_B}\right]$$

From equation B.1 we know at equilibrium

$$d\left[b(z)\left(1 - F_{[B]}(z)\right)^{N_B}\right] = \Pi^{[B]}_{IM}(z)d\left(1 - F_{[B]}(z)\right)^{N_B}$$

So

$$E\Pi^{[B]\prime}_{IM}(z, c_B) = \left[\Pi^{[B]}_{IM}(c_B) - \Pi^{[B]}_{IM}(z)\right] d\left(1 - F_{[B]}(z)\right)^{N_B}$$

Note that $E\Pi^{[B]\prime}_{IM}(z, c_B) \lesseqgtr 0$ if $c_B \lesseqgtr z$. So $E\Pi^{[B]}_{IM}(z, c_B)$ is indeed maximized at $z = c_B$ Finally, we have to show that $b^{[B]\prime}_{IM}(c_B) < 0$. To show this, first, we do an integration-by-parts and get

$$b^{[B]}_{IM}(c_B) = \Pi^{[B]}_{IM}(c_B) + \frac{1}{\left(1 - F_{[B]}(c_B)\right)^{N_B}} \int_{c_B}^{\bar{c}_B} \Pi^{[B]\prime}_{IM}(z) \left(1 - F_{[B]}(z)\right)^{N_B} dz$$

Therefore,

$$b^{[B]\prime}_{IM}(c_B) = \left[\int_{c_B}^{\bar{c}_B} \Pi^{[B]\prime}_{IM}(z) \left(1 - F_{[B]}(z)\right)^{N_B} dz\right] \left[\frac{d}{dc_B} \frac{1}{\left(1 - F_{[B]}(c_B)\right)^{N_B}}\right]$$

First note that as $\Pi^{[B]\prime}_{IM}(z) < 0$ we have $\int_{c_B}^{\bar{c}_B} \Pi^{[B]\prime}_{IM}(z) \left(1 - F_{[B]}(z)\right)^{N_B} dz < 0$. And second $\frac{d}{dc_B} \frac{1}{\left(1-F_{[B]}(c_B)\right)^{N_B}} > 0$. So, $b^{[B]\prime}_{IM}(c_B) < 0$ as desired.