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Abstract

The equation of exchange is well-known as a quantitative expression of money circulation, but it has a defect in that the relation between the velocity of money and the situation of economic agents is not clear. This paper attempts to found the velocity which pays attention to movement of money.

For that purpose, this paper shows a money circulation equation in which agents of the whole society are unified. If this equation has a unique solution, the velocity of money is reduced to the expenditure rate of the whole society. Thereby, the defect of the equation of exchange can be remedied. Our attempt can be interpreted as connecting the velocity of money with the multiplier analysis.

Success or failure of the trial depends on its solvability. This solvability problem of the money circulation equation is closely related to the missing problems of the monetary budget constraint. This paper also attempts to explain the missing problems in the case of the budget constraint of the whole society. This paper explains that a time irreversible disposal solves those problems by using an analogy.

Keywords: Equation of Exchange, Money Circulation, Budget Constraint.

1. Introduction

The equation of exchange has been known as a quantitative method to represent money circulation since the olden days. The equation is denoted as MV=PT, where M is the money stock, V is the velocity of money, P is the price level, and T is the real gross transactions.

Irving Fisher, who spread it widely, regarded Simon Newcomb as a pioneer of the algebraic statement of the equation.[1] The work which Newcomb showed in *The Principle of Political Economy* was published in 1885,[2] but in fact the equation had been known before Newcomb's work.

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[1] Cf. Fisher [1922] p.25.

[2] Cf. Newcomb [1966] pp.320-328.

As far as the author knows, the first writer who grasped the concept of the velocity of money was William Petty, who was a British economist in the seventeenth century.[3] Moreover, according to Reghinos Theocharis, the first writer who used an algebraic statement of the equation of exchange was Claus Kröncke, who was a German economist in the early nineteenth century. Joseph Lang in Germany[4] and Samuel Turner in Britain also seem to have used this equation before Newcomb.[5]

The concept which characterizes the equation of exchange is the velocity of money. However, this concept is not related to the situation of economic agents, thus an economic meaning of the concept is not so clear. This is a defect of the equation of exchange.

As an attempt to overcome the defect, Arthur Cecil Pigou suggested that the velocity of money ought to be reduced to the demand for legal tender money.[6] This approach is known as the Cambridge cash-balance approach nowadays.

If this is correct as an interpretation of the velocity of money, the concept is founded by subjective intention. However, we think that it ought to be founded by objective movement of money.

Note that Mária Augustinovics in Hungary showed a money circulation equation in Augustinovics [1965] and the current author modified it in Miura [2014b]. The money circulation equation can found the velocity of money by an objective movement. However, the equations in the above papers have shown are expressed in a form that a society is separated into plural elements. Therefore, the relationship between the equation of exchange and the money circulation equation may be difficult to understand.

In order to overcome the difficulty, this paper aims to show a money circulation equation in which agents of the whole society are unified. Then, we will clarify that the velocity of money can be reduced to the whole expenditure rate.

Moreover, the author's preceding paper proved that the solvability of the money circulation equation is guaranteed if money is transferred time irreversibly.[7] However, since the proof is mathematically a little advanced, some readers may find it difficult to understand. But in fact, it is based on a simple idea. In order to inform this simplicity, this paper shows an analogical explanation of the proof.

This solvability problem is closely related to the missing problems of the

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[3] Cf. Holtrop [1929], Deane [1968] p.67, Humphrey [1993] pp.2-5. Roncaglia [2008] p.390.

[4] Joseph Lang is remarkable as an early researcher of an economic circulation. Cf. Theocharis [1958], Uebe [1992], Uebe [1992].

[5] Cf. Theocharis [1983] pp.102-110, pp.120-121, Humphrey [1984] pp.14-16, Ardor [2002] pp.145-146.

[6] Cf. Pigou [1952].

[7] A similar solvability theorem exists in the economic input-output equation invented by Wassily Leontief. Cf. Miura [2014a].

monetary budget constraint shown in Miura [2015b]. This paper also attempts to explain these problems in the case of the budget constraint of the whole society. The analogical explanation will also promote an understanding of the problems and their solutions.

2. Unified Money Circulation Equation

To start with, we aim to formalize a unified money circulation equation.

We decide that a group of target agents for description is called the relevant society and a target term for description is called the relevant term. We assume that the relevant term is a finite length. The sphere that satisfies both the relevant society and the relevant term is called the relevant space-time.

We define expenditure as transferring money to the relevant space-time, and revenue as money being transferred from the relevant space-time. There is a possibility that a money transfer occurs between the relevant society and its outside, but transferring money to the outside is not called expenditure and money being transferred from the outside is not called revenue.

Then, we consider the sources of money possession in the relevant space-time. Revenue is one source, but the possession at term beginning, production and being transferred from the outside of the relevant society are other sources. We decide that the sources excluding revenue are collectively called the beginning money.

On the other hand, we consider the results of money possession in the relevant space-time. Expenditure is one result, but the possession at term end, disappearance and transferring to the outside of the relevant society are other results. We decide that the results excluding expenditure are collectively called the end money.

Let X be the expenditure quantity in the whole relevant space-time; Y be the revenue quantity in the whole relevant space-time; Ψ be the quantity of the beginning money in the whole relevant space-time; and Ω be the quantity of the end money in the whole relevant space-time.

The gross source of money in the relevant space-time is the sum of the beginning money and revenue, whereas the gross result of money in the relevant space-time is the sum of expenditure and the end money. These quantities are equal. That is, $X+\Omega=\Psi+Y$ holds. We call this the law of gross disposal.

Each source and result is non-negative from their economic meaning. That is, $X\geq 0$, $Y\geq 0$, $\Psi\geq 0$, and $\Omega\geq 0$ hold. Therefore, the gross source and the gross result are also non-negative, but we suppose that they are positive in order to simplify the description. That is, $X+\Omega=\Psi+Y>0$ is supposed. If they are zero, we cannot regard that a monetary economy exists in the relevant space-time. Hence, this supposition does not make our theory lose effectiveness as an analysis of monetary economy.

Next, we define the whole expenditure rate as the percentage of expenditure in the gross result. If we let θ be the whole expenditure rate, it is defined as $\theta=X/(X+\Omega)$ using symbols.

Here, we confirm the range of the whole expenditure rate. Since $X\geq 0$ and $X+\Omega>0$ hold, $\theta\geq 0$ is satisfied. Further, since $1-\theta=\Omega/(X+\Omega)$ and $\Omega\geq 0$ and $X+\Omega>0$ hold, $\theta\leq 1$ is satisfied.

Multiplying both sides of the definition formula of the whole expenditure rate by $X+\Omega$, we can derive $X=\theta(X+\Omega)$. Substituting the law of gross disposal into the equation, we can derive $X=\theta(\Psi+Y)$. We call this the circular disposal formula.

The circular disposal formula expresses a money flow from revenue to expenditure. In a monetary economy, however, expended money becomes revenue for somebody. The received money repeats being expended, and becomes revenue again. Then, an agent who receives in this case may be an agent who originally expended. In other words, there exists a money flow from expenditure to revenue into the formula. We called this flow the expenditure reflux.[8] Money circulation consists of the two money flows between expenditure and revenue. By incorporating the two flows, we can manage to express money circulation.

Recall the definition of expenditure and revenue. Expenditure is defined as transferring money to the relevant space-time, and revenue is defined as money being transferred from the relevant space-time. Hence, they are the same entity named money transfer grasped from different viewpoints. Accordingly, their quantities must be equal in the whole society. That is, $X=Y$ holds. We call this the law of transfer equality. Then, the law can be interpreted as a quantitative expression of the expenditure reflux in the whole society.[9] That is, the whole expenditure causes the same amount of the whole revenue.

Note that, from the law of transfer equality and the law of gross disposal, we can derive $\Psi=\Omega$. This means that the beginning money and the end money in the relevant space-time are equal. We call this the law of money conversation. This law reflects a fact that transfer does not change the money stock in the whole relevant society.

In order to represent money circulation completely, we must consider not only the flow from revenue to expenditure but also the flow from expenditure to revenue. For the purpose, we substitute the law of transfer equality $Y=X$ into the circular disposal formula $X=\theta(\Psi+Y)$. Then, we can obtain $X=\theta(\Psi+X)$. Hereby, money circulation can be expressed completely. Transposing this, we can derive $(1-\theta)X=\theta\Psi$. This is a unified money circulation equation.[10]

As confirmed above, the whole expenditure rate is limited to $0\leq\theta\leq 1$. If it satisfies $0\leq\theta<1$, we can divide both sides of the equation by $1-\theta>0$. Mathematically, this means that the unified money circulation equation has

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[8] Cf. Miura [2015a] p.25.

[9] Cf. Miura [2015b] p.98.

[10] This equation is a special case of the money circulation equation shown in Miura [2014b] p.191 where the relevant society consists of only one agent.

a unique solution. In this case, the solution is $X=\{\theta/(1-\theta)\}\Psi$. Due to the law of transfer equality, $Y=\{\theta/(1-\theta)\}\Psi$ also holds. We can see that, if we can solve the unified money circulation equation, expenditure and revenue can be calculated by the beginning money with the help of the whole expenditure rate.

Here, we remember the equation of exchange MV=PT and compare it with our unified money circulation equation.

M in the equation of exchange refers to the money stock in the whole society. On the other hand, if we do not consider its variation in the relevant term, the money stock seems to be equal to the beginning money Ψ. Therefore, M corresponds to Ψ.

Moreover, P in the equation of exchange refers to the price level and T in the equation refers to the real gross transactions. Therefore, PT refers to the gross money transfer quantity of the relevant space-time as long as money transfer is used only for exchange with real commodities. On the other hand, the gross money transfer quantity is equal to the gross quantity of expenditure X in our equation. By the law of transfer equality, it is also equal to the gross quantity of revenue Y. Hence, PT corresponds to X and Y.[11]

Therefore, V corresponds to $\theta/(1-\theta)$. We can see that, if the unified money circulation equation has a unique solution, the velocity of money can be reduced to the whole expenditure rate. Hereby, we can understand the velocity of money with being related to expenditure behavior of agents.

By this correspondence, someone may feel that this is an attempt to connect the velocity of money with the multiplier analysis. Such an attempt has often been executed until now.[12] Above all, an idea suggested by Yougei Wang, Yan Xu and Li Liu is similar to ours.[13] If we regard their concepts of "marginal propensity to expend with respect to wealth" and "marginal propensity to expend with respect to income" as an equivalent concept of our expenditure rate, their foundation of the velocity of money is identical with ours.

However, we do not agree their method to derive the equality between expenditure and revenue. They derived the equality between expenditure and revenue as follows. At the aggregate level, the current expenditure is equal to the revenue of next period. Then, since the aggregate revenue reaches a steady level when the system gets to its equilibrium state, the current revenue is equal to the revenue of next period. As a result, the

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11 In reality, since a money transfer is not limited to the usage for exchange with real commodities, PT does not always equate with X and Y.

12 Cf. Neisser [1936], Kahn [1936], Maculup [1939], Samuelson [1942] pp.601-605, Anderson [1945], Turvey [1948], Goodwin [1950] pp.487-489, Ackley [19551], Lutz [1955], Archibald [1956], Millar [1956], Tsiang [1956] pp.555-564, Mayer [1964], Gechert [2014] pp. 81-87. It is remarkable that time issue has been emphasized after Maculap [1939]. Recently, Moore [1988], Cottrell [1994], Moore [1994], Dalziel [1996] and Moore [2008] discussed this issue while connecting it with endogeneity of money.

13 Cf. Wang & Xu & Liu [2010] pp.1708-1711.

current expenditure is equal to the current revenue.

This seems to be an unnecessarily redundant justification. The first and second propositions are not always satisfied, whereas the third proposition always holds. As mentioned above, expenditure and revenue are the same entity named money transfer grasped from different viewpoints. Therefore, expenditure of a period must be equal to revenue of the same period regardless of "equilibrium". The validity of the law of transfer equality is absolute in such a meaning.

3. Solvability Problems of the Unified Money Circulation Equation and the Missing Problems of the Whole Monetary Budget Constraint

Note that the unified money circulation equation has a unique solution only in the case where the whole expenditure rate satisfies $0 \leq \theta < 1$. If $\theta=1$, it is impossible. In this case, the equation becomes $0X=\Psi$. In order to satisfy this equation, $\Psi=0$ must be hold. By the law of money conservation, $\Omega=0$ must also be hold.

The conclusion $\Psi=\Omega=0$ can also be derived another way. The equation does not have a unique solution when $\theta=1$ holds. By the definition of the expenditure rate, $\theta=1$ is equivalent to $X/(X+\Omega)=1$. This holds if and only if $\Omega=0$. By the law of money conservation, this is also equivalent to $\Psi=0$. Conversely, $0 \leq \theta < 1$ is equivalent to $\Psi=\Omega>0$. Eventually, the unified money circulation equation has a unique solution if and only if the beginning money and the end money are positive. We call these solvability conditions the space-time openness conditions. $\Psi>0$ refers to the openness for the source direction, and $\Omega>0$ refers to the openness for the result direction.

We will qualitatively consider the meaning of a situation where the space-time openness conditions are not satisfied. $\Psi=0$ represents that money possessed at term beginning, produced and transferred from the outside does not exist at all. Further, $\Omega=0$ represents that money possessed at term end, disappearing, transferring to the outside does not exist at all. In these cases, it seems that money does not exist in the relevant space-time. Nevertheless, positive expenditure is still permitted under the equation $0X=0$. This seems to refer to a situation that money can be expended even though money does not exist. This is felt a strange situation.

Moreover, $0X=0$ is satisfied no matter how large expenditure is. In other words, it permits infinite expenditure. If we suppose that the beginning money is an infinite quantity, it is a natural conclusion that expenditure is also infinite. If we put a supposition that the relevant term has an infinite length, infinite expenditure would also be no wonder. However, the conclusion is derived without these suppositions. Infinite expenditure is permitted under finite beginning money, in a finite term. This is strange but also an epistemological aporia because we cannot observe it empirically.

In the case that the unified money circulation is not solvable, these two strange situations occur. Therefore, we feel this is an impossible case. However, a reason of the impossibility is not shown yet. This is a solvability problem of the unified money circulation equation.

Here, we change a topic. In the preceding section, the equation X+Ω=Ψ+Y is called the law of gross disposal. Note that the budget consists of the beginning money and revenue. Then, it is disposed only as expenditure and the end money. Hence, this equation can also be interpreted as a budget constraint of the whole society.

Suppose that total budgets of all agents are expended under this constraint. In this case, Ω=0 must hold. Based on the meaning of the end money, this represents that money does not disappear and is not transferred to the outside. Therefore, money ought to exist in the relevant space-time, but money does not exist at the term end because the end money is zero.

Then, where does money exist? Money in the relevant space-time is all missing. This conclusion is unnatural based on our common sense, but the preceding simple budget constraint cannot deny a possibility that this unnatural situation occurs. This is the first missing problem of the whole monetary budget constraint. This problem represents that the simple budget constraint is incorrect as an objective monetary budget constraint of the whole society.

What is a defect of the simple budget constraint? As mentioned in the preceding section, money circulation consists of two flows, the money flow from revenue to expenditure and the money flow from expenditure to revenue. Nevertheless, the simple constraint reflects only the former flow. It does not reflect the latter flow, the expenditure reflux. Therefore, the simple constraint does not still express money circulation completely. For the purpose, we have to incorporate the expenditure reflux into the budget constraint.

Also mentioned in the preceding section, the expenditure reflux of the whole society is expressed by the law of transfer equality. Therefore, we can obtain the reflux budget constraint of the whole society by incorporating the law of transfer equality Y=X into the simple budget constraint X+Ω=Ψ+Y. Then, we can obtain Ω=Ψ as the reflux budget constraint of the whole society. This is substantially the same as the law of money conversation.

This constraint does not include expenditure. Hence, even if agents expend their budgets under this constraint as much as possible, the end money is never missing provided that the beginning money is positive. The reflux budget constraint seems to have succeeded in solving the first missing problem.

However, the budget constraint is originally a thing which constrains expenditure. Nevertheless, the whole reflux budget constraint does not include expenditure. This implies that an upper limit of the constraint is missing and expenditure can be infinite under the constraint. We feel this conclusion strange because expenditure in a finite term ought to be a finite quantity. This is the second missing problem of the whole monetary budget constraint.

Careful readers will notice that the missing problems are similar to the solvability problem of the money circulation equation. Insolubility of the money circulation equation implies that infinite expenditure is permitted. This is the same as the second missing problem. Further, the space-time openness condition, which is an equivalent condition for the solvability of the money circulation equation, is not satisfied if and only if the end money is zero. This is a content of the first missing problem. That is, the solvability problem of the unified money circulation equation and the missing problems of the whole monetary budget constraint are the same problems grasped from different viewpoints.

Judging by our common sense, the beginning money and the end money ought to exist as far as money can be expended, and infinite expenditure should not be permitted. They ought to be impossible situations. Then, how can we found the impossibility?

4. Analogical Explanation for a Solution by Time Irreversibility

Note that, judging by our common sense about time, money cannot be disposed from future revenue to past expenditure. Moreover, money received at a certain time cannot be expended at exactly the same time. In other words, money can be disposed from revenue of the past to expenditure of the future. We call this the disposal irreversibility principle.

In fact, if money is disposed time irreversibly, the beginning money and the end money are always positive. Moreover, expenditure must be finite in this case. Therefore, time irreversible disposal guarantees the solvability of the money circulation equation and solves the missing problems of the monetary budget constraint.

The disposal irreversibility principle is essentially the same as the impossibility of a time travel into the past and an exactly simultaneous teleportation. Since modern science does not usually support the possibilities of these phenomena, the disposal irreversibility principle seems to be a universal truth.[14] If this universality is certainly true, the solutions of the two problems are guaranteed universally.

We entrust the mathematically strict proofs of these solutions to the preceding papers.[15] These solutions are indeed based on a simple idea. This paper will clarify this simplicity by the following explanation using an analogy.

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[14] Cf. Gott [2002], Davies [2002], Nemiroff & Wilson [2013].

[15] Cf. Miura [2014b] pp.192-197 regarding the solvability of the money circulation equation. Cf. Miura [2015b] pp.93-97 regarding the solution of the missing problems of the monetary budget constraint.

We wind up a finite length tube such that it can be seen as circular from above, and we put a ball in it. Further, we suppose that the ball in the tube move only one way. Then, how many times does the ball circulate seen from above?

If we do not join two mouths, the exit of the ball is open. Since the tube has a finite length, the ball will leave the tube after circulating finite times. On the other hand, if we join two mouths of the tube together, the exit of the ball is closed. Since the ball cannot leave the tube, it continues circulating eternally. Hence, the number of circulations is infinite.

Assume that the tube corresponds to the relevant space-time and the ball corresponds to money. Further, we regard the vertical direction as an analogy of time and the horizontal direction as that of space. Finite length of the tube is prepared as an analogy of the finite relevant space-time.

Then, we can see that the analogy shows that money circulates finitely if the relevant space-time is open and that money circulates infinitely if the relevant space-time is closed. This explains why the space-time openness is an equivalent condition for the solvability of the money circulation equation.

Then, in what case are the mouths of the tube open? There are various cases. For example, if we wind up the tube in a spiral so as to be seen as circular from above, mouths cannot be closed because each part of the tube has a different position in a vertical direction. Hence, the ball in the spiral tube can circulate only finitely.

Note that the vertical direction refers to time and the horizontal direction refers to space in this analogy. Since the spiral tube is vertically one-way and horizontally circular, it expresses an irreversible circulation.

This analogy represents the following truth.

If money can circulate time reversibly, it can continue to circulate eternally in a temporally closed place. Since the place is closed, the beginning money and the end money cannot exist there. Then, this eternal circulation implies that expenditure is an infinite quantity in a finite term.

However, if we are allowed to assume a time irreversible disposal, it has an ability to open a space-time. In this case, the beginning money and the end money must exist. Further, money is impossible to continue eternal circulation in a temporally closed place, and then expenditure is a finite quantity in a finite term. In this way, we can see that the solvability problem of the money circulation equation and the missing problems of the monetary budget constraint can be solved by time irreversible disposal.

We earlier interpreted that the absence of the beginning money and the end money represents the absence of money. Due to this interpretation, we derived a strange conclusion that money can be expended even though money does not exist. The origin of this judgment was our comparison between the equation of exchange and the money circulation equation, in which we regarded the money stock in the former equation as the beginning money and the end money in the latter equation.

However, the analogy also teaches us that, strictly speaking, this is an incorrect interpretation. If the tube is not wound up spirally, the same ball can appear multiple times at one point on the vertical axis even though only one ball is put in the tube. Therefore, the number of the ball which exists in the tube does not accord with the number of the ball which exists at one point on the vertical axis. Due to a similar reason, if we permit the possibility of a time reversible disposal, the money stock in each period is not equal to the quantity of the beginning money and the end money.[16]

As its expansion, money can be expended even if the beginning money and the end money do not exist. In the preceding analogy, this corresponds to a situation that, if the tube is closed, the ball circulates eternally without entering from the outside and exiting to the outside. In this case, where does the ball enter from? Where does the ball exit to? These questions do not have any meaning in the reversible world. Our common sense that the ball should enter from somewhere and exit to somewhere does not hold in this world.[17]

This paper does not intend to judge whether a time reversible movement can be realized or not. However, we should recognize that our common sense is often formed by an assumption of time irreversibility and we must not apply the common sense to a time reversible world.

5. Concluding Comments

We can found the solvability of the money circulation equation by time irreversibility, which is a purely objective principle irrelevant to subjective intention. Although we do not intend to deny that the expenditure rate is affected by an intention, its effect is limited to which value the expenditure rate takes in $0 \leq \theta < 1$. The effect does not reach whether $\theta = 1$ or not, which is a life line of the equation because it connects with its solvability. Since the cash balance approach does not recognize the importance of time irreversibility, the equation of exchange should not be founded by the approach. We ought to recognize the superiority of physical environment over mental intention in our world.

We can also solve the missing problems of the monetary budget constraint. The author intends to release a new paper that deals with money circulation optimization theory, which unifies recognition of money circulation and an optimization method. Especially the solution of the second missing problem is too important for the theory. A time irreversible disposal is needed to derive its optimal solution. Although an optimization in a social science is based on subjective utility, the optimal solution is guaranteed by an objective physical principle, the irreversibility disposal principle. This also makes us reconfirm the superiority of environment over intention in our society.

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[16] An explanation relevant to a time travel in Davies [2002] pp.111-113 seems to be a good reference to this issue.

[17] The discussion regarding the jinn particle in a physics world may promote an understanding of this issue. Cf. Lossev & Novikov [1992], Gott [2002] pp.20-24.

We must not forget a fact that human beings are always living with being constrained by physical environment.

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