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# Forecasting German Car Sales Using Google Data and Multivariate Models

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## Abstract

Long-term forecasts are of key importance for the car industry due to the lengthy period of time required for the development and production processes. With this in mind, this paper proposes new multivariate models to forecast monthly car sales data using economic variables and Google online search data. An out-of-sample forecasting comparison with forecast horizons up to 2 years ahead was implemented using the monthly sales of ten car brands in Germany for the period from 2001M1 to 2014M6. Models including Google search data statistically outperformed the competing models for most of the car brands and forecast horizons. These results also hold after several robustness checks which consider nonlinear models, different out-of-sample forecasts, directional accuracy, the variability of Google data and additional car brands.

*Keywords:* Car Sales, Forecasting, Google, Google Trends, Global Financial Crisis, Great Recession.

*JEL classification:* C22, C32, C52, C53, L62.

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# 1 Introduction

Long-term forecasting of car sales plays an important role in the automobile industry. Accurate predictions allow firms to improve market performance, minimize profit losses, and plan manufacturing processes and marketing policies more efficiently.

Tough competition, significant investments, and the need for quick model updates are the specifics of the automotive industry which make forecasting an element of key importance for the sales and production processes. Like other complex industries, it can be characterized by long product development cycles varying from 12 up to 60 months. An effective planning of the production therefore requires accurate long-term sales forecasts. Inaccurate forecasts may result in several negative consequences, such as overstocking or shortage of production supplies, high costs for different workforce activities, loss of reputation for the manufacturer and even bankruptcy.

There are several economic factors affecting the automobile industry, and they can be broadly divided into three groups. The first group incorporates the technological aspects of the products: quality, innovation and technology, performance and economy of the engine, functionality, safety, space management, design and aesthetics (Lin and Zhang, 2004; Sa-ngasoongsong and Bukkapatnam, 2011). The second group comprises promotion and sales factors, including wholesale and retail prices, customer service, advertising campaigns, and brand image (Landwehr, Labroo, and Herrmann, 2011). These factors are significant, but usually do not have a long-term effect and automobile producers in most cases can manage and control them (Dekimpe, Hanssens, and Silva-Risso, 1998; Nijs, Dekimpe, Steenkamp, and Hanssens, 2001; Pauwels, Hanssens, and Siddarth, 2002; Pauwels, Silva-Risso, Srinivasan, and Hanssens, 2004). The third group includes various political, economic and social environmental factors which are generally beyond the control of manufacturers, such as organizational issues, political issues, global economic growth, ecological and physical forces, socio-cultural effects and consumer behavior. The use of these factors for car sales forecasting has been rather limited, see Brühl, Borscheid, Friedrich, and Reith (2009), Shahabuddin (2009), Wang, Chang, and Tzeng (2011) and Sa-ngasoongsong, Bukkapatnam, Kim, Iyer, and Suresh (2012). Moreover, most previous studies have focused on the dynamics of car sales in the short-term, with forecast horizons usually less than 4 months, whereas car sales forecasting requires time scales with duration up to one year or more.

Following the growing number of Internet users (International Telecommunications Union, 2014) and the increasing popularity of Google as a search engine for obtaining information about cars, we propose the use of Google search data as a leading indicator for the long-term forecasting of car sales. In this regard, Google Search holds the world leadership among all search engines with a 54% market share (Net Applications, 2014). Since 2004, it has offered a tool called Google Trends, which provides information on the relative interest of users in a particular search query, at a given geographic region and at a given time (the data are available on a weekly or even daily basis). Moreover, Google Trends can attribute queries to different search categories (Autos, Computers, Finance, Health and others). In recent years, researchers worldwide have begun to use online search data to produce real-time forecasts where information from official sources is released with a lag (such as ‘nowcasting’), or simply as an additional variable for forecasting purposes, see Choi and Varian (2012), Askatas and Zimmermann (2009), Suhoy (2009), Ginsberg, Mohebbi, Patel, Brammer, Smolinski, and Brilliant (2009), Da, Engelberg, and Pengjie (2011), D’Amuri and Marcucci (2013) and Fantazzini and Fomichev (2014) for some recent applications.

With this in mind, we propose a set of models for the long-term forecasting of car sales in Germany, which consider both economic variables and online search queries. Germany is the third biggest car producer in the world (about 14 million vehicles in 2013 and 20% of the total world production) and the absolute leader in Europe (31% of the total European production), see the reports by the German Association of the Automotive Industry (GTAI, 2014) and the Germany Trade and Invest Organization (VDA, 2014) for more details. As for Internet users, Germany has the second highest number of users in Europe (12.3% of all European users) and the 7th in the world. In June 2014, more than 71 million people in Germany visited the Web at least once a month, representing 88.6% of the adult population (Internet World Stats, 2014).

The first contribution of this paper is a set of multivariate models which include both Google data and economic variables. So far, the vast majority of the literature has used Google data as an exogenous variable in univariate models for short-term forecasting. Given that the car industry is interested in long-term forecasting, simple univariate models are not sufficient, and multivariate models are required to produce multi-step ahead forecasts for all variables, Google data included. Moreover, we consider

multivariate models for both deseasonalized data, the usual approach in the economic literature, and for data not seasonally adjusted, which is more common in practice, since planning and production departments tend to work with raw data<sup>1</sup>.

The second contribution of our paper is a large-scale forecasting exercise for ten car brands in Germany, where we compute out-of-sample forecasts ranging from 1 month to 24 months ahead. Our results show that models including car sales, Google data and economic variables outperform the competing models in the medium term for most of the car brands, while multivariate models including only car sales and Google data outperform the other models for long-term forecasts up to 24 steps ahead. The use of parsimonious models is crucial to obtain precise forecasts in the long run, and the use of Google search data represents a simple and powerful way to summarize the large amount of information available (see also Fantazzini and Fomichev, 2014).

The third contribution of the paper is a set of robustness checks to verify that our results also hold when considering nonlinear models, different out-of-sample forecasts, the use of directional accuracy as the main evaluation tool, Google data downloaded on different days, and additional car brands.

The paper is organized as follows. Section 2 describes the data and the in-sample analysis, and the forecasting models and their out-of-sample performance are reported in Section 3. Robustness checks are discussed in Section 4, and Section 5 briefly concludes.

## 2 Data and In-Sample analysis

We analyze new car registrations in the Federal Republic of Germany, as provided in press releases by the Federal Motor Transport Authority (Kraftfahrt-Bundesamt). These data cover the period from January 2001 to June 2014, for a total of 162 observations. The data consist of monthly numbers of new vehicle registrations by vehicle type and new registrations of passenger cars by brand starting from 2001. For different reasons, the information for some car brands was truncated: certain brands were present only after 2001; others stopped being observed well before 2014; or the registration statistics were not published due to the small number of registrations per month. Our car brands were selected based on the availability of a long time series for new car registrations and their presence in the “Vehicle Brands” Google subcategory. Moreover, car brands were chosen to reflect both foreign and domestic car producers.

There were only 22 brands which had both monthly data continuously available since 2001 and were present in Google Trends. We divided these brands into clusters by taking the average sales for each brand and using the method of k-means with Euclidian distance. We wanted to determine large, medium and small car manufacturers, and assign all brands into three clusters. The method of k-means allowed us to define the number of clusters a priori and minimize the within-cluster distance while maximizing the between-cluster distance (see e.g. Hartigan (1975)). The initial  $k$  cluster centers are chosen to maximize the initial distance. The data are arranged to the nearest cluster center, therefore  $k$  clusters are formed. Next, new cluster centers are chosen as centers of mass for the clusters. After recalculation, the data are again assigned to the nearest cluster centers. The procedure ends when all centers of mass are stabilized. We found three clusters consisting of the following brands:

- Large sellers: Volkswagen, Opel, Ford, BMW, Audi (average monthly sales between 19523 and 53820);
- Medium-sized sellers: Renault, Toyota, Peugeot, Hyundai, Fiat, Mazda, Citroen, Nissan (average monthly sales between 4976 and 14074);
- Small sellers: Jaguar, Kia, Land Rover, Porsche, Subaru, Honda, Volvo, Mitsubishi, Suzuki (average monthly sales between 355 and 3351).

We also used the method of k-means with the monthly sales data from January 2001 to June 2014 and we obtained the same division into three clusters.

For the sake of space, interest and to keep the empirical analysis computationally tractable, throughout the paper we will consider three large sellers (Volkswagen, Opel, BMW), three medium-sized sellers (Toyota, Fiat, Citroen), and four small sellers (Jaguar, Kia, Mitsubishi, Suzuki). The remaining 12 brands will be examined as a robustness check in section 4.5.

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<sup>1</sup>The authors wish to thank an anonymous director of marketing and sales for pointing out this issue.

The plots of the monthly sales are reported in Figure 1 (right vertical axis). Car sales are subject to seasonal fluctuations and all car brands tend to show several peaks during the year, with the biggest one taking place at the end of spring. In general, car sales decline during winter. The Census X-12 tests for seasonality detected that all brands exhibit stable seasonality, with no evidence of moving seasonality.

The second source of data consists of Google Trends data, which can be downloaded from [www.google.com/trends/](http://www.google.com/trends/), using the specific “**Autos and Vehicles**” category and its “**Vehicle Brands**” subcategory. The Google Index (GI) is the ratio of the number of queries relative to a particular category (in our case the car brand), with respect to all queries in the selected region at a given point of time. The data were collected for the whole of Germany for the period January 2004 - June 2014. The data have a weekly frequency and were converted to a monthly series by taking average values. While the GIs for a keyword are normalized to be bounded between 0 to 100, where 100 is the peak of the search queries, the GIs for a category are expressed in terms of percentage change from their first observation in January 2004, so that they can be both positive and negative. Their plots are reported in Figure 1 (left vertical axis): it is interesting to note that the turning points in the GIs anticipate those in the car sales for all car brands. This initial evidence suggests that Google data may be of some help for medium- and long-term forecasting.

Additionally, we included a number of economic variables related to car sales, based on recent works by Shahabuddin (2009) and Sa-ngasoongsong, Bukkapatnam, Kim, Iyer, and Suresh (2012). These variables are assumed to reflect the state of the national economy, and the factors that can influence a consumer’s decision to purchase a car. The selected economic variables and their descriptions are presented in Table 1. The data were collected for the period January 2001 to June 2014. All data, with the exception of building construction orders (which were available only seasonally adjusted), show some form of seasonality, with peaks during the summer season and troughs at the end of the year. The quarterly GDP data were converted to monthly data via the quadratic match average procedure, while the daily data for Euribor rates were transformed into monthly data by taking their average. Their plots are reported in Figure 2.

Economic variable	Frequency	Seasonally adjusted	Source	Explanation
Building Construction (BC)	M	yes	GFB	Volume index of new orders for residential buildings construction
Consumer Confidence Indicator (CCI)	M	no	DG ECFIN	Consumer survey that reflects consumer expectations
Consumer Price Index (CPI)	M	no	FSO	Measure of the ratio of a price of fixed set of consumer goods and services in current period to its price in a basic period
Euro Interbank Offered Rate (EURIBOR)	D	no	EBF	Calculated as an average rate of lending rate of the banks which participate in the survey. For the current research EURIBOR for long-term credits (1 year) is considered
Gross Domestic Product (GDP)	Q	no	FSO	Market value of all goods and services produced within a country. In the present work GDP in nominal billions Euro was taken
Production Index (PI)	M	no	FSO	Production Index for durable goods
Unemployment Rate (UR)	M	no	FEA	The registered unemployed population as a percentage of the civilian labor force
Petrol Price (PP)	M	no	FSO	Consumer price for petrol, price index

Table 1: Description of economic variables used in the analysis. The second column reports the frequency of publishing: M - monthly data, Q - quarterly data, D - daily data. The abbreviations used in the fourth column represent the data sources: GFB - German Federal Bank (Deutsche Bundesbank), DG ECFIN -Directorate General for Economic and Financial Affairs, FSO - The Federal Statistical Office (Statistisches Bundesamt), EBF - The European Banking Federation, FEA - The Federal Employment Agency (Bundesagentur für Arbeit).

Data with seasonal behavior were seasonally adjusted with the Census X-12 adjustment program developed by US Census Bureau. However, we also considered the raw data, since they are more common in practice and of greater interest for production planners and marketing managers, who base their decisions on real data which exhibit seasonality.

All data were transformed into logarithms to reduce variability and convert nonlinear patterns to

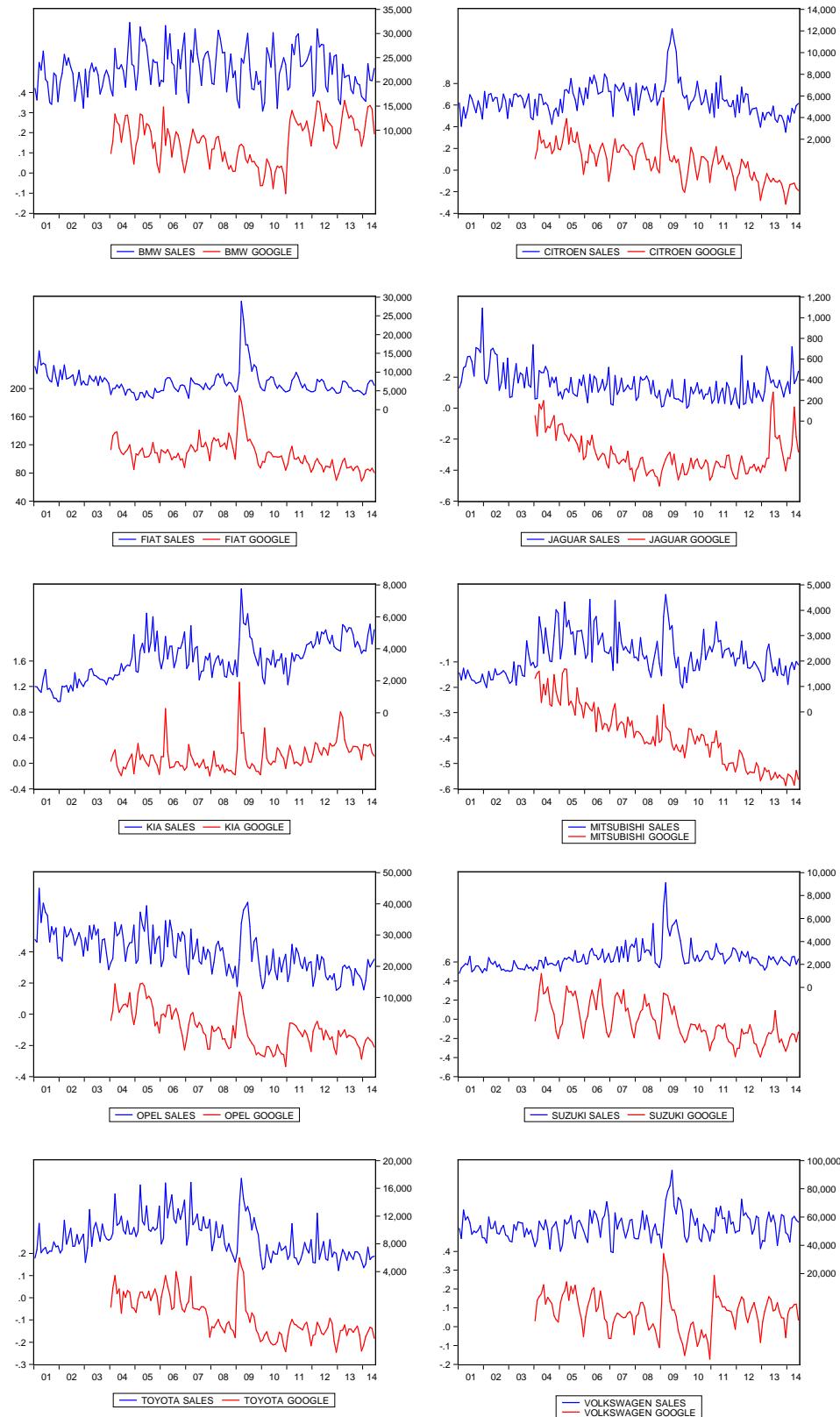


Figure 1: Car sales (right vertical axis) and relative GIs (left vertical axis) - not seasonally adjusted.  
Sample: 2001M1 - 2014M6.

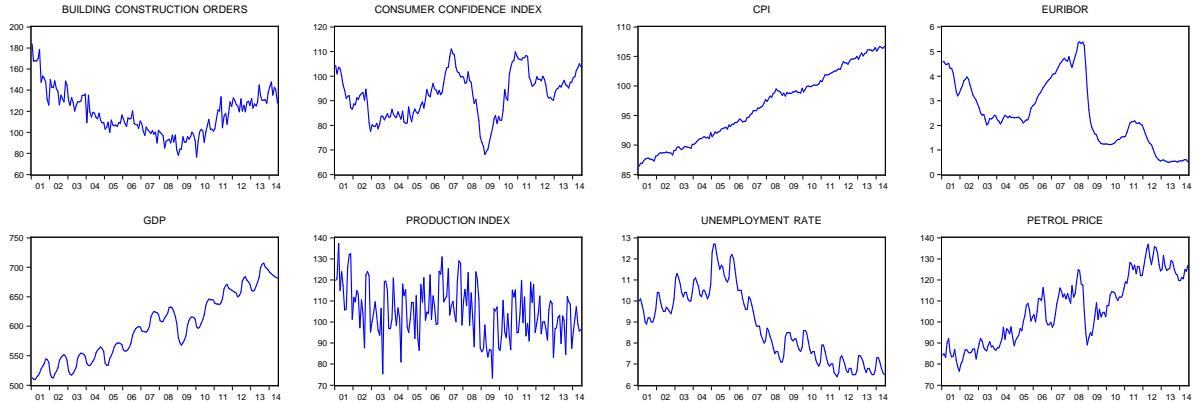


Figure 2: Economic variables - not seasonally adjusted. Sample: 2001M1 - 2014M6

linear patterns<sup>2</sup>(see Sa-ngasoongsong, Bukkapatnam, Kim, Iyer, and Suresh (2012)). The descriptive statistics for the car registrations, the Google data and the economic variables (both seasonally adjusted and raw data) are not reported for the sake of space and are available from the authors upon request.

To select the best multivariate model for each car brand, we follow the structural relationship identification methodology discussed by Sa-ngasoongsong, Bukkapatnam, Kim, Iyer, and Suresh (2012) for the case of the US car market. Briefly, the first step is to identify the order of integration using unit root tests; if all variables are stationary, VAR and VARX (Vector Autoregressive with exogenous variables) models are used. The second step determines the exogeneity of each variable using the sequential reduction method for weak exogeneity by Hall, Henry, and Greenslade (2002), who consider weakly exogenous each variable for which the test is not rejected and re-test the remaining variables until all weakly exogenous variables are identified. For non-stationary variables, cointegration rank tests are employed to determine the presence of a long-run relationship among the endogenous variables: if this is the case, VECM or VECMX (Vector Error Correction model with exogenous variables) models are used, otherwise VAR or VARX models in differences are applied. The last step is to compute the impulse response functions from the chosen model to trace the effect of a unit shock in one of the variables on the future values of car sales, and to compute out-of-sample forecasts (see Sa-ngasoongsong, Bukkapatnam, Kim, Iyer, and Suresh (2012) for more details). Our approach differs from the one proposed by Sa-ngasoongsong, Bukkapatnam, Kim, Iyer, and Suresh (2012) in two respects: first, we employ unit root tests and cointegration tests allowing for structural breaks, given the possible break in the years 2008-2009 during the global financial crisis. Second, we employ the previous identification methodology for both the seasonally adjusted data and the raw data.

## 2.1 Stationarity

### 2.1.1 Seasonally Adjusted data

The stationarity of our variables is analyzed using several unit root tests allowing for potential endogenous structural break(s), both under the null of a unit root and under the alternative. We justify this choice considering the strong influence the global financial crisis in the years 2007-2009 had on the German economy, which is visible when looking at Figures 1 and 2. As for the Google data, we remark that the statistical effects of dividing the original search data by the total number of web searches in the same week and area are unknown, so that we cannot say a priori whether they are stationary or not (see also Fantazzini and Fomichev (2014) for a discussion on this issue). More specifically, we employed four unit root tests: the Lee and Strazicich (2003) unit root tests allowing for one and two breaks, respectively, and the Range Unit Root (RUR) and the Forward-Backward RUR tests suggested by Aparicio, Escribano, and Garcia (2006), which are non-parametric tests robust against nonlinearities, error distributions, structural breaks and outliers. A brief description of these tests is reported in the Technical Appendix

<sup>2</sup>The GIs were linearly re-scaled to positive numbers and then transformed into logarithms.

A accompanying this paper and can be found on the authors' websites.

	RUR Test statistic	FB Test statistic	LS 1 break Test statistic	LS 2 breaks Test statistic	The null hypothesis is rejected by all tests?
<i>Car sales</i>					
BMW	0.71 *	1.16	-5.08 *	-11.14 *	no
Citroen	1.34	1.95	-5.12 *	-6.09 *	no
Fiat	0.79 *	1.89	-4.75 *	-6.31 *	no
Jaguar	0.87 *	1.39	-4.47	-6.98 *	no
Kia	1.42	2.01	-4.94 *	-5.89 *	no
Mitsubishi	0.79 *	1.34	-5.05 *	-5.79 *	no
Opel	0.87 *	1.56	-6.17 *	-6.87 *	no
Suzuki	1.02 *	1.67	-4.91 *	-6.47 *	no
Toyota	1.50	1.95	-4.92 *	-5.86 *	no
Volkswagen	0.87 *	1.73	-6.66 *	-7.52 *	no
<i>Economic variables</i>					
BUILD	1.34	2.17	-2.33	-8.68 *	no
CCI	1.18	2.23	-3.60	-4.07	no
CPI	9.14 *	13.15*	-3.53	-4.10	no
EURIBOR	3.07	3.73 *	-3.46	-4.29	no
PP	2.68	3.96 *	-3.65	-5.26	no
GDP	6.30 *	8.75 *	-3.67	-4.53	no
PI	1.42	1.67	-3.88	-4.80	no
UR	5.28 *	7.30 *	-3.42	-5.66	no
<i>Google data</i>					
BMW GI	1.34	1.77	-5.24 *	-8.59 *	no
Citroen GI	1.97	2.34	-5.98 *	-6.71 *	no
Fiat GI	1.43	2.34	-4.59 *	-7.07 *	no
Jaguar GI	1.52	1.90	-7.12 *	-8.10 *	no
Kia GI	0.80 *	1.39	-7.45 *	-8.12 *	no
Mitsubishi GI	2.68	2.97	-9.26 *	-9.83 *	no
Opel GI	1.25	2.53	-4.51 *	-5.24	no
Suzuki GI	1.88	2.09	-7.18 *	-8.24 *	no
Toyota GI	1.34	1.90	-4.67 *	-5.17	no
Volkswagen GI	1.34	1.83	-4.96 *	-5.55	no

Table 2: Unit root tests: RUR = Range Unit Root test by Aparicio, Escribano, and Garcia (2006); FB = Forward-Backward RUR test by Aparicio, Escribano, and Garcia (2006); LS = Unit Root test by Lee and Strazicich (2003). Null hypothesis: the time series has a unit root. \* Significance at the 5% level.

The results in Table 2 show that the majority of our time series are not stationary. However, the Lee and Strazicich (2003) tests show a stronger evidence of unit roots for economic variables, while the Aparicio, Escribano, and Garcia (2006) tests show the same for car sales and Google data. If we follow a conservative approach and analyze when all four tests reject the null hypothesis (see the last column in Table 2), then all car brands can be deemed non-stationary.

### 2.1.2 Raw data

To test the null hypothesis of a periodic unit root, we follow the two-step strategy suggested by Boswijk and Franses (1996) and Franses and Paap (2004). In the first step, a likelihood ratio test for testing a single unit root in a Periodic Auto-Regressive (PAR) model of order  $p$  is performed. Since there is no version of this test with endogenous breaks, we estimated it both with the full sample starting in 2001, and with a smaller sample starting in 2008. The year 2008 was chosen following the previous evidence of a possible break in this year, which emerged with the unit root tests allowing for breaks in the case of seasonally adjusted data. If the null of a periodic unit root cannot be rejected, Boswijk and Franses (1996) and Franses and Paap (2004) suggest to test in a second step whether the process contains a non-periodic unit root equal to 1 for all seasons. A description of these tests is reported in the Technical Appendix B.

Table 3 shows that car sales offer different results depending on the sample used: if the full sample is considered, non-stationarity is rejected for all car brands but BMW (for which the estimates did not reach numerical convergence); if the smaller sample starting from 2008 is used, the test failed to converge for several brands, while for two brands (Citroen and Kia) the null of a non-periodic unit root cannot be rejected. This evidence again highlights the possible presence of a structural break in 2008 during the global financial crisis. Economic variables and GIs are mostly non-stationary with a non-periodic unit root and the results do not change substantially with the sample used.

	Sample: 2001-2014		Sample: 2008-2014	
	1st step $H_0:$ periodic unit root	2nd step $H_0:$ non periodic unit root	1st step $H_0:$ periodic unit root	2nd step $H_0:$ non periodic unit root
	<i>Car Sales</i>			
BMW	NC	NC	NC	NC
Citroen	18.66*	/	7.21	0.46
Fiat	16.60*	/	4.43	<b>0.00</b>
Jaguar	42.41*	/	NC	NC
Kia	10.46*	/	4.96	0.08
Mitsubishi	22.97*	/	16.96*	/
Opel	15.38*	/	10.66*	/
Suzuki	24.85*	/	15.95*	/
Toyota	10.19*	/	15.81*	/
Volkswagen	58.20*	/	NC	NC
<i>Economic Variables</i>				
BUILD	7.99	0.09	2.32	0.11
CCI	3.23	0.06	1.02	0.14
CPI	0.13	<b>0.00</b>	0.30	0.44
EURIBOR	0.37	0.66	1.99	0.15
PP	1.97	0.88	1.36	0.10
GDP	0.01	<b>0.00</b>	0.15	<b>0.00</b>
PI	36.79*	/	22.07*	/
UR	0.52	0.56	NC	NC
<i>Google data</i>				
BMW GI	8.93	0.49	2.71	0.53
Citroen GI	4.90	0.47	4.46	0.13
Fiat GI	4.47	<b>0.04</b>	1.84	0.11
Jaguar GI	12.02*	/	5.17	<b>0.01</b>
Kia GI	16.82*	/	8.07	0.76
Mitsubishi GI	3.91	0.99	2.19	0.35
Opel GI	6.06	0.64	6.69	0.53
Suzuki GI	3.60	<b>0.02</b>	3.63	<b>0.04</b>
Toyota GI	5.86	0.46	5.15	<b>0.01</b>
Volkswagen GI	11.20*	/	5.38	0.39

Table 3: Periodic Unit root tests by Boswijk and Franses (1996) and Franses and Paap (2004).

\* Significance at the 5% level. NC = Not Converged. The second step is performed only if the first step numerically converged and did not reject the null hypothesis.  $p$ -values smaller than 0.05 are in bold.

## 2.2 Weak Exogeneity and Cointegration Tests

### 2.2.1 Seasonally Adjusted data

The next step in the structural relationship identification methodology discussed by Sa-ngasoongsong, Bukkapatnam, Kim, Iyer, and Suresh (2012) is to determine the exogeneity of each variable using the sequential reduction method for weak exogeneity proposed by Hall, Henry, and Greenslade (2002). This method exogenizes all weakly exogenous variables and re-tests the remaining variables until all weakly exogenous variables are identified. The variables that reject the null of weak exogeneity after re-testing are reported in Table 12 in Appendix A: the Euribor series can be considered weakly exogenous for four car brands, while almost all other variables are deemed endogenous (with some exceptions for Mitsubishi).

We then proceeded to test for cointegration using the variables which were deemed endogenous according to the previous sequential test procedure by Hall, Henry, and Greenslade (2002). We test for cointegration using a set of cointegration tests allowing for the presence of structural break(s):

- Gregory and Hansen (1996) single-equation cointegration test allowing for one endogenous break;
- Hatemi (2008) single-equation cointegration test allowing for two endogenous breaks;
- Johansen, Mosconi, and Nielsen (2000) multivariate test allowing for the presence of one or two exogenous break(s), where the dates of the breaks are the ones selected by the Gregory and Hansen (1996) and Hatemi (2008) tests, respectively.

A description of these cointegration tests is reported in the Technical Appendix C. For the sake of generality, we also considered the multivariate cointegration test by Johansen (1995) without breaks. The main advantage of single-equation approaches is that they allow for endogenous breaks. However, these tests are not suitable when the right-hand variables in the cointegration vector are not weakly exogenous (as in our case) and when there is more than one cointegrating vector. In this case, multivariate cointegration tests should be used. The only problem with the multivariate tests by Johansen, Mosconi,

and Nielsen (2000) is that they allow only for exogenous breaks. Accordingly, we followed a 2-step strategy: we first estimated the single-equation tests to obtain an indication of the structural break dates. We then used these dates to compute the tests by Johansen, Mosconi, and Nielsen (2000). Finally, we remark that the number of lags for the Johansen tests were chosen to minimize the Schwartz criterion and to make the residuals approximately white noise.

Single-Equation cointegration tests				
	Gregory and Hansen (1996)		Hatemi (2008)	
	one(endogenous) break	Break date	two(endogenous) breaks	Break dates
	Z-t statistic		Z-t statistic	
BMW	-10.61*	2010M02	-11.14*	2006M09 2008M07
Citroen	-7.38*	2009M02	-8.35	2005M08 2007M07
Fiat	-7.54*	2006M01	-8.27	2005M11 2007M08
Jaguar	-14.54*	2012M09	-14.30*	2007M10 2011M02
Kia	-8.27*	2006M09	-8.61	2006M09 2011M01
Mitsubishi	-10.98*	2009M03	-10.79*	2008M04 2008M12
Opel	-8.72*	2009M02	-7.60	2009M09 2010M10
Suzuki	-10.85*	2009M02	-10.14	2006M09 2007M06
Toyota	-7.95*	2009M12	-8.40	2006M09 2009M07
Volkswagen	-9.96*	2009M03	-9.35	2005M08 2007M08

Multivariate cointegration tests				
Johansen (1995)	Johansen, Mosconi, and Nielsen (2000)		Johansen, Mosconi, and Nielsen (2000)	
No Breaks	one(exogenous) break	two (exogenous) breaks	N. of CEs at 5% level	Break dates (H,2008)
	N. of CEs at 5% level	Break date (GH,1996)		
BMW	5 CE	2010M02	5 CE	2006M09 2008M07
Citroen	5 CE	2009M02	5 CE	2005M08 2007M07
Fiat	7 CE	2006M01	7 CE	2005M11 2007M08
Jaguar	5 CE	2012M09	5 CE	2007M10 2011M02
Kia	5 CE	2006M09	4 CE	2006M09 2011M01
Mitsubishi	4 CE	2009M03	NC	2008M04 2008M12
Opel	5 CE	2009M02	5 CE	2009M09 2010M10
Suzuki	5 CE	2009M02	NC	2006M09 2007M06
Toyota	5 CE	2009M12	5 CE	2006M09 2009M07
Volkswagen	5 CE	2009M03	5 CE	2005M08 2007M08

Table 4: Single-equation and multivariate cointegration tests with and without structural break(s) for seasonally-adjusted data. The null hypothesis for all tests is the absence of cointegration. The tests considered the case of a level shift. The table cells for the Johansen tests report the number of CEs selected at the 5% level. NC=not converged. \* Significance at the 5% level.

Table 4 shows that there is strong evidence for cointegration for all considered car brands. However, structural breaks seem to have a non-negligible effect, particularly when considering Johansen multivariate tests. Moreover, the effects of breaks appear to be much stronger for foreign brands than for domestic brands (BMW, Volkswagen and, to a lesser extent, Opel), for which the cointegration tests do not change substantially when breaks are taken into account.

### 2.2.2 Raw data

To determine the exogeneity of variables with potential seasonal behavior, we extend the previous sequential reduction method for weak exogeneity by including centered seasonal dummies: they sum to zero over time and therefore do not affect the asymptotic distributions of the tests (see Johansen (1995, 2006)). The variables that reject the null of weak exogeneity after re-testing are reported in Table 13 in Appendix A: the results for raw data are not too dissimilar to the seasonally-adjusted data, even though there are less variables which are weakly exogenous. We then tested for cointegration using the variables which were found to be endogenous, and the previous cointegration tests augmented with centered seasonal dummies, see Table 5.

In the case of raw data, the evidence for cointegration appears to be quite similar to that of seasonally-adjusted data, particularly when considering the Johansen test without breaks and with one break. Moreover, the fact that the Johansen test with two breaks failed to converge for some car brands indicates that our sample is too small for two breaks and that only tests with one break should be considered.

Periodic cointegration tests using all variables could not be implemented due to the high number of parameters being estimated (the so-called “curse of dimensionality”). However, we wanted to consider a restricted bivariate periodic error correction model including only car sales and Google data. Even though such a specification is definitely biased – missing several important economic variables – this

Single-Equation cointegration tests				
Gregory and Hansen (1996) one (endogenous) break			Hatemi (2008) two (endogenous) breaks	
Z-t statistic	Break date	Z-t statistic	Break dates	
BMW	-10.78*	2010M02	11.35*	2006M09 2008M07
Citroen	-7.70*	2009M02	8.60	2005M08 2007M07
Fiat	-7.63*	2005M10	8.64	2005M10 2007M08
Jaguar	-13.10*	2006M11	NC	NC
Kia	-8.71*	2006M09	9.25	2009M09 2011M01
Mitsubishi	-11.54*	2009M02	10.88*	2008M03 2008M12
Opel	-8.48*	2009M02	7.30	2009M09 2010M12
Suzuki	-11.00*	2009M02	9.64	2006M09 2007M07
Toyota	-7.44*	2009M12	8.03	2009M10 2010M12
Volkswagen	-10.67*	2009M02	9.63	2005M08 2007M07

Multivariate cointegration tests					
Johansen (1995) No Breaks		Johansen, Mosconi, and Nielsen (2000) one(exogenous) break		Johansen, Mosconi, and Nielsen (2000) two(exogenous) breaks	
N. of CEs at 5% level	N. of CEs at 5% level	Break date (GH,1996)	N. of CEs at 5% level	Break dates (H,2008)	
BMW	5 CE	4 CE	2010M02	5 CE	2006M09 2008M07
Citroen	5 CE	5 CE	2009M02	5 CE	2005M08 2007M07
Fiat	5 CE	6 CE	2005M10	7 CE	2005M10 2007M08
Jaguar	3 CE	0 CE	2006M11	NC	NC
Kia	5 CE	5 CE	2006M09	5 CE	2009M09 2011M01
Mitsubishi	4 CE	4 CE	2009M02	NC	NC
Opel	5 CE	4 CE	2009M02	5 CE	2009M09 2010M12
Suzuki	5 CE	6 CE	2009M02	NC	NC
Toyota	5 CE	5 CE	2009M12	5 CE	2009M10 2010M12
Volkswagen	5 CE	6 CE	2009M02	6 CE	2005M08 2007M07

Table 5: Single-equation and multivariate cointegration tests with and without structural break(s) for raw data. The null hypothesis for all tests is the absence of cointegration. The tests considered the case of a level shift. The table cells for the Johansen tests report the number of CEs selected at the 5% level. NC=not converged. \* Significance at the 5% level.

parsimonious model can nevertheless be of interest for forecasting purposes. Moreover, the capacity of Google data to summarize a wealth of information should not be underestimated. In this regard, we implemented the single-equation periodic cointegration test discussed in Franses and Paap (2004), which is an extension of the Boswijk (1994) cointegration test. The null hypothesis is the absence of cointegration against the alternative of periodic cointegration and the right-hand variables should be weakly exogenous. A description of this test as well as the test for weak exogeneity in the case of periodic variables by Boswijk (1994) is reported in the Technical Appendix D. Since we are not aware of any extension of this test allowing for structural breaks, we estimated it using both the full sample and a reduced sample starting in 2008 to take any potential break into account and the results are reported in Table 14 in Appendix A: the evidence in favor of periodic cointegration is fairly strong, but the results of the Boswijk test statistics change partially when the smaller sample starting in 2008 is considered. Caution should therefore be exercised when dealing with this restricted model. Interestingly, the GIs are weakly exogenous with respect to car sales for almost all brands at the 5% level and this outcome does not change substantially with the sample used.

### 2.3 Impulse Response Functions

After the VECM (or VECMX) models were selected for each car brand, we proceeded to compute the impulse response functions (IRFs) in order to trace the effects of a one-time shock in one of the variables on current and future values of car sales. More specifically, we computed the generalized impulse response functions by Pesaran and Shin (1998), which do not depend on the ordering of the variables. For the sake of interest and space, we report here only the IRFs for the seasonally-adjusted sales data (Figure 3) with respect to a generalized one standard deviation innovation in the Google Indexes. Moreover, we report in Table 6 the estimated long-run parameters in the cointegration equations and their adjustment coefficients for the Volkswagen car sales equation, noting that Volkswagen is the biggest car maker and seller in Germany. A battery of misspecification tests computed on the VECMX model residuals is reported in the same table as well: we computed multivariate LM test statistics for residual serial correlation up to a specified order, univariate and multivariate Jarque-Bera residual normality tests, and the multivariate White heteroskedasticity test (see Johansen (1995) and Lutkepohl (2005) for more

details about these tests). The full results are available from the authors upon request.

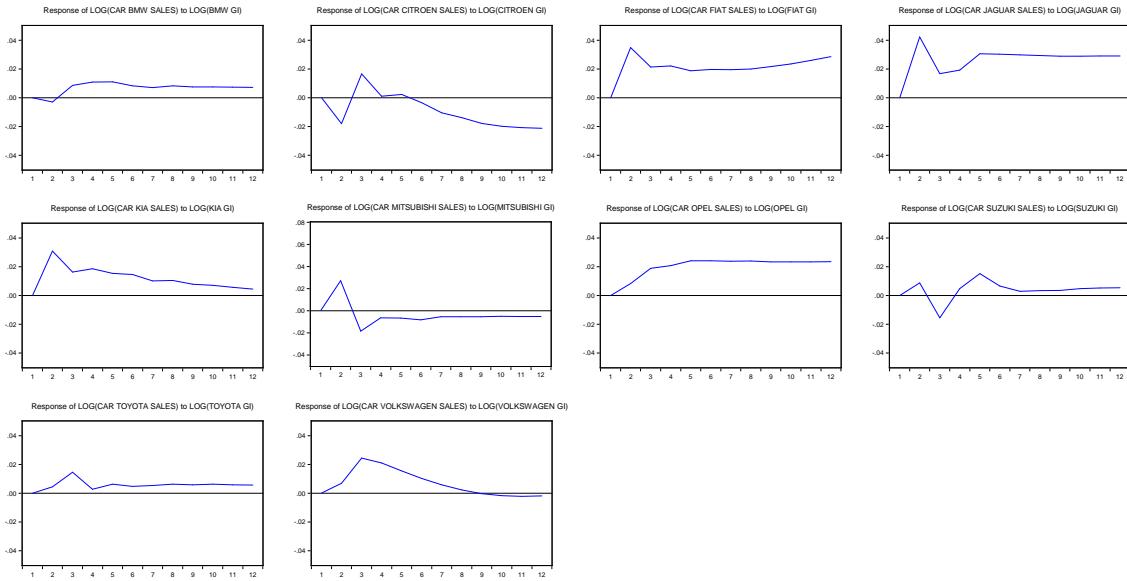


Figure 3: Impulse response functions: response of car sales (in logs) to generalized one standard deviation innovations in the Google Indexes.

	Long-run parameters ( $\beta$ )					Misspecification tests		
	CE 1	CE 2	CE 3	CE 4	CE 5	p-values		p-values
Log(SALES(-1))	1	0	0	0	0	Multi. LM(1)	0.06	Uni. JB test
Log(BC(-1))	0	1	0	0	0	Multi. LM(2)	0.76	SALES
Log(CCI(-1))	0	0	1	0	0	Multi. LM(3)	0.22	BC
Log(CPI(-1))	0	0	0	1	0	Multi. LM(4)	0.35	CCI
Log(EURIBOR(-1))	0	0	0	0	1	Multi. LM(5)	<b>0.02</b>	CPI
Log(PP(-1))	0.11 [ 0.35]	-0.71 [ -2.63]	-0.29 [ -0.91]	-0.03 [ -3.34]	0.77 [ 1.20]	Multi. LM(6)	0.65	EURIBOR
Log(PI(-1))	2.03 [ 5.19]	1.90 [ 5.60]	1.57 [ 3.97]	0.14 [ 14.10]	-8.97 [ -11.10]	Multi. LM(7)	0.75	PP
Log(UR(-1))	0.98 [ 3.51]	-1.12 [ -4.63]	-0.27 [ -0.95]	0.05 [ 6.33]	6.53 [ 11.31]	Multi. LM(8)	0.09	PI
Log(GOOGLE(-1))	-1.95 [ -6.82]	-0.77 [ -3.08]	0.08 [ 0.26]	-0.03 [ -3.83]	-3.16 [ -5.34]	Multi. LM(9)	0.52	UR
Log(GDP(-1))	2.16 [ 2.91]	-2.96 [ -4.61]	-0.83 [ -1.11]	-0.46 [ -24.16]	19.78 [ 12.95]	Multi. LM(10)	0.41	GOOGLE
Constant	-27.59 [ -6.64]	14.91 [ 4.13]	-4.73 [ -1.13]	-2.14 [ -20.05]	-89.07 [ -10.39]	Multi. LM(11)	0.06	GDP
	Adjustment coefficients ( $\alpha$ ) - car sales equation					Multi. LM(12)	0.33	Multi. JB test
	-0.72 [ -6.33]	-0.04 [ -0.23]	0.64 [ 4.60]	5.55 [ 1.70]	0.18 [ 4.25]			Multi. White

Table 6: Long-run parameters and adjustment coefficients for the Volkswagen car sales equation (left table). Misspecification tests on the residuals from the Volkswagen VECMX model (right table). *t*-statistics are reported in brackets, while *p*-values smaller than 5% are reported in bold.

As expected, a unit shock in the Google Index has a rather long and positive effect for almost all car brands. Similarly, the model estimates in Table 6 show that the Google Index enters almost all cointegration equations with significant positive coefficients<sup>3</sup>, while the residual tests do not signal any serious misspecification.

<sup>3</sup>The signs of the long-run parameters in Table 6 are switched due to the error correction representation.

### 3 Out-of-Sample Forecasting Analysis

The last step in the structural relationship identification methodology discussed by Sa-ngasoongsong, Bukkapatnam, Kim, Iyer, and Suresh (2012) is to compare the forecasting performances of the selected VECM (or VECMX) models with a set of competitors.

#### 3.1 Seasonally Adjusted data

We compared a set of 34 models, which allow for different degrees of model flexibility, parsimonious specifications and numerical tractability. More specifically, three types of multivariate models were employed:

- *Vector Error Correction (VEC) models:* We considered both VECM and VECMX models, as well as models with and without Google data, to better examine their effects on forecasting performance. The number of lags was selected to minimize the Schwartz criteria and to make the residuals approximately white noise. We also considered a set of parsimonious bivariate specifications including only car sales and Google data, which may be of interest for long-term forecasting.
- *Vector Auto-Regressive (VAR) models:* We considered VAR models with variables in log-levels and in log-differences, to consider both cases of stationarity and non-stationarity. Moreover, models with and without exogenous variables and with and without Google data were also considered. Finally, a set of parsimonious bivariate VAR models including only car sales and Google data was included.
- *Bayesian Vector Auto-Regressive (BVAR) models:* When there are a lot of variables and a high number of lags, estimating the parameters of a VAR model can be very difficult, if not impossible. One way to solve this issue is to shrink the parameters using Bayesian methods. Bayesian VAR models have recently enjoyed a lot of success in macroeconomic forecasting (see Koop and Korobilis (2010) for a recent review and Fantazzini and Fomichev (2014) for a recent application with Google data). In this regard, we used the so-called Litterman/Minnesota prior, which was developed by researchers at the University of Minnesota and at the Federal Reserve Bank of Minneapolis, and which is a common choice in empirical applications due to its computational speed and forecasting success (see Doan, Litterman, and Sims (1984), Litterman (1986) and Koop and Korobilis (2010)). A brief description of BVAR models can be found in the Technical Appendix E. Similarly to the VAR and VECM models, we considered models with and without exogenous variables, with and without Google data and with variables both in log-levels and in log-differences.

Besides these models, we also considered a set of standard univariate time series models:

- The Random Walk with drift;
- An AR(12) model for the log-returns of car sales.

Moreover, all models without Google data were estimated using both a long sample starting in 2001 and a short one starting in 2004, in the hope that this will show more clearly the advantages of Google data. The full details of all 34 multivariate models are reported in Table 7. For ease of reference, we also report in the sixth column a short-cut notation for identifying each model in the tables reporting the models forecasting performances.

We used the data between 2001M1 and 2008M9 as the first initialization sample for the models without Google data, and data from 2004M1 till 2008M9 for the models with Google data and those without Google data but estimated on a shorter sample. The evaluation period ranged from 2008M10 till 2014M6 and was used to compare forecasts from 1 step ahead up to 24 steps ahead. The top three models in terms of the Mean Squared Prediction Error (MSPE) for each forecasting horizon and each car brand are reported in Table 15, while the full results are available from the authors upon request.

Table 15 shows that there is no single model which outperforms all competitors for all horizons and all car brands. However, some general indications can be retrieved:

- The MSPEs of the competing models with forecasting horizons up to 8-10 steps ahead are relatively close (results not reported) and the Random Walk and the AR(12) models are sometimes ranked among the top three models;

Type	Log-levels / log-returns	Exogenous variables	Google data	Notes	Short cut notation (seas. adj. data)	Short cut notation (raw data)
<b>VEC MODELS</b>						
VECM	Log-lev/log-ret	no	yes		VECM	VECMP
VECMX	Log-lev/log-ret	yes	yes		VECMX	VECMXP
VECM	Log-lev/log-ret	no	no		VECMNOGO	VECMPNIGO
VECM	Log-lev/log-ret	no	no	Sample starts in 2004	VECMNOGO4	VECMPNIGO4
VECMX	Log-lev/log-ret	yes	no		VECMXNOGO	VECMXPNOGO
VECMX	Log-lev/log-ret	yes	no	Sample starts in 2004	VECMXNOGO4	VECMXPNOGO4
VECM	Log-lev/log-ret	no	yes	Only sales and GI. Lags: 1,12	VECongo112	VEPongo112
VECM	Log-lev/log-ret	no	yes	Only sales and GI. Lags: 1-12	VECongo12	VECPongo12
<b>VAR MODELS</b>						
VAR	Log-levels	no	yes		VAR	VARP
VAR	Log-returns	no	yes		VARD	VARPD
VAR	Log-levels	yes	yes		VARX	VARXP
VAR	Log-returns	yes	yes		VARXD	VARXPD
VAR	Log-levels	no	no		VARNOGO	VARPNIGO
VAR	Log-levels	no	no	Sample starts in 2004	VARNOGO4	VARPNIGO4
VAR	Log-returns	no	no		VARDNOGO	VARPDNOGO
VAR	Log-returns	no	no	Sample starts in 2004	VARDNOGO4	VARPDNOGO4
VAR	Log-levels	yes	no		VARXNOGO	VARXPNOGO
VAR	Log-returns	yes	no	Sample starts in 2004	VARXNOGO4	VARXPNOGO4
VAR	Log-returns	yes	no		VARXDNNOGO	VARXPDNNOGO
VAR	Log-returns	yes	no	Sample starts in 2004	VARXDNNOGO4	VARXPDNNOGO4
VAR	Log-levels	no	yes	Only sales and GI. Lags: 1,12	VARongo112	VARongo112
VAR	Log-levels	no	yes	Only sales and GI. Lags: 1-12	VARongo12	VARongo12
VAR	Log-returns	no	yes	Only sales and GI. Lags: 1,12	VADongo112	VADongo112
VAR	Log-returns	no	yes	Only sales and GI. Lags: 1-12	VADongo12	VADongo12
<b>BVAR MODELS</b>						
BVAR	Log-levels	yes	yes		BVAR	BVARP
BVAR	Log-returns	yes	yes		BVARD	BVARPD
BVAR	Log-levels	yes	no		BVARNOGO	BVARPNIGO
BVAR	Log-levels	yes	no	Sample starts in 2004	BVARNOGO4	BVARPNIGO4
BVAR	Log-returns	yes	no		BVARDNOGO	BVARPDNOGO
BVAR	Log-returns	yes	no	Sample starts in 2004	BVARDNOGO4	BVARPDNOGO4
<b>UNIVARIATE TIME SERIES MODELS</b>						
AR(12)	Log-returns	no	no		AR12	AR12
AR(12)	Log-returns	no	no	Sample starts in 2004	AR124	AR124
R. w.	Log-returns	no	no		RW	RW
R. w.	Log-returns	no	no	Sample starts in 2004	RW4	RW4
<b>PERIODIC ERROR CORRECTION MODELS</b>						
Periodic ECM	Log-lev/log-ret	no	yes	Only sales and GI. Lags: 1-12	/	PECM

Table 7: Models used for forecasting (baseline case).

- Bayesian VAR models, particularly in differences and without Google data, perform rather well across all car brands and for short and medium forecasts (up to 12 steps ahead);
- Bivariate models including only car sales and Google models and using only the first and the 12th lags perform extremely well across most of the car brands examined, particularly for long-term forecasts. The parsimonious specifications of these models clearly allow for efficiency gains where forecasting is of concern.
- The forecasting power of the best models using Google data increases with the length of the forecast horizon, particularly with forecast horizons higher than 12 steps ahead. This evidence is similar to that found in D’Amuri and Marcucci (2013) and Fantazzini and Fomichev (2014).
- Models without Google data estimated with the long sample starting in 2001 tend to perform better than those estimated with a shorter sample starting in 2004.
- There are no particular differences between large, medium-sized and small sellers and between foreign and German manufacturers.

So as to provide an idea about how prediction errors evolve over time, Figure 4 (columns 1 and 2 for seasonally adjusted data) shows the ratios of the MSPE of the best model with Google data and the Random Walk model across all forecasting horizons, together with the ratios of the MSPE of the best

model without Google data and the Random Walk model. We remark that the best models tend to vary across different horizons.

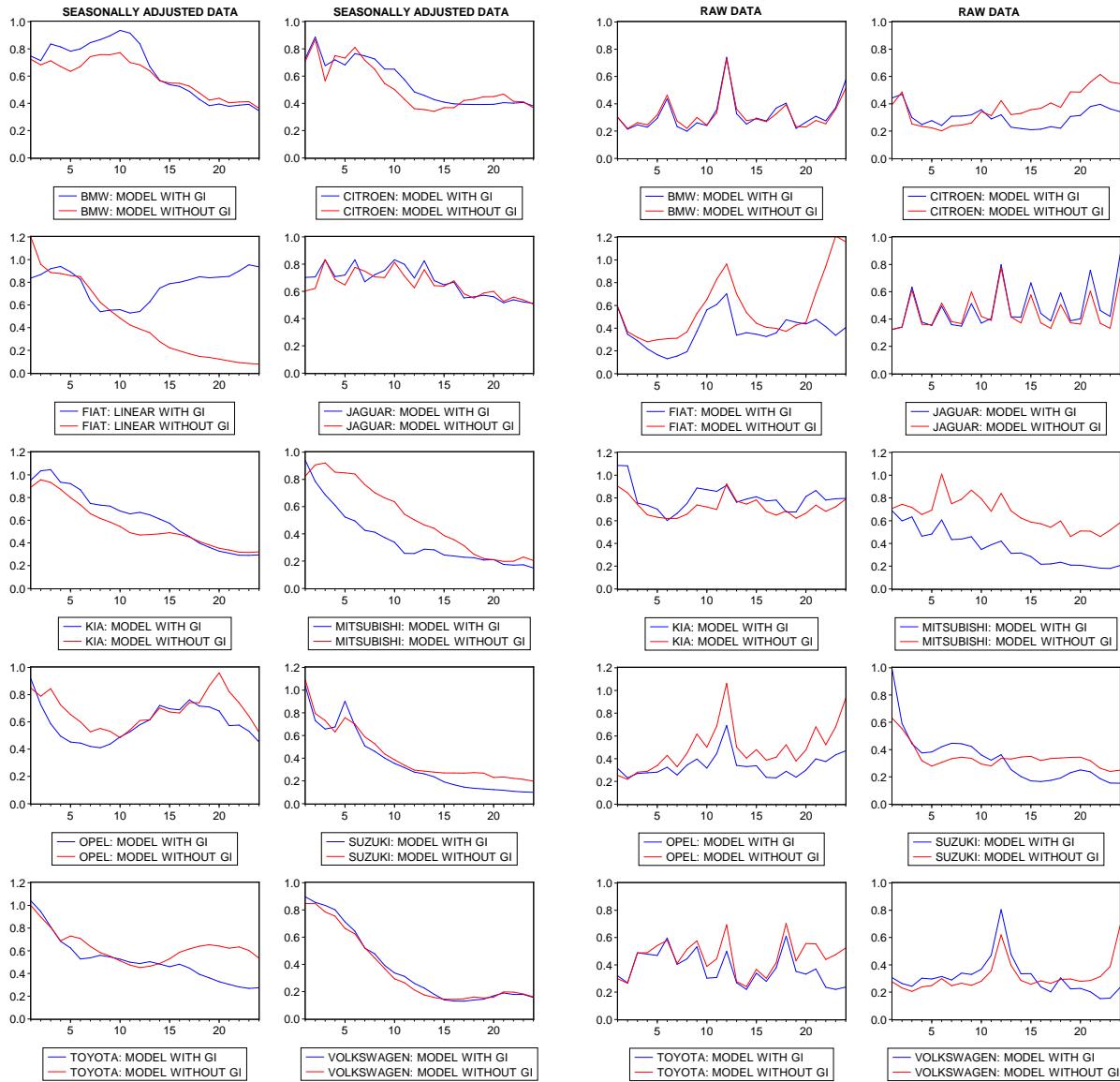


Figure 4: Ratios of the MSPEs of the best models with and without Google data and the Random Walk model across all forecasting horizons. The first two columns show results for seasonally-adjusted data, and the last two for raw data.

The ratios in Figure 4 show that it is difficult to outperform the random walk model in the case of short-term forecasts. Moreover, the best models without Google data tend to perform better than the best models with Google data for short and medium forecasts, whereas in general models using Google data show lower MSPEs for long-term forecasts with horizons higher than 12 steps ahead. This evidence suggests that potential gains in terms of forecasting performance may be achieved by using forecast combination methods. The development of these methods is beyond the scope of this paper and will be the subject of future studies.

Model rankings in terms of the MSPE do not show whether the competing forecasts are statistically different or not. We therefore tested for significant differences in forecast accuracy using the Model Confidence Set (MCS) approach proposed by Hansen, Lunde, and Nason (2011). The MCS is a sequential test of equal predictive ability, with the starting hypothesis that all models considered have equal forecasting performance. Given an initial set of forecasts, it tests the null that no forecast is distinguishable from any other and discards any inferior forecasts if they exist. The MCS procedure yields a model confidence set containing the best forecasting models at a given confidence level. Since our dataset is not too large and the number of forecasting models is moderate, we employed the semiquadratic test statistic ( $T_{SQ}$ ), which is more computationally intensive but more selective, see e.g. Rossi and Fantazzini (2014). The loss function used was the MSPE, while the  $p$ -values for the test statistic were obtained using a stationary block bootstrap with a block length of 12 months and 1000 re-samples. If the  $p$ -value was lower than a defined confidence level  $\alpha$ , the model was not included in the MCS and viceversa. A brief description of the MCS approach is reported in the Technical Appendix F.

The models included in the MCS at the 10% level for all car brands and forecast horizons are reported in Table 16<sup>4</sup>: for the sake of space and interest, we report only the total number of selected models, the total number of selected Google-based models, and whether the Random Walk model was included or not. The full set of results is available from the authors upon request.

Table 16 shows that most, if not all, models are selected in the case of forecasts up to 10-12 steps ahead for five car brands out of ten: the differences in forecasting performances are not large enough to distinguish between them, meaning that the MCS contains a large number of models. Moreover, the Random Walk model is often included. Instead, for long-term forecasts (12 steps ahead and higher), only a small number of models is selected, most of them bivariate models including only car sales and GIs, Bayesian VARs with GIs and sometimes the AR(12). Besides, the Random Walk model is seldom included. Here, the data are much more informative and it is possible to select a limited number of models which statistically outperform their competitors.

### 3.2 Raw data

We compared the same 34 models used for seasonally-adjusted data, but augmented with centered seasonal dummies to model potential seasonal behavior. Moreover, we also considered the bivariate Periodic Error Correction Model PECM(1,12) which includes only car sales and Google data, as discussed in section 2.2.2. To account for the possible endogeneity of regressors and improve the efficiency of the parameter estimates in small samples, we estimated the error correction term using the method of dynamic OLS (see Boswijk and Franses (1995), Hayashi (2000) and Franses and Paap (2004)). A short-cut notation for identifying each model in the subsequent tables reporting their forecasting performances is reported in the last column of Table 7.

We used the data between 2001M1 and 2009M6 as the first initialization sample for the models without Google data, while we used the initialization sample 2004M1-2009M6 for the models with Google data and for those without Google data but estimated on a shorter sample. The evaluation period ranged from 2009M7 till 2014M6 and was used to compare forecasts from 1 step ahead up to 24 steps ahead. The top three models in terms of the Mean Squared Prediction Error (MSPE) for each forecasting horizon and each car brand are reported in Table 17, while a summary of the models included in the MCS is reported in Table 18. The ratios of the MSPE of the best model with Google data and the Random Walk model across all forecasting horizons, together with the ratios of the MSPE of the best model without Google data and the Random Walk model are shown in the last two columns of Figure 4.

The results are somewhat similar to those which emerged from seasonally-adjusted data, but there are also some important differences. Models without Google data now perform better, with respect to the case of seasonally-adjusted data. Moreover, the number of models selected in the MCS is now much smaller (often no more than 2-6 models): Bayesian VARs (with and without Google data) and parsimonious bivariate models including only sales and GIs again represent the majority of models included in the MCS at the 10% level.

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<sup>4</sup>We set  $\alpha = 0.10$  as in Hansen, Lunde, and Nason (2011).

## 4 Robustness Checks

We wanted to verify that the superior performance of Google-based models also holds under alternative forecasting. We performed a series of robustness checks, considering alternative nonlinear models, alternative out-of-sample intervals, evaluating the directional accuracy of the competing forecasting models, checking whether Google data downloaded on different days can affect the models' forecasting performances, and examining additional car brands.

### 4.1 Nonlinear Models

A part of the economic and financial literature has suggested the use of nonlinear models for forecasting purposes (for instance, see Franses and Dijk (2000) and Terasvirta, Tjostheim, and Granger (2011) for a discussion at the textbook level). Given this evidence, we estimated a set of nonlinear models and compared their forecasting performances with the models in section 3. More specifically, we considered three nonlinear models:

- the SETAR model with 2 regimes (see Tong (1990) for a discussion at the textbook level);
- the logistic smooth transition autoregressive (LSTAR) model, which is a generalization of the SETAR model (see Tong (1990));
- the additive autoregressive model (AAR), also known as generalized additive model (GAM), since it combines generalized linear models and additive models (see Wood (2006) for a discussion at the textbook level).

A description of these nonlinear models is given in the Technical Appendix G. See D'Amuri and Marcucci (2013) and Fantazzini and Fomichev (2014) for a discussion of robustness checks using these nonlinear models.

The top three models in terms of the MSPE for each forecasting horizon and each car brand are reported in Table 19 for seasonally-adjusted data and in Table 21 for raw data. A summary of the models included in the MCS is reported in Table 20 for seasonally-adjusted data and in Table 22 for raw data.

In general, nonlinear models are very competitive, thus confirming past literature dealing with car sales forecasting (see Da, Engelberg, and Pengjie (2003), Kunhui, Qiang, Changle, and Junfeng (2007), Brühl, Borscheid, Friedrich, and Reith (2009), Hulsmann, Borscheid, Friedrich, and Reith (2012)). Particularly, parsimonious AAR and SETAR models involving only a few lags are often ranked among the top models in terms of MSPE. Moreover, AAR models with log-prices performed very well for medium- and long-term forecasts, similarly to what was found in Fantazzini and Fomichev (2014) when forecasting the real price of oil. However, nonlinear models were difficult to estimate, and specifications with a large number of lags failed to converge. Particularly, the LSTAR proved to be the most challenging and computationally intensive (see Franses and Dijk (2000) for a discussion of this issue). The results of the MCS confirm this evidence and most of the models included at the 10% level are nonlinear, whereas the only selected linear models are mostly Google-based. This evidence therefore seems to suggest that Google data may explain a good portion of the nonlinearity displayed by sales data.

In the case of raw data, nonlinear models are less competitive than linear models, particularly for forecasting horizons up to 12 steps ahead, whereas Bayesian VAR models and bivariate linear models including car sales and GIs are often the top ranked models across most of the car brands. However, for long-term forecasts, more than half of the models included in the MCS are nonlinear, while the remaining selected models are mainly bivariate Google-based models.

Tables 8-11 report the MSPEs, rankings, and eventual inclusion in the MCS of the best models in the case of 6, 12, 18, 24 step-ahead forecasts, respectively, for four model classes: linear models with GI, linear models without GI, nonlinear models and Random Walk models. Parsimonious bivariate models including only car sales and GIs are the best in the first class; AR(12) models and Bayesian models usually top the second class, while AAR and SETAR models with few lags are the best nonlinear models. The Random Walk has low rankings in long-term forecasts, but fares better for short-term forecasts.





The previous evidence is confirmed and summarized by Figure 5 in Appendix C, which shows the ratios of the MSPEs of the best models with and without Google data with those of the Random Walk model, together with the ratios of the MSPEs of the best nonlinear models and the Random Walk model across all forecasting horizons: nonlinear models tend to perform better with seasonally adjusted data and medium- and long-term forecasts.

Finally, for the sake of interest (given the importance of long-term forecasts for car manufacturers) and space, we report in Tables 23 and 24 the list of models included in the Model Confidence Set for each car brand for 24 step-ahead forecasts, for seasonally-adjusted data and raw data, respectively. In the latter case, the number of models selected is higher on average than for seasonally-adjusted data, which was expected given the more noisy nature of raw data.

## 4.2 Alternative Out-of-Sample Periods

Our baseline out-of-sample interval includes the global financial crisis which started in 2007 and had a strong effect on car sales. Moreover, our in-sample analysis highlighted a potential structural break in the years 2008-2009. Therefore, we want to verify that our results continue to hold with different business cycle conditions, as recently highlighted by D'Amuri and Marcucci (2013). We considered the following two alternative out-of-samples:

- 2008M10-2009M6: this sample includes the official period of recession in Germany.
- 2009M7-2014M6: this sample starts after the end of the recession.

Due to the dimensionality of these new out-of-samples, we considered forecasts up to only 8 steps ahead. Moreover, this robustness check was performed only with seasonally-adjusted data, since the first forecast with raw data takes place after the end of the recession<sup>5</sup>. The top three models in terms of the MSPE for each forecasting horizon and each car brand are reported in Table 25 for the recession period, and in Table 26 for the expansion period.

The results are somewhat mixed and change substantially according to the car brand which is examined. However, some general indications can still be gained: Google-based models and linear models without Google data were the best models during the recession, while Google-based models and nonlinear models performed (slightly) better during the economic expansion. These results therefore provide further evidence of a structural break in the years 2008-2009. In general, Google-based models had forecasting performances which were more robust across different business cycles than their competitors, thus confirming similar evidence found by D'Amuri and Marcucci (2013) and Fantazzini and Fomichev (2014).

## 4.3 Directional Accuracy

The analysis has so far only considered the accuracy of forecasts in terms of magnitude, but directional accuracy is also important: forecasts with the correct direction of change may still provide useful information even with large forecast errors. This is particularly important when predicting a turning point, which is a special case of directional accuracy and represents a change in the direction of movement of the analyzed variable (Theil (1961) and Naik and Leuthold (1986)).

The top three models in terms of average directional accuracy (in %) for each car brand, for short-term forecasts (1-6 steps ahead), medium-term forecasts (7-12 steps ahead), and long-term forecasts (13-24 steps ahead) are reported in Table 27 (top part) for seasonally adjusted data and in Table 27 (bottom part) for raw data.

In the case of seasonally-adjusted data, parsimonious bivariate models, including only car sales and GIs, as well as AAR models had the higher percentage of correct forecasts of the direction of change for most of the car brands and forecasting horizons. As for raw data, similarly to what we saw in section 4.1, nonlinear models are, in general, less competitive than linear models. More specifically, linear models without Google data performed better than with seasonally-adjusted data (particularly for short-term directional forecasts), while nonlinear models were competitive only for medium- to long-term directional accuracy. Instead, Google-based models performed relatively well and simple bivariate models with car

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<sup>5</sup>Raw data required a larger initialization sample due to the inclusion of centered seasonal dummies.

sales, GIs and centered seasonal dummies provided very precise forecasts of the direction of change for most of the car brands and forecasting horizons.

The somewhat differing results between seasonally-adjusted data and raw data could be due to two reasons. Firstly, the procedure of seasonal adjustment changes the statistical properties of the data and can affect considerably the models' forecasting performances (Zellner (1978) and Franses and Paap (2004)). Secondly, Boivin and Ng (2006) and Stock and Watson (2006) have shown that small models may outperform models with a larger number of parameters because they allow for a better extraction of relevant signals than models overloaded with parameters and complex specifications. In this regard, Google data allow us to summarize a lot of information and reduce model complexity.

#### 4.4 Sampling Variability of Google Data

Google data does not refer to the population of searches, but only to a sample. As a consequence, the time series of Google data can vary substantially from one download to another<sup>6</sup>. We downloaded the GIs for a number of subsequent days to check how sampling variability can affect the models' forecasting performances. More specifically, we compared the forecasts computed in our baseline case with GIs downloaded on 15/08/2014, with forecasts computed with the average GIs downloaded between the 15/08/2014 and the 02/09/2014. We used the average GIs following the approach recently proposed by Carriere-Swallow and Labb   (2013). Table 28 shows the average ratio – averaged across all forecasting horizons – of the MSPE for the forecasts computed with GIs downloaded on the 15/08/2014, with respect to MSPE for the forecasts computed with the average GIs downloaded between 15/08/2014 and 02/09/2014.

Almost all models have ratios close to 1, with the notable exception of high-dimensional VEC models, which did not reach convergence for a couple of car brands (Toyota and Kia). The large variance of estimators for cointegrated models in small-medium samples is a well known issue in the econometric literature (Stock and Watson (1993), Maddala and Kim (1998)(section 5.7) and Hayashi (2000)(section 10.4)): most likely, the sampling noise of Google data exacerbates this inference problem. Using average GIs can solve this issue to some extent, but not completely: the high-dimensional VECM models still did not reach convergence in some cases. Moreover, the rankings of Google-based models in the case of averaged data are very close, if not identical, to the rankings of Google-based models in the baseline case for all car brands (results not reported). Therefore, the most advisable solution is probably either to use parsimonious VEC models or revert to Bayesian methods.

#### 4.5 Additional Car Brands

In the baseline section, we analyzed 10 car brands out of the 22 car brands which both have monthly data continuously available since 2001 and are present in Google Trends. We briefly examine here the forecasting performances of the remaining 12 car brands:

- Large sellers: Ford, Audi;
- Medium-sized sellers: Hyundai, Mazda, Nissan, Peugeot, Renault;
- Small sellers: Honda, Land Rover, Porsche, Subaru, Volvo.

Table 29 and 30 report the top three models in terms of MSPE for each forecasting horizon and each car brand, in the case of seasonally-adjusted data and raw data, respectively.

The results are similar to those of the baseline case: parsimonious bivariate linear models involving only GIs and car sales and nonlinear models (with few lags) are the best models for all brands examined. Bayesian models are a valid alternative for short-term forecasting in the case of seasonally-adjusted data.

### 5 Conclusions

This paper proposed a set of multivariate models for forecasting car sales using both Google data and economic variables. Moreover, we considered multivariate models for both deseasonalized data and for raw data. We performed a forecasting exercise for ten car brands in Germany, and we computed

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<sup>6</sup>The authors want to thank an anonymous referee for pointing out this issue.

out-of-sample forecasts ranging from 1 month to 24 months ahead. Our results showed that Bayesian VAR models performed rather well for all car brands and for short- and medium-term forecasts, while parsimonious bivariate models including only car sales and Google models outperformed the competing models in the case of long-term forecasts for several brands. Furthermore, the forecasting power of the best Google-based models increased with the length of the forecast horizon, particularly with forecast horizons higher than 12 steps ahead. Apart from this, no particular differences between large, medium-sized and small sellers and between foreign and German manufacturers were found. In case of raw data, models without Google data performed better than in the case of seasonally-adjusted data. However, Bayesian VARs (with and without Google data) and parsimonious bivariate models including only sales and Google data represented again the majority of models included in the MCS at the 10% level. Finally, we performed a set of robustness checks to verify that our results also hold under different forecasting setups. We found out that nonlinear AAR and SETAR models were very competitive and were included in the MCS together with Google-based models, thus suggesting that Google data may explain a part of the nonlinearity displayed by sales data. However, nonlinear models were difficult to estimate and on several occasions failed to converge. Alternative out-of-sample intervals highlighted that Google-based models performed better during the recession (which is of particular importance for car manufacturers) and, in general, they had forecasting performances which were more robust across different business cycles than their competitors. Our previous results also held in the case of directional accuracy, which showed that Google-based models provided the most precise forecasts of the direction of change. We found that the sampling variability of Google data can be problematic for high-dimensional VEC models. Using the averaged Google data over several days can solve this issue to some extent, but parsimonious VEC models and Bayesian methods are valid alternatives as well. The results in the baseline case also held for twelve additional car brands.

Even though we considered a very large set of models, we had to restrict their potential range in order to keep the forecasting exercise computationally tractable. An avenue of future research would be to consider additional models such as fractional cointegration, exponential smoothing methods in state space form, and many others.

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## A In-sample Analysis

	BC	CCI	CPI	EURIBOR	PP	GDP	PI	UR	Google
BMW	V	V	V		V	V	V	V	V
Citroen	V	V	V	V	V	V	V	V	V
Fiat	V	V	V	V	V	V	V	V	V
Jaguar	V	V	V		V	V	V	V	V
Kia	V	V	V		V	V	V	V	V
Mitsubishi	V		V			V	V	V	V
Opel	V	V	V	V	V	V	V	V	V
Suzuki	V	V	V	V	V	V	V	V	V
Toyota	V	V	V	V	V	V	V	V	V
Volkswagen	V	V	V	V	V	V	V	V	V

Table 12: Weak exogeneity of seasonally-adjusted data: variables for which the null hypothesis of weak exogeneity can be rejected after re-testing at the 5% probability level.

	BC	CCI	CPI	EURIBOR	PP	GDP	PI	UR	Google
BMW	V	V	V		V	V	V	V	V
Citroen	V	V	V	V	V	V	V	V	V
Fiat	V	V	V	V	V	V	V	V	V
Jaguar	V		V			V	V	V	V
Kia	V	V	V	V	V	V	V	V	V
Mitsubishi	V	V	V	V	V	V	V	V	V
Opel	V	V	V	V	V	V	V	V	V
Suzuki	V	V	V	V	V	V	V	V	V
Toyota	V	V	V	V	V	V	V	V	V
Volkswagen	V	V	V	V	V	V	V	V	V

Table 13: Weak exogeneity of raw data: variables for which the null hypothesis of weak exogeneity can be rejected after re-testing at the 5% probability level.

	Sample: 2001-2014		Sample: 2008-2014	
	Boskwaik joint Wald test statistic	Weak exogeneity test of GIs (p-value)	Boskwaik joint Wald test statistic	Weak exogeneity test of GIs (p-value)
BMW	46.90*	0.86	39.08*	0.85
Citroen	45.75*	0.63	58.38*	0.30
Fiat	65.87*	0.15	116.11*	<b>0.00</b>
Jaguar	22.26	0.05	19.83	0.49
Kia	48.45*	0.06	31.83*	0.95
Mitsubishi	37.46*	0.66	29.17	0.62
Opel	49.26*	0.53	29.70	0.54
Suzuki	41.72*	0.61	67.46*	0.25
Toyota	69.64*	<b>0.01</b>	47.02*	<b>0.02</b>
Volkswagen	41.10*	0.56	48.33*	0.37

Table 14: The null hypothesis is the absence of cointegration against the alternative of periodic cointegration. The Boskwaik test considered the case with seasonal intercepts. \* Significance at the 5% level. p-values smaller than 5% are reported in bold.

















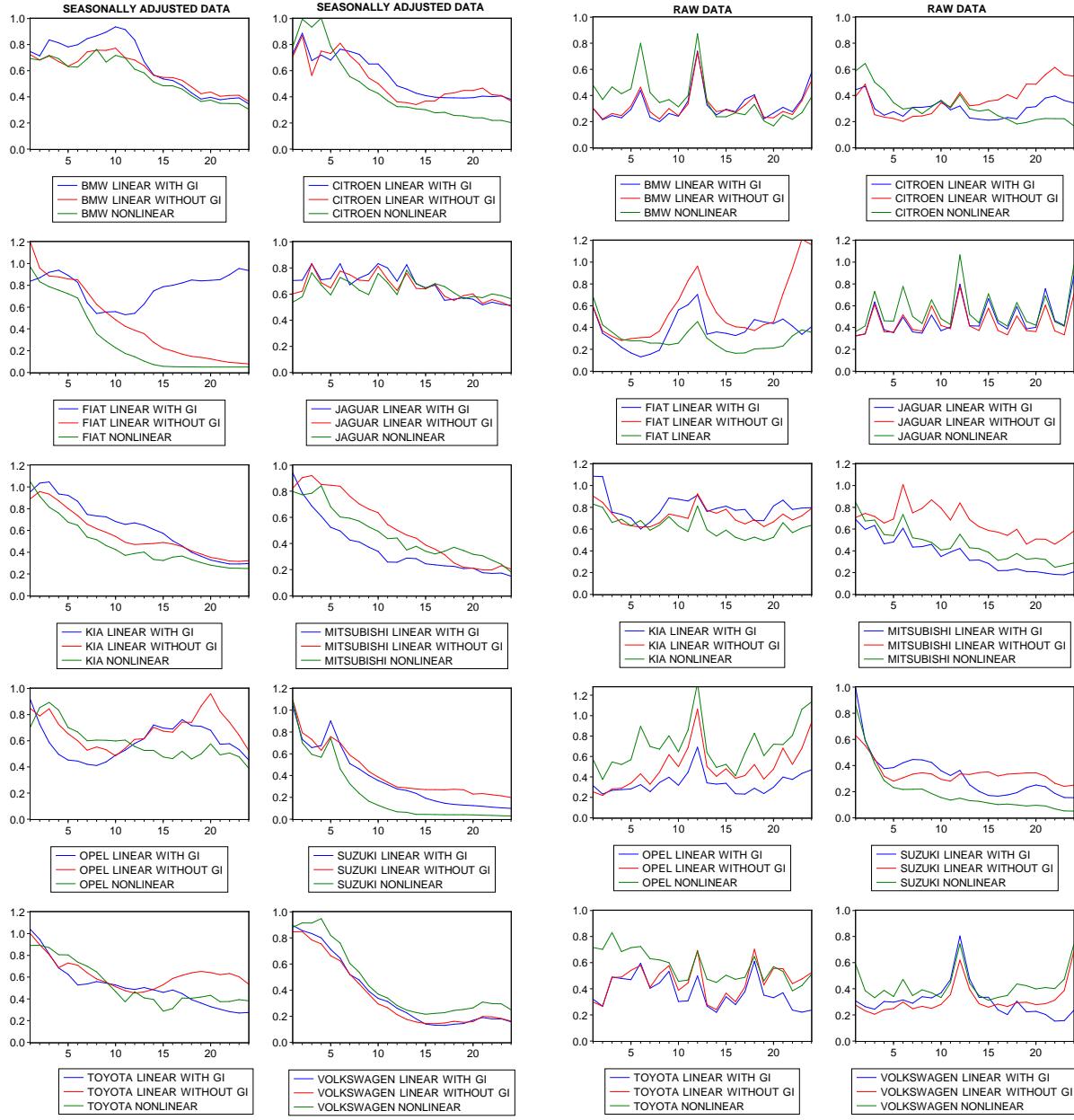


Figure 5: Ratios of the MSPEs of the best models with and without Google data with those of the Random Walk model, together with the ratios of the MSPEs of the best nonlinear models and the Random Walk model across all forecasting horizons. The first two columns show results for seasonally-adjusted data, and the last two for raw data.

### C.3 List of Models included in the Model Confidence Set. Forecast horizon: 24 steps ahead. Seasonally Adjusted data

SEASONALLY ADJUSTED DATA									
BmwBMW	Citroen	Fiat	Jaguar	Kia	Mitsubishi	Opel	Suzuki	Toyota	Volkswagen
AAR(1)log	AAR(1)log	AAR(1)log	AR12	SETAR(6)log	VECongo1112	SETAR(8)dlog	SETAR(2)log	VECongo1112	VARongo1112
VARongo1112	SETAR(4)log	AR124	VECongo1112	LSTAR(1)log	SETAR(1)log	VECongo1112	AAR(1)log	SETAR(6)dlog	AR124
AR124	SETAR(3)log	ARI24	VARongo1112	SETAR(8)log	VECMNOGO4	SETAR(7)dlog	SETAR(3)log	SETAR(1)dlog	AR12
AR12	LSTAR(3)log	VARongo1112	SETAR(5)log	SETAR(1)log	VARongo12	SETAR(5)dlog	AAR(3)log	SETAR(7)dlog	AAR(1)log
AAR(4)log	SETAR(5)log	AAR(2)log	SETAR(2)log	VARongo1112	AAR(8)dlog	SETAR(4)log	SETAR(3)dlog		
BVARNOGO	AAR(2)log	SETAR(1)log	SETAR(7)log	LSTAR(1)log0	VECMNOGO		SETAR(4)dlog		
		AAR(6)log	SETAR(1)log	SETAR(1)log					
		AAR(1)log	SETAR(9)log		AR12				
		LSTAR(1)log	SETAR(1)log		VECongo12				
		AAR(7)log	SETAR(3)log		VADong012				
		AAR(3)log		LSTAR(1)log1					
		SETAR(3)log		LSTAR(1)log2					
		AAR(5)log		ARI24					
		AAR(4)log		SETAR(4)log					
		VECongo12		AAR(5)log					
		LSTAR(4)log		AAR(3)dlog					
		LSTAR(5)log		VECMNOGO					
		LSTAR(3)log							
		SETAR(4)log							
		VARongo12							
		SETAR(1)log							
		VECMNOGO							
		VECMNOGO4							

Table 23: List of Models included in the Model Confidence Set for each car brand. Forecast horizon: 24 steps ahead.

#### C.4 List of Models included in the Model Confidence Set. Forecast horizon: 24 steps ahead. Raw data

RAW DATA									
BmwBMW	Citroen	Fiat	Jaguar	Kia	Mitsubishi	Opel	Suzuki	Toyota	Volkswagen
LSTAR(8)log	AAR(3)log	LSTAR(5)log	VARPNOGO	AAR(9)log	VARRongo1112	VARRongo1112	AAR(2)log	VARRongo1112	VECPongo12
LSTAR(1)log1	SETAR(2)dlog	AAR(3)log	VARPNOGO4	AAR(8)log	VEPongo1112	VEPongo1112	AAR(1)log	VEPongo1112	VEPongo1112
SETAR(1)log	LSTAR(2)dlog	AAR(4)log	VEPongo1112		AAR(1)log	VECMNPGO	SETAR(7)log	VADongo12	
SETAR(3)log	SETAR(1)log	SETAR(4)log	BVARPDNOGO4		AAR(2)log	RW	SETAR(3)log	SETAR(6)log	
LSTAR(1)log0	LSTAR(1)dlog	VARRongo1112	BVARPD		LSTAR(5)log		SETAR(2)log	SETAR(8)log	
SETAR(1)log	SETAR(3)log	LSTAR(4)log	VADongo12		LSTAR(4)log		SETAR(4)log	SETAR(3)log	
LSTAR(7)log	LSTAR(4)dlog	SETAR(1)log	VARXPNOGO		VECPongo12		AAR(3)log	VECMNPGO	
SETAR(7)log	SETAR(3)dlog	AAR(1)log	BVARPDNOGO		LSTAR(3)log		AAR(4)log	SETAR(5)log	
LSTAR(6)log	SETAR(4)dlog	LSTAR(3)log	VADongo1112		SETAR(5)log		SETAR(8)log	SETAR(4)log	
AR12	VARRongo1112	SETAR(5)log	VECPongo12		AAR(3)log		SETAR(6)log	BVARPD	
SETAR(5)log	LSTAR(3)dlog	AAR(2)log	LSTAR(1)log0		AAR(4)log		AAR(7)dlog	SETAR(9)log	
AAR(1)log	SETAR(6)log	LSTAR(2)log	LSTAR(9)log		SETAR(4)log		BVARPDNOGO4	VECMXPNOGO	
SETAR(6)log	SETAR(4)log	AAR(6)log	BVARPNOGO		LSTAR(7)log		SETAR(6)log	BVARPDNOGO	
AR124	LSTAR(4)log	SETAR(1)log	AAR(3)log		LSTAR(2)log		AAR(8)log	SETAR(1)log	
VARPNOGO	LSTAR(6)log	LSTAR(1)log	SETAR(9)log		SETAR(2)log		RW4		
LSTAR(1)log	SETAR(7)log	AAR(5)log	SETAR(1)log		SETAR(6)log		SETAR(4)log	SETAR(1)dlog	
SETAR(8)log	LSTAR(8)log	SETAR(2)log	RW		SETAR(5)log		SETAR(2)log	SETAR(4)log	
SETAR(1)log	LSTAR(3)log	SETAR(3)log	AR12		AAR(6)log		VADongo1112	VADongo1112	
SETAR(4)log	LSTAR(7)log	LSTAR(1)log1	AAR(4)log		SETAR(2)log		LSTAR(1)log	LSTAR(4)log	
AAR(3)log	AAR(4)log	VEPongo1112	LSTAR(1)log		LSTAR(4)log		LSTAR(6)log	BVARP	
VECMNPGO	SETAR(2)log	VARXPDNOGO	AAR(1)log		LSTAR(5)log		RW	SETAR(2)dlog	
VECMNPGO4	SETAR(8)log	AR12	LSTAR(3)log		SETAR(3)log		SETAR(1)log		
LSTAR(4)log	SETAR(1)dlog	SETAR(1)log	SETAR(5)log		SETAR(7)log				
AAR(8)log	LSTAR(5)log	LSTAR(6)log	SETAR(3)log		AAR(6)log				
LSTAR(2)log	AAR(1)log		SETAR(4)log		AAR(5)log				
AAR(2)log	VEPongo1112		SETAR(6)log		AAR(7)log				
LSTAR(5)log	SETAR(5)log	LSTAR(5)log	LSTAR(4)log		LSTAR(6)log				
LSTAR(3)log	BVARPNOGO	LSTAR(5)log	LSTAR(5)log		LSTAR(5)log				
VADongo1112	AAR(6)dlog		BVARP		SETAR(6)log				
AAR(5)log	LSTAR(5)log	LSTAR(6)log	SETAR(6)log		LSTAR(5)log				
AAR(4)log	AAR(8)log	AAR(2)log	AAR(2)log		PECM				
LSTAR(9)log	LSTAR(2)log	SETAR(7)log	SETAR(7)log		SETAR(5)log				
SETAR(2)log	BVARPNOGO4	SETAR(2)log	SETAR(2)log		BVARP				
AAR(6)log	VARRongo12	SETAR(8)log	SETAR(8)log		AAR(3)log				
BVARPNOGO	AAR(2)dlog	LSTAR(7)log	LSTAR(7)log		AAR(4)log				
VARPNOGO4	AAR(9)log	AR124	SETAR(1)log		SETAR(1)log				
BVARPNOGO4	AAR(7)log	LSTAR(2)log	LSTAR(2)log		SETAR(1)log				
AAR(7)log	LSTAR(8)log	LSTAR(8)log	AAR(2)log		SETAR(1)log				
SETAR(9)log	LSTAR(11)log	AAR(2)log	SETAR(4)log		AAR(2)log				
SETAR(5)log	LSTAR(9)log	BVARPNOGO4	LSTAR(4)log		LSTAR(4)log				
VEPongo1112	VARXPDNOGO4	VECMNPGO4	LSTAR(3)log		LSTAR(3)log				
BVARP		VECMNPGO	LSTAR(2)log		LSTAR(2)log				
BVARPD		SETAR(8)log	SETAR(8)log		SETAR(8)log				
BVARPDNOGO		SETAR(5)log	SETAR(5)log		SETAR(5)log				
BVARPDNOGO4		VARP	VARP		SETAR(3)log				
AAR(9)log		VARongo12							
PECM		VECM							
VECM		VARPD							
VADongo12		AAR(8)log							
RW		VARPD							
RW4									
AAR(5)dlog									
AAR(7)dlog									
VARPD									
SETAR(4)dlog									
SETAR(6)dlog									
VECMXP									
VECMXPNOGO4									
VARXP									

Table 24: List of Models included in the Model Confidence Set for each car brand. Forecast horizon: 24 steps ahead.



## E Robustness Checks: Directional Accuracy

SEASONALLY ADJUSTED DATA						
	Steps 1-6	%	Steps 7-12	%	Steps 13-24	%
<b>BMW</b>	AR124	68.21%	AR124	66.18%	AAR(1)log	80.51%
	SETAR(3)log	66.70%	AAR(3)log	65.88%	AR124	80.28%
	SETAR(8)log	66.20%	LSTAR(1)log	65.38%	VARongo1112	77.58%
<b>OPEL</b>	VADongo12	68.95%	VADongo12	69.02%	SETAR(11)dlog	77.55%
	SETAR(11)dlog	66.51%	SETAR(11)dlog	68.13%	LSTAR(8)dlog	75.06%
	VECM	65.15%	LSTAR(8)dlog	67.88%	BVARNOGO	74.12%
<b>VOLKSWAGEN</b>	AR124	74.53%	AR124	82.88%	VARongo1112	84.59%
	VARongo1112	73.79%	VARongo1112	80.79%	AR124	84.25%
	AR12	72.28%	AR12	80.66%	AR12	82.25%
<b>CITROEN</b>	VECMXNOGO4	65.59%	VEC Congo1112	67.69%	VEC Congo1112	75.12%
	VECMNOGO4	65.59%	VAR	67.31%	AAR(4)dlog	66.61%
	VECM	65.15%	VARX	67.31%	AAR(5)dlog	61.16%
<b>FIAT</b>	VEC Congo1112	61.02%	VEC Congo1112	74.54%	AAR(1)dlog	85.16%
	VADongo12	60.41%	VEC Congo12	67.31%	AAR(2)dlog	83.39%
	VARongo1112	59.99%	LSTAR(12)dlog	64.59%	AAR(3)dlog	83.21%
<b>TOYOTA</b>	VEC Congo1112	66.56%	VEC Congo12	71.59%	BVAR	74.53%
	VARongo1112	65.27%	VADongo12	71.57%	BVARNOGO4	72.44%
	VECMXNOGO	64.97%	BVARNOGO4	71.06%	VADongo12	71.03%
<b>JAGUAR</b>	LSTAR(3)log	68.15%	SETAR(4)log	66.42%	VARongo1112	70.79%
	AAR(4)log	68.13%	LSTAR(3)log	66.17%	AR12	70.10%
	LSTAR(4)log	68.12%	LSTAR(4)log	65.63%	VARongo12	69.98%
<b>KIA</b>	AAR(7)log	67.52%	AAR(2)log	71.68%	SETAR(5)log	78.77%
	AAR(4)log	66.06%	AAR(3)log	71.40%	SETAR(6)log	78.77%
	VARXNOGO4	65.80%	AAR(6)log	71.11%	AAR(2)log	78.46%
<b>MITSUBISHI</b>	VEC Congo1112	69.07%	VEC Congo12	81.34%	VEC Congo1112	84.41%
	VEC Congo12	67.31%	VARongo12	77.16%	VEC Congo12	82.43%
	AAR(4)dlog	65.86%	VEC Congo1112	77.16%	VARongo12	79.91%
<b>SUZUKI</b>	AAR(1)log	71.10%	SETAR(3)log	75.20%	AAR(1)log	84.56%
	SETAR(3)log	68.56%	SETAR(2)log	74.67%	SETAR(2)log	79.60%
	SETAR(2)log	68.08%	AAR(1)log	73.82%	SETAR(3)log	77.50%
RAW DATA						
	Steps 1-6	%	Steps 7-12	%	Steps 13-24	%
<b>BMW</b>	VADongo1112	84.45%	VADongo1112	82.27%	LSTAR(10)log	81.52%
	VARongo12	84.38%	BVARPD	79.96%	LSTAR(11)log	80.57%
	BVARPD	83.89%	LSTAR(11)log	79.80%	LSTAR(9)log	80.44%
<b>OPEL</b>	VADongo1112	87.02%	VEPongo1112	81.69%	VARongo1112	81.99%
	VARongo1112	86.41%	VADongo1112	81.13%	VADongo1112	77.82%
	VADongo12	85.48%	VADongo12	79.77%	VEPongo1112	77.76%
<b>VOLKSWAGEN</b>	BVARPNOGO	82.85%	VEPongo1112	78.75%	VECPongo12	84.64%
	VECMP	82.61%	BVARPNOGO	78.42%	VEPongo1112	81.13%
	VECMXP	82.61%	VECMP	76.48%	VARongo1112	77.99%
<b>CITROEN</b>	VARPNOGO4	80.54%	VEPongo1112	76.72%	AAR(3)log	78.02%
	VARXPNOGO4	80.54%	BVARPNOGO4	76.52%	AAR(4)log	77.40%
	VARPNOGO	80.29%	BVARPNOGO	76.49%	LSTAR(3)dlog	76.97%
<b>FIAT</b>	VARongo1112	85.77%	PECM	84.58%	VARongo1112	83.02%
	VADongo1112	84.65%	VADongo12	79.70%	VEPongo1112	80.62%
	BVARP	84.37%	VADongo1112	78.25%	VADongo12	79.48%
<b>TOYOTA</b>	VARongo1112	81.73%	BVARP	79.23%	VEPongo1112	79.42%
	VEPongo1112	79.64%	VADongo12	78.56%	VARongo1112	79.03%
	BVARPNOGO4	77.02%	VECPongo12	78.52%	BVARP	75.00%
<b>JAGUAR</b>	VARXPNOGO4	80.60%	VARXPNOGO4	79.57%	VARPNOGO	80.45%
	VARXPNOGO	80.05%	VARPNOGO	79.12%	VARPNOGO4	77.87%
	AR12	80.04%	VARPNOGO4	77.58%	AR12	77.32%
<b>KIA</b>	BVARPDNOGO	72.52%	VARPNOGO4	67.93%	VARPNOGO4	70.62%
	VARongo1112	71.97%	VARXPNOGO4	67.93%	VARXPNOGO4	70.62%
	VADongo1112	71.96%	VEPongo1112	66.56%	RW	69.21%
<b>MITSUBISHI</b>	VEPongo1112	75.45%	VEPongo1112	77.87%	VEPongo1112	82.60%
	VARongo1112	74.56%	VECPongo12	76.31%	VECPongo12	82.07%
	VECPongo12	72.98%	LSTAR(2)log	73.21%	VARongo1112	80.77%
<b>SUZUKI</b>	VARongo1112	75.88%	SETAR(6)log	77.95%	AAR(2)log	78.95%
	BVARPNOGO	73.27%	SETAR(5)log	77.55%	SETAR(8)log	78.90%
	AR12	72.68%	AAR(4)log	76.77%	VARongo1112	78.75%

Table 27: Top three models in terms of average directional accuracy (in %) for each car brand, for short-term forecasts (1-6 steps ahead), medium-term forecasts (7-12 steps ahead), and long-term forecasts (13-24 steps ahead).

## F Robustness Checks: Variability of Google Data

	SEASONALLY ADJUSTED DATA									
	BMW	Opel	Volkswagen	Citroen	Fiat	Toyota	Jaguar	Kia	Mitsubishi	Suzuki
VECM	1.08	3.65	0.94	1.00	0.65	NC	1.03	0.95	1.06	1.95
VECMX	1.07	3.65	0.94	1.00	0.65	NC	0.85	NC	0.77	1.95
VAR	0.92	0.66	1.00	1.00	1.15	0.96	1.00	0.98	0.95	1.18
VARD	1.14	1.61	0.79	0.22	1.49	1.16	0.94	0.92	0.99	1.26
VARX	0.97	0.66	1.00	1.00	1.07	0.86	1.42	0.97	0.95	1.18
VARXD	1.07	1.61	0.79	1.00	1.41	1.05	1.00	1.28	1.00	1.26
VECongo1112	1.02	0.94	1.03	0.99	0.99	1.13	1.02	1.05	0.95	1.05
VECongo12	1.00	0.99	1.04	1.05	1.48	0.41	0.87	1.14	1.01	1.10
VARongo1112	1.00	1.02	0.98	0.99	1.03	1.04	0.98	1.04	1.03	0.89
VARongo12	1.01	0.97	1.04	1.09	1.01	1.19	0.83	1.05	1.03	0.91
VADongo1112	1.02	0.97	1.00	1.00	0.99	1.03	0.96	2.15	1.00	0.98
VADongo12	1.08	0.95	1.04	0.95	0.97	1.02	1.13	3.58	0.93	1.19
BVAR	1.00	1.00	1.00	1.00	0.99	1.00	1.00	0.97	1.00	1.00
BVARD	1.00	0.99	1.00	1.00	1.00	1.00	1.00	2.06	0.99	0.99
	RAW DATA									
	BMW	Opel	Volkswagen	Citroen	Fiat	Toyota	Jaguar	Kia	Mitsubishi	Suzuki
VECMP	0.61	0.99	1.54	1.20	0.25	NC	1.03	NC	1.57	0.58
VECMXP	1.16	0.99	1.54	1.20	0.25	NC	0.81	NC	1.10	0.58
VEPongo1112	1.00	0.99	0.97	0.98	0.97	0.83	1.04	1.00	0.96	1.04
VECPongo12	0.96	0.93	0.92	1.59	1.27	0.90	1.04	1.07	1.06	2.73
VARP	0.96	0.97	0.98	1.07	0.96	0.99	0.97	1.00	1.08	0.95
VARPD	1.06	1.00	0.93	1.01	1.17	0.57	1.00	0.80	1.12	1.22
VARXP	1.12	0.97	0.98	1.07	0.96	0.88	0.98	1.00	0.97	0.95
VARXPD	1.09	1.00	0.93	1.01	1.17	1.26	1.00	0.80	1.25	1.22
VADongo1112	1.02	1.00	0.96	1.00	0.96	1.49	1.00	1.00	1.00	1.05
VADongo12	1.04	0.91	1.04	1.21	0.98	1.10	0.98	1.03	1.00	3.69
VARongo1112	0.94	0.96	0.96	0.99	0.96	0.88	1.00	0.98	0.99	1.04
VARongo12	1.02	0.93	1.08	1.47	1.00	1.25	0.97	1.02	0.97	1.20
BVARP	1.00	1.00	1.00	1.00	0.99	0.95	1.00	1.00	1.00	1.00
BVARPD	1.00	1.00	1.00	1.00	0.98	0.99	1.00	1.00	1.00	1.00
PECM	0.97	1.46	1.70	1.13	1.00	3.10	1.04	0.63	1.10	1.25

Table 28: Average ratio – averaged across all forecasting horizons – of the MSE for the forecasts computed with GIs downloaded on the 15/08/2014, with respect to MSE for the forecasts computed with the average GIs downloaded between 15/08/2014 and 02/09/2014. NC=not converged.





# **Forecasting German Car Sales Using Google Data and Multivariate Models**

*Technical appendix - not for publication. To be posted on the authors'  
web sites.*

Dean Fantazzini      Zhamal Toktamyssova

# A Unit Root Tests allowing for Structural Breaks

## A.1 Minimum LM Unit Root test by Lee and Strazicich (2003)

Unit root tests where the break dates are data-driven are called unit root tests with endogenous structural breaks. Zivot and Andrews (1992) consider the case of one break, while Lumsdaine and Papell (1997) of two breaks. Both tests assume no break under the null of a unit root. However, if the data generating process has a unit root with break(s), then these tests exhibit size distortions. In this regard, Nunes, Newbold, and Kuan (1997) and Lee and Strazicich (2001) point out that this frequently leads to a spurious rejection of the null hypothesis, so that the case of stationarity with break(s) is accepted too often. Moreover, the break point is often incorrectly determined one period prior to the true break date (Lee and Strazicich (2001)). The one-break and two-break Langrange Multiplier (LM) unit root tests by Lee and Strazicich (2003) allow for structural breaks under the null and the alternative, and the rejection of the null unambiguously implies stationarity.

Lee and Strazicich (2003) consider the model  $y_t = \delta' Z_t + X_t$ , where  $X_t = \beta X_{t-1} + \varepsilon_t$  and  $Z_t$  is a vector of exogenous variables. Their LM test has variations for different types of breaks (change in intercept, change in trend slope, and both). For example their model C (change in both intercept and trend) has the following components:

$$Z_t = [1, t, D_{1t}, D_{2t}, DT_{1t}, DT_{2t}]'$$

where  $D_{jt} = 1$  for  $t \geq T_{B_j+1}$  and 0 otherwise,  $DT_{jt} = t - T_{B_j}$  for  $t \geq T_{B_j+1}$ , and 0 otherwise,  $j = 1, 2$ . The two break points are denoted as  $T_{B_1}$  and  $T_{B_2}$ . When the differencing operator is utilized, then  $\Delta Z_t = [1, B_{1t}, B_{2t}, D_{1t}, D_{2t}]'$  with  $B_{jt} = \Delta D_{jt}$ ,  $D_{jt} = \Delta DT_{jt}$  for  $j = 1, 2$ .

The test statistic of the two-break LM unit root test is obtained by using the LM (score) principle from the regression:

$$\Delta y_t = \delta' \Delta Z_t + \phi \tilde{S}_{t-1} + \sum_{i=1}^k \gamma_i \Delta \tilde{S}_{t-i} + u_t,$$

where  $\tilde{S}_{t-1} = y_t - \tilde{\psi}_x - Z_t \tilde{\delta}$ ,  $t = 2, \dots, T$  with  $\psi_x = y_1 - Z_1 \tilde{\delta}$ , where  $\tilde{\delta}$  are coefficients in the regression of  $\Delta y_t$  on  $\Delta Z_t$ , and  $y_1$  and  $Z_1$  are the initial observations of  $y_t$  and  $Z_t$ , respectively.  $\Delta \tilde{S}_{t-i}$ ,  $i = 1, \dots, k$  are augmented terms to correct for autocorrelated errors. The LM-test statistic  $\tilde{\tau}$  is the  $t$ -statistic testing for  $\phi = 0$  (which corresponds to the null hypothesis that a unit root exists).

The optimal number of lags  $k$  is determined from Ng and Perron (1995)'s "general to specific" procedure. It starts with  $k = 8$ . If the last term is not significantly different from zero at the 10% significance level, then the number of lags  $k = 7$  is considered and the procedure is repeated again. If the last term significantly differs from zero or  $k = 0$ , the procedure stops. Practically, at first, the optimal number of lags  $k$  is determined for each possible combination of break points. Then, these combinations are examined on the time interval  $[0.1T; 0.9T]$ . The break dates are determined by the points where the LM  $t$ -test statistic is minimized. Critical values are obtained from 20,000 replications of the model with a  $T = 100$  sample size.

## A.2 Range Unit Root Test and Forward Backward Range Unit Root Test by Aparicio et al. (2006)

Standard unit root tests do not take into account the fact that real macroeconomic data are exposed to structural breaks, outliers and nonlinearity. In such a situation, the power and size of unit root tests can be strongly affected, see e.g. Perron (1990), Perron and Volgelsang (1992) and Perron (2006). The presence of additive outliers affects the size of the test and the null of a unit root can be mistakenly rejected, see Franses and Haldrup (1994). To deal with these problems, Aparicio et al. (2006) suggested the Range unit root (RUR) test, which has a number of advantages: it is invariant to monotonic transformations and model errors, is robust to parameter shifts and structural breaks, and it outperforms standard unit root tests in terms of power when the process is stationary with a near unit root.

For a given time series  $x_t$ , Aparicio et al. (2006) consider  $i$ -th extremes defined as  $x_{1i} = \min(x_1, \dots, x_i)$  and  $x_{ii} = \max(x_1, \dots, x_i)$  and constructed a sequence of running ranges  $R_i^{(x)} = x_{ii} - x_{1i}$ , for  $i = 1, \dots, n$ , where  $n$  is the sample size. To test for the null hypothesis of unit root the following RUR statistic is suggested:

$$RUR \equiv J_0^{(n)} = \frac{1}{\sqrt{n}} \sum_{i=2}^n \mathbf{1}(\Delta R_i^{(x)} > 0),$$

where  $\mathbf{1}()$  is the indicator function and  $\Delta$  is the differencing operator.

Under  $H_0$ , the test-statistic  $J_0^{(n)}$  converges weakly to a non-degenerate unimodal random variable. Under the alternative of stationarity,  $J_0^{(n)}$  converges to 0 in probability. Consequently, the left tail of  $J_0^{(n)}$  distribution can be used to distinguish between  $I(0)$  and  $I(1)$  time series without trend, and the right tail for a case of a linear trend.

When additive outliers are considered, Aparicio et al. (2006) proposed an extension of the RUR test known as the Forward-Backward Range unit root (FB-RUR) test. It reduces the size distortion of the test when additive outliers are situated in the beginning of the sample and improves the power compared to the RUR test. For this, the reversed time series  $x'_t = x_{n-t+1}$ , for  $t = 1, \dots, n$ , are considered, and the analogous sequence of running ranges  $R_t^{(x')}$  is constructed as before. The FB RUR test statistic is then as follows:

$$FBRUR \equiv J_*^{(n)} = \frac{1}{\sqrt{2n}} \sum_{i=1}^n (\mathbf{1}(\Delta R_i^{(x)} > 0) + \mathbf{1}(\Delta R_i^{(x')} > 0)).$$

Critical values of  $J_0^{(n)}$  are obtained from 10,000 replications for different sample sizes and critical levels of the null model with normally distributed errors.

## B Periodic Unit Root Tests by Boswijk and Franses (1996) and Franses and Paap (2004)

We report below the case of quarterly data for simplicity, but the results can be extended to monthly data. Consider a Periodic Auto-Regressive (PAR) model of order 1 for a quarterly time series:

$$y_t = \alpha_s y_{t-1} + \varepsilon_t,$$

where  $\varepsilon_t$  are normally distributed error terms and  $s$  corresponds to four seasons,  $s = 1, 2, 3, 4$ . Denote  $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$  and  $g(\boldsymbol{\alpha}) = \alpha_1 \alpha_2 \alpha_3 \alpha_4$ . The process  $y_t$  is called stationary when  $|g(\boldsymbol{\alpha})| < 1$ . If  $g(\boldsymbol{\alpha}) = 1$ , then a unit root exists. We need to test the hypothesis

$$H_0 : g(\boldsymbol{\alpha}) = 1$$

against the alternative  $H_1 : |g(\boldsymbol{\alpha})| < 1$ . The unrestricted model is given by

$$y_t = \sum_{s=1}^4 \alpha_s D_{s,t} y_{t-1} + \varepsilon_t,$$

where  $D_{s,t}$  are seasonal dummy variables. When the null condition is imposed, i.e.  $\alpha_1 \alpha_2 \alpha_3 \alpha_4 = 1$ , the restricted regression is given by

$$y_t = \alpha_1 D_{1,t} y_{t-1} + \alpha_2 D_{2,t} y_{t-1} + \alpha_3 D_{3,t} y_{t-1} + (\alpha_1 \alpha_2 \alpha_3)^{-1} D_{4,t} y_{t-1} + \varepsilon_t.$$

The parameters  $\alpha_s$  can be estimated by ordinary least squares from the unrestricted model and with nonlinear least squares from the restricted model. If we define  $RSS_0$  and  $RSS_1$  as the residual sum of squares for the unrestricted and restricted models, respectively, then the test-statistic of the likelihood ratio test can be computed as

$$LR = n \log \frac{RSS_0}{RSS_1}.$$

Asymptotically, this  $LR$  test statistic is distributed as Johansen's trace statistic, and the critical values are tabulated in Osterwald-Lenum (1992), Table 1.1. This test can be generalized to periodic auto-regressions of higher order  $p$ , where  $p$  is usually determined by using information criteria and checking that the residuals behave approximately as a white noise. For details, see Boswijk and Franses (1996) and Franses and Paap (2004).

If the null hypothesis  $\alpha_1 \alpha_2 \alpha_3 \alpha_4 = 1$  of a periodic unit root is not rejected, then we can test two types of parameter restrictions in a second step:

$$H_0 : \alpha_s = 1, s = 1, 2, 3;$$

$$H_0 : \alpha_s = -1, s = 1, 2, 3.$$

If the first  $H_0$  is not rejected, then  $\alpha_4 = 1$  and we have a non-periodic unit root, so that the periodic differencing filter can be simplified to  $1 - L$ , where  $L$  is lag operator. If the second  $H_0$  cannot be rejected, then the differencing filter equals  $1 + L$  and we have a seasonal unit root. In both cases, the resulting process is a PARI(1), periodically integrated autoregressive model of order 1. Monte-Carlo simulations by Franses and Paap (1994) showed that the maximum-likelihood statistics for testing these  $H_0$  hypotheses follow a standard F-distribution under the null.

## C Cointegration Tests allowing for Structural Breaks

### C.1 Cointegration test allowing for one break by Gregory and Hansen (1996)

The residual-based cointegration tests by Engle and Granger (1987) and Phillips and Ouliaris (1990) are built without considering any structural break(s). When this is the case, these tests have low power. Gregory and Hansen (1996) proposed a cointegration test which allows for a single endogenous regime shift. The starting point is a model with a standard cointegrating equation (model 1):

$$y_{1t} = \mu + \alpha^T y_{2t} + e_t,$$

where  $y_{1t}$  is real-valued,  $y_{2t}$  is a  $I(1)$   $m$ -dimensional vector and  $e_t$  is  $I(0)$ ,  $t = 1, \dots, n$ . To develop a model which allows for structural change, a dummy variable is introduced :

$$\phi_{t\tau} = \begin{cases} 0, & \text{if } t \leq [n\tau], \\ 1, & \text{if } t > [n\tau], \end{cases},$$

where  $\tau \in (0, 1)$  is a parameter for timing the change point, while the brackets denote the integer part. Gregory and Hansen (1996) considered several specifications, allowing for a change in the intercept  $\mu$ , in the slope  $\alpha$  and with a time trend:

$$y_{1t} = \mu_1 + \mu_2 \phi_{t\tau} + \alpha^T y_{2t} + e_t \quad \text{Level shift (model 2)} \quad (1)$$

$$y_{1t} = \mu_1 + \mu_2 \phi_{t\tau} + \beta t + \alpha^T y_{2t} + e_t \quad \text{Level shift with trend (model 3)} \quad (2)$$

$$y_{1t} = \mu_1 + \mu_2 \phi_{t\tau} + \alpha_1^T y_{2t} + \alpha_2^T \phi_{t\tau} y_{2t} + e_t \quad \text{Regime shift (model 4).} \quad (3)$$

(4)

The null hypothesis is no cointegration, while the alternative is cointegration with possible regime shifts (one of the models 2-4). A model is estimated with OLS for each possible  $\tau$ , and the resulting residuals  $\hat{e}_{t\tau}$  are used to compute the first

order serial correlation coefficient  $\hat{\rho}_\tau$ . The second-stage residuals  $\hat{v}_{t\tau} = \hat{e}_{t\tau} - \hat{\rho}_\tau \hat{e}_{t-1\tau}$  are then obtained and their long-run variance  $\hat{s}_\tau^2$  is estimated (see Gregory and Hansen(1996) for details). The bias-corrected first-order serial correlation coefficient  $\hat{\rho}_\tau^*$  is calculated as follows:

$$\hat{\rho}_\tau^* = \left( \sum_{t=1}^{n-1} \hat{e}_{t\tau} \hat{e}_{t+1\tau} - \hat{\lambda}_\tau \right) / \sum_{t=1}^{n-1} \hat{e}_{t\tau}^2,$$

where  $\hat{\lambda}_\tau$  is the estimation of the weighted sum of autocovariances of  $\hat{v}_{t\tau}$ . The following test statistics are obtained:

$$\begin{aligned} Z_\alpha(\tau) &= n(\hat{\rho}_\tau^* - 1), \\ Z_t(\tau) &= (\hat{\rho}_\tau^* - 1)/\hat{s}_\tau^2, \end{aligned}$$

where  $\hat{s}_\tau^2 = \hat{\sigma}_\tau^2 / \sum_{t=1}^{n-1} \hat{e}_{t\tau}^2$ . The third statistic considered by Gregory and Hansen (1996) is the ADF test statistic from an augmented regression of the residuals  $\Delta \hat{e}_{t\tau}$  on  $\hat{e}_{t-1\tau}$  and lagged first-differences residuals. It is the  $t$ -statistic of the regressor  $\hat{e}_{t-1\tau}$ :

$$ADF(\tau) = tstat(\hat{e}_{t-1\tau}).$$

Since small values of these test statistics are evidence against the null of no cointegration, the smallest values across all possible break points  $\tau \in T$  are taken :

$$\begin{aligned} Z_\alpha^* &= \inf_{\tau \in T} Z_\alpha(\tau), \\ Z_t^* &= \inf_{\tau \in T} Z_t(\tau), \\ ADF^* &= \inf_{\tau \in T} ADF(\tau) \end{aligned}$$

Critical values for models 2-4 and different significance levels are provided via simulation methods in Gregory and Hansen (1996).

## C.2 Cointegration test allowing for two breaks by Hatemi (2008)

The Hatemi (2008) cointegration test is an extension of Gregory and Hansen's (1996) cointegration test, and it allows for two endogenous breaks. To take two possible changes into account, two dummy variables are introduced:

$$D_{1t} = \begin{cases} 0, & \text{if } t \leq [n\tau_1], \\ 1, & \text{if } t > [n\tau_1], \end{cases},$$

and

$$D_{2t} = \begin{cases} 0, & \text{if } t \leq [n\tau_2], \\ 1, & \text{if } t > [n\tau_2], \end{cases},$$

where  $\tau_1 \in (0, 1)$  and  $\tau_2 \in (0, 1)$  are unknown parameters for determining the timing of the change points, while the brackets denote the integer part. Considering the case of a level shift, we have the following model:

$$y_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \beta'_0 x_t + u_t.$$

The three residual-based test statistics  $Z_\alpha$ ,  $Z_t$  and  $ADF$  are obtained in a similar way to Gregory and Hansen (1996), and their smallest values provide evidence against the null:

$$\begin{aligned} Z_\alpha^* &= \inf_{(\tau_1, \tau_2) \in T} Z_\alpha(\tau_1, \tau_2), \\ Z_t^* &= \inf_{(\tau_1, \tau_2) \in T} Z_t(\tau_1, \tau_2), \\ ADF^* &= \inf_{(\tau_1, \tau_2) \in T} ADF(\tau_1, \tau_2). \end{aligned}$$

Asymptotic critical values for these tests are obtained by simulation methods and are tabulated in Hatemi (2008).

## C.3 Cointegration test by Johansen, Mosconi, and Nielsen (2000) with exogenous structural breaks

The distribution of the standard cointegration test by Johansen (1995) changes if interventional dummies are considered. Johansen et al. (2000) therefore modify this approach, allowing for trend and level breaks at known dates.

Suppose that  $Y_t$  is a  $p$ -vector process, and that without structural breaks the VECM can be formulated as follows:

$$\Delta Y_t = \Pi Y_{t-1} + \Pi_1 t + \mu + \sum_{i=1}^{k-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t,$$

where  $\varepsilon_t \sim MN(\mathbf{0}, \Sigma)$ . The hypothesis of cointegration can be reformulated as a reduced rank problem of the  $\Pi$  matrix, with  $\Pi = \alpha\beta$ , where  $\alpha$  and  $\beta$  are  $(p \times r)$  full rank matrices. In the case that none of the  $p$  time series has a quadratic trend, we have that  $\Pi_1 = \alpha\gamma'$ , where  $\gamma$  is a  $(1 \times r)$  full rank matrix.

If there are  $q - 1$  breaks in the VECM which occur at the dates  $T_1, T_2, \dots, T_{q-1}$ , then the initial sample can be divided into  $q$  sample periods  $1 = T_0 < T_1 < T_2 < \dots < T_q = T$ . Here  $T_j$  is the last observation of the  $j$ -th sub-sample,  $j = 1, \dots, q$ . To account for the break dates,  $q - 1$  intervention dummy variables are introduced,

$$D_{jt} = \begin{cases} 0, & \text{if } T_{j-1} + 1 \leq t \leq T_j, \\ 1, & \text{otherwise} \end{cases},$$

as well as  $q - 1$  indicator dummy variables

$$I_{jt} = \begin{cases} 0, & \text{if } t = T_{j-1} + 1 \\ 1, & \text{otherwise} \end{cases}$$

Furthermore, we can define the matrices  $D_t = (1, \dots, D_{q,t})'$ ,  $\mu = (\mu_1, \dots, \mu_q)'$  and  $\gamma = (\gamma'_1, \dots, \gamma'_q)'$ , of dimensions  $(q \times 1)$ ,  $(p \times q)$ ,  $(q \times r)$ , respectively. The cointegrated VECM model with  $q - 1$  exogenous breaks can then be written as

$$\Delta Y_t = \alpha \begin{pmatrix} \beta \\ \gamma \end{pmatrix}' \begin{pmatrix} Y_{t-1} \\ tD_{t-k} \end{pmatrix} + \mu D_{t-k} + \sum_{i=1}^{k-1} \Gamma_i \Delta Y_{t-i} + \sum_{i=0}^{k-1} \sum_{j=2}^q \kappa_{j,i} I_{j,t-i} + \varepsilon_t,$$

where  $\kappa_{j,i}$  are  $(p \times 1)$  vectors and  $\varepsilon_t$  is a Gaussian white noise vector. For testing the hypothesis of at most  $r$  cointegrating relations, the likelihood ratio test statistic is used:

$$LR = -T \sum_{i=r+1}^p \ln(1 - \hat{\lambda}_i),$$

where  $\hat{\lambda}_i$  are the squared sample canonical correlations of  $\Delta Y_t$  and  $(Y'_{t-1}, tD'_{t-k})$  corrected for intervention dummies, indicator dummies and augmented terms. Johansen et al. (2000) consider three cases:

1. None of the  $p$  time series have a deterministic trend, but the intercepts in the cointegrating relations can vary between sub-samples. The model is denoted  $H_c(r)$ . The asymptotic distribution of the test-statistic is approximated by a Gamma-distribution.
2. Some or all of the time series follow a trending pattern, the cointegrating relations are stationary in the sub-samples, non-stationary time series and cointegrating relations may have trend breaks. This model is denoted  $H_l(r)$ .
3. Some or all of the time series follow a trending pattern, the cointegrating relations are stationary in the sub-samples and may have an intercept break, but only non-stationary series may have trend breaks. The model is denoted  $H_{lc}(r)$ .

The critical values for the models  $H_c(r)$ ,  $H_l(r)$  and  $H_{lc}(r)$  depend on the number of non-cointegrating relations  $p - r$  and the relative location of the breaks. See section 3.4 of Johansen et al. (2000) for more details.

## D Periodic Cointegration Tests

Franses and Paap (2004) and Boswijk (1994) suggested a single-equation approach for testing periodic cointegration. For the sake of simplicity, we consider here the bivariate case with two monthly time series  $y_{1,t}$  and  $y_{2,t}$ , where  $y_{2,t}$  is weakly exogenous (however,  $y_{2,t}$  can easily be extended to a vector of regressors). The conditional periodic error correction model for  $y_{1,t}$  is then given by

$$\Delta_{12}y_{1,t} = \gamma_{1s}(y_{1,t-12} - \kappa_s y_{2,t-12}) + \sum_{j=1}^{p-12} \phi_{1s} \Delta_{12}y_{1,t-j} + \sum_{j=0}^{p-12} \phi_{2s} \Delta_{12}y_{2,t-j} + \varepsilon_{1,t},$$

where  $\Delta_{12}$  is the seasonal difference operator. If needed, the  $\Delta_{12}y_{1,t-j}$  and  $\Delta_{12}y_{2,t-j}$  variables on the right-hand side can be replaced by  $\Delta_1 y_{1,t-j}$  and  $\Delta_1 y_{2,t-j}$  variables, see Franses and Paap (2004, p. 111-117) for a discussion. If we define a vector variable  $w_t$  which contains the various differenced variables, denote  $\delta_{1s} = \gamma_{1s}$ ,  $\delta_{2s} = -\gamma_{1s}\kappa_s$ , and  $\delta_s = (\delta_{1s}, \delta_{2s})$ , then the model can be rewritten as

$$\Delta_{12}y_{1,t} = \sum_{s=1}^{12} (\delta_{1s} D_{s,t} y_{1,t-12} - \delta_{2s} D_{s,t} y_{2,t-12}) + \phi' w_t + \varepsilon_{1,t}.$$

Franses and Paap (2004) extended the Boswijk (1994) cointegration test to test the null hypothesis of no cointegration against the alternative of periodic cointegration and considered two variants: the first one is a Wald test for periodic cointegration in season  $s$ , where the null is  $\delta_s = 0$  against the alternative of  $\delta_s \neq 0$ ; the second test is a joint Wald test testing the null  $\delta_s = 0$  for all seasons  $s$  against the alternative that at least one  $\delta_s \neq 0$ .

Denote  $\hat{\delta}_s$  the OLS estimator of  $\delta_s$  and  $\hat{V}(\hat{\delta}'_s)$  its estimated covariance matrix. Also, define  $\hat{\delta} = (\hat{\delta}_1, \hat{\delta}_2, \dots, \hat{\delta}_{12})$  and  $\hat{V}(\hat{\delta}')$  its covariance matrix estimator. Then, the two Wald test statistics are given by

$$Wald_s = \hat{\delta}'_s (\hat{V}(\hat{\delta}'_s))^{-1} \hat{\delta}_s,$$

$$Wald = \hat{\delta}' (\hat{V}(\hat{\delta}'))^{-1} \hat{\delta}.$$

Moreover, define  $RSS_{0s}$  and  $RSS_0$  as the residual sum of squares under the null for the season-specific and the joint test, respectively, while  $RSS_1$  the residual sum of squares under the alternative for both tests. Let  $l$  be the number of estimated parameters. Then, the two test statistics can be re-written as follows:

$$Wald_s = (n - l) \frac{RSS_{0s} - RSS_1}{RSS_1},$$

$$Wald = (n - l) \frac{RSS_0 - RSS_1}{RSS_1}.$$

The previous tests can be easily extended to the case involving seasonal intercepts and trends, see Franses and Paap (2004) for details. The asymptotic distribution of the test statistics is non-standard and critical values are reported in Table C.1 in Franses and Paap (2004).

The previous test for periodic cointegration assumes that  $y_{2,t}$  is weakly exogenous. To test this hypothesis, Boswijk (1994) suggests adding the periodic error terms  $D_{s,t}(y_{1,t-12} - \hat{k}_s y_{2,t-12})$  to an autoregressive model for  $\Delta_{12}y_{2,t}$  which also includes lags of  $\Delta_{12}y_{1,t}$ . Under the null of weak exogeneity, the Likelihood Ratio test that the parameters of the periodic error terms are zero for all  $s$  is asymptotically  $\chi^2(12)$  distributed. When the null hypothesis of weak exogeneity is rejected, alternative methods have to be used (for example, dynamic-OLS; see Boswijk and Franses (1995) for details).

## E Bayesian VARs

Bayesian methods treat the true value of the unknown parameter vector  $\theta$  as a probability distribution  $\pi(\theta|y)$ , which is the called posterior distribution of  $\theta$  given data  $y$ . The prior distribution,  $\pi(\theta)$ , is set externally and reflects the researcher's beliefs on the unknown parameter of interest, while  $l(y|\theta)$  is the likelihood distribution, which depends on the information from the given data  $y$ . Bayes' theorem links all these distributions through this formula:

$$\pi(\theta|y) = \frac{\pi(\theta)l(y|\theta)}{\int \pi(\theta)l(y|\theta)d\theta}.$$

Given that the denominator is a normalizing constant, the posterior is proportional to the product of the likelihood and the prior, i.e.  $\pi(\theta|y) \propto \pi(\theta)l(y|\theta)$ .

Consider the following VAR model of order  $p$  for the  $m$ -dimensional vector  $y_t$ :

$$y_t = a_0 + \sum_{j=1}^p A_j y_{t-j} + \varepsilon_t, \quad \varepsilon_t \sim i.i.d.N(0, \Sigma_\varepsilon)$$

for  $t = 1, \dots, T$ , where  $\varepsilon_t$  is an error vector term. In matrix-vector notation, it takes the form

$$y = (I_m \otimes X)\theta + e,$$

where  $I_m$  is an  $m \times m$  identity matrix,  $X = (x_1, \dots, x_T)'$  is a  $T \times (mp+1)$  matrix for  $x_t = (1, y'_{t-1}, \dots, y'_{t-q})$ ,  $\theta = \text{vec}(A)$ , and  $e \sim N(0, \Sigma_\varepsilon \otimes I_T)$ .

In this work, we used the Litterman/Minnesota prior, which is a family of priors where  $\Sigma_\varepsilon$  is known and replaced with an estimated  $\hat{\Sigma}_\varepsilon$ . The Minnesota prior assumes that  $\theta \sim N(\theta_0, V_0)$ , where  $\theta_0 = \mu_1 \cdot j_{mp}$ ,  $j_{mp}$  is an  $mp$ -element unit vector, and  $\mu_1 = 0$  is a hyper-parameter.  $V_0$  is a non-zero covariance matrix constructed as follows: the elements of  $V_0$  which correspond to exogenous variables are set to infinity, and the remaining part is a diagonal matrix with the following diagonal elements:

$$v_{ij}^l = \begin{cases} \left(\frac{\lambda_1}{l^{\lambda_3}}\right)^2 & \text{for } i = j \\ \left(\frac{\lambda_1 \lambda_2 \sigma_i}{l^{\lambda_3} \sigma_j}\right)^2 & \text{for } i \neq j \end{cases},$$

where  $l = 1, \dots, p$  and  $\sigma_i$  is the  $i$ -th diagonal element of  $\hat{\Sigma}_\varepsilon$ . The scalars  $\lambda_1, \lambda_2, \lambda_3$  control the overall tightness, relative cross-variable weight and the decay of lag coefficients, respectively. We chose  $\lambda_1 = 0.1, \lambda_2 = 1, \lambda_3 = 1$ . Given the Minnesota prior, the posterior distribution of the parameter  $\theta$  is given by

$$\theta \sim N(\bar{\theta}, \bar{V}),$$

where

$$\bar{V} = \left(V_0^{-1} + (\hat{\Sigma}_\varepsilon^{-1} \otimes X'X)\right)^{-1},$$

$$\bar{\theta} = \bar{V} \left(V_0^{-1} \theta_0 + (\hat{\Sigma}_\varepsilon^{-1} \otimes X)' y\right).$$

## F The Model Confidence Set

The Model Confidence Set (MCS) approach by Hansen et al. (2011) can be used to select the best forecasting models among a set of models, given a confidence level  $\alpha$ .

First, the MCS procedure applies an equivalence test  $\delta_M$  to the set of forecasting models  $M = M_0$ , to test the null hypothesis of equal forecasting accuracy,

$$H_{0,M} = E(d_{ij,t}) = 0, \quad \forall i, j \in M,$$

where  $d_{ij,t} = L_{i,t} - L_{j,t}$  is the sample loss differential between forecasting models  $i$  and  $j$  and  $L_{i,t}$  stands for the loss function of model  $i$  at time  $t$ . The alternative hypothesis  $H_{A,M}$  is that  $E(d_{ij,t}) \neq 0$  for some  $i, j \in M$ . If the null cannot

be rejected, then  $\widehat{M}_{1-\alpha}^* = M$ . When the null is rejected, it indicates that some of the models of the set  $M$  have worse sample performance than others. Therefore, the elimination rule  $e_M$  is used to remove these models from the set  $M$ . The procedure is repeated until the null cannot be rejected, and the resulting models define the model confidence set  $\widehat{M}_{1-\alpha}^*$ .

Hansen et al. (2011) proposes different equivalence tests and we discuss here the Semi-Quadratic statistic which we used in the paper. First, the following  $t$ -statistics are computed:

$$t_{ij} = \frac{\bar{d}_{ij}}{\widehat{var}(\bar{d}_{ij})}, \quad \text{for } i, j \in M,$$

with  $\bar{d}_{ij} = T^{-1} \sum_{t=1}^T d_{ij,t}$ . Then, the semi-quadratic statistic,  $T_{S,Q}$ , is computed as follows:

$$T_{S,Q} = \sum_{i,j \in M} t_{ij}^2.$$

The distribution of this test statistic is non-standard and is estimated using bootstrapping methods, see Hansen et al. (2011) for details. For all tests, the same significance level  $\alpha$  is used, which asymptotically guarantees that  $\Pr(M^* \subset \widehat{M}_{1-\alpha}^*) \geq 1 - \alpha$ , where  $\widehat{M}^*$  is the set of models with a given confidence level. If only one model is included in  $M^*$ , we have  $\lim_{n \rightarrow \infty} \Pr(M^* = \widehat{M}_{1-\alpha}^*) = 1$ .

## G Nonlinear Models

We considered three nonlinear models: the first one was the SETAR model with 2 regimes:

$$Y_t = \begin{cases} \phi_{0,1} + \phi_{1,1} Y_{t-1} + \dots + \phi_{1,p} Y_{t-p} + \varepsilon_t, & \text{if } Y_{t-1} \leq c \\ \phi_{0,2} + \phi_{1,2} Y_{t-1} + \dots + \phi_{1,p} Y_{t-p} + \varepsilon_t, & \text{if } Y_{t-1} > c \end{cases}$$

where  $c$  is a threshold to be estimated and which identifies the two regimes. We allowed the number of lags  $p$  to vary between 1 and 12, while  $Y_t$  was either the log-prices or the log-returns, for a total of 24 models.

The second nonlinear model was the logistic smooth transition autoregressive (LSTAR) model, which is a generalization of the SETAR model:

$$\begin{aligned} Y_t = & (\phi_{0,1} + \phi_{1,1} Y_{t-1} + \dots + \phi_{1,p} Y_{t-p})[1 - G(Y_{t-1}, \gamma, c)] + \\ & + (\phi_{0,2} + \phi_{1,2} Y_{t-1} + \dots + \phi_{1,p} Y_{t-p})[G(Y_{t-1}, \gamma, c)] + \varepsilon_t \end{aligned}$$

where  $G(Y_{t-1}, \gamma, c) = [1 + \exp(-\gamma(Y_{t-1} - c))]^{-1}$  is the first order logistic transition function, bounded between 0 and 1,  $\gamma$  is the slope parameter and  $c$  is the location parameter. Differently from SETAR models, the LSTAR model assume that the change between the two regimes is gradual and smooth, see Tong (1990) for a discussion at the textbook level. We allowed again the number of lags  $p$  to vary between 1 and 12, while  $Y_t$  was either the log-prices or the log-returns, for a total of additional 24 nonlinear models.

Finally, we considered the additive autoregressive model (AAR), also known as generalized additive model (GAM), since it combines generalized linear models and additive models:

$$Y_t = \phi_0 + s_1(Y_{t-1}) + \dots + s_p(Y_{t-p}) + \varepsilon_t$$

where  $s_i$  are smooth functions represented by penalized cubic regression splines, see Wood (2006) for a discussion at the textbook level. The number of lags  $p$  varied between 1 and 12, while  $Y_t$  was either the log-prices or the log-returns, for a total of additional 24 models.