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Abstract: The present paper discusses the transition from linear modelling to the first nonlinear models in economic analysis. In this vein, an important contribution was J. Hicks's *A Contribution to the Theory of the Trade Cycle* where he developed his own endogenous model of the cycle. Hicks thought that fluctuations in investment, – caused by nonlinear changes in autonomous investment and the acceleration principle governing induced investment – led to an adjustment process taking place throughout many periods. In this paper we introduce some modifications regarding the econometric estimation of Hicks's nonlinear model and an empirical application for Greece (1960-2007) takes place demonstrating the almost ideal fit of the model.

Keywords: Linear, Nonlinear, dynamics, Hicks, economics.

1. Introduction

Modern economic analysis draws heavily on the tools of mathematics ([7], [9]). In fact, the tools of nonlinear analysis are regarded as promising ways towards overcoming the problems associated with the traditional approaches and have developed through different strands of thought and across diverse disciplines ([5], [8], [20], [21], [22]).

An important contribution was Hicks's *A Contribution to the Theory of the Trade Cycle* where he developed his own endogenous model of the cycle. In this paper we introduce a novel approach regarding the econometric estimation of Hicks's nonlinear model. Next, an empirical application for the Greek economy using real world data for the time period 1960-2007 takes place and shows the ideal fit of the model.

The paper is structured as follows: section 2 deals with the transition from early linear to the first nonlinear model in economics; section 3 presents the Hicks model; section 4 describes the proposed methodological framework for its empirical estimation; section 5 presents the empirical analysis for the case of Greece; section 6 concludes the paper.

2. From linear to nonlinear models in Economics: A Review

This section, which draws heavily on [17], deals with the development of early nonlinear models in economics. Early empirical efforts, focused on time series models whose *ad hoc* nature was (and still is) criticised by many researchers, who insisted on the importance of adopting 'structural' models [12]. It should be stressed that before the linear (typically Keynesian) approach dominated economic theory, numerous economists were involved in the development of nonlinear models ([3], [6], [25]). This trend was associated with the study of business fluctuations, a very hot topic on most economists' agenda [18]. Consequently, the study of business fluctuations came up because several economies were unstable.

One of the seminal papers was Frisch's 1933 *Propagation problems and impulse problems in dynamics economics*. According to it, the study of cycles was dynamic, and could be decomposed into two aspects: propagation and impulse. Frisch tried to think of several ways to make his system more realistic. One idea was to produce oscillations through the introduction of time-lags in the investment function [10]. In this context, Frisch constructed a mixed differential-difference equation system which was capable of producing cycles. However, the model's oscillations died out in a finite time. This situation made him add an exogenous shock to his model [10]. However, several other papers criticised it for not including the Keynesian multiplier, an omission that prevented it from being a purely endogenous model of the cycle [23].

Another seminal contribution in the development of nonlinear dynamic modelling in economic analysis is Samuelson's famous model [19]. Actually, while Frisch constructed a dynamic system based on investment lags, Samuelson used time-lags in his model in a different way. However, the dynamic behaviour expressed by his model was, practically, devoid of meaning [19]. As a result, the model would only reproduce damped oscillations and thus one had to also introduce exogenous shocks [2].

In this line of thought, Kalecki ([15], [16]) constructed a model focusing on the time lag between investment decisions and the production of capital goods. Kalecki's model was not based on the acceleration principle to generate cycles. Rather, the thrust of his model's theoretical idea was that the rate of investment decisions was a positive function of the gap between the rate of profit and the rate of interest, influenced by lenders' confidence as well as the money market. Both the prospective rate of profit and the rate of interest were in turn a function of investment and the stock of capital. [16].

In 1940, Kaldor [14] employed nonlinearities to account for economic fluctuations. The model was different from the previous ones since the author did not make it dynamic by utilising a time-lag structure on the investment function. Rather, Kaldor followed a different strategy: his model was dynamic in nature because *ex ante* Savings differed from Investment, which together with the multiplier-accelerator, yielded a model that produced endogenous cyclical fluctuations.

Harrod stressed the fact that oscillations were produced by the lag itself rather than by mechanisms endogenous to the system. In Harrod's model dynamics were not caused by the lags, but rather depended on the internal structure of the model [11]. Harrod proposed a very simple model, based on the multiplier-accelerator mechanism and derived his well-known equation. He also came to show that for a static equilibrium, the process was unstable. Moreover, he showed that instability was, at least partly, responsible for generating cycles.

Harrod's model was criticised for being simplistic by Baumol [3] and Alexander [1]. A fundamental contribution in this vein was Hicks's *A Contribution to the Theory of the Trade Cycle* [13] where he defended a combination of both approaches. Hicks thought that fluctuations in investment led to a process taking place throughout many periods, because of lags in the multiplier. Of course, Hicks came to the standard solutions of a higher order linear system. Also, another crucial aspect of the model was the introduction of nonlinearities in the investment function.

3. The Hicks Model

According to Hicks the consumption function is a linear function of Y_{t-1}

$$C_t = (1-s)Y_{t-1} \tag{1}$$

where 0 < *1-s*< 1 is the so-called marginal propensity to consume, $1/s$ is the multiplier and Y_{t-1} denotes output one period back. In the Samuelson business cycle model, investment was determined by the growth in output, through the so-called *acceleration principle*. Analytically, investment was assumed to be proportional to the change in output, or:

$$I_t = u(Y_{t-1} - Y_{t-2})$$

where I_t denotes investment in time period *t*, Y_{t-1} and Y_{t-2} output one and two periods back, respectively, and *u* (>0) is the *accelerator*. However, Hicks thought that fluctuations in investment, caused by nonlinear changes in autonomous investment and the acceleration principle governing induced investment, led to an adjustment process taking place throughout many periods. In this spirit, Hicks introduced autonomous expenditures which may be growing exponentially. Investment has two components: autonomous investment growing at a constant rate *g*:

$$A_t = A_0(1+g)^t$$

where A_0 is the autonomous investment. Of course, there is also the induced part of the investment which responds to changes in output. Thus:

$$I_t = A_0(1+g)^t + u(Y_{t-1} - Y_{t-2}) \tag{2}$$

Hicks modeled the growth process of a closed economy without government sector, within the Keynesian framework. In this context:

$$Y_t = C_t + I_t \tag{3}$$

In what follows, we introduce a novel approach for the econometric estimation of Hicks's nonlinear model and apply it for the Greek economy in the time period 1960-2007 using data from the AMECO database.

4. Econometric Estimation

The estimation of the consumption function (1) is straightforward using the 2 Stages Least Squares (2SLS) method. This method is typically used for the estimation of multiplier–accelerator models [4].

For the estimation of the consumption function we use 2SLS:

$$C_t = C_0 + (1 - s)Y_{t-1} + v_t \quad (4)$$

where C_0 is the intercept of the regression, and v_t is the random error term, independent and identically distributed (*i.i.d.*).

In what follows, the proposed methodology estimates the Hicks model, based on nonlinear investments. It is the case that the LS estimation principle applies for deriving estimators, ones that are called Nonlinear Least Squares (NLS). Unlike Ordinary Least Squares (O.L.S.) estimators, NLS estimators cannot be obtained analytically. However, the minimization of the Sum of Squared Residuals (SSR) is a well-defined optimization problem that can be solved numerically by iterating on a solution:

Step 1: Let $g \in [\alpha, \beta], \alpha, \beta \in \Re : \alpha < \beta$ be drawn from a uniform distribution.

Step 2: Estimate A_0 >0 and *u*>0 in the following equation using O.L.S.:

$$I_t = A_1 + A_0(1+g)^t + u(Y_{t-1} - Y_{t-2}) + e_t \quad (5)$$

where A_1 is the intercept of the estimation and e_t is the random error (*i.i.d.*).

Step 3: Compute the Sum of Squared Residuals *SSR(i)* for the equation.

Step 4: Repeat for $i = 1,..,I$ and select the value $\overline{g}$ that yields the minimum SSR(*i*), subject to A_0 >0 and *u*>0 being statistically significant.

Step 5: Given the value of $\overline{g}$ estimated in the previous step, use 2SLS to estimate the linearized equation (5) and keep the values of the parameters.

Given that *g* is the economy's growth rate in autonomous investment it should, normally, be positive over time and range between 0% and 15%. Also, other criteria for model selection include the SIC (*Schwartz Information Criterion*) and the AIC (*Akaike Information Criterion*).

5. Empirical Analysis: The Greek Economy (1960-2007)

5.1 Estimation Results

The estimation of equation (4) by means of 2SLS yields:

$$C_t = -4.46 + 0.83Y_{t-1} + v_t$$

(-2.80) (49.79) R^2 =0.98, S.S.E.=3.99

The equation (5) by means of the procedure developed above, yields:

$$I_t = 11.54 + 2.69(1+0.05)^t + 0.49(Y_{t-1} - Y_{t-2}) + e_t$$

(7.45) (8.37) (1.99) R^2 =0.68, S.S.E.=5.20

Values in parentheses are t-statistics which imply that all estimated parameters are statistically significant. It should be stressed that for $\overline{g}$ =0.05 the estimation of equation (5) yields the minimum value of SSR, under which A_0 and u are statistically significant. Thus s=0.17, C_0 = -4.46, u=0.49, A_0 =2.69, A_1 =11.54 and $m = 1 + g$ =1.05.

5.2 *Solutions and Stability*

By substituting equations (4) and (5) into (3) and rearranging we get

$$Y_t - (1 - s + u)Y_{t-1} + uY_{t-2} = A_0(1+g)^t + A_1 + C_0 \qquad (6)$$

Also, λ_1 and λ_2 are the roots of the characteristic equation

$$\lambda_1, \lambda_2 = \frac{1}{2}\left[1 - s + u \pm \sqrt{(1 - s + u)^2 - 4u}\right] \qquad (7)$$

The complete solution for (6) is:

$$Y(t) = Y_c(t) + Y_e(t) \qquad (8)$$

where $Y_c(t)$ is the complementarity function and $Y_e(t)$ is the particular integral. We can show that the particular integral is equal to

$$Y_e(t) = Y_p(t) = \frac{(A_0 m^2) m^t}{m^2 - (1 - s - u)m + u} + \frac{A_1 + C_0}{s} \qquad (9)$$

So, the solution which clearly implies a “moving equilibrium” output is

$$Y_e(t) = 14.36 \cdot (1.05)^t + 41.65 \qquad (10)$$

As is well known, the solution depends on the discriminant $\Delta = (1 - s + u)^2 - 4u$. Given that in our case the discriminant takes a negative value ($\Delta < 0$), the complementarity function is

$$Y_c(t) = r^t (B_1 \cos \vartheta t + B_2 \sin \vartheta t) \qquad (11)$$

where:

$$r = \sqrt{a^2 + b^2} \qquad (12)$$

$$\vartheta = \tan^{-1}(b/a) \qquad (13)$$

$$\lambda_{1,2} = \alpha \pm bi \tag{14}$$

Substitution in (7), given that $\Delta < 0$, yields

$$\lambda_{1,2} = 0.66 \pm 0.23i$$

Which implies

$$r = 0.7 \text{ and } \vartheta = 0.34 .$$

By substituting these values into equation (11) we get

$$\mathrm{Y}_c(t) = 0.7^t (B_1 \cos 0.34t + B_2 \sin 0.34t)$$

So

$$\mathrm{Y}(t) = 0.7^t (B_1 \cos 0.34t + -B_2 \sin 0.34t) + 14.36 \cdot (1.05)^t + 41.65 .$$

Finally, given the two initial conditions (i.e. actual values for Y(0) and Y(1)) we get the values for the arbitrary constants

$$B_1 = -31.00 \text{ and } B_2 = -33.57 .$$

Conslusively

$$\mathrm{Y}(t) = 0.7^t (-31 \cos 0.34t + -33.57 \sin 0.34t) + 14.36 \cdot (1.05)^t + 41.65 .$$

This is the analytical solution for $Y(t)$, i.e. Gross Domestic Product (GDP) and by substituting the values of *t* we get the estimated values of $Y(t)$. The estimated GDP is illustrated in the same plot with the real GDP values and the calculated correlation coefficient ($r_{correlation} = 0.99$) implies an ideal fit.

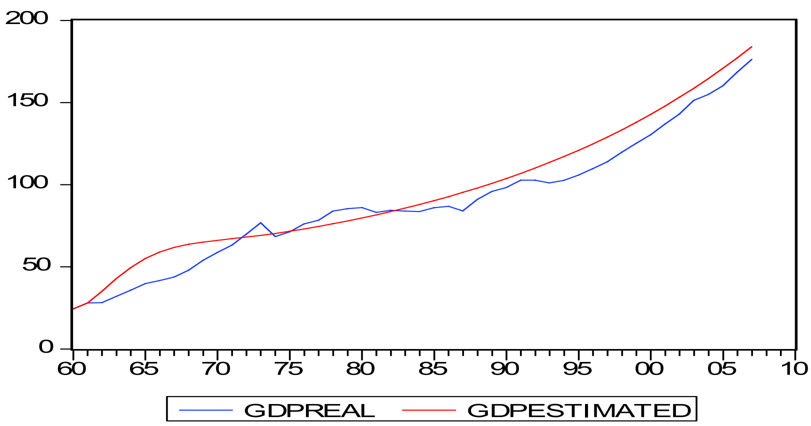

Figure 1. Estimated GDP vs real GDP for the Greek economy (1960-2007)

Finally, stability depends on the discriminant $\Delta = (1 - s + u)^2 - 4u$. Since $\Delta < 0$, r^t (with $|r| < 1$) will dampen the fluctuations caused by the $(B_1 \cos 0.34t + B_2 \sin 0.34t)$. In another formulation, since $(1 - \sqrt{s})^2 < u < 1$ or 0.35<0.49<1, the solution is periodically convergent, i.e. stable [24].

6. Conclusions

In this paper, after discussing the transition from early linear modelling to the first nonlinear models in economics, we proposed a novel approach for estimating, in a Keynesian system of equations, the non-linear Trade (Business) Cycle model developed by Hicks. The proposed methodology yields very satisfactory results when fitted to GDP data for the Greek economy over the time period 1960-2007. We believe that the results of this paper suggest that the modified Hicks model with its generality, conformity with theory and simplicity of structure is an appropriate vehicle for testing, expanding and improving conventional business cycle theory in empirical applications. Meanwhile, preliminary results of the proposed approach from other countries are extremely encouraging. Clearly, future and more extended research on the subject would be of great interest.

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