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17 October 2015

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October 17, 2015

Abstract

The catchup and convergence of developing economies with the Western world is a major experience in modern history. In this paper we explore the role played jointly by technological imitation and trade liberalization in a North-South endogenous growth model. We prove that a gradual trade liberalization between South and North will promote convergence *at any level of initial trade costs* if the Southern economies are fully industrialized, their R&D potential is relatively low and the elasticity of substitution among manufactures is high enough.

1 Introduction and Related Literature

Empirically the patterns of North-South trade show a division of world production where the North keeps the production of new, unstandardized manufactures while relegating the South to making standardized, older products at lower cost. The pioneering study of Vernon (1966) and the followup literature on product cycles analysis shed a lot of light in understanding such patterns of trade and technology transfer.

Krugman (1979) advanced a seminal exogenous growth model, describing the determinants of international convergence as a function of the strength of innovation in the North and technological imitation in the South. His work allowed Grossman and Helpman (1991, henceforth GH), Lai (1995) or Chui et al. (2001) to endogenize the forces leading to such innovation and imitation rates. Over time, the literature on endogenous product cycles has further extended GH's model. For instance, Mondal (2008) undertook a local stability analysis of the steady states in GH. The realities of FDI and world migration were incorporated as determinants of product cycles and technology transfer in

Mondal and Gupta (2008). Importantly, both the original GH model and all its extensions assumed that the economies were either autarkic or trade was perfectly free.

Dinopoulos and Segerstrom (2006), Cristobal Campoamor (2009) and Gustaffson and Segerstrom (2010) showed that trade liberalization was potentially important for North-South convergence in a product cycle setting. But these authors evaluated the effect of a marginal rise in trade openness on real convergence, *once trade openness was almost complete to start with.* In this paper we attempt to fill this gap by generalizing the conditions for real-wage convergence, since we allow for any possible initial level of trade openness. In particular, we prove that the convergence phenomenon arises if the size and R&D productivity of the North is sufficiently large, relative to the South, and the elasticity of substitution among manufactures is high enough.

2 Environment

2.1 Endowments

As in GH (1991), we consider two countries: North and South. The population of both countries is exogenously given (being L_s for the South and L_n for the North). There are three productive factors: labor, researchers and financial capital. Labor is employed in manufacturing, whereas researchers are employed in a competitive R&D sector.

Given the assumed Northern comparative advantage to innovate, researchers in the North are used to conceive new varieties; researchers in the South can only replicate the existing ones to produce them in the South at lower cost.

2.2 Preferences

Any representative household living in location k maximizes, in every period t, an intertemporal utility function W_t^k such as

$$W_t^k = \int_t^{\infty} e^{-\rho(s-t)} \log \left[U_s \left(X_s^k \right) \right] ds \tag{1}$$

This function shows the discounted utility flow that the household k expects to obtain from period t onwards by acquiring manufactures, grouped into the composite X. The composite of manufactures X_s is a Dixit-Stiglitz subutility function over the aggregate mass of varieties invented up to period

s.

$$X_s = \left[\int_0^{n(s)} x_j^{\alpha}(s)\, dj \right]^{\frac{1}{\alpha}} \tag{2}$$

where $0 < \alpha < 1$ is a direct measure of the substitutability between varieties and $x_j(s)$ quantifies the household demand for variety j at time s. These preferences imply an appreciation of manufacturing diversity, since utility grows as a given expenditure is more thinly split into an increasing number of varieties.

2.3 Technologies

In the global economy there is a continuum of industrial varieties with measure n, and $n = n_n + n_s$ (the sum of the Northern and Southern masses of varieties). The degree of product variety expands over time due to innovation. Moreover, an increase in the measure of manufactures enlarges the stock of public knowledge and reduces future R&D costs. GH's local stocks of public knowledge are equal to n in the North, since all patents were originally made up there, and n_s in the South.

The production function for every particular manufacture is identical and very simple: one unit of labor produces one unit of final output. Prior to the production of any manufacture it is necessary to incur a fixed cost to invent or imitate the corresponding patent. By perfect competition and free entry in the innovative and imitative activities, such a fixed cost is at least equal to the market value of the patent. This value decreases with the local stock of public knowledge as follows:

$$\begin{aligned} v_s &\leq \tfrac{a_s w_s}{n_s}, \text{ with equality when } \dot{n}_s > 0 \\ v_n &\leq \tfrac{a_n w_n}{n}, \text{ with equality when } \dot{n} > 0 \end{aligned} \tag{3}$$

where v_s and v_n denote the values of Southern and Northern patents, respectively. $\frac{a_s}{n_s}$ and $\frac{a_n}{n}$ stand for the number of researchers needed to imitate a Northern patent in the South and to create a new variety in the North. Our variables w_s and w_n denote the nominal wage in the South and the North, respectively. Consequently, when both innovation and imitation are active we can conclude that

$$w_s = \frac{n_s v_s}{a_s};\ w_n = \frac{n v_n}{a_n} \tag{4}$$

Southern researchers need to incur the previous fixed cost in order to replicate a Northern patent, while Northern researchers do it to invent one from scratch. On the other hand, we assume that our parameter $\tau \geq 1$ introduces the classical iceberg notion of trade costs for manufactures: it is necessary to buy τ units of the good abroad to consume one unit at home.

2.4 Static optimization

Productive firms must decide which price to quote in every period to maximize profits. On the other hand, consumers in any location not only decide how much to save, which equity to buy and which varieties to consume, but also choose their job: they become either manufacturing workers or researchers.

The function W_s^k is intertemporally maximized with respect to its ultimate arguments ($x_j(s)$, $\forall j$, $\forall s \geq t$) at every period t, taking as given the expected temporal paths of $v_n(s)$, $v_s(s)$, n, n_s, $p_j(s)$ $\forall j$ $\forall s \geq t$. This problem can be decomposed into two parts:

- The static allocation of a given per-household expenditure $E_k(s)$ among all kind of manufactures, which gives rise to a demand function for each of these commodities.

- The choice of an optimal path for $E_k(s)$, given the possibility of saving and investing in equity of Northern and Southern firms.

Let us denote by E the aggregate world expenditure and by E_n and E_s the part spent by people from the North and the South, respectively, which are endogenous variables. Considering that the demand for any variety comes from both Northern and Southern consumers who face different c.i.f. prices, we can derive the aggregate demand for any Northern (x_n) and Southern manufacture (x_s), taking into account (1) and (2) as follows:

$$x_n = p_n^{-\sigma}\left[\frac{E_n}{n_n p_n^{1-\sigma} + \delta n_s p_s^{1-\sigma}} + \frac{\delta E_s}{\delta n_n p_n^{1-\sigma} + n_s p_s^{1-\sigma}}\right] \tag{5}$$

$$x_s = p_s^{-\sigma}\left[\frac{\delta E_n}{n_n p_n^{1-\sigma} + \delta n_s p_s^{1-\sigma}} + \frac{E_s}{\delta n_n p_n^{1-\sigma} + n_s p_s^{1-\sigma}}\right] \tag{6}$$

where $\sigma = \frac{1}{1-\alpha}$ and $\delta = \tau^{1-\sigma}$ $(0 \leq \delta \leq 1)$ is a measure of trade openness in the global economy with respect to manufactures.

Firms maximize profits at any period s taking into account a demand of the type (5) or (6) and the simple production function described above. As a result, both utility and profit maximization from expressions (2), (5) and (6) result in an unconstrained markup over marginal costs, common for all manufacturing firms in location k: $p_k^u = \frac{w_k}{\alpha}$, for k =North, South.

2.5 Dynamic optimization under perfect mobility of financial capital

We have to deal now with the intertemporal allocation of expenditure and savings. Such allocation serves two basic purposes: the distribution of consumption along the time horizon and the financing of new startups in the North and the South. During that process, the household needs to consider that a share $m = \frac{\dot{n}_s}{n_n}$ of the Northern mass of varieties is copied by Southern imitators. The previous owners of these firms will consequently lose part of their equity.

Under perfect international mobility of financial capital, our dynamic optimization problem leads to the following non arbitrage condition, to be satisfied period by period:

$$\frac{\dot{E}}{E} = \frac{\dot{E}_s}{E_s} = \frac{\dot{E}_n}{E_n} = \frac{\pi_n}{v_n} - m - \rho + \frac{\dot{v}_n}{v_n} = \frac{\pi_s}{v_s} - \rho + \frac{\dot{v}_s}{v_s} \tag{7}$$

In order to characterize below our dynamical system, we also need to follow the evolution of the aggregate mass of manufactures in the South and in the global economy ($\frac{\dot{n}_s}{n_s}$ and $\frac{\dot{n}}{n}$), whose path will be determined by the labor market clearing conditions in both countries:

$$\begin{aligned} L_s &= a_s \frac{\dot{n}_s}{n_s} + n_s x_s \\ L_n &= a_n \frac{\dot{n}}{n} + n_n x_n \end{aligned} \tag{8}$$

2.6 Our key endogenous variables

We are going to group our set of endogenous variables into three key ratios, which will be constant in the steady state of our dynamical system. Let us define now these three basic endogenous variables (b, c and d):

$$b \equiv \frac{E}{w_n}; c \equiv \frac{E}{w_s}; d \equiv \frac{n_s}{n}$$

It will also be useful to define the Southern relative wage as $\omega \equiv \frac{w_s}{w_n} = \frac{b}{c}$.

3 North-South convergence analysis of the wide gap case

The wide gap case is a situation in which the Southern firms can quote their unconstrained markup $\left(\frac{w_s}{\alpha}\right)$ in both markets. This happens because the Southern equilibrium wage is so low that any potential Northern competitor will never try to undercut the Southern firms. More specifically, from now on we will only consider the case in which

$$\omega < \alpha/\tau = \frac{\sigma - 1}{\sigma\tau} = \left(\frac{\sigma - 1}{\sigma}\right)\delta^{\frac{1}{\sigma-1}} \tag{9}$$

Lai (1995; 2008) argues that this is a reasonable assumption, given the international patterns of income disparity between the West and less developed countries.

3.1 Derivation of our system of differential equations

First of all, we are going to solve for the share of Northern expenditure $\frac{E_n}{E}$. That will help us obtain the system of differential equations in terms of our three endogenous variables and the parameters of the model: L_n, L_s, a_n, a_s, δ, σ and ρ.

As in Mondal (2008), we will use a trade balance condition in both countries. Such balance implies that

$$E_n = n_n p_n x_n \text{ and } E_s = n_s p_s x_s \tag{10}$$

That is, the local value of production must be equal to the local value of expenditure in both locations.

Let us now denote by q the following endogenous variable:

$$q \equiv \frac{d}{1-d}\omega^{1-\sigma} = \frac{d}{1-d}\left(\frac{b}{c}\right)^{1-\sigma} \tag{11}$$

After plugging (11) into (5), considering (10) and rearranging, we are finally able to get that

$$\frac{E_n}{E} = \frac{1+\delta q}{q^2+2\delta q+1} \text{ and } \frac{E_s}{E} = \frac{q\,(\delta+q)}{q^2+2\delta q+1} \tag{12}$$

The dynamics of the system will be explored by solving for $\frac{\pi_n}{v_n}$, $\frac{\pi_s}{v_s}$, $n_n x_n$ and $n_s x_s$ in terms of b, c and d. If we consider our expression (4) and use the labor market clearing conditions in (8), from (7) and (12) it is possible to derive a 3x3 dynamical system of nonlinear differential equations as follows:

$$\left\{\begin{array}{rcl} \frac{\dot{b}}{b} & = & -(\rho+\frac{L_n}{a_n}) - \frac{d}{1-d}\frac{L_s}{a_s} + \frac{d}{(1-d)(q^2+2\delta q+1)\sigma}\left[b\,(1+\delta q) + c\,(\sigma-1)\,q\,(\delta+q)\right] \\ \frac{\dot{c}}{c} & = & -(\rho+\frac{L_s}{a_s}) + \frac{cq(q+\delta)}{q^2+2\delta q+1} \\ \frac{\dot{d}}{d} & = & \left(-\frac{L_n}{a_n}+\frac{L_s}{a_s}\right) + \frac{(\sigma-1)}{(q^2+2\delta q+1)\sigma}\left[-cq\,(\delta+q) + b\,(1+\delta q)\right] \end{array}\right\} \tag{13}$$

Obtaining this system paves the way to possible extensions of our work. In this paper we will limit ourselves to some steady-state analysis; however, it is possible to explore the local and global stability of the system. Another very interesting task would be evaluating the welfare implications of trade liberalization for both the Northern and Southern representative consumers. That would also require the analyisis of the transitional dynamics.

3.2 Convergence effects of trade liberalization in steady state

It is already possible to solve for the steady state values of our three endogenous variables: b^*, c^* and d^*. By setting the left-hand side of our three differential equations equal to zero, it is straightforward to come up with:

$$\begin{aligned} b^* &= \frac{\left(q^2+2\delta q+1\right)\left(\frac{L_s}{a_s}+\rho\right)}{q(q+\delta)} \\ c^* &= \frac{\left(q^2+2\delta q+1\right)}{(1+\delta q)}\left(\frac{\sigma}{\sigma-1}\frac{L_n}{a_n}+\rho-\left(\frac{1}{\sigma-1}\right)\frac{L_s}{a_s}\right) \\ d^* &= \frac{\frac{L_n}{a_n}+\rho}{\frac{L_s}{a_s}+2\rho-\frac{\sigma}{\sigma-1}\left(\frac{L_s}{a_s}-\frac{L_n}{a_n}\right)} \end{aligned} \tag{14}$$

Obtaining the ratio of w_s over w_n,

$$\omega = \frac{b^*}{c^*} = \bar{K}\frac{q\left(q+\delta\right)}{(1+\delta q)} \tag{15}$$

where

$$\bar{K} \equiv \frac{L_n - \frac{a_n}{\sigma a_s}L_s + \frac{\sigma-1}{\sigma}a_n\rho}{\frac{\sigma-1}{\sigma}\left(\frac{L_s+a_s\rho}{a_s}\right)} \tag{16}$$

As we can observe in (14), both nominal wages depend in steady state on the level of trade openness. Nevertheless, the local shares of varieties are not affected by any trade liberalization. The steady-state growth rate would not suffer any modification either. Notice that our comparative statics exercise will gain in predictability if there exists a unique steady-state for the system under the wide gap case. Such condition will be guaranteed by the following Lemma.

Lemma 1.There exists a unique steady state equilibrium in our North-South economy under costly trade.

Proof. The right hand side of (15) is positive and strictly decreasing in ω, going to zero as ω tends to infinity and to infinity as ω tends to zero. Therefore, the continuous and differentiable function $G\left(\omega\right) = \bar{K}\frac{q(q+\delta)}{(1+\delta q)}$ has a unique fixed point. That means the steady state equilibrium exists and is also unique.

We will introduce now an auxiliary lemma that will be useful to derive our Proposition 1 below.

Lemma 2. Let us consider the following equation, which defines an implicit function for q: $q^{\sigma}\left(\frac{q+\delta}{1+\delta q}\right)^{\sigma-1}=\frac{d}{1-d}\bar{K}^{1-\sigma}$. We claim that $q>(<)\,1$ if and only if $\frac{d}{1-d}\bar{K}^{1-\sigma}>(<)1$.

Proof. See the Appendix.

We are already able to obtain our main results, spelled out in Propositions 1 and 2.

Proposition 1. The nominal relative wage of the South (ω) will rise in response to higher trade openness, if and only if $\frac{1}{1-\frac{\frac{1}{\sigma-1}\left(\frac{L_s}{a_s}-\frac{L_n}{a_n}\right)}{\left(\frac{L_n}{a_n}+\rho\right)}}<\left[\frac{L_n-\frac{a_n}{\sigma a_s}L_s+\frac{\sigma-1}{\sigma}a_n\rho}{\frac{\sigma-1}{\sigma}\left(\frac{L_s+a_s\rho}{a_s}\right)}\right]^{\sigma-1}$.

Such condition does not depend on the level of initial trade costs.

Proof. Let us now denote by $\omega_{\delta}\equiv\frac{d\omega}{d\delta}$. Using the implicit function in (15) and our definition in (11), it is possible to differentiate and get that, over the steady state,

$$\frac{\omega_{\delta}}{\omega}=\frac{\left(1-q^{2}\right)}{\sigma\left(1+\delta q\right)\left(q+\delta\right)+\left(1-\delta^{2}\right)\left(\sigma-1\right)q} \tag{17}$$

This last expression involves that higher trade openness will increase (decrease) the Southern relative wage only if q is lower (higher) than one. Since the expressions (11) and (15) need to hold, we can derive the following expression:

$$q^{\sigma}\left(\frac{q+\delta}{1+\delta q}\right)^{\sigma-1}=\frac{d}{1-d}\bar{K}^{1-\sigma} \tag{18}$$

We know from Lemma 2 that

$$q>(<)\,1 \text{ if and only if} \frac{d}{1-d}\bar{K}^{1-\sigma}>(<)1 \tag{19}$$

By (16), (17) and our last condition in (19), we can conclude that our nominal, relative wage ω will be increasing (decreasing) in δ if and only if

$$\frac{1}{1-\frac{\left(\frac{L_s}{a_s}-\frac{L_n}{a_n}\right)}{(\sigma-1)\left(\frac{L_n}{a_n}+\rho\right)}}<(>)\left[\frac{L_n-\frac{a_n}{\sigma a_s}L_s+\frac{\sigma-1}{\sigma}a_n\rho}{\frac{\sigma-1}{\sigma}\left(\frac{L_s+a_s\rho}{a_s}\right)}\right]^{\sigma-1} \tag{20}$$

But we should pay attention to the ratio of *real* wages as well, taking the local price indices into account. The Southern and Northern price indices (I_s and I_n) are given by the following well known expressions:

$$I_s=\left(n_s p_s^{1-\sigma}+n_n p_n^{1-\sigma}\delta\right)^{\frac{1}{1-\sigma}}\ ;\ I_n=\left(\delta n_s p_s^{1-\sigma}+n_n p_n^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{21}$$

We know that the local real wage is equivalent to the indirect utility. Therefore, after some simplifications, we can see that

$$\text{Relative Southern } \textit{real} \text{ wage} \equiv \omega_R = \omega \frac{I_n}{I_s} = \omega \left(\frac{q+\delta}{1+\delta q} \right)^{\frac{1}{\sigma-1}} = \bar{K} q \left(\frac{q+\delta}{1+\delta q} \right)^{\frac{\sigma}{\sigma-1}} \tag{22}$$

Proposition 2. It is possible to check that ω_R will also be increasing in δ if and only if $q < 1$. That implies that our crucial condition (20) will be necessary and sufficient for *real* convergence as well.

Proof. See the Appendix.

In expression (20) we can see that ω_δ is more likely to be positive the higher is the innovative potential in the North and the lower is the imitative capacity in the South. The R&D potential in each block will be determined by the population employed in the research sector and the R&D costs.

That is true because, as the size and productivity of the R&D sector in the North increases, the Northern share of world manufactures will grow and more world demand will be channeled to the North. That will increase the demand for labor in the North and the aggregate Northern income.

Once trade openness rises, initially there would be an upward swing in the net exports of the South, since the aggregate income of the North is higher and imports become more attractive in both countries. In fact, all Southern and Northern consumers will increase their imports by the same percentage.

However, the trade balance condition means that net exports are always equal to zero in both countries and, therefore, local wages need to adjust in favor of the South. As we can observe in (20), this is only possible if σ is high enough. Otherwise the aggregate expenditure on imports would not increase sufficiently.

For instance, such aggregate expenditure would remain completely unaltered if $\sigma = 1$, which would prevent convergence. That is true because the right hand side of (20) would be equal to one and lower than the left hand side.

Here we have provided a condition for convergence valid for the whole possible range of trade openness, and not only for the limiting case in which $\delta \to 1^-$. An interesting task would be exploring its robustness as well. For instance, we could try to see if either occupational choice or the existence of a stagnant (agricultural) sector would make convergence depend on the initial level of trade costs.

4 Conclusions

Some crucial historical events, like China's accession to the World Trade Organization, have brought the international distribution of production to the forefront of economic analysis. Therefore, economists have devoted much attention to the analysis of the product cycle in the context of free trade, exploring the effects of new intellectual property rights or the incorporation of new countries to the global economy. However, the technical difficulty of the problem often prevented the analysis of product cycles with costly trade. Here we tried to offer a first step in that direction by looking for international convergence implications over the steady state. Our main contribution to such debate is a novel, closed-form condition for convergence in response to freer trade, at any level of initial trade openness.

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6 Appendix

Details on dynamic optimization under perfect mobility of financial capital

Let us denote by π_n and π_s the operating profits of any Northern and Southern firm, respectively. At every period φ, a representative household from location k owns a mass $\beta_{nk}(\varphi)$ and $\beta_{sk}(\varphi)$ of Southern firms, respectively. Moreover, $f_{nk}(\varphi)$ stands for the share of gross savings devoted to buying Northern equity. We will explore now the properties of an interior equilibrium in which all consumers finance new startups in both countries (i.e. $0 < f_{nk} < 1$).

Our control variables will be E_k (our household's expenditure) and f_{nk}, whereas the state variables are β_{nk} and β_{sk}. Then, the present value Hamiltonian faced at time t by any household for the period φ can be specified as follows:

$$\begin{aligned} H_k(\varphi) &= e^{-\rho(\varphi-t)} \log E_k(\varphi) \\ &+\mu_{nk}(\varphi)\left[\frac{(w_k+\beta_{nk}\pi_n+\beta_{sk}\pi_s-E_k)\, f_{nk}(\varphi)}{v_n} - m\beta_{nk}\right] \\ &+\mu_{sk}(\varphi)\left[\frac{(w_k+\beta_{nk}\pi_n+\beta_{sk}\pi_s-E_k)\,(1-f_{nk}(\varphi))}{v_s}\right] \end{aligned}$$

The first order condition corresponding to an interior solution for f_{nk} $(0 < f_{nk} < 1)$ is the following one:

$$e^{-\rho(\varphi-t)}\frac{1}{E_k(\varphi)} = \frac{\mu_{nk}(\varphi)}{v_n(\varphi)} = \frac{\mu_{sk}(\varphi)}{v_s(\varphi)},\ \forall\varphi \tag{23}$$

By differentiating in (23) and using the first order conditions with respect to the state variables, we can conclude that

$$\frac{\dot{E}}{E} = \frac{\dot{E}_s}{E_s} = \frac{\dot{E}_n}{E_n} = \frac{\pi_n}{v_n} - m - \rho + \frac{\dot{v}_n}{v_n} = \frac{\pi_s}{v_s} - \rho + \frac{\dot{v}_s}{v_s}$$

The last expression shows how the profitability of Northern and Southern firms must satisfy a non arbitrage condition period by period. That condition immediately makes both local expenditure levels grow at the same rate. In turn, such an identical growth rate in local expenditures immediately requires a permanent trade balance, since otherwise no country would repay their debt while spending permanently as fast as the rest of the world.

Details on the derivation of our system of differential equations

Northern and Southern expenditure shares

Taking into account the demand function for every Northern variety, after some algebraic transformations we can obtain from (5) and (6) that

$$n_n x_n = \frac{E_n \left(\frac{w_n}{\alpha}\right)^{-\sigma}}{\left(\frac{w_n}{\alpha}\right)^{1-\sigma} + \delta \frac{d}{1-d} \left(\frac{w_s}{\alpha}\right)^{1-\sigma}} + \frac{\delta E_s \left(\frac{w_n}{\alpha}\right)^{-\sigma}}{\delta \left(\frac{w_n}{\alpha}\right)^{1-\sigma} + \frac{d}{1-d} \left(\frac{w_s}{\alpha}\right)^{1-\sigma}} \tag{24}$$

Let us now rearrange the last expression to get that

$$\frac{\alpha E_n}{w_n} = \alpha b \frac{E_n}{E} = n_n x_n = \frac{\left(E_n/E\right) \left(\frac{w_n}{\alpha E}\right)^{-\sigma}}{\left(\frac{w_n}{\alpha E}\right)^{1-\sigma} + \delta \frac{d}{1-d} \left(\frac{w_s}{\alpha E}\right)^{1-\sigma}} + \frac{\delta \left(E_s/E\right) \left(\frac{w_n}{\alpha E}\right)^{-\sigma}}{\delta \left(\frac{w_n}{\alpha E}\right)^{1-\sigma} + \frac{d}{1-d} \left(\frac{w_s}{\alpha E}\right)^{1-\sigma}}$$

and hence

$$\frac{E_n}{E} = \frac{\frac{E_n}{E}}{1+\delta q} + \frac{\delta \frac{E_s}{E}}{\delta + q} \tag{25}$$

Similarly, we can derive that

$$\frac{E_s}{E} = 1 - \frac{E_n}{E} = \frac{\delta \frac{E_n}{E}}{1+\delta q} + \frac{\frac{E_s}{E}}{\delta + q} \tag{26}$$

Using simultaneously our equations (25) and (26), we can finally conclude that

$$\frac{E_n}{E} = \frac{1+\delta q}{q^2 + 2\delta q + 1} \text{ and } \frac{E_s}{E} = \frac{q\left(\delta + q\right)}{q^2 + 2\delta q + 1}$$

Non-arbitrage conditions

Since all firms from both countries will be able to quote the unconstrained markup, per period operating profits for any manufacturing firm in location k are

$$\pi_k = \left(\frac{1-\alpha}{\alpha}\right) w_k x_k, \text{ for } k = \text{ North, South.} \tag{27}$$

Therefore, we can develop the expression (7) by using (4), (7) and (27) as follows:

$$\frac{\pi_n}{v_n} = \left(\frac{1-\alpha}{\alpha a_n}\right) n x_n = \frac{1-\alpha}{\alpha a_n \left(1-d\right)} n_n x_n \tag{28}$$

And now, from (10), (28) and (12)

$$\frac{\pi_n}{v_n} = \frac{1-\alpha}{a_n(1-d)}\frac{E}{w_n}\left(\frac{E_n}{E}\right) = \frac{1}{\sigma(1-d)}b\left(\frac{1+\delta q}{q^2+2\delta q+1}\right) \tag{29}$$

By the same token, from (10) and (12),

$$\begin{aligned} n_s x_s &= \alpha\left(\frac{E}{w_s}\right)\left(\frac{E_s}{E}\right) = \alpha c a_s\left(\frac{q(\delta+q)}{q^2+2\delta q+1}\right); \\ n_n x_n &= \alpha b a_n\left(\frac{1+\delta q}{q^2+2\delta q+1}\right) \end{aligned}$$

And then we are ready to obtain that

$$m = \frac{\dot{n}_s}{n_n} = \frac{\dot{n}_s}{n_s}\frac{n_s}{n_n} = \left[\frac{L_s}{a_s} - \frac{n_s x_s}{a_s}\right]\frac{d}{1-d} = \frac{d}{1-d}\left[\frac{L_s}{a_s} - \frac{\alpha q(\delta+q)}{(q^2+2\delta q+1)}c\right] \tag{30}$$

As a result, from (7), (29) and (30), we can derive the first of our three differential equations:

$$\begin{aligned} \frac{\dot{b}}{b} &= \frac{\dot{E}}{E} - \frac{\dot{n}}{n} - \frac{\dot{v}_n}{v_n} = \frac{1}{\sigma(1-d)}b\left(\frac{1+\delta q}{q^2+2\delta q+1}\right) - \frac{d}{1-d}\left[\frac{L_s}{a_s} - \frac{\alpha q(\delta+q)}{(q^2+2\delta q+1)}c\right] - \\ &\quad -\rho - \frac{L_n}{a_n} + \left(\frac{\sigma-1}{\sigma}\right)b\left(\frac{1+\delta q}{q^2+2\delta q+1}\right) \end{aligned} \tag{31}$$

And finally,

$$\frac{\dot{b}}{b} = -\rho - \frac{L_n}{a_n} - \frac{d}{1-d}\frac{L_s}{a_s} + \frac{d}{(1-d)(q^2+2\delta q+1)\sigma}\left[b(1+\delta q) + c(\sigma-1)q(\delta+q)\right] \tag{32}$$

It is possible to conduct a very similar analysis with respect to our second endogenous variable c, which results in the following differential equation:

$$\frac{\dot{c}}{c} = \frac{\dot{E}}{E} - \frac{\dot{n}_s}{n_s} - \frac{\dot{v}_s}{v_s} = -\rho - \frac{L_s}{a_s} + \frac{cq(q+\delta)}{(q^2+2\delta q+1)} \tag{33}$$

Dealing with our labor market clearing conditions, we can also derive the last differential equation of our dynamical system:

$$\frac{\dot{d}}{d} = \frac{\dot{n}_s}{n_s} - \frac{\dot{n}}{n} = \left(-\frac{L_n}{a_n} + \frac{L_s}{a_s}\right) + \frac{(\sigma-1)}{(q^2+2\delta q+1)\sigma}\left[-cq(\delta+q) + b(1+\delta q)\right] \tag{34}$$

Proof of Lemma 2.

Assume that $q < 1$. We are going to prove that then, necessarily, $\frac{d}{1-d}\bar{K}^{1-\sigma} < 1$. It is straightforward to observe that $\left(\frac{q+\delta}{1+\delta q}\right)$ is increasing in δ when $q < 1$. Therefore, the maximum value that $\left(\frac{q+\delta}{1+\delta q}\right)$ can take is one, which implies that our left-hand side is equal to $q^\sigma < 1$ and hence the equality above implies that $\frac{d}{1-d}\bar{K}^{1-\sigma} < 1$.

Assume now that $\frac{d}{1-d}\bar{K}^{1-\sigma} < 1$. We are going to show that then, necessarily, $q < 1$. If $q \geq 1$, then our factor $\left(\frac{q+\delta}{1+\delta q}\right)$ is decreasing in δ and, therefore, the minimum value it can take is one. As a result, our left hand side will be $q^{\sigma} \geq 1$, which contradicts our initial equality. Therefore, our conclusion is that $q < 1$ if and only if $\frac{d}{1-d}\bar{K}^{1-\sigma} < 1$.

We can proceed in an analogous way to prove the second inequality.

Proof of Proposition 2.

Our starting point is

$$\omega_R = \omega \frac{I_n}{I_s} = \omega \left(\frac{q+\delta}{1+\delta q}\right)^{\frac{1}{\sigma-1}} = \bar{K} q \left(\frac{q+\delta}{1+\delta q}\right)^{\frac{\sigma}{\sigma-1}} \tag{35}$$

Let us first differentiate with respect to δ the expression $\left(\frac{q+\delta}{1+\delta q}\right)$. If we denote by $\left(\frac{q+\delta}{1+\delta q}\right)_{\delta} \equiv \frac{d\left(\frac{q+\delta}{1+\delta q}\right)}{d\delta}$,

$$\frac{\left(\frac{q+\delta}{1+\delta q}\right)_{\delta}}{\left(\frac{q+\delta}{1+\delta q}\right)} = \frac{q_{\delta}\left(1-\delta^2\right) + \left(1-q^2\right)}{\left(1+\delta q\right)\left(q+\delta\right)} \tag{36}$$

Then, from (17), (35) and (36)

$$\begin{aligned}
\frac{\omega_{R\delta}}{\omega_R} &= \frac{\omega_{\delta}}{\omega} + \frac{1}{(\sigma-1)}\left\{\frac{q_{\delta}\left(1-\delta^2\right) + \left(1-q^2\right)}{\left(1+\delta q\right)\left(q+\delta\right)}\right\} = \\
&= \left(1-q^2\right)\left(\frac{q+\phi}{(\sigma-1)\left(1+\delta q\right)\left(q+\delta\right)q}\right)
\end{aligned} \tag{37}$$

where

$$\phi \equiv \frac{\left(1+\delta q\right)\left(q+\delta\right) - q\left(1-\delta^2\right)}{\sigma\left(1+\delta q\right)\left(q+\delta\right) + q\left(1-\delta^2\right)} > 0 \tag{38}$$

Then, we can see from (37) and (38) that $\omega_{R\delta} > 0$ iff $q < 1$. In terms of the parameters, $\omega_{R\delta} > 0$ iff (20) holds.