Why Has the U.S. Current Account Deficit Persisted? International Sustainable Heterogeneity under Floating Exchange Rates

Harashima, Taiji

Kanazawa Seiryo University

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Taiji Harashima*

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Abstract

Under floating exchange rates, there is a mechanism that generates persistent current account imbalances at a competitively achieved steady state. Even if preferences are heterogeneous, sustainable heterogeneity in which all optimality conditions of all heterogeneous households are satisfied can be achieved if relatively advantaged households do not behave unilaterally. Even if households of a relatively advantaged country behave unilaterally, households in less advantaged countries can counter the unilateral behavior under floating exchange rates and achieve optimality even with persistent current account surpluses or deficits. The observed persistent current account imbalances in many countries in recent decades are shown to be basically consistent with the predictions of this model.

JEL Classification code: E10, F30, F31
Keywords: Current account imbalance; Floating exchange rate; Heterogeneity

*Correspondence: Taiji HARASHIMA, Kanazawa Seiryo University, 10-1 Goshomachi-Ushi, Kanazawa-shi, Ishikawa, 920-8620, Japan.
Email: harashim@seiryo-u.ac.jp or t-harashima@mve.biglobe.ne.jp.

1 INTRODUCTION

If there is no heterogeneity across countries, their current account balances should be stationary and have a zero mean. However, current account balances have shown persistent large deficits or surpluses in many countries in recent decades.[1] For example, the U.S. has shown persistent large current account deficits since the 1980s. Therefore, there must be some important persisting heterogeneity across countries, but what type of heterogeneity exists? Hervey and Merkel (2000) categorized commonly mentioned explanations for the U.S. current account deficit into three types: the consumption boom hypothesis, the safe haven hypothesis (Hervey and Kouparitsas, 2000), and the technological change hypothesis. They concluded that the technological change hypothesis (i.e., heterogeneity in productivity) is the most likely cause of the U.S. deficit. Obstfeld and Rogoff (2005, 2007), Edwards (2005), Engel and Rogers (2006), and Cooper (2008) argued that the present U.S. current account deficits necessitate future depreciation of the U.S. dollar. These studies imply that heterogeneity in prices across countries is the main cause of the persistent deficit.

Harashima (2009b, 2009c 2010) showed that heterogeneity in preferences can generate persistent current account imbalances at steady state and on a balanced growth path. When the representative households across countries are heterogeneous in the rate of time preference (RTP) or the degree of risk aversion (DRA), there is a state in which all optimality conditions of the representative households in all countries are satisfied. This state is called "sustainable heterogeneity." Under sustainable heterogeneity, the current account balances of countries with relatively less patient or risk averse households (i.e., those with a lower RTP or DRA) show persistent deficits at steady state and on a balanced growth path and vice versa. An important nature of heterogeneity in preferences is that preferences generally persist although they may occasionally change; thus, heterogeneity in preferences may cause persistent current account deficits/surpluses.

However, sustainable heterogeneity is not necessarily naturally achieved. If relatively more advantaged households behave unilaterally, it is not achieved under fixed exchange rates unless an authority forces them not to behave unilaterally. Therefore, the question arises, can sustainable heterogeneity be achieved internationally even if a supranational authority does not exist? Because there is actually no supranational authority that has a right to impose taxes on some countries and distribute the revenues to other countries, international sustainable heterogeneity may depend mostly on the generosity of households of relatively more advantaged countries.

In this paper, I present a model in which there is a mechanism that achieves international sustainable heterogeneity even when households of relatively more advantaged countries behave unilaterally if exchange rates are allowed to float. With floating exchange rates, prices of products can change differently across countries. In contrast, prices of products change equally within a country (if various minor local divergences are ignored) because the currency is identical within a given country. The key point is that countries have different and independent currencies. If households in a relatively less advantaged country can exploit their currency's independence, they may be able to counter the unilateral behavior of the households of a relatively more advantaged country. In addition, I discuss examples of current account deficits or surpluses in several countries and regions relative to the model's predictions.

2 SUSTAINABLE HETEROGENEITY

[1] International Monetary Fund, *World Economic Outlook Database* http://www.imf.org/external/pubs/ft/weo/2015/01/weodata/index.aspx

2.1 *Multilateral balanced growth*

2.1.1 Sustainable heterogeneity

Three heterogeneities in preferences are examined—heterogeneous RTP, DRA, and productivity that indicates total factor productivity excluding technology (see Section A1.2.3 in Appendix A). The model presented in Harashima (2010) is used; it is shown in the Appendix. Suppose that there are $H(\in N)$ countries that are identical except for the RTP, DRA, or productivity of the representative household of each country. The representative household is also the representative laborer who is one of factors that determine productivity of a country. The population growth rate is zero in all countries. The countries are fully open to each other, and goods, services, and capital are freely transacted among them, but labor is immobilized in each country. The countries use different currencies.

The model indicates that if and only if

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right)^{-1}\left\{\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha}-\frac{\sum_{q=1}^{H}\theta_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right\} \tag{1}$$

for any country i (= 1, 2, … , H), all the optimality conditions of all heterogeneous countries' representative households are satisfied at steady state, and

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}}=\lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}}=\lim_{t\to\infty}\frac{\frac{d\int_0^t\tau_{i,j,s}ds}{dt}}{\int_0^t\tau_{i,j,s}ds}$$

for any i and j ($i\neq j$) where $c_{i,t}$, $k_{i,t}$, and $y_{i,t}$ are per capita consumption, capital, and output of country i in period t, respectively; θ_i, ε_i, and ω_i are RTP, DRA, and productivity of country i, respectively; A_t is technology in period t; and α, m, v, and ϖ are constants. In addition, $\tau_{i,j,t}$ is the current account balance of country i with country j. Equation (1) is identical to equation (A33) in the Appendix. The state satisfying this condition is called "sustainable heterogeneity."

Note that the concept of heterogeneity in RTP is relevant to sustainable heterogeneity in both exogenous technology and endogenous growth models, whereas heterogeneity in DRA and productivity is irrelevant to sustainable heterogeneity in exogenous technology models (see Harashima, 2010).

2.1.2 Balance of payments of heterogeneous countries

Suppose for simplicity that there are only two countries—country 1 and country 2—that are identical except for RTP, DRA, or productivity. This type of two-country model can be easily extended to a multi-country model, and the essential results are identical in both two- and multi-country models (see Harashima, 2010).

2.1.2.1 Heterogeneous time preference

Suppose that the RTP of the representative household of country 1 is lower than that of country 2 (i.e., $\theta_1 < \theta_2$), and the other elements are identical between the two countries. The model indicates that $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}} = \lim_{t\to\infty}\frac{\tau_t}{k_{2,t}} = \Xi$ and $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}} = \lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}} = \Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$ if sustainable heterogeneity is achieved, and

$$\Xi = \frac{\theta_1 - \theta_2}{2}\left\{\varepsilon\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \frac{\theta_1+\theta_2}{2}\right]^{-1} - 1\right\}^{-1}$$

where τ_t is the current account balance in country 1 in period t; thus, the current account balance in country 2 is $-\tau_t$ (see Section A2.4.1 in the Appendix). In addition, if $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}[1-(1-\alpha)\varepsilon] < \frac{\theta_1+\theta_2}{2}$, then $\Xi < 0$. Because $k_{i,t}$ is positive, $\Xi < 0$ indicates that the current account of country 1 shows persistent deficits and that of country 2 shows persistent surpluses if sustainable heterogeneity holds. The condition $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}[1-(1-\alpha)\varepsilon] < \frac{\theta_1+\theta_2}{2}$ is generally satisfied for reasonable parameter values.

2.1.2.2 Heterogeneous risk aversion

Suppose that countries 1 and 2 are identical except for the DRA of the representative household such that $\varepsilon_1 < \varepsilon_2$. The model indicates that

$$\Xi = \frac{(\varepsilon_1 - \varepsilon_2)\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right]}{(\varepsilon_1+\varepsilon_2)\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right]^{-1} - 1\right\}}$$

for this heterogeneity in DRA (see Section A2.4.2 in the Appendix). In addition, if $1-\theta\left(\frac{\varpi\alpha}{mv}\right)^{-\alpha}(1-\alpha)^{-1+\alpha} < \frac{\varepsilon_1+\varepsilon_2}{2}$, then $\Xi < 0$. Hence, the current account of country 1 shows persistent deficits and that of country 2 shows persistent surpluses if sustainable heterogeneity holds. The condition $1-\theta\left(\frac{\varpi\alpha}{mv}\right)^{-\alpha}(1-\alpha)^{-1+\alpha} < \frac{\varepsilon_1+\varepsilon_2}{2}$ is generally satisfied for reasonable parameter values.

2.1.2.3 Heterogeneous productivity

Suppose that countries 1 and 2 are identical except for the productivity of the representative household (laborer) such that $\omega_1 < \omega_2$. Unlike the cases of heterogeneity in RTP and DRA, sustainable heterogeneity can be achieved even if country 1 behaves unilaterally. Therefore, the balance of payments in the case of heterogeneity in productivity is different from those of RTP and DRA. The model indicates that if sustainable heterogeneity is achieved, either $\lim_{t\to\infty}\tau_t = 0$

and $\lim_{t\to\infty}\int_0^t \tau_s ds = 0$ or $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}$ (see Section A2.4.3 in the Appendix). On the former path, $\Xi = 0$ and heterogeneous productivity does not result in permanent trade imbalances. However, on the latter path, trade imbalances usually grow at a higher rate than consumption. Hence, the latter path will generally not be selected (see Section A2.4.3 in the Appendix). In addition, country 1 will not prefer to behave unilaterally in this case (see Section A3.3 in the Appendix). Hence, in the case of heterogeneity in productivity, the current accounts of both countries 1 and 2 will generally be balanced.

2.2 Unilateral balanced growth

Sustainable heterogeneity is not naturally achieved. Whether it is achieved depends on the behavior of relatively more advantaged countries (see Section A3 in the Appendix). If they behave unilaterally without considering the optimality of relatively less advantaged countries, sustainable heterogeneity is not achieved.

In the two-country model of heterogeneous RTP ($\theta_1 < \theta_2$), all the optimality conditions of economy 1 can be satisfied only if either

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} \tag{2}$$

or

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} \tag{3}$$

(see Section A3.1 in the Appendix). Equations (2) and (3) are identical to equations (A36) and (A37) in the Appendix, respectively. Equations (2) and (3) indicate that country 1 has two paths on which all of its optimality conditions are satisfied. On the other hand, country 2 can achieve optimality only when country 1 chooses the path of equation (2). Equation (2) corresponds to sustainable heterogeneity. If country 1 chooses the path of equation (3), that is, behaves unilaterally without considering the optimality of country 2, however, country 2 cannot achieve optimality. Country 2 will eventually lose ownership of all capital, and the state shown by Becker (1980) will be observed.

The same result is obtained for heterogeneity in DRA ($\varepsilon_1 < \varepsilon_2$) (see Section A3.2 in the Appendix). Country 1 has two paths for optimality, but country 2 has only one path of sustainable heterogeneity for optimality. However, heterogeneity in productivity can be sustainable for country 2 even if country 1 behaves unilaterally (see Section A3.3 in the Appendix).

3 CURRENT ACCOUNT BALANCES

As with Section 2, suppose for simplicity that there are only two countries that are identical except for RTP or DRA such that $\theta_1 < \theta_2$ or $\varepsilon_1 < \varepsilon_2$. Heterogeneity in productivity can be ignored because it is generally irrelevant to current account imbalances as shown in Section 2.1.2.3. It is assumed for simplicity that there is no overall inflation or deflation in both countries. Nevertheless, changes in relative prices across goods and services can occur within a country. In addition, the two countries use different currencies.

As shown in Section 2.1.2, $\lim_{t\to\infty}\int_0^t \tau_s ds = 0$ is impossible to satisfy at steady state and on a balanced growth path because of heterogeneity in RTP or DRA. Therefore, even if sustainable heterogeneity is achieved, $\lim_{t\to\infty}\int_0^t \tau_s ds \neq 0$ and persistent current account imbalances must exist.

3.1 Current account balances under a fixed exchange rate

An important point of fixed exchange rates is that if country 1 behaves unilaterally, households of country 2 have no tool to counter the unilateral behavior of country 1. Therefore, sustainable heterogeneity cannot be achieved because country 2 cannot achieve optimality and has to experience accelerating current account deficits as discussed in Section 2. In this circumstance, households of country 2 can only hope that households of country 1 do not behave unilaterally or that some outside authority prohibits households of country 1 from behaving unilaterally, e.g., through various regulations.

3.2 Current account balances under floating exchange rates

3.2.1 The countermeasure

If the exchange rate floats, however, households of country 2 have a tool to achieve optimality. The countermeasure consists of the following two pillars:

(a) reduce the overall level of consumption from the initially expected (planned) level, and
(b) reduce the level of consumption of products imported from country 1 from the initially expected (planned) level.

Implementing the countermeasure means that demands of country 2's households are deliberately (or "strategically") reduced relative to their preferred levels; that is, the households deliberately deviate from the consumption path that is consistent with the Euler equation calculated based on their preferences. Pillar (2) may be seen as a type of "patriotic" behavior, for example, the concept behind "Buy American." The question arises, however, is this behavior rational? This behavior is in fact quite rational because the only alternative is that country 2's households can never achieve optimality if country 1's households behave unilaterally. In other words, rationality requires the households of country 2 to not follow their own preferences.

In economics, rationality usually means that, given the available information, optimal decisions are made to achieve an objective (i.e., achieve optimality). In this case, because of the unilateral behavior of country 1's households, households of country 2 are placed in a situation where, to achieve optimality, they have to deviate from their preferred path. Deciding to use the countermeasure therefore indicates that preference (i.e., RTP or DRA) succumbs to rationality when the two conflict.

3.2.2 Depreciating the currency of country 2

Assume for simplicity that both countries produce only one type of product, the quality of which is identical. The two countries are completely open and products of the two countries are freely transacted and sold in both countries (i.e., identical products are imported and exported to and from each country). Suppose that households of country 1 choose a unilateral path for their expected stream of floating exchange rate. Households in country 2 will therefore have to deviate from their preferences to survive.

3.2.2.1 The countermeasure under a fixed exchange rate

First, I examine the use of the countermeasure under a fixed exchange rate for the sake of comparison with that under a floating exchange rate. Let c^*_t be the amount of reduction in country 2's consumption of country 1's products in period t because of the countermeasure. Because of the second pillar of the countermeasure, a part of country 1's products that would be exported to country 2 (i.e., c^*_t) is not purchased by country 2. Firms of country 1 are aware of pillar (2) of the countermeasure; thus, they instead sell c^*_t in their home country. Country 1's households will purchase these products because prices in both countries are unchanged under a fixed exchange rate. Country 1's households therefore replace the consumption of products imported from country 2 with that of their own country by c^*_t. Because of this simple replacement, the level of consumption and formation of capital in country 1 is unchanged from the initially expected path.

In country 2, exports decrease by c^*_t because of the replacement of consumption by the households of country 1, and c^*_t becomes dead stock for country 2's firms. A part of c^*_t may be consumed by country 2's households, but this additional consumption is merely replaced by other dead stock because the overall level of consumption is being reduced according to pillar (1) of the countermeasure. Therefore, firms of country 2 have to possess dead stock under a fixed exchange rate, and country 2 has no power to counter the unilateral behavior of country 1.

3.2.2.2 The countermeasure under floating exchange rates

An essential difference between the case with a fixed exchange rate and that of floating rates is that prices (i.e., the exchange rate in this case) can decrease if excess supply exists. By the same reasoning as in the case of a fixed exchange rate, firms of country 2 have to possess dead stock because of the countermeasure. However, if the currency of country 2 depreciates, the dead stock can be sold in country 1 at a cheaper price in the currency of country 1 without lowering the price in the currency of country 2. As a result of the cheaper prices, households of country 1 will not replace the consumption of country 2's products with that of their own country's products. If the currency of country 2 depreciates, therefore, firms of country 2 can eliminate the dead stock. Taking this into consideration, firms of country 2 will agree to export the dead stock to country 1 at the depreciated exchange rate even though the revenue from the sales in country 1 will be lower in the currency of country 2. Arbitrators in foreign currency exchange markets know firms in country 2 will take this action and thus will buy and sell both countries' currencies at the depreciated exchange rate. Other market participants also are aware of the behaviors of country 2's firms and arbitrators, and thus their expectations of the future exchange rate are modified to the depreciated rate as compared with the initially expected exchange rate. As a result, under floating exchange rates, a force to depreciate the currency of country 2 is naturally generated by the countermeasure.

Note that if the currency of country 2 is expected to depreciate, investors of country 1 will not invest in capital in country 2 unless the yield on investment in country 2 evaluated in country 1's currency exceeds that evaluated in country 2's currency plus the expected rate of depreciation of country 2's currency. Hence, investments by firms of country 1 will not be implemented as initially expected before the countermeasure was taken.

There is another type of countermeasure that has the same effect as the countermeasure discussed, and it may be implementable even under a fixed exchange rate. It has been assumed hitherto that there is no overall inflation or deflation in both countries. However, if country 2 can manage to reduce its overall inflation rate (e.g., produce deflation), the currency of country 2 in essence depreciates even under a fixed exchange rate. However, households of country 2 cannot manipulate overall inflation or deflation by themselves. Therefore, this type of countermeasure will not be implementable by households alone. A central bank and government may, however, use this countermeasure, for example, by deliberately contracting domestic demand by raising nominal interest rates under a fixed exchange rate.

3.2.3 The balanced growth path

The depreciation of country 2's currency makes the initially expected future economic path of country 1 impossible to actualize because country 1 cannot satisfy equation (3), which is an optimality condition when country 1 behaves unilaterally. Because $\int_0^t \tau_s ds$ evaluated in the currency of country 1 does not accumulate as initially planned because of the depreciation, equation (3) cannot be achieved; that is, the following inequality

$$\lim_{t \to \infty} \frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} < \left(\frac{\varpi \alpha}{m v}\right)^{\alpha} (1-\alpha)^{1-\alpha}$$

that is evaluated in the currency of country 1, is always satisfied.

Faced with the country 2's countermeasure, households of country 1 may adjust their consumption plan to achieve unilateral optimality again by resetting their initial level of consumption to a lower level. However, households of country 2 will then reset their own initial level of consumption. As a result, if country 2 remains determined to continue to strengthen the countermeasure as long as country 1 behaves unilaterally, households of country 1 can never achieve unilateral optimality under floating exchange rates. To achieve optimality, households of country 1 have to renounce their unilateral behavior and agree to follow the path of sustainable heterogeneity. Unlike the case of a fixed exchange rate, therefore, the countermeasure is very effective under floating exchange rates and sustainable heterogeneity is naturally achieved.

4 DISCUSSION

Sustainable heterogeneity is competitively achieved at steady state and a balanced growth path under floating exchange rates also in multi-country models because, if a country behaves unilaterally, its currency is forced to appreciate by the same mechanism shown in Section 3 for a two-country model. However, international sustainable heterogeneity may not be clearly observed in the current account balances of many developing countries because generally they are not perfectly under floating exchange rates. In addition, many strict regulations in financial and other markets exist in these countries. In developed countries, however, the effect of international sustainable heterogeneity can be clearly observed.

4.1 Persistent current account deficits in the U.S.

Before the floating exchange rate system was introduced for the U.S. dollar (i.e., before the mid-1970s), the U.S. current account balance generally showed surpluses, although the surplus to GDP ratios were very small compared with the current account deficit to GDP ratio at present. After floating exchange rates were introduced, U.S. current account balances have continued to show large deficits. This trend is quite consistent with what the model discussed in Section 2 and the mechanism shown in Section 3 predict.

RTP has been observed to be negatively correlated to incomes (e.g., Lawrance, 1991);

thus, it is highly likely that RTP is negatively correlated to productivity. The U.S. has one of the highest productivities in the world, and it is therefore likely that its RTP is one of the lowest. Hence, the RTP of the U.S. will be lower than the RTPs of many of its trade partners. The model therefore predicts that the U.S. will experience large current account deficits because sustainable heterogeneity is the competitively achieved steady state (or balanced growth path) under floating exchange rates. In addition, it is highly likely that the DRA of the U.S. is lower than that of Japan, which is one of major trade partners of the U.S. (see Harashima, 2009c). Because of the difference in the DRA between the two countries, the model predicts that the U.S. will experience large current account deficits with Japan, which have in fact been observed.

The observed persistent large current account deficits of the U.S. are therefore a natural consequence of its low RTP relative to its many trade partners and low DRA to Japan under floating exchange rates. The deficits are justifiable, normal, and sustainable because international sustainable heterogeneity is achieved.

The U.K., Canada, and Australia have also generally experienced current account deficits since the 1980s. The economic and cultural characteristics of these countries are very similar to those of the U.S. Hence, it is likely that they also have similar preferences and levels of productivity. The persistent current account deficits observed in these countries are therefore also consistent with the model's predictions.

According to the model, the current account surpluses to GDP ratios of the U.S. before the floating exchange rate system was introduced should have been much larger than those that were actually observed in the 1950s and 1960s because sustainable heterogeneity would not hold. During that period, U.S. households may not have behaved strictly unilaterally, but a more likely reason for the smaller than predicted surpluses is that international trade and finance were strictly regulated by governments and thus current account balances were biased in many countries during that period. In addition, domestic financial markets were also strictly regulated. Since the 1970s, however, many of those regulations have been lifted in most developed countries.

4.2 Persistent current account surpluses of the eurozone

The overall current account balance of the Eurozone countries has shown surpluses except a few years after the creation of euro and a few years after the global financial crisis of 2008. Recently, surpluses have markedly increased. The eurozone consists of countries with various kinds of traditions, cultures, and levels of productivity. It includes some of the highest productivity countries in the world as well as some countries with relatively lower levels of productivity. The average productivity of the eurozone therefore is lower than that of the U.S.; thus, the average RTP of households in the eurozone will be higher than that of U.S. households. Hence, the observed current account surpluses of the eurozone are consistent with the predictions of the model discussed in Section 2 and the mechanism in Section 3. In this respect, persistent surpluses in the eurozone are not necessarily unjustifiable but may be a natural and normal consequence of its relatively higher RTP as compared with that of the U.S.

Note that three non-eurozone Nordic countries (Norway, Sweden, and Demark) are some of the wealthiest countries and thus likely to have the highest productivity in the world. Therefore, they are likely to have the lowest RTP in the world, but they all have experienced large current account surpluses since the late 1990s. These surpluses appear to be inconsistent with the model's predictions. However, it is likely that these surpluses were affected by Norway's large increases in oil exports since the late 1990s. Oil exports consist of more than half of Norway's exports, and Norway is the major trade partner of neighboring Sweden and Demark. Because of this oil factor, the effect the model predicts may be unclear in the current account balances of these countries.

4.3 Persistent current account deficits of Greece

If a country is a member state of a monetary union, the countermeasure is meaningless within the union because being a member of a monetary union indicates that there is no exchange rate within the union. If relatively more advantaged member states (i.e., those with relatively lower RTP or DRA) behave unilaterally, sustainable heterogeneity within a monetary union is impossible.

Greece is a part of the eurozone, but its productivity will be lower than average in the Eurozone from data on GDP per capita; thus, it is highly likely that its RTP is higher than the average. Therefore, Greece is one of relatively less advantaged member states in the eurozone. If relatively more advantaged member states behave unilaterally, Greece will experience large current account deficits and eventually fall into a situation where all capital is owned by relatively more advantaged member states, as Becker (1980) discussed. The observed persistent large current account deficits of Greece since the euro's creation are consistent with the model predictions. The deficits of Greece therefore should be judged as unjustifiable imbalances resulting from the unilateral behavior of households in relatively more advantaged member states.

The persistent current account deficits of Greece signify the inadequacy of the countermeasure discussed in this paper in a euro-like monetary union. To achieve sustainable heterogeneity within a monetary union, some additional mechanism is needed, for example, systematic fiscal transfers from more advantaged member states to less advantaged ones.

Harashima (2011, 2015) showed another source of imbalances within member states of the eurozone. Because the European central bank cannot separately control the behaviors of each of the member states' governments, inflation differentials and current account imbalances are generated. Unless a proper mechanism of fiscal transfers among member states is introduced, the economies of relatively less advantaged states are eventually devastated. Therefore, Greece is suffering from the two unilateral behaviors. One is the unilateral behavior of households of relatively more advantaged member states (by the mechanism shown in this paper), and the other is the unilateral behavior of governments of relatively more advantaged member states that refuse to introduce a systematic fiscal transfer mechanism.

4.4 Persistent current account surpluses of Japan

Japan has experienced persistent and large current account surpluses since the 1980s. As discussed in Section 4.1, the average DRA of the Japanese is considerably higher than that of people in the U.S and many other countries (see Harashima, 2009c). Because of this, the model predicts that, under floating exchange rates, Japan will experience persistent large current account surpluses (see also Harashima, 2009c). On the other hand, the RTP of Japan may be almost as low as that of the U. S. The low RTP means that Japan may experience persistent large current account deficits like the U.S. The observed persistent large current account surpluses imply that the effect of the high DRA is much larger than that of the low RTP.

4.5 Persistent current account surpluses of China

China's current account balances have shown large surpluses since the 1990s. China's productivity is clearly lower than those of most developed countries from data on GDP per capita. Therefore, China's RTP will be higher than those of most developed countries. The model indicates that, if households of China use the countermeasure, persistent large current account surpluses will be observed (see also Harashima, 2009b). However, unlike the U.S., countries in the eurozone, Japan, and other developed countries, it may be difficult to simply apply the model to the current account balance of China because it is markedly different from these developed countries in several important aspects: (1) China has a socialist market economy, (2) it is a developing country, and (3) it is a non-democratic country.

With these unique features, the persistent and large current account surpluses of China may be generated by different mechanisms from the one the model describes. First, China's exchange rate system is a managed system that is largely controlled by the Chinese government. If a currency is managed to be kept artificially and persistently at a lower level than the level that would be generated competitively in markets, then current account balances will show persistent surpluses.

In addition, because China is a socialist market economy, strict regulations in financial and many other markets exist. International transactions are also strictly regulated so it is not a typical market-oriented economy. Demand and supply can be controlled by the government to some extent so they are not necessarily realized fully competitively through markets. Therefore, the current account balances of China may not necessarily correctly reflect competitive equilibria in markets. In particular, many regulations in China are designed to encourage exports and discourage imports of consumption goods. As a result, the persistently large current account surpluses may have been artificially generated.

State-owned enterprises are also dominant in the Chinese economy. Their activities are particularly important because investment makes up about 50% of China's GDP (whereas consumption makes up only about 40% of GDP), and many investments are undertaken by state-owned enterprises. In many industrialized economies, in contrast, investment and consumption consist of about 20% and 60% of GDP, respectively.[2] This feature implies that it is the state-owned enterprises and not households that are implementing the countermeasure through investments. They may purchase domestic investment goods to the extent possible and produce export goods without considering domestic household demand. Even if households dislike or complain about this behavior, they may be silenced by the Chinese government. In that sense, state-owned enterprises of China may be behaving "patriotically", as discussed previously, and this will have the same effect as when households implement the countermeasure. As a result, persistently large current account surpluses may have been generated.

Whatever the reason for the creation of the surpluses, an important point is that the current account surpluses of China have generally been tolerated internationally. Some critics, however, and particularly those in the U.S., have criticized some of China's actions as being unfair. That said, no severe international sanctions have been imposed on China, which may mean that China's persistently large surpluses are implicitly regarded to be consistent with international sustainable heterogeneity.

4.5 Current account balances in developing countries

4.5.1 An insufficient countermeasure

As discussed in Section 3.2.1, implementing the countermeasure indicates that rationality prevails over preference. Conversely, if rationality cannot sufficiently prevail over preference, the effect of the countermeasure will be limited. For example, country 2's households may not resist buying goods and services even if they aware of the ensuing eventual non-optimal state. If the countermeasure is insufficient, the currency will not depreciate enough to stop the unilateral behavior of the households of country 1.

Productivities of developing countries are generally lower than those of developed countries, and their RTPs will therefore generally be higher. If a developing country's countermeasure is insufficient, its current account balances will show accelerating deficits because sustainable heterogeneity has not been achieved.

4.5.2 Borrowing from foreign countries

If the government of country 2 (a developing country) borrows a huge amount of money from

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[2] United Nations, *National Accounts Main Aggregates Database*
http://unstats.un.org/unsd/snaama/Introduction.asp

country 1 (a developed country) in the currency of country 1, the government of country 2 will not want its currency to depreciate because depreciation will make it more expensive to pay back country 1 in the currency of country 1. Therefore, the government of country 2 may hinder its households from implementing the countermeasure. For example, the government (together with the central bank) of country 2 may keep interest rates much higher than usual to prevent the currency from depreciating. The households of country 1 can therefore behave unilaterally without anxiety, and households of country 2 have to endure eventual non-optimality. As a result, the current account balances of country 2 will show accelerating deficits. A default on the loan by the government of country 2 or debt forgiveness by country 1 may be the only practical ways to resolve the problem for country 2.

4.5.3 The impact of regulation

In most developed countries, many regulations in financial markets have been lifted or greatly reduced since the 1980s. Therefore, current account balances will basically reflect increasingly unbiased transactions among households and firms. However, in many developing countries, there are still strict regulations in essential economic activities and international transactions. As a result, current account balances as well as the behaviors of households and firms are biased by the regulations. Hence, the model predictions may not be clearly observed in the current account balances of those countries.

4.5.4 Raw materials

In many developing countries, the main exports are raw materials. An important feature of raw materials is large fluctuations in their international prices. Current account balances in countries whose main exports are raw materials are greatly affected by those fluctuations. When prices are relatively high, current account balances will generally show surpluses, but if prices fall, deficits will often be observed. Therefore, in such countries, the mechanism of the model may not be clearly distinguished in the current account balances.

Note that if a relatively small country can enjoy some kind of economic rent in a broad sense such as being oil-rich or having some geopolitical or tax advantages, the effect the model predicts will be also unclear in the current account balances regardless of whether it is a developing or developed country (e.g., Kuwait, Norway, Singapore, Switzerland).

5 CONCLUDING REMARKS

If there is no sustainable heterogeneity across countries, current account balances should be stationary with a zero mean in any country. However, current account balances actually often show persistent deficits or surpluses in many countries. Some heterogeneity must have played an important role in these imbalances. Harashima (2009b, 2009c, 2010) showed that heterogeneity in preferences across countries can generate persistent current account imbalances at steady state and on a balanced growth path. Under sustainable heterogeneity, the current account balances of countries with relatively less patient or risk averse households show persisting deficits at steady state and on a balanced growth path and vice versa. However, sustainable heterogeneity is not naturally achieved.

Here, I have presented a mechanism by which international sustainable heterogeneity can be naturally achieved if exchange rates are allowed to float because of a countermeasure used by households in the less advantaged country. The countermeasure is to (1) reduce the overall level of consumption and (2) reduce the level of consumption of products imported from more advantaged countries relative to the initially expected amounts. With the implementation of the countermeasure, sustainable heterogeneity can be achieved and persistent current account deficits or surpluses are generated at a competitively achieved steady state (or balanced growth path). This countermeasure is the only feasible measure for less advantaged countries' households to escape from a non-optimal state. Therefore, implementing this countermeasure is fully rational and optimal for households of less advantaged countries.

The observed persistent current account deficits or surpluses in recent decades in the world seem to be basically consistent with the predictions of the model with some notable exceptions.

APPENDIX

A1 The model

A1.1 The base model

In this paper, sustainability of heterogeneity is examined in the framework of endogenous growth, but most endogenous growth models commonly have problems with scale effects or the influence of population growth (e.g., Jones, 1995a, b). Hence, this paper uses the model presented by Harashima (2010, 2013), which is free from both problems (see also Jones, 1995a; Aghion and Howitt, 1998; Peretto and Smulders, 2002). The production function is $Y_t = F(A_t, K_t, L_t)$, and the accumulation of capital is

$$\dot{K}_t = Y_t - C_t - v\dot{A}_t \quad , \tag{A1}$$

where Y_t is outputs, A_t is technology, K_t is capital inputs, L_t is labor inputs, C_t is consumption, $v(>0)$ is a constant, and a unit of K_t and $\frac{1}{v}$ of a unit of A_t are equivalent: that is, they are produced using the same quantities of inputs. All firms are identical and have the same size, and for any period,

$$m = \frac{M_t}{L_t} \quad , \tag{A2}$$

where M_t is the number of firms, and $m(>0)$ is a constant. In addition,

$$\frac{\partial Y_t}{\partial K_t} = \frac{\varpi}{M_t} \frac{\partial Y_t}{\partial (vA_t)} \quad ; \tag{A3}$$

thus,

$$\frac{\partial y_t}{\partial k_t} = \frac{\varpi}{mv} \frac{\partial y_t}{\partial A_t} \tag{A4}$$

is always kept, where y_t is output per capita, k_t is capital per capita, and $\varpi(>1)$ is a constant. For simplicity, the period of patent is assumed to be indefinite, and no capital depreciation is assumed. ϖ indicates the effect of patent protection. With patents, the income is distributed to not only capitals and labors but technologies. Equation (A2) indicates that population and number of firms are positively correlated. Equations (A3) and (A4) indicate that returns on investing in K_t and in A_t are kept equal and that a firm that produces a new technology cannot obtain all the returns on an investment in A_t. This means that investing in A_t increases Y_t, but the investing firm's return on the investment in A_t is only a fraction of the increase of Y_t, such that $\frac{\varpi}{M_t} \frac{\partial Y_t}{\partial (vA_t)} = \frac{\varpi}{mL_t} \frac{\partial Y_t}{\partial (vA_t)}$ because of uncompensated knowledge spillovers to other firms and complementarity of technologies.

A part of the knowledge generated as a result of an investment made by a firm spills over to other firms. Researchers in firms as well as universities and research institutions could not effectively generate innovations if they were isolated from other researchers. They contact and stimulate each other. Probably, mutual partial knowledge spillovers among researchers and firms give each other reciprocal benefits. Researchers take hints on their researches in exchange for spilled knowledge. Therefore, even though the investing firm wishes to keep its knowledge secret, some parts of it will spill over. In addition, many uncompensated knowledge spillovers occur because many technologies are regarded as so minor that they are not applied for patents and left unprotected by patents. Nevertheless, even if a technology that was generated as a byproduct is completely useless for the investing firm, it may be a treasure for firms in a different industry. A_t includes all these technologies, and an investment in technology generates many technologies that the investing firm cannot protect by patents.

Broadly speaking, there are two types of uncompensated knowledge spillovers: intra-sectoral knowledge spillovers (i.e., Marshall-Arrow-Romer [MAR] externalities; Marshall, 1890; Arrow, 1962; Romer, 1986) and inter-sectoral knowledge spillovers (i.e., Jacobs externalities; Jacobs, 1969). MAR theory assumes that knowledge spillovers between homogenous firms work out most effectively and that spillovers will therefore primarily emerge within one sector. As a result, uncompensated knowledge spillovers will be more active if the number of firms within a sector is larger. On the other hand, Jacobs (1969) argues that knowledge spillovers are most effective among firms that practice different activities and that diversification (i.e., a variety of sectors) is important for spillovers. As a result, uncompensated knowledge spillovers will be more active if the number of sectors in the economy is larger. Nevertheless, if all sectors have the same number of firms, an increase in the number of firms in the economy results in more active knowledge spillovers in any case, owing to either MAR externalities or Jacobs externalities.

Furthermore, as the volume of uncompensated knowledge spillovers increases, the investing firm's returns on the investment in A_t decrease. $\frac{\partial Y_t}{\partial A_t}$ indicates the total increase in Y_t in the economy by an increase in A_t, which consists of increases in both outputs in the firm that invested in the new technologies and outputs in other firms that utilize the newly invented technologies, whether the firms obtained the technologies by compensating the originating firm or by using uncompensated knowledge spillovers. If the number of firms becomes larger and uncompensated knowledge spillovers occur more actively, the compensated fraction in $\frac{\partial Y_t}{\partial A_t}$ that the investing firm can obtain becomes smaller, and the investing firm's returns on the investment in A_t also become smaller.

Complementarity of technologies also reduces the fraction of $\frac{\partial Y_t}{\partial A_t}$ that the investing firm can obtain. If a new technology is effective only if it is combined with some particular technologies, the return on the investment in technology will belong not only to the investing firm but to the firms that hold these particular technologies. For example, an innovation in software technology generated by a software company increases the sales and profits of computer hardware companies. The economy's productivity increases because of the innovation but the increased incomes are attributed not only to the firm that generated the innovation but also to the firms that hold complementary technologies. A part of $\frac{\partial Y_t}{\partial A_t}$ leaks to these firms. For them, the leaked income is a kind of rent revenue unexpectedly become obtainable thanks to the innovation. Most new technologies will have complementary technologies. In addition, as the number of firms increases, the number of firms that holds complementary technologies will also increase, and thereby these leaks will also increase.

Because of the uncompensated knowledge spillovers and the complementarity of technologies, therefore, the fraction of $\frac{\partial Y_t}{\partial A_t}$ that an investing firm can obtain on average will be comparatively small, i.e., ϖ will be far smaller than M_t except that M_t is very small,[3] and the fraction will decrease as M_t increases.

The production function is specified as $Y_t = A_t^{\alpha} f(K_t, L_t)$ where α $(0 < \alpha < 1)$ is a constant. Let $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $c_t = \frac{C_t}{L_t}$, and $n_t = \frac{\dot{L}_t}{L_t}$, and assume that $f(K_t, L_t)$ is homogenous of degree one. Thus $y_t = A_t^{\alpha} f(k_t)$ and $\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t$. By equation (A4), $A_t = \frac{\varpi \alpha f(k_t)}{mvf'(k_t)}$ because $\frac{\varpi \partial y_t}{mv\partial A_t} = \frac{\partial y_t}{\partial k_t} \Leftrightarrow \frac{\varpi \alpha}{mv} A_t^{\alpha-1} f(k_t) = A_t^{\alpha} f'(k_t)$.

A1.2 Models with heterogeneous households

Three heterogeneities—heterogeneous time preference, risk aversion, and productivity—are examined in endogenous growth models, which are modified versions of the model shown in Section A1.1. First, suppose that there are two economies— economy 1 and economy 2—that are identical except for time preference, risk aversion, or productivity. The population growth rate is zero (i.e., $n_t = 0$). The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy.

Each economy can be interpreted as representing either a country (the international interpretation) or a group of identical households in a country (the national interpretation). Because the economies are fully open, they are integrated through trade and form a combined economy. The combined economy is the world economy in the international interpretation and the national economy in the national interpretation. In the following discussion, a model based on the international interpretation is called an international model and that based on the national interpretation is called a national model. Usually, the concept of the balance of payments is used only for the international transactions. However, because both national and international interpretations are possible, this concept and terminology are also used for the national models in this paper.

A1.2.1 Heterogeneous time preference model

First, a model in which the two economies are identical except for time preference is constructed.[4] The rate of time preference of the representative household in economy 1 is θ_1 and that in economy 2 is θ_2, and $\theta_1 < \theta_2$. The production function in economy 1 is $y_{1,t} = A_t^{\alpha} f(k_{1,t})$ and that in economy 2 is $y_{2,t} = A_t^{\alpha} f(k_{2,t})$, where $y_{i,t}$ and $k_{i,t}$ are, respectively, output and capital per capita in economy i in period t for i = 1, 2. The population of each economy is $\frac{L_t}{2}$; thus, the total for both is L_t, which is sufficiently large. Firms operate in both

[3] If M_t is very small, the value of ϖ will be far smaller than that for sufficiently large M_t, because the number of firms that can benefit from an innovation is constrained owing to very small M_t. The very small number of firms indicates that the economy is not sufficiently sophisticated, and thereby the benefit of an innovation can not be fully realized in the economy. This constraint can be modeled as $\varpi = \tilde{\varpi}\left[1 - \left(1 - \tilde{\varpi}^{-1}\right)^{M_t}\right]$ where $\tilde{\varpi}(\geq 1)$ is a constant. Nevertheless, for sufficiently large M_t (i.e., in sufficiently sophisticated economies), the constraint is removed such that $\lim_{M_t \to \infty} \tilde{\varpi}\left[1 - \left(1 - \tilde{\varpi}^{-1}\right)^{M_t}\right] = \tilde{\varpi} = \varpi$.

[4] This type of endogenous growth model of heterogeneous time preference was originally shown by Harashima (2009b).

economies, and the number of firms is M_t. The current account balance in economy 1 is τ_t and that in economy 2 is $-\tau_t$. Because a balanced growth path requires Harrod neutral technological progress, the production functions are further specified as

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha} \quad ;$$

thus, $Y_{i,t} = K_{i,t}^{1-\alpha} \left(A_t L_t\right)^{\alpha} \left(i = 1,2\right)$.

Because both economies are fully open, returns on investments in each economy are kept equal through arbitration such that

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\varpi}{2mv} \frac{\partial \left(y_{1,t} + y_{2,t}\right)}{\partial A_t} = \frac{\partial y_{2,t}}{\partial k_{2,t}} \; . \tag{A5}$$

Equation (A5) indicates that an increase in A_t enhances outputs in both economies such that $\frac{\partial Y_{i,t}}{\partial K_{i,t}} = \frac{\varpi}{M_t} \frac{\partial \left(Y_{1,t} + Y_{2,t}\right)}{\partial \left(vA_t\right)}$, and because the population is equal ($\frac{L_t}{2}$), $\frac{\partial Y_{i,t}}{\partial K_{i,t}} = \frac{\partial y_{i,t}}{\partial k_{i,t}} = \frac{\varpi}{M_t} \frac{\partial \left(Y_{1,t} + Y_{2,t}\right)}{\partial \left(vA_t\right)} = \frac{\varpi}{mL_t} \frac{\partial \left(y_{1,t} + y_{2,t}\right)}{\partial \left(vA_t\right)} \frac{L_t}{2} = \frac{\varpi}{2mv} \frac{\partial \left(y_{1,t} + y_{2,t}\right)}{\partial A_t}$. Therefore,

$$A_t = \frac{\varpi \alpha \left[f\left(k_{1,t}\right) + f\left(k_{2,t}\right) \right]}{2mvf'\left(k_{1,t}\right)} = \frac{\varpi \alpha \left[f\left(k_{1,t}\right) + f\left(k_{2,t}\right) \right]}{2mvf'\left(k_{2,t}\right)} \quad .$$

Because equation (A5) is always held through arbitration, equations $k_{1,t} = k_{2,t}$, $\dot{k}_{1,t} = \dot{k}_{2,t}$, $y_{1,t} = y_{2,t}$ and $\dot{y}_{1,t} = \dot{y}_{2,t}$ are also held. Hence,

$$A_t = \frac{\varpi \alpha f\left(k_{1,t}\right)}{mvf'\left(k_{1,t}\right)} = \frac{\varpi \alpha f\left(k_{2,t}\right)}{mvf'\left(k_{2,t}\right)} \quad .$$

In addition, because $\frac{\partial \left(y_{1,t} + y_{2,t}\right)}{\partial A_{1,t}} = \frac{\partial \left(y_{1,t} + y_{2,t}\right)}{\partial A_{2,t}}$ through arbitration, then $\dot{A}_{1,t} = \dot{A}_{2,t}$ is held.

The accumulated current account balance $\int_0^t \tau_s \, ds$ mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}} \right)$ are returns on investments, $\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds$ and $\frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$ represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance on goods and services of economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds - \tau_t$$

is that of economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that

$$\tau_t = g(k_{1,t}, k_{2,t}) \quad .$$

The representative household in economy 1 maximizes its expected utility

$$E\int_0^{\infty} u_1(c_{1,t})\exp(-\theta_1 t)dt \quad ,$$

subject to

$$\dot{k}_{1,t} = y_{1,t} + \frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s \, ds - \tau_t - c_{1,t} - v\dot{A}_{1,t}\left(\frac{L_t}{2}\right)^{-1} \quad , \tag{A6}$$

and the representative household in economy 2 maximizes its expected utility

$$E\int_0^{\infty} u_2(c_{2,t})\exp(-\theta_2 t)dt \quad ,$$

subject to

$$\dot{k}_{2,t} = y_{2,t} - \frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds + \tau_t - c_{2,t} - v\dot{A}_{2,t}\left(\frac{L_t}{2}\right)^{-1} \quad , \tag{A7}$$

where $u_{i,t}$, $c_{i,t}$, and $\dot{A}_{i,t}$, respectively, are the utility function, per capita consumption, and the increase in A_t by R&D activities in economy i in period t for i = 1, 2; E is the expectation operator; and $\dot{A}_t = \dot{A}_{1,t} + \dot{A}_{2,t}$. Equations (A6) and (A7) implicitly assume that each economy does not have foreign assets or debt in period t = 0.

Because the production function is Harrod neutral and because $A_t = \frac{\varpi \alpha f(k_{1,t})}{mvf'(k_{1,t})} = \frac{\varpi \alpha f(k_{2,t})}{mvf'(k_{2,t})}$ and $f = k_{i,t}^{1-\alpha}$, then

$$A_t = \frac{\varpi\alpha}{mv(1-\alpha)} k_{i,t}$$

and

$$\frac{\partial y_{i,t}}{\partial k_{i,t}} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} \quad .$$

Since $\dot{A}_{1,t} = \dot{A}_{2,t}$ and $\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\partial y_{2,t}}{\partial k_{2,t}}$, then

$$\dot{k}_{1,t} = y_{1,t} + \frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{v\dot{A}_t}{2}\left(\frac{L_t}{2}\right)^{-1}$$
$$= \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} k_{1,t} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{\varpi\alpha}{mL_t(1-\alpha)}\dot{k}_{1,t}$$

and

$$\dot{k}_{1,t} = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} k_{1,t} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t - c_{1,t}\right] .$$

Because L_t is sufficiently large and ϖ is far smaller than M_t, the problem of scale effects vanishes and thereby $\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha} = 1$.

Putting the above elements together, the optimization problem of economy 1 can be rewritten as

$$Max\ E\int_0^\infty u_1(c_{1,t})\exp(-\theta_1 t)dt \quad ,$$

subject to

$$\dot{k}_{1,t} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} k_{1,t} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t - c_{1,t} \quad .$$

Similarly, that of economy 2 can be rewritten as

$$Max\ E\int_0^\infty u_2(c_{2,t})\exp(-\theta_2 t)dt \quad ,$$

subject to

$$\dot{k}_{2,t} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} k_{2,t} - \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds + \tau_t - c_{2,t} \quad .$$

A1.2.2 Heterogeneous risk aversion model

The basic structure of the model with heterogeneous risk aversion is the same as that of heterogeneous time preference. The two economies are identical except in regard to risk aversion.[5] The degree of relative risk aversion of economy 1 is $\varepsilon_1 = -\frac{c_{1,t}\, u_1''}{u_1'}$ and that of

[5] This type of endogenous growth model of heterogeneous risk aversion was originally shown by Harashima (2009c).

economy 2 is $\varepsilon_2 = -\frac{c_{2,t}\,u_2''}{u_2'}$, which are constant, and $\varepsilon_1 < \varepsilon_2$. The optimization problem of economy 1 is

$$Max\ E\int_0^\infty u_1(c_{1,t})\exp(-\theta t)dt \quad ,$$

subject to

$$\dot{k}_{1,t} = \left(\frac{\varpi\alpha}{mv}\right)^\alpha (1-\alpha)^{-\alpha} k_{1,t} + \left(\frac{\varpi\alpha}{mv}\right)^\alpha (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds - \tau_t - c_{1,t} \quad ,$$

and that of economy 2 is

$$Max\ E\int_0^\infty u_2(c_{2,t})\exp(-\theta t)dt \quad ,$$

subject to

$$\dot{k}_{2,t} = \left(\frac{\varpi\alpha}{mv}\right)^\alpha (1-\alpha)^{-\alpha} k_{2,t} - \left(\frac{\varpi\alpha}{mv}\right)^\alpha (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds + \tau_t - c_{2,t} \quad .$$

A1.2.3 Heterogeneous productivity model

With heterogeneous productivity, the production function is heterogeneous, not the utility function. Because technology A_t is common to both economies, a heterogeneous production function requires heterogeneity in elements other than technology. Prescott (1998) argues that unknown factors other than technology have made total factor productivity (TFP) heterogeneous across countries. Harashima (2009a) argues that average workers' innovative activities are an essential element of productivity and make TFP heterogeneous across workers, firms, and economies. Since average workers are human and capable of creative intellectual activities, they can create innovations even if their innovations are minor. It is rational for firms to exploit all the opportunities that these ordinary workers' innovative activities offer. Furthermore, innovations created by ordinary workers are indispensable for efficient production. A production function incorporating average workers' innovations has been shown to have a Cobb-Douglas functional form with a labor share of about 70% (Harashima 2009a), such that

$$Y_t = \bar{\sigma}\omega_A\omega_L A_t^\alpha K_t^{1-\alpha} L_t^\alpha \quad , \qquad \text{(A8)}$$

where ω_A and ω_L are positive constant parameters with regard to average workers' creative activities, and $\bar{\sigma}$ is a parameter that represents a worker's accessibility limit to capital with regard to location. The parameters ω_A and ω_L are independent of A_t but are dependent on the creative activities of average workers. Thereby, unlike with technology A_t, these parameters can be heterogeneous across workers, firms, and economies.

In this model of heterogeneous productivity, it is assumed that workers whose households belong to different economies have different values of ω_A and ω_L. In addition, only productivity that is represented by $\bar{\sigma}\omega_A\omega_L A_t^\alpha$ in equation (A8) is heterogeneous between the two economies. The production function of economy 1 is $y_{1,t} = \omega_1^\alpha A_t^\alpha f(k_{1,t})$ and that of economy 2 is $y_{2,t} = \omega_2^{\alpha} A_t^{\alpha} f(k_{2,t})$, where $\omega_1 (0 < \omega_1 \le 1)$ and $\omega_2 (0 < \omega_2 \le 1)$ are constants and $\omega_2 < \omega_1$. Since $\frac{\partial Y_{i,t}}{\partial K_{i,t}} = \frac{\partial y_{i,t}}{\partial k_{i,t}} = M_t^{-1} \frac{\partial (Y_{1,t} + Y_{2,t})}{\partial (vA_t)} = \frac{\varpi}{mL_t} \frac{\partial (y_{1,t} + y_{2,t})}{\partial (vA_t)} \frac{L_t}{2} = \frac{\varpi}{2mL_t} \frac{\partial (y_{1,t} + y_{2,t})}{\partial A_t}$ by equation (A5), then

$$A_t = \frac{\varpi \alpha [\omega_1^{\alpha} f(k_{1,t}) + \omega_2^{\alpha} f(k_{2,t})]}{2mv\,\omega_1^{\alpha} f'(k_{1,t})} = \frac{\varpi \alpha [\omega_1^{\alpha} f(k_{1,t}) + \omega_2^{\alpha} f(k_{2,t})]}{2mv\,\omega_2^{\alpha} f'(k_{2,t})} \quad . \tag{A9}$$

Because equation (A5) is always held through arbitration, equations $k_{1,t} = \frac{\omega_1}{\omega_2} k_{2,t}$, $\dot{k}_{1,t} = \frac{\omega_1}{\omega_2} \dot{k}_{2,t}$, $y_{1,t} = \frac{\omega_1}{\omega_2} y_{2,t}$, and $\dot{y}_{1,t} = \frac{\omega_1}{\omega_2} \dot{y}_{2,t}$ are also held. In addition, since $\frac{\partial (y_{1,t} + y_{2,t})}{\partial A_{1,t}} = \frac{\partial (y_{1,t} + y_{2,t})}{\partial A_{2,t}}$ by arbitration, $\dot{A}_{1,t} = \frac{\omega_1}{\omega_2} \dot{A}_{2,t}$ is held. Because of equation (A9) and $f = \omega_i^{\alpha} k_{i,t}^{1-\alpha}$, then $A_t =$ $\frac{\varpi \alpha}{2mv(1-\alpha)\omega_1^{\alpha}} \left(\omega_1^{\alpha} k_1 + \omega_2^{\alpha} k_1^{\alpha} k_2^{1-\alpha}\right) = \frac{\varpi \alpha}{2mv(1-\alpha)\omega_2^{\alpha}} \left(\omega_1^{\alpha} k_1^{1-\alpha} k_2^{\alpha} + \omega_2^{\alpha} k_2\right)$, $\frac{\omega_1^{\alpha} k_1 + \omega_2^{\alpha} k_1^{\alpha} k_2^{1-\alpha}}{\omega_1^{\alpha}} = \frac{\omega_1^{\alpha} k_1^{1-\alpha} k_2^{\alpha} + \omega_2^{\alpha} k_2}{\omega_2^{\alpha}}$, and $\frac{\partial y_{i,t}}{\partial k_{i,t}} = \left(\frac{\varpi \alpha}{2mv}\right)^{\alpha} (1-\alpha)^{1-\alpha} \left(\omega_1^{\alpha} k_1 + \omega_2^{\alpha} k_1^{\alpha} k_2^{1-\alpha}\right)^{\alpha} k_1^{-\alpha} = \left(\frac{\varpi \alpha}{2mv}\right)^{\alpha} (1-\alpha)^{1-\alpha} \left(\omega_1^{\alpha} k_1^{1-\alpha} k_2^{\alpha} + \omega_2^{\alpha} k_2\right)^{\alpha} k_2^{-\alpha}$. Since $\frac{\omega_2}{\omega_1} k_{1,t} = k_{2,t}$, then $\frac{\omega_1^{\alpha} k_1 + \omega_2^{\alpha} k_1^{\alpha} k_2^{1-\alpha}}{\omega_1^{\alpha}} = \frac{\omega_1^{\alpha} k_1 + \omega_2^{\alpha} k_1^{\alpha} \left(\frac{\omega_2}{\omega_1}\right)^{1-\alpha} k_1^{1-\alpha}}{\omega_1^{\alpha}} = k_1 \left(1 + \omega_1^{-1} \omega_2\right)$ and $\frac{\omega_1^{\alpha} k_1^{1-\alpha} k_2^{\alpha} + \omega_2^{\alpha} k_2}{\omega_2^{\alpha}} = \frac{\omega_1^{\alpha} k_1^{1-\alpha} \left(\frac{\omega_2}{\omega_1}\right)^{\alpha} k_1^{\alpha} + \omega_2^{\alpha} \frac{\omega_2}{\omega_1} k_1}{\omega_2^{\alpha}} = k_1 + \frac{\omega_2}{\omega_1} k_1 = k_1 \left(1 + \omega_1^{-1} \omega_2\right) = k_2 \left(1 + \omega_1 \omega_2^{-1}\right)$. Hence,

$$A_t = k_1 \frac{\varpi \alpha \left(1 + \omega_1^{-1} \omega_2\right)}{2mv(1-\alpha)} = k_2 \frac{\varpi \alpha \left(1 + \omega_1 \omega_2^{-1}\right)}{2mv(1-\alpha)} \quad ,$$

and

$$\frac{\partial y_{i,t}}{\partial k_{i,t}} = \left(\frac{\omega_1 + \omega_2}{2}\right)^{\alpha} \left(\frac{\varpi \alpha}{mv}\right)^{\alpha} (1-\alpha)^{1-\alpha}$$

for i = 1, 2. Because $\dot{A}_{1,t} = \left(\frac{\omega_2}{\omega_1}\right)^{-\frac{1}{\alpha}} \dot{A}_{2,t}$ (i.e., $\dot{A}_t = \dot{A}_{1,t} + \dot{A}_{2,t} = \left(1 + \omega_1^{-1} \omega_2\right) \dot{A}_{1,t}$) and $\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\partial y_{2,t}}{\partial k_{2,t}}$, then

$$\dot{k}_{1,t} = y_{1,t} + \frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t - c_{1,t} - v\dot{A}_{1,t} \left(\frac{L_t}{2}\right)^{-1}$$

$$= y_{1,t} + \frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t - c_{1,t} - v\dot{A}_t \left(1 + \omega_1^{-1}\omega_2\right)^{-1} \left(\frac{L_t}{2}\right)^{-1}$$

$$= \omega_1^{\alpha} \left[\frac{\left(1+\omega_1^{-1}\omega_2\right)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha} k_{1,t} + \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{\varpi\alpha}{mL_t(1-\alpha)} \dot{k}_{1,t} \quad ,$$

and

$$\dot{k}_{1,t} = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha} \left\{ \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha} k_{1,t} + \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds - \tau_t - c_{1,t} \right\} \quad .$$

Because L_t is sufficiently large and ϖ is far smaller than M_t and thus $\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\varpi\alpha} = 1$, the optimization problem of economy 1 is

$$Max \; E\int_0^{\infty} u_1\left(c_{1,t}\right)\exp(-\theta t)dt \quad ,$$

subject to

$$\dot{k}_{1,t} = \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha} k_{1,t} + \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds - \tau_t - c_{1,t} \quad ,$$

and similarly, that of economy 2 is

$$Max \; E\int_0^{\infty} u_2\left(c_{2,t}\right)\exp(-\theta t)dt \quad ,$$

subject to

$$\dot{k}_{2,t} = \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha} k_{2,t} - \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds + \tau_t - c_{2,t} \quad .$$

A2 Sustainability of heterogeneity

Heterogeneity is defined as being sustainable if all the optimality conditions of all heterogeneous households are satisfied indefinitely. Although the previously discussed state of Becker (1980) is Pareto efficient, by this definition, the heterogeneity is not sustainable because only the most patient household can achieve optimality. Sustainability is therefore the stricter criterion for welfare than Pareto efficiency.

In this section, the growth path that makes heterogeneity sustainable is examined. First, the basic natures of the models presented in Section A1 are examined and then sustainability is examined.

A2.1 The consumption growth rate

A2.1.1 Heterogeneous time preference model

Let Hamiltonian H_1 be

$$H_1 = u_1(c_{1,t})\exp(-\theta_1 t) + \lambda_{1,t}\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_{1,t} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t - c_{1,t}\right\} ,$$

where λ_{1t} is a costate variable. The optimality conditions for economy 1 are

$$\frac{\partial u_1(c_{1,t})}{\partial c_{1,t}}\exp(-\theta_1 t) = \lambda_{1,t} , \tag{A10}$$

$$\dot{\lambda}_{1,t} = -\frac{\partial H_1}{\partial k_{1,t}} , \tag{A11}$$

$$\dot{k}_{1,t} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_{1,t} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t - c_{1,t} \text{ , and} \tag{A12}$$

$$\lim_{t\to\infty}\lambda_{1,t}\, k_{1,t} = 0 . \tag{A13}$$

Similarly, let Hamiltonian H_2 be

$$H_2 = u_2(c_{2,t})\exp(-\theta_1 t) + \lambda_{2,t}\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_{2,t} - \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds + \tau_t - c_{2,t}\right\},$$

where λ_{2t} is a costate variable. The optimality conditions for economy 2 are

$$\frac{\partial u_2(c_{2,t})}{\partial c_{2,t}}\exp(-\theta_1 t) = \lambda_{2,t} , \tag{A14}$$

$$\dot{\lambda}_{2,t} = -\frac{\partial H_2}{\partial k_{2,t}} , \tag{A15}$$

$$\dot{k}_{2,t} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_{2,t} - \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s\, ds + \tau_t - c_{2,t}\text{, and} \tag{A16}$$

$$\lim_{t\to\infty}\lambda_{2,t}\, k_{2,t} = 0 . \tag{A17}$$

By equations (A10), (A11), and (A12), the consumption growth rate in economy 1 is

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \frac{\partial\tau_t}{\partial k_{1,t}} - \theta_1\right] , \tag{A18}$$

and by equations (A14), (A15), and (A16), that in economy 2 is

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}} + \frac{\partial\tau_t}{\partial k_{2,t}} - \theta_2\right] , \tag{A19}$$

where $\varepsilon = -\frac{c_{1,t}\, u_1''}{u_1'} = -\frac{c_{2,t}\, u_2''}{u_2'}$ is the degree of relative risk aversion, which is constant. A constant growth rate such that $\frac{\dot{c}_{1,t}}{c_{1,t}} = \frac{\dot{c}_{2,t}}{c_{2,t}}$ is possible if

$$\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left[\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} + \frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right] - \left(\frac{\partial \tau_t}{\partial k_{1,t}} + \frac{\partial \tau_t}{\partial k_{2,t}}\right) = \theta_1 - \theta_2 \qquad \text{(A20)}$$

is satisfied.

A2.1.2 Heterogeneous risk aversion model

By using similar procedures as were used with the heterogeneous time preference model, the consumption growth rate in economy 1 in this model is

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon_1^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}} - \theta\right] , \qquad \text{(A21)}$$

and that in economy 2 is

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon_2^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}} + \frac{\partial \tau_t}{\partial k_{2,t}} - \theta\right] . \qquad \text{(A22)}$$

A constant growth rate such that $\frac{\dot{c}_{1,t}}{c_{1,t}} = \frac{\dot{c}_{2,t}}{c_{2,t}}$ is possible if

$$\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left[\varepsilon_2\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} + \varepsilon_1\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right] + (\varepsilon_2 - \varepsilon_1)\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right]$$
$$= \varepsilon_2\frac{\partial \tau_t}{\partial k_{1,t}} + \varepsilon_1\frac{\partial \tau_t}{\partial k_{2,t}} \qquad \text{(A23)}$$

is satisfied.

A2.1.3 Heterogeneous productivity model

By similar procedures, the consumption growth rate in economy 1 in this model is

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha} + \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}} - \theta\right\} , \qquad \text{(A24)}$$

and that in economy 2 is

$$\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}-\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{2,t}}+\frac{\partial\tau_t}{\partial k_{2,t}}-\theta\right\} \quad . \tag{A25}$$

A constant growth rate such that $\frac{\dot{c}_{1,t}}{c_{1,t}}=\frac{\dot{c}_{2,t}}{c_{2,t}}$ is possible if

$$\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}(1-\alpha)\left(\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}+\frac{\partial\int_0^t\tau_s ds}{\partial k_{2,t}}\right)=\frac{\partial\tau_t}{\partial k_{1,t}}+\frac{\partial\tau_t}{\partial k_{2,t}} \tag{A26}$$

is satisfied.

A2.2 Transversality conditions

A2.2.1 Heterogeneous time preference model

Transversality conditions are satisfied if the following conditions are satisfied.

Lemma 1-1: In the model of heterogeneous time preference, unless $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}<-1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}<-1$, the transversality conditions (equations [A13] and [A17]) are satisfied if

$$\lim_{t\to\infty}\left\{\left(\frac{\partial\tau_t}{\partial k_{1,t}}-\frac{\tau_t}{k_{1,t}}\right)-\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left[\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1,t}}-\frac{\int_0^t\tau_s ds}{k_{1,t}}\right]-\frac{c_{1,t}}{k_{1,t}}\right\}<0 \tag{A27}$$

and

$$\lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2,t}}-\frac{\partial\tau_t}{\partial k_{2,t}}\right)-\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left[\frac{\int_0^t\tau_s ds}{k_{2,t}}-\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2,t}}\right]-\frac{c_{2,t}}{k_{2,t}}\right\}<0 \quad . \tag{A28}$$

Proof: See Harashima (2010).

A2.2.2 Heterogeneous risk aversion model

Lemma 1-2: In the model of heterogeneous risk aversion, unless $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}<-1$,

$\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}<-1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}<-1$, the transversality conditions are satisfied if

$$\lim_{t\to\infty}\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}\left[\frac{\int_0^t\tau_s ds}{k_{1,t}}-\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1,t}}\right]-\left(\frac{\tau_t}{k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}\right)-\frac{c_{1,t}}{k_{1,t}}\right\}<0$$

and

$$-\lim_{t\to\infty}\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}\left[\frac{\int_0^t\tau_s ds}{k_{2,t}}-\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2,t}}\right]-\left(\frac{\tau_t}{k_{2,t}}-\frac{\partial\tau_t}{\partial k_{2,t}}\right)+\frac{c_{2,t}}{k_{2,t}}\right\}<0 \cdot$$

A2.2.3 Heterogeneous productivity model

Lemma 1-3: In the model of heterogeneous productivity, unless $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}<-1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}<-1$, the transversality conditions are satisfied if

$$\lim_{t\to\infty}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{\int_0^t\tau_s ds}{k_{1,t}}-\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}\right)-\left(\frac{\tau_t}{k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}\right)-\frac{c_{1,t}}{k_{1,t}}\right\}<0$$

and

$$-\lim_{t\to\infty}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{\int_0^t\tau_s ds}{k_{2,t}}-\frac{\partial\int_0^t\tau_s ds}{\partial k_{2,t}}\right)-\left(\frac{\tau_t}{k_{2,t}}-\frac{\partial\tau_t}{\partial k_{2,t}}\right)+\frac{c_{2,t}}{k_{2,t}}\right\}<0 \quad \cdot$$

In all three models, the occurrence of $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}<-1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}<-1$ is extremely unusual, and these cases are excluded in the following discussion.

A2.3 Sustainability

Because balanced growth is the focal point for the growth path analysis, the following analyses focus on the steady state such that $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$, $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}$, $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}$, and $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}$ are constants.

A2.3.1 Heterogeneous time preference model

The balanced growth path in the heterogeneous time preference model has the following properties.

Lemma 2-1: In the model of heterogeneous time preference, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, then

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t\tau_s ds\right)}{dt}}{\int_0^t\tau_s ds} \ .$$

Proof: See Harashima (2010).

Proposition 1-1: In the model of heterogeneous time preference, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ $=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$constant, all the optimality conditions of both economies are satisfied at steady state.

Proof: By Lemma 2-1, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant,

$$\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\Xi \quad ,$$

where Ξ is a constant. In addition, because $\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t\tau_s ds\right)}{dt}}{\int_0^t\tau_s ds}=\lim_{t\to\infty}\frac{\tau_t}{\int_0^t\tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$,

$$\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}}=\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{2,t}}=\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1} \ .$$

Thus, $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{2,t}}$ and $\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}}=\lim_{t\to\infty}\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}=$ $\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{2,t}}=\lim_{t\to\infty}\frac{\partial\int_0^t\tau_s ds}{\partial k_{2,t}}$, and

$$\lim_{t\to\infty}\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}\left[\frac{\int_0^t \tau_s ds}{k_{1,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}\right]-\left(\frac{\tau_t}{k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right)-\frac{c_{1,t}}{k_{1,t}}\right\}=-\lim_{t\to\infty}\frac{c_{1,t}}{k_{1,t}}<0 \quad ,$$

and

$$-\lim_{t\to\infty}\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}\left[\frac{\int_0^t \tau_s ds}{k_{2,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right]-\left(\frac{\tau_t}{k_{2,t}}-\frac{\partial \tau_t}{\partial k_{2,t}}\right)+\frac{c_{2,t}}{k_{2,t}}\right\}=-\lim_{t\to\infty}\frac{c_{2,t}}{k_{2,t}}<0 \quad .$$

Hence, by Lemma 1-1, the transversality conditions are satisfied while all the other optimality conditions are also satisfied.

On the other hand, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} \neq \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, then $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} \neq \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$. Thus, by Lemma 1-1, for both economies to satisfy the transverality conditions, it is necessary that $\lim_{t\to\infty}\frac{c_{1,t}}{k_{1,t}}=\infty$ or $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{k_{2,t}}=\infty$, which violates equation (A12) or (A16). ■

The path on which $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant has the following properties.

Corollary 1-1: In the model of heterogeneous time preference, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, then

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\text{constant.}$$

Proof: See Harashima (2010).

Note that the limit of the growth rate on this path is

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\frac{\theta_1+\theta_2}{2}\right] \qquad \text{(A29)}$$

by equations (A18) and (A19).

Corollary 2-1: In the model of heterogeneous time preference, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=$

$\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant,

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}$$

$$=\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\text{constant.} \tag{A30}$$

Proof: By Lemma 2-1, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds}$. Therefore, by Corollary 1-1, equation (A30) holds. ■

Because current account imbalances eventually grow at the same rate as output, consumption, and capital on the multilateral path, the ratios of the current account balance to output, consumption, and capital do not explode, but they stabilize as shown in the proof of Proposition 1-1; that is, $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\Xi$.

On the balanced growth path satisfying Proposition 1-1 and Corollaries 1-1 and 2-1, heterogeneity in time preference is sustainable by definition because all the optimality conditions of the two economies are indefinitely satisfied. The balanced growth path satisfying Proposition 1-1 and Corollaries 1-1 and 2-1 is called the "multilateral balanced growth path" or (more briefly) the "multilateral path" in the following discussion. The term "multilateral" is used even though there are only two economies, because the two-economy models shown can easily be extended to the multi-economy models shown in Section A2.6.

Because technology will not decrease persistently (i.e., $\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}>0$), only the case such that $\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}>0$ (i.e., $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}>0$ on the multilateral path by Corollary 1-1) is examined in the following discussion.

A2.3.2 Heterogeneous risk aversion model

On the multilateral path in the heterogeneous risk aversion model, the same Proposition, Lemmas, and Corollaries are proved by arguments similar to those shown in Section A2.3.1.

Lemma 2-2: In the model of heterogeneous risk aversion, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}.$$

Proposition 1-2: In the model of heterogeneous risk aversion, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, all the optimality conditions of both economies are satisfied at steady state.

Corollary 1-2: In the model of heterogeneous risk aversion, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} = \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} = \text{constant.}$$

Corollary 2-2: In the model of heterogeneous risk aversion, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant,

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}$$
$$= \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} = \text{constant.}$$

On the balanced growth path satisfying Proposition 1-2 and Corollaries 1-2 and 2-2, heterogeneity in risk aversion is also sustainable by definition because all the optimality conditions of the two economies are indefinitely satisfied, and this path is the multilateral path.

A2.3.3 Heterogeneous productivity model

Similar Proposition, Lemmas, and Corollaries also hold in the heterogeneous productivity model. However, unlike heterogeneous preferences, $\lim_{t\to\infty}\tau_t = 0$ and $\lim_{t\to\infty}\int_0^t \tau_s ds = 0$ are possible even if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$ as equations (A24) and (A25) indicate. Therefore, the case of $\lim_{t\to\infty}\tau_t = 0$ and $\lim_{t\to\infty}\int_0^t \tau_s ds = 0$ will be dealt with separately from the case of $\lim_{t\to\infty}\tau_t \neq 0$ and $\lim_{t\to\infty}\int_0^t \tau_s ds \neq 0$ if necessary.

Lemma 2-3: In the model of heterogeneous productivity, if $\lim_{t\to\infty}\tau_t = 0$ and $\lim_{t\to\infty}\int_0^t \tau_s ds = 0$, then if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} \quad ,$$

and if $\lim_{t\to\infty}\tau_t \neq 0$ and $\lim_{t\to\infty}\int_0^t \tau_s ds \neq 0$, then if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}$$

and

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha} \quad .$$

By Lemma 2-3, if all the optimality conditions of both economies are satisfied, either

$$\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}} = \lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}} = 0 \tag{A31}$$

or

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha} \quad . \tag{A32}$$

Proposition 1-3: If and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, all the optimality conditions of both economies are satisfied at steady state.

Corollary 1-3: In the model of heterogeneous productivity, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} =$ $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\text{constant.}$$

Corollary 2-3: In the model of heterogeneous productivity, if $\lim_{t\to\infty}\tau_t\neq 0$ and $\lim_{t\to\infty}\int_0^t\tau_s ds\neq 0$, then if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\text{constant}$$

and

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t\tau_s ds\right)}{dt}}{\int_0^t\tau_s ds}=\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\quad .$$

On the two balanced growth paths satisfying Proposition 1-3 and Corollaries 1-3 and 2-3, heterogeneity in productivity is sustainable by definition because all the optimality conditions of the two economies are indefinitely satisfied.

By equations (A24) and (A25), the limit of the growth rate on these sustainable paths is

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}-\theta\right\}\quad .$$

A2.4 The balance of payments

A2.4.1 Heterogeneous time preference model

As shown in the proof of Proposition 1-1, $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\Xi$ and $\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}}$ $=\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{2,t}}=\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$ on the multilateral path. Because $k_{i,t}$ is positive, if the sign of Ξ is negative, the current account of economy 1 will eventually show permanent deficits and vice versa.

Lemma 3-1: In the model of heterogeneous time preference,

$$\Xi = \frac{\theta_1 - \theta_2}{2}\left\{\varepsilon\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \frac{\theta_1 + \theta_2}{2}\right]^{-1} - 1\right\}^{-1} .$$

Proof: See Harashima (2010).

Lemma 3-1 indicates that the value of Ξ is uniquely determined on the multilateral path, and the sign of Ξ is also therefore uniquely determined.

Proposition 2-1: In the model of heterogeneous time preference, $\Xi < 0$ if $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}[1-(1-\alpha)\varepsilon] < \frac{\theta_1 + \theta_2}{2}$.

Proof: For $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$ to be satisfied,

$$\lim_{t\to\infty}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}}\right] < 0$$

by equations (A18) and (A19). Here, $\lim_{t\to\infty}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}}\right] =$ $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1} - \Xi = \Xi\left[\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - 1\right] < 0$. Since the limit of the growth rate is $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \frac{\theta_1+\theta_2}{2}\right]$, $\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - 1$ $= \frac{\varepsilon\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}}{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \frac{\theta_1+\theta_2}{2}} - 1$. Therefore, if $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}[1-\varepsilon(1-\alpha)] < \frac{\theta_1+\theta_2}{2}$, then $0 < \left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - 1$ and $\Xi < 0$. ■

Proposition 2-1 indicates that the current account deficit of economy 1 and the current account surplus of economy 2 continue indefinitely on the multilateral path. The condition $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}[1-(1-\alpha)\varepsilon] < \frac{\theta_1+\theta_2}{2}$ is generally satisfied for reasonable parameter values.

Conversely, the opposite is true for the trade balance.

Corollary 3-1: In the model of heterogeneous time preference, $\lim_{t\to\infty}\left(\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s ds\right) > 0$ if $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}[1-(1-\alpha)\varepsilon] < \frac{\theta_1+\theta_2}{2}$.

Proof: See Harashima (2010).

Corollary 3-1 indicates that, on the multilateral path, the trade surpluses of economy 1 continue indefinitely and vice versa. That is, goods and services are transferred from economy 1 to economy 2 in each period indefinitely in exchange for the returns on the accumulated current account deficits (i.e., debts) of economy 1.

Nevertheless, the trade balance of economy 1 is not a surplus from the beginning. Before Corollary 3-1 is satisfied, negative $\int_0^t \tau_s ds$ should be accumulated. In the early periods, when $\int_0^t \tau_s ds$ is small, the balance on goods and services of economy 1 ($\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s ds$) continues to be a deficit. After a sufficient negative amount of $\int_0^t \tau_s ds$ is accumulated, the trade balances of economy 1 shift to surpluses.

A2.4.2 Heterogeneous risk aversion model

Similarly, the value of Ξ in the heterogeneous risk aversion model is uniquely determined on the multilateral path.

Lemma 3-2: In the model of heterogeneous risk aversion,

$$\Xi = \frac{(\varepsilon_1-\varepsilon_2)\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\theta\right]}{(\varepsilon_1+\varepsilon_2)\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\theta\right]^{-1}-1\right\}}.$$

Proposition 2-2: In the model of heterogeneous risk aversion, $\Xi < 0$ if $1-\theta\left(\frac{\varpi\alpha}{mv}\right)^{-\alpha}(1-\alpha)^{-1+\alpha} < \frac{\varepsilon_1+\varepsilon_2}{2}$.

The condition $1-\theta\left(\frac{\varpi\alpha}{mv}\right)^{-\alpha}(1-\alpha)^{-1+\alpha} < \frac{\varepsilon_1+\varepsilon_2}{2}$ is generally satisfied for reasonable parameter values.

Corollary 3-2: In the model of heterogeneous risk aversion, $\lim_{t\to\infty}\left(\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s ds\right) > 0$.

By Lemma 3-2 and equations (A21) and (A22), the limit of the growth rate on the multilateral path is

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\theta\right] .$$

A2.4.3 Heterogeneous productivity model

As Lemma 2-3 shows, on the multilateral path, either $\lim_{t\to\infty}\tau_t=0$ and $\lim_{t\to\infty}\int_0^t\tau_s ds=0$ or $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t\tau_s ds\right)}{dt}}{\int_0^t\tau_s ds}=\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}$. On the former path, $\varXi=0$ and heterogeneous productivity does not result in permanent trade imbalances. However, on the latter path, trade imbalances usually grow at a higher rate than consumption, because usually

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t\tau_s ds\right)}{dt}}{\int_0^t\tau_s ds}=\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}>\varepsilon^{-1}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}-\theta\right\}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} ;$$

thus, $\varXi$ explodes to infinity. Hence, the latter path will generally not be selected. The question of which path is selected is examined in detail in Section A3.3.

A2.5 A model with heterogeneities in multiple elements

The three heterogeneities are not exclusive. It is particularly likely that heterogeneities in time preference and productivity coexist. Many empirical studies conclude that the rate of time preference is negatively correlated with income (e.g., Lawrance, 1991; Samwick, 1998; Ventura, 2003); this indicates that the economy with the higher productivity has a lower rate of time preference and vice versa. In this section, the models are extended to include heterogeneity in multiple elements.

Suppose that economies 1 and 2 are identical except for time preference, risk aversion, and productivity. The Hamiltonian for economy 1 is

$$H_1=u_1(c_{1,t})\exp(-\theta_1 t)+\lambda_{1t}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}k_{1,t}+\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t\tau_s ds-\tau_t-c_{1,t}\right\} ,$$

and that for economy 2 is

$$H_2=u_2(c_{2,t})\exp(-\theta_2 t)+\lambda_{2,t}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}k_{2,t}-\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t\tau_s ds+\tau_t-c_{2,t}\right\} .$$

The growth rates are

$$\frac{\dot{c}_{1,t}}{c_{1,t}}=\varepsilon_1^{-1}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}+\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}-\theta_1\right\} ,$$

and

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon_2^{-1}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha} - \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\int_0^t \tau_s ds}{\partial k_{2,t}} + \frac{\partial \tau_t}{\partial k_{2,t}} - \theta_2\right\} .$$

Here, $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}} = \Xi$, $\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}} = \frac{\omega_1}{\omega_2}\Xi$, $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}} = \Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$, and $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}} = \frac{\omega_1}{\omega_2}\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$ at steady state. Thus, $\Xi = \dfrac{\frac{\varepsilon_2}{\varepsilon_1}\theta_1 - \theta_2 - \left(\frac{\varepsilon_2}{\varepsilon_1}-1\right)\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}}{\left(\frac{\varepsilon_2}{\varepsilon_1}+\frac{\omega_1}{\omega_2}\right)\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1} - 1\right\}}$,

and the limit of the growth rate on the multilateral path is

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \left(\frac{\varepsilon_1\omega_1+\varepsilon_2\omega_2}{\omega_1+\omega_2}\right)^{-1}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha} - \frac{\theta_1\omega_1+\theta_2\omega_2}{\omega_1+\omega_2}\right\} .$$

Clearly, if $\varepsilon_1=\varepsilon_2$ and $\omega_1=\omega_2=1$, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon_1^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \frac{\theta_1+\theta_2}{2}\right]$; if $\theta_1=\theta_2$ and $\omega_1=\omega_2=1$, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta_1\right]$; and if $\theta_1=\theta_2$ and $\varepsilon_1=\varepsilon_2$, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon_1^{-1}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha} - \theta_1\right\}$ as shown in Sections A2.3 and A2.4.

The sign of Ξ on the multilateral path depends on the relative values between θ_1 and θ_2, ε_1 and ε_2, and ω_1 and ω_2. Nevertheless, if the rate of time preference and productivity are negatively correlated, as argued above (i.e., if $\theta_1<\theta_2$ and $\omega_1>\omega_2$ while $\varepsilon_1=\varepsilon_2$), then by similar proofs as those presented for Proposition 2-1 and Corollary 3-1, if $\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}\left[1-(1-\alpha)^{1-\alpha}\varepsilon_1\right] < \frac{\omega_1\theta_1+\omega_2\theta_2}{\omega_1+\omega_2}$, then $\Xi<0$ and $\lim_{t\to\infty}\left(\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s ds\right) > 0$ on the multilateral path; that is, the current account deficits and trade surpluses of economy 1 continue indefinitely. The condition $\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}\left[1-(1-\alpha)^{1-\alpha}\varepsilon_1\right] < \frac{\omega_1\theta_1+\omega_2\theta_2}{\omega_1+\omega_2}$ is generally satisfied for reasonable parameter values.

A2.6 Multi-economy models

The two-economy models can be extended to include numerous economies that have differing degrees of heterogeneity.

A2.6.1 Heterogeneous time preference model

Suppose that there are H economies that are identical except for time preference. Let θ_i be the rate of time preference of economy i and $\tau_{i,j,t}$ be the current account balance of economy i

with economy j, where i = 1, 2, … , H, j = 1, 2, … , H, and $i \neq j$. Because the total population is L_t, the population in each economy is $\frac{L_t}{H}$. The representative household of economy i maximizes its expected utility

$$E\int_0^{\infty} u_i(c_{i,t})\exp(-\theta_i t)dt \quad ,$$

subject to

$$\dot{k}_{i,t} = y_{i,t} + \sum_{j=1}^{H} \frac{\partial y_{j,t}}{\partial k_{j,t}} \int_0^t \tau_{i,j,s} ds - \sum_{j=1}^{H} \tau_{i,j,t} - c_{i,t} - v\dot{A}_{i,t}\left(\frac{L_t}{H}\right)^{-1}$$

for $i \neq j$.

Proposition 3-1: In the multi-economy model of heterogeneous time preference, if and only if

$$\lim_{t\to\infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \frac{\sum_{q=1}^{H}\theta_q}{H}\right]$$

for any i, all the optimality conditions of all heterogeneous economies are satisfied at steady state, and

$$\lim_{t\to\infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \lim_{t\to\infty} \frac{\dot{k}_{i,t}}{k_{i,t}} = \lim_{t\to\infty} \frac{\dot{y}_{i,t}}{y_{i,t}} = \lim_{t\to\infty} \frac{\dot{A}_t}{A_t} = \lim_{t\to\infty} \frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}} = \lim_{t\to\infty} \frac{\frac{d\int_0^t \tau_{i,j,s} ds}{dt}}{\int_0^t \tau_{i,j,s} ds}$$

for any i and j ($i \neq j$).
Proof: See Harashima (2010).

A2.6.2 Heterogeneous risk aversion model

The heterogeneous risk aversion model can be extended to the multi-economy model by a proof similar to that for Proposition 3-1. Suppose that H economies are identical except for risk aversion, and their degrees of risk aversion are ε_i (i = 1, 2, … , H).

Proposition 3-2: In the multi-economy model of heterogeneous risk aversion, if and only if

$$\lim_{t\to\infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \left(\frac{\sum_{q=1}^{H}\varepsilon_q}{H}\right)^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right]$$

for any i, all the optimality conditions of all heterogeneous economies are satisfied at steady state, and

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}}=\lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}}=\lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_{i,j,s}ds}{dt}}{\int_0^t \tau_{i,j,s}ds}$$

for any i and j ($i \neq j$).

A2.6.3 Heterogeneous productivity model

The heterogeneous productivity model can also be extended by a proof similar to that for Proposition 3-1. Suppose that H economies are identical except for productivity, and their productivities are ω_i (i = 1, 2, … , H). Note that, because $k_{1+2,t} = k_{1,t}+k_{2,t}=k_{2,t}\left[\frac{\omega_1}{\omega_2}+1\right]$, the productivity of economy 1+2 is $y_{1+2,t}=A_t^{\alpha}\left(\omega_1^{\alpha}k_{1,t}^{1-\alpha}+\omega_2^{\alpha}k_{2,t}^{1-\alpha}\right)=\left(\omega_1+\omega_2\right)^{\alpha}A_t^{\alpha}k_{1+2,t}^{1-\alpha}$.

Proposition 3-3: In the multi-economy model of heterogeneous productivity, if and only if

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\varepsilon^{-1}\left\{\left[\frac{\left(\sum_{q=1}^{H}\omega_q\right)\varpi\alpha}{Hmv(1-\alpha)}\right]^{\alpha}-\theta\right\}$$

for any i, all the optimality conditions of all heterogeneous economies are satisfied at steady state, and

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}}=\lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}$$

for any i and j ($i \neq j$).

A2.6.4 Heterogeneity in multiple elements

Similarly, the multi-economy model can be extended to heterogeneity in multiple elements, as follows.

Proposition 3-4: In the multi-economy model of heterogeneity in multiple elements, if and only if

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right)^{-1}\left\{\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha}-\frac{\sum_{q=1}^{H}\theta_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right\} \tag{A33}$$

for any i (= 1, 2, … , H), all the optimality conditions of all heterogeneous economies are satisfied at steady state, and

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}}=\lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}}=\lim_{t\to\infty}\frac{\frac{d\int_0^t\tau_{i,j,s}ds}{dt}}{\int_0^t\tau_{i,j,s}ds}$$

for any i and j ($i \neq j$).

Proposition 3-4 implies that the concept of the representative household in a heterogeneous population implicitly assumes that all households are on the multilateral path.

A2.7 Degeneration to an exogenous technology model

The multilateral paths in the endogenous growth models imply that similar sustainable states exist in exogenous technology models. However, this is true only for the heterogeneous time preference model, because, in exogenous technology models, the steady state means that $\frac{\partial y_t}{\partial k_t}=\theta$; that is, the heterogeneity in risk aversion is irrelevant to the steady state, and the heterogeneous productivities do not result in permanent trade imbalances due to $\frac{\partial y_{1,t}}{\partial k_{1,t}}=\frac{\partial y_{2,t}}{\partial k_{2,t}}$. Thereby, only heterogeneous time preference is relevant to sustainable heterogeneity in exogenous growth models.

If technology is exogenously given and constant (A_t = A), Hamiltonians for the heterogeneous time preference model shown in Section A1.2.1 degenerate to

$$H_1=u_1(c_{1,t})\exp(-\theta_1 t)+\lambda_{1,t}\left[A^\alpha k_{1,t}^{1-\alpha}+(1-\alpha)A^\alpha k_{1,t}^{-\alpha}\int_0^t\tau_s ds-\tau_t-c_{1,t}\right]$$

and

$$H_2=u_2(c_{2,t})\exp(-\theta_2 t)+\lambda_{2,t}\left[A^\alpha k_{2,t}^{1-\alpha}-(1-\alpha)A^\alpha k_{2,t}^{-\alpha}\int_0^t\tau_s ds+\tau_t-c_{2,t}\right] .$$

By equations (A10), (A11), and (A12), the growth rate of consumption in economy 1 is $\frac{\dot{c}_{1,t}}{c_{1,t}}=\varepsilon^{-1}\left\{(1-\alpha)A^\alpha k_{1,t}^{-\alpha}+(1-\alpha)A^\alpha k_{1,t}^{-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}-\alpha(1-\alpha)A^\alpha k_{1,t}^{-\alpha-1}\int_0^t\tau_s ds-\frac{\partial\tau_t}{\partial k_{1,t}}-\theta_1\right\}$. Hence,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\varepsilon^{-1}\lim_{t\to\infty}\left\{(1-\alpha)A^\alpha k_{1,t}^{-\alpha}+(1-\alpha)A^\alpha k_{1,t}^{-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}-\alpha(1-\alpha)A^\alpha k_{1,t}^{-\alpha-1}\int_0^t\tau_s ds-\frac{\partial\tau_t}{\partial k_{1,t}}-\theta_1\right\}$$

= 0 and thereby $\lim_{t\to\infty}(1-\alpha)A^\alpha k_{1,t}^{-\alpha}[1+(1-\alpha)\Psi]-\Xi-\theta_1=0$, where $\Psi=\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}}$ and Ψ is constant at steady state and $\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=0$. For Ψ to be constant at steady state, it is necessary that $\lim_{t\to\infty}\tau_t = 0$ and thus $\Xi = 0$. Therefore, $\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}[1+(1-\alpha)\Psi]-\theta_1 = 0$, and $\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}[1-(1-\alpha)\Psi]-\theta_2 = 0$ because

$$\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon^{-1}\lim_{t\to\infty}\left\{(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}-(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\frac{\partial\int_0^t\tau_s ds}{\partial k_{2,t}}+\alpha(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha-1}\int_0^t\tau_s ds+\frac{\partial\tau_t}{\partial k_{2,t}}-\theta_2\right\}$$

= 0.

Because $\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}[1+(1-\alpha)\Psi] = \theta_1$, $\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}[1-(1-\alpha)\Psi] = \theta_2$, and $\frac{\partial y_{1,t}}{\partial k_{1.t}} = \frac{\partial y_{2,t}}{\partial k_{2.t}} = A^{\alpha}k_{1,t}^{-\alpha} = A^{\alpha}k_{2,t}^{-\alpha}$, then

$$\Psi = \frac{\theta_1-\theta_2}{2(1-\alpha)\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}}} \,. \tag{A34}$$

By $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\lim_{t\to\infty}\left\{\frac{\partial y_{1,t}}{\partial k_{1.t}}+\frac{\partial y_{1,t}}{\partial k_{1.t}}\frac{\partial\int_0^t\tau_s ds}{\partial k_{1,t}}-\alpha\frac{\partial y_{1,t}}{\partial k_{1.t}}\frac{\int_0^t\tau_s ds}{k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}-\theta_1\right\} = 0$ and equation (A34), then $\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}}+\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}}(1-\alpha)\Psi = \theta_1$; thus,

$$\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}} = \frac{\theta_1+\theta_2}{2} = \lim_{t\to\infty}\frac{\partial y_{2,t}}{\partial k_{2.t}} \,. \tag{A35}$$

If equation (A35) holds, all the optimality conditions of both economies are indefinitely satisfied. This result is analogous to equation (A29) and corresponds to the multilateral path in the endogenous growth models. The state indicated by equation (A35) is called the "multilateral steady state" in the following discussion.

If both economies are not open and are isolated, $\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}} = \theta_1$ and $\lim_{t\to\infty}\frac{\partial y_{2,t}}{\partial k_{2.t}} = \theta_2$ at steady state instead of the conditions shown in equation (A35). Hence, at the multilateral steady state with $\theta_1 < \theta_2$, the amount of capital in economy 1 is smaller than when the economy is isolated and vice versa. As a result, output and consumption in economy 1 are also smaller in the multilateral steady state with $\theta_1 < \theta_2$ than when the economy is isolated. Furthermore, $\Psi = \frac{\theta_1-\theta_2}{2(1-\alpha)\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}}} = \frac{\theta_1-\theta_2}{(1-\alpha)(\theta_1+\theta_2)} < 0$ by equation (A35). Thus, by

$$\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}} = \Psi < 0,$$

$$\lim_{t\to\infty}\int_0^t\tau_s ds < 0 \;;$$

that is, economy 1 possesses accumulated debts owed to economy 2 at steady state, and economy 1 has to export goods and services to economy 2 by

$$\left|(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\int_0^t \tau_s ds\right|$$

in every period to pay the debts. Nevertheless, because $\lim_{t\to\infty}\tau_t = 0$ and $\Xi = 0$, the debts do not explode but stabilize at steady state.

In the multilateral steady state, all the optimality conditions of both economies are satisfied, and heterogeneity is therefore sustainable. However, this state will be economically less preferable for economy 1 as compared with the state of Becker (1980), because consumption is smaller and debts are owed. Which state should economy 1 select? A similar dilemma—whether to give priority to simultaneous optimality with economy 2 or to unilaterally optimal higher utility—will also arise in the endogenous growth models; this is examined in the following sections.

A3 Unilateral balanced growth

The multilateral path satisfies all the optimality conditions, but that does not mean that the two economies naturally select the multilateral path. Ghiglino (2002) predicts that it is likely that, under appropriate assumptions, the results of Becker (1980) still hold in endogenous growth models. Farmer and Lahiri (2005) show that balanced growth equilibria do not exist in a multi-agent economy in general, except in the special case that all agents have the same constant rate of time preference. How the economies behave in the environments described in Sections A1 and A2 is examined in this section.

A3.1 Heterogeneous time preference model

The multilateral path is not the only path on which all the optimality conditions of economy 1 are satisfied. Even if economy 1 behaves unilaterally, it can achieve optimality, but economy 2 cannot.

Lemma 4-1: In the heterogeneous time preference model, if each economy sets τ_t without regarding the other economy's optimality conditions, then it is not possible to satisfy all the optimality conditions of both economies.
Proof: See Harashima (2010).

Since $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}\left(\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}\right)^{-1} - \lim_{t\to\infty}\frac{\tau_t}{k_{1,t}} - \theta_1\right\}$

at steady state, all the optimality conditions of economy 1 can be satisfied only if either

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\dfrac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} \tag{A36}$$

or

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}=\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} \quad . \tag{A37}$$

That is, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ can be constant only when either equation (A36) or (A37) is satisfied. Conversely, economy 1 has two paths on which all its optimality conditions are satisfied. Equation (A36) indicates that $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=$ constant, and equation (A37) indicates that $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left(\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}\right)^{-1}-1=0$ for any $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}$. Equation (A36) corresponds to the multilateral path. On the path satisfying equation (A37), $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\neq\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$, and $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$. Here, by equations (A6) and (A7),

$$c_{1,t}-c_{2,t}=2\left(\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds-\tau_t\right)=2\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds-\tau_t\right] ,$$

and

$$\lim_{t\to\infty}\left(c_{1,t}-c_{2,t}\right)=0$$

is required because $\lim_{t\to\infty}\frac{\tau_t}{\int_0^t \tau_s ds}=\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}$. However, because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, economy 2 must initially set consumption such that $c_{2,0}=\infty$, which violates the optimality condition of economy 2. Therefore, unlike with the multilateral path, all the optimality conditions of economy 2 cannot be satisfied on the path satisfying equation (A37) even though those of economy 1 can. Hence, economy 2 has only one path on which all its optimality conditions can be satisfied—the multilateral path. The path satisfying equation (A37) is called the "unilateral balanced growth path" or the "unilateral path" in the following discussion. Clearly, heterogeneity in time preference is not sustainable on the unilateral path.

How should economy 2 respond to the unilateral behavior of economy 1? Possibly, both economies negotiate for the trade between them, and some agreements may be reached. If no agreement is reached, however, and economy 1 never regards economy 2's optimality conditions, economy 2 generally will fall into the following unfavorable situation.

Remark 1-1: In the model of heterogeneous time preference, if economy 1 does not regard the optimality conditions of economy 2, the ratio of economy 2's debts (owed to economy 1) to its consumption explodes to infinity while all the optimality conditions of economy 1 are satisfied.

The reasoning behind Remark 1-1 is as follows. When economy 1 selects the unilateral path and sets $c_{1,0}$ so as to achieve this path, there are two options for economy 2. The first option is for economy 2 to also pursue its own optimality without regarding economy 1: that is, to select its own unilateral path. The second option is to adapt to the behavior of economy 1 as a follower. If economy 2 takes the first option, it sets $c_{2,0}$ without regarding $c_{1,0}$. As the proof of Lemma 4-1 indicates, unilaterally optimal growth rates are different between the two economies and $\frac{\dot{c}_{1,t}}{c_{1,t}} > \frac{\dot{c}_{2,t}}{c_{2,t}}$; thus, the initial consumption should be set as $c_{1,0} < c_{2,0}$. Because $\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\varpi}{2mv}\frac{\partial\left(y_{1,t}+y_{2,t}\right)}{\partial A_t} = \frac{\partial y_{2,t}}{\partial k_{2,t}}$ and $k_{1,t} = k_{2,t}$ must be kept, capital and technology are equal and grow at the same rate in both economies. Hence, because $c_{1,0} < c_{2,0}$, more capital is initially produced in economy 1 than in economy 2 and some of it will need to be exported to economy 2. As a result, $\frac{\dot{c}_{1,t}}{c_{1,t}} > \frac{\dot{k}_{1,t}}{k_{1,t}} = \frac{\dot{k}_{2,t}}{k_{2,t}} > \frac{\dot{c}_{2,t}}{c_{2,t}}$, which means that all the optimality conditions of both economies cannot be satisfied. Since $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} > \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} > \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, capital soon becomes abundant in economy 2, and excess goods and services are produced in that economy. These excess products are exported to and utilized in economy 1. This process escalates as time passes because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} > \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} > \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, and eventually almost all consumer goods and services produced in economy 2 are consumed by households in economy 1. These consequences will be unfavorable for economy 2.

If economy 2 takes the second option, it should set $c_{2,0} = \infty$ to satisfy all its optimality conditions, as the proof of Lemma 4-1 indicates. Setting $c_{2,0} = \infty$ is impossible, but economy 2 as the follower will initially set $c_{2,t}$ as large as possible. This action gives economy 2 a higher expected utility than that of the first option, because consumption in economy 2 in the second case is always higher. As a result, economy 2 imports as many goods and services as possible from economy 1, and the trade deficit of economy 2 continues until $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds = \tau_t$ is achieved; this is, $\frac{\dot{\tau}_t}{\tau_t} = \frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$ is achieved. The current account deficits and the accumulated debts of economy 2 will continue to increase indefinitely. Furthermore, they will increase more rapidly than the growth rate of outputs ($\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}$) because, in general, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} < \lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}$; that is, $(1-\varepsilon)\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} < \theta_1(<\theta_2)$. If no disturbance occurs, the expansion of debts may be sustained forever, but economy 2 becomes extremely vulnerable to even a very tiny negative disturbance. If such a disturbance occurs, economy 2 will lose all its capital and will no longer be able to repay its debts. This result corresponds to the state shown by Becker (1980), and it will also be unfavorable for economy 2. Because $\lim_{t\to\infty}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t\right] = 0$, inequality (A27) holds, and the transversality condition for economy 1 is satisfied by Lemma 1-1. Thus, all the optimality conditions of economy 1 are satisfied if economy 2 takes the second option.

As a result, all the optimality conditions of economy 2 cannot be satisfied in any case if economy 1 takes the unilateral path. Both options to counter the unilateral behavior of economy 1 are unfavorable for economy 2. However, the expected utility of economy 2 is higher if it takes the second option rather than the first, and economy 2 will choose the second option. Hence, if economy 1 does not regard economy 2's optimality conditions, the debts owed by economy 2 to economy 1 increase indefinitely at a higher rate than consumption.

A3.2 Heterogeneous risk aversion model

The same consequences are observed in this model.

Lemma 4-2: In the model of heterogeneous risk aversion, if each economy sets τ_t without regard for the other economy's optimality conditions, then all the optimality conditions of both economies cannot be satisfied.

Therefore, heterogeneity in risk aversion is not sustainable on the unilateral path.

Remark 1-2: In the model of heterogeneous risk aversion, if economy 1 does not regard economy 2's optimality conditions, the ratio of economy 2's debts (owed to economy 1) to its consumption explodes to infinity while all the optimality conditions of economy 1 are satisfied.

A3.3 Heterogeneous productivity model

Unlike the heterogeneous preferences shown in Sections A3.1 and A3.2, heterogeneity in productivity can be sustainable even on the unilateral path.

Lemma 4-3: In the heterogeneous productivity model, even if each economy sets τ_t without regard for the other economy's optimality conditions, it is possible that all the optimality conditions of both economies are satisfied if

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}=\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha} \ .$$

Proof: See Harashima (2010).

All the optimality conditions of economy 1 can be satisfied only if either equation (A31) or (A32) holds, because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ can be constant only when equation (A31) or (A32) holds. Equation (A31) corresponds to the multilateral path, and equation (A32) corresponds to the unilateral path. Unlike the heterogeneity in preferences, Lemma 4-3 shows that, even on the unilateral path, all the optimality conditions of both economies are satisfied because the limit of both economies' growth rates is identical on the path of either equation (A31) or (A32), such that $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\left\{\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}-\theta\right\}$. Therefore, heterogeneity in productivity is sustainable even on the unilateral path.

Nevertheless, on the unilateral path, current account imbalances generally grow steadily at a higher rate than consumption; this is not the case on the multilateral path. How does economy 1 set τ? If economy 1 imports as many goods and services as possible before reaching the steady state at which $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}=\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^{\alpha}(1-\alpha)^{1-\alpha}$ (i.e., if it initially sets τ_t as $\tau_t<0$ and $\tau_t-\frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s ds<0$), the expected utility of economy 1 will be higher than it is in either case where $\tau_t>0$ or in the multilateral path. However, the debts economy 1 owes to economy 2 will grow indefinitely at a higher rate than consumption, and the ratio of debt to consumption explodes to infinity. If there is no disturbance, this situation will be sustained forever, but economy 1 will become extremely vulnerable to even a very tiny negative disturbance. Hence, the unilateral path will not necessarily be favorable for economy 1 although all its optimality conditions are satisfied on this path, and economy 1 will prefer the multilateral path.

Remark 1-3: In the heterogeneous productivity model, even though economy 1 does not regard economy 2's optimality conditions, the multilateral balanced growth path will be selected.

Hence, the state shown by Becker (1980) will not be observed in the case of heterogeneous productivity.

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